Agent–Based Keynesian Macroeconomics - An Evolutionary Model Embedded in an Agent–Based Computer Simulation

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AGENT–BASED KEYNESIAN MACROECONOMICS
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AN EVOLUTIONARY MODEL EMBEDDED IN AN AGENT–BASED COMPUTER SIMULATION

INAUGURAL DISSERTATION
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To Eva–Maria.

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List of Abbreviations

ACE:	Agent–based computational economics
AD:	Agent Dollar
ANOVA:	Analysis of variances
CAPM:	Capital Asset Pricing Model
c.p.:	Ceteris paribus
CPI:	Consumer price inflation
EBIT:	Earnings before interest and taxes
DoE:	Design of Experiments
CD:	Compact Disc
e.g.:	‘Exempli gratia’ (Latin) alias ‘for example’
ERP:	Enterprise resource planing
EU:	European Union
EUR:	Euro (currency of the European Union/Eurozone)
GDP:	Gross domestic product
GE:	General Equilibrium
i.e.:	‘Id est’ (Latin) alias ‘that is’
IMF:	International Monetary Fund
IT:	Information technology
MOA:	Medium of account
MOE:	Medium of exchange
NOLH:	Nearly Orthogonal Latin Hypercube
OECD:	Organisation for Economic Co–operation and Development
SCM:	Supply Chain Management
SeSAm:	Shell for Simulated Agent Systems
TFP:	Total factor productivity
U.K.:	United Kingdom
UOA:	Unit of account
U.S.:	United States of America
viz.:	‘Videlicet’ (Latin) alias ‘namely’
vs.:	Versus

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Introduction

The foundation of macroeconomics, as a separate branch of economics, was laid down by John Maynard Keynes (1883 – 1946). Since the 1970s, probably encouraged by the 'Lucas Critique', many macroeconomists insist on a explicitly modeled 'microfoundation' of macroeconomics—as opposed to 'Keynesian' macroeconomics, where an explicit model is only existing on the aggregate level. This development resulted in the status quo of macroeconomic research: Since the early 1990s almost all important developments in the branch of macroeconomics were made by research based upon 'Walrasian microfoundation' (typically 'Dynamic Stochastic General Equilibrium' models)[1]. The central problem of this approach, as we see it, is the relation between micro and macro structure: In the overwhelming majority of applications, the 'microfoundation' of 'General Equilibrium' models is (according to simplification) built on the *aggregate* level. Obviously, this does not solve the essential problem of macroeconomics, namely how individual (i.e. microeconomic) behavior generates the dynamics on the aggregate (i.e. macroeconomic) level.

As an alternative approach, in recent years the agent–based simulation technique has emerged. This was enabled by a rapid improvement of computing power of IT systems and by the development of sophisticated programing languages. As a result of this development, the question arose, what is the main difference between the traditional 'General Equilibrium' framework in contrast to this new approach? We see the borderline between both approaches in the fact that agent–based macroeconomic models are built bottom–up, while 'orthodox' models are, as stated above, designed top–down on the macro level. Opposed to that, agent–based models are designed on the micro level. They contain about several thousand individual agents, and the researcher usually does *not*

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[1] The 'General Equilibrium' framework was initially developed by Léon Walras. The modern, i.e. dynamic, interpretation of the 'General Equilibrium' approach is provided by a model developed jointly by Kenneth Arrow and Gerard Debreu. See the discussion in chapter 1, and especially footnote 17.

constrain the macro level through specifications, which are necessary to compute (or better to run) the model (or the simulation).[2] The modeler of an agent–based computer simulation only observes the generated macro dynamics of the simulation, while he designs the model solely on the basis of individual behaviors and interactions.[3] Basically, this approach is related to the theory of 'complex systems'. The named 'complex system' consists of interconnected parts; its properties, as a whole, are not necessarily represented by the properties of the individual parts. Interestingly, older neoclassics (foremost represented by Alfred Marshall) thought of economics as a representation of a 'complex system'—but they did not possess the mathematical tools to solve dynamic applications of such 'complex systems'. This situation has changed by the advent of agent–based computer simulations.

This description leads us to the phenomenon that the benefits of agent–based modeling, which stem from its flexibility, are sometimes challenged by economists: The often heard criticism is that a scientific theory must be based upon 'abstraction', and that the agent–based modeling opens a door for the (more or less detailed) 'replication' of reality. But such a 'replication' would overload an economic model. It would lead to complex interrelations which cannot build the solid groundwork for economic theory–building. This would oppose the idea of 'abstraction' as a basis of scientific research. The key criticism is therefore that scientific models should be far less complex than reality. We want to survey the relation between 'abstraction' and agent–based modeling in a different perspective. One can define 'abstraction' as the process (or result) of generalization by reducing the information content of a problem the researcher is interested in. It is crucial that this reduction takes place in order to *retain* only that information which is *relevant* for the particular purpose. Therefore, our central question should be, which information is of pivotal importance in the context of macroeconomics?

Macroeconomic research, as we see it, should retain the emergence of macro structure out of micro behaviors and interactions. This macroeconomic emergence and the according theory of 'complex

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[2] Such a macro constraint is the general equilibrium. It is imposed to the aggregate level of a model, and it is necessary to solve it.

[3] In a further step he uses the observed macro outcomes in order to adjust the model. This is encompassed by the 'validation' of the model. See the discussion below.

systems' should be central issues in each complete macroeconomic investigation. Unfortunately, until now almost no research is carried out with respect to this research question in the field of dynamic monetary macroeconomics.[4] The present study aims to close this gap. Hence, we designed an agent–based macroeconomic model that is structured bottom–up, so that its aggregate dynamics develop out of both micro behaviors and micro interactions. As we will see, this leads to complex and non–linear micro–macro interactions. In this sense, our approach is related to Joshua M. Epstein's notion coined by the expression: "If you didn't grow it [author's note: the macro model], you didn't explain its emergence" (Epstein, 2006a, p. 9). Against this background, it is not legitimate to conclude that the complex micro–macro interrelations account for an unfavorable departure from 'abstraction'. The mentioned complexity is, in our view, the crucial feature of a macroeconomic system. It is therefore not legitimate to dispose these characteristics by 'abstraction', as usually done in 'orthodox' economics.

Objectives of the Study

The present study can be placed into the field of agent–based computational economics. As we will discuss in chapter 1, the agent–based technique enables a flexible way of designing, simulating and analyzing a particular model structure. In here, the structure of the model represents an intuitive analogy to reality. In addition, the benefit of flexibility induces the question, as to what extend the generated model is the 'right' one for a defined purpose? This is the subject of the model 'validation'. According to this, 'validation' is the key issue in agent–based research. Most importantly, our main purpose is therefore to develop a *reasonably validated* agent–based macroeconomic simulation model. Moreover, we have to outline the objectives the model is built for: The presented model needs to be a dynamic macro model. Its main innovation with respect to agent–based modeling is its 'monetary circuit' or 'monetary sphere'. As opposed to other agent–based research, the presented model belongs to the field of *monetary* macroeconomics. Equally important, the model has to contain 'Keynesian' and 'Wicksellian' elements. The former elements indicate several important 'Keynesian' properties, such as the importance of the demand side, the 'paradox of thrift', and so on. The latter elements impliy the role of the central bank and monetary policy. Accordingly, the

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[4] In the state–of–the–art framework of 'New Keynesian' macroeconomics, the economy is modeled on the aggregate level by a 'representative agent'. See, for example, (Woodford, 2003), and the discussion in this study below.

presented model contains a central bank agent that conducts monetary policy through an interest rate instrument. Thereby, the basic framework is constituted by Knut Wicksell's idea of a monetary transmission mechanism.

The second aim of this study is closely connected to the first one. The objective to construct a first agent–based monetary macro model causes the problem that we cannot use any existing framework. Therefore, the second purpose of this study is to develop a guideline for future work in this field. Here, the focus lies (i) on *methodological* aspects. As we will see, agent–based computational economics constitutes an IT–based tool, which enables to simulate a certain model structure—it is not a methodological basis for the model structure. Consequently, we have to define a methodological framework for the modeling. According to the important role of the 'validation' task, we must, in addition, elaborate an appropriate 'validation' methodology. Those two methodological questions have to be answered. (ii) Secondly, our guideline focuses on the theoretical aspects of the model. Therefrom, it is our aim to refer to the theoretical roots of the presented model—especially in context of its 'monetary circuit'. On the other part, we do not want to discuss all technical aspects, which are needed to conduct an agent–based research in principle. (iii) Thirdly, we identify some pitfalls that one could experience in carrying out research such as the presented one. Therefore, we will give advice how to identify possible sources of problems.

Structure of the Study

The structure of this study is straightforward: Chapter 1 gives a propaedeutic survey of the main topics of agent–based research. One challenge is thereby is to discuss the methodological aspects, such as the basic methodologies of the modeling and 'validation' approaches. The subsequent chapter establishes the conceptual model. It gives an detailed overview of the theoretical roots and antecedents of the model, and it outlines the reasons for the chosen design. We will also address problems of model design in this context. The study finishes with a comprehensive model 'validation' in chapter 3. This is executed in several stages, which are built on each other. The methodology of this 'validation' procedure is prepared in chapter 1. The study ends with concluding remarks.

Chapter 1

A Road Map to an Agent–Based Computational Macro Model

An economy is an evolving, complex, adaptive, and dynamic system. Other scientific fields than economics made much progress in the study of similar systems, which feature the same basic elements, such as heterogenous and autonomous entities (agents) that are engaged in complex interaction profiles, while the macro behavior of the system as a whole emerges out of micro structures, micro behaviors and micro interactions. The aggregate behavior emerges bottom–up. Such approaches are found in the fields of medicine and brain research, logistics, ecology and biology. Within those fields, computer modeling and experimentation is widely accepted (without much question) as valuable tools. On the contrary, to this date agent–based analysis did not attract great attention in economics, and in macroeconomics in particular. This can be due to the fact that macroeconomists are averse to agent–based approaches (Leijonhufvud, 2006a). The reasons for this phenomenon are shrewdly characterized by Axel Leijonhufvud:

> “The apparent threat of cognitive loss is perhaps steeper in macro than in other areas. Each generation of scholars inherits a knowledge base of theory, of empirically confirmed ‘facts’ and of investigative techniques. Inherent in this base are directions for future work—which problems are interesting and which ones not, what facts are puzzling and which ones can be taken for granted, what methods of investigation are approved and not approved, and so forth. The macroeconomics of the last century, from Lucas through Presccot to Woodford, has been strongly wedded to stochastic general equilibrium theory.

> It is the well–developed knowledge base with which the last couple of generations of macroresearchers have been equipped. Acquiring it required a large investment. But then recruits to this research program are confident that their technical equipment is the best in the business." (Leijonhufvud, 2006a, p. 1627)

The objective of this chapter is to discuss an alternative framework based upon the agent–based simulation technique. Hence, this chapter illustrates the main aspects of the approach of agent–based computational economics (ACE) and its advantages compared to 'General Equilibrium' (GE) theory. In the last section, we will describe a suitable 'validation' framework for the development of an agent–based macroeconomic model. As we will see, 'validation' is the core issue within agent–based research. Moreover, this chapter defines the main concepts of agent–based models, which are in turn necessary to develop and validate the model throughout the remainder of this study.

1.1 What is Agent–Based Computational Macroeconomics?

Imagine the total number of economic processes, such as producing and trading, happening in any economy in reality. They are usually driven by the actions of hundreds of thousand individuals, social groupings or institutions. In many circumstances information technology systems (IT systems)[1] support the execution of such actions. The basic idea of an IT system is to map real actions, facts and circumstances into digital data. Especially firms utilize IT systems to improve the efficiency of business processes: Suppose a supplier in the automotive industry, where an 'enterprise resource planning system' (ERP system in brief) collects the data of production and logistic processes. This system provides suitably prepared and presentable data in order to allocate business resources (materials, employees), for example through the scheduling of new orders or the minimization of inventory costs. Inevitably, the operations of the ERP system requires the interconnection between the real business processes and the respective data inventory within the IT system. Hence, there have to be some exogenous actions affecting the ERP system. This means, for example, that the data inventory has to be updated on condition that the stock of inventory of the automotive supplier has changed. Such maintenance can be operated manually by the users of the IT system,

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[1] IT defines the study, design, development, implementation, support or management of computer–based information systems, particularly software applications and computer hardware. IT deals with the use of electronic computers and computer software to convert, store, protect, process, transmit, and securely retrieve information.

as well as semi or fully automatically. In summary: The ERP system supplies information and data about business resources to the automotive supplier.

However, some systems—such as complex 'Supply Chain Management' (SCM) systems—contain fully automated processes due to the use of robots. These robots react automatically to a change in the data. For example, provided that the stock of inventory of an intermediate product needed in the production process of an automotive supplier (such as the stock of inventory of unmachined engine hoods) falls short of a certain level (e.g. 1,000 engine hoods), the robotic agent starts a fully automatic digital procurement process via a network (presumably via the internet). This means that the software agent executes a routinized search for suitable offer(s) in one or more online trading platforms, where suppliers and buyers of certain intermediate products meet. Such processes can appear on several stages of a vertical value added chain in a more or less automatic sense. A SCM system therefore collects, maintains and delivers data—but it can also feature automated elements where, for example, robotic trading happens. As a consequence, real business processes are affected by the information system automatically through robots, causing true interaction between real processes and the IT system. It is important that such an active role of the IT system must be guided by a rule–based or routinized behavior of the software agents. This behavior can even represent some kind of 'artificial intelligence'.

In a next step, we can reveal the basic idea of agent–based simulation[2] technique by using these introductory explanations: Like ERP or SCM systems, an agent–based computer simulation collects first of all digital data. It is populated by many agents, and each of these agents features a certain data set. The point is that the data set is not a direct representation of facts or information about reality as is the case in an ERP or SCM system. Rather, the data inventory of agents represents an abstract model, which is in turn the simplified representation of certain relationships known from reality. Accordingly, agent–based computational economics build upon the construction of an

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[2] A simulation is a certain type of modeling, whereas a model is a simplification of reality. Such a simplification implies a smaller, less detailed, or less complex representation of real processes or relationships. It thus builds on 'abstraction'. Similar to statistical models, simulation output is produced during a simulation run. This output depends on certain inputs (Gilbert and Troitzsch, 2005). We will investigate inputs and outputs of the presented model later on in this study. For a detailed discussion of simulation techniques in social sciences see Gilbert and Troitzsch, 2005.

artificial world, in which all actions are completely endogenous. This world covers special aspects of the real world we are interested in. The present study is interested in the behavior of a closed economy, i.e. the subject of the study is an an artificial world which represents an extremely simplified national economy encompassing the basic economic sectors. Within this artificial world, data are permanently generated, collected, and manipulated endogenously on the micro level. The key difference between the common (every–day) usage of information technology (e.g. as represented by an ERP system) and an agent–based computer simulation is that in the former at least some degree of interaction between reality and the information system is necessary, whereas in the latter all decisions, actions, and processes are fully automated—the agent world is autarchic.[3] This implies that an agent–based computer simulation contains agents, which are routinized robots, and which stand for the actors in the real processes we are interested in. This, in fact, represents basically the intuitive modeling approach of agent–based computer simulations. Moreover, such simulations are somewhat similar to complex SCM systems, in which robot agents are employed: If an agent simulation is started, each robot behaves exclusively according to the programed routines, so that no connection between the real world (e.g. the designer) and the simulation (run) prevails. To sum up, an agent–based computational simulation contains an autarchic artificial world containing robot agents represented by a set of data and rules (or routines). In the following paragraph we illustrate such an artificial world representing the subject of the present study.

Imagine the artificial world of Agent Island. Agent Island is a autarchic world populated by firm and household robot agents. If the computer simulation is started, the population arrives on Agent Island. Upon arrival each agent receives his personal data and instruction booklet: This booklet contains a set of rules and restrictions the agent has to follow as well as the initial data set. If the agent is trading any goods or services throughout the simulation, he has to register the movements in the data entries in his booklet. The agent–based simulation technique therefore supplies all possible data (individual, aggregate or otherwise manipulated data) to the researcher. The researcher can request the data entries in the booklets of those agents he is interested in. Data entries in the booklet of all agents are the basis for the routinized decisions and behaviors of the agents. That is,

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[3] Indeed, it is imaginable that there may be also some kind of human action or interaction in an agent–based model. Throughout this study, we are not interested in such approaches.

an agent uses these data together with the routines in his booklet in order to operate decisions and actions. Routines define therefore the processes of the agent (e.g. production or trading processes). Thereby, routines need not be static, insofar as they can evolve over time—again according to simplified and routinized adaption behavior. In addition, we use a round–based simulation approach, and the agents employ data to their routines once a round. If all routinized decisions and actions are conducted, the economy on Agent Island enters the next round. At the end of each round we collect data on aggregate levels, because the business cycle dynamics of the Agent Island economy is the topic we are ultimately interested in.

As suggested by intuition, we have to design the individual sets of data and rules for all relevant aspects of the model—for each agent of the Agent Island population. To give an idea of such an design, the following subsections highlights some important aspects of ACE. The next subsection illustrates the main conceptual building blocks. Thereafter, we describe which research objectives can be pursued within such a model, and which ingredients are necessary. Finally, the introduction closes with the discussion of the methodological relevance of ACE.

1.1.1 Conceptual Building Blocks

Agent–based models can be characterized by several concepts. However, this subsection does not give an in–depth review of these theoretical concepts; the objective is rather to outline the relevant building blocks of an agent–based computational model and relate them to the framework of Agent Island. We will discuss in section 1.2 the virtues of agent–based computational economics by comparing the 'orthodox' framework of macroeconomics with the possibilities of ACE. Thereby, we will take up the conceptual building blocks again and deal with them in somewhat greater detail. The following overview therefore summarizes the main building blocks of ACE in brief:

Bottom–up perspective and macroeconomic emergence Traditional 'neoclassical' models follow a top–down perspective, where the aggregate level typically comprises a 'representative agent'. In contrast, agent–based models build on an environment, in which micro entities engage in repeated interactions. As in reality, the dynamic on the macro level emerges from the behavior of the basic entities on the micro level (Windrum and Moneta, 2007; Pyka and

Giorgio, 2005; Tesfatsion, 2003). It is thus intuitive that Agent Island is designed bottom–up. This corresponds to the assumption that the agents, upon arriving on Agent Island, receive a personal data and instruction booklet. The macro behavior of the economy of Agent Island emerges from repeated individual actions and interactions according to the instructions and data in the booklets. Such an approach allows us to investigate the relationship between micro and macro dynamics. This is done during the 'validation' process in chapter 3. The relationship between micro and macro properties is of particular importance, when one is interested in the analysis of 'fallacies of composition' in economics.[4]

Heterogeneity Agents might be heterogenous in almost all characteristics, i.e. with respect to data or behavior. The former might be defined through varying variables or initial values of some variables (Pyka and Giorgio, 2005). The latter is based upon varying behavioral rules or, at least, levels of behavioral parameters within one rule. According to that, the personal data and instruction booklets of the population of Agent Island reflect this heterogeneity. In here, we simplify by the assumption that agents of the same type (households, consumer goods firms, capital goods firms) receive the same rules, but the level of the parameters in the rules can vary.

Network direct interactions: Interactions among agents are direct and inherently non–linear. This means that the decisions of an agent depend to some extend on the past and present choices made by all other agents (Pyka and Giorgio, 2005). Moreover, in ACE the trading and procurement processes are usually modeled explicitly, which implies that the institution of the 'Walrasian auctioneer' is not mandatory (Tesfatsion, 2006). Consequently, it is possible to employ various forms of procurement processes within an agent–based model. In particular, ACE enables 'face–to–face' interactions within a procurement process. We will explain below that such a 'face–to–face' procurement process is adopted in the market for capital goods on the island. Then again, the consumer goods market is working simplified in institutional analogy to 'orthodox' economics (viz. by employing implicitly some kind of auctioneer).

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[4] A 'fallacy of composition' could arise when one infers that something is true for the whole from the fact that it is true for some part of the whole. We will refer to this concept, and explain it with respect to a relevant application in chapter 2. See also Stützel, 1978, for an extensive discussions of such 'fallacies of composition' in economics (especially based upon flow–of–funds accounting).

Bounded rationality By its nature, the environment on Agent Island is too complex to apply hyper–rationality. This is for example apparent in the context of expectation formation, because agents on Agent Island are not able to derive rational expectation outcomes, as in 'orthodox' models. Rather, one has to apply routinized outcomes of myopic optimizations in combination with adaptive expectations. The latter is necessary, because agents face 'true uncertainty'[5] so that expectations cannot be rational as assumed by 'orthodox' economic theory. According to this, the agents on Agent Island face 'true uncertainty', so that they do not know (and cannot calculate) the future outcome of economic interactions on the island. This must affect the formation of expectations in such a way that expectations are adaptive.

Learning Behavior In many ACE models sophisticated learning algorithms are implemented (Tesfatsion, 2006; Windrum and Moneta, 2007).[6] Not so in the present study. In a first step of the development of the model, we have employed such a complex and sophisticated learning algorithm. As suggested by Tesfatsion, 2006, we have applied it to the supply decision of consumer goods firms. Unfortunately, this design produced undesired effects on the macro level, i.e. the assumed 'Phillips curve' relationship (viz. the positive correlation between output gaps and inflation rates) was upside down. Therefore we abandoned this approach and have adopted a more suitable approach for the supply decisions, as it will be described in subsection 2.2.2. In this approach, firms adopt their behavior to a change in the environment on Agent Island, but a complex learning algorithm is absent.

1.1.2 Objectives

The following description illustrates four main objectives of agent–based research. If necessary, we extend each description by a short link to the objectives of the present study:[7]

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[5] Here, 'true uncertainty' means 'Knightian uncertainty' (Knight, 1921), i.e. situations which cannot be described with a certain probability of occurrence. This 'true uncertainty' is different from risk. The latter is usually employed in 'orthodox' economic models, where it is necessary to assign probabilities of occurrences in order to handle this kind of uncertainty (i.e. risk) in expected utility functions.

[6] For a discussion of several learning algorithms see Brenner, 2006.

[7] Tesfatsion, 2003, gives a review of the agent–based literature and relates the models to certain ACE topics. Insofar as none of these models fall into the field of monetary macroeconomics, we do not refer to them here explicitly. So far as we know, the only agent–based model that can be placed into the field of monetary economics is an older one, created by Bruun, 1995. Hence, we do not present an introductory literature review. Nevertheless, we will refer to specific ACE research throughout the representation of the model in chapter 2.

Empirical understanding In this case the researcher has to investigate the question, why certain empirical phenomena or regularities evolve. They seek for causal explanations for such phenomena through agent–based environments (Tesfatsion, 2006). Based upon empirical understanding an agent–based simulation can deliver predictions of future tendencies or events (Gilbert and Troitzsch, 2005).

Normative understanding An agent–based model can deliver normative insights as well (Tesfatsion, 2006). It is certainly possible to compare various policies (e.g. various central bank strategies) based upon a *valid*[8] agent–based model. The crucial point is the 'validation' of the ACE model. Even though we do not chase after any normative objectives, our analysis could to some extend be useful for further normative postulates. It delivers a correctly validated model, which is necessary to conduct a normative analysis.[9] Our objective is to deliver such a model: This could be a starting point for normative analyses in the future or, at least, a foundation for the further development of a valid monetary macro model that in turn could be used for a normative analysis.

Methodological advancement The question of interest is, how best to provide agent–based researchers with a suitable methodology needed to undertake a study of the economic system. Thereby, researches need to model structural, institutional and behavioral characteristics of the economic system; they ought to evaluate the logical validity of their model through computer experiments, and test their theories against real–world data (Tesfatsion, 2006). Due to the flexibility of agent–based models, those requirements can be fulfilled through a variety of ways. If the researcher is able to find a proper way for doing this, he develops further methodological insight with respect to the topic of interest. In the context of the present study, this is one aim. We strive for the development of a reasonable validated agent–based monetary

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[8] See subsection 1.3 for the notion behind this term.

[9] Economists make a distinction between positive and normative that closely parallels Karl Popper's view of philosophy of science (Popper, 2005). See also Friedman, 1953 for a comprehensive discussion of this point. A positive statement is a statement about what is, and that contains no indication of approval or disapproval. Notice that a positive statement can be wrong. "The earth is made of chocolate" is incorrect, but it is a positive statement, because it is a statement about what exists. Then again, a normative statement expresses a judgment about whether a situation is desirable or undesirable: "The world would be a better place, if it were made of chocolate" is a normative statement, because it expresses a judgment about what ought to be. Notice that there is no way of disproving this statement. If you disagree with it, you have no sure way of convincing someone who believes in the statement that he is wrong. Along those lines of philosophy of science it is possible to divide the objectives of agent-based research into positive and normative groups.

macro model. This should become the basis for further analysis of monetary policy issues. In addition, we apply a 'validation' framework developed in the field of computer science (see section 1.3.3), which has never been applied to an economic issue until now. Accordingly, we wish to deliver a suitable framework for further research in monetary macroeconomics within the field of agent–based computational economics.

Qualitative insight and theory generation Through research in agent–based models one can gather new insights about an economic issue of interest. An agent–based simulation can be used as a method of theory development, in order to improve the understanding of phenomena of the social world (Gilbert and Troitzsch, 2005). Consequently, a well–designed and suitable agent–based world can improve the understanding of the dynamic behavior of a complex economic system. Usually, this objective is based upon the systematic examination of simulation inputs[10] (initial values, behavioral and structural parameters, etc.) and their impact on simulation outputs of interest (Tesfatsion, 2006).

The last point expounds the idea that ACE has the potential to assist in the discovery and formalization of theories. Researchers can investigate theories in the artificial agent world they have built. In order to do this, the researchers have to take theories expressed in textual or conceptual form and formalize them into a specification which can be programed into the computer. According to this, the theory will be precise, coherent and complete. In this respect agent–based computer simulations could feature a similar role in social sciences, comparable to that of mathematics in the physical science (Gilbert and Troitzsch, 2005). On the contrary, mathematics have been widely used as a means of formalization in economics and econometrics. In fact, there are several reasons why agent–based simulations are more appropriate to social science than mathematics (Gilbert and Troitzsch, 2005). We will explain these main virtues of agent–based computational economics in section 1.2, and, in addition, compare them to 'orthodox' economic modeling (which is solely based on the mathematical framework of 'optimal control theory'). Inevitably, the presented model of Agent Island illustrates how the formalization of an agent–based monetary macro model can look like.

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[10] See footnote 2 for an explanation of simulation input. We will explain the detailed role of inputs later on.

1.1.3 Ingredients

The following overview contains a broad set of ingredients, each agent–based computational model consists of (see Pyka and Giorgio, 2005):

Time As an agent–based model is by its nature a dynamic model, we have to define the time perspective of the model. As we will see, the model is round–based, i.e. it evolves in discrete time steps, which we define as periods. Next to this period time ($T = 1, 2, ...$), there exists an intra–period time. The sequence of decisions and actions within one period is based upon the concept of intra–period time. Hence, when one period ends, the intra–period sequence restarts.

Agents Each agent–based simulation is populated by a set of agents. The term 'agent' refers to bundled data and methods (or routines). It represents an entity constituting a part of a world constructed by computation. Agents can be (i) individuals (e.g. consumer, workers), (ii) social groupings (e.g. families, firms, government agencies), (iii) institutions (e.g. markets), (iv) biological entities (e.g. livestock, forest), and physical entities (e.g. weather, geographical regions) (Tesfatsion, 2006). In context of the present task, viz. the development of a monetary macro model, agents represent the actors within the opted framework, viz. households (i.e. consumers/workers), firms (i.e. consumer goods and capital goods firms) and the central bank. It should be noted that we assume a constant set of agents. The existing agents do not die (drop out), and no new agents are born during a simulation run. Thus, the once initialized population outlasts the whole simulation run. In general, agents are supposed to be (i) autonomous entities (i.e. the state of the agent and its actions are first of all independent from its environment or other agents), (ii) social entities (i.e. agents are able to interact with other agents), (iii) reacting entities (i.e. agents are able to perceive their environment, which usually leads to a reaction), (iv) active entities (i.e. agents are able to initiate actions themselves) (Pyka and Giorgio, 2005).

Micro variables Each agent is characterized by a vector of microeconomic (state) variables. Those variables are usually supposed to be modified endogenously throughout the simulation. In our model such microeconomic variables are, for example, the net financial wealth (or net debt) of a household agent, or the real capital stock of a firm agent, or the produced/supplied output of firms, and so on. During the 'validation' of the model it is one task to define reasonable initial values of several microeconomic variables (such as the initial capital stock of firms).

Micro parameters Next to the micro variables each agent is characterized by a vector of microeconomic parameters. Parameters are variables that cannot be endogenously adapted throughout a simulation run. Typically, such parameters describe the behavior of the agent (behavioral parameters) or certain restrictions (structural parameters). For example, the supply decision of a consumer goods firm is defined via a behavioral parameter. This parameter connects the produced/supplied output of the present period to the marginal profitability of one output unit in the last period. Moreover, this supply decision is restricted by a structural parameter characterizing the production function. To highlight the important micro parameters of the model we label them through lower case Greek letters.

Macro parameters The system as whole is characterized by a vector of macroeconomic parameters. Similar to micro parameters, macro parameters cannot be modified endogenously, i.e. once fixed to a certain level, these values remain unchanged. In the present model, the technological progress is represented through a 'random walk process' defined by two parameters, namely by a 'drift term' and the variance of the 'white noise' term. Such a technical progress is constituted on the global level (i.e. for the whole economy) and on individual firm levels. A combination of both figures constitutes the individual technical change of a firm. Besides this, on the global level the 'drift term' and the variance are defined by two macro parameters. We call such macro parameters also global parameters. To highlight the important macro parameters of the model we characterize them also through lower case Greek letters.

Macro (or aggregate) variables Finally, there exists a set of macroeconomic variables. Usually, such variables (such as the GDP) emerge through some kind of aggregation of micro variables. Other macro variables are by nature defined on the macro level (e.g. the credit interest rate). We call macro variables also global variables.

Interaction structure The interaction structure controls the flow of information between agents. Consider firm agents that are trading on the capital goods market. Provided that two specific agents close a contract for the sale of a capital good (i.e. a machine), the seller updates his order book, while the buyer books a purchase order. Simultaneously, the account is settled by the buyer. According to that, the cash reserve of the buyer decreases, while the cash reserve of the seller increases by the same amount. Besides this, there is a third party involved in this payment process, as we apply a banking system to the model. Thereby, subsequent actions of each of the parties (in the next period) can be affected by that trading. According to this rather simple example, on can imagine that relatively complex interaction structures emerge on Agent Island.

Micro decisions rules Each agent is endowed with a set of decision rules. Such rules are routines, which map observable figures (past micro variables and macro variables or parameters) into present micro variables. Such a mapping process is based upon the micro parameters (i.e. behavioral or structural parameters) of the individual agent. It can also contain stochastic elements, if necessary. The concept of decision rules is crucial to agent–based models. It mirrors the notion of routinized behavior, known from 'evolutionary' economics (see explanations below). As we will discuss later on, micro decision rules based upon micro parameters define the 'genes' of the agents.

Space In principle, it is possible that an agent–based computational model features a spatial dimension. For example, the real map of a landscape could serve as the environment, in which agents live, produce and trade. This enables a more specific perspective on trading and other interactions. However, for the sake of simplicity we do not integrate such a spatial dimension to Agent Island.

1.1.4 Methodology vs. IT–Based Tool

According to the descriptions mentioned so far, one could assume that agent—based computational economics constitutes a methodology—such as the 'Walrasian' GE approach defines the methodological framework of modern 'neoclassical' macroeconomics. This, however, is not true. Agent–based computer simulations are a tool, viz. an IT–based technique of simulating a certain model. In here, an agent–based model features a general structure as described in the last subsections. According to this notion, it is not surprising that an agent–based framework would in principle allow the analysis of a GE model.[11,12] In fact, this would lead to the degeneration of the virtues of an agent–based technique. Consequently, it is interesting to see whether an alternative methodological framework for ACE is existing: We prefer the framework of 'evolutionary' economics.[13] The following table 1.1 illustrates, why the assumptions or concepts of 'evolutionary' economics fit very well into the agent–based approach. As the reader can see, the agent–based simulation technique is an ideal tool for the analysis of 'evolutionary' economics. The table should in addition compare the assumptions of 'neoclassical' and 'evolutionary' methodology.

Assumptions	Neoclassical	Evolutionary
System behavior	Can be derived from micro level Time and place independent Need not be dynamic	Not deducible from micro level Time and place dependent Has to be dynamic
Individual behavior	Optimizing Mechanical	Satisficing[1] Rules–of–thumb & routines
Interactions	Perfect capabilities & information Actors are substitutable	Imperfect capabilities & information Actors are not substitutable Learning, path dependency, co–evolution
Actors	Hyper–rational agents No history Often homogenous	Boundedly rational robots History existing Typically heterogenous

Sources: See Alkemade, 2004; Arnold and Boekholt, 2002; Jaffe1 et al., 2002; Nelson and Winter, 1982. Note: 1) The term 'satisficing' is coined by Herbert Simon. The tendency to satisfice shows up in many cognitive tasks such as playing games, solving problems, and making decisions where people typically do not or cannot search for the optimal solutions (Simon, 1982).

Table 1.1: Comparison of methodologies – neoclassical vs. evolutionary economics

This review should render a better understanding of the elements of an 'evolutionary' model. The concrete meaning will become clear throughout the remainder of this study. However, the main differences and virtues of 'evolutionary' economics based upon an agent–based environment will be worked out through the next section. Within the following brief description of the basic concepts

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[11] For an illustration of a 'Walrasian' agent–based computational model see Gintis, 2007.

[12] The main problem of such an approach would be the calculation of rational expectations in a forward–looking framework. However, when the model is completely developed within the boundaries of GE models (e.g. by the application of one 'representative agent'), one could handle this problem in the same way as 'orthodox' economics does, so that the 'representative agent' knows all structural equations of the mechanical system. As a result, he could calculate the rational expectations outcomes of the economy far into the future.

[13] In the context of macroeconomics the term 'evolutionary' economics goes back to the seminal work of Nelson and Winter, 1982. The framework of Nelson and Winter follows the 'Schumpeterian' view of capitalism as an engine of progressive change. This view is connected to the problem of economic agents concerning the future, viz: The key character of progressive change is that it seems impossible for agents to calculate the right thing to do. What is an appropriate action and what not, will be only revealed by future events (see also Knight, 1921).

of ‘evolutionary’ economics, we link them to the agent–based approach of the present study (see Nelson and Winter, 1982):

Routines The set of routines of an agent describes the way the agent is doing things and the ways he determines what he has to do. Hence, the concept of routines covers the more ‘orthodox’ notions of capabilities (budget constraints) and choice (maximization). Behavior defined through routines does neither mean that agents behave irrational nor that their behavior is unchanging. Moreover, the concept of routines links the present behavior of the agent to the actions the agent (or its environment) is taking or has recently undertaken. Even though the basic flexibility of routinized behavior is limited, we can extend the framework so that a changing environment can force agents to modify their routines (see the next point). The concept of routines is basically one of the most important concepts used throughout the development of the model in this study. Each agent decides and behaves according to routines. Usually, a routine links past data (macro or micro data) to present decisions and actions. For example, each period the supply decisions of capital goods firms is delivered through a routine. Capital goods firm calculate their individual offer price through a ‘mark–up’ calculation, i.e. via a given percentage ‘mark–up’ over given marginal costs. This routine delivers the supply price of capital goods firms. Importantly, routines are the genes of an ‘evolutionary’ theory.

Search This concept contains all activities which are associated with the evaluation and potential modification of routines. The point is that such activities are themselves routinized and predictable. Then again, they can also have a stochastic character. To use the example of the supply decisions of capital goods firms above, the firm modifies its supply decision each period through the adjustment of price ‘mark–ups’. This is in turn a routinized activity, as one can see within the next point.

Selection environment The ‘selection environment’ is the ensemble of conditions outside or inside the agent, which affects its well–being or success. Such conditions can be delivered on the micro level (of the respective agent for example) or on an aggregate level (for example on the industry level). For instance, the above discussed supply decision of capital goods firms is determined via ‘mark–up’ pricing. As stated, this as well as the adaption of ‘mark–ups’ is routinized. Importantly, the adaption of ‘mark–ups’ (i.e. the ‘search process’ for a better ‘mark–up’)

> is defined via the 'selection environment' of the agent. This is given through past supply decisions, the resulting sales and profit figures and the conditions of the capital goods market. The 'selection environment' defined through these conditions gives the basis for the routinized adaption of present price 'mark–ups' in a rational way.

The term 'genes' within that description sheds light on the analogy between 'evolutionary' economics and biology.[14] According to this, there a is link between the genotypic level (i.e. behavioral patterns, technologies, policies etc.) and the entities (i.e. the agents) accommodating these genes (Dosi and Nelson, 1994). In fact, this notion mirrors exactly the notion of agent–based modeling. It is thus not surprising that several examples of agent–based models based upon the methodological framework of 'evolutionary' economics exist (for example Dosi et al., 2005; Dosi et al., 2006; Dosi et al., 2008).[15]

1.2 Virtues of Agent–Based Computational Macroeconomics

In this section we review the weakness of the orthodox approach to macroeconomics, and confront these weaknesses with the virtues of agent–based computational economics. In here, we subsume both the (neoclassical) 'Walrasian' GE approach and the 'New Keynesian' framework[16] of monetary theory under the term 'orthodox' economics. In fact, all modern models that belong to the group of 'orthodox' economics are rooted in the Walras or Arrow–Debreu framework.[17] This section reviews some assumptions and aspects of these models—namely those aspects which are subject to criticism. In order to illustrate the main positions of orthodox economics and compare them to the agent–based approach, we introduce a nearby island to Agent Island. The artificial economy of this neighbor

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[14] Before the development of 'evolutionary' economics Alfred Marshall in fact states "that the Mecca of economics [lies] in economic biology rather than economic mechanism" (Marshall, 1948, p. xiv).

[15] For more information on the link between 'evolutionary' economics and agent–based computational economics see, among others, Dosi and Winter, 2002; Tesfatsion, 1997; Dosi and Nelson, 1994.

[16] See Woodford, 2003, for an introduction to the 'New Keynesian' framework. It is derived as the so–called 'New Neoclassical Synthesis' from the 'New Keynesian' paradigm (see e.g. Mankiw and Romer, 1991) and 'Real Business Cycle' models (see e.g. King and Rebelo, 1999). Woodford calls his approach also 'Neo–Wicksellian', because it builds on the distinction between the natural rate of interest and the money or credit interest rate (Woodford, 2003). We will explain both concepts in the following chapter.

[17] The 'Arrow–Debreu' framework is the modern successor of the original Walras model (see the original paper of Arrow and Debreu, 1954). It is the groundwork for all 'Dynamic Stochastic General Equilibrium' (DSGE) models, which were mentioned in the last subsection. The key is that it extends the static framework of Walras by introducing so–called 'Arrow-Debreu securities'. The notion of such securities draws on the concept of risk, i.e. that future states of the world could be defined through probabilities. If a certain state occurs, only that specific 'Arrow–Debreu security' assigned to this specific state pays out. All other 'Arrow–Debreu' securities pay zero return.

island is built upon a different structure compared to Agent Island. The following paragraphs illustrate that.

Population

The economy of the neighbor island of Agent Island is constituted by a 'representative agent'.[18] Now, what is, or rather what does the 'representative agent' in the artificial island economy? Gun, 2004, characterizes the idea of the the 'representative agent' unequivocally:

> "However, the representative agent of new macroeconomics is not 'representative' in this way [note of the author: here, 'this way' means representing a lot of different people]: He is identical with the people he 'represents'—because only identical persons are considered. Why are only identical persons considered? Because aggregation of non–identical agents creates problems. But, if people are identical, they have no reason for trading (exchange results from differences, in tastes, endowments, technologies): the situation is exactly the same if there is one or 'many identical' persons. 'Representative agent' is, thus, another name for Robinson Crusoe: new macroeconomics is 'Crusoe microeconomics' and, therefore, devoid of usefulness—it is even a regression in comparison with the 'old' (IS–LM) macroeconomics. Moreover, it is nonsense. New macroeconomists probably feel this, as they practically *never* try to justify the representative agent assumption. In the alphabetical index, at the end of their books or textbooks, they often 'forget' to mention him (as also happens with the 'auctioneer', in the index of microeconomic textbooks)." (Gun, 2004, p. 120)

Thereby the crucial point of the assumption that such an economy is populated by many identical households is not the word 'many'—rather, the key word is 'identical' (Gun, 2004). This notion implies that the many agents can be represented by one single agent. For this reason, we call this island subsequently Robinson Crusoe Island. The need for the modeling of the 'representative

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[18] The 'representative agent' framework, as applied in almost every modern application of the 'orthodox' framework, goes back to Ramsey, 1928, and Cass, 1965. It should be noted that these seminal papers were normative studies, i.e. they search for economy's best path. Accordingly, it would be ideal, if aggregate savings behaved according to the constrained optimization of an aggregate utility function. However, in many modern applications (within the 'orthodox' branch of economics) the idea of the original normative 'representative agent' model is applied to positive models (Gun, 2004). This is a substantial chance, because the notion of the 'representative agent' approach could be seen as an ideal (efficient) outcome of barter. But it could be hardly seen as a good positive representation of reality.

agent', which is indeed a pretty strong simplification, lies in its simplicity: It reduces the complexity of the orthodox framework in order to get stable and unique equilibria (Fagiolo and Roventini, 2008). Another study describes the failure of modern 'representative agent' macroeconomics in the following way:

> "[...] it seems worthwhile to review why Walrasian microfoundations should be considered as the wrong answer to what is probably the most stimulating research question ever raised in economics, that is to explain how a completely decentralized economy composed of millions of (mainly) self–interested people coordinate actions." (Gaffeo et al., 2007, p. 91)

Hence, the 'representative agent' living on Robinson Crusoe Island represents not a component, simpler than the system of which he is part (Leijonhufvud, 2006a). This would be an intuitive assumption of an economy and its parts. The idea that the whole system is more complex than the part it is made up of, is one core assumption of 'complex system theory'. In addition, such a system consists of interrelated components. Not so the economy of Robinson Crusoe Island. Its economy is reduced to a unique single agent. But this contradicts the very essence of microeconomics, because without diversity of agents, there cannot be any exchange (Gun, 2004).

A good critical review of the 'representative agent' approach is delivered by Kirman, 1992. He finds at least five major aspects of criticism to the 'representative agent', which summarize the core problem of this approach: (i) Individual rationality does not imply aggregate rationality. This means that one cannot provide any formal justification for the assumption that the maximizing individual behavior could be applied to the aggregate level. (ii) The reaction of the 'representative agent' to shocks cannot coincide with the aggregate micro reactions of individuals. (iii) Even if the above mentioned problems are solved, other cases are existing where out of two given situations x and y, the 'representative agent' would prefer x, while all the individual agents would prefer y. (iv) There appears an additional problem at the empirical level. If one tests a theory delivered by a 'representative agent' model, one is also jointly testing the 'representative agent' hypothesis. (v) Finally, in case of heterogenous agents, it is implied that basic properties of linear dynamic micro properties are not preserved by aggregation. For example, the aggregation of *static* micro–equations could produce *dynamic* macro equations (Froni and Lippi, 1997).

We want to finish the discussion of the 'representative agent' living on Robinson Crusoe Island by a pointed picture delivered by Gun, 2004:

> "But, at the same time, they present representative models as positive models, and try to fit the model with existing data (through 'calibration' and other techniques): observed GDP, employment, consumption, investment of a country during, say, 10 years, are thus compared with what a representative agent's intertemporal choice would be–taking into account observed 'shocks'. This is total nonsense: How can any reasonable person admit that, for example, the evolution of the US aggregates' results from decisions made by a single individual who owns all factories and who decides how much to produce, how much labor to use, how production will be distributed between consumption and investment, and so on? It is quite incredible that the majority of a profession (which pretend to be 'scientific') readily indulges in this kind of absurdity, teaches it, and does a lot of 'research' on it—with maths, statistics, and computers—attempting to specify the representative agents 'parameter' (that is, coefficients in his utility and production functions) which allow good fits with observed data." (Gun, 2004, p. 121)

In contrast to this view, agent–based computational economics enables maximum flexibility in the design of heterogeneity. The artificial economy of Agent Island is populated by many agents, and these agents might be heterogenous in many dimensions (such as endowments, technology, tastes, behavior, etc.). We have already explained this issue. It is the difficult task of the model design and its 'validation' process to find a reasonable specifications for the heterogeneity. However, the role of heterogeneity is not as trivial as one might expect. It is not a mere extension of the homogenous agent framework: If heterogeneous agents (e.g. heterogenous with respect to behavior) adjust continually to the overall situation they create together, then they adapt within an environment they created together. And in so adapting, they change that environment (which could also be termed 'ecology'). According to this, 'evolution' (in the sense of 'evolutionary' economics) is used in the broadest sense of the word, which can be interpreted as elements adapting their state to the situation they together create (Arthur, 2006). We see that in this sense our adopted framework of ‘evolutionary’ economics emerges naturally from the very construction of the modeling in the agent–based framework. It need not be added as an adjunct.

Against the background of those explanations, it should be clear that the artificial economy of Agent Island emerges bottom–up; it is not constructed top–down as the Robinson Crusoe economy. We start from individual choices, whereas the latter takes as its starting point observed relations between aggregates. In general, agent–based computational are characterized in the following way:

> “There is no central, or ‘top down’, control over individual behavior in agent–based models. Of course, there will generally be feedback between macrostructures and microstructures, as where newborn agents are conditioned by social norms or institutions that have taken shape endogenously through earlier agent interactions. In this sense, micro and macro will, in general, co–evolve. But as a matter of model specification, no central controllers (e.g., Walrasian auctioneers) or higher authorities are posited ab initio.” (Epstein, 2006b, p. 1588)

Consequently, the present analysis is able to investigate the true relationship between micro behavior and macro dynamics, which is not possible in ‘representative agent’ models. This will ultimately enable the discussion concerning ‘fallacies of composition’.

Behavior

Next to the problematic aspect that Robinson Crusoe Island is populated by just one single representative inhabitant, the behavior of this Robinson Crusoe agent is furthermore quite unrealistic: Its basic structure is defined through a fundamental abstraction. It covers intertemporal choice, according to which an intertemporal expected utility (in case of uncertainty) is maximized subject to the budget constraint(s). This is a typical dynamic programing problem, known from ‘control theory’—an interdisciplinary branch of engineering and mathematics.[19] It is, for example, similar to the program that an engineer has to solve in order to determine the best path for the flight of a rocket (e.g. with minimum use of fuel), given its target (Gun, 2004):

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[19] Such models belong to the field of ‘dynamic stochastic optimal control theory’ (Colander, 2006).

> “It is a problem for an engineer, *not* for an economist. And, it can be very complicated to solve (as always with non–linear programs). Indeed, generally, it is not possible to find the exact optimal path, but only successive approximations of it (using computer and so on). So, the door is open to a lot of of ‘work’, and ‘papers’, about maths and econometric techniques that to get an ‘as good as possible’ approximation for the optimal path, with different kinds of utility and production functions, and ‘shocks’. As unkowns (paths) are sequences of functions (and not numbers, as in common micro problems), Hamiltonians replace Lagrangians, and first order condictions take the form of differential equations; as there is an unlimited horizon, ‘transversality’ conditions exclude infinite solutions, and so on. These are very complicated problems; but they are Robinson Crusoe’s problems—not ours!” (Gun, 2004, p. 122)

In addition, there are further problems concerning the behavior of the ‘representative agent’: For example, one can state that he is in fact ‘schizophrenic’, because he is at the same time a firm and a household: He employs himself and sells (buys) to himself. He pays himself (and earns) a wage equal to the marginal productivity of labor, and pays (again to himself) an interest rate equal to marginal productivity of capital (Gun, 2004). Finally, such models rest usually on the inconsistency that all firm and household agents are price takers. Firms and household treat prices as given in their optimization problem (in case of perfect competitive markets). But if prices are given for everyone, who sets those prices? See the following statement of Hal Varian:

> “The biggest problem is one that is the most fundamental, namely the paradoxical relationship between the idea of competition and price adjustment: if all economic agents take market prices as given and outside their control, how can prices move? Who is left to adjust prices?” (Varian, 1992, p. 397)

Within ‘orthodox’ theory this puzzle is solved by the concept of the ‘Walrasian auctioneer’, who searches for prices that solve the mutual optimization problem of all agents in the economy (see the ‘formal view’ of the GE framework below). Hence, he matches demand and supply schedules in all relevant markets of the economy of Robinson Crusoe Island. We will discuss below the implications of this abstraction, and its impact on interaction of agents and the role of information. However, we know that Agent Island is populated by a plurality of agents that employ rule–based or routinized behavior. This is a strong deviation from the ‘orthodox’ framework: It enables a maximum flexibility to the design of agent behavior. This leads to a more realistic modeling of interaction, information processing, uncertainty, and so on. The researcher is not caught in the narrow ‘prison’ of ‘Walrasian’ economics; however, if necessary, he can ‘borrow’ some aspects form ‘orthodox’ economics. The point is that the modeling features the flexibility to move as near as necessary to the relevant behavior of agents. Nevertheless, agent–based models possess a likewise high level of ‘abstraction’. In contrast to some critics of agent–based research we can state that ‘abstraction’ in ACE prevails as the core concept of scientific research. Agent–based modeling, however, opens the possibility to adjust the degree of abstraction perfectly to the needs of the research field or the investigated topic as opposed to ‘orthodox’ economics, where the degree of abstraction is unchangeably defined by the strict assumptions of the GE framework.

Another important issue of agent behavior is the role of uncertainty. We know that Robinson Crusoe (of ‘orthodox’ economics) follows the notion of far forward–looking rational expectations[20]. To understand this idea, we have to make some preliminary considerations: In this context the differentiation between ‘risk’ and ‘true uncertainty’ is important. This topic is introduced by Knight, 1921, whereby ‘risk’ refers to situations where an agent can assign mathematical probabilities to the randomness which he is facing. On the contrary, in case of ‘true uncertainty’, the existing randomness can *not* be expressed in terms of probabilities. This phenomenon lies in the mere complexity to *assign* probabilities to events. Because of the complexity of an economy as a whole and because of the interacting behaviors, it seems to be impossible for any economic agent to calculate probabilities for relevant states. This point was already emphasized by John Maynard Keynes in his ‘response’ to his critics in 1937 where he states:

> “By ‘uncertain’ knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again,

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[20] The term ‘rational expectations’ is coined by Muth, 1961. It implies that agents’ expectations are *correct on average*. To put it differently, although the future is not fully predictable, agents’ expectations are assumed not to be systematically biased. The agents use all relevant information in forming expectations of economic variables. The notion behind that is the ‘best guess’ of the future or ‘the optimal forecast’ of future outcomes.

> the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know." (Keynes, 1973, p. 113-114)

In addition, Keynes added:

> "[...] the hypothesis of a calculable future leads to a wrong interpretation of the principles of behavior." (Keynes, 1973, p. 122)

The concept of rational expectations applied to the optimization problem on Robinson Crusoe Island is therefore questionable. It should thus not surprise that Agent Island is subject to 'true uncertainty': The islanders cannot calculate any rational expectations outcomes for the future values of economic variables. They simply do not know the true 'meta model' of the economy. The researcher does not know this model neither. He rather endows the agents with data and routinized behavior, but he has no 'meta model' of how the interaction of many thousands of such routines operates. If he had, he would not need to conduct agent–based computer simulations. Lastly, it is important to examine the behavior of agents under 'true uncertainty'. According to Keynes two 'factors' influence the expectation formation of an agent (Keynes, 1936): (i) current facts, and (ii) the expectations of other agents. Keynes argues that expectations formed under 'true uncertainty' tend to be to a considerable degree backward–looking as they project past or current situational 'factors' into the future instead of being exclusively forward–looking, as suggested by the rational expectations hypothesis. According to this notion, it is apparent that the inhabitants of Agent Island form backward–looking expectations, because they are not able to forecast the true future outcomes of the model.

In brief, the Robinson Crusoe agent is endowed with a sort of hyper–rationality or 'olympic' rationality (Fagiolo and Roventini, 2008). Its rationality knows no bounds (Leijonhufvud, 2006a).

In contrast, the artificial economy of Agent Island is built upon simplified rule–based or routinized behavior; agents are thus subject to the concept of ‘bounded rationality’. This ‘bounded rationality’ has two components (Epstein, 2006b): Agents have neither global information nor infinite computational power.

Economic Interactions

One important source of interaction is rooted in the role of uncertainty. According to Pesaran, 1987, decision making under uncertainty can be described by a process in which an agent is not perfectly aware of the consequences of his action. When one examines the role of uncertainty in economics, two different sources of uncertainty can be identified (Pesaran, 1987): (i) ‘exogenous uncertainty’, and (ii) ‘endogenous uncertainty’. ‘Exogenous uncertainty’ covers uncertainty due to exogenous ‘factors’ (in the context of macro dynamics this is for example described by the role of exogenous disturbances like wars or other political events). In contrast, ‘endogenous uncertainty’ can be attributed to the impact of economic actions chosen by some other agents. Therefore, ‘endogenous uncertainty’ is also characterized as ‘behavioral uncertainty’, because it arises endogenously from the behavior of other market participants. Following ‘endogenous’ or ‘behavioral uncertainty’ the occurrence of a certain state is not an invariant result of the agent’s own behavior. The existence and prevalence of ‘behavioral uncertainty’ is rather due to the capacity of individuals to adapt and react to another in a non–negligible manner (Pesaran, 1987). Consequently, the degree of ‘behavioral uncertainty’ is related to the extent to which individuals are be able to influence the actions of others by their own actions, or conversely, to what extent they are themselves influenced by the actions of other agents. Therefore, Pesaran, 1987, concludes that in reality all decentralized systems of economic decision making are subject to ‘behavioral uncertainty’.

This point highlights the weakness of the ‘microfoundation’ in the case of Robinson Crusoe: In this approach ‘endogenous’ or ‘behavioral uncertainty’ is not comprised. Agents do not react explicitly to the behavior of other agents. They rather do constraint optimizations in order to obtain their individual demand or supply schedules. All ‘interactions’ are considered via the ‘Walrasian auctioneer’:

> “The most salient structural characteristic of Walrasian equilibrium is its strong dependence on the Walrasian Auctioneer pricing mechanism, a coordination device that eliminates the possibility of strategic behavior. All agent interactions are passively mediated through payment systems; ‘face–to–face’ interactions are not permitted. [...] The equilibrium values for the linking price [...] variables are determined by market clearing conditions imposed through the Walrasian Auctioneer pricing mechanism; they are not determined by the actions of consumers, firms, or any other agency supposed to actually reside in the economy. Walrasian equilibrium is an elegant affirmative answer to a logically posed issue: can efficient allocations be supported through decentralized market prices? It does not address, and was not meant to address, how production, pricing, and trade actually take place in real–world economies through various forms of procurement processes. [...] What happens in a standard Walrasian equilibrium if the Walrasian Auctioneer pricing mechanism is removed and if prices and quantities are instead required to be set entirely through the actions of firms and consumers themselves? Not surprisingly, this ‘small’ perturbation of the Walrasian model turns out to be anything but small. [...] As elaborated by numerous commentators, the modeler must now come to grips with challenging issues such as asymmetric information, strategic interaction, expectation formation on the basis of limited information, mutual learning, social norms, transaction costs, externalities, market power, predation, collusion, and the possibility of coordination failure.” (Tesfatsion, 2006, p. 833–835)

Against the background of the stated notion of behavioral uncertainty it becomes obvious, that the economy of Crusoe Island does not feature such behavioral uncertainty or any forms of interactions beyond the price mechanism governed by the ‘Walrasian auctioneer’. In addition, according to the application of the ‘representative agent’ in the household and firm sectors, the logic of ‘behavioral uncertainty’ within these sectors is totally factored out. In contrast, trading on Agent Island is governed by procurement processes. Evidence from real world implies that:

> “[...] customers and suppliers must identify what goods and services they wish to buy and sell, in what volume, and at what prices. Potential traders must be identified, offers to buy and sell must be prepared and transmitted, and received offers must be

> compared and evaluated. Specific trade partners must be selected, possibly with further negotiation to determine contract provisions, and transactions and payment processing must be carried out." (Tesfatsion, 2006, p. 834)

This observation is the basis for the design of interaction through 'face–to–face trading' and procurement processes on Agent Island.[21] In case of removing the 'Walrasian auctioneer' and turning to more realistic procurement processes, we have to apply a minimal requirement: This implies that individual agents on Agent Island have to be endowed with rules, which satisfy that (i) terms of trades (prices and production levels) are defined, (ii) a seller–buyer–matching is described (defined through search routines), (iii) actual trade is conducted, (iv) settlement is fulfilled, and (v) a rationing mechanism is defined in case of excess demand (Tesfatsion, 2006). Furthermore, generating and processing information takes up an important role with respect to the interaction of agents. The following paragraphs discuss that point.

Information

By now it should become clear that the access to information and the possibility to process the available information is crucial to the structure of the economy. The economy of Robinson Crusoe Island is based upon some strict and special assumptions concerning the treatment of information. In principle, there are two alternative views concerning the role of information in the 'Walrasian' framework:

Informal view According to the 'informal view' the process of price determination is not defined clearly. It is the vague notion stated above, namely that all agents are price takers, and prices are set by an unknown process. According to Kirman, 2006, this characterization of the 'Walrasian' system characterizes an uncontrolled system, where equilibrium prices are found through a undefined bargaining process. The crucial point is that according to the 'informal view', the Robinson Crusoe agent possesses a great deal of information (effectively all information existing in the whole economy). Moreover, the agent must exhibit all calculation capabilities to process the information. Consequently, the 'informal view' represents the idea of hyper–rationality or 'olympic' rationality—as stated above. The key problem of the 'informal

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[21] This is especially true for the capital goods market, which features 'monopolistic competition'.

view' is that in reality agents are endowed with limited computing power and they have access to limited information only (Colander, 2006). Thus, one can assume that no real–world agent can do such complex calculations into the far future, for which economist have to use complex computer–based approximation algorithms. Note that the GE framework is, however, far less complex than reality. Hence, in reality, agents would need much more computational power to solve 'Walrasian' optimization problems.

Formal view In contrast to the 'informal view', there exists the more abstract and precise view, which is near the original view of Léon Walras or Kenneth Arrow and Gerard Debreu. The 'formal view' assumes that the amount of information that individuals have to know and process is negligible (Kirman, 2006). All the agents need is the current vector of prices and their opportunity set. Given those facts, 'Walrasian' agents have to calculate and announce their excess demand to a central institution. But agents have to know nothing about the generation of equilibrium prices. The mechanism behind the generation of prices is the 'Walrasian auctioneer'. Assuming this, little information is needed by individual agents, because the relevant information is processed for them. The key problem to this central price setting mechanism is the fact that the application of this central auctioneer to reality is impossible. Such a central institution would need an infinite amount of information in order to bring the system into equilibrium (Kirman, 2006).

In both views the treatment of information is crucial. Both views represent pretty unrealistic assumptions on generating and processing information. In contrast, on Agent Island, agents do not posses perfect information, nor is there a central planer that is needed to calculate equilibrium prices for the markets.[22] Accordingly, some individual data (i.e. information) are designated as publicly accessible to all other agents, some are designated as private and therefore not accessible by any other agents, and some are designated as protected from access by all but a specified subset of other agents (Tesfatsion, 2006). Figure 1.1 illustrates the role of information within economic interactions on Agent Island.

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[22] As already noted, the consumer goods market on Agent Island is cleared by a central institution that collects individual supply and demand schedules. This is for the sake of simplicity. But this is not the case in the capital goods market.

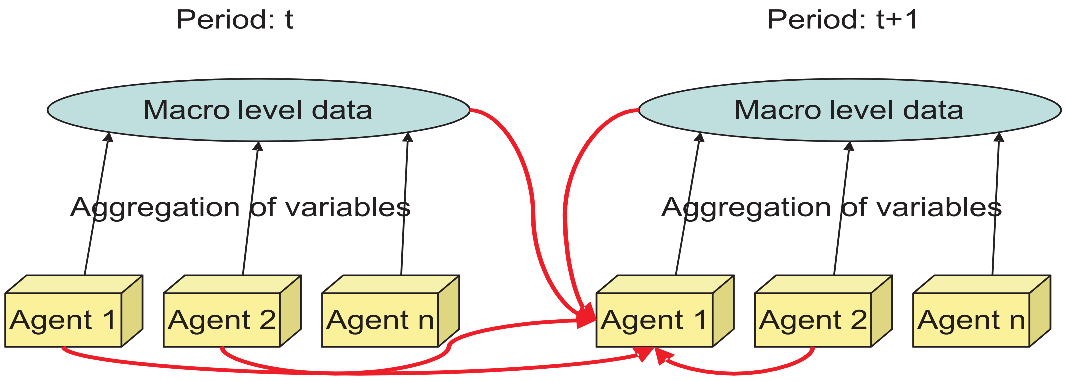

Figure 1.1: Interaction profile represented by the information flow

Figure 1.1 depicts the fact that aggregate information emerges bottom up by aggregating individual information (e.g. by aggregating individual incomes to GDP). This aggregate information set is used in the following period by agents, which is illustrated through agent 1 in the figure: He receives information from the last–period aggregate level (e.g. the last–period's consumer goods price) and from the present aggregate level (e.g. the present credit interest rates). In addition, he receives information from other agents (e.g. the individual supply prices for goods he demands), generated in the present or in the previous period. Finally, agent 1 receives own information generated in the last period, such as the last–period's disposable income. Hence, there are many sources of information. In sum, this enables complex interaction effects within the model. One can state, as Leight Tesfatsion does, that "everything seems to depend on everything else" (Tesfatsion, 2006, p. 861). This is obviously a major difference between Robinson Crusoe Island and Agent Island.

Role of Money

Both artificial island economies are designed to perform monetary policy analysis[23]—but on Robinson Crusoe Island money is not decisive. This means that the economy is based upon the real exchange of goods and services: All economic transactions are rooted in their real parts. If at all, money is *added* rather than *integrated* into the model. In addition, the islanders of Robinson Cursoe Island have perfect calculation capacities, so that the main functions of money (such as the reduction of transaction cost through fewer relative prices) are obsolete:

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[23] In this case Robinson Crusoe Island represents the 'New Keynesian framework' as outlined by Woodford, 2003.

> “However the structure of the dominant macro– and microeconomic theories of our time, which are built upon the modern version of Walrasian general equilibrium theory, ignores the financial dimensions of capitalist economies. [...] The postulate of general equilibrium theory which ensures that money and finance are excluded from the core of the theory is that variables in preferences systems are goods and services.” (Minsky, 1995, p. 197 and 207)

On the contrary, the economy of Agent Island is endowed with a central bank that issues means of payment. Money is decisive on Agent Island, and it is integrated in each economic process. Accordingly, there exists a ‘unit of account’, a ‘medium of account’, and a ‘medium of exchange’ on Agent Island.[24] This enables, for example, the storage of wealth through financial assets. Finally, the model of Agent Island features a stock–flow consistent framework of an economy. There exists a double–entry book keeping system, so that, for example, ‘flow–of–funds accounting’ is possible. It is therefore possible to analyze systematically the ‘monetary circuit’ of Agent Island. In each computational step of the simulation run the researcher has access to any accounting data, such as flow–of–funds data of each individual agent, or on each level or aggregation. This is obviously an advantage over ‘orthodox’ economics.

Macroeconomic Dynamics

The artificial economy of Robinson Crusoe Island moves even beyond perfect information by adding stochastic risk (defined through probability distributions) to the general equilibrium optimization problem over time. Accordingly, the behavior of Crusoe becomes a gigantic ‘dynamic stochastic optimal control problem’ (Colander, 2006). One key difference of Agent Island and Robinson Crusoe Island lies therefore in the notion of equilibrium:

> “One key departure of ACE modeling from more standard approaches is that events are driven solely by agent interactions once initial conditions have been specified. Thus, rather than focusing on the equilibrium states of a system, the idea is to watch and see if some form of equilibrium develops over time.” (Tesfatsion, 2006, p. 843)

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[24] This terms are defined at the end of chapter 2.

On Robinson Crusoe Island the equilibrium is exogenously imposed on the economy, i.e. the researcher searches for simultaneous equilibrium prices in all relevant markets. This searching is conducted by the 'Walrasian auctioneer'.[25] In contrast, no exogenous equilibrium concept is *imposed* on the Agent Island economy. The aggregate behavior and dynamic of the economy evolves bottom–up out of individual actions and interactions. It is one topic of the 'validation' procedure to find an appropriate equilibrium concept, which can be applied to our simulation model.

Lastly, the artificial economy on Robinson Crusoe Island does not contain an explicit theory of business cycle dynamics, because the economy rests in the steady state unless it is hit by some *exogenous* stochastic shocks. It does therefore not explain the movements of the business cycle endogenously. It rather generates its dynamics with a sort of 'deus–ex–machina mechanism' (Fagiolo and Roventini, 2008). 'Walrasian' researches ask the question, how can deviations from the equilibrium take place? They search for shocks that account for such fluctuations. In addition, they usually add 'imperfections' (such as nominal price rigidities) to the system, which account for the fact that shocks are not perfectly dampened (Colander, 2006). For example, on 'Woodford Island' (the island of the 'New Keynesian' macroeconomics, the state–of–the–art in modern monetary theory) real effects of monetary policy are exclusively based upon the existence of price rigidities. If they were absent, the model would immediately fall back into a new flexible–price equilibrium state, after the occurrence of an exogenous shock. In this equilibrium state the real interest rate is the outcome (i.e. the equilibrium price) of the market for savings and investment. In contrast to that view, the artificial economy of Agent Island has a tendency to chaotic behavior, which is kept under control by institutions. The important question is therefore, why is there as much stability in an economy as there is? Thus, one difference between the perspective of Crusoe Island and that of Agent Island lies in the instability of the system. For Agent Island, "what is unusual about the macroeconomy is not that it exhibits instability; it is that it is not in total chaos" (Colander, 2006, p. 10).

Drawbacks of Agent–Based Computer Simulations

According to Tesfatsion, 2006, one can identify (of course) some drawbacks of the agent–based computer simulation approach: (i) An agent–based model requires a dynamically *complete* modeling.

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[25] This is the logic of the above described 'formal view' of the 'Walrasian' GE.

This implies that starting from the setting of the model, the model must permit and fully support the playing out of agent interaction *without* further intervention from the researcher. Due to complex interactions and feedback loops, this initial adjustment of the model is a really difficult task. (ii) According to this requirement of *complete* modeling, the researcher has to consider all possible cases (outcomes or states); otherwise the model could stop, provided that a state occurs which is not considered in advance (such as the capital stock of a firm is falling to zero). (iii) In the next place, it is not clear how well agent–based models will be able to scale up to provide empirically and practically useful models of large–scale systems with many thousand agents. (iv) Lastly, the major problem is the 'validation' of the agent–based model, i.e. the adjustment of the model settings against empirical data. This last point is the central issue of the following section.

1.3 Validation Framework

It is the main purpose of the present study to deliver a reasonable validated macroeconomic model. 'Validity' is thereby the key property of an agent–based simulation model (Klügl, 2008b). It means that the 'right' model is used with respect to the intention of the researcher (Balci, 1994). Hence, validity of an agent–based model is necessary for any normative analysis: A valid model produces reliable results, and only a valid model is able to answer questions directed at the original system. Therefore, 'validation' can be defined as "the process of determining whether a simulation model is an accurate representation of the system, for the particular objectives of the study" (Law, 2005, p. 24). In general, there exists a variety of 'validation' types. For example 'validation' can be empirical, or statistical; 'validation' can cover the theory, the conceptional model, or the program code, and so on.[26] In this study we follow the 'validation' approach suggested by Klügl (Klügl, 2008a; Klügl, 2008b), which is developed in the field of computer science. Figure 1.2 illustrates the framework of this approach.

Before we describe the single steps depicted in figure 1.2 during the following subsections, we want to illustrate some problems regarding the 'validation' process. ACE is an interesting framework due to the intuitive structure of the models based on the analogy between agents and the active

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[26] See for example Sargent, 2007, for a review of the various 'validation' types.

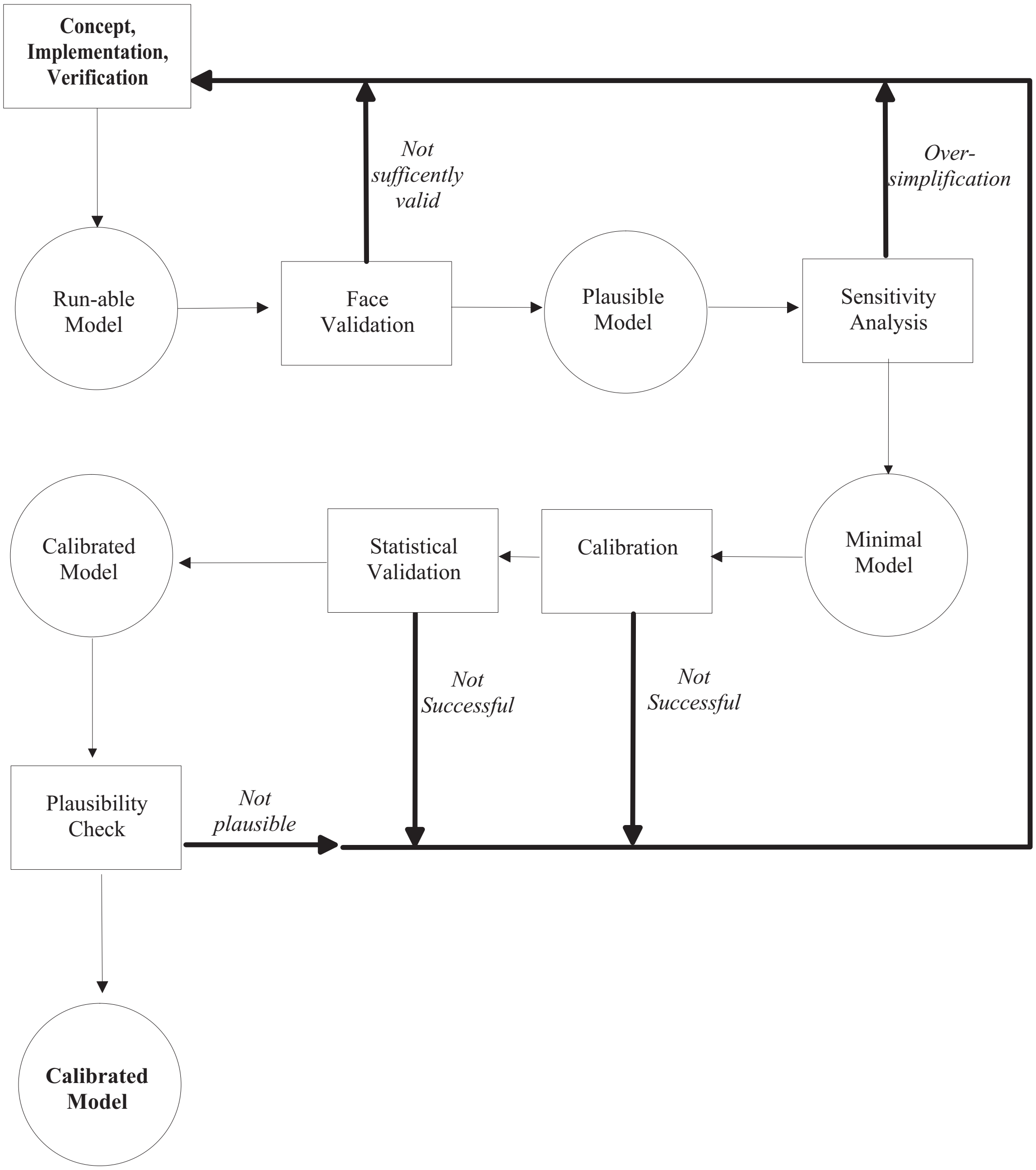

Figure 1.2: Validation framework; Source: Klügl, 2008

elements in the ‘original system’. The ‘validation’ of such an ACE model is an essential task, which features several problems (see Klügl, 2008b): For example, empirical and statistical ‘validation’ is only possible, if characteristic figures can be defined that describe the system in a correct way. The topic of the present analysis treats time series data on the aggregate level. In this circumstance, it will be necessary to compress the time series data to some individual figures, e.g. expected value and standard deviation. Moreover, agent–based simulations are adequate for studying transient dynamics. It is therefore of interest whether the dynamics of a system can or cannot lead to a steady state. In addition, the ‘validation’ task is further complicated through complex interaction effects, feedback loops and non–linear effects of parameter changes on simulation outputs.

Furthermore, it is problematic that the ‘validation’ process is optimally conducted on multiple levels. In effect, this means that input–output relations have to be investigated on the aggregate level, but also on some disaggregate levels—down to the individual agent level (Klügl, 2008b). In the context of the ‘validation’ of the present model, we forgo such a ‘validation’ on multiple levels of the model. According to our framework, we validate the model exclusively to phenomena perceived on the macro level: Thereby the parameters on micro level have to be adjusted in such a way that the macro output matches that of the ‘original system’. We call this ‘original system’ also the ‘reference system’. The approach of validating a model only on the macro level is along the lines of the concept of ‘generative sufficiency’, which is explained in the subsequent paragraph. Besides this, ‘validation’ on the individual level would be a difficult (if not impossible) task. This is at least due to lacking individual micro data. We will see later on that most empirical studies are conducted on the aggregate level (such as the estimations of aggregate savings’ rates, or aggregate production functions, and so on). Empirical studies based on micro data are quite rare. Finally, there exists the problem of over–parametrization: Suppose that the model contains to many ‘degrees of freedom’. In this case an automatic optimizing calibration tool will be always able to fit the model to the data.

As a consequence of multiple problems concerning ‘validation’, the concept of ‘generative sufficiency’ was introduced by Epstein, 2006a:

> “Agent-based models provide computational demonstrations that a given microspecification is in fact *sufficient to generate* a macrostructure of interest. Agent–based modelers may use statistics to gauge the generative sufficiency of a given microspecification—to test the agreement between real-world and generated macro structures. [...] A good fit demonstrates that the target macrostructure—the *explanandum*—be it a wealth distribution, segregation pattern, price equilibrium, norm, or some other macrostructure, is effectively attainable under repeated application of agent-interaction rules: It is *effectively computable by agent society.* [...] Indeed, *this demonstration is taken as a necessary condition for explanation itself.* [...] Thus, the motto of *generative social science*, if you will, is: If you didn’t grow it, you didn’t explain its emergence. [...] In summary, if the microspecification m does not generate the macrostructure x, then m is not a candidate explanation. If m does generate x, it is a candidate. If there is more than one candidate, further work is required at the micro–level to determine which m is the most tenable explanation empirically” (Epstein, 2006a, p. 8 and p. 9)

According to ‘generative sufficiency’, the focus of our model development lies in the ability to reproduce observed aggregate phenomena based upon individual agents and their interactions. We apply this notion and integrate it into the ‘validation’ framework of Klügl, 2008b: As explained above, the intention of ‘validation’ is to verify that the ‘right’ model is used for the purpose of interest. The point is, what the word ‘right’ implies? In the present context we use the concept of ‘generative sufficiency’ to concretize the notion behind the term ‘right model’. Accordingly, our model is the ‘right’ one, if it is able to reproduce (bottom up) the macro behavior of the ‘reference system’. Because it is our aim to develop a micro structure that generates the macro phenomena we are interested in, the present model belongs to the group of ‘generative social sciences’—and the ‘validation’ approach of figure 1.2 guarantees that the model is able to fit to the macro phenomena. Conversely, we exclude the formal testing of any ‘micro validity’. The following subsections explain briefly the ‘validation’ steps indicated by figure 1.2.

1.3.1 Conceptual Model

The basic building block of an agent–based simulation is the 'conceptual model'. Constructing an ACE model gives the researcher a sense of playing God in his own artificial world. As explained throughout section 1.1, the researcher has to define a number of agents with characteristic variables, a set of decision rules or routines, and an environment in which interaction takes place. Those definitions are constituted in the 'conceptual model'. In case of the presented model, the programing took place in the SeSAm programing environment.[27]

For the initial construction of the model we followed a three–step approach (see Bruun, 1995). In the first step, we have to find macro–bindings, which are relevant to our macro system. We thus define that the model of Agent Island has to feature the following aspects on the aggregate level:

1. The Agent Island economy is a closed economy without a government sector (i.e. government expenditures do not take place). The aggregate national income equation for the Agent Island economy is $Y = C + I$.
2. We design a 'perfect competitive' consumer goods market and a capital goods market featuring 'monopolistic competition'. In addition, a labor market must be integrated, in which a central bargaining between a labor union and an employer association takes place at the beginning of each period.
3. Agent Island has a closed 'monetary circuit' without cash money, i.e. there is a perfect bookkeeping system for monetary flows and stocks.
4. Economic policy on Agent Island is executed through interest rate policy of a central bank. This implies the existence of monetary 'transmission channel(s)'. In here, we use a 'Wicksellian' framework.

The identification of macro–bindings does not imply that we should model those macro–bindings first, but they assist us in the subsequent steps. In the second step, we consider the micro analysis:

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[27] SeSAm stands for 'Shell for Simulated Agent Systems'. The reader can find the SeSAm software, tutorials, as well as additional material on http://www.simsesam.de/. The installer for the presented simulation model is located on the CD in appendix C.

This covers the design of individual behavior of agents, i.e. how they act and react. As mentioned, we have to bear in mind the above mentioned macro restrictions when modeling behavior on the micro level. Accordingly, the Agent Island economy encompasses markets for consumption goods and capital goods. The latter is necessary because of the intertemporal characteristics of the model, i.e. according to to the central bank interest rate policy. Moreover, the central bank policy must affect some decisions of the agents, so that there is at least one 'transmission channel' of monetary theory. The financial settlement of transactions is conducted through a book keeping system. Finally, we assume 'perfect competition' in the consumer goods market, 'monopolistic competition' in the capital goods market, and central bargaining in the labor market. We will explain these assumptions during chapter 2. However, the design of the markets affects the individual routines of all agents. We have to keep these points in mind throughout the design of any transaction. The third and final step of the basic model design is the simulation. It combines macro and micro perspectives. This implies the interaction of micro behaviors and the macro bindings within the programed computer simulation.

1.3.2 Face Validation

The 'validation' process described in figure 1.2 starts with a run–able model. This implies that simulation output can be generated through simulation runs. It is important to note that this does not mean that 'validation' is irrelevant in earlier phases of the model development. In fact, the opposite is true: If not, at first, the conceptual 'validation' is considered, the subsequent steps considered in the 'validation' framework do not make sense (Klügl, 2008b). We define 'face validation' in accordance to (Klügl, 2008b, p. 3): "All tests based on reviews, audits, involving presentation and justification of assumptions and model structure are used for reaching this form of plausibility". Importantly, 'face validation' takes place on several aggregation levels, i.e. it can be applied on the macro or individual level (Klügl, 2008b). Usually we are interested in aggregate model outputs given through absolute values, relations between different values, and the dynamics of certain variables. In here, the researcher (i.e. a human expert) has to evaluate whether the simulation behaves like the 'original system'. In context of the model of Agent Island, the process of 'face validating' the model has taken up several months. In our view, it is maybe the most helpful (or effective) device within the whole 'validation' process. Without intensive 'face validation', we could have never developed and validated the model in a reasonable way.

1.3.3 Sensitivity Analysis

Within the present framework depicted in figure 1.2 the results of the sensitivity analysis delivers a minimal model to be investigated in the further 'validation' process. This implies that parameters without significant impact on model output drop out from further investigations. Equally important, the sensitivity analysis is used to verify the assumed relationships between micro parameters and macro output. Accordingly, we use the sensitivity analysis to develop a basic understanding of our simulation model (Kleijnen et al., 2003). In this context the present subsection should give some basic methodological guidelines for a sensitivity analysis and computer experiments. The latter is necessary, because the data used in the sensitivity analysis are generated through computer experiments. Hence, we need to discuss some basics in 'experimental design' (usually termed 'Design of Experiments', or DoE in brief) as well. Before we turn to these specific topics, we have to define some terminology (see Fang et al., 2006):

Experiment, 'Design of Experiments' An experiment is the methodical configuration of a systematic scientific inquiry.[28] Such an experimental inquiry can be conducted physically or as a computer experiment. Within an experiment there is at least one experimental 'factor' (see the next point) which is varied, in order to investigate the systematic effect on the 'response(s)' of the experiment (see again below). The configuration of the variation of 'factors' is subject to the 'Design of Experiment'. Thus, DoE indicates how to vary the settings of the 'factors' to see whether and how they affect the 'responses'.

Factor A 'factor' is a controllable parameter (such as structural or behavioral parameter, but also initial values of endogenous variables) that is of interest in the experiment. In general, a 'factor' may be quantitative or qualitative. Except for one case we treat only quantitative 'factors'. The only qualitative 'factor' leads to a distinction of cases within our analysis. In a computer experiment (as in the present study), a 'factor' is often called 'input variable' as

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[28] In general, an experiment is a method of investigating less known fields, solving practical problems, and proving theoretical assumptions.

well. Henceforward, we will use both terms (i.e. ‘factor’ and ‘input variable’) as synonyms.

Experimental domain, level, and scenario The experimental domain is the space where the ‘factors’ or the input variables take values. Within computer experiments this is also called ‘input variable space’. A ‘factor’ may be chosen to have few or many specific values, at which the ‘factor’ is tested. We call these selected values the levels of the ‘factor’. In addition, a level combination defines an investigated scenario, i.e. it defines a certain point in the ‘input variable space’ or the ‘experimental domain’. Sometimes it is also called ‘experimental point’.

Run A run defines the implementation of a scenario (or level combination) in a computer experiment. Multiple runs within the same scenario (i.e. replications or reruns) reproduce the same results, when the model is deterministic; or they produce various results, when the model exhibits stochastic elements. The latter is the case within the model of the present study.

Response The ‘response’ defines the results (outputs, or outcomes) of a simulation run based on the purposes of the experiment. Usually, the ‘response’ is a quantitative measure, but it can be also qualitative or categorial. We concentrate on quantitative ‘responses’ throughout this study.

Factorial design A ‘factorial design’ is a set of level combinations with the main objective of estimating the effects of the ‘factors’ on the ‘response(s)’. It is the topic of DoE to find an appropriate ‘factorial design’. In some cases, it is possible and appropriate to investigate the total experimental domain in an experiment. Such a design is called ‘full factorial design’. On the other hand, in a ‘fractional factorial design’ only a subset of all level combinations (i.e. the entire input variable space) is investigated.

In the next step we explain, at first, the opted ‘experimental design’. This gives the basis of the experimental investigations in section 3.3. Thereafter, we give a short illustration of the statistical methodology applied in the sensitivity analysis of section 3.3. As explained above, the sensitivity analysis is based upon data generated through computer experiments.

Design of Experiments: Nearly Orthogonal Latin Hypercube

Considering the model of this study, it will be necessary to design the experiments in an appropriate way, because we have to investigate many ‘factors’, and the underlying processes are assumed to be complex and non–linear. The following statement should illustrate the rationale for designing experiments and what happens if this is not done:

> “Instead of using even a simple experimental design, many analysts end up making runs to measure performance for only a single system specification, or they choose to vary a handful of the many potential ‘factors’ one–at–a–time. Their efforts are focused on building, rather than analyzing, the simulation model. DoE benefits can be cast in terms of achieving gains (e.g., improving average performance by using DoE instead of a trial–and–error approach to finding a good solution) or avoiding losses (e.g., obtaining an optimal result with respect to one specific environmental setting may lead to disastrous results when implemented).” (Kleijnen et al., 2003, p. 2)

As explained above, DoE covers the variation of the experimental ‘factors’, i.e. it treats the factorial design. Suppose a sensitivity analysis with n ‘factors’. We can therefore define a ‘design matrix’ F_X for experiment X:

$$F_X = \begin{pmatrix} f_{1,1} & f_{1,2} & \dots & f_{1,n} \\ f_{2,1} & & & \\ \dots & & & \\ f_{m,1} & & & f_{m,n} \end{pmatrix}$$

In general, the columns of the ‘design matrix’ correspond to ‘factors’, and the entries within a column represent settings (or levels) for the corresponding ‘factors’. The rows represent a particular level combination, scenario, or design point. The levels may be coded (e.g. ‘+’ for the high level, ‘0’ for the medium level, and ‘-’ for the low level in a three–level design). Considering the present study, we do not use coded levels, i.e. we use the ‘natural levels’ of the ‘factors’. The matrix above is arranged in the following way: $f_{1,2}$ delivers the level of ‘factor’ 2 (second column) in the first of m scenarios (first row). In general, the ‘design matrix’ is spanned over a set of n ‘factors’, and m scenarios. This gives a $m \times n$ ‘design matrix’. Besides this, we assume r replications of the m

scenarios. Accordingly, we need to conduct $r \times m$ total runs in the experiment. Next to the 'factor matrix' F_X, the 'responses' of experiment X are captured in a 'response matrix' R_X (including l 'responses'):

$$R_X = \begin{pmatrix} r_{1,1} & r_{1,2} & \dots & r_{1,l} \\ r_{2,1} & & & \\ \dots & & & \\ r_{m,1} & & & r_{m,l} \end{pmatrix}$$

This matrix is arranged in analogy to the 'design matrix' above. Again, as we apply r replications, we obtain r replications of this 'response matrix'. The description so far will give us a rough idea of 'experimental design'. However, there is an obvious problem: When the number of 'factors' becomes large, the data requirement grows exponentially. For example, if we investigate only two 'factors' with two levels, we have to conduct $2^2 = 4$ runs. If we expand the levels from 2 to 10, this number rises to $10^2 = 100$ scenarios. If we investigate 10 'factors' with just 2 levels, $2^{10} = 1,024$ runs must be conducted and investigated. Finally, if we are interested in 10 'factors' each of them comprising 10 levels, this amounts to 10 billion simulation runs! Thereby, no replications are assumed. In anticipation of the description of the conceptual model and the according parameters, we must state that we are interested in more than 10 'factors', each of them spanned over a broad domain.

As a consequence, we have to apply a smart DoE method in order to conduct the sensitivity analysis (see Sanchez, 2006): We apply the design of a 'Nearly Orthogonal Latin Hypercube' (NOLH).[29] This design provides a flexible way of constructing an efficient design for computer experiments with many 'factors'. In particular, a NOLH features good 'space filling' properties. See the scatterplots in figure 1.3 for a comparison of a 'full 5^4 factorial design'[30] and its NOLH counterpart for 4 'factors'. In the 'full factorial matrix' each of the $5^4 = 625$ 'design points' have to be investigated. Unlike the 'full factorial design', the NOLH design employs only few design points: In case of the 4 'factor' design (see figure 1.3), we just need 17 'design points'. Thereby, the design is called 'Latin Hypercube', because it requires that there is only one 'design point' in each row and one in each column. This

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[29] This design goes back to Cioppa, 2002.

[30] 5^4 means 4 'factors' each of them comprising 5 levels.

gives a 'Latin Hypercube'. Moreover, this notion is extended by the the concept of 'orthogonality', which implies that the entire 'input variable space' is sampled evenly. As a consequence of the NOLH design, we require 65 'design points' for the investigation of an experiment comprising 16 'factors'.[31] As we will see in section 3.3, we are interested exactly in 16 'factors'.

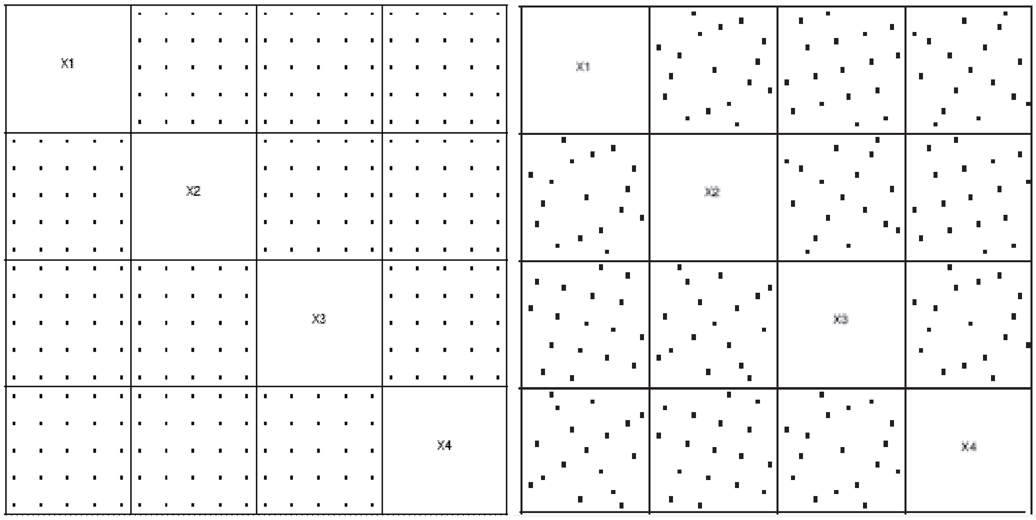

Figure 1.3: Scatterplot matrices of (i) a full 5^4 factorial design (left panel) vs. (ii) an Orthogonal Latin Hypercube design with 4 continuous factors (right panel)

Gaussian Kriging

The sensitivity analysis in section 3.3 is based upon the estimation of a 'meta model'. Suppose that $x_i, i = 1, \ldots s$ are design points over an s–dimensional experimental domain, and suppose a response y. In addition, we assume that there exists a true 'meta model' (of the economy of Agent Island) that describes the connection between inputs and the response (see, for example, Fang et al., 2006). This 'meta model' is represented by a real–valued function:

$$y = f(x_1, \ldots x_s) = f(x),\ x = (x_1, \ldots x_s)' \in T.$$

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[31] The Excel spreadsheet is available on http://diana.cs.nps.navy.mil/seedlab/software.html. It delivers the specification of the 65 scenarios. In general, the spreadsheet supplies 'nearly orthogonal designs' up to 29 'factors'. The obtained outputs constitute a 'Nearly Orthogonal Latin Hypercube' design in the units of the problem.

In here, T defines the entire input variable space. The scenarios investigated in the sensitivity experiments deliver the data required to estimate an approximation $\hat{y} = g(x)$ of this (true) 'meta model' $y = f(x)$. In this context we use a 'Gaussian Kriging'[32] model, which belongs to the family of linear least squares estimation algorithms. The goal of the 'Gaussian Kriging' model is to estimate the value of the unknown true function, f, at a point x^*, given the values of the function at some other points $x_1, \ldots x_s$. The crucial point is that the 'Kriging' method is usually applied, if one is interested in a large number of inputs, and if one is able to investigate a rather small subset of the entire input variable space (as suggested above by the NOLH design): Therefore, 'Kringing' provides a sophisticated method to *interpolate* the value of a random field at an *unobserved* location from observations of its value at *nearby* locations.

The 'Gaussian Kriging' model is defined by Fang et al., 2006, as

$$\hat{y} = g(x) = \sum_{j=0}^{L} \beta_j B_j(x) + z(x),$$

whereby $B_j(x), j \in [0, \ldots, L]$ is a set of 'basis functions' defined on the investigated subset of the input variable space. The figure z constitutes the random error. In case of the 'ordinary Kriging', this equation (i.e. the 'basis functions') is simplified to

$$\hat{y} = g(x) = \mu + z(x),$$

As opposed to the usually applied IID assumption, i.e. that the random error is independent and identically distributed, the 'Kriging' method rather assumes that $z(x)$ is a 'Gaussian process'.[33] This rather assumes that the realization of y (i.e. the 'response') is normally distributed with mean μ and variance σ^2. The variance–covariance matrix is represented by $\sigma^2 R$, and the R matrix is composed of elements r_{ij}. The used JMP statistical software package applies a product exponential correlation function with a power of 2 as the estimated model:

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[32] The word 'Kriging' is synonymous with 'optimal prediction'. The method was originally proposed by the geologist D.G. Krige. See Krige, 1951.

[33] A 'Gaussian process' is a stochastic process which generates samples $\{X_t\}_{t \in T}$ in any set T such that no matter which finite linear combination of the X_t one takes, these linear combination will be normally distributed. Two 'Gaussian processes' that deliver equal functions for the expected value and the covariance are distributed equally.

$$r(\theta; i, j) = Corr(z(i), z(j)) = exp\{-\sum_{k=1}^{s} \theta_k (x_{ik} - x_{jk})^2\},$$

The parameters μ, σ and θ_k are all fitted in the JMP software via 'maximum likelihood'. It reports the figure ($-2 \times log$Likelihood), that is minus 2 times the natural log of the likelihood function evaluated at the best–fit parameter estimates. As a consequence, smaller values produce better fits. See also the explanations below in section 3.3.

1.3.4 Calibration

Agent–based simulation can be a valuable tool for studying real world economies. Thereby, calibration can be a useful device: Within the calibration procedure selected parameters of the model are varied in such a way that the model output resembles in sufficient detail the output of the original system. Hence, calibration is a computer experiment, in which an optimization is applied (Klügl, 2008b). One important topic within the calibration of the model is therefore the selection of the parameters as well as a quantitative measure of goodness, i.e. an 'objective function' that indicates how good or bad the simulation model matches the original system. We will explain in section 3.4, which measure characterizes the behavior of the original system in a satisfactory and appropriate manner. Equally important, it is necessary to identify significant parameters within the previous sensitivity analysis, i.e. parameters which influence the model outputs of interest. Conversely, insignificant parameters drop out from further analysis and from the calibration procedure. The remaining parameters constitute the 'minimal model' to be adjusted within the calibration procedure (see the description in figure 1.2).

We develop the simulation within the SeSAm programming environment. SeSAm also enables the development of computer experiments based upon the programed model. Besides this, there are several 'plug–ins' available for SeSAm, among others, a calibration 'plug–in'. This employs an optimization method that minimizes (or maximizes) a quantitative 'objective function'.[34] Importantly, the calibration tool searches for a parameter combination (a scenario) that minimizes the 'objective function' by employing the 'simulated annealing' optimization method. In the following

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[34] For a detailed description of the used calibration tool see Fehler, 2008, or Fehler et al., 2005; Fehler et al., 2004.

we want to give a short description of this optimization method: 'Simulated annealing' is a heuristic optimization method that searches for a global maximum or global minimum of an 'objective function'. The method is usually applied, if the relationship between the 'objective function' and parameters is complex, and if a basic trial and error process cannot be applied because of too many possible parameter combinations. Consequently, 'simulated annealing' constitutes a method for the *approximation* of a global minimum (or maximum). Other heuristic optimization methods can get stuck in a local minimum (or maximum)—not so the 'simulated annealing' method: It usually finds a way out of a local minimum (or maximum).

The basic idea of this method is derived from the annealing method in metallurgy.[35] Accordingly, it is a technique where controlled heating and cooling of a piece of metal is used to increase the size of its crystals and reduce their defects: When the metal is heated, its atoms can move freely. When the temperature of the metal is slowly reduced, atoms can move in order to adopt a more stable orientation. Finally, when the metal is cooled slowly enough, the atoms are able to relax in the most stable orientation. In analogy to this physical process, in each step of the 'simulated annealing' algorithm, the current 'temperature' of the optimization method is gradually reduced. The implication of this cooling will become clear immediately (Kirkpatrick et al., 1983; Fang et al., 2006): Suppose an 'objective function' $O(\vec{x})$ which should be minimized by varying the parameters described by the vector $\vec{x}$. The process starts with an initial 'temperature' T (in case of the calibration in the present study $T = 500$) and an initial level combination, which defines the initial vector $\vec{x}_0$. Now a small random variation $\Delta\vec{x}$ (in the investigated parameters) is initiated. If the resulting value of the function $O(\vec{x} + \Delta\vec{x})$ is smaller than $O(\vec{x})$, the new position $(\vec{x} + \Delta\vec{x})$ is chosen. However, this algorithm can become stuck in a local minima, because the values of the function $O(\vec{x})$ would be non–increasing. That is, the algorithm searches for smaller values of $O(\vec{x})$, while temporarily rising $O(\vec{x})$ is ruled out. In order to circumvent such a lock–in effect in a local minimum, 'simulated annealing' must contain a further instruction. Here, the 'temperature' T comes into play: 'Simulated annealing' allows to jump to higher values of the function (which should be minimized) conditional to the 'temperature' of the simulation (which is exogenously annealing). Such an 'uphill' movement depends on the probability p, given through $p = exp\frac{O(\vec{x}) - O(\vec{x} + \Delta\vec{x})}{T}$. Obviously, the

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[35] See Kirkpatrick et al., 1983, for the original illustration of this method and its analogy to metallurgy.

probability p depends on the 'temperature' T in such a way that a very high 'temperature' allows (c.p.) 'uphill' moves in the value of the 'objective function' $O(\vec{x})$ with a higher probability. But, the longer the simulation runs, the lower the 'temperature' gets. Accordingly, the probability that 'uphill' moves are allowed decreases (c.p., i.e. for given $\Delta O(\vec{x})$). In brief, 'simulated annealing' requires four ingredients (Kirkpatrick et al., 1983):

1. A description of the parameters of the system;
2. A random generator for moves in the parameters of the system;
3. A quantitative 'objective function';
4. An exogenous annealing scheduling of the 'temperature', and the simulation length.

Within the calibration 'plug–in' we fix the maximum (i.e. the start) 'temperature' to 500, and the minimum 'temperature' to 50. Moreover, there is a fixed process of annealing the 'temperature', which is based on the simulation length. We fix the latter to a maximum of 1,000 simulation runs. Usually, the calibration procedure is finished after 400 to 700 single runs. This method is used to generate a calibrated model (see figure 1.2). In the last step, one has to verify the calibrated model through statistical 'validation'.

1.3.5 Statistical Validation

In the last building block of the 'validation' framework, the statistical 'validation' has to be conducted. This is necessary to verify the results of the calibration procedure. If one would not apply statistical 'validation', the model could be merely tuned through calibration to reproduce given facts. Hence, we employ several statistical analyses, i.e. we compare some descriptive statistics of the simulation output with statistics of the original system. Thereby, we use different data than in the calibration process (Klügl, 2008b). Again, we investigate only outputs on the aggregate level.[36] After the successful statistical 'validation', we add 'plausibility' checks of aggregate model outputs (delivered by several 'face validation' runs). Accordingly, we review the obtained data *qualitatively*

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[36] In contrast to this, during 'face validation' we also investigate data of individual agents. This is necessary to work out some regularities of the model. But we do not apply systematic 'validation' methods on the level of individual agents.

against the background of our model and the intended relationships. If, after all of these 'validation' steps, the model produces reasonable results on the aggregate level, we conclude that it is validated by the macro dynamics of the 'original system'. 'Validation' is then finished.

1.4 Conclusion

This section explains several basic aspects of agent–based computational economics. It should become clear:

1. What elements an agent–based model must contain;
2. For what reasons a researcher may prefer agent–based computational simulation technique over the orthodox framework;
3. How a reasonable validated agent–based macroeconomic model can be obtained.

According to that, we have a starting point for the following chapters. There, the conceptual model will be illustrated in detail, and the 'validation' procedure of the model will be discussed.

Chapter 2

Conceptual Model of Agent Island

This chapter describes a monetary macro model developed in an agent–based environment, and furthermore presents a road map for the design of such a model. As one major aim of this study is to deliver a first agent–based simulation approach in the field of monetary macroeconomics, the model features some monetary aspects that are new in agent–based research, e.g. nominal prices, inflation, financial assets as well as the central bank. The central bank agent sets nominal interest rates, in order to control output and inflation in analogy to the simplified situation in the real world. From this perspective the model pertains to the field of monetary theory. In the following description we present all relevant components of the model, so that the reader gets an idea of the model itself, but also how to design an agent–based monetary macro model. We will see through this conceptual model that some pitfalls can occur in developing such a model: The implementation of the monetary circuit and the financial system is easily done, but how to deal with problems of large inflation and deflation is yet more complex. We will address these topics respectively during the next two chapters. Some of these problems may appear strange to economists unfamiliar with the agent–based technique. However, they have to be tackled, when one tries to validate such a model. The description of this chapter therefore gives some guidelines pertaining to further research in this field.

Another contribution of this model is the development of an environment that comprises the main advantages of agent–based simulations. To point out these advantages, we repeat some basic theoretical points of chapter 1 with special regard to the model of Agent Island. Chapter 1 identifies the most important virtues of the agent–based simulation technique. It is at the very heart of every agent–based computational model that heterogenous agents influence each other through various interaction–channels and that this micro–interaction is guided by the notion of emergence from the bottom–up. The following overview stresses these points in the context of the artificial economy of Agent Island:

Heterogenous agents The model contains a variety of heterogeneity in many dimensions. Above all, the business sector of the model contains two totally heterogenous subsectors. Consumer goods firms produce goods (by using the production factors labor and capital) and sell them to private households in perfectly competitive markets. Each of the produced homogenous consumer goods, i.e. hash and beans, are sold for one market price during one period. Moreover, the consumer goods firms are owned by private households. Hence, these firms pay capital incomes to the private household sector. In contrast to this, the capital goods sector produces machines by applying only the labor factor to the production process. Capital firms sell their products to firms in the consumer goods sector in a market exhibiting 'monopolistic competition'. We thus assume that capital goods are differentiated products (i.e. no perfect substitute for each other), and that no single market price for all goods exists. In addition, prices lie above marginal costs due to markup calculations of capital firms. This leads to the insight that capital firms always generate profits[1], whereas consumer goods firms can undergo periods of losses. After all, capital goods firms distribute their profits to their owners; these owners are not private households but consumer goods firms. This is a short review of the basic heterogeneity in the business sector. In addition, the private household sector can be divided into two subgroups: (i) There are households without any financial wealth at the beginning of the simulation. These agents constitute the main fraction of all private households. (ii) Some private households exhibit a large initial financial wealth. Consequently, it is an important feature of the private household sector that agents within this sector exhibit various income patterns. We will explain this point below.

Beside the above mentioned basic heterogeneity between sectors or subsectors, the agents

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[1] This takes place, because capital goods prices are always above marginal costs. Fixed costs do not exist.

within each (sub)sector of the model can exhibit heterogenous behaviors as well. We call this sectorial heterogeneity: As noted, the consumer goods industry is divided into two subsectors, the hash producing firms and the beans producing firms. Firms in each subgroup produce the single good according to an idiosyncratic 'Cobb–Douglas' production technology with two production factors, labor and capital. Thereby, output elasticities with respect to labor and capital differ among these firms. The production technologies are characterized by 'Hicks–neutral' exogenous technical change, and the growth rates of 'total factor productivity' (TFP) are heterogenous, because they follow an idiosyncratic 'random walk with drift' on the level of each firm. In addition, the quantities supplied by each consumer goods firm are heterogenous according to an idiosyncratic supply rule; behavioral parameters in these rules differ among firms. Capital firms behave also differently in their supply decisions. Besides this, they produce machines with respect to an idiosyncratic TFP and growth rates of the TFP.

The private household sector is heterogenous in many points as well: Households differ in their preferences in both consumption goods as well as in their preferences between consumption today and in the future, i.e. consumption and savings behaviors are heterogenous. Note that it is a basic feature of the model that household agents exhibit heterogenous income patterns according to initial financial wealth. Besides this, every household is assigned exclusively to one firm as an employee (and maybe as a shareholder) through the whole simulation run. Thus, the labor (capital) income varies due to the income generated by the firm, the household agent is employed by (the household agent owns): As employers alter their produced output, the labor income of the employees alters with the output. The same is true for the profits of firms, and therefore the capital income of the shareholders. Consequently, the labor and capital incomes of household agents assigned to different firms vary among each other, and the labor and capital incomes of one and the same household varies between periods; the incomes of workers employed at the same firm vary only with respect to their interest incomes. On Agent Island household agents have the obligation to work, whereby the supplied quantity of labor depends on the total labor demand of the employer (and the number of employees).

Interaction profile It is the very nature of an agent–based macro model that interaction between agents takes place, especially between the main sectors of the model. On the one hand, capacity utilizations in the consumer and the capital goods industries as well as the dynamics of wages influence the labor income of worker agents. On the other hand, the consumption and savings behavior of these workers operating as consumers influences the revenues of consumer goods firms. But heterogeneity enables interaction between firms as well: Well earning and highly utilized consumer goods firms buy new machines from the capital goods sector as long as machine prices are attractive. This is the major interaction within the firm sector. The profit of each capital firm is distributed to its owner each period. We construct the model in such a way that the owner of each capital firm must be a specific consumer goods firm.[2] As mentioned above, high earnings of hash or beans firms make new investments possible—but this is only possible, when earnings yield as much as the credit interest rate. Finally, it is an important feature of the interaction profile that the central bank influences via interest rate policy (i) the investment behavior of consumer goods firms, (ii) the savings behavior of households, and (iii) the interest incomes (or expenditures) of all agents.

Macro structure emerging bottom–up The main parts of section 2.2 present the micro structure of the model. That is, we illustrate the behavior of household and firm agents on the individual level. Moreover, the macro structure is not (and can not be) modeled in detail—it rather emerges bottom–up. As issues of monetary theory are the main purpose of this study, the macroeconomic role of the central bank agent is discussed in detail in subsection 2.2.4. Finally, subsection 2.2.5 summarizes the resulting macro view of the Agent Island model. Chapter 3 analyzes this macro structure in depth, i.e. we will discuss several implications of the micro structure on the macro level. This is done through sensitivity analyses, statistical investigations, and 'plausibility checks'. The main objective in this context is the connection between the central bank, the financial system and the targets of monetary policy on the one hand, and the micro structure of firm and household agents on the other hand.

In sum, there are many aspects of heterogeneity and interaction embodied in the model of Agent Island. Beyond this contribution, it is the aim of the study to develop a monetary model. According

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[2] This design stems from the calculation of the return on assets. We will explain this point below in greater detail.

to the novelty of this approach in ACE, it is necessary to highlight some specific features which account for a monetary model. Basically, it is necessary to think about the banking system as well as the role of the central bank within the banking system. To include all these points in a straightforward way, the model exhibits only one single bank, the central bank. We thus follow the notion of a 'pure credit economy', where the central bank also fulfills the role of private banks. It is (i) granting credits to the private non–financial sector, and (ii) collecting private deposits in its accounts. These basic mechanisms constitute straightforward channels of monetary transmission:

Investment–channel The firms in the consumer goods sector generate profits. These profits enable the calculation of a return on assets on the firm level. It is one condition for expansion investment that this yield is above the interest rate paid to the creditor, i.e. the central bank. Thus, the central bank can trigger the expansion of the capital stock through interest rate policy.

Savings–channel Savings behavior of household agents is, among other aspects, captured by individual time–variant savings rates. Moreover, the savings rates are adjusted over time according to a rule that treats the deviation of the present real interest rate form its historical average and the growth rate of disposable income as its determinants. By its influence on both magnitudes, the central bank can coordinate financial savings.[3]

It is obviously an interesting question, to what extend a control of the business cycle of Agent Island can be exercised through these channels.[4] Another interesting point of monetary analysis is the insight into causal relations that drive inflation. Among other things, we conduct several sensitivity analyses (in section 3.3), in which we identify the main determinants of inflation in our agent–based framework.

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[3] In contrast to reality, the model does not afford real investments of household agents, as for example in real estate. Hence, if private households agents are saving, the saving takes place exclusively through financial assets in central bank accounts.

[4] As explained during subsection 2.2.5, the model features also the 'expectations–channel' of monetary transmission in its simplest specification—the 'wage–price spiral'.

2.1 Overview

The structure of the economy of Agent Island is straightforward. It contains two main economic sectors including private households and firms; the firm sector is consisting of three subsectors (hash, bean and capital firms), and each of the sectors (or subsectors) comprises agents, which can be heterogenous or homogenous. Hence, the model is composed of: (i) consumers, (ii) hash firms, (iii) bean firms, and (iv) capital firms. In the following the variables H, J, K, and C define the number of agents within each sector (or subsector), and the control variables h, j, k and c define any agent within the sets of agents:

- Consumers: $h \in [1, \dots, H]$
- Hash firms: $j \in [1, \dots, J]$
- Bean firms: $k \in [1, \dots, K]$
- Capital firms: $c \in [1, \dots, C]$

Next to these sectors the central bank operates—technically—as a single agent. In the following subsections we explain some basic aspects of the model including its theoretical roots (subsection 2.1.1), its basic composition (subsection 2.1.2), and its time and sequence structure (subsection 2.1.3).

2.1.1 Theoretical Roots and Antecedents

The summary of this subsection highlights only some basic links to literature. It is thus a mere introduction to the underlying literature of the present model—detailed links to literature are presented in the following section. In general, it is complex and difficult to develop an agent–based macro model. One difficulty is due to the fact that unlike in 'orthodox' macroeconomics, until now no single accepted methodology exists in the agent–based framework. If available, one could use some existing ACE models as guidelines. This is a strong contrast and a disadvantage of ACE compared to the construction kit of GE and its variety of applications. One problem of this study is that it copes with the task of building a novel monetary ACE model, and that one can find only a few guidelines in ACE literature for this purpose. By consequence, this study builds in many aspects which are primarily non–ACE based. A second important consequence is that the description of the present model has to be more in–depth than in case of a GE model. This stems from the novelty of the present approach and the variety of options offered by agent–based modeling.

The present model follows the 'Keynesian' notion that the core market in the economy is the goods market: In a reversal of 'Say's Law', Keynes in essence argues that demand creates its own supply. Accordingly, business cycle dynamics are mainly determined by aggregate demand conditions (Keynes, 1936), the demand for consumer and investment goods is the driving forces for important aggregate magnitudes, such as GDP and inflation. The model of Agent Island features this notion. In contrast to 'Keynesian' models, labor markets and and wages dominate in 'neoclassical' models. According to our approach, consumer goods demand determines consumer output, and in turn the level of employment in the consumer goods industry: Higher demand induces higher production, higher production induces higher incomes, which in turn enhances demand, and so on. In this manner the present model is a true 'Keynesian' model. Besides this, several behavioral approaches in the design of savings and consumption behavior are applied, which are features of 'Keynesian' economics as well.

Next to this 'Keynesian' feature the consumer goods market is basically 'neoclassical'. This is based upon the supply behavior, which follows the seminal notion of Alfred Marshall—a famous neoclassical economist. The difference of Marshall's 'neoclassical' framework compared to the 'orthodox' approach (i.e. the 'Walrasian' GE models) is characterized by Axel Leijonhufvud:

> "The allegiance of modern macroeconomics is also very much fortified by a strong sense of tradition, of carrying on an economics that was always built on 'rational choice', on 'optimizing behavior', on equilibrium, a tradition that you stray from at your peril. But this sense of tradition is in large measure based on a misreading (or, more likely perhaps, a lack of reading) of the history of our subject. What is today commonly thought of as neoclassical economics is really the hypertrophy of optimizing choice theory—the branch of neoclassicism which at one stage in the development of economics happened to be the most easily formalized. There is an earlier tradition of neoclassical economics, in some

> respects a more interesting one, which could not be adequately formalized and therefore gradually fell into neglect. This tradition could be revived with agent–based methods. It would be worth doing." (Leijonhufvud, 2006a, p. 1628)

Hence, what is today commonly thought of as 'neoclassical' economics (termed as 'orthodox' economics by us) is the efficient allocation of scare resources fulfilled by choice theory, formalized in terms of constrained optimization (Leijonhufvud, 2006a). Thereby, the maximization of utilities or profits constitutes the actions of the agents. In clear contrast to this, early neoclassicals—like Marshall—thought that the micro–behavior of agents is 'adaptive'.[5] The remarkable difference compared to the 'orthodox' optimization approach is the role of utility maximization: In the sense of Marshall, the maximization of utility or profit was a proposition about motivation within adaptive behavior—rather than a constitution of actions as in 'orthodox' economics. No claims were made about the 'rationality' of individuals. Equally important, Marshall and his followers (presumably also Keynes) had a strong affinity to 'complex system theory', whereas 'orthodox neoclassicals' were biased towards the mechanical concept of GE.

To understand the contrast between constrained optimization and Marshall's notion, it is helpful to define 'complex system theory'. A 'complex system' exhibits the following properties: (i) The system is composed of interacting units; (ii) the system exhibits emergent properties that are arising from the interactions of the units; (iii) but these emergent properties are not properties of the individual units themselves (Flake, 1998). In general, 'complex systems' consist of interrelated modules, which can be systems themselves. They are, in general, hierarchical, with multiple layers, which typically work on various time scales. According to this perspective laid down by older neoclassicism (headed by Alfred Marshall) the agent–based technique is an ideal candidate for the construction of an economic model in the sense of a 'complex system'. We will see during the conceptual model that we adapt Marshall's 'laws of motion' in constructing the behavior of agents in the consumer goods market. These laws are 'adaptive' rules, and the variety of agents applying those 'adaptive' rules

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[5] Imagine a collection of decision rules that dictate actions to be taken in given situations, and imagine a set of preferences used to evaluate the outcomes arising from particular situation–action combinations. These decision rules are continuously under review and revision through the decision maker. He tries and tests decisions against experience, and evaluates rules that produce desirable outcomes superior to those that do not. This process of 'trial–and–error' through which our modes of behavior are determined is described by the term 'adaptive' behavior (Lucas, 1986).

constitute the ‘complex system’. According to this perspective, our model contains many (older) ‘neoclassical’ aspects, next to its basic ‘Keynesian’ elements.

Furthermore, it is necessary to incorporate real capital investment into the model. The agent–based model of Dosi et al. gives us some guidelines for this purpose. They construct an agent–based business cycle model that features several empirical facts. Among others, their model captures the evidence that investment is considerably more volatile than output (on the macro level), and that investment is lumpy (on the micro level) (Dosi et al., 2005; Dosi et al., 2006; Dosi et al., 2008). We follow their approach for a (S,s) investment routine for the determination of expansion investment. In addition, we integrate a ‘Wicksellian’ element into the investment demand. According to the monetary theory of Knut Wicksell,[6] the difference between the natural interest rate and the credit interest rate is important: When the natural rate is higher than the credit interest rate, the price level is rising—and vice versa (Wicksell, 1898). One important point of Wicksell’s approach is the concept of the natural interest rate. We treat the return on assets of a firm as the individual natural rate. Accordingly, the firm may invest, if its return on assets is larger than the credit interest rate. In this case the firm generates extra profits and enlarges its business.[7] If the return on assets is below the credit rate, no net investments take place. By combining these two approaches, i.e. the (S,s) investment model of Dosi et al. and the ‘natural rate vs. credit rate’ model of Knut Wicksell, we construct the investment demand of the model. This enables the ‘investment–channel’ of monetary transmission on Agent Island.

The present design includes, in addition, the classical ‘investment accelerator’, which goes back to Clark, 1917. According to this view, production is a multi–level process: Production facilities are necessary to produce consumption goods. The production facilities are part of the real capital stock of the respective industry, and the capital stock is used up over time (e.g. over ten years). Moreover, it is necessary to replace the depreciated capital stock to keep the capacity constant. Hence, replacement investment constitutes only a small part of the capital stock (e.g. about a tenth of the present capital stock). But, if capacity of the consumer goods industry should be enlarged,

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[6] See section 2.2.4 for a short review of the ‘Wicksellian’ monetary theory.

[7] Net investment depends, in fact, on additional conditions.

investment demand enlarges additionally. For example, it is doubled through expansion investment, if the capital stock is enlarged to an additional 10% of its actual level (i.e. 10% replacement investment plus 10% expansion investment) (Stützel and Grass, 1983). It is a consequence of this vertical relationship in production that a rather small variation in consumption demand could cause relatively large variations in investment demand. This phenomenon is called ‘investment accelerator’ or ‘business accelerator’ (Clark, 1917; Stützel and Grass, 1983). It constitutes a feature of the artificial island economy.

Finally, the financial system of Agent Island is based upon the idea of a ‘pure credit economy’. This goes again back to Wicksell, 1898. At this point it is sufficient to state that the central bank of the agent economy is the ‘mono–bank’ within a ‘pure credit system’. This facilitates a financial system with one financial asset, which allows the central bank to accomplish monetary policy. Thus, from the perspective of the financial system, Agent Island is designed straightforward.

2.1.2 Markets, Transactions and Financing Contracts

Before turning to the detailed description of the model we want to give a general overview of the sectors (subsectors) and their interactions. First, figure 2.1 presents an overview of transactions in the model, whereby the figure depicts only monetary streams, but not the opposite streams of real goods and services. It should be noted that all real transactions within the model are immediately settled through bank accounts at the ‘mono–bank’. Additionally, for households the terms (i) ‘expenses’ (‘proceeds’), (ii) ‘expenditures’ (‘receipts’), and (iii) ‘disbursement’ (‘payment’) have the same meaning, and we use them as synonyms.[8] In the following we will concentrate on the system of receipts and expenditures in the agent economy, and for household agents we will also use the term ‘income’ analogical to ‘receipts’. Finally, for any agent in the model we do not differentiate between receipts and payments, i.e. financial assets and money are the same.

As usual, a macro model comprises a private household sector. The agents within the private

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[8] These terms describe transactions that (i) change the net worth, (ii) the net financial assets (or liabilities), and (iii) the net stock of instruments of payment (i.e. money) of an agent.

household sector receive labor income (red lines) and capital incomes (dotted black lines). Then again, the private households allocate consumption expenditures to hash and bean firms (blue lines). According to central bank deposits, household agents receive interest incomes (yellow lines).[9] Hash and bean firms constitute the consumption goods sector. Next to consumer goods markets, capital goods markets are constituted through the trading of new machines produced through capital goods firms; accordingly, investment expenditures occur (green lines). As mentioned, consumer goods firms employ both factors in the production process; whereby capital goods firms produce investment goods, using labor only. Moreover, as explained by figure 2.2, the capital firm agents are not owned by the private households, but by hash and bean firms. Accordingly, they distribute capital profits to hash and bean agents (dotted black lines).

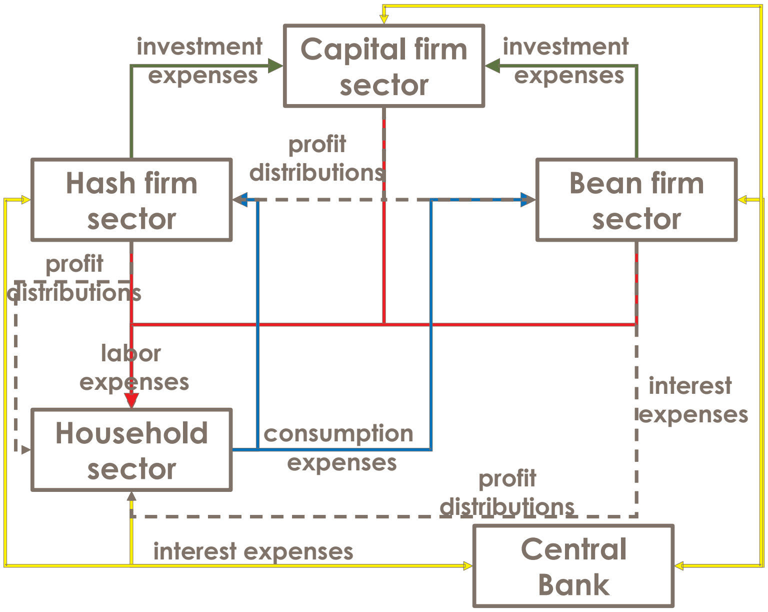

Figure 2.1: Markets and transactions on Agent Island

In accordance to the net position of each agent vis–à–vis the central bank, he receives interest receipts or pays interest expenditures. According to our abstraction of a ‘pure credit economy’, the

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[9] When the household is a net debtor, he pays interest income to the central bank according to outstanding debt.

central bank constitutes the financial sector of the model as a whole: Private agents can accumulate financial wealth solely in central bank deposit accounts; on the other hand, agents can borrow form the central bank (blue lines in figure 2.2). Figure 2.2 gives a overview of the various financing contracts between sectors.

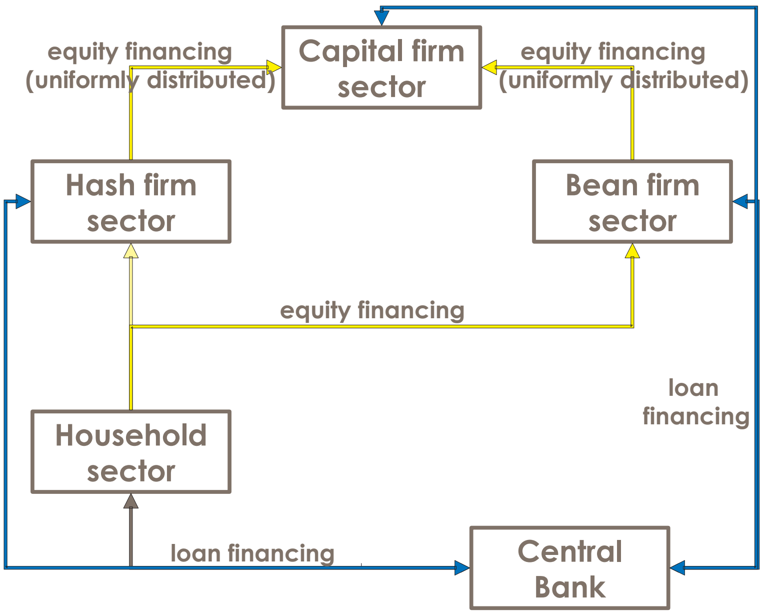

Figure 2.2: Financing contracts on Agent Island

Next to the role of the central bank as a financial intermediary, equity financing does occur in the model as well (yellow lines). But this takes place in a passive manner: In the initial state of the simulation the ownership of hash, bean and capital firms is specified, and unchanged throughout one simulation run. We assume that each hash or bean firm is owned by the workers that are assigned to it. Besides this, each of the shareholders holds exactly the same amount of equity. In contrast, the ownership of capital firms is randomly distributed among hash or bean firms:[10] This is due to the fact that capital firms use only labor in the production process, and therefore it is impossible to calculate the return on assets for these firms. We have to integrate, however, the profits of capital goods firms into this calculation. In the process of 'face validation' it becomes clear that this is

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[10] In here, we apply a uniform distribution.

necessary. Only then it is possible to relate the aggregate profits of the firm sector to its aggregate capital stock. As a results, we construct corporate groups controlled by hash or bean firms; and we randomly assign each capital goods firm to one single hash or bean firm, so that each hash or bean firm can be the parent company of one or more capital goods firms. Again, this aspect characterizes the heterogeneity of the model, and in addition, it reflects some degree of corporate integration, known from real world economies.

2.1.3 Time and Sequence Structure

This subsection discusses the sequence of decisions and actions within one period and the question of period length. In each agent model the sequence of actions has to be defined in a reasonable way, so that the agents possess all relevant information to take the next step in the simulation. For example, a private household agent needs the relevant information concerning its period income before he can choose between its desired saving and consumption. In addition, the capital goods market transactions cannot be executed until capital demand and supply schedules have been determined, and so on. Hence, a detailed plan is necessary for scheduling agent activities. The second topic of this section, i.e. the question of period length, should give an idea about the counterpart of a model period in reality. We thus review some points that give a clue as to how long a model period should last in the abstract sense of our model.

Flow of Activities

Figure 2.3 depicts the flow of activities within one period. Different colors are assigned to each of the sectors, viz.: The consumer goods sector is marked blue, the capital goods sector is green, the private household sector is red, the central bank is pink, and overall activities[11] are black. According to figure 2.3, every round is arranged by a certain sequence of activities: The round starts with the interest rate set by the central bank (see subsection 2.2.4 for details). In the next step, the

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[11] In the framework of the SeSAM programing environment these overall activities are assigned to the ‘world agent’. In the present model, market clearing activities or the calculation of aggregate magnitudes are assigned to the ‘world agent’. Technically, we assign the central bank agent also to the ‘world agent’.

market wage for labor is set. This is modeled via labor agreements between the employer association (representing firms) and the labor union (representing employees). In fact, the overall wage agreement is modeled via a rule known from the 'Phillips curve' approach (see subsections 2.2.2 and 2.2.3 for details). With this information at hand, firms in the consumer goods industries decide the adjustment of output through a 'Marshallian' adaptive rule (see subsection 2.2.2 for details). In combination with the budget constraint of each single firm (i.e. the production function) this enables the calculation of required labor resources on the individual level. This way, the payment of labor incomes and imputed capital interests (via firms equity) to workers and shareholders can be calculated and fulfilled. Within this sequence each consumer goods firm plans replacement and expansion investment, conditional to several requirements (see subsection 2.2.2 for details). Simultaneously, the capital goods firms set their supply prices . Lastly, the capital market is cleared and individual contracts are signed (see subsection 2.2.3 for details).

After the allocation of capital goods, firms in the capital goods industry produce the ordered machines (see again subsection 2.2.3 for details), which will be added to the capital stock in the consumer goods industries at the end of the period. Accordingly, the new machines will be added to the capital stock after producing the consumer goods of the present period, i.e. they will be available as soon as the production in the following period takes place. During the next step, after receiving all incomes, private households calculate their disposable period income. The basic decision of household agents is to decide between savings and consumption (see subsection 2.2.1 for details). This ends up in the clearing of the consumer goods market (see subsection 2.2.2 for details). It should be noted that in contrast to capital market clearing no individual contracts are closed in the consumer goods market. Because the goods are homogenous and one single market price exists for each good, the transactions can be simplified by a middleman. After the settlement of all market transactions, firms calculate their profits on the individual level (see subsection 2.2.2 for details). In the end, several magnitudes are aggregated on the 'world–level'. Between the present and the following period, i.e. 'over–night', the central bank pays and receives interest payments according to account balances. In addition, firms distribute dividends to their shareholders.

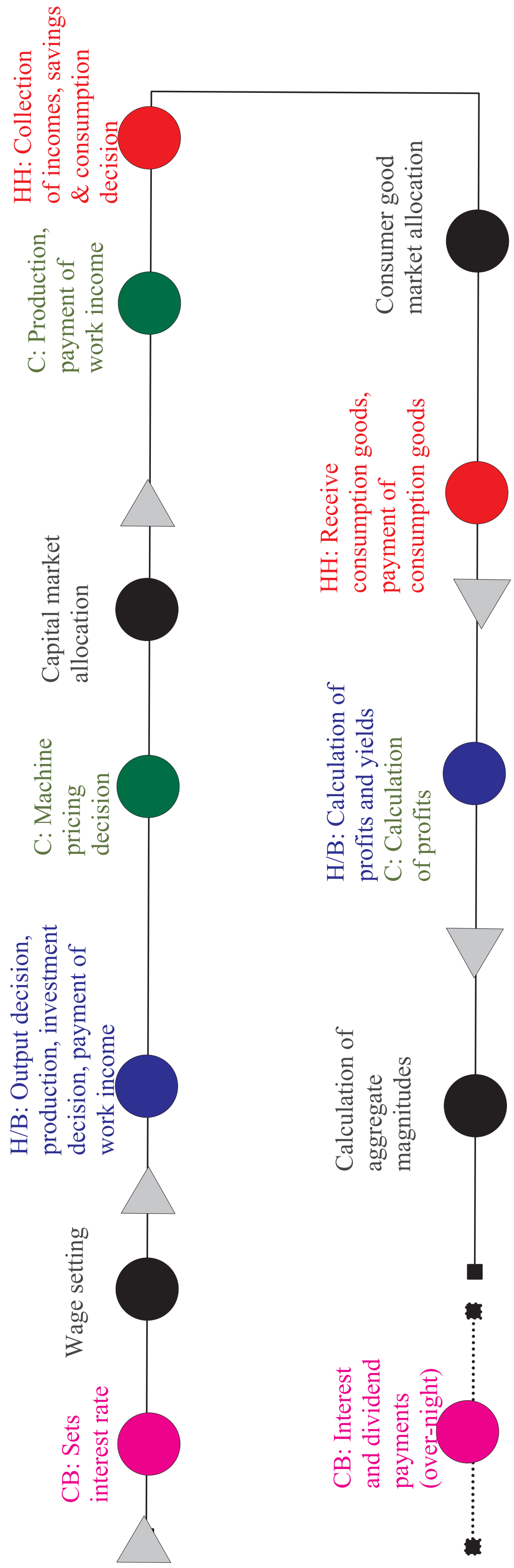

Figure 2.3: Intra–period sequence of decisions and actions

Period Length

The definition of the length of the model–period is a complex and difficult task—similar to the construction of an agent–based model itself. In the present model, overall wages are set according to a certain rule representing labor market agreements between the employer association and the labor union. This rule connects the wage growth to period inflation, and output gap. In real world economies this connection is often characterized by wage rigidities, i.e. by wages that are fixed in contracts for at least one year. From this perspective it is reasonable to assume a period of at least one year on Agent Island. By the same token, the calculation and distribution of profits indicate a period of one year: Corporate entities generally close their business and accounting period after one year, and therefore, capital incomes are often distributed once a year. Hence, the accounting year indicates a model–period of one year as well, albeit it would be possible to use a shorter period (e.g. a quarter), and distribute capital incomes at the end of the accounting year (e.g. after four quarters). But as a simplification, a model period that coincides with the accounting year in the real world is adequate.

In the present model the capital stock is constant during each period, and firms can only adjust it between periods. It is moreover an empirical fact that investment behavior of firms is not continuous—it rather exhibits complex patterns over time.[12] Provided that the adjustment of the capital stock is costly in terms of time[13], firms will be able to increase their capacity only within a reasonable time–frame. We assume that this time–frame is one year. When a firm agent decides at the beginning of period 1 to expand its capital stock by one machine, the new machine will be available for production purposes in the beginning of period 2. For complex production facilities this seems to be a quite reasonable assumption.

According to our 'Marshallian' goods–market–approach, prices adjust to a short–term equilibrium level *within* a period, while output adjusts *between* periods. This gives an idea as to how long one period should be: In real world economies the adjustment of output and capital is due to technology and institutional arrangements of production. Certainly, there are some industries that

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[12] See, for example, Doms and Dunne, 1998, for the time pattern of firm–level investment.

[13] For an investigation of the adjustment speed of capital stock see, e.g., Roberts, 2003.

can adjust production within a very short time–frame, such as one week. For example, an industry that employs low–qualified workers and little capital in the production process can adjust production immediately, provided that the institutional arrangements of the labor market allow short–term employment. This can be the case in the farm business, for example. Then again, there are certainly some industries that are not able to adjust output within weeks: Think about the semiconductor industry, which employs well–educated workers, sophisticated production technologies, and large amounts of real capital. When a semiconductor production facility is working close to the limit of its capacity, it is impossible to increase output within one month. Such an adjustment could take years. In sum, it would be realistic to design various time layers for the output adjustment in the various industries. But in order to keep our design as simple as possible, we employ only one common time layer for the output adjustment of all consumer goods firms. Accordingly, the adjustment of output in the model is only possible between two years—not within one year. This can be justified by an inflexible labor market, where short term employment is impossible, and by a sophisticated production technology.

Finally, the topic of designing the period length can be treated from the view of monetary transmission. Actually, the purpose of the present analysis is to analyze issues of monetary theory in an agent–based framework, and hence the model should follow some stylized facts of the transmission process. According to empirical investigations of the monetary transmission process, there are long and variable lags to which monetary transmission is subject. Consequently, if the central bank reacts to movements in the target values with its instrument, it can take quite a long time for a policy to show the desired effects. This fact is crucial to the question, of whether monetary policy is able to provide countercyclical economic management. In case of too long time lags, policy can become procyclical. Several studies indicate average lags of 12 months in the case of restrictive monetary policy, and average lags of up to 18 months when pursuing expansive monetary policy.[14] According to that, it seems reasonable to construct the model periods in such a way that these time lags work adequately. The simplest approach would be to construct a period with a length of one year, where the present stance of monetary policy affects output in the next period. This is, however, not

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[14] See Bofinger, 2001, for a discussion of this issue.

perfectly adopted in the present model.[15]

2.2 Model of Agent Island

This section gives a detailed description of the conceptual model of the economy of Agent Island and of the process that leads to the development of the given design. Consequently, we make several references to the 'face validation' of the model, which takes up a prominent part within the overall 'validation'. In the case of the present model, this stage of the project lasted about several months. Next to the intensive discussion of the micro structure, the final subsections 2.2.4 and 2.2.5 summarize the main macro aspects of the model. We use this macro structure in chapter 3 when validating Agent Island. The interested reader can investigate the programed model developed in the SeSAm software environment. See the CD in appendix C for the the installation suite needed to implement and run the model.

2.2.1 Households

According to the scheduling of decisions, explained in the last section, it becomes apparent that household agents make their decisions after firms. Nevertheless, the decisions of household agents are central to the model and rather complex. Therefore, we start with analyzing the household sector. During the simulation, private household agents face two decisions: (i) The decision of dividing present disposable income (or better receipts) into consumption expenditures and financial savings. (ii) Furthermore, they choose to allocate the consumption expenditures to several goods. Consequently, an agent can choose how much income he would like to save in one risk–less financial asset and how much income (or wealth) he should spend to buy consumption goods. We should bear in mind that individual savings behavior should be linked to interest rate movements. This is one implication of the macro bindings of Agent Island (see section 1.3), as it is our aim to implement monetary transmission channels into the model. This implies a reaction of individual savings behavior to the interest rate policy of the central bank.

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[15] In fact, the monetary transmission process in the model works faster: Changes of the credit interest rate influence decisions of consumer goods firms in the same period. Hence, monetary transmission is already effective within one year. See section 3.4.3 for a discussion of this topic.

The examination of the first decision, the decision of saving, is one of the most complex tasks one can undertake in economic theory. This is also true in the context of designing an agent–based model. In constructing such a model one should follow the aim of designing valid agents, by taking into consideration the complexity of the whole simulation. The latter is one restriction. In order to reveal the complex process of designing the savings decision, we first give a brief review of the various perspectives that have been developed during the 'history of economic thought'. In a next step, we connect these findings to our model, in which we have the results of 'face validation' in mind. In doing so, we find out that some special restrictions have to be incorporated into the model. Throughout this subsection we illustrate the theories we adopt, and we explain why we rule others out. This can be also helpful in further research on this topic.

Brief Review of the Theory of Savings in the History of Economic Thought

The first theoretical analysis of the savings decision takes place at the end of the nineteenth and the beginning of the twentieth century, by the advocates of 'neoclassical' theory. They built their perception of savings on the notion of 'intertemporal substitution' and the process of balancing marginal utility over time.[16] According to this view, the straightforward interpretation of savings is the postponing of today's consumption into the future. This is called 'intertemporal substitution'. Moreover, it is assumed that present consumption is evaluated in a superior manner when compared to future consumption. This point is captured by the time preference rate $\rho > 0$. Discounting further consumption by $(1+\rho)^T$ yields the results that a certain amount of one consumption good tomorrow delivers less utility than the same amount today. In order to fulfill 'intertemporal substitution', a household seeks for a fee, i.e. he asks for an 'intertemporal price' as a compensation for the postponing of consumption somewhat into the future: A higher interest rate induces cheaper future consumption relative to present consumption. In this respect a change in the interest rate activates the 'intertemporal substitution effect'. Then again, a higher interest rates induces higher future income, if the agent is a net creditor. This indicates that the agent needs less savings in order to fulfill a given future consumption, which is called the 'intertemporal income effect'. In case of a creditor, both effects (the substitution and the income effect) aim in opposite directions,

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[16] See Böhm-Bawerk, 1921; Fisher, 1930.

insofar it is theoretical a priori ambiguous whether a higher real interest rate causes increasing or decreasing savings. Additionally, if the agent is a net debtor, substitution and income effect aim in the same direction. In addition to the substitution and income effects, rates of return also effect savings through a 'wealth effect': A higher real interest rate reduces the present value of future income streams. Hence, consumption is depressed even if the substitution and income effects cancel each other out (Schmidt-Hebbel et al., 1992). This short review describes the underlying interest–rate–mechanism identified by neoclassic economics. We will refer to this point below.

In the 1930s the emerging 'Keynesian' theory replaced the 'neoclassical' view of savings and their utility oriented perspective. In his macroeconomic theory Keynes, 1936, replaced the microeconomic view by a more heuristic approach, which is described as the 'absolute income hypothesis'. His theory centered on the marginal propensity to consume: Households employ a marginal rate of consumption to their disposable income. With the help of his theory building, it is possible to derive a residual savings function. Several followers of Keynes developed this behavioral theory of consumption further in various directions: According to Duesenberry, 1967, the savings and consumption behavior is interdependent within households. This idea of a 'relative income hypothesis' is related to the 'habit persistence hypothesis'. Under 'habit persistence', the level of consumption (and savings) is determined by lagged consumption (Brown, 1952). Intuitively the habit persistence theory says, the more the consumer eats today, the more hungry he wakes up tomorrow. We will apply the basic 'Keynesian' consumption function, the 'habit persistence' approach as well the 'relative income hypothesis'. Especially the last two theories mark an important detail in our framework as worked out in the process of 'face validation': They guarantee smooth consumption profiles over time, which are necessary to match the empirical data. At the end of the day, these 'Keynesian' approaches differ from older and from newer theories of consumption, as they build on 'psychological factors' rather than on an optimal 'intertemporal choice'.

The 'Keynesian' approach is exposed to criticism of the so–called 'Lucas Critique', which is the initial point of 'modern' macroeconomics that builds on explicit 'microfoundation'. The 'Lucas Critique' says that it seems to be naive to predict the effect of a policy experiment based purely on correlations in historical data. Especially, predictions through high–level aggregated historical data seem doubtful (Lucas, 1976). The Lucas Critique implies that if we want to predict the effect of a policy experiment, we must model the 'deep parameters', i.e. preferences, technology and resource constraints, which govern individual behavior. Then it is, in turn, possible to predict what individuals will do, conditionally on the change in policy, and add up individual behaviors to calculate the macroeconomic outcome. Surely, an agent–based model builds on individual behavior and macroeconomic emergence from the micro structure. Consequently, the core of the 'Lucas Critique' does not apply to our model of Agent Island, because we do not specify the parameters of any aggregate relationship, such as an aggregate consumption function. However, individual behavior on Agent Island is designed through routines, which usually represent rules–of–thumb; these rules reflect individual preferences differently compared to 'orthodox' economics. So far it is for example not absolutely clear, as to how human savings behavior and maybe the underlying preferences change in reaction to the environment. What we know from empirical investigations is that savings behavior depends on several determinants—but we cannot identify the preferences of agents. As we will see, our model characterizes an interactive savings behavior based upon the behavioral approach of 'Keynesian' economics. Hence, we do not model the 'deep parameters' of private household agents, as orthodox economics does. We simply do not know them.

In response to these 'Keynesian' approaches, a renaissance of the 'neoclassical' perspective emerged in the fifties. These authors, rooted in neoclassical tradition, pick up the seminal idea of Fisher, 1930, and build their theory on the basis of the maximization of 'intertemporal utility': The 'permanent income hypothesis' of Friedman and the 'life–cycle hypothesis' of Modigliani and Brumberg (Friedman, 1957; Modigliani and Brumberg, 1955) comprise the view that rational agents smooth consumption over time. The basic version of Friedman's 'permanent income hypothesis' sets consumption equal to the annuity value of the sum of assets and the discounted present value of expected future work income (Flavin, 1981). Such a result can be derived from the maximization under uncertainty of a quadratic intertemporally additive utility function under the assumptions that the real interest rate and the time preference rate are constant and equal to one another (Deaton, 1992). If the horizon is assumed to be infinite, Flavin's version of the 'permanent income hypothesis' takes the form

$$c_T = \frac{r}{1+r}(A_T + \sum_{k=0}^{\infty} \frac{\mathbb{E}_T y_{T+k}}{(1+r)^k}),$$

where r is the real interest rate, c_T is the real consumption in period T, y_{T+k} is the real work income in year $T+k$, whose expectation conditional on information available at time T is $\mathbb{E}_T y_{T+k}$, and A_T is the real value of the single asset whose return is defined as r (Flavin, 1981). For current purposes the equation can be rewritten in order to obtain a savings function: Savings are defined as the difference between disposable income and consumption. We assume that disposable income is the sum of earnings and asset income. By putting the pieces together one obtains the following function of savings in period T (for its derivation see Campbell and Deaton, 1989):

$$s_T = -\sum_{1}^{\infty} \mathbb{E}_T \frac{\Delta y_{T+k}}{(1+r)^k}. \tag{2.2.1}$$

It should be noted that both equations, defining c_T as well as s_T, are precisely equivalent. Usually this savings function is labeled as 'saving for a rainy day' or the 'rainy day' equation. This characterization marks the notion that savings equal the discounted present value of expected future *falls* in income. According to this, individuals are saving today for bad (or 'rainy') times in the future. Thereby, equation (2.2.1) is derived from the assumption underlying the 'permanent income hypothesis', i.e. that optimal consumption is flat over time. The important insights of this analysis are (i) that likewise flat earnings imply no need to save, and (ii), if earnings are growing, households should dissave in order to enhance their current consumptions to the optimal flat levels. Thus, equation (2.2.1) indicates that the 'permanent income hypothesis' is strongly questionable in growing economies, where one can identify a positive relationship between income growth and savings (Deaton, 1997).[17] In fact, this 'positive' relationship grounds on an insight of the empirical findings in the literature on savings behavior.[18] Campbell and Deaton summarize these points as

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[17] The hypothesis that consumption responds to predictable income movements is referred to as 'excess sensitivity' of consumption. According to the view of the 'permanent income hypothesis', consumption should respond to the changes in permanent income signaled by innovations in the current income. If, in addition, consumption responds to current income beyond that attributable to the role of current income signaling changes in permanent income, this is termed 'excess sensitivity' of consumption to current income. Thus, the 'permanent income hypothesis' suggests an 'excess sensitivity' of zero. According to Flavin, there is substantial evidence against the 'permanent income hypothesis' in terms of this 'excess sensitivity' analysis (Flavin, 1981).

[18] For a short review of the literature and the positive influence of income growth on saving see table 2.2.1.

follows:

> "We believe that the answer to the question with which we began, 'Why is consumption so smooth?' lies not in the truth of permanent income hypothesis, but rather in its inadequacy. Whatever it is that causes changes in consumption to be correlated with lagged changes in income, whether it is that measured consumption and income are time averages of continuous processes, or that real interest rates vary, or that the marginal utility of consumption depends on other variables besides consumption, or that some consumers are liquidity constrained or that consumers adjust slowly through inertia or habit formation, the same failure of the model is responsible for the smoothness of consumption relative to permanent income." (Campbell and Deaton, 1989, p. 372)

To sum up these findings concerning the 'permanent income hypothesis', we find that it does not feature in a prominent role in our framework, because we want smooth consumption behavior within a framework of growing income patterns. Moreover, the savings behavior rooted in the 'permanent income hypothesis' implies complex calculations as well as far–forward looking and 'rational expectations'[19], which in turn mark its foundation as unsuitable for our agent–based model. Note that agents face 'true uncertainty', so that it is clearly impossible to calculate at any date T the 'rational expectations' outcomes for dates $T+1$ to $\bar{T}$ in an agent–based environment. In general, it would be pretty difficult to form reasonable expectations for income growth, inflation and nominal interest until the termination of the simulation. Hence, we evaluate the 'permanent income hypothesis' as an suboptimal candidate for the design of individual savings behavior on Agent Island.

At the end of the day, our model touches the 'permanent income hypothesis' at one point, where the role of 'windfall profits' are easy to determine. According to the permanent income hypothesis 'windfall profits' do not enhance present consumption. Within all income sources the interest income class exhibits clear properties of 'windfall profits': If the present nominal interest rate is

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[19] Not all applications of the 'permanent income hypothesis' build on 'rational expectations' and on far–forward locking behavior. In fact, Milton Friedman himself initially states: "The permanent income component is not to be regarded as expected lifetime earnings. [...] It is to be interpreted as the mean income at any age regarded as permanent by the consumer unit in question, which in turn depends on its horizon and foresightedness" (Friedman, 1957, p. 93). But all modern interpretations of the theory build on 'rational expectations' and on far–forward locking behavior. It could be the task for further work in agent–based research to design and incorporate a reasonable 'myopic permanent income hypothesis'.

relatively large to its mean, agents receive relatively large incomes from financial assets. But this can be expected to be short–lived. Then again, in this class it is an easy task to define a proxy for 'permanent interest income', namely the historical average real interest rate applied to the present stock of financial wealth; the difference between this 'proxy permanent interest income' and the 'present interest income' is treated as 'windfall interest income', which does not influence present consumption. This perspective mirrors the idea of the 'permanent income hypothesis'. In contrast, it would be much more complex to find that part of labor income, which is due to 'windfall profits'. If the human capital of an agent had to be determined, it would be an easy task to calculate an approximation of 'permanent labor income', and via that the 'windfall labor income'. But the difficult task would be to find a plausible proxy for the human capital, i.e. the present value of all future labor income streams.[20] This is again due to the fact that agents are exposed to 'true uncertainty' when evaluating the future path of their income growth.

Finally, in the process of 'face validation' it became obvious that there was some contradiction in the model: In an early version of Agent Island, private household agents calculate their savings and consumption based upon present interest income rather than permanent interest income. In this case, inflation is rising up to hyperinflation, in situations of high nominal and real interest rates. By contrast, evidence from central banking practices as well as from monetary theory suggests that rising interest rates, nominal and especially real interest rates imply downside pressure on inflation. Knut Wicksell's monetary theory gives a clear notion of the reasoning behind this mechanism.[21] Apparently, the contradictory phenomenon of our early design is a result of the fact that rising nominal interest rates induce rising present interest incomes, which in turn induce strong nominal consumption growth (based upon present interest incomes) and thus inflation. This induces again rising interest rates, and so on. From this perspective it is imperative to cancel out that effect by employing 'proxy permanent interest income' instead of actual interest income. In this case implications of short–term movements of the nominal interest rate are cancel out.

Similar to the 'permanent income hypothesis' the 'life–cycle hypothesis' is not a good candidate

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[20] Note that interest rates do not feature a long term trend, while labor income patters can do.

[21] See section 2.2.4 for a description of the 'Wicksellian' theory.

for the design of a savings behavior within the simulation. Both hypotheses imply complex calculations and far–forward looking and 'rational expectations'. Again, it would be too complex for our framework to form expectations for income growth as well as life–cycle income patterns, inflation and nominal interest rates. Besides this, on condition that ever–working and infinitive living agents are assumed, the retirement of households is ruled out. In this case, the underlying perspective of the 'life–cycle hypothesis', i.e. the existence of hump–shaped income profiles, is not given. In sum, we did not treat the integration of a 'life–cycle perspective', because we identify already many difficulties in the chosen—much simpler—approach. The 'life–cycle hypothesis' would therefore overload the complexity of the model additionally. Moreover, the empirical evidence of the 'life–cycle hypothesis' (such as that of the 'permanent income hypothesis') is questionable: For example, the interesting study of Carroll and Summers investigates both hypotheses intensively with cross–country and cross–sectional data. According to this study, both hypotheses have to be rejected. They come to the results that "there is clear evidence that consumption and income growth are much more closely linked than those theories predict" (Carroll and Summers, 1991, p. 305). However, both theories would be good candidates for the savings design in our model with respect to incorporation of the real interest rate. But we do not integrate them according to the reasoning stated above.

By far, the most active area of research (within the field of 'orthodox' economics) in recent years has been the move away from 'life–cycle models' towards a 'richer' class of models, still within the general framework of 'intertemporal choice' under uncertainty. In 'discrete time', optimization follows the maximization of an expected utility function in the form of e.g. $\mathbb{E}U = \mathbb{E}_0 \sum_{T=0}^{\infty} 1/(1+\rho)^T u(C(T))$ with $u(C(T)) = C(T)^{1-\theta}/(1-\theta)$ (Romer, 1996). Thereby θ is the coefficient of 'constant relative–risk–aversion' (the inverse of the 'elasticity of substitution' between consumption at different dates); it determines the households willingness to shift consumption between periods. That is, the smaller θ is, the more willing the private household is to allow its consumption to vary over time. In addition, savings are decreasing in the real interest rate if θ is larger than 1, and increasing in if θ is below 1. In the first case, if θ is large, the individual has a strong preference for similar levels of consumption in the different periods and the 'income effect' dominates. In the second case, with θ less than 1, the 'substitution effect' of a rise in the real interest rate dominates the ‘income effect’ (Romer, 1996).

Turning back to the ‘expected utility function’: The maximization is subject to an intertemporal budget constraint, which leads to the well–known ‘Euler equation’ as the optimality condition. This approach generates ‘precautionary motives’ for savings. Usually the optimization problem must be solved by ‘dynamic programming’, i.e. ‘backward induction’. As mentioned above in chapter 1, it can be assumed that households do not follow such a hyper–rational optimization: This can be due to limited computational power, or at least due to the fact that in reality agents face ‘true uncertainty’—so that they cannot calculate expected utility. Finally, individuals can have various savings objectives, which differ from consumption smoothing. For example bequest motives, prestige, economic power, or safety can also justify the accumulation of wealth (Frietsch, 1991). Usually, models of optimal ‘intertemporal choice’ do not incorporate such motives. Moreover, macroeconomic and microeconomic evidence offer some support for the view that individuals follow rule–of–thumb behavior (Shefrin and Thaler, 1988; Campbell and Mankiw, 1989; Loewenstein, 1988). Several experimental studies identify that individuals do not perform ‘backward induction’ to solve such complex ‘intertemporal’ decision problems in a rational way (Anderhub, 1998; Carbone and Hey, 1997; Hey and Dardanoni, 1988). The most important reason is that people have limited computational power. Hence there is empirical and experimental evidence that justifies that we do not employ hyper–rational optimizing behavior: We follow this intention and design the savings behavior through rule–of–thumb behavior. Surely, it could be the task for further research to develop a more sophisticated yet better manageable approach. To note, it could be fruitful to use the results of future experimental studies in order to define reasonable savings heuristics.

Last but not least, in recent years some new behavioral approaches with strong links to psychological research have emerged within the literature on consumption and savings behavior. For example, the basic ‘life–cycle hypothesis’ has been expanded to a ‘behavioral life–cycle hypothesis’.[22] The ‘behavioral’ literature offers some interesting perspectives on the treatment of income sources: According to the theory of ‘mental accounting’, different income sources are treated differently by individuals. That is, the most tempting income class accounts for large parts of consumed income

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[22] For an overview of the named literature see Friedel, 2004.

(Thaler, 1999; Shefrin and Thaler, 1988). Importantly, we apply this notion as we define different income classes for each agent, which are treated differently with respect to consumption and savings behavior. Before turning to the question, how the consumption and savings behavior of household agents is designed on Agent Island, the following table summarizes the discussed consumption theories briefly. It should be noted that the last column represents the relationship between the named theory and our model. There, the term ‘restriction’ means that the theory in question is incorporated as an additional restriction to the behavior of household agents.

Description	Author	Crucial variable	Relation to Agent Island
Classical consumption function	Classics and followers	Real interest rate	Basic saving rule
Absolute income hypothesis	Keynes	Current income	Basic saving rule
Relative income hypothesis	Duesenberry	Relative income	Restriction, constraint
Habit–persistence	Brown	Consumer habits	Restriction, constraint
Life–cycle hypothesis	Modigliani–Brumberg	Life–time resources	None
Permanent income hypothesis	Friedman	Permanent income	Permanent interest income

Table 2.1: Savings/Consumption theories and their relation to the present model

Design of the Saving Decision

Henceforward, the assignment of (Greek) letters to variables/parameters restarts. The household sector on Agent Island is populated by agents, who (i) are the suppliers of labor to firm agents (hash, bean and capital firms), (ii) are creditors or debtors vs. the central bank agent, and (iii) are the shareholders (owners of the equity) of hash and bean firm agents. For this reason, a household agent h generates in period T income out of three sources: (i) wage income $Y^L_{h,T}$ from labor, (ii) interest income $Y^I_{h,T}$ from financial assets, and (iii) profit or capital income from equity ownership. The latter is divided into two subgroups, namely ‘imputed capital interest’ $Y^C_{h,T}$ (i.e. regular capital income), which is immediately distributed in the beginning of each period, and dividends $Y^D_{h,T}$ (i.e. extra capital income), which are distributed at the end of each period. Thereby the first subscript labels the agent and the second the period. Capital T denotes the time period, i.e. $T = 0, 1, 2 \ldots \bar{T}$, where $\bar{T}$ defines the terminal simulation period; furthermore lower case t denotes intra–period–time. It is worth noting that we treat ‘imputed capital interest’ as ‘imputed entrepreneurial profit’. In addition, we combine income from 'imputed entrepreneurial profit' and from labor income to one group called work income, $Y^W_{h,T} = Y^L_{h,T} + Y^C_{h,T}$. It should be noted, that household agents generate substantial work incomes over time, which is verified by 'face validation' runs.

The specification of the household agents' savings decision requires some initial explanations. We will see that agents employ different savings rates to different sources of income (as defined above). This can be justified by the theory of 'mental accounting' known from 'behavioral economics'. If we otherwise treated all income sources in the same way (i.e. by applying the same savings rate to each source), this would produce undesired effects on the macro level, such as high inflation. We verified this in 'face validations' runs.[23] According to our 'mental accounting' framework, income from work $Y^W_{h,T}$ is the most tempting income class. Savings out of other income sources have to be defined in relation to the work income. It is important to note that we do not stress the role of equity ownership throughout the model. Household agents are owners of the capital stock, i.e. they own hash and beans firm agents. But the shares of those assets are not traded during the simulation.[24] Consequently, households save a part of their receipts in a financial asset, while expenditures, due to purchasing any new real assets, do not take place. Household agents therefore possess a constant stock of real capital.[25] The key to the understanding of our savings design is that the term 'savings' means '*financial* savings' during this study; and the term 'savings rate' means '*financial* savings rate'. Usually, in economic studies, the term 'savings' stands for savings in real *and* financial assets. This is important because in reality the financial savings rate can become negative (such as in the U.S. in recent years), while the total savings rate (in financial & real assets) is in most cases positive.[26]

The basic framework of the savings decision follows a rule–of–thumb or behavioral approach

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[23] See also the explanation in the box below. In case of one savings rate applied to all income sources, the cause of high inflation goes back to role of interest incomes. If they are relatively high, the central bank's interest rate policy (defined by a 'Taylor rule') is destabilizing: A rising interest rate induces rising interest incomes, which affects inflation conditional upon the savings rate. According to that, we employ different savings rates to different income sources. This enables the monitoring of consumed interest incomes.

[24] In some rare cases, the household makes payments due to equity. This does not imply that the agent buys new shares; the payment is rather due to negative equity of the owned firm. We will discuss that point below.

[25] The valuation of this real capital is excluded from our investigation. We implicitly assume constant share prices.

[26] See for example the (aggregate) financial savings rates for the U.S. private household sector between 1998 and 2007. Financial savings rates were always negative, with a peak of about -8% in 2006. In contrast, the total savings rate was always positive, but in 2006 pretty close to 0%.

based upon the seminal notion of John Maynard Keynes (Keynes, 1936). Agents employ a time–variant and idiosyncratic savings rates to the parts of their disposable income. Disposable income of agent h is defined as $Y_{h,T} = Y^W_{h,T} + Y^D_{h,T} + \tilde{Y}^I_{h,T}$. Importantly, Keynes' theory centers on the marginal propensity to consume:

> "The fundamental psychological law [...] is that men are disposed, as a rule and on the average, to increase their consumption as their incomes increases, but not by as much as the increase in their income." (Keynes, 1973, p. 95)

The reception of Keynes transforms this psychological law into two variants (Frietsch, 1991): (i) A linear consumption equation $C = a + bY$, with b defining the marginal propensity to consume. Moreover, $s = 1 - b$ describes the marginal propensity to save, which represents the savings rate. (ii) A continuously decreasing marginal propensity to consume, which is pictured by a concave function, e.g. $C = dY^{1/e}$. Even though the latter variant is more compelling, the former is the usual way to describe the 'Keynesian' consumption function. Both variants reflect that savings fulfill the character of luxury goods, which increase with growing incomes. It should be noted that the notion of the second function incorporates an anticyclical reaction of the savings rate: In a cyclical upswing the savings rate tends to increase, and vice versa (Frietsch, 1991); such a reaction of the savings behavior stabilizes cyclical fluctuations automatically. The present study employs both ideas of the 'Keynesian' consumption function. At first, the period consumption function of agent h, i.e. $C_{h,T}$, is a linear function of incomes (i.e. receipts). In addition, the savings rates in this linear function evolve over time according to a basic law of motion, which comprises income growth as one determinant. The levels of consumption expenditures $C_{h,T}$ and financial savings $S_{h,T}$ of agent h in period T are accordingly described by the following equations:

$$
\begin{aligned}
C_{h,T} &= (1 - s^W_{h,T})(Y^W_{h,T}) + (1 - s^D_{h,T})\max(0, Y^D_{h,T}) + (1 - s^I_{h,T})\max(0, \tilde{Y}^I_{h,T}) - Ex_{h,T} \qquad (2.2.2)\\
\tilde{Y}^I_{h,T} &\equiv \bar{r}_T \overline{FA}_{h,T-1},\\
S_{h,T} &= \sum_x Y^x_{h,T} - C_{h,T} \text{ with } x \in [W, D, I]. \qquad (2.2.3)
\end{aligned}
$$

According to these equations the household agent schedules in nominal terms. Then again, consumption plans are regularly supposed to be defined in real terms, otherwise the phenomenon of 'money illusion' could arise.[27] In GE models, 'money illusion' does not appear, because the decision of agents are based upon real magnitudes. However, this is only possible, because the agents do not schedule *concrete* quantities for their demand or supply; agents in GE models rather define *price–dependent* quantities. They have a system of price–quantity combinations in mind. Importantly, the aggregate system of such price-dependent demand and supply schedules is solved by the 'Walrasian auctioneer'. Trading happens in the equilibrium simultaneously, with equilibrium quantities and prices. Hence, the agents in a GE model need not know prices in advance of trading—unlike in the present agent–based model: In here, the sequence of decisions and actions generates the basic problem. Usually, agent–based models do not build upon the mechanism of a general equilibrium, where the 'Walrasian auctioneer' pauses trading until all schedules are matched to one another by equilibrium prices. In agent–based models agents face 'true uncertainty'.[28] That is, consumer good prices are not available prior to trading, so that the scheduling of consumer goods demand with 'true prices' is not possible. One has to apply (i) expectations about prices (e.g. constituted by last period's prices), or (ii) nominal scheduling. We think that, as a simplification, the scheduling in nominal terms is adequate. It simplifies the problem. Conversely, 'money illusion' would be a serious problem, if inflation and nominal income growth drifted apart. But this is not the case in the present model. Labor income growth is driven by a strong connection between inflation and wage growth; the nominal wage is at least constant, and usually it grows at the same rate (or a slightly higher rate) as consumer goods prices grow. From this perspective it is not decisive that an agent uses past prices to determine the present purchasing power of his income.[29] Finally, this simplifies the savings and consumption decisions, as defined by equations 2.2.2 and 2.2.3.

Importantly, equation 2.2.2 captures the notion that different savings rates are applied to different sources of receipts. Basically, we apply the following approach: Each agent receives receipts from work, which have to be non–negative. Usually, the agent decides to save some of those receipts in financial assets (in rare cases the option of negative financial savings is chosen). On the other

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[27] 'Money illusion' refers to the phenomenon that individuals think of the value of money (or monetary payments) in nominal terms, rather than in real terms.

[28] It should be noted that in the strict rational perspective the scheduling of present consumption cannot be separated from future consumption. Hence, for scheduling consumption today, it is necessary to know the real quantities one can buy for one Agent Dollar in the future. Thus, agents have to 'know' future prices, or have to form expectations about future prices. In agent–based environments this is difficult, especially for the remote future.

[29] In the 'General Theory', Keynes circumvents this problem by assuming constant prices (Keynes, 1936).

hand, incomes from financial assets and from equity could become negative; this means that the agent could make expenditures according to debt[30] or according to negative equity[31]. In this case, the agent lowers the planned consumption goods expenditures out of work receipts by that amount; he uses that part of work receipts, which he does not want to save, in order to finance the named expenditures. In such a case the variables $\tilde{Y}^I_{h,T}$ and $Y^D_{h,T}$ in equation 2.2.2 are fixed to 0 (see the max–functions in the equation), and the non–consumption expenditure of agent h becomes positive (i.e. $Ex_{h,T} > 0$). Accordingly, the scheduled consumption expenditures out of work income are reduced by the amount of non–consumption expenditures, and the residuum is consumed. Then again, provided that interest and dividend income are positive, the agent employs a financial savings rate to those incomes as well. If the agent employs a positive savings rate to his incomes (i.e. receipts), and if the agent has no expenditures due to debt or negative equity, the expenditures of the agent are in most cases below his receipts. This design could appear complicated to researchers unfamiliar with agent–based models. But within the present framework one has to consider, and design accordingly, all possible states and outcomes of the model. As stated in chapter 1, one has to design a dynamically *complete* model. Otherwise, the model would come to a halt or, equally bad, the result would not make any sense for future computations: For example, if we did not integrate the term ‘non–consumption expenditures’ in the consumption function, incomes could become negative (e.g. $Y^I_{h,T} < 0$). This would imply that we apply a (positive) savings rates to a negative income, which would not make any sense.

In addition, the variable $\tilde{Y}^I_{h,T}$ is defined as the ‘myopic permanent real interest income’ that can be expected from financial wealth. We define the disposable income not via actual interest income, but rather through the historical average real interest rate applied to the end–of–last period’s stock of financial assets[32], i.e. $\tilde{Y}^I_{h,T} \equiv \bar{r}_T \overline{FA}_{h,T-1}$. This notion is a contribution to the idea of ‘permanent income’, insofar as this design should dampen the changes in consumption expenditures based upon

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[30] Interest expenditures according to debt: As private households can become debtors, they have to make interest payments (expenditures) to their creditor (i.e. to the central bank).

[31] Expenditures according to negative equity: In some circumstances shareholders have to make subsequent payments (if the equity of the owned firm is negative), which results in negative dividend income. This phenomenon stems from the role of the credit worthiness of firms, to be discussed in subsection 2.2.4.

[32] The variable $\overline{FA}_{h,T-1}$ describes the financial assets at the end of period $T-1$ and $\underline{FA}_{h,T}$ the financial assets at the beginning of period T. The difference between both represents the over–night interest and dividend payments between $T-1$ and T.

'windfall profits'. We treat the average real interest income as a proxy for the 'permanent real income'. As mentioned, the deviation of present income from 'permanent income' is called 'windfall profits' or 'transitory income', and the 'marginal propensity to consume' out of the 'transitory interest income' is 0 in the 'permanent income hypothesis' (Frietsch, 1991). Insofar as the 'permanent income' derived via the historical average interest rate approximates the 'permanent income' only for the near future, such as the next three years, this can be regarded as a 'myopic' interpretation of the 'permanent income hypothesis'. Finally, the reason to adopt the average interest income instead of the actual income is due to the fact that the actual interest income varies strongly with the nominal interest rate: A high nominal interest rate in the last period coincides with high interest incomes in the present period. This, in turn, implies upside risks to inflation. However, monetary theory assumes, in contrast, that rising short term interest rates coincide with falling inflation rates.[33] Finally, undesired effects of high interest rates on consumption expenditures are reduced by the chosen design represented by $\tilde{Y}^I_{h,T}$ in the disposable income.[34]

In the next step we describe the law of motion for the financial (reference) savings rate of household agent h. The variable $\tilde{s}_{h,T}$ stands for this reference savings rate.[35] The defining rule contains two determining factors: (i) The real interest rate $i_T - \mathbb{E}_T\pi_T$, controlled by the central bank, and (ii) the real growth rate of disposable income $g_{h,T}$. We hence assume the following law of motion for $\tilde{s}^W_{h,T}$:

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[33] There exists a rich empirical literature that documents the 'price puzzle', which describes the positive response of prices to a 'restrictive monetary policy shock'. For empirical examinations of this positive relationship between short term interest rates and inflation, see for example Balke and Emery, 1994, or Castelnuovo and Surico, 2006.

[34] In fact, this design eliminates the income effect of a high interest rate, but not its wealth effect. Consequently, this design rules out the short term effects of a high interest rate on consumption expenditures, but not its medium and long term effects via growing wealth.

[35] All other savings rates develop according to this reference rate. See the following explanations.

$$
\begin{array}{rcl}
\tilde{s}_{h,T} & = & \tilde{s}_{h,T-1} + \eta_h^{RR}(i_T - \mathbb{E}_T\pi_T - \bar{r}_T) + \eta_h^{IC} g_{h,T}, \\
\text{with } \mathbb{E}_T\pi_T & \equiv & \pi_{T-1}, \\
g_{h,T} & \equiv & \dfrac{Y_{h,T} - Y_{h,T-1}}{Y_{h,T-1}} - \mathbb{E}_T\pi_T, \\
-0.05 \le & \Delta s_{h,T} & \le +0.05, \\
\Delta s_{h,T} & = & \tilde{s}_{h,T} - \tilde{s}_{h,T-1}, \\
-0.2 \le & \tilde{s}_{h,T} & \le 0.5.
\end{array}
\tag{2.2.4}
$$

The savings rate $\tilde{s}_{h,T}$ is the reference rate, the other savings rates develop according to this reference rate, as explained below. The figures -0.2 and 0.5 define the lower and upper boundaries for the financial savings rate $\tilde{s}_{h,T}$, and -0.05 and 0.05 describe the boundaries for the period–to–period change of the savings rate (expressed in percentage points). As a result, the financial savings rate cannot move more than 5 percentage points up or down; the maximum financial savings rate is 50% of the receipts, and the minimum one is -20%. We have already justified a possibly negative financial savings rate. Moreover, the influence of the deposit interest rate on savings is modeled via the historical average real interest rate $\bar{r}_T$. Accordingly, the savings rate rises, (c.p.) if the expected real interest rate $i_T - \mathbb{E}_T\pi_T$ lies above its historical average $\bar{r}_T$. The sensitivity of this reaction is defined through the parameter η_h^{RR}; and the sensitivity of the savings rate to income growth is given by η_h^{IC}. For example, if both behavioral parameters are set to 0.1 (i.e. $\eta_h^{RR} \equiv \eta_h^{IC} \equiv 0.1$), and if the real interest rate lies 5 percentage points above its historical average (i.e. $i_T - \mathbb{E}_T\pi_T - \bar{r}_T = 0.05$), and if the real growth rate of disposable income is 4% (i.e. $g_{h,T} = 0.04$), the savings rate of agent h rises 0.9 percentage points between period T and $T-1$ (i.e. $0.009 = 0.1[0.05] + 0.1[0.04]$). However, especially the sensitivity of savings rates to real interest rates is theoretically ambiguous and potentially subject to offsetting ‘substitution’ and ‘income effects’, as discussed above in this subsection. A variety of empirical studies examines the connection between (aggregate) savings rates and real interest rates as well as between (aggregate) savings rates and income growth rates. Table 2.2 summarizes these results.

At first, we should note that all studies listed in table 2.2 examine relationships on aggregate

Study	Investigated countries/regions, period	IV^{1,2}	DV^{3,4}
RWI, 2007	Germany, 1993-2007	RR	Household savings rate (++)
Paul, 2004	U.S., U.K., Canada & Japan, 1974-1999	RR	National savings rate (++)
Hussain and Brookins, 2001	104 countries, 1965-1994	RR	National savings rate (+)
Berubé and Côté, 2000	Canada, 1963-1997	RR	Household savings rate (++)
Loyaza et al., 1998	150 countries, 1965-1994	RR	Private savings rate (-)
Callen and Thimann, 1997	21 OECD countries, 1974-1995	RR	Household savings rate (+)
Bundesbank, 1996	Germany, 1975-1994	RR	Household savings rate (+)
Edwards, 1996	36 countries, 1970-1992	RR	Private savings rate (-)
Thomas and Towe, 1996	Canada, 1963-1992	RR	Household savings rate (++)
Masson et al., 1995	21 industrial countries, 1971-1993	RR	Private savings rate (++)
Masson et al., 1995	64 developing countries, 1971-1993	RR	Private savings rate (-)
Liu and Woo, 1994	13 countries, 1975-1985	RR	Private savings rate (- -)
Schmidt-Hebbel et al., 1992	7 developing countries, 1970-1985	RR	Household savings rate (-)
Giovannini, 1983	7 Asian countries, 1962-1972	RR	National savings rate (-)
Fry, 1978	7 Asian countries, 1962-1972	RR	National savings rate (++)
Hussain and Brookins, 2001	104 countries, 1965-1994	IC	National savings rate (+)
Loyaza et al., 1998	150 countries, 1965-1994	IC	Private savings rate (++)
Edwards, 1996	36 countries, 1970-1992	IC	Private savings rate (++)
Callen and Thimann, 1997	21 OECD countries, 1975-1995	IC	Household savings rate (++)
Masson et al., 1995	64 developing countries, 1971-1993	IC	Private savings rate (++)
Masson et al., 1995	21 industrial countries, 1971-1993	IC	Private savings rate (-)
Liu and Woo, 1994	13 countries, 1975-1985	IC	Private savings rate (++)
Schmidt-Hebbel et al., 1992	7 developing countries, 1970-1985	IC	Household savings rate (++)
Giovannini, 1983	7 Asian countries, 1962-1972	IC	National savings rate (++)
Fry, 1978	7 Asian countries, 1962-1972	IC	National saving Rate (++)

Note: 1) The shortcut 'IV' stands for 'independent variable'. 2) The studies treat, among others, the following 'independent variables': 'RR' = the real interest rate, 'IC' = the income growth rate. 3) The shortcut 'DV' stands for 'dependent variable'. 4) The studies identify the following influence(s) of 'IV' on 'DV': (++) = significantly positive influence, (+) = weakly positive influence (possibly insignificant), (-) = weakly negative influence (possibly insignificant), (- -) = significantly negative influence. A weak positive or negative effect can be interpreted as almost no effect.

Table 2.2: Empirical results of the influence of both income growth rates and real interest rates on aggregate savings rates

levels. In contrast, the construction of an agent–based model requires the determinants of savings on individual levels. Secondly, we use financial savings rates, whereas the listed studies investigate total savings rates. Due to a lack of knowledge of *individual financial* savings rates, we have to use the results of these studies. With reference to table 2.2 the effect of income growth on savings rates is quite obvious: Rising incomes should induce higher savings rates. In addition, in a non–econometric analysis of German time series between 1950 and 1988 Frietsch identifies the growth rate of real disposable income per capita as the adequate income category for the determination of the private household savings rate (Frietsch, 1991). According to these results, we incorporate an (unambiguously) positive relationship between both variables on the individual level ($\eta_h^{IC} > 0$) in equation 2.2.4.

According to table 2.2, empirical evidence indicates an ambiguous impact of real interest rates on savings rates. Deaton summarizes the empirical results in the following way: "My reading of this literature is that the empirical results are as ambiguous as is the theory" (Deaton, 1992, p. 60). In addition, Frietsch, 1991, and McKinnon, 1991, treat the subject from a non–econometric perspective: Frietsch identifies that for some periods the German private household savings rate parallels the (ex post) real interest rate (Frietsch, 1991). McKinnon identifies a strong correlation between real interest rates and the growth rates of financial assets in groups of developing countries in the period 1971 to 1980. That is, in the group where countries with high positive real interest rates are summarized, the growth rate of financial assets is substantial higher when compared to the group with lower positive real interest rates. The lowest growth rates of financial assets are realized in the group with strongly negative real interest rates (McKinnon, 1991). In sum, the empirical literature punctuates the theoretical ambiguous effect of real interest rates on savings rates—with a slight tendency to positive correlations between both figures. Even though these findings are ambiguous, we suppose, at first, a positive relationship between both variables on the individual level, i.e. $\eta_h^{RR} > 0$. At first sight, this seems necessary in order to guarantee a control of the business cycle through the central bank, i.e. this should guarantee the existence of the 'savings–channel' of monetary transmission. However, we will treat this subject in a special way: In section 3.3 this connection will be subject to a sensitivity analysis, i.e. within several hundred experiments we examine whether inflation rates are notably sensitive to η_h^{RR}. Thereby, we will also allow negative levels for

η_h^{RR}.[36] If we find out that this parameter impacts inflation[37], we will minimize the level of η_h^{RR} during the subsequent calibration procedure of section 3.4. This minimization is due to the ambiguity of interest rate effects on savings rates in reality. Importantly, the used calibration tool of the SeSAm programming environment enables such a minimization during calibration experiments. By applying this two–level process (first, sensitivity experiments, and second, calibration with optional minimized η_h^{RR}) it should be guaranteed that the real interest rate is incorporated into individual savings behavior in the right way.

With the law of motion for the basic savings rate at hand, we are able to go one step further: Note that we apply different savings rules to different income sources, as it is known from the 'mental accounting' literature. This modeling is necessary in order to treat the different income sources differently. Otherwise, hyperinflation could occur. As we will see below, the disposition of interest incomes is of particular interest. However, the 'mental accounting' literature identifies three different sources of wealth: (i) 'Cash on hand' and other liquid accounts, (ii) other current wealth and (iii) future income. These accounts are sorted hierarchically, which means that liquid accounts are the most tempting class, and future income is the least tempting category (Thaler, 1999; Shefrin and Thaler, 1988). Against this theoretical background we sort the resources of agent h in the following way: (i) Work income is the most tempting class, (ii) incomes from financial wealth and from dividends is less tempting, and (iii) the stock of financial wealth and future incomes are the least tempting category. In the model, this notion is, at first, modeled by employing different savings rates to different income sources. The main linkage to 'mental accounting' are the following definitions: $s_{h,T}^{I} \equiv \chi_h^I s_{h,T}^{W}$ and $s_{h,T}^{D} \equiv \chi_h^D s_{h,T}^{W}$. Thereby, the 'mental accounting' parameters $\chi_h^I, \chi_h^D \geq 1$ constitute the 'mark–up' of $s_{h,T}^{I,D}$ over $s_{h,T}^{W}$. Hence, the idea of 'mental accounting' is incorporated by the fact that the savings rates applied to interest and dividend incomes have to be larger than the savings rate of work income. Agents therefore tend to reinvest interest and dividend incomes.

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[36] See the NOLH in appendix B, which defines the basis for the sensitivity analysis in subsection 3.3. The NOLH design contains some negative entries in the column representing the investigated levels of η_h^{RR}.

[37] This would be due to central bank interest rate policy: If inflation is high, the central bank lifts real interest rates. If, in addition, the impact of real interest rates on savings rates is assumed to be positive, this would imply that a higher η_h^{RR} dampens inflation. If we assume, in contrast, a negative influence of real interest rates on savings rates, a lower η_h^{RR} dampens inflation.

Next to these behavioral rules, we apply three restrictions. First, when the mode of the private household changes from creditor to debtor, the agent raises his work savings rate. This takes place in order to exit indebtedness: The consumption and savings decision of the present model is based upon financial savings rates and current incomes (receipts). There exists, first of all, no direct linkage between financial stocks and savings behavior. Moreover, the repayment of debt through an amortization schedule is not explicitly incorporated into the model. Insofar, household agents schedule to pay interest, not the principal, to their creditor. This is a rather unrealistic assumption. To incorporate the repayment of debt without any amortization schedules, we lift the savings rate of work income in the case of indebtedness. This increase of the savings rate depends on the level of indebtedness. When the indebtedness of an agent enlarges, he must enhance his savings rate in a linear way up to an upper boundary.[38] This is modeled in the following way: If agent h reaches a level of indebtedness, which equates the sum of his work incomes in the last three periods $(Y^W_{h,T} + Y^W_{h,T-1} + Y^W_{h,T-2})$, the basic savings rate is doubled. Up to this level, the basic savings rate enlarges linear with respect to indebtedness. This increase of financial savings represents an implicit amortization schedule of agent h. The following equations describe this design:

$$s^W_{h,T} \quad = \quad \tilde{s}_{h,T}[1 + \min(1, \frac{\max(0, \overline{LL}_{h,T-1})}{Y^W_{h,T} + Y^W_{h,T-1} + Y^W_{h,T-2}})].$$

The min–function in the equation guarantees that the basic savings rates is at most doubled. The max–function delivers only positive values for the liabilities $\overline{LL}_{h,T-1}$; the variable is positive, when agent h is a debtor at the end of period $T-1$. When the agent, in turn, leaves the debtor status and becomes a creditor (which implies $\overline{LL}_{h,T-1} = -\overline{FA}_{h,T-1} < 0$)[39], the expression $\max(0, \overline{LL}_{h,T-1})$ becomes 0, and the increase is abandoned. The work savings rate falls back to $\tilde{s}_{h,T}$. Hence, if agent h is a creditor, the work income savings rate and the basic savings rate are identical, i.e. $\tilde{s}_{h,T} = s^W_{h,T}$. As explained, the rate $\tilde{s}_{h,T}$ is the basic savings rate of the model; it evolves according to the law of motion described by equation (2.2.4). In addition, $s^W_{h,T}$ and the 'mental accounting' parameters

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[38] If the agent is a creditor, the upper boundary for the basic savings rate $\tilde{s}_{h,T}$ is fixed to a level of 50%. If the agent is a debtor, the savings rate can, in principle, be doubled up to its natural upper boundary of 100%.

[39] The variable $\overline{LL}_{h,T-1}$ represents the liabilities of agent h at the end of period $T-1$, and the variable $\overline{FA}_{h,T-1}$ stands for his financial wealth. Therefore, we can define the following connection: $\overline{LL}_{h,T-1} = -\overline{FA}_{h,T-1}$. Technically, both variables are expressed by one single variable in the simulation code. Throughout this study we use both expressions, as defined in this footnote.

$\chi_h^{D,I}$ deliver the other two savings rates $s_{h,T}^D$ and $s_{h,T}^I$.

One could question, how an agent could get indebted provided that he regularly applies positive savings rates to all income sources? Additionally, would it not be reasonable that consumers have some 'subsistence level' of consumption? Therefore, we define the following restriction: When the consumption expenditures of agent h, derived by equation (2.2.2), fall below a minimum level, he consumes this minimum level. This impacts the consumption out of the agent's financial wealth; or if financial wealth is lacking, consumption is financed through new debt (i.e. based upon future incomes). From this perspective, the financial wealth accumulated in normal times is a 'buffer stock' preventing low consumption in bad times. This is the saving for 'rainy days', as mentioned above. In this extreme case, the agent consumes out of the least tempting income source, viz. out of financial wealth or future incomes. It should be noted that the subsistence–rule dominates other rules modeled within this subsection.[40]

At this point we should mention one deficit of the model. In fact, no direct borrowing constraints are employed through credit markets (as we will see in the discussion of the central bank below). First of all, household agent h can accumulate as much debt as necessary to fulfill his consumption plans:[41] The change of debt (or financial assets) reflects his balance between receipts and expenditures. When a household agent is able to find a seller supplying sufficient goods, he can buy as many products as he desires, based upon his scheduled consumption expenditures. Thereby, the payment is immediately settled through the central bank—without any further check of creditworthiness. This is an extreme simplification in the model. It constitutes a weakness of the financial markets on Agent Island. In fact, financial markets are totally passive.[42] One justification of this simplification is the fact that household agents on Agent Island have the obligation to work. Insofar it is not possible that household agents stop supplying labor, and therefore, stop receiving labor incomes. It prevents that they stop working, and that they finance their total consumption expenditures exclusively through debt. This case is ruled out due to the obligation to work. However,

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[40] It is therefore a 'quasi budget constraint' for agent h. We will explain that point below.

[41] The same is in principle also true for the indebtedness of individual firms. But there, we introduce a certain mechanism, which prevents over-indebtedness of firms. This is described in section 2.2.4, where the financial conditions of the model are discussed.

[42] This implies that the credit supply of the central bank is not explicitly modeled.

the case that agent h finances his consumption totally through debt is in principle possible, but only if his employer does not demand any labor at all due to a zero–production plan. This implies a collapse of the consumer goods market. Such a breakdown case will be avoided by model 'validation'.

It is an important feature that the subsistence level is defined in relation to the average per capita consumption expenditures in the last period. See equation (2.2.5) below. Thereby, the variable $\hat{C}_{T-1}$ describes this average per capita consumption expenditures in the last period; and the subsistence level is given through $\nu_h^{lower}\hat{C}_{T-1}$ (with $0 < \nu_h^{lower} < 1$). This way of modeling consumption behavior is related to the work of Duesenberry. He suggests a 'relative income hypothesis': A consumer wants to consume in such a way that his consumption pattern is close to the consumption pattern of other agents (Duesenberry, 1967). Moreover, one can argue that the average per capita consumption expenditures serve as a reference point for the consumption decision of agent h, so that the lower bound of present consumption expenditure is defined via such a reference point.[43] This leads to the third restriction of the consumption behavior. Finally, we introduce an upper bound in a similar way. One can explain such an upper bound through the fact that a saturation level of consumption exists, where the marginal utility of consumption approaches to 0, and the agent does not want to consume any further goods. This restriction ensures that consumption expenditures do not rise uncontrolled. Again, we connect the level of saturation to the average per capita consumption expenditures of the last period via ν_h^{upper}. By combining the lower and upper bounds, we can define the domain of $C_{h,T}$:

$$C_{h,T} \in [\nu_h^{lower}\hat{C}_{T-1}, \dots \ \nu_h^{upper}\hat{C}_{T-1}], \text{ with } 0 < \nu_h^{lower} < 1, \text{ and } \nu_h^{upper} > 1. \tag{2.2.5}$$

Equation (2.2.5) serves also as a 'quasi intertemporal budget constraint' of agent h; henceforth we call it the intertemporal budget constraint. Consumption expenditures are able to vary according to several behavioral assumptions made until now, but they have to stay within the bounds defined through the intertemporal budget constraint. Besides this, the intertemporal budget constraint implies a 'quasi borrowing constraint' as well. For example, consider a quite low disposable income of agent h, e.g. close to zero. It is possible to calculate the maximum negative balance between receipts (in this case $Y_{h,T} \approx 0$) and consumption expenditures defined by the subsistence

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[43] See Loewenstein, 1988, for the role of reference points in intertemporal choice.

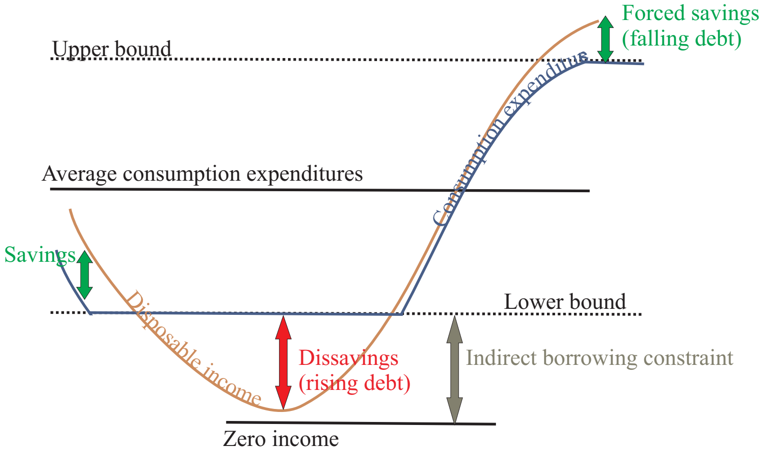


Figure 2.4: Quasi intertemporal budget constraint and borrowing constraint

level ($C_{h,T} = \nu_h^{lower}\hat{C}_{T-1}$). This balance is negative and approaches $-\nu_h^{lower}\hat{C}_{T-1}$ from above. Figure 2.4 illustrates this situation: (i) Initially, the disposable income lies above the consumption expenditures; the agent is saving. (ii) Thereafter, his income falls close to zero and the agent makes consumption expenditures amounting to the lower bound. We could argue that $\nu_h^{lower}\hat{C}_{T-1}$ constitutes a 'quasi borrowing constraint', so that the agent h can borrow as much as given through the difference between this level and his income (if and only if his disposable income falls short of the level $\nu_h^{lower}\hat{C}_{T-1}$). (iii) One can assume that his work income will rise again in the following periods, so that he is subsequently able to repay his debt bit by bit. Thereby, the average per capita consumption expenditures serve as a proxy for average per capita disposable income. It is implicitly assumed that the income of agent h will return to this average income. (iv) Moreover, his income can rise further on. In figure 2.4 it is illustrated that the disposable income of household h rises subsequently above the upper bound of consumption expenditures. In this case the agent is *forced* to save, and therefore, to repay his debt. To summarize, we assume that the work income of any agent h returns to the average level soon. Insofar, he is allowed to borrow from the central bank, if his income falls below the subsistence level. Consequently, equation (2.2.5) defines an intertemporal budget constraint of agent h, and the lower bound of this budget represents a 'quasi borrowing constraint', implicitly imposed by the central bank.

At this point the design of households savings and consumption behavior is finished. It guarantees smooth consumption behavior. Furthermore, the quite complex approach (encompassing many routines) stems from the causes of hyperinflation. In earlier stages of the model development, the design of savings behavior was much simpler: For example, the model did not relate different savings behaviors to different income sources. But in this simpler setting hyperinflation occurred in many simulation runs. The following box characterizes this special topic.

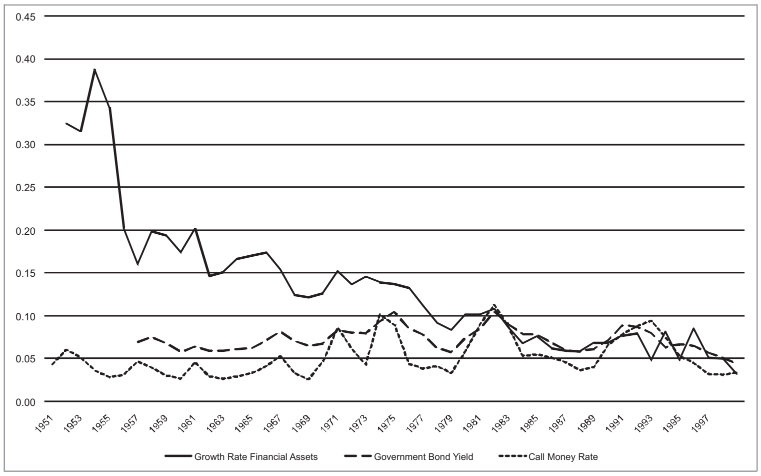
Box: Managing the Causes of Hyperinflation

During early 'face validation' runs the problem of hyperinflation occured. Together with the problem of deflation, we identify this as the core issue in the whole model development process: In a model encompassing only real variables, several restrictions (especially resource constraints) prevent the explosion of real variables. In a monetary model these restrictions are not decisive for the monetary sphere, and the enduring rise (or fall) of nominal variables becomes the main topic. It took several weeks to identify the sources of such an explosion in case of the present study. Finally, we found the cause: Provided that private household agents consume their interest incomes exclusively in accordance with their individual savings rates, i.e. without any further restrictions (e.g. given by ν^{upper} and ν^{lower}), this leads over the time frame of about 20 to 30 periods to hyperinflation. The reason for this failure lies in the fact that growing financial wealth induces large interest incomes. In this case rising inflation appeared, triggered through consumed interest incomes.

The interesting point is that the central bank tries to counteract inflation by increasing interest rates, which again enhances interest incomes of agents further. This boosts again consumption and inflation, and so on. The identification and monitoring of this 'vicious circle' was one major task in the development of the present model. In real–world economics such a process is unlikely to happen, because in reality (i) credit expansion of firms is restricted (credit constraints are certainly more restrictive than on Agent Island), and (ii) debtors are supposed to react faster to high interest rates (by dampening economic activity) as creditors do (e.g. through expansion of consumption expenditures). Importantly, on Agent Island the government sectors as a main debtors lacks.

Before discussing a suitable solution to this practical problem, we want to look to some stylized facts: Figure 2.5 shows the growth rates of private households interest–bearing financial assets in Germany in the years 1951 to 1998. In the present model, the accumulation of household savings can be achieved exclusively through financial assets—real savings are not considered. By consequence, we can compare the data in figure 2.5 with savings behavior on Agent Island. In addition, the nominal government bond interest rate is illustrated. The interesting facts displayed in this figure are that in the early years of the development of the German economy after World War II, (i) the growth rates of financial assets were substantial higher than the interest rates, and (ii) over the years they were falling towards the level of nominal interest rates. 30 years later in the beginning 1980s, both values settled around the same range. Since this time the average growth rate of financial assets became relatively stable and coincided with the average nominal interest rates. Against the background of the present study the interpretation of these facts are straightforward: (i) If nothing else but the complete interest earnings are saved every year by households, the growth rate of their financial assets coincides with the nominal interest rates. (ii) If, in addition, positive (or negative) savings are conducted out of other income sources, the growth rates of financial assets lies somewhat above (or below) the nominal interest rate. (iii) The crucial point is that reinvested interest earnings are withdrawn from the 'income circuit' (Reich, 1998). If they are withdrawn in such a way, the dynamics of inflation rates is not affected by those reinvested interest incomes (Reich, 1998). The reinvestment of interest incomes characterizes the mechanism of a savings book, which is a quite popular form of savings among German households—especially during that time. Consequently, the above mentioned 'vicious circle' cannot occur on Agent Island provided that interest incomes are reinvested by household agents in each period (like in a savings book). Conversely, this can produce deflationary effects, and it can result in exponentially growing financial assets (as identified in reality as well).

However, after discovering this fact it becomes obvious that one possible scenario for the present simulation is as follows: The savings rate for interest income (and dividend income) of household



Note: The investigated time span ends in 1998 because of a lack of suitable data. The methodology of the European System of National Account (ESA) was updated in 1995 (to the new ESA 1995). The data of the previous ESA is only available till 1998. In ESA 1995 especially the classification of the private household sector does not fit optimally to our objectives here. Moreover, in the new flow–of–fund data it is more difficult to identify interest–bearing financial assets. Hence, we take the data from the older ESA, which ends in 1998.

Figure 2.5: Growth rates of private households' interest–bearing financial assets and nominal interest rates; Germany, data from 1951 to 1998; Sources: Bundesbank and IMF, International Financial Statistics

agents is fixed to 1, i.e. $s_h^{I,D} \equiv 1$. Besides this, household agents are endowed with an initial financial wealth that corresponds to the situation in Germany in the early 1980s. This second point is constituted through the ratio $\frac{\sum_H \underline{FA}_{h,0}}{GDP_0}$, which gives the ratio of the sum total of the initial wealth of all agents at the beginning of period 0 to the GDP in the initial period 0.[44] This ratio should correspond to the data in Germany in the beginning 80s. Moreover, we apply an extremely uneven distribution of initial financial wealth among private households, that is, financial assets are initially allocated among $\frac{1}{5}$ to $\frac{1}{30}$ of the population.[45] Those agents stand for the extreme wealthy part of the population in the economy. Consequently, we investigate two scenarios for the design of the savings decision throughout this study. It will be one task of the 'validation' procedure to identify the better suited of the following scenarios:

1. **Baseline model:** This contains the general framework described in the present subsection, according to which savings behavior for all income sources is determined via several behavioral parameters. In the following we call this the 'baseline case'.

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[44] Note that $\underline{FA}_T$ represents the financial stocks at the beginning of T, and $\overline{FA}_T$ that at the end of T.

[45] The concrete value of this figure is governed by the parameter ϖ. We explain that below in chapter 3.

2. **Ponzi scheme model:** This builds upon the general framework where, in addition, the savings rates of interest and dividend income are exogenously fixed to 1, i.e. $s_h^I \equiv 1$ and $s_h^D \equiv 1$. In this case the parameters χ^I and χ^D drop out. We call this scenario the 'Ponzi case'.[46]

Design of the Consumption Decision

At the beginning of this section we mentioned that agent h faces a two–level decision process. The last paragraphs discuss in length the complex (first–level) savings decision. In the next step, on the second level, the agent decides on the make–up of his basket of consumed goods. Henceforth, we follow the agent–based approach suggested by Tesfatsion (Tesfatsion, 2006): She constructs a hash–and–bean macroeconomy. The consumption goods sector on Agent Island accordingly consists of two subsectors, each consisting of hash and bean producing firms. Each firm, within one of these subsectors, produces a single good. Thus, agent h can decide among the consumption of hash and beans according to his period utility function:

$$U_{h,T} = h_{h,T}^{\alpha_h} b_{h,T}^{(1-\alpha_h)}. \tag{2.2.6}$$

Agent h maximizes a 'Cobb–Douglas' utility function[47] subject to the budget constraint given by the consumption expenditures derived through the savings decision. The parameter α_h defines the idiosyncratic preferences between hash and beans, the variables $h_{h,T}$ and $b_{h,T}$ define the amount of hash and beans consumed in period T by agent h. The period (or static) budget constraint for the consumption decision is described by

$$C_{h,T} = p_T^{hash} h_{h,T} + p_T^{bean} b_{h,T}, \tag{2.2.7}$$

whereas the budget $C_{h,T}$ (i.e. the consumption expenditure) is defined through equation (2.2.2) in combination with the equation (2.2.5) and all other restrictions described above. As a result of this 'Cobb–Douglas' specification, agent h divides his budget in two parts by applying the preference

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[46] The reason for this labeling will be explained in section 3.

[47] See the definition of the 'Cobb–Douglas' production function below.

parameter α_h to the budget. When the price for hash is rising, the agents still allocates the same budget to hash expenditures. Hence, hash consumption expenditures are given through $\alpha_h C_{h,T}$, and bean consumption expenditures through $(1 - \alpha_h)C_{h,T}$. Consequently, the valuation of hash and beans through agent h are described in the following way:

$$\begin{aligned} p_T^{hash} &= \frac{\alpha_h C_{h,T}}{h_{h,T}}, \\ p_T^{bean} &= \frac{(1-\alpha_h)C_{h,T}}{b_{h,T}}. \end{aligned}$$

Both, p_T^{hash} and p_T^{bean}, describe the consumer goods valuations of consumer h in period T, i.e. they define the maximum price the consumer is willing to pay for one unit of hash or bean. In the following section, all aspects of the consumer goods sector as well as the market clearing process are illustrated.

2.2.2 Consumer Goods Firms and Markets

We assume that the consumption goods market is competitive, i.e. that it is populated by a large number of firms, which supply consumption goods. The design of these consumer goods firms follows the proposal of Axel Leijonhufvud, 2006a. Based upon the ‘theory of complex systems’, Leijonhufvud’s model introduces the seminal notion of Alfred Marshall into the field of agent–based macroeconomics. Marshall designs the behavior of firms in an adaptive manner, which can be described adequately by difference equations. The combination of several of such difference equations make the system highly non–linear and therefore complex. In Marshall’s time, the analytical techniques to handle such systems were not available (Leijonhufvud, 2006a). Nowadays, with the development of advanced programing languages and increasing computing power, it is possible to handle such non–linear ‘complex systems’.

It is of some interest to note that according to Marshall the adjustment of prices, output and capital stock operates on different time scales, that is, each of these equations is ranked from the fastest to the slowest (Leijonhufvud, 1973). In the context of the present model this means that prices adjust qualitatively faster than output and the capital stock.[48] Accordingly, prices find their

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[48] In Marshall’s short–term perspective the adjustment speed of prices is infinite, whereas the adjustment speed of

equilibrium level within each period, but output is adjusted in a sequence of periods. Furthermore, some basic institutional arrangements are necessary to implement such an approach into an agent–based framework (Leijonhufvud, 2006a): Leijonhufvud assumes a market where in each period T, a certain amount of goods is produced by each firm, i.e. $h^s_{j,T}$ and $b^s_{k,T}$. The former is the supply of hash by the hash firm j in period T; the latter is the corresponding supply of the bean firm k. Both goods are non–storable, i.e. the produced quantities have to be sold within the present period. As already mentioned, capital T denotes a time period and the lower case t an intra–period–time. In the following we concentrate on the decision of a hash firm j, in order to keep the model description as short as possible. Thus, bean firms are not discussed. But, in principle, the specifications of hash firm behavior can be transferred to bean firms without special adjustments.

Adaptive Behavior and Short–Term Equilibrium

The following 'Marshallian' rules describe the behavior of agents in the consumer goods markets (Leijonhufvud, 2006b):

1. **For consumers:** If the demand price exceeds the market price, the consumer enhances purchases; and in the opposite case, he reduces purchases.
2. **For suppliers:** If the supply price exceeds the market price, the supplier reduces the output; and in the opposite case, he expands the output.

As stated, the first rule works on a faster time–scale than the second rule. Accordingly, the consumer rule determines the short term, intra–period, equilibrium, so that within one period prices find their equilibrium levels and the supplied quantity is held constant. Between periods suppliers are able to adjust output according to the second rule. We describe these mechanisms in the following paragraphs.

In doing so, it is necessary to define the process of price formation. This process model is not literally found in Marshall's 'Principles', but it gives a good description of his ideas (Leijonhufvud,

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capital stock is zero. Thus, in the very short–term, the capital stock is a parameter (Leijonhufvud, 1973).

2006b). In order to fulfill this task, stationary market demand schedules for hash and beans must be available. Such stationary market demand functions in the form of $h^d = D(p^h)$ are derived by aggregation of individual demand functions (over all consumers $h \in [1, \dots, H]$):

$$h_T^d = \sum_{h=1}^{H} h_{h,T} = \frac{\sum_{h=1}^{H} \alpha_h C_{h,T}}{p_T^{hash}}.$$

In addition, the total industry output produced at day T, $h_T^s = \sum_{j=1}^{J} h_{j,T}^s$, is auctioned off. The according price formation is well described by the following 'tâtonnement process' (Leijonhufvud, 2006a):

$$\begin{aligned} p_{t+1}^{hash} &= f[D(p_t^{hash}) - h_T^s] + p_t^{hash}; \\ p_{t+2}^{hash} &= f[D(p_{t+1}^{hash}) - h_T^s] + p_{t+1}^{hash}; \\ &\text{etc.} \end{aligned} \tag{2.2.8}$$

The demand schedule $D(p)$ assumes that demanders have moved to equate their demand prices to the market price. The function $f(\cdot)$ aggregates all the relevant market information into a well defined excess demand. This adjustment function, $f(\cdot)$, is assumed to have the standard properties: $f(0) = 0$; $f(x > 0) > 0$ and $f(x < 0) < 0$. As assumed above, the price finds its equilibrium level within one period, that is, the system of equations (2.2.8) converges on:

$$p_T^{hash*} = p^*(h_T^s). \tag{2.2.9}$$

Equation (2.2.9) describes the short–term goods market equilibrium on Agent Island. This point attractor defines the intra–period equilibrium (Leijonhufvud, 2006b). In fact, we do not model the 'tâtonnement process' explicitly; we rather derive the intra–period equilibrium price by plugging the inelastic aggregate supply function into the aggregate demand function, and solving for the price p_T^{hash*}.[49] This is given by

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[49] This is basically the equilibrium price known from 'orthodox' economics. However, in the next step we depart from the 'orthodox' perspective.

$$p_T^{hash*} = \frac{\sum_{h=1}^{H} \alpha_h C_{h,T}}{h_T^s}. \tag{2.2.10}$$

One can describe this logic as a 'pricing–to–market' behavior. An interesting effect of this design is that a change in the aggregate demand schedule has a marked effect on the market price compared to an upward supply schedule. Figure 2.6 illustrates this situation. In sum, the consumer goods market exhibits rather perfect competition for homogenous goods. In analogy to real–world markets, we think that consumer goods markets are rather saturated, and therefore the technology requirements for firms in these markets tend to be lower than in the capital goods market. Consequently, we prefer the perfect competition framework, where a single market–clearing price for a homogenous goods emerges. Under such conditions, firms tend to be similar in their technological development, which will be of interest below.

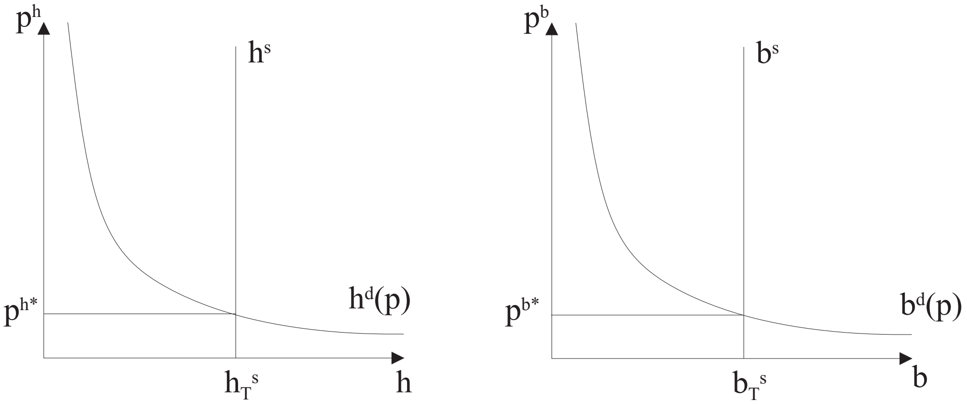

Figure 2.6: Equilibrium prices in aggregate consumer goods markets

Output Adjustment and Production Technology

To following description highlights the difference of the adopted 'Marshallian' approach compared to 'orthodox' models: So far the model describes the mechanism in the goods market within one period. According to this process, prices adjust faster and converge to the short–term equilibrium depicted by the equation (2.2.10). In the next step we analyze the adjustment of output between periods: Output starts in period 0 with the initial value $h_{j,0}^s$. This initial value for firm j is calculated via random initial capacity utilizations; accordingly, the initial level is drawn from a uniform distribution between 50% and 100% capacity utilizations. In addition, output develops from $T-1$ to T according to a difference equation that comprises marginal cost and the market price in $T-1$:[50]

$$h^s_{j,T} \quad = \quad h^s_{j,T-1}[1+\theta_j \frac{p^{hash}_{T-1} - MC_{j,T-1}}{MC_{j,T-1}}]. \tag{2.2.11}$$

Equation (2.2.11) defines the law of motion for hash firm j. The variable $MC_{j,T-1}$ describes the marginal costs of firm j in the last period. The mechanism of this rule is as follows: For example, when the last period equilibrium prices lies 20% above marginal costs of producing the last period's output, and when the behavioral parameter θ_j is set to 0.1, hash firm j enlarges its output by 2%.[51]

In order to calculate the marginal cost it is necessary to know the individual production function of firm j. This is a quite complex task. As pointed out by Griliches and Mairesse, 1983, the basic production function model is at best an approximation to a much more complex and changing reality at the firm, product, and factory floor level. This is also true, if the approach would be augmented by additional variables and non–linear terms (Griliches and Mairesse, 1983). Nevertheless, we had to incorporate some production technology in order to describe the production process, the productivity of factors and so on. Hence, we apply a 'Cobb–Douglas' production function that features 'Hicks–neutral' disembodied technical change:[52]

$$h_{j,T} \quad = \quad A_{j,T}F(L_{j,T}, K_{j,T}) = A_{j,T}L^{\beta_j}_{j,T}K^{(1-\beta_j)}_{j,T}. \tag{2.2.12}$$

The variable $A_{j,T}$ and the parameter β_j describe the production technology of firm j. The variable β_j gives the 'elasticity of output to labor'. As usual for 'Cobb–Douglas' functions, we assume

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[50] Again we follow Leijonhufvud (2006a). But in the present case we adjust his linear difference equation in such a way that we apply the percentage deviation of prices from marginal costs–whereas Leijonhufvud uses the absolute deviation of prices from marginal costs.

[51] This is given through $h^s_{j,T} = h^s_{j,T-1}[1 + 0.1 \times 0.2] = 1.02h^s_{j,T-1}$.

[52] 'Hicks–neutral' technical change goes back to the work of John Hicks (Hicks, 1966). It is defined in the following way: A change is considered to be Hicks neutral, if the change does not affect the balance of labor and capital in the products production function. More formally, given the Solow model production function $Y = AF(K, L)$, a 'Hicks–neutral' change is one which only changes A. Recent work suggest that the aggregate production function shifts in a non–neutral way (Castro and Coen-Pirani, 2005; Dupuy and Marey, 2004). A new paper from Dupuy points to the direction that both neutral and non–neutral technical changes appear in the time frame from 1948 to 1999 in the U.S. (Dupuy, 2006). Nevertheless, this research is conducted on the aggregate level, whereas the design of the present model appears on micro level. For the sake of simplicity, we follow the notion of 'Hicks–neutral' technical change for all firms, i.e. for hash, bean as well as capital firms.

‘constant returns to scale’ (so that the function is ‘homogenous of degree 1’). It is worth noting that β_j and $A_{j,T}$ are allowed to vary from firm to firm in the scenario with sectoral heterogeneity. The technical parameter $A_{j,T}$ is initially equal for all consumer goods firms. This may be satisfied by the assumption of perfect competition in the consumer goods market, as mentioned above. In this case, firms tend to feature similar technological levels, which accounts for the initial equal level of $A_{j,T}$ for all hash firms. Moreover, the variable $A_{j,T}$ is defining a time–variant technology variable, which evolves according to

$$\begin{aligned} A_{j,T} &= 0.5A_{world,T} + 0.5[A_{j,T-1}(1 + \varrho_j + \epsilon_{j,T})], \\ A_{world,T} &= A_{world,T-1}[1 + \varrho_{world} + \epsilon_{world,T}]. \end{aligned}$$

Following these equations, $A_{j,T}$ develops according to a ‘random walk with drift’. The development of $A_{world,T}$ describes the technological progress of the Agent Island economy as a whole. Consequently, there is individual technological progress (on the firm level) as well as social technological progress (on the island or ‘world’ level). The (equally weighted) combination of both defines the technical parameter relevant for the individual production technology of firm j. The incorporation of a ‘world’ level technological progress models the idea of technology diffusion. The determinants of the technical progress are as follows: ϱ_j defines the ‘drift parameter’ of agent j; $\epsilon_{j,T}$ describes the level of the ‘white noise’ term in period T (as is usually assumed with $\mathbb{E}[\epsilon_j] \equiv 0$). The variance of the ‘white noise term’ ($Var[\epsilon_j]$) has to be determined in the ‘validation’ procedure of chapter 3. In general, it is not a topic of the present study to investigate (i) sources of technological progress, or (ii) the impacts of technological progress on the business cycle in detail. Thus, we will treat the ‘drift parameter’ as exogenous; furthermore this parameter will not be investigated in the sensitivity analysis of chapter 3. But, as a note in the margin, we will apply different growth rates for hash, bean and capital industries, and we will investigate different variances of ϵ_j (i.e. different supply–shock–scenarios) in the sensitivity analysis of subsection 3.4.

If we rearrange the production function for the amount of labor $L_{j,T}$ and plug the resulting equation into the variable cost function $VC_{j,T} = w_T L_{j,T}$, we can derive the marginal costs of hash firm j by differentiating the variable cost function with respect to output $h_{j,T}$. This delivers finally the marginal costs

$$MC_{j,T} = \frac{w_T}{\beta_j}(\frac{1}{A_{j,T}})^{\frac{1}{\beta_j}}(\frac{1}{K_{j,T}})^{\frac{1-\beta_j}{\beta_j}}(h_{j,T})^{\frac{1}{\beta_j}-1}. \quad (2.2.13)$$

As explained above, the calculation of marginal costs is required for the supply decision of firm j. Thereby, the partial derivatives of $MC_{j,T,}$ with respect to (i) w_T and (ii) $h_{j,T}$ are positive; and that with respect to (iii) $A_{j,T}$, (iv) β_j, and (v) $K_{j,T}$ are negative.

Capacity Limit

Following the seminal idea of Alfred Marshall, namely that prices, the output and the capital stock adjust according to different time scales, we assume that within one period the capital stock is treated as a parameter (Leijonhufvud, 1973). Accordingly, consumer goods firms decide to buy new machines and expand the capital stock at the beginning of each period—but the ordered new machines arrive after producing the output of the current period, so that the capital stock is fixed between the decision of output adjustment (equation (2.2.11)) and the production of output. Hence, the production function (equation (2.2.12)), the definition of the parameter A_{jT} and the capital stock $K_{j,T}$ are given. This enables the calculation of labor demand of firm j in the next section.

In addition, the maximum labor supply of the workers of each firm (hash, bean or capital) is represented by the amount of workers assigned to each firm times the working hours per period. In the case of a period length of one year, we assume 52 weeks times 40 hours per week, i.e. one worker is assumed to work a maximum of $2,080$ hours per period. This delivers the maximum labor supply to firm j: $\check{L}_{j,T} = 2,080\varpi$, with $\varpi = \frac{H}{J+K+C}$. The parameter ϖ depicts the number of workers assigned to one firm during the simulation run, given by the ratio of the number of household agents H to the number of firm agents $(J + K + C)$. Using this maximum labor supply $\check{L}_{j,T}$, one can derive the capacity limit of hash firm j through $\check{h}_{j,T} = A_{j,T}\check{L}_{j,T}^{\beta_j}K_{j,T}^{(1-\beta_j)}$. Note that the technical parameter is exogenous and the capital stock can be treated as a parameter within a period. We will need this definition of the capacity limit of firm j later on in this chapter.

Labor Demand, Labor Supply and Wage Dynamics

As mentioned above, rearranging the production function delivers the labor demand of hash firm j:

$$L^D_{j,T} \quad = \quad (\frac{h_{j,T}}{A_{j,T}K^{(1-\beta_j)}_{j,T}})^{\frac{1}{\beta_j}}.$$

In our framework, households have the obligation work, and therefore, to supply the demanded quantity of labor to the firms they are assigned to. In contrast to 'neoclassical' economics, the labor market of the present model is therefore totally passive. This stems from the objective of the present study: The analysis concentrates on inflation, its sources and its possible control through monetary policy. Hence topics concerning the labor market in general, the research on unemployment and its sources, and so on, are beyond our subject. Moreover, we assume that the supplied labor is uniformly distributed among the ϖ workers of firm j. Each worker of firm j receives a wage payment of $\frac{PR_{j,T}}{\varpi}$; whereby each firm engages ϖ workers, and $PR_{j,T}$ gives the total payroll of firm j. Additionally, we assume an immediate settlement of wages via central bank accounts.[53] In sum, the total payroll (PR) of firm j is given by

$$PR_{j,T} \quad = \quad w_T L^D_{j,T}. \tag{2.2.14}$$

For purposes of monetary theory and policy, the important aspect of the labor market is the dynamics of wages. The 'expectations–channel' of monetary transmission assumes that wage dynamics are an important determinant of overall inflation (Bofinger, 2001), which is in turn the main objective of monetary policy. With respect to the determination of wage growth we assume that the supply side in the labor market is represented by a labor union, which negotiates the nominal wage for all of its members. According to that, all household agents receive the same wage per hour independent of the firm they work for. The union and the representatives of the firm sector negotiate the new nominal wage w_T at the beginning of each period T. Importantly, the mechanism of this wage negotiation is described by a difference equation, which defines the negotiated nominal wage in period T as a function of the nominal wage in period $T-1$. We apply an approach used in the

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[53] Similar to wages, the 'imputed capital interests' are paid immediately at the beginning of each period, just after the production decision and production itself. In contrast, dividend and interest incomes are settled over–night, i.e. between periods.

deviation of the 'Phillips curve' based upon 'labor–market rigidities'.[54] The point is that within this approach the nominal wage is set according to a rule that includes (i) the expected inflation rate and (ii) the expected state of the business cycle (represented by the output gap) (Bofinger, 2001):

$$\begin{aligned} w_T &= w_{T-1}(1 + \mathbb{E}_T\pi_T + \lambda\mathbb{E}_T y_T), \qquad (2.2.15)\\ \text{with}\,\mathbb{E}_T\pi_T &= \pi_{T-1},\\ \mathbb{E}_T y_T &= y_{T-1},\\ w_T &\geq w_{T-1}. \end{aligned}$$

Equation (2.2.15) connects the nominal wage in period T to its lagged level in $T-1$ via the expectation for inflation in T (at the beginning of period T) and the expectation for the output gap y in T (again at the beginning of period T). It should be clear that intra–period timing is an important aspect of our agent–based model, and that inflation and output gap figures of the present period are not available at the beginning of period T. Both figures are naturally the outcome of the economic interactions in period T. According to that, it is necessary to build the expectation for both variables. We employ (throughout this study) expectation formation of an adaptive style, i.e. the expectations of the variables are described by their own (one period) lagged values. Finally, we define that the newly negotiated wage has to be at least as large as the previous period wage, so that nominal wage growth has to be non–negative. Moreover, the wage negotiation builds upon the parameter λ, which in turn defines the power of the labor union in the negotiation process. That is, the higher the union power the higher is λ and the higher is in turn the growth of wages due to positive output gaps. Remember, negative nominal wage growth is ruled out by definition (see equations above). The mechanism of the wage growth equation is straightforward: Provided that the labor union expects an inflation rate of 3% and an output gap of 4%, and that the labor union has enough power to enforce a parameter $\lambda = 0.25$ in the negotiation, nominal wages grow by 4%.[55]

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[54] See for example the approach of Taylor, 1979, and Taylor, 1980. To highlight the difference between the role of rigidities in the 'orthodox General Equilibrium approach' and the present one, it is important to understand that in 'New Keynesian' models the rigidities are necessary to incorporate any real impact of monetary policy at all. In the present model this is basically not necessary. We define the negotiation of wages for a period of one year, because a year defines the length of the model period: Shorter negotiation periods are not possible in our framework. Longer periods would simply enlarge the dynamics of upside pressure on wages via expected inflation. But the effect of monetary policy on real activity is not directly influenced by these rigidities.

[55] This is given through $w_T = w_{T-1}(1 + 0.03 + 0.25 \times 0.04) = 1.04w_{T-1}$.

Last but not least, the household agents receive 'imputed capital interests' from the consumer firms as well. We already mentioned that consumer firms are owned by the workers assigned to them: Workers own their employer in equal shares, and therefore, they receive next to labor income also 'imputed capital interest' from the firm.[56] The amount of distributed 'imputed capital interest' of hash firm j at the beginning of period T is defined through $\sum_{h=1}^{\varpi} Y_{h,T}^{C} = \max(0, \bar{i}_T \overline{E}_{j,T-1})$. Thereby, the left hand side of the equation describes the sum of all imputed capital interest of the ϖ shareholders of firm j. Firm j is forced to distribute this income. The right hand side defines the computation of the sum of 'imputed capital interests' of firm j, given by the product of historical average nominal credit interest rate $\bar{i}_T$ (this represents the regular yield or the risk–free rate of interest)[57] times $\overline{E}_{j,T-1}$ (i.e. the equity of firm j at the end of the previous period). We assume that 'imputed capital interests' have to be non–negative. This figure defines the yield the hash firm j has to pay to its shareholders each period. Finally, the 'imputed capital interest' of a single household agent h is delivered by rearranging the last equation to $Y_{h,T}^{C} = \frac{\max(0, \bar{i}_T \overline{E}_{j,T-1})}{\varpi}$, i.e. the total distributed capital interests are divided through the headcount ϖ of firm j.

Profit Figures, Distributed Dividends and Investment Demand

Before we analyze the demand for investment goods (i.e. the demand for new machines) of firm j it is necessary to define some additional profit variables. Every period, after the execution of all market transactions, firms have to calculate profit figures. First of all, the definition of 'business profit' of hash firm j is straightforward:

$$\Phi_{j,T}^{business} = p_T^{hash} h_{j,T} - PR_{j,T} - D_{j,T} - i_{T-1}\overline{LL}_{j,T-1} + \sum_{n=1}^{N} \Phi_{n,T}^{business}. \tag{2.2.16}$$

Equation (2.2.16) depicts that the 'business profit' of a hash firm in T calculated as the difference between revenues and costs of firm j plus the 'business profit' of its N subsidiaries. As mentioned above, capital firms do not utilize capital in the production process. In this respect it is impossible

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[56] In addition, shareholders can receive dividends. Firms distribute dividends according to extra (i.e. economic) profits. See the description below.

[57] According to the 'capital asset pricing model' (CAPM), the yield of the secure investment is the basis for the calculation of equity prices (and therefore equity yields) of corporate enterprises. We follow the approach of the CAPM insofar as firms have to pay at least the (average) risk–free rate of interest $\bar{i}_T$ to their shareholders.

to calculate the return on assets for a capital firms, because the capital stock, necessary to calculate the return on assets, does not exist. However, the return on assets is an important figure within the model, so that we treat capital firms as subsidiaries of hash or bean firms. Such corporate groups are defined randomly at the beginning of each simulation run.

Equation (2.2.16) shows that the revenues of hash firms are the product of the hash market price times the quantity sold in period T. In addition, possible sources of costs are: (i) cost of labor $PR_{j,T}$, (ii) nominal depreciation $D_{j,T}$, and (iii) interest expenditures $i_{T-1}\overline{LL}_{j,T-1}$. Payroll ($PR_{j,T}$) has been defined above by equation (2.2.14). The depreciation of the capital stock $D_{j,T}$ is designed by the concept of 'physical depreciation' of capital. This works as follows: For each firm the invested real capital is collected in an 'asset ledger'. In this ledger a data set for every machine exists. Any machine z is represented in the ledger by three entries: (i) The maximum number of work hours ($Maximum\ hours$)[58], (ii) the number of served work hours ($Served\ hours_{z,j,T}$), and (iii) the purchase costs of the machine ($p^{machine}_{z,\tilde{T}}$). Thereby $\tilde{T}$ depicts the period the machine was purchased.

For machine z, in each subsequent period the variable ($Served\ hours_{z,j,T}$) is increased by the amount of hours, which are necessary to produce the period output. This increase represents the capital stock of machine z used in period T, i.e. $K_{z,j,T} = (Servedhours_{z,j,T}) - (Servedhours_{z,j,T-1})$. The remaining work hours of a machine z can be derived from the data entries in the 'asset ledger', i.e. through $(Remaining\,hours_{z,j,T}) = (Maximum\,hours) - (Served\,hours_{z,j,T})$. Consequently, the present value of machine z in the balance sheet of hash firm j at the end of period T, (i.e. $\ddot{p}^{machine}_{z,j,T}$), and the (physical) depreciation in period T (i.e. $D_{z,j,T}$), are given by

$$\begin{aligned}
\ddot{p}^{machine}_{z,j,T} &= p^{machine}_{z,\tilde{T}} \frac{(Remaining\ hours_{z,j,T})}{(Maximum\ hours)}, \\
D_{z,j,T} &= p^{machine}_{z,\tilde{T}} \frac{K_{z,j,T}}{(Maximum\ hours)} = \ddot{p}^{machine}_{z,j,T} \frac{K_{z,j,T}}{(Remaining\ hours_{z,j,T})}.
\end{aligned}$$

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[58] As in the case of the calculation of the maximum quantity of labor per agent per period, we assume a maximum number of 2,080 work hours per machine per period. Hence, we do not assume a three–shift production process. In case of such a three–shift production, the maximum number of work hours per machine and period would be $3 \times 2,080 = 6,240$. Again, this assumption is due to simplification.

Finally, the total capital stock represented in the production function of firm j is represented by $K_{j,T} = \sum_{z=1}^{Z} K_{z,j,T}$, and the (physical) depreciation of the total capital stock is given through $D_{j,T} = \sum_{z=1}^{Z} D_{z,j,T}$, whereby Z indicates the total number of machines in the 'asset ledger' of firm j. Moreover, the interest expenditures in equation (2.2.16) are defined through the product of the interest rate i_{T-1} and the liabilities $\overline{LL}_{j,T-1}$.[59] It is an important fact that the central bank pays and receives the interest over–night, and it is assumed that the interest payments between period $T-1$ and T enter the calculation of the profit in period T. Consequently, the assets and liabilities at the end of each period $(T-1)$ are the basis for interest expenditures in the income statement in the subsequent period (T). In addition to the 'business profit' of hash firm j defined by equation (2.2.16), the respective 'economic profit' is given through

$$\Phi_{j,T}^{economic} \quad = \quad \Phi_{j,T}^{business} - \max(0, \bar{i}_T \overline{E}_{j,T-1}). \tag{2.2.17}$$

The 'economic profit' differs from 'business profit' by 'imputed capital interest', which are defined by $\bar{i}_T E_{T-1}$; as mentioned above, this stands for 'imputed entrepreneurial profit' as well. We assume that 'imputed capital interest' must be non–negative. Next to 'imputed capital interest', hash firm j distributes a part of its 'economic profit' to shareholders via dividends. These dividends are calculated via extra profits (i.e. via 'economic profits'), i.e. dividends of firm j are given through $\Phi_{j,T}^{economic} \times POF_j$. Thereby, POF_j stands for the 'pay–out–factor' of the 'economic profit', which naturally lies between 0 and 1. It is interesting to note that we calculate dividends via 'economic profits', and not via 'business profits'. On the one hand, 'imputed capital interest' defines the regular capital profits, on the other hand, dividends are generated through excess profits in the sense of $\Phi_{j,T}^{economic} > 0$.

An important determinant of the investment demand of hash firm j is its return on assets. In order to calculate this figure it is necessary to define the firm's 'business profit before interest

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[59] In order to prevent confusion, $\overline{LL}_{j,T-1}$ stands for the liabilities of firm j at the end of period $T-1$, i.e. when the firm is a net debtor this figure is positive. In the other case, when firm j is a net creditor (i.e. $-\overline{FA}_{j,T-1} = \overline{LL}_{j,T-1} < 0$), the firm does not have any interest expenditures, but collects interest receipts. As a simplification, we add those incomes to the business profit.

payments' (EBIT)[60]. EBIT is defined by the 'business profit' plus interest expenditures:[61]

$$\Phi^{EBIT}_{j,T} \quad = \quad \Phi^{business}_{j,T} + i_{T-1}\overline{LL}_{j,T-1}. \tag{2.2.18}$$

With this result in hand it is easy to calculate the return on assets $i^{assets}_{j,T}$:

$$i^{assets}_{j,T} \quad = \quad \frac{\Phi^{EBIT}_{j,T}}{A_{j,T}}. \tag{2.2.19}$$

Thereby $A_{j,T}$ stands for the assets of firm j, which is the sum of present value of the Z machines in the 'asset ledger', i.e. $\sum_{z=1}^{Z} \ddot{p}^{machine}_{z,j,T}$.

So far we have described some figures that play an important role in the condition for investment demand. In general, hash and bean firms can generate investment demand out of two sources: (i) Replacement of written–off machines m^R, and (ii) expansion investments m^E (i.e. increasing the capital stock). Note that $K_{j,T}$ represents the capital stock that is used each period. The total available capital stock $\check{K}_{j,T}$ is the sum of this figure over all periods the existing machines are working. It can be derived through the entries for all machines Z in the 'asset ledger' of firm j by $\check{K}_{j,T} = \sum_{z=1}^{Z}(Remaining\ hours)_{z,j,T}$. Replacement investments m^R are subject to the above explained mechanics of depreciation. Each time a machine is canceled out from the firm's 'asset ledger' a replacement is required, so that the capital stock remains (at least) constant. It is natural that the market price for machines $\hat{p}^{machine}$ influences investment demand.[62] In the following, we employ the restriction that the last period's market price for machines, $\hat{p}^{machine}_{T-1}$, should not be higher than the 'mean shadow value' of a new machine. We approximate this 'mean shadow value' ($MSV^R_{g,j,T}$) for the replacement of the used–up machine z through a new machine g in period T by the following expression:

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[60] The term EBIT stands for 'earnings before interest payments and taxes'. Insofar as taxes are not considered in the present model, EBIT stands in fact for 'earnings before interest payments'.

[61] If firm j is a net creditor, we also exclude the respective income form EBIT.

[62] Below, we will explain the market clearing process for machines. According to our approach of 'monopolistic competition', the price for machines sold in one period varies from capital firm to capital firm. Thus, there does not exist one market price, as in case of (homogenous) consumption goods. The price $\hat{p}^{machine}_{t-1}$ is therefore the average price of all machines traded in period $T-1$.

$$MSV^{R}_{g,j,T} \quad = \quad \frac{h_{j,T}}{K_{j,T}} \Delta \check{K} p^{hash}_{T-1}.$$

The notion behind this equation is that the 'shadow value' is given by the expected output of the new machine ($\frac{h_{j,T}}{K_{j,T}} \Delta \check{K}$) evaluated through the expected price of the output (p^{hash}_{T-1}). The former is delivered by the 'mean output per unit of capital'[63] (expressed in hash units per capital working hour) times the change of the capital stock $\Delta \check{K}$ (expressed in total working hours of the new machine g); therefore, $\Delta \check{K}$ is represented by $\Delta \check{K} = (Maximum\, hours)$, i.e. the working hours any machine is able to serve *totally.* This approach implicitly assumes that (i) the output produced by the capacity of the new machine can be sold with a price equal to the last period's market price p^{hash}_{T-1}; and (ii) that this output will be sold immediately. In fact, the new machine will produce output at least over the next 20 periods, so that (i) the market price will certainly change over time; and (ii), equally important, that the output will not be sold immediately. It would therefore be correct to discount the future turnover to its present value. For the sake of simplicity this is not done. Nevertheless, the magnitude of $MSV^{R}_{g,j,T}$ delivers an approximation of the upper bound for the machine price firm j is willing to pay (for the replacement of a machine z). When the average machine price in the last period $\hat{p}^{machine}_{T-1}$ exceeds this upper bound, the consumer firm will not replace the worn–down capital stock. It will rather postpone the replacement investment to a (unknown) time in the future, where this particular condition is fulfilled. That is, the average capital price $\hat{p}^{machine}$ has to fall, or $MSV^{R}_{g,j,T}$ has to rise in order to satisfy replacement investment in the future. Consequently, postponed replacement investment will never expire in the model—firms just wait for suitable price–conditions.

Importantly, hash firm j expands its capital stock, when the following conditions are fulfilled *simultaneously*:

1. **Profit condition:** General firm profitability (return on assets) is large enough to satisfy opportunity costs, i.e. $i^{assets}_{j,T-1} \geq i_T$. This design characterizes the 'Wicksellian' perspective of our model;

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[63] In contrast to this, the expected output of a new machine due to expansion investments are given by the marginal output of one machine—and not by the mean output as in case of replacement investment. See the descriptions below.

2. **Capacity condition:** Capacity limit $\check{l}_j$ is reached, i.e. $l_{j,T} \geq \check{l}_j$;
3. **Market price condition:** 'Shadow value' of a new machine f is at least as high as last period's average machine price, i.e. $MSV^E_{f,j,T} \geq \hat{p}^{machine}_{T-1}$.

In the following paragraphs we will explain these three conditions in detail. The first two conditions are rooted in economic literature. The first is a strong link to the monetary theory of Knut Wicksell. In his seminal work 'Geldzins und Güterpreise' (1898) Wicksell describes a theory of price level dynamics that grounds in the difference between the natural interest rate and the credit interest rate. According to his theory the influence of the credit interest rate on nominal prices is as follows: If the natural rate is larger than the credit interest rate, the price level is rising—and vice versa. In this context Wicksell's concept of the natural interest rate is important: Suppose an imaginary world where no money exists at all. The credit interest rate that would emerge in such an environment through the interaction of capital supply and demand would give the natural interest rate. In such a world a credit would not be defined and settled in monetary terms—it would rather be defined and settled in real output terms (i.e. credit would be defined as a payment in kind). The price that would emerge in this market for real–capital–lending would be the basis for the natural rate in the original sense of Knut Wicksell. If this natural rate was larger than the real credit interest rate, firms would generate extra profits. Consequently, they would enlarge their business and the price level would in turn rise. If the natural rate was below the credit rate, the opposite process would occur (Wicksell, 1898).

We view our monetary theory in this 'Wicksellian' tradition. Hence, we construct the theory of capital demand with respect to the relationship between credit interest rate and natural rate. In the model the role of the credit interest rate is obvious—it is the interest rate paid for central bank credits (adjusted by inflation). Moreover, we replace the construct of the natural rate through the real return on assets of individual firms. Consequently, a firm expands its capital stock, when the real return on assets is larger than the real credit interest rate. As an approximation, we switch form real rates to nominal rates by applying the fact both rates (the nominal credit interest rate as well as the nominal return on assets) are given by the respective real rate plus inflation.[64] According

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[64] According to Irving Fisher the real rate can be derived via $1 + i = (1 + \pi)(1 + i)$. By rearrangement this delivers

to that, the decision based upon nominal rates delivers the same result as the decision based upon nominal rates. This delivers the first condition $i_{j,T-1}^{assets} \geq i_T$.

The second condition goes back to the work of Dosi et al. (Dosi et al., 2005; Dosi et al., 2006; Dosi et al., 2008), who emphasize that investment expenditures are 'lumpy'. This stands in contrast to the standard neoclassical view of investment, where the assumptions of 'convex adjustment cost' and 'reversibility' produce smooth and continuous adjustments of the capital stock over time. Such standard models fail to explain investment behavior that can be identified on the micro–level (particulary for plant–level data). A good descriptive study of the investment behavior of 13,700 U.S. manufacturing plants for the period 1972–1988 is made by Doms and Dunne, 1998. By describing the investment activities of those firms they find that many plants alter their capital stocks in a 'lumpy' fashion. Due to the necessity to construct agent models bottom–up, Doms and Dunne, 1998, laid down the foundation for the agent–based work in the field of investment models.[65] To summarize, several empirical micro–level studies highlight two main findings:[66] (i) Investments by manufacturing plants are characterized by periods of intense investment activity and periods of very low activity; (ii) episodes of intense activity are responsible for a significant part of aggregate investment fluctuations.

Earlier than those empirical findings, several authors reexamined the unrealistic assumptions of 'convex adjustment costs'[67] and reversibility[68], which lead to adjustment of capital in discrete bursts (Doms and Dunne, 1998). This lumpiness can be interpreted as the rational outcome of optimizing behavior in a so-called (S,s) investment model. In such a framework firms face the problem of choosing the optimal level of capital that maximizes the flow of profits. Firms invest as long as they are able to recover 'capital adjustment costs' on condition that desired capital is lager than actual capital. If these adjustment costs present non–convexities, firms will invest up to some 'optimal

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$r = i - \pi - \pi r$. Insofar as (πr) is relatively small, this term drops out and the real rate is approximately given by $r \approx i - \pi$. Consequently, one can add inflation to any real interest rate and obtain (approximately) the nominal rate. This mechanism is applied here.

[65] For instance, Dosi et al., 2005; Dosi et al., 2006; Dosi et al., 2008 use the findings and the stylized facts delivered by Doms and Dunne, 1998 as a benchmark for validating their agent–based model with respect to investment behavior.

[66] See, for instance, Caballero et al., 1995; Power, 1994; Cooper and Haltiwanger, 1993; Doms and Dunne, 1998.

[67] Usually it is assumed that non–convex capital adjustment costs cause a non–linear investment behavior. See, among others, for the theory of non–convex adjustment costs and the non–linearities in investment behavior Abel and Eberly, 1994; Nickell, 1978; Rothschild, 1971.

[68] See, for example, Dixit, 1992; Dixit and Pindyck, 1994; Pindyck, 1991.

target level' (S) only if capital drops to a 'trigger threshold' (s). In this case investment of size $S - s$ takes place. The structure of the (S, s)–rule makes it an ideal guideline for the routinized investment behavior in the present model: Here, the 'trigger threshold' (s) for hash firm j is given by $\check{l_j}$. Admittedly, our model exhibits a different perspective. Investment takes place (given the other conditions are fulfilled), when capacity utilization exceeds some capacity limit ('trigger')–level, and not if capital falls to the trigger level. The perspective is therefore twisted—we focus on the actual and the 'trigger' capacity limit instead of on the optimal capital stock and its 'trigger' level. To sum up, in our framework investment takes place (given the other conditions are fulfilled), when the present capacity utilization $(l_{j,T})$ exceeds some 'trigger' level $(\check{l_j})$. Besides this, the expression $\psi_j \check{K}_{j,T}$ gives the size of the expansion investment, whereby the behavioral parameter ψ defines this size in relation to the existing capital stock in period T (i.e. $\check{K}_{j,T}$).

Finally, similar to the case of replacement investment, the price conditions are effective: This notion is again modeled via the 'shadow value' of a new machine f, which enlarges the capital stock. In accordance to the evidence from firm–level data expansion investment is 'lumpy', and more volatile than replacement investment. We therefore model the 'shadow value' of a new machine f somewhat different compared to expansion investment:

$$MSV^{E}_{f,j,T} \quad = \quad \frac{\partial h_{j,T}}{\partial K_{j,T}} \Delta \check{K} p^{hash}_{T-1}.$$

According to this equation, the 'shadow value' $MSV^{E}_{f,j,T}$ of a new machine f depends on the marginal output of the the capital stock, i.e. $\frac{\partial h_{j,T}}{\partial K_{j,T}}$. If $MSV^{E}_{f,j,T}$ exceeds the last market price of machines $\hat{p}^{machine}_{T-1}$, firm j can expand its capital stock as suggested through $\psi_j \check{K}_{j,T}$. Otherwise, and in contrast to replacement investment, the order (delivered by the other two conditions) is canceled and not postponed into the next period. In the following period firm j has to restart the checking of all conditions of expansion investment. The present design is born out of experience from 'face validation' runs. There, we found out that it is reasonable to define the 'shadow value' of replacement investment via mean output of one capital unit, and the 'shadow value' of expansion investment via the marginal output of capital. Due to the structure of the production function the latter is always smaller than the former. Thus, the restriction for expansion investment is more restrictive. Moreover, it seems reasonable that the replacement of an old machine depends on the mean productivity of the existing capital stock, whereas the expansion depends on the marginal productivity of one new machine. 'Face validation' also suggests the postponing of replacement investment into the following period, provided that the capital price $\hat{p}_{T-1}^{machine}$ is too high. Conversely, a possible postponing of expansion investment would accumulate a pile of open expansion investment orders—over time. If then then market price of capital falls in the subsequent periods, the aggregate sum of all expansion investments would possibly explode—which is not desired. Thus, we do not allow the postponing of expansion investment, if machine prices are too high.

In brief, firm j plans to expand its capital stock,(i) when its return on assets in the last period is larger than the present credit interest rate set by the central bank, (ii) when its level of capacity utilization in the present period l_T is larger than a certain 'trigger' level $\check{l_T}$, and (iii) when $\hat{p}_{T-1}^{machine}$ is smaller than $MSV_{f,j,T}^{E}$. For hash firm j the capacity utilization in period T is defined by $l_{j,T} = h_{j,T}/\check{h}_{j,T}$; $\check{h}_{j,T}$ is the capacity limit of firm j, as derived above. By definition, this figure lies between 0 and 1. In addition, the 'trigger' level specifies a capacity utilization near the limit, where the firm needs a capacity expansion in order to expand production substantially. This limit $\check{l_T}$ lies below, but close to 1. If all three conditions are fulfilled, expansion investment is triggered. In this view, the expansion of the capital stock does not appear gradual, but rather in jumps (represented by ψ_j) from one level to another. By this mechanism, hash firm j calculates its demand for expansion investment in period T ($m_{j,T}^{E}$). In sum, the total investment demand is given by $m_{j,T}^{D} = m_{j,T}^{E} + m_{j,T}^{R}$. It should be noted that all investment demand figures are integers, i.e. whole numbers. Last but not least, we define that investment demand has to be larger than 1: Whatever the conditions for replacement and expansion investment are, the lower bound for machine demand is given by 1. This design builds again upon the empirical evidence of plant level investment behavior. The interesting fact in this context is that even though a significant portion of investment occurs in relatively few periods, plants still invest in every period (see the enumeration below).

Another interesting explanation of the 'lumpy' investment behavior on the aggregate level is based upon the vertical relationship of the production scheme. The resulting phenomenon is described in older economic literature as the 'investment accelerator':[69] Accordingly, the two–level production scheme in the present model (i.e. the economic connection between consumer goods and capital goods firms) produces the effect that a rather small enhancement in consumption demand, can cause relatively large upswings in the investment demand. Evidence from GDP data highlights that the implications of the 'investment accelerator' work in reality. The idea of the 'accelerator' is therefore transferred to our investment framework as well. Hence, consumer firms regularly invest in the replacement of depreciated machines without doing any expansion investment; but from time to time (especially in situations of economic upswings) capacities have to be enlarged in order to satisfy conditions of consumer goods demand. In this case, a rather small increase in the consumer goods demand, so that the capacity utilization of presumably many consumer firms approaches its limit, can in turn boost investment demand. This accounts for the effect that investment is 'lumpy' on the aggregate level.

A further consequence of such a model design is the circuit of 'feedback loops': An initial boost in the investment demand produces positive effects on labor demand in the capital industry, which in turn induces rising labor incomes and consumer expenditures of the workers in that industry. This in turn drives the capacity utilization of the consumer goods industry upward, and additionally, consumer goods firms tend to yield a higher return on assets. Therefore, new expansion investment is also more likely, which perpetuates the 'feedback loop'. In principle, it is imaginable that such an effect works also the other way around, even though, active de–investment of firms is not incorporated into the model. Nevertheless, such 'feedback loops' can produce destabilizing effects, if they are not counteracted by an opposite force.

Finally, the following short review highlights some ideas that may be beneficial for the process of calibrating and validating the present model in chapter 3. In fact, those micro and macro findings were already guiding the construction of the investment behavior from the very beginning of the project. It should be noted that we investigate those points in chapter 3 only marginally.[70] Empirical

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[69] See for example Clark, 1917; Stützel and Grass, 1983 for an explanation and examples of the effect.

[70] The macro facts are integrated in the 'validation' process. However, the micro facts were guidelines in the construction of the micro behavior; but we do not use those facts in chapter 3 explicitly.

literature delivers the following findings on aggregate and individual investment behavior:

- On the aggregate level investment behavior is 'lumpy' and more volatile than consumer demand, which is rather smooth.
- Individual expenditures for replacement investments vary over time. They are not, as one could assume, a constant proportion of the capital stock. Nevertheless they are a much more constant proportion than were expenditures for expansion investments.[71]
- While varying less, replacement investments are not a stabilizing substitute for expansion investments, but rather move up and down with expansion investments.[72]
- While a significant portion of investments occurs in relatively few periods, plants still invest in every period.[73]
- Tremendous heterogeneity occurs in the capital accumulation patterns across firms (or plants).[74]

2.2.3 Capital Goods Firms and Markets

According to the previous design of consumer goods firms, we derive an individual demand for machines, which is quasi price–elastic. As explained above, investment demand is conditioned by past machine prices. However, if we treat these results strictly, individual demand schedules delivered in the last section section are represented by quantities *only*, which are not directly related to (present) market prices for machines.[75] From the point of view of capital goods firms, the (aggregate) demand does not relate the demanded quantities to machine prices. Moreover, we assume, according to the macro bindings of Agent Island, that the demand side of the market is confronted with supply conditions based upon 'monopolistic competition'. As a result, we cannot apply 'orthodox' frameworks for the market clearing process within 'monopolistic competition', because such approaches require a direct price–elastic aggregate demand schedule. The following paragraphs describe the notion of 'monopolistic competition' in our context, i.e. based upon 'face–to–face' trading. Thereafter, the

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[71] See Eisner, 1972; Feldstein and Foot, 1971.

[72] See Eisner, 1972.

[73] See Doms and Dunne, 1998.

[74] See Doms and Dunne, 1998.

[75] As we will see in this subsection, the individual search and trading process in the machine market is governed by chance and *prices*. Accordingly, machine prices are an important determinant of aggregate demand as well as for individual trading.

explicit supply conditions and the market clearing process are illustrated. The subsection closes with a short description of labor demand and profit figures of capital goods firms. Whenever possible in this subsection, we (implicitly) use results already represented in the description of hash firms during the last section.

Monopolistic Competition

The following mechanism describes the aggregate machine demand: During each period a new $V-$dimensional demand vector emerges; it describes the quantities of machines demanded by J hash and K bean firms. The sum of the V ($V = J + K$) elements of this vector determines the aggregate quantity of machines demanded by the consumer goods sector. The aggregate demand side for machines is confronted with supply schedules that are in accord with the theory of 'monopolistic competition':[76] According to this theory, a large number of firms produce differentiated products. A differentiated product is one that buyers consider to be a good, but not perfect substitute for another good. In 'monopolistic competition' the industry has enough firms that when one firm cuts its prices, every other firm loses only a small quantity of sales. Additionally, two distinct features of 'monopolistic competition'—in contrast to perfect competition—are (i) the existence of price 'mark–ups', i.e. supply prices are larger than the firms' marginal costs, and (ii) the existence of excess capacities in the market.

We are convinced that the framework of 'monopolistic competition' fits to the capital goods industry on Agent Island: We assume that producing capital goods is more sophisticated than producing consumer goods. For example, in reality plant constructing tends to be a high–tech business, which demands for a well–educated workforce and expensive research & development efforts. This is due to complex environments for the application of the products. Finally, this tends to result in less competition and higher rents in the market.[77] The same applies to other high–tech branches like business software, biotechnology, or the medical equipment industries. Hence, the existence of a

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[76] See, among others, Obstfeld and Rogoff, 1996; Obstfeld and Rogoff, 1995, for the theory of monopolistic competition.

[77] According to the research of Michael Porter five forces dominate the profitability of a branch. That are employees, substitutes, victualers, potential competitors and existing competitors. If those forces a rather low, the market tends to exhibit larger profitability and rents (Porter, 1992). In the context of the present model the conditions in the capital industry on Agent Island result in higher barriers for market entrance, lower competition, and therefore, higher rents.

sophisticated production environment accounts for 'monopolistic competition' (which in turn leads to outstanding profitable firms).

Finally, we assume a 'face–to–face' trading based upon 'make–to–order' production schemes: With the demanded number of new machines in hand each consumer goods firm searches within the market for one or more suitable suppliers. This 'search process' is notably driven by chance and prices. This means that a capital firm c, supplying machines for a rather low supply price, can generate rather large sales—but it need not be the case. To be sure, our design of the capital market features the characteristics of 'monopolistic competition': (i) The supply prices are higher than the 'marginal costs', i.e. capital firms apply a 'mark–up' calculation; (ii) firms usually have excess capacities; and (iii) if a capital firm cuts its price, others firms lose only a small quantity of their sales. As to how these results are obtained through the design of the capital supply and market clearing, is described in the following paragraphs.

The Investment Supply and Market Clearing

Production technology of capital firm c is described by the production function

$$m_{c,T} \quad = \quad A_{c,T}F(L_{c,T}) = A_{c,T}\gamma_c L_{c,T}^{\beta_c}. \tag{2.2.20}$$

The parameter γ should only make the scaling of the production function easier. It has just a technical meaning, and one can integrate γ into the initial value A_0 as well. It should be noted that we treat the productivity of capital goods firms differently compared to consumer goods firms. In the consumer goods sector we apply an equal initial value for A, as a result of the assumption of 'perfect competition'. Accordingly, consumer goods firms can differ (i) in the 'factor elasticities' (i.e. in the relationship between α and $(1-\alpha)$), (ii) in technological progress, and (iii) in the initial capital stock. Insofar, initial capacities of hash and bean firms vary some percent among each other (due to rather small variations of the initial capital stocks). That is, initially firms tend to be rather equal in the consumer goods market. On the contrary, capital goods firms exhibit more heterogeneity, due to the sophisticated production process for machines and less competition in the market. We model this stronger initial heterogeneity through the parameter β. In the framework with heterogenous agents within one sector we allow β to vary between 0.4 and 0.6. When we assign 20 workers to each capital goods firm, the respective capacities vary with the factor 8. Such large dispersions in productivity are identified in several studies examining longitudinal micro–level data.[78] Next to the heterogeneity in the parameter β, the initial value $A_{c,0}$ is equal for all firms. Over time $A_{c,T}$ evolves according to an exogenous process equal to that of consumer goods firms. Finally, the size of scaling parameter γ is equal for all firms.

In the next step, we plug the maximum amount of labor $\check{L}_j$ into the production function (i.e. equation (2.2.20)). This delivers the maximum capacity of firm c:

$$\check{m}_{c,T} \quad = \quad A_{c,T} F(\check{L}_{c,T}) = A_{c,T} \gamma_c \check{L}_{c,T}^{\beta_c}.$$

Both values, the maximum capacity of firm c as well as any produced output of machines $m_{c,T}$, are integers, i.e. whole numbers; machines are not separable. Each period, the aggregate demand for machines and the maximum aggregate capacity of all capital firms are compared. When the latter is higher than the former, supply constraints demand. In this case our market clearing process indicates random rationing driven by chance and by machine prices. In the 'normal' case, if the maximum aggregate capacity is larger than aggregate demand, regular trading (without rationing) takes place. This trading happens on the foundation of a 'made–to–order' production scheme, where consumer goods firms search for one or more suitable suppliers of machines. In order to enable this, we collect the supply side conditions in a $C \times 5$ matrix. Row c of the matrix represents capital goods firm c (with $c \in [1, \ldots, C]$). This delivers the C rows of the matrix. The five columns are determined by the following five variables:

This means that row c and column 1 labels the capacity limit $\check{m}_{c,T}$ of capital goods firm c. Column 2 indicates how many orders have been filed with capital firm c by its customer(s); column

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[78] See, among others, the studies of Dhrymes, 1991; Dwyer, 1996; Olley and Pakes, 1996; Oulton, 1998.

Capacity	Number of orders	Ordered machines	Supply price	Probability
$\check{m}_{c,T}$	$0, 1, 2, \ldots n$	$o_{c,T}$	$p_{c,T}^{machine}$	$\frac{1/(1+p_{c,T}^{machine})}{\sum_{\tilde{c}=1}^{C} 1/(1+p_{\tilde{c},T}^{machine})}$

Table 2.3: Columns of the supply matrix in the capital goods market

3 depicts the quantity of ordered machines, and column 4 shows the supply price $p_{c,T}^{machine}$ of one machine offered by firm c. This price is the basis for every ('face–to–face') contract between capital firm c and a potential machine buyer. Column 5 defines a probability measure that constitutes the 'search process'. This round–based 'search process' works as follows: The demand vector and the supply matrix are given. When the capital market opens, in turn each consumer goods firm v draws randomly a supply firm from the supply matrix. The order of the V consumer goods firms in the 'demand queue' (i.e. in the demand vector) is ordered by the size of individual machine demand (i.e. the largest individual demand is the first element in the demand vector). Thereby the measure

$$\frac{1/(1+p_{c,T}^{machine})}{\sum_{\tilde{c}=1}^{C} 1/(1+p_{\tilde{c},T}^{machine})} \tag{2.2.21}$$

indicates the probability that capital firm c is drawn by firm v. The numerator of the measure is the reciprocal of the supply price of c plus 1. In here, the mechanism is straightforward: The selected measure in the numerator gets large, when the individual supply price of c is small. When the price is very low (e.g. little above 1 AD), the measure converges to 1. When the supply price is very high (e.g. $1,000,000$ AD), the measure converges to 0. The adjustment by one in the numerator takes place in order to prevent the disturbing effect of very small prices between 0 and 1 AD. In addition, the denominator is the sum of the just described measure over all capital firms. In effect, if all firms offer machines at low prices, but approximately at the same level, every firm features approximately the same probability to be drawn in this mechanism. But, if only some firms feature a quite low supply price, while all others ask for higher prices, the probabilities of the few firms to be drawn is quite large.

On the assumption that firm c is drawn as a supplier, the respective consumer firm closes a contract with firm c. That is, it places an order at firm c, which covers the demanded quantity of new machines. Hence, the variable $o_{c,T}$ is adjusted from zero to the sales number, and the field 'number of orders' is enhanced by one, so that firm c drops out from market-clearing round 0. Provided that the present capacity of the supplier (given by $\bar{m}_{c,T} - o_{c,T}$) is smaller than the demanded quantity, the demander continues with an equal search process in order to satisfy its demand. Besides this, the payment of the new machines is settled immediately after ordering through central bank accounts. As trading takes place in rounds, which are indicated by column 2 in the supply matrix, all supply firms with 0 orders stay within round 0. After each ordering process the supplier is deleted from round 0, i.e. the following demander in the queue draws one supplier out of the set of firms remaining in round 0. When all suppliers close one order, the mechanism steps into the next round, where the process starts from the beginning. It is obvious that a consumer goods firm that satisfies its demand is deleted from the demand vector. When the vector is empty, the process comes to halt and the trading is finished. At the end of this search process the aggregate amount of supplied (or ordered) machines $\sum_{c=1}^{C} m^s_{c,T} (= \sum_{c=1}^{C} o_{c,T})$ equates the aggregate demand of hash and bean firms $\sum_{j=1}^{J} m^d_{j,T} + \sum_{k=1}^{K} m^d_{k,T} = \sum_{v=1}^{V} m^d_{v,T}$.[79]

The described process features all characteristics of 'monopolistic competition': (i) Usually there is excess capacity in the market. (ii) Furthermore, due to the probability $\frac{1/(1+p^{machine}_{c,T})}{\sum_{\tilde{c}=1}^{C} 1/(1+p^{machine}_{\tilde{c},T})}$ a lower supply price of a capital firm c influences the sales of other firms only by a small quantity. In a simulation run the total number of consumer goods firms $V = J + K$ is larger than the total number of capital goods firms C, i.e. in all simulation runs we follow a ratio of $\frac{V}{C} = \frac{600}{400}$. Thus, after one round in the search and trading process at least 1/3 of the demanders (i.e. $\frac{200}{600}$) are left.[80] As a result, in the following second round about 50% of the capital firms (i.e. $\frac{200}{400}$) receive a second order, whereby the remainder do not. In some cases, a third round is necessary, due to some backlog of orders. Nevertheless, the third round is always the last.[81] With a lower supply price, the probability to belong to the 'lucky half' that obtains one more order, is higher. Consequently, low market prices

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[79] Only in some rare cases (without overall rationing) the total number of ordered machines is smaller than the aggregate demand for machines, even though the aggregate capacity of the capital sector would be large enough to deliver all demanded machines. In this case the demand of some consumer firms is not meet, due to the allocation process. This goes back to the number of rounds, which must be defined in advance. In this rare case the unsatisfied demand is postponed into the next period.

[80] In some cases, one demander closes contracts with more than one capital firm, and therefore, more than 200 firms enter the next round.

[81] See footnote 79 for a description what happens, if three rounds do not suffice to satisfy aggregate market demand.

of some firms affect the sales of other firms to some (marginal) extend. (iii) Finally, the characteristic of 'monopolistic competition' that supply prices are larger than 'marginal costs' is satisfied, because we employ a 'mark–up' calculation. The dynamics of those 'mark–ups' is defined in the next section.

Price Mark–up Dynamics

By rearranging the production function, the variable costs VC of producing output $m_{c,T}$ through capital firm c are given by

$$VC_{c,T} = w_T L_{c,T} = w_T\left(\frac{m_{m_{c,T}}}{A_{c,T}\gamma_c}\right)^{\frac{1}{\beta_c}}.$$

Accordingly, the last period's marginal costs per output unit, $MC_{c,T-1}$, are defined as $\frac{\partial VC_{c,T-1}}{\partial m_{c,T-1}}$. This delivers

$$MC_{c,T-1} = \frac{w_{T-1}}{\beta_c}\left(\frac{1}{A_{c,T-1}\gamma_c}\right)^{\frac{1}{\beta_c}}(m_{c,T-1})^{\frac{1-\beta_c}{\beta_c}}. \tag{2.2.22}$$

Based upon this figure capital firm c can calculate the present supply price through a 'mark–up' calculation:

$$p_{c,T}^{machine} = MC_{c,T-1}(1+\mu_{c,T}).$$

Hence, the supply price of firm c in period T is given by last period's marginal costs adjusted by a time–variant 'mark–up' $\mu_{c,T}$. We assume that the mark–up evolves over time with respect to (i) the state of the business cycle in the capital market, i.e. the output gap $y_{capital,T-1}$[82], and (ii) with respect to a measure that indicates individual marginal profits (via the relation between marginal revenues and marginal costs). Insofar we employ (i) an indicator on the industry level (macro indicator) and (ii) an indicator on the firm level (micro indicator).

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[82] In section 3.1, we discuss whether the connection between mark–ups an output gaps is positive or negative according to empirical evidence. Thus, we have to define whether markups behave procyclical or countercyclical.

In order to calculate the micro indicator, we require the marginal revenues of firm c. We know from the 'monopoly theory' that the 'Amoroso–Robinson' equation describes the demand side conditions, i.e. the relationship between marginal revenues on the one hand, and price as well as 'direct price elasticity' on the other hand. The 'Amoroso–Robinson' relation for firm c is represented by

$$MR_{c,T-1} = p^{machine}_{c,T-1}(1 + \frac{1}{\varepsilon_{p,T-1}}). \tag{2.2.23}$$

Thereby, $\varepsilon_{p,T-1}$ indicates the 'elasticity' of machine demand with respect to its price $p^{maschine}$ between period $T-2$ and $T-1$. The simulation provides the necessary data to determine this 'elasticity' via sales data from period $T-2$ and $T-1$. The values of both the 'elasticity' $\varepsilon_{p,T-1}$ and the supply price in $T-1$ enable the calculation of marginal revenues $MR_{c,T-1}$. From 'monopoly theory' it is well known that firms set their prices in order to equate marginal revenues with marginal cost. Marginal costs for period $T-1$ are explicitly given by equation (2.2.22), and marginal revenues by equation (2.2.23). We define the routine for the 'mark–up' dynamics of capital firm c with respect to the macro and micro indicators in the following way:

$$\mu_{c,T} = \mu_{c,T-1}[1 + \omega^{macro}_c y_{capital,T-1} + \omega^{micro}_c(\frac{MR_{c,T-1} - MC_{c,T-1}}{MC_{c,T-1}})].$$

Again, as in case of consumer goods firms, capital goods firms do not chose an optimal supply schedule. They rather adapt their price in a rational fashion in order to approach the optimal situation where marginal revenues equate marginal costs. In addition, the macro conditions of the market are of interest as well. Lastly, we illustrate the logic of this rule by an example: Provided that the output gap of the capital sector in $T-1$ was 5% and the marginal revenues of firm c in $T-1$ were 10% larger than its marginal costs, and that the behavioral parameters of c are fixed to 0.1 (i.e. $\omega^{macro}_c \equiv \omega^{micro}_c \equiv 0.1$), the price 'mark–up' of firm c is rising by 1.5% between period $T-1$ and T.[83]

Labor Demand, Profit Figures and Dividends

Equation (2.2.20) delivers the production function of capital firm c. According to the make–to–order process in a 'monopolistic competition' environment firm c receives a quantity of orders $o_{c,T}$

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[83] This is given through $\mu_{c,T} = \mu_{c,T-1}(1 + 0.1 \times 0.05 + 0.1 \times 0.1) = 1.015\mu_{c,T-1}$.

in period T, which are produced and supplied, i.e. $m_{c,T} = m^s_{c,T} = o_{c,T}$. Consequently, it is possible to derive the labor demand of firm c:

$$L^D_{c,T} = (\frac{m_{c,T}}{A_{c,T}\gamma_c})^{\frac{1}{\beta_c}}.$$

Additionally, the total payroll of firm c is given through

$$PR_{c,T} = w_T L_{c,T}.$$

Due to the fact that capital goods firms do not employ any capital in the production process, the total payroll plus interest expenditures equals the total costs. Moreover, the wage is the same as in the consumer goods industry (see equation (2.2.15)). Finally, the profit of a capital firm c is given by

$$\Phi^{business}_{c,T} = \Phi^{economic}_{c,T} = p^{machine}_{c,T} m_{c,T} - PR_{c,T} - i_{T-1}\overline{LL}_{c,T-1}.$$

Insofar as capital is not employed in production, economic profit equals business profit. Hence, capital firm c does not pay any 'imputed capital interest' to its shareholder. Nevertheless, it generates profits, and distributes them to its owner (which is a hash or bean firm). Thereby, the distribution of profits is restricted to the fact that liabilities of capital firm c are not allowed to exceed three times the present turnover.

2.2.4 Monetary Circuit and the Central Bank

This subsection characterizes the central aspects of the monetary theory of the present model. This requires a shift from the micro perspective of the previous sections to a macro perspective. In fact, the main difference between a monetary and a real model builds upon the consequences resulting from the integration of financial assets. This leads to the notion of the 'monetary circuit'. It is therefore necessary to integrate some institutional arrangements into the model, which satisfy that money serves as (i) 'means of payment', (ii) a 'unit of account', and in particular (iii) a 'store of wealth'. In the present subsection these points are described. Firstly, we have to discuss some issues concerning definition. Secondly, the basic mechanism of the simplified 'monetary system', or the 'monetary circuit', of Agent Island is analyzed. Thirdly, we deliver arguments, why the present model belongs to the group of 'Wicksellian' monetary models, and as to how monetary transmission works. Finally, the interest rate rule applied by the central bank of Agent Island is discussed.

Definitions

The following enumeration defines the 'concept' of money in three ways:

Medium of exchange Terminologically, an economy's money is, by "traditionally usage", its tangible 'medium of exchange' (MOE)—it is an item which is generally acceptable in payment for any goods or services (McCallum, 2004). Conventionally, claims to such a primary MOE (i.e. bank deposits) are considered as part of the money stock, if they are convertible on demand. By its role as a MOE money satisfies its function as a 'means of payment' (McCallum, 2004). For example, in the U.S., Federal Reserve banknotes as well as bank deposits, convertible into banknotes, constitute the MOE.

Unit of account Next to this role, it is often emphasized that money servers also in a function role as a 'unit of account' (UOA). The MOE is a tangible object by its traditional usage, but the UOA is an intangible unit of measurement. For example, the UOA in the U.S. is the US Dollar, i.e. all nominal prices are expressed in terms of Dollars.

Medium of account In addition, one can distinguish the UOA and the 'medium of account' (MOA), because the latter is a particular commodity or commodity bundle, and the former is specified via some quantity of the latter (McCallum, 2004). Hence, the MOA is a good (or a collection of goods), some unit which is used as the base for prices quoted by sellers in the economy. In addition, sometimes the term 'numeraire' is used as a synonym for the MOA (Niehans, 1978). For example from 1901 to 1932 the MOA in the U.S. economy was gold, and the UOA was the dollar, where 0.04838 ounce of gold defined one dollar. Nowadays, the MOA for the US economy are Federal Reserve Notes (and claims to them), with the Dollar serving as the UOA.[84] From this perspective the basis for the accumulation of financial wealth is the

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[84] As a note in the margin: McCallum points out that there is no necessity for a MOE to be the MOA (McCallum, 2004).

MOA, i.e. financial assets are quoted in terms of the MOA, denominated in the UOA.

The following enumeration defines the four main types of payment systems which are imaginable. The present model of Agent Island has to belong to one of them:

Barter economy Commodities are directly exchanged without any intermediate conversion into money.

Monetary system of exchange In contrast to a 'barter system' of exchange, a 'monetary system of exchange' is one in which the majority of transactions involves money on one side (McCallum, 2004). If in such a system all monetary transactions are conducted through 'paper' or 'commodity money', we call this a 'pure currency system'. If in such a system the central bank issues 'credit money' and requires that commercial banks hold a certain fraction of this 'central bank credit money' as reserves, we call this a 'fractional reserve banking' system.

Accounting system of exchange This is a theoretical system. McCallum defines it as a non–monetary barter system:

> "[...] an accounting system of exchange is one in which there is no money but exchanges are conducted by means of signals to an accounting network, with debits and credits to the wealth accounts of buyers and sellers being effected with each exchange. ... I will classify the latter type of system (author's note: the 'accounting system of exchange') as non–monetary. In effect, an accounting system of exchange is a highly efficient form of barter."(McCallum, 2004, p.1)

Moreover, McCallum (1985) highlights that the presence of demand deposits is not decisive for a 'monetary system' (McCallum, 1985). The point of McCallum's definition is that in a 'monetary system' deposits have to be claims on tangible currency. As McCallum notes, this definition is terminological—it refers to money in the sense of a 'paper' or 'commodity currency'. We will see below that there exists another view, which defines money via a 'theory of credit'. From such a perspective, an 'accounting system' could be a 'monetary system' in its very sense. Besides this, the institutional arrangement, described by McCallum, matches the accounting system of 'Walrasian' models: It is a purely theoretical system, where one does not need an UOA, a MOA, or a MOE. In an 'accounting system of exchange', in principle, all transactions could be settled through book–keeping entries based upon real variables via a system of relative prices—as implicitly conducted in 'Walrasian' models. Nevertheless, an 'accounting system' could be a 'monetary system' as well, as explained in the next point.

Pure credit system Similar to the 'accounting system of exchange', a 'credit system' provides an institutional arrangement with a clearing system through book–keeping entries. In this case book–keeping entries have to be expressed in monetary terms. Consequently, there is the need for an UOA, a MOA, and a MOE. We call a system a 'pure credit economy', when next to this system of book–keeping entries no other payment system exists, i.e. when 'paper money' for example is absent. The money medium in such a system is credit. Such a system in its pure form does not exist in the real world, though it would be hypothetically possible. In reality, there is a drive towards such a 'cashless economy', in which all money transactions are settled through bank accounts.[85]

Lastly, we want to define a **'monetary circuit'**: It is well described by a 'monetary system' that features a set of stock–flow accounting rules (i.e. double–entry book keeping), which imply that every monetary flow comes from somewhere and goes somewhere, so that there are no 'black holes'. In the German literature, the book keeping system of flow–of–funds accounting, and the interrelation between these flows and financial stocks goes back to the seminal work of Wolfgang Stützel. He worked out that each economic transaction based upon a real economic activity, exhibits an immediate and directly opposed net flow of funds: For an individual, the balance between the incoming flow of funds (i.e. receipts) and the outgoing flow of funds (i.e. expenditures) defines the change of his stock of financial assets. On the aggregate level, the sum of all flow of funds (as well as the sum of all stocks of financial assets) is—per definition—zero in a closed system (Stützel, 1978). In such a 'closed monetary circuit', several interesting interactions occur—see again Stützel, 1978, for a comprehensive discussion of that. We will build upon those phenomena in section 2.2.5, where the macro perspective of the present model is discussed.

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[85] For a discussion of a 'cashless economy' see especially McCallum, 2004, and Woodford, 2003, for an application in the 'New Keynesian' framework.

Credit Theory of Money

For our purposes, one especially interesting approach is to define money via credit. Wicksell (1898) lays a foundation for the theory of 'credit money' by introducing a 'pure credit system'. Before Wicksell, MacLeod introduces the concept of money as a debt rather than money as a 'medium of exchange'. He rejects the idea of treating money as a commodity. According to him, "money does not represent commodities at all, but only debt, or services due, which have not yet received their equivalent in commodities" (MacLeod, 1876, II, p. 245). Ludwig von Mises identified three definitions of money. The first bases upon the fact that a physical commodity is money (the 'commodity money'). According to the second view, money is defined via legal characteristics (the 'fiat money') (Mises, 1912). The last definition builds upon the credit theory of money (the 'credit money'): "A third category may be called credit money, this being that sort of money which constitutes a claim against any physical or legal person"(Mises, 1912, I.3.24-25). Schumpeter goes one step further as he centers the theory of capitalist finance around a clearing system that features claims and debts. We will adopt this approach below. In such a system, (if at all) currency money payments constitute a special case without any fundamental importance (Schumpeter, 1954). Schumpeter states that "practically and analytically, a credit theory of money is possibly preferable to a monetary theory of credit" (Schumpeter, 1954, p. 717).[86]

The Role of Money on Agent Island

Due to the 'credit theory of money', credit money is monetized debt or claims, i.e. money is an asset to one agent, but a liability to another agent in the system. We define—in contrast to McCallum above—that intangible bank deposits constitute the MOE and the MOA within a 'monetary accounting system' where no tangible 'paper' or 'commodity currency' exists.[87] The agent economy is therefore a monetary, but cashless, economy. It is a 'pure credit system' where the central bank (the monetary authority) serves as the 'mono–bank' in a one–level banking system. In addition, we define the 'Agent dollar' (AD) to be the opted UOA of the monetary system, and furthermore, claims to the central bank (i.e. central bank demand deposits denoted in AD) define the MOA and

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[86] For a comprehensive review of the history of the 'credit theory' of money, see Bruun, 1995.

[87] In contrast to McCallum's definition of a monetary system, the deposits of the present model do not define a claim on some 'tangible paper' or 'commodity currency'.

the MOE.

It is the intrinsic character of a ‘monetary system’ that an UOA exists, based upon some MOA. Such an institutional arrangement reduces the complexity of exchange compared to a ‘barter economy’: In a world with n goods the usage of an UOA reduces the number of relative prices from $\frac{n(n-1)}{2}$ to $n-1$ (Bofinger, 2001). For example, in the present model the prices of hash or beans, as well as the wage per hour, and so on, are expressed in monetary terms. In addition, the existence of a ‘means of payment’ simplifies trade between agents, because the number of markets is reduced: Hence, markets for the exchange of hash for labor, or hash for machines, and so on, do not exist. Each transaction is based upon an exchange between real goods or services and monetary units. That is, machines are sold for money, and so on. Consequently, one aspect of the monetary system of Agent Island is to facilitate trade through a reduction of transaction costs. Certainly, this is not its main aspect. However, it should be noted that the existence of money simplifies also the design of the agent–based model somewhat: One can state that the ‘design cost’ of the model is somewhat lower, because it features only four markets where the trading of real goods and services for money takes place—instead of six markets for trading those goods without money. Money therefore simplifies the program design, just like it facilitates trade in reality. On the other hand, the integration of money produces the critical effect that some decisions are based upon monetary terms: For example, in the present model households decide about nominal consumption expenditures—and not about real consumption demand. In addition, agents save exclusively in financial assets and decide about their savings behavior—to some extend—in nominal terms. It would be rather complex to model all that decisions based upon real, or nearly real, terms. To some extend the household agents suffer therefore from ‘money illusion’.[88] This effect is supposed to become large, as inflation becomes pretty high.

The most interesting aspect is that money is considered to fulfill the function of a ‘store of value’ or a ‘value reserve’. Consequently, monetary and financial stocks emerge due to the desire to save.

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[88] We already gave a short definition of ‘money illusion’ in footnote 27. Evidence indicates that ‘money illusion’ is existing in the real world. For example, it is shown by experiments that people generally perceive a 2% cut in nominal income as unfair, but (surprisingly) see a 2% rise in nominal income where there is 4% inflation as fair, despite them being almost rational equivalents. See Shafir et al., 1997 for the empirical evidence of ‘money illusion’.

They constitute the link between present and future in the economy on Agent Island. Insofar as household agents do not directly invest in real capital, financial wealth and inflation are in particular critical for the accumulation of individual wealth. A further topic will be of interest in subsection 2.2.5, where macroeconomic issues of the model are discussed: We will see that the investigated equilibrium concept bases upon the interactions between flows of funds (induced through receipts and expenditures). The critical aspect of this equilibrium is the relation between scheduled outflows of funds (into the stock of financial assets) and scheduled inflows of funds (out of the existing stock of financial assets). However, several complex and non–trivial interactions between flow of funds (induced by real economics transactions) and financial stocks are at work in the model. We will highlight those points throughout the following paragraphs.

The Pure Credit System of Agent Island

The design of the 'monetary circuit' of Agent Island follows the idea of a 'pure credit system' introduced by Knut Wicksell (Wicksell, 1898).[89] Within this system the central bank of Agent Island features the role of the 'mono–bank'. Moreover, it is guaranteed that the 'monetary circuit' of the model is closed through a system of double–entry accounting or book keeping. Accordingly, flow–of–funds transactions are settled exclusively through the accounts of the 'mono–bank'. The general mechanism of the 'mono–bank–based monetary circuit' is in close accordance to the work of Wolfgang (Stützel, 1978): Stützel defines that a net flow–of–funds transaction is a transaction, through which the net assets (or the net debt) of an agent are manipulated. When net assets are enlarged (or net debt is reduced), we call this net non–financial sources of funds. In the opposite case, when net assets decrease (or net debt is enlarged), we call this net non–financial borrowing of funds. Such transactions change the net wealth of the agent, and therefore, its bank account. Basically, such transactions must involve the delivery of some real goods or services, viz. all net flow–of–funds transactions between two dates mirror the creation of receipts or expenditures. Finally, the difference between the stocks of financial assets (or liabilities) of two dates (i.e. the change of the account balance) mirrors the balance between the receipts and expenditures of the respective period.

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[89] Some of Wicksell's ideas as well as the theory of 'credit money' were already introduced by Henry Thornton in the beginning 18th century (Thornton, 1802).

Insofar as solely financial transactions (changing the composition of the portfolio of financial assets or liabilities, but not the stock of net assets or liabilities) are ruled out in the model (due to the fact that the model features just one class of financial assets), all model transactions fall into the class of net flow–of–funds transactions. That is, flow–of–funds transactions that are *not* net flow–of–funds transactions are ruled out. Hence, all flow–of–funds transactions fall either in the class of net non–financial sources of funds (for an agent that generates receipts) or net non–financial borrowing of funds (for an agent that generates expenditures). This perspective is in the tradition of Josef Schumpeter. He highlights that the banking system as a whole is supposed to be the 'book keeper' of the national accounts of an economy (Schumpeter, 1964). The central bank of Agent Island is in fact such a 'book keeper'. In addition, Schumpeter thinks of a 'superbank' (namely the central bank) to be the 'clearinghouse' of the banking system. It settles transactions between banks (Schumpeter, 1970). In our 'pure credit system', private banks do not exists so that the central bank agent constitutes the national clearing house for all monetary transactions (i.e. net flow–of–funds transactions) between agents. Finally, the central bank of Agent Island serves four functions: (i) The settlement of transactions via its accounts, and accordingly the funding of agents through debt, (ii) the double–entry book keeping of all flows–of–funds transactions, (iii) the accumulation of financial wealth in its accounts due to net non–financial sources of funds, and (iv) the execution of monetary policy.

The 'monetary circuit' of the artificial economy of Agent Island works as follows: At the initialization of each simulation run, the central bank opens an account for each agent, i.e. for each hash, bean, and capital firm as well as for each consumer agent. Such an account can be a credit or a deposit account. According to section 2.2.1, we endow household agents initially with financial wealth (i.e. a deposit), and firm agents with initial debt (i.e. a credit). The endowment with initial assets thereby mirrors the cumulative (overall) net non–financial sources of funds of all previous periods; and the initial debt mirrors the cumulative net non–financial borrowing of funds in the past (Stützel, 1978). For example, figure 2.7 depicts a central bank balance sheet at the beginning of any period T. It identifies that all, except one consumer agent are creditors. The opposite is true for firms. Most of them are debtors, but one is a creditor. This mirrors the fact that most firms engaged in net non–financial borrowings of funds in the past, while most consumers exhibit net non–financial sources of funds. The account of the central bank itself, its equity or financial assets account, is balanced at the beginning of each period. Thus, the central bank plays the role of a clearing house, without any assets of its own.

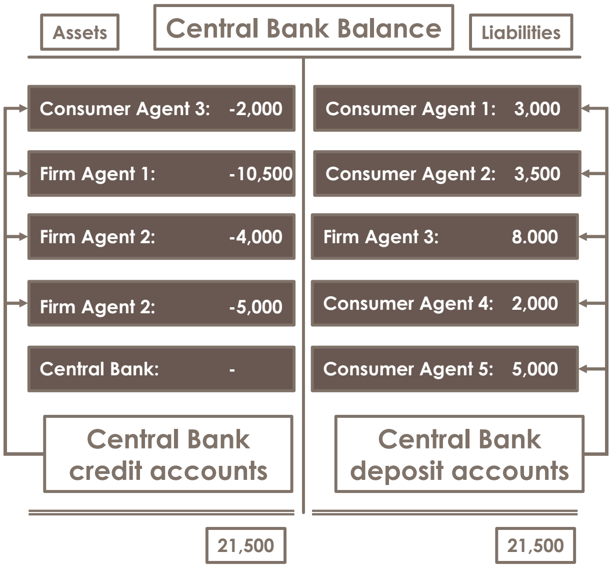

Figure 2.7: Central bank balance sheet at the beginning of period T

According to the perspective of figure 2.7, the central bank seems to be a middleman between the creditors and the debtors.[90] Consequently, the credit supply by the central bank could be constructed along the patterns of the a credit operation between two private agents (Schumpeter, 1954). In addition, Schumpeter outlines a different—a modern—perspective for the role of credit creation by the banking system:

> "It is much more realistic to say that the banks 'create credit', that is, that they create deposits in their act of lending, than to say that they lend the deposits that have been entrusted to them. And the reason for insisting on this is that depositors should not be invested with the insignia of a role which they do not have."(Schumpeter, 1954, p. 1114)

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[90] This notion is part of some older theories of bank credit. See Schumpeter, 1954, for a review of these theories.

This is an important insight into the functioning of the 'monetary circuit' within our model, and to the role of the 'mono–bank'. The point is that the 'mono–bank' creates money through credit,[91] that is, if the 'mono–bank' grants credits to a buyer of goods or services, the amount of deposits (i.e. the amount of financial assets) of the transaction counterparty is (in certain cases) enhanced immediately. Whether the creation of deposits takes place or not, depends on the net financial situation of the transaction counterparty (i.e. the seller) (Stützel, 1978): Deposits are only created, if the transaction counterparty will not pay back his debt. This depends on whether the counterparty is currently a net creditor or a net debtor. According to that, the initial granting of a credit to the buyer in a transaction creates deposits for the counterparty (i.e. enhances his net assets), when the counterparty is a net creditor. On the other hand, when the counterparty is a net debtor, the credit is finally used to repay existing debts. In this case, an initial credit does not enhance deposits in the central bank accounts. See for example, the transaction between firm agent 1 and firm agent 2 in figure 2.8 below, where capital goods are paid.

In the following, we turn back to the relation between the two counterparties of any real transaction settled through central bank accounts; we assume a transaction, where credit and deposits are increased: In this context it would be counterfactual to say that the transaction counterparty (i.e. the seller in the real transaction) funds the credit granted to the debtor (the buyer)—as suggested by the older view of credit. Even though the 'mono–bank' balance sheet points in this direction: One can argue that the immediate creation of deposits requires that both, the creation of credit and of deposits, are one and the same thing—two sides to a coin. But there is a hidden causality between credits and deposits. When a buyer in a real economic transaction is currently a net debtor, he needs an additional credit to finance the transaction in advance. Thus, the bank initially decides to create a credit. Furthermore, without the funding, the potential buyer would not carry out the real transaction. Moreover, the transaction counterparty (i.e. the seller) does not take the role of financier—he rather expects the monetary payment of the delivered goods (or services). Hence, at

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[91] Before Schumpeter, the idea that money is created through credit goes back to Albert Hahn. He introduced the idea that the deposit business of the banking system is a reflex to its lending business. Hence the deposit business 'follows' the lending business (Hahn, 1930). A connection between an 'orthodox view' and such a 'modern view' on bank credits is made by Wolfgang Stützel. Stützel points out that the deposit business need not be a reflex to the lending business in all circumstances, i.e. in some cases the deposit business is, in fact, the cause of the lending business (Stützel, 1978, p. 213 ff.).

least on Agent Island, the financing through credit has to be logically prior to the accumulation of payments by the seller.[92] In this sense, our model builds upon a credit theory of money (represented, among others, by Hahn, 1930; Schumpeter, 1954).

The most important aspect of the 'monetary circuit' in the model is the influence of expenditures on the economic activity on Agent Island: That is, each round starts with hash and bean firms scheduling labor and investment expenditures (via scheduling and producing output). Thereafter, capital goods firms and households receive receipts (more or less) passively. The receipts to the capital goods firms induce in turn labor expenditures (again via producing output). Households collect all receipts and dispose in turn of consumption goods expenditures. Finally, the state of the consumption goods market, driven by supply decision and consumption expenditures, influences the profitability of consumer goods firms and, therefore, output and investment decisions in the subsequent period. The circuit therewith starts anew. There is a permanent sequence of choosing expenditures, which constitute the receipts of other sectors; and these receipts account for the decisions of making subsequent expenditures, and so on. We will see throughout section 2.2.5, as to how the willingness to make expenditures drives the business cycle and real economic activity.

We must note that household agent behavior is especially important for the business cycle, because households (as opposed to consumption goods firms) are able to plan expenditures after receiving receipts. Thus, households are able to plan, whether they create net non–financial sources or borrowing of funds. If households lift expenditures, this can facilitate credit expansion, and the accumulation of financial assets of the complementary group. In fact, these higher expenditures propagate into higher expenditures of the complementary group in the following periods. Over the periods, this tends to increase the receipts of all agents, which in turn lifts financial savings and debts. Besides this, the willingness of firms to make expenditures has the same effect on household receipts and the accumulation of financial assets for the household sector: The accumulation of savings through households depends upon the willingness of the complementary group to make

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[92] Again, as emphasized in footnote 91, it should be noted that the creation of a credit need not—in principle—cause the creation of deposits. In some circumstances deposits cause credits (Stützel, 1978). But, insofar as the present model does not contain other financial assets than bank deposits, the explanation of Stützel (1978) does not apply to the present model.

expenditures and to produce net non–financial borrowing of funds. This willingness is driven by complex interaction effects. There is a direct interaction effect between the ability to save in financial assets and the will to make expenditures in the complementary group. But there is also the indirect effect that expenditures on their own, propagate into further expenditures of the complementary group. For example, in the short run, households can plan lower expenditures—e.g. far below their present receipts. But, over time, this restrains their ability to receive further receipts from the firm sector. Suppose a situation, where household agents plan to save about all of their receipts in financial assets. This would have markedly deflationary effects on the consumption goods market. The real output growth would become extremely negative, consumer goods prices as well as work incomes would fall, and the capital stock would supposedly decline likewise. Finally, this would lead to negative *real* savings of the whole economy (to be discussed in the next paragraph).[93] Such deflationary trends are subject to the 'Keynesian equilibrium' concept, discussed in section 2.2.5.

Importantly, one can expect that there is an impact of overall expenditures on real savings (i.e. net investments) as well. In the long run the investment decisions of firms as well as technical progress affect the real savings abilities of the economy. Real savings facilitate higher future output. The point is, if consumption expenditures (and other expenditures) are rising, the aggregate amount of tangible assets is expected to rise too: The economy 'is able' to safe more in real terms, if households (and other sectors) schedule to save less in financial assets. This 'paradox of thrift' is indeed a 'Keynesian' feature of the present model.[94,95] Moreover, we will furthermore investigate in the final chapter, how the nominal worth of the capital stock develops in comparison to the stock of debt (or financial assets). This should give us an idea as to how the real savings of an economy depend on financial savings.

Turning back to double–entry accounting by the central bank: Figure 2.8 depicts the settlement of some typical transactions throughout period T. Obviously, the settlement of transactions does

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[93] The described phenomena are aspects of the 'paradox of thrift'.

[94] The 'paradox of thrift' is a 'fallacy of composition', as financial savings are beneficial to each individual, but on the whole economy they can be harmful.

[95] See subsection 2.2.5 for a discussion of the 'Keynesian' theory of a business cycle equilibrium, and subsection 3.4.2 for simulation experiments concerning this perspective.

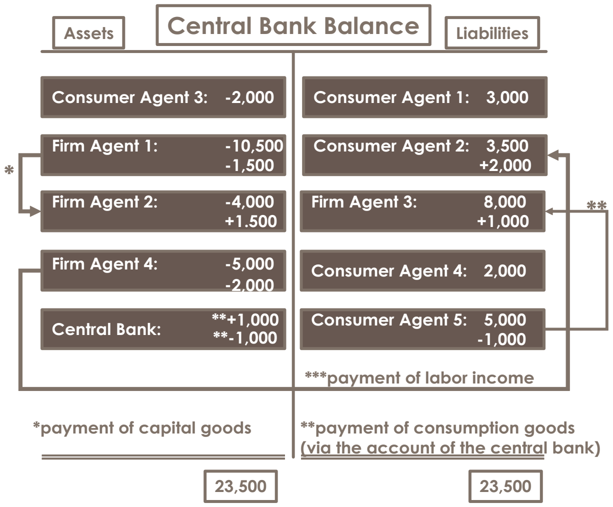

Figure 2.8: Central bank balance sheet movements during period T

not require any commodity money or currency—the payment is accomplished by transferring the appropriate sum of money in the books of the central bank. In addition, figure 2.8 depicts the process of credit (and deposit) creation: For example, within the transaction between firm agent 4 and consumer agent 2, the central bank increases the credit account of firm agent 4, so that it is able to pay labor income to consumer agent 2. Through this process—in contrast to the other transactions displayed in figure 2.8—the amount of total financial assets within the simulation rises from 21.500 AD (in figure 2.7) to $23,500$ AD (in figure 2.8). Equivalently, the amount of outstanding debts grows by the same rate. Both values, the total quantity of outstanding debts and of financial assets, are indicated by the balance sheet total.

Finally, it should be noted that for the sake of simplification some payments are distributed through the central bank: In case of consumer goods transactions (i.e. hash or bean transactions), one market price exists for all homogenous consumer goods traded within one period. Hence, it is in principle possible that buyers pay to a middleman, who distributes the sales to the respective firms. The advantage is obvious: When we insert a middleman, it is not necessary to identify the counterparties of each transaction. In fact, we do not look at any individual transaction in the consumer goods market. Consumer agents rather make a payment to the account of the central bank (who acts as a middleman), and thereafter, the central bank distributes payments to consumer firms. After all, at the end of the period, the account of the central bank itself is balanced again. For example, in figure 2.8, the sales of consumer goods between firm agent 3 and consumer agent 5 demonstrate the role of a middleman.

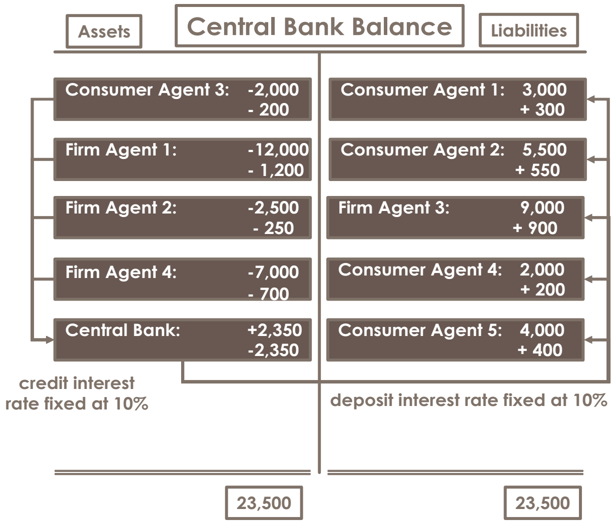

Figure 2.9: Central bank balance sheet at the end of a period T, plus over–night interest payments

See figure 2.9 for the state of the central bank balance sheet at the end of period T. Figure 2.9 shows also the over–night interest payments: Between the end of period T and the start of period $T+1$, the central bank pays interest incomes to creditors, and receives interest incomes from debtors. It would be correct to use the average values of the balance sheet entries during period T for the calculation of the interest incomes. However, as a simplification, the state of the balance sheet at the end of period T is used as the basis for the calculation of the over–night interest payments. Because the credit and deposit interest rates are equal, the central bank account has to be balanced again after these payments. Thus, at the beginning of each period the central bank account balance is 0. After all, this mechanism captures the idea of Schumpeter that the central bank is the national ‘clearing house’ of an economy.

Importantly, money is completely endogenous on Agent Island, created through book–keeping entries in the accounts of the 'mono–bank'. In principle, such a system exhibits no limit for the creation of money through credit. This is certainly one effect of the present model: In the 'pure credit system' of Agent Island, financial assets (and liabilities) can approach infinity over the long run. This could, in turn, influence inflation in the long run, provided that financial assets or the respective interest incomes pour into the 'income circuit'. We have already discussed such inflationary effects during section 2.2.1. This indicates a loose link between the present 'Wicksellian' model and the 'quantity theory of money', which shows that monetary aggregates determine inflation in the long run. Compared to reality this may constitute a rather unlikely case. However, a pretty large stock of financial assets relative to real output produces—at least—the danger of boosting inflation. The answer to the question, whether such inflation takes places or not, lies in the individual consumption behavior of agents.

Monetary Transmission on Agent Island: A Wicksellian Perspective

Economic theory captures the impact of monetary impulses on economic activity by the theory of 'monetary transmission' (Bofinger, 2001). The aim of the present agent–based model is to facilitate the discussion of monetary policy issues within a 'Wicksellian' framework, i.e. within a framework of interest rate targeting through the central bank. Wicksell outlines his notion of the origin of price dynamics as follow:[96]

> "[...] der Regulator der Geldpreise – im Gegensatz zu dem der relativen Preise – kann niemals aus den Verhältnissen des Warenmarktes (oder der Güterproduktion) selbst hervorgehen, er muss vielmehr in den Beziehungen dieses Marktes zum Geldmarkte, im weitesten Sinne des letzteren Wortes, gesucht werden."(Wicksell, 1898, p. 22)

It is a major aim of this study to integrate the 'Wicksellian' perspective into the present agent–based macroeconomic model. Before turning to the 'Wicksellian' aspects of the model, it is necessary to analyze the sources of inflation: According to the design of the consumer goods market, it is obvious that consumer goods prices are driven by consumption expenditures. In turn, consumption

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[96] *[...] money prices, as opposed to relative prices, can never be governed by the conditions of the commodity market itself (or of the production of goods); it is rather in the relations of this market to the money market, in the widest sense of the term, that it is necessary to search for the causes that regulate money prices.*

expenditures are governed by households' incomes and their savings behaviors: (i) When incomes are growing and overall savings behavior is constant, or (ii) when savings rates are declining and incomes are constant, or (iii) when saving rates are declining and incomes are growing, the consumer expenditures are consequently growing and (c.p., i.e. given a constant supply of goods) consumer goods prices are rising. That is the direct source of inflation.

Equally important, consumer goods firms lift output (in accordance to the 'Marshallian' supply rule), if last period's marginal profits were positive. This process counteracts inflationary pressure. But, to complicate the inflation dynamics even further, such an increase of production enhances labor income of workers, which in turn tends to result in rising consumption expenditures and (c.p.) rising inflation. Finally, it should be mentioned that inflation is self–energizing, because of the 'wage–price spiral'. Hence, the present model features a 'feedback loop' due to the wage setting rule:[97] According to this, the nominal wage grows with inflation plus the output gap times a weighting parameter λ. A positive inflation induces therefore a higher wage in the next period, which in turn induce c.p. higher demand and inflation, and so on. Consequently, inflation is self–energizing.

One main question is, as to how can the artificial economy of Agent Island be aligned to Wicksell's notion of price–level dynamics? In general, the model exhibits two main 'channels of monetary transmission': (i) an 'investment–channel', and (ii) a 'savings–channel'. The direction of the 'savings–channel' is theoretically and empirically ambiguous. Moreover, the sensitivity analysis in the next chapter puts the impact of savings behavior (via savings rates) on inflation and output gaps into question. Insofar, we do not discuss this channel further. According to that, monetary transmission has to build upon the 'investment–channel', governed by a 'Wicksellian' mechanism. As explained, the dynamics of the general price level in Wicksell's 'cumulative process' depends on the relative level of the credit interest rate to the 'natural rate': Is the former lower than the latter, the general price level is rising, and vice versa. If the unlikely case appears that both rates coincide for a while, the 'cumulative process' comes to halt so that the price level rests at the present point. Basically, in a 'pure credit economy' there is no limit to how long the 'cumulative process'

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[97] This mechanism is captured by the 'expectations–channel' of monetary policy. It will be discussed in section 2.2.5.

can continue, insofar as the ‘mono–bank’ can keep the credit rate below or above the ‘natural rate’ in perpetuity (Wicksell, 1898).[98]

Wicksell operates with an underlying ‘real world’ that is not disturbed by monetary magnitudes, that is, the ‘natural rate’ of interest is independent of the monetary sphere. We try to integrate this concept by the return on assets. Its value relative to the credit interest rate determines one condition of a net investment. Therefore, the credit interest rate is for each hash or bean firm a ‘reference point’ in the investment decision. Thereby, Wicksell bases his concept of the ‘natural rate’ along the lines of Eugen von Böhm–Bawerk. According to this view, the interest rate has its source in the ‘roundabout process’ (Wicksell, 1898; Böhm-Bawerk, 1921). Wicksell defines the upper bound of the natural rate in the following way:[99]

> “Es ist dies der Ueberschuss, welchen die Produktion (oder der Erlös derselben in anderen Waren) über die Summe der ausgezahlten Löhne, Grundrenten u.s.f. zu liefern vermag.”(Wicksell, 1898, p. 95)

Against this background, we are convinced that the return on assets is a good proxy for the natural rate, because it captures the yield of the ‘roundabout process’, as described by Wicksell and Böhm–Bawerk. The return on assets, as defined previously in this chapter, delivers the ratio between the EBIT and the assets of a firm agent. Thereby, the EBIT matches the notion described by Wicksell. According to that, the comparison of the return on assets and the credit interest rate delivers a result comparable to Wicksell’s theory.

In the end, the impact of a restrictive interest rate impulse through the ‘investment–channel’ is obvious: If the central bank lifts the credit interest rate above the average rate of return in the firm sector, expansions investment for many firms is prevented. If the central bank sets the rate far above the average rate of return, expansion investment in almost every firm is prevented. Lower investment demand in turn reduces the capacity utilization in the capital goods industry, what impairs labor

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[98] In a ‘currency system’, or in a ‘credit system’ with restrictions on the volume of credit creation through the central bank, there is a limit as to how long the two rates can differ, because there are limits to the credit creation abilities of the banking system.

[99] *It is the surplus that production (or the revenues of production quoted in other goods) over the sum of paid out wages, economic rents and so on, may deliver.*

incomes and c.p. the consumption expenditures of workers employed in this industry. This in turn influences the incomes generated in the consumer goods industry. Finally, inflationary pressure is reduced and the output gaps of both, the consumer goods and capital goods industry, should decline. For a positive monetary impulse, the opposite effect arises. In addition, it is an important feature of the model that the market for investment goods is, among other things, amplified by an 'investment accelerator', producing another 'feedback loop', which should enlarge the impact of monetary policy additionally.[100]

Notes on Real Interest Rate Targeting

'Classical' and 'neoclassical' models of the economic process were in all essentials barter models. As Schumpeter states in his posthumous 'History of Economic Analysis':

> "Practically all the most valuable work of the period [author's note: 1870 to 1914]—so far as it was not concerned with specifically monetary problems—was Real Analysis, even where it expressed its concepts in terms of money."(Schumpeter, 1954, p. 1088)

According to the 'neoclassical' perspective, money is 'neutral', or, 'money is a veil'.[101] As natural, modern GE models stand in the 'neoclassical' tradition: The real interest rate is determined by the intertemporal preferences of individuals, and by the 'marginal productivity of capital'. It expresses the exchange ratio of present and future goods and services—it is therefore the price that clears the market for intertemporal exchange. It is a straightforward consequence of this perspective that a central bank can control nominal rates and inflation—but cannot control real rates or any other real variables. To circumvent this peculiarity of GE models, the 'New Keynesian' framework tackles price rigidities. Through the introduction of such nominal rigidities, monetary policy exhibits *short–term* real effects and therefore an impact on real interest rates as well.[102] Against this background, the question, as to how the central bank on Agent Island controls real interest rates, is a serious topic. In this respect, 'orthodox' economists could 'attack' the present model, because it does not build explicitly on nominal rigidities. We can counter this potential criticism in the following ways:

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[100] See at the end of subsection 2.2.2 for a short discussion of this topic.

[101] See Schumpeter, 1954 or Green, 1992, for a discussion of this point. As an example, the 'quantity theory of money' is based upon on the assumption of the 'neutrality of money' (Bofinger, 2001).

[102] See Woodford, 2003, for a comprehensive discussion of that point within the 'New Keynesian' theory.

In Wicksell's rule nominal interest rates matter as well At first, we want to emphasize that the control of investment demand in the model does not require the control of real interest rates by the central bank agent. Firms compare their nominal return on assets with the nominal interest rate, because this delivers a good approximation for the comparison of the respective real rates. This is especially true, if inflation rates are quite low. Hence, it is adequate for the central bank to lift *nominal* credit rates above the *nominal* return on assets.

Matter of programming In an agent–based model based upon discrete periods it is necessary to construct the time schedule of a period in detail. According to our approach, the central bank fixes the nominal credit rate at the beginning of each period—whereas goods market trading takes place throughout the period. At the end of each period, after all transactions are conducted, the period inflation rate, individual profits and yields are calculated. Consequently, there exists no direct (or mechanical) connection between (i) the nominal credit interest rate, (ii) inflation, and (iii) the real return on assets, as in GE models.[103] The main reason for an absence of a mechanical connection is the fact that the decisions and actions delivering those figures do not take place at the same time—they are separated from one another. For example, (i) is determined by an interest rate rule of the central bank, (ii) is determined through market–clearing in the consumption goods market, and (iii) is determined by individual profits of heterogenous firms (or corporate groups). Thus, the real credit interest rate deviates from the 'marginal productivity of capital', or—as suggested by our 'Wicksellian' approach—from the individual real return on assets.

Timing and information This original notion of Wicksell is similar to the last point. According to his dynamic method of analysis, the decisions of individuals do not happen at any given moment in time with full information. For example, in Wicksell's model the following aspects emerge: When workers decide on their labor supply, they cannot know what their real wage will be; when banks decide on the money rate of interest, they cannot know what the real rate of interest will be. If they did, if everything took place in any given moment with full information, it would not be possible for entrepreneurs to gain profit (Wicksell, 1898). The present agent–based model reproduces this situation described by Wicksell properly: (i) The

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[103] In fact, the central bank connects these figures—to some extend—through its interest rate rule.

decisions of agents do not take place in one logical moment; (ii) there is rather a sequence of decisions; and (iii) agents do not possess perfect information—rather they exhibit imperfect information.

Monetary economy The most important point is highlighted by Knut Wicksell himself: "Interest on money and profit on capital are not the same thing, nor are they immediately connected with each other; if they were, they could not differ at all, or could not differ a certain amount at every time. There is no doubt some connecting link between them, but the proper nature and extent of this connection is not very easy to define" (Wicksell, 1907, W. 6).[104] The central reason for a possible deviation of the interest on money (credit interest rate) and the interest on capital (return on assets) lies in the nature of the monetary system. The following distinctions clarify: (i) A 'perfect barter economy' would correspond with the results of 'neoclassical' analysis, where the credit interest rate and natural interest rate coincide. (ii) In a 'monetary system' without banks as financial intermediaries, where credit transactions between individuals take place, both rates would obviously be connected. A lender tends to receive a yield that corresponds to the profit of investment—after allowance for risk, and so on (Wicksell, 1907). (iii) In a 'fractional reserve system', i.e. a monetary system as known form reality, there is surely a connection between both rates. But this connections is neither direct nor obvious. (iv) In a 'pure credit economy', as outlined in the present model, there is, in principle, no direct connection between the yield of real capital and the credit interest rate. Accordingly, the 'mono–bank' can set the latter (in nominal or real terms) wherever preferred—without any *necessary* linkage to the profitability of firms. As a result, the traditional approach of GE models (i) and our approach (iv) can be treated as 'corner solutions'. In case (i), the central bank has no influence on the real interest rates, whereas in case (iv) the central bank exhibits nearly perfect control on real credit interest rates. The latter is only restricted by the insecure inflation of the present period, i.e. by inflation expectations. Of course, the mechanism working in reality lies somewhere between both 'corner solutions'.

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[104] It should be noted that in GE approaches the savings decision and the investment decision are literally the same issue. Due to the fact that such models are regularly real models, this approach is correct. But in a 'monetary system' the situation changes, and the savings decision of households (in financial assets) and the investment decision of firms (in real assets) are not the same decision—they are separated. The linkage between both is constituted through financial intermediation.

Interest Rate Rule

It is a familiar fact that the real–world interest rate policy can be well approximated through a 'Taylor rule'.[105] Therefore, we opt for a 'Taylor rule' as a suitable and easy to compute interest rate routine for the central bank agent:

$$r^*_T = \bar{r}_{T-1} + \tau(\mathbb{E}\pi_T - \pi^*) + \varsigma\mathbb{E}y_T.$$

The 'Taylor rule' indicates that if both the expected output gap $\mathbb{E}y_T$ as well as the expected inflation gap $(\mathbb{E}\pi_T - \pi^*)$ are 0%, the central bank sets the real credit interest rate equal to the 'neutral' real credit interest rate. The latter is usually computed through a historical average, i.e. by $\bar{r}_{T-1}$. According to the discussion of the 'Wicksellian' transmission process above, it would be an option to use the historical average real return on assets (as an average over all consumer goods firms). Insofar as we try to validate and adjust the model to real–world behavior, we prefer the average real credit interest rate. Moreover, the expected values are determined by last period's magnitudes, i.e. $\mathbb{E}\pi_T \equiv \pi_{T-1}$ and $\mathbb{E}y_T \equiv Ey_{T-1}$. In case of inflation or output missing their optimum values, the real interest rate reacts to those deviations weighted by τ and ς:[106] In case of positive output or inflation gaps, the central bank sets the real interest rate above its historical average value. In a next step we express the 'Taylor rule' in nominal terms by adding expected inflation rates on both sides:

$$i^*_T = r^*_T + \mathbb{E}\pi_T = \mathbb{E}\pi_T + (\bar{r}_{T-1} - \tau\pi^*) + \tau\mathbb{E}\pi_T + \varsigma\mathbb{E}y_T. \tag{2.2.24}$$

As a second routine, we apply an interest rate 'smoothing rule'. Since the early 1990s it is observed that central banks tend to move policy interest rates in small steps. This leads to inertia in the policy instrument (Rudebusch, 2002). Accordingly, the actual interest rate r_T adjusts to the the target interest rate r^*_T according to the following 'smoothing rule':

$$r_T \quad = \quad (1-\rho)r^*_T + \rho r_{T-1}.$$

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[105] See the original study by John B. Taylor (Taylor, 1993). For a discussion of the 'Taylor rule' see Bofinger, 2001. A review of several estimations of various 'Taylor rule' types for the Euro area is given by Sauer and Sturm, 2007.

[106] The original 'Taylor rule' assumes equal weights that add to one, i.e. $\tau = \varsigma = 0.5$. See Taylor, 1993.

Or, more suitable, expressed in nominal terms:

$$i_T = r_T + \mathbb{E}\pi_T = (1-\rho)r_T^* + \rho r_{T-1} + \mathbb{E}\pi_T. \tag{2.2.25}$$

Thereby the parameter ρ is the 'smoothing parameter', i.e. the higher ρ, the higher is inertia of credit interest rates. It should be noted that the magnitude of smoothing naturally decreases with the investigated time frame, viz.: ρ is larger in monthly data compared to quarterly data. Insofar as our approach comprises periods of one year length, the level of ρ has to be significantly smaller than those values identified with monthly or quarterly data. Finally, we plug equation (2.2.25) into the 'Taylor rule' (equation (2.2.24)), which delivers the following interest rate (reaction) function:

$$\begin{aligned} i_T &= (1-\rho)(\tau\mathbb{E}\pi_T + \varsigma\mathbb{E}y_T) + (1-\rho)(\bar{r}_{T-1} - \tau\pi^*) + \rho r_{T-1} + \mathbb{E}\pi_T, \\ i_T &\geq 0. \end{aligned} \tag{2.2.26}$$

Inequation (2.2.26) defines the zero lower bound restriction of monetary policy: In case of strong deflationary pressure, the central bank is not able to react, because nominal interest rates are already at zero–level. Subsequently, real interest rates are expected to rise, instead of falling. As we will see in the final chapter, this is a serious problem in our model.

Creditworthiness of Firms

As a last topic in this subsection we have to discuss the creditworthiness of firms: Normally, banks analyze the creditworthiness of potential debtors and, occasionally, deny agents with bad creditworthiness (or bad credit rating) a new credit. In our simplified framework the central bank agent does not run a creditworthiness–check; in fact, it grants credit to each demand. However, this would normally (i.e. in reality) induce problems of loan default or credit loss. For this reason, some firms (or consumers) can accumulate outstanding amounts of debt, which is first of all a rather unreasonable assumption. This became apparent during 'face validation' runs.

It is one aim of our approach to model the economic process as simple as possible. Nevertheless, the model becomes more complicated step by step. As a last step, it would be possible to incorporate credit supply restrictions based upon profit figures of firms[107] or income prospects of consumers[108]. But we do not follow such a modeling because of the already rather complicated design, and the resulting problems in the 'validation' process: An additional restriction would further complicate the named 'validation' procedure, and it may induce additional deflationary pressure. As we will see in the next chapter, the monitoring of deflation on Agent Island becomes the main topic of the final 'validation'. Consequently, we decide on a pretty simple approach. This mechanism characterizes the over–indebtedness of consumer goods firms. As explained, consumer goods firms are owned by households, and these owners collect dividend payments from the firms. In case of over–indebtedness of a hash firm j (so that equity becomes negative: $E_{j,T} < 0$), the owners of j are obliged to make a subsequent payment to firm j. Hence, shareholders hold 'assessable stocks'. Such stocks require subsequent payments of all shareholders of firm j defined through $(0 - E_{j,T})$, when equity is negative $(E_{j,T} < 0)$. Technically, this payment is treated as a negative 'dividend'. Accordingly, dividend incomes of the respective households can become negative—see the non-consumption expenditures of household h $(Ex_{h,T})$, in equation (2.2.2).

2.2.5 Macro Perspective

This subsection summarizes several macroeconomic topics concerning the model and its 'validation' process. We start with the review of the main decisions and actions of the model from an aggregate perspective, and thereafter, with the definition of some aggregate variables. In a next step, we analyze the flow–of–funds accounting within the 'monetary circuit' of Agent Island. Thereafter, the equilibrium concept of the model is illustrated, and finally, we mention the 'expectations–channel' of monetary transmission, which is assumed to work in the model as well.

Review of Decision and Actions on the Aggregate Level

The following chronological review focuses the decisions and actions on Agent Island from a macroeconomic perspective:

1. **Start new round**

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[107] This would incorporate the 'balance sheet channel' of monetary transmission. See Bofinger, 2001.

[108] In fact, we apply such a credit restriction for households. See the discussion of the intertemporal budget constraint of households in section 2.2.1.

2. **Central bank:** The central bank sets the credit interest rate according to the last period's inflation and output gap as well as the smoothing of credit interest rates. All following payments are settled through central bank accounts.

3. **Labor union and employer association:** Overall wage growth is bargained. This is based upon inflation and output gap expectations, given according to their last period's figures.

4. **Consumer goods firms:** They schedule and produce the supplied quantities, delivered by an adaptive rule based upon last period's marginal profits. After individual decisions are made, the individual outputs add up to the aggregate quantity of the supplied consumer goods. Accordingly, the labor market opens for workers employed in the capital goods industry; work incomes are generated and paid.

5. **Consumer goods firms:** The firms decide individual investment demand based upon (i) the depreciation of the capital stock, (ii) the comparison of last period's average machines price and the 'shadow machine values', (iii) the comparison of the capacity utilization and its trigger level, (iv) the comparison of the individual return on assets and the credit interest rate, and (v) the expansion rate of the capital stock. Individual quantities of demanded machines add up to the aggregate demand vector.

6. **Capital goods firms:** The individual growth rate of price 'mark–up' is driven by micro and macro indicators. The micro indicator is delivered through last period's marginal profits; the macro indicator through last period's output gap of the capital goods industry. The firms decide on individual machine supply prices (due to a 'mark–up pricing'). The aggregate supply matrix is generated.

7. **Capital market clearing:** The capital market opens. The 'make–to–order production' is conducted by a random, round–based search model encompassing 'face–to–face trading'. The random search process is driven by individual supply prices and by chance. The individually demanded quantities are ordered and paid.

8. **Capital goods firms:** Firms produce the ordered quantities of machines. Accordingly, the labor market opens for workers employed in the capital goods industry. Machines are delivered.

Effectively, machines enlarge the capital stock of consumer goods firms in the next period. Labor incomes are generated and paid.

9. **Households:** Households collect their incomes (i.e. work income, (over–night) interest and dividend incomes). The agents compute their disposable period income. The individual savings rates are manipulated through (i) income growth and (ii) the real interest rate gap. Thereafter, households split disposable incomes into consumption expenditures and financial savings via savings rates. Besides this, upper and lower bounds for consumption expenditures are applied. Finally, the aggregate expenditures for hash and beans are computed, which are given through the aggregation of individual consumption expenditures.
10. **Consumer goods market:** The consumer goods market opens. A market clearing on the aggregate level takes place, viz. aggregate nominal consumption expenditures are confronted with aggregate supplied quantities of hash and beans. Through a 'pricing–to–market' mechanism, the equilibrium prices are computed. Thereafter, the supplied quantities are delivered to households, and sales are paid to consumer goods firms.
11. **All firms:** All markets are closed. The firms calculate their profit figures. Each capital firm distributes its profit to the parent company. Each consumer goods firm calculates its return on assets. The distribution of dividends to shareholders is determined.
12. **World:** Aggregate measures (inflation rates, output gaps, GDP, and so on) are calculated.
13. **All private agents & central bank:** The central bank calculates the interest receipts and expenditures via the period credit interest rate and individual account balances. Interest and dividend incomes are paid.
14. **End round**

Calculation of some Main Macro Indicators

In the next step, we illustrate the calculation of inflation rates and output gaps. Inflation rates are calculated via the price index for a basket of consumer goods. This basket contains *one* unit of hash and *one* unit of bean throughout the whole simulation. In the initial period of each simulation run, the price of the basket is calculated by

$$p_0^{basket} = 1 \times p_0^{hash} + 1 \times p_0^{bean}.$$

The price of the basket in any period T is calculated accordingly, with the quantities still fixed to 1. The 'Laspeyres price index'[109] in any period T is given through

$$LI_T = \frac{p_T^{basket}}{p_0^{basket}}.$$

The growth rate of the price index between to dates $T-1$ and T delivers the consumer price inflation in T:

$$\pi_T = \frac{LI_T - LI_{T-1}}{LI_{T-1}}.$$

The price index serves also as the GDP deflator. Moreover, we calculate the inflation of capital goods as well. Here, we take as an approximation the average price of one machine in period $T-1$ and in period T. The growth rate of $\hat{p}^{machine}$ indicates the capital goods inflation. Finally, we use output gap figures throughout this study. The output gap is normally given through the deviation of the GDP from its potential level, viz.

$$y_T = \frac{Y_T - Y_T^{pot}}{Y_T^{pot}}.$$

The difficulty lies in the determination of the potential GDP. This figure is often calculated through historical trends and their extrapolation. We apply, in contrast, a straightforward approach. First, we calculate the capacity utilization of an industry. That is, the capacity utilization of the consumer goods industry (i.e. $l_{consumer,T}$) is given through

$$l_{consumer,T} = \frac{\sum_{j=1}^{J} h_{j,T} + \sum_{k=1}^{K} b_{k,T}}{\sum_{j=1}^{J} \check{h}_{j,T} + \sum_{k=1}^{K} \check{b}_{k,T}}.$$

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[109] The 'Laspeyres index' is based upon the quantities of the base period. Because the basket of goods is unchanged during the simulation, we have to apply the 'Laspeyres index'.

Thereby, the variables in the denominator ($\check{h}_{j,T}$ and $\check{b}_{k,T}$) give the capacity limits of hash firm j and bean firm k. In a second step, we calculate the potential capacity utilization of the consumer goods industry through the historical (arithmetic) average capacity utilization, i.e. $\bar{l}_{consumer,T}$. The subscript '$consumer'$' stands for the consumer goods industry (and '$capital'$' for the capital goods industry). Hence, the output gap of the consumer goods industry is delivered by

$$y_{consumer,T} = \frac{l_{consumer,T} - \bar{l}_{consumer,T}}{\bar{l}_{consumer,T}}.$$

The capital goods industry output gap $y_{capital,T}$ is delivered by the same procedure. Basically, the total output gap of the economy is given by averaging both output gaps:

$$y_T = \frac{J+K}{J+K+C} y_{consumer,T} + \frac{C}{J+K+C} y_{capital,T}.$$

Finally, the nominal GDP in period T is given by $Y_T = C_T + I_T$, which is the sum total of all (nominal) consumption expenditures plus the sum total of all (nominal) investment goods expenditures. By deflating these nominal figures through the price index LI_T, one obtains the respective real levels for Y, C and I. Moreover, one can calculate the growth rates for nominal and real Y, C, and I. The descriptions so far should only summarize the main macro indicators of the model. In principle, every aggregate or disaggregate figure, based upon the individual variables incorporated into the model, could be calculated. In fact, the model developed in SeSAm contains many more aggregate figures, than are described here.

Flow–of–Funds Accounting

In the next step, we investigate the 'monetary circuit' of Agent Island. We know that every monetary flow comes from somewhere and goes somewhere so that there are no 'black holes' in the model. The present model features a basic set of accounting rules, which satisfy such a consistency of monetary stocks and flows.[110,111] Table 2.4 illustrates the 'flow–of funds–accounts' of the present model (on

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[110] Nevertheless, the model also features the consistency of stocks and flows of real assets. This is especially relevant for the specification of the 'asset ledger' of consumer goods firms.

[111] One can find the framework of stock–flow consistency especially in the 'Post Keynesian theory'. For example, see Lavoie, 2008, or Dos Santos, 2006.

the sectoral level). The variables in a column indicate the inflows of funds into the respective sector (i.e. according to receipts). The variables in a row indicate the outflow of funds (i.e. according to expenditures). Accordingly, the last cell in a column adds up all inflows of funds of a sector, and the last cell in a row adds up all outflows. The balance between the last cell in the column of a sector and the last cell in the respective row of this sector defines the change in financial wealth of the sector (i.e. the net non–financial sources or borrowing of funds). Insofar as the financial wealth of agents is accumulated exclusively in central bank accounts, such a balance (i.e. between the sum of a column and the sum of a row for one sector) is mirrored by the change of the central bank accounts of all agents in a sector between two dates. In the case of the central bank, this balance is per definition always 0.

inflows to→ outflows from ↓	Consumer goods ind.	Capital goods ind.	Private households	Central bank	Sum
Consumer goods ind.		I	$Y^{L,C}$ (Y^D)	(Y^I)	$\sum$Expenditures
Capital goods ind.	$\Phi^{business}$		Y^L	(Y^I)	$\sum$Expenditures
Private households	C (Y^D)			(Y^I)	$\sum$Expenditures
Central bank	(Y^I)	(Y^I)	(Y^I)		$\sum$Expenditures
Sum	$\sum$Receipts	$\sum$Receipts	$\sum$Receipts	$\sum$Receipts	0

Note: The head of a column indicates the sector that receives inflows of funds. The rows describe the outflows of funds of each sector. The value in each cell is the sum over all flow of funds in this sector (for columns) or out of this sector (for rows). In case of households' dividend income Y^D or interest income Y^I of any sector, we illustrate the effect that both can represent receipts or expenditures through brackets.

Table 2.4: Flow–of–funds accounting matrix on the sectoral level

Such an illustration is in the tradition of the work of Wolfgang Stützel. In accordance to Stützel (Stützel, 1978), our analysis builds upon the flow of funds, and not on income flows, as is usually done in national accounting. National accounting is based upon income generation and income streams. Thereby, incomes are defined as the sum of consumption and wealth growth. Wealth growth is, in turn, defined through the growth of financial assets plus the growth of tangible (i.e. real) assets (Stützel, 1978).[112] The point is that the former is naturally expressed in nominal terms, whereby the latter is expressed in real terms, so that one has to transform it to nominal terms.

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[112] We will use this expression below, where we define the aggregate profits of the firm sector.

But this requires a general accepted rule for the ‘valuation of tangible assets’—which is definitely not existing. Hence, the definition of the ‘flow–of–funds accounting’ rules is clear–cut; whereas the definition of income streams is to some extend based upon additional assumptions (Stützel, 1978). Copeland emphasizes this point:

> “Moneyflows accounting patterns are especially well adapted for analyzing business conditions [...] National income patterns include various accrual and imputed items. Many of these represent transactions a transactor enters into with himself; group upon another such items are not clearly relevant.” (Copeland, 1952, p. 34)

This view is likewise facilitated by Wassily Leontief, who characterizes the accounting of receipts and expenditures and its balances as “the entire balance of trade of the individual enterprise (or household)” (Leontieff, 1951, p. 12). We therefore build our analysis upon such balances of trade on the individual levels. Equally important, the accounting of flow of funds lays down the foundation for a stock–flow–consistent model. It constitutes the connection between flow of funds and financial stocks (or between money flows and money stocks). Consequently, and importantly, we concentrate on the analysis of ‘flow–of–funds accounting’—rather than on income streams. This approach seems clear–cut; moreover, it benefits the investigation of a monetary macroeconomic model, such as that of Agent Island.

Importantly, one is able to connect the ‘flow–of–funds accounting’ and the ‘income accounting’ system. This is one key to the understand the mechanism of the present model. Again, Stützel derives the link between both systems through the following system of equations (Stützel, 1978):

These equations account for the fact that individual savings of agent y (i.e. $S_{y,T}$) can differ from the individual change in tangible assets ($I_{y,T}$) via the net non–financial sources (or borrowing) of funds ($\Delta FA_{y,T}$). But if we sum up all agents Y (with $Y = H + J + K + C$), total savings are equal to the change in tangible assets (i.e. net investments). This gives the well–known ex post identity of national income accounting in a closed economy: $S = I$ (see the last line in the system of equations above). The same idea is represented by the aggregate identity of ‘flow–of–founds accounting’. Namely, that the sum total of changes in financial wealth has to be 0 within a closed economy. The

$$\begin{array}{ccccccc}
\multicolumn{5}{l}{\text{Individual savings (as a change in wealth)}} & & \begin{array}{c}\text{Individual change in tangible assets}\\ \text{(as net investments)}\end{array}\\
S_{1,T} & = & I_{1,T} & + & \Delta FA_{1,T} & & I_{1,T}\\
S_{2,T} & = & I_{2,T} & + & \Delta FA_{2,T} & & I_{2,T}\\
\vdots & = & & & & & \vdots\\
S_{Y,T} & = & I_{Y,T} & + & \Delta FA_{Y,T} & & I_{Y,T}\\
\hline
\sum_{y=1}^{Y} S_{y,T} & = & \sum_{y=1}^{Y} I_{y,T} & + & \sum_{y=1}^{Y} \Delta FA_{y,T}(=0) & = & \sum_{y=1}^{Y} I_{y,T}\\
S_T & = & I_T & & & = & I_T
\end{array}$$

link between both identities is thereby also clear–cut: When the sum total of all net flow–of–funds transactions is 0, total savings equate *inevitably* total net investments (Stützel, 1978). This is a key insight for our analysis.

Finally, this mechanism leads us to the 'Keynesian' aggregate business profit equation (Stützel, 1978): Aggregate ex post profits (net income) of the firm sector (i.e. $\Phi_{firm,T}^{Business}$) are defined through the sum of (i) net investments of the firm sector, (ii) transfers to other sectors (i.e. consumption of the firm sector), and (iii) the net non–financial sources (borrowing) of funds by the firm sector as a whole. This definition delivers the change of net worth of the firm sector as a whole. Besides this, we apply the fact that the net non–financial sources (or borrowing) of funds $\Delta FA_{firm,T}$ of the firm sector have to be *necessarily* as large as the net non–financial borrowing (sources) of funds of the household sector (i.e. of the complementary group) $\Delta FA_{house,T}$. This fact is given through the 'flow–of–funds accounting' identity (for the Agent Island economy):

$$\begin{aligned}
\Delta FA_T = \Delta FA_{firm,T} \quad &+ \quad \Delta FA_{house,T} \equiv 0,\\
\Delta FA_{firm,T} \quad &= \quad -\Delta FA_{house,T}.
\end{aligned}$$

In addition, the transfers of the firm sector to the household sector are defined through the dividend payments from consumer goods firms to the household sector in the last 'night'; this gives

$Y_T^D = \sum_{h=1}^{H} Y_{h,T}^D$.[113] Finally, this delivers the aggregate ex post firm profits on Agent Island:

$$\Phi_{firm,T} = I_T + Y_T^D - \Delta FA_{house,T}.$$

We will use this equation at the end of chapter 3. It illustrates the 'Keynesian' notion, already described (above) within the analysis of the 'monetary circuit': The willingness of the household sector to make expenditures, possibly above its own receipts (i.e. $\Delta FA_{house,T} < 0$), enlarges the profitability of the firm sector as a whole. This enhances, in turn, investment expenditures of consumer goods firms, which determine real savings capabilities of the economy as a whole. Consequently, the intention of the household sector to produce net non–financial borrowing of funds (i.e. the intention of financial 'dissavings'), improves aggregate *real* savings of the economy.[114] It is this 'Keynesian paradox of thrift' that drives the business cycle on Agent Island—but it is in strong contrast to the 'neoclassical' perspective that savings lifts investment *mechanically.*

Keynesian Business Cycle Equilibrium

As initially stated in this study, the present agent–based model is not constrained by an exogenous equilibrium concept—as GE models are. If there is a macroeconomic equilibrium, this emerges bottom–up, out of individual interactions. Hence, Agent Island may be in some kind of equilibrium, or in disequilibrium. Insofar as the model is designed as a 'Keynesian' business cycle model within an agent–based environment, we investigate the 'Keynesian business cycle equilibrium', or its disequilibrium states. The latter are the well–known 'inflationary' or 'deflationary gaps'. We are convinced that the model should feature these concepts.

However, against the background of the described mechanism of the 'monetary circuit' (and its virtues compared to the 'income circuit'), it seems reasonable to build the 'business cycle equilibrium' theory upon the 'monetary circuit' (and not upon the 'income circuit'). As a result, we use the 'flow–of–funds accounting' system instead of the 'income accounting' system. Again, Wolfgang Stützel

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[113] By definition, the 'over–night' dividend payments belong, in fact, to the following period.

[114] In reality, this perspective is broadened as the government sector and the foreign sector are present. Both could deliver additional sources for firm profits.

describes, as to how the *scheduled* flow–of–funds transactions affect the business cycle behavior and its dynamics:[115]

> "Das Problem der Konjunkturtheorie würde z.B. auftauchen, sobald wir untersuchen, wie sich Gesamtausgaben wohl verändern, wenn in der Gesamtwirtschaft die Summe der einzelnen *Pläne*, Geldvermögen zu bilden, kleiner ist als die Summe der *Pläne*, Geldvermögen zu verringern, also Ausgabenüberschüsse zu bilden. [...] Allgemein sei hier nur festgestellt: *Tendenzen* zur Divergenz zwischen Angebot und Nachfrage nach *Geldvermögen* sind ein Problem der Konjunkturtheorie, führen zu Veränderungen der Gesamtausgaben/Periode." (Stützel, 1978, p. 82–83)

This description of Stüzel translates the notion of a 'Keynesian inflationary gap' from 'income accounting' into 'flow–of–funds accounting'. Hence, it illustrates exactly the same situation, where scheduled savings are smaller than scheduled net investments ($S^P < I^P$). Stützel points out that a divergence between scheduled savings and net investments is possible, if and only if the sum of all scheduled net flow–of–funds transactions is *not* equal to zero. If they were equal to zero, the total planned savings would equate total planned net investments *by definition.* See the descriptions of this mechanism above, i.e. the system of equations describing individual and aggregate savings, and its interpretation. Finally, Stützel describes the equilibrium concept in question:[116]

> "Die Gleichheit von Angebot und Nachfrage nach Geldvermögen, also eine Situation, in der keine *unfreiwilligen* Geldvermögensumschichtungen stattfinden, ist identisch mit Kreislaufgleichgewicht." (Stützel, 1978, p. 83)

Thus, the 'circuit equilibrium' is the concept that has to be analyzed within the 'validation' of the model of Agent Island. It should be noted that so far, according to our knowledge, no agent–based study examined the 'Keynesian equilibrium' concept. We are be able to investigate this issue in a suitable environment, where many heterogenous agents are interacting, where the macro behavior emerges bottom–up, where a basic but well–defined 'monetary circuit' satisfies at least the

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[115] *The problem of the business cycle theory would emerge for example by investigating, as to how total expenditures would change in case of the sum total of individual* ***planned*** *net non–financial sources of funds being smaller than* ***planned*** *net non–financial borrowings of funds, i.e. generating a surplus of expenditures. [...] In general, it should be noted that* ***tendencies*** *of diverging demand and supply of* ***financial assets*** *are an issue of the business cycle theory, causing changes in aggregate expenditures.*

[116] *The equation of supply and demand of financial assets, viz. a situation where no* ***unintended*** *net non–financial sources (or borrowings) of funds take place, is identical to circuit equilibrium.*

requirements of stock–flow–consistency, where (as in 'Keynesian' theory) the household sector, the consumption goods sector and the capital goods sector are present (each populated by many agents), and where a monetary authority conducts monetary policy through interest rate adjustment.[117] This gives a suitable framework to tackle the question of a 'Keynesian business cycle equilibrium'.

Insofar, we can try to 'falsify'[118] the 'Keynesian' theory in the sense of its equilibrium concept. The (several hundred) 'validation' experiments within section 3.4.2 in the following chapter feature therefore two (quite opposite) aims: (i) Validating the model of Agent Island with respect to the 'Keynesian' notion of a 'circuit equilibrium', and (ii) falsifying the 'Keynesian' notion of a 'circuit equilibrium' within agent–based experiments (on Agent Island). Both aspect point in opposite directions: On the assumption that the business cycle dynamics of the presented model represent the features of a 'circuit equilibrium', the model is (i) validated in terms of the equilibrium concept identified by John Maynard Keynes. At the same time, this 'Keynesian equilibrium' concept can not (ii) be 'falsified' (in the sense of Karl Popper, see footnote 118) by our agent–based experiments, if point (i) is fulfilled. Conversely, it is thinkable that the present model does not represent characteristics of the 'circuit equilibrium' so that it (the model) cannot be validated. But this does not inevitably lead to a 'falsification' of the 'Keynesian equilibrium' concept. Possibly, the micro behaviors of agents are not valid, so that they do not represent the original system. Such a falsification could be misleading.

Table 2.5 illustrates the planned flow–of–funds transactions (marked by a P superscript) or, rather, transactions where the ex ante figures are able to diverge from ex post figures (according to our assumptions of course). Thereby, we simplify the task of defining planned receipts and expenditures of the sectors: As explained, household agents know their disposable income (their receipts) prior to planning. Moreover, planed expenditures equate ex post expenditures. Hence, the balance of planned receipts and expenditures of the household sector is therefore equal to the ex post balance. As a consequence, the inflows and outflows of funds of the household sector drop out from

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[117] In 'Keynesian' theory the government sector takes an important role. We focus exclusively on monetary issues, and therefore, we leave the government sector and the fiscal policy aside.

[118] Falsifiability is the logical possibility that an assertion can be shown false by a physical experiment, for example. Falsifiability is an important concept in science and in the 'philosophy of science', introduced by Karl Popper. He asserted that a hypothesis, proposition or theory is scientific only if it is falsifiable. Hence, no number of experiments can ever prove a theory, but a single experiment can contradict one (Popper, 2005).

inflows to→ outflows from ↓	Consumer goods ind.	Capital goods ind.	Private households	Central bank	Sum
Consumer goods ind.		I^P			$\sum$Expenditures^P
Capital goods ind.					$\sum$Expenditures
Private households	C^P				$\sum$Expenditures
Central bank					
Sum	$\sum$Receipts^P	$\sum$Receipts^P	$\sum$Receipts		Equilibrium: 0

Note: The head of a column indicates the sector that receives inflows of funds. The rows describe the outflows of funds of each sector. The value in each cell is the sum over all flow of funds in this sector (for columns) or out of this sector (for rows). In fact, only those variables are illustrated, where a difference between ex ante and ex post figures appear.

Table 2.5: Difference between ex ante and ex post flow–of–funds accounting matrix on the sectoral level

table 2.5. Besides this, we assume that the ex post and ex ante dividend and interest expenditures of firms are likewise equal.[119] Therefore, they also drop out from table 2.5. By another simplifying assumption, the dividend receipts of the corporate groups in the consumer goods sector also drop out. Finally, only (i) the consumption receipts of consumer goods firms, (ii) the investment receipts of the capital goods industry, and (iii) the investment expenditures of the consumer goods industry can represent deviations between ex ante and ex post figures. See table 2.5. Based upon these facts, we want to derive a 'business cycle indicator'. At first, we define the following equations, which represent the insights of the 'flow–of–funds accounting' matrix:

$$\Delta FA^P_{house,T} = \Delta FA_{house,T}, \tag{2.2.27}$$

$$\Delta FA^P_{firm,T} = C^P_{consumer,T} + I^P_{capital,T} - I^P_{consumer,T} - \sum Y^{W,D,I}_{firm,T}. \tag{2.2.28}$$

Equation (2.2.27) illustrates the fact that ex post net non–financial sources (or borrowing) of funds through households $\Delta FA_{house,T}$ equal the ex ante (or planned) figure $\Delta FA^P_{house,T}$. Furthermore, equation (2.2.28) represents the ex ante net non financial sources (or borrowing) of funds by the firm sector as a whole. Thereby, as explained, flow of funds due to expenditures of firms to the household sector and the central bank are incorporated by realized (ex post) figures (see the last

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[119] Remember, these payments take place 'over night', i.e. previous to the present period.

term in equation (2.2.28)). Thus, as just mentioned, the deviation of $(\Delta FA^{P}_{house,T} + \Delta FA^{P}_{firm,T})$ from its equilibrium (i.e. from 0, see the lower right 'cell' in table 2.5) bases exclusively upon the levels of (i) $C^{P}_{consumer,T}$ (consumption goods receipts of the consumer goods industry), (ii) $I^{P}_{capital,T}$ (investment goods receipts of the capital goods industry), and (iii) $-I^{P}_{consumer,T}$ (investment goods expenditures of the consumer goods industry). In a next step, we define the expectation formation of these figures:

- **Receipts of the consumer goods industry:** This is given by hash (bean) price (from the last period) times present aggregate supply of hash (beans), i.e. $C^{P}_{consumer,T} \equiv p^{hash}_{T-1}h_T + p^{bean}_{T-1}b_T$.
- **Receipts of the capital goods industry:** This is given by the receipts of the capital goods industry (from the last period), i.e. $I^{P}_{capital,T} \equiv I_{capital,T-1}$.
- **Investment expenditures of the consumer goods industry:** This is given by the average machine price (from the last period) times present aggregate demand for machines by the consumer goods industry, i.e. $I^{P}_{consumer,T} \equiv m^{D}_{T}\overline{p}^{machine}_{T-1}$.

By plugging these equations into equation 2.2.28 we obtain:

$$\Delta FA^{P}_{firm,T} = p^{hash}_{T-1}h_T + p^{bean}_{T-1}b_T + I_{capital,T-1} - m^{D}_{T}\overline{p}^{machine}_{T-1} - \sum Y^{W,D,I}_{firm,T}. \tag{2.2.29}$$

Finally, we define a 'circuit equilibrium indicator' (CEI) through equation (2.2.30) below: It is the sum total of planned net non–financial sources and borrowings of funds of all agents in the model, i.e. it is the sum of equation (2.2.27) and equation (2.2.29). Accordingly, the following equation (2.2.31) represents a routine for the calculation of the indicator in the model.

$$\begin{aligned} CEI &= \Delta FA^{P}_{House,T} + \Delta FA^{P}_{Firm,T}, && (2.2.30)\\ &= \Delta FA_{House,T} + p^{hash}_{T-1}h_T + p^{bean}_{T-1}b_T + I_{capital,T-1} - m^{D}_{T}\overline{p}^{machine}_{T-1} - \sum Y^{W,D,I}_{firm,T} && (2.2.31)\end{aligned}$$

As a last step, we describe the assumed implications of this indicator: In 'circuit equilibrium', the indicator has to be 0 (or close to 0). When CEI is larger than 0, the sum total of all planned receipts and expenditures features an excess of expected receipts. This defines the 'Keynesian deflationary gap'. In the opposite case, when CEI is below 0, there are planned excess expenditures. This leads to the 'Keynesian inflationary gap'. The following table summarizes these results. The critical aspect of this equilibrium is therefore the relation between (i) scheduled outflows of funds out of the 'income circuit' into the stock of financial assets/debts, and (ii) scheduled inflows of funds out of the existing stock of financial assets/debts into the 'income circuit'. When the aggregate plans of such outflows and inflows are in equilibrium, no upside or downside pressure on inflation and the business cycle are expected.

Deflationary gap	Circuit equilibrium	Inflationary gap
Expected excess receipts	No expected excess	Expected excess expenditures
$CEI > 0$	$CEI = 0$	$CEI < 0$

Table 2.6: Keynesian deflationary vs. inflationary gap

Finally, it should be noted that a negative or positive indicator should propagate itself into the future. This characterizes the notion of the 'Keynesian' multiplier. In section 3.4.2 we will draw on these results, as we analyze the simulation data with respect to the 'circuit equilibrium'. We expect that the model features all 'Keynesian' aspects discussed so far.

Expectations–Channel of Monetary Transmission

Finally, the present model should feature on the aggregate level the 'Phillips curve' relationship. This topic goes back to an empirical phenomenon discovered by Alban Phillips (Phillips, 1958): Originally, it shows the labor market based relationship between wage growth and unemployment. In recent years this connection is modified insofar as the wage growth is substituted by inflation, and unemployment by output gap. Hence, this 'New Phillips curve' connects the two targets of monetary policy in such a way that inflation is rising, if the output gap is rising too—and vice versa. Behind this connection stands the intuitive notion that in an economic upswing, accompanied by rising output gaps, labor markets are cleared, which induces higher wages and (via a 'mark–up' price calculation of firms) upside pressure on inflation (Bofinger, 2001). From this perspective, rising output gaps cause rising inflation. This is also a basic notion of 'New Keynesian' macroeconomic models, according to which the central bank controls inflation indirectly via the output gap (based upon the 'Phillips curve' relationship).[120] Moreover, the 'Phillips curve' mechanism defines the 'expectations–channel' of monetary transmission. This channel is described by Bofinger in the following way:

> "[...] it is obvious that central banks that are committed to price stability have very often been able to achieve low and stable inflation rates. This indicates that there must be a relatively simple and robust mechanism for the control of the inflation rate. The expectations channel comes very close to this requirement, especially for relatively large currency areas where the impact of exchange rate changes on the price level can be neglected. The essential relationship [...] shows that, once a low inflation rate has been reached, it is automatically translated into the future because wage agreements are normally based on adaptive expectations. This explains why 'credibility', i.e. the public's belief that a central bank is able and willing to achieve price stability, became one of the key words in the discussion on monetary policy in the 1990s." (Bofinger, 2001, p. 114)

With respect to our model the essential point is the way, past inflation is translated into the future due to backward–looking expectations and wage agreements: This aspect is integrated into the wage growth rule of the present model. We call this mechanism also the 'wage–price spiral'. In fact, it goes back to the 'expectations–channel' of monetary policy. Another interesting point of Bofinger's description is the 'credibility' of the central bank. However, this notion is not separately incorporated in the model, because the structure of expectation formation on Agent Island is trivial: Agents use last period's inflation data when forming present inflation expectations. This basic approach does not incorporate any actions of the central bank. As a result, it is not explicitly modeled, how the central bank was counteracting inflation in the past. Therefore, 'credibility' of the central bank is not a specific issue of the present study.

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[120] See again Woodford, 2003, for a comprehensive representation of this perspective.

2.3 Conclusion

By its nature, the construction of an agent–based computational model happens bottom–up. We carry out this bottom–up approach by constructing heterogenous sectors populated by heterogenous agents. This results in an intensive micro structure of the model: In the initial steps of the model development, we construct the consumption goods market and the according behavior of firm and household agents. But this was not sufficient for discussing issues of monetary theory and policy. We had to integrate intertemporal aspects, viz: A central bank that is controlling credit interest rates, firms that exhibit a real capital stock, and therefore the capital goods industry. By consequence, the model becomes complex and quite large with respect to micro behavior based upon routines.

Starting with the discussion of the 'monetary circuit' of the model, we left the microeconomic 'world' and shift to the macroeconomic perspective. The macro structure will be of particular interest during the next chapter, where the 'validation' of macro facts takes place. According to the discussions in the last sections, expenditures constitute the driving force of the business cycle. The 'Keynesian equilibrium' illustrates this fact. Hence, we will validate Agent Island with respect to this equilibrium concept in the next chapter. In general, the model should feature several 'Keynesian' elements, such as the 'Keynesian circuit equilibrium', the 'Keynesian' multiplier, and (most importantly) the 'Keynesian paradox of thrift'. Additionally, we assume that the model exhibits several feedback loops, such as the 'investment multiplier' and the 'wage–price spiral'. To sum up, there are complex self–energizing effects, which are supposed to generate complex and non–linear business cycle patterns on the aggregate level.

In contrast to 'Keynesian' theory, the present model features a basic 'monetary circuit', which includes just one financial asset. Conversely, John Maynard Keynes suggests a financial system exhibiting two financial assets (money and bonds). This is the groundwork of his 'liquidity preference theory'. According to this, his theory of interest rates rests upon (i) the portfolio decision between bonds and money, and (ii) the exogenous money supply through the central bank. In contrast, our approach builds upon a single financial asset (central bank deposits), and a completely endogenous money stock. Hence, our interest rate theory cannot build upon the concept of 'liquidity preference theory'. As stated, the present approach is basic: In accordance to Knut Wicksell, the monetary authority in our framework controls the nominal interest rate directly and freely in a 'pure credit system' (with the central bank representing the 'mono–bank'). Besides this, household agents need not select their financial portfolio. The main purpose of the central bank is straightforward too. Its objective is to control the business cycle via an interest rate policy. The basic mechanism fulfilling this aim is the 'Wicksellian' monetary transmission process, i.e. the 'Wicksellian cumulative process'. It is incorporated into the model in the context of the firms' investment decision. Finally, the 'perfect credit system' of Agent Island is featuring a double–entry book keeping system, which guarantees the stock–flows–consistency of the model.

Chapter 3

Validation of the Model of Agent Island

It is the task of validation to obtain a reasonably adjusted agent-based computational model. The conceptual model of the last chapter lays down a set of decision rules and structural equations that allows the functioning of the artificial economy of Agent Island. Thereby, chapter 1.3 delivers the methodological validation framework. This includes several steps: The model needs to pass (i) 'face validation', (ii) sensitivity analysis, (iii) calibration experiments, (iv) statistical 'validation', and (v) a final 'plausibility check'. This chapter characterizes the last four points, whereas questions of 'face validation' were already discussed during the description of the conceptual model in the last chapter. Before we start the 'validation' procedure, we have to discuss the settings of the model (such as behavioral parameters, initial values, structural parameters) within section 3.1. In addition, we classify 'peripheral' and 'main settings' of the model. The latter defines those parameters that are of special interest during the sensitivity and calibration experiments. 'Main parameters' are given by the most important behavioral and structural parameters of the model. In the case of the present rather complex model, the choice of these parameters is crucial and needs some further explanations. The following brief review of the sequence of actions identifies them:

1. **Central bank:** The central bank sets the credit interest rate. The parameters τ, ς and ρ determine this decision.
2. **Labor union and employer association:** Overall wage growth is bargained. This is modeled via the bargaining power of the labor union, determined by λ.

3. **Consumer goods firms:** The technical progress of firms is determined. Thereby, the 'random walk' term ϵ influences the supply side of the consumer market in the sense of a stochastic supply shock. Accordingly, the variance of this 'random walk' term (i.e. $Var[\epsilon]$) is of special interest. It measures the overall magnitude of stochastic supply shocks. Thereafter, firms schedule and produce output; this requires the behavioral parameter θ and the structural parameter of the production function β.
4. **Consumer goods firms:** Firms decide their individual investment demand. This is executed by the behavioral parameters ψ and ι.
5. **Capital goods firms:** Firms decide individual machine supply prices due to a 'mark–up' pricing. In advance, 'mark–ups' are manipulated via the behavioral parameters ω^{Micro} and ω^{Macro}.
6. **Capital goods market clearing:** The capital goods market opens and clears.
7. **Capital goods firms:** The firms produce the ordered quantities of machines and pay wages.
8. **Households:** Household agents compute their disposable period income. Thereafter, individual savings rates are manipulated through η^{IC}, η^{RR}, and χ. The intertemporal budget constraint is defined through ν^{upper} and ν^{lower}. Finally, consumption expenditures and financial savings are calculated.
9. **Consumer goods market:** The consumer goods market opens and clears. Goods are paid.
10. **And so on,... the round starts anew.**

The parameters represented in this scheme constitute the 'input variables' of the 'validation' process. They are supposed to have significant impact on individual decisions, driving the business cycle. Moreover, we identify the global parameter ϖ, i.e. the number of workers assigned to each firm, as another important 'factor'. See the explanations in section 3.2, where we conduct an initial investigation. In section 3.1, we will specify the 'input variable space' for the named parameters. Subsequently, these parameters have to be investigated during the whole 'validation' process: It is the task of the 'validation' to find the best level combination out of the defined 'input variable space'.

Due to the complex and non–trivial interaction effects in the model, as well as the large set of important 'factors', this sounds like the proverbial 'search for a needle in a haystack'. In fact, it is like that.

As a first step, we reduce in section 3.1 the in principle infinite 'domain' of each 'main parameter' to a reasonable range. This is based upon a review of the empirical economic literature. Unfortunately, empirical data is not available for each 'main parameter'. Nevertheless, we have to define the 'input variable space' for the subsequent analysis. Furthermore, we determine all 'peripheral settings'. 'Peripheral settings' are parameters (initial values, subordinated behavioral and structural parameters), which are of subordinate importance for our analysis. Importantly, it is also necessary to define some reasonable output figures for the sensitivity analysis in section 3.3; they represent the 'responses' in the sensitivity experiments. Insofar as the model should be characterized by typical dynamics identified on the aggregate level, we choose inflation and output gap time series as proxies for those dynamics.[1] Furthermore, we compress the data of both time series to single figures, hence we use the expected value and the standard deviation of both series. This gives four 'responses': (i) expected value of inflation (μ^{π}) and (ii) expected value of the output gap (μ^{y}) time series; as well as (iii) standard deviation of inflation (σ^{π}), and (iv) standard deviation of output gap (σ^{y}) time series.

The remainder of this chapter is organized as follows: We start with the discussion of the 'main' and 'peripheral settings' of the model in section 3.1. Section 3.2 investigates some basic features of the model concerning the stability of the simulation output. It features also a comparison between scenarios with 'homogenous' vs. 'heterogenous sectors'. The following section 3.3 discusses the sensitivity experiments. The chapter closes with the calibration procedure in section 3.4 combined with statistical 'validations' and 'plausibility checks' of the model.

3.1 Peripheral and Main Settings of the Model

This section elaborates on the general description of the 'main model parameters' and their 'domains' (see subsection 3.1.2). Besides this, several 'peripheral parameters' have to be fixed in advance (see subsection 3.1.1) in order to conduct simulation runs. It should be noted that there are complex

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[1] Both time series are important target values of monetary policy (see the definition of the 'Taylor rule').

interaction effects within the model. Consequently, the influence of a certain parameter on the model output depends on the levels of probably many other parameters. Insofar as some of the decision rules of agents are not investigated in the empirical economic literature, it is not possible to find empirical verified ‘domains’ (or levels) for all ‘main’ (or ‘peripheral’) parameters; some have to be defined ad hoc—primarily based upon ‘face validation’ runs. But as explained above, one (ad hoc defined) parameter is supposed to influence the reasonable levels of other parameters indirectly. For example, the supply decision of consumer firms features a sensitivity of output growth to marginal profit, defined through the parameter θ. This parameter is supposed to affect consumer goods markets, and therefore future incomes and future investment demand as well. However, this parameter has not been investigated in *any* empirical study until now. This uncertainty propagates, i.e. it produces undesired interaction effects on other parameters. For instance, it affects the behavioral parameters of the investment decisions of firms, or parameters defining the savings and consumption decisions of households. Additionally, feedback–loops are operating. In the example, the consumption decision, in turn, influences the subsequent supply decisions of consumer goods firms, and hence θ. To sum up, the complexity of the artificial economy of Agent Island indicates that one cannot cut a single parameter out of the model, investigate its level in reality through empirical data, and apply the obtained levels to the simulation model without consideration for the other parameters. As stated in chapter 1, ‘everything seems to depend on everything else’.

In addition, the definition of some parameters are limited by technical considerations, such as the volume of the simulation: For example, the total number of agents in the simulation is limited by the available computing power, i.e. by hardware resources. Finally, some parameters do not have a concrete ‘counterpart’ in reality. Consider the *behavioral* parameter ν, which defines subsistence and saturation levels of consumption. At the same time, it defines the intertemporal budget constraint of the household agent, which is a *structural* parameter. This indicates, we will hardly find an exact counterpart of the behavioral and structural parameter ν in reality. According to that, some parameters are difficult to define, and there are many interdependencies. Insofar, the review of the relevant literature is not and cannot be perfect and complete. If the reader does not bear this fact in mind, he could easily expect a high degree of accuracy of the literature review with respect to the 'settings' and 'domains' of the model parameters. This, however, cannot be achieved, otherwise it would lead to a 'pseudo–accuracy'. Finally, it was necessary to recheck the 'domains' and 'levels' of the model parameters in many 'face validation' runs.

3.1.1 Settings of the Peripheral Parameters

In principle, the model features 'global', 'firm', and 'household' parameters. For example, the 'period length' or the 'working time per period' are global parameters. They are equal for all households and firm agents. Besides this, there are individual or micro parameters; they could be equal for all agents, or not. The former means that the individual levels of the parameters are deterministic, whereas the latter implicates that the parameters are stochastic: Imagine 10,000 household agents. It is surely not possible to adjust each agent by hand. One has to use a 'creation routine'[2] to produce 'instances' of agents in each simulation run. Within this routine any deterministic or random process could be used to create household agents. Consequently, if we decide that agents should be heterogenous, this must be incorporated into the 'creation routine'. Such an automatically created heterogeneity of 10,000 agents is usually executed through a stochastic process.

We will see below that 'main parameters' are stochastic in case of 'heterogenous sectors'. See the discussion in the next subsection for the differences between the scenarios with 'homogenous' and 'heterogenous sectors'. At this point, it should be sufficient to know that in case of 'heterogenous sectors', an individual 'main parameter' of one certain agent is defined as a continuous random variable, drawn from a certain uniform distribution. Otherwise, with 'homogenous sectors', the individual parameters are deterministic. We will also refer to both cases (if necessary) in the following description of the 'peripheral parameters'.

Global Settings

Number of agents Y ($Y = J + K + C + H$): The quantities of hash firms (J), bean firms (K) and capital firms (C) are important parameters in the model, which have to be set almost freely.

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[2] In the initial step of each simulation run all agents have to be 'created': This implies that they are endowed with initial values for variables, with decision rules, and with behavioral as well as structural parameters. This takes place through a 'creation routine', which is part of the programed model.

Usually, we investigate models with a sum total ($J+K+C$) of 10 to 500 firm agents.[3] Thereby, we apply a fixed segmentation in hash firms ($0.3 \times [J+K+C]$), bean firms ($0.3 \times [J+K+C]$), and capital goods firms ($0.4 \times [J+K+C]$). Hence, the ratio is $3:3:4$. It will be the task of the following initial analysis in section 3.2, to find a reasonable value for the sum total of firm agents ($J+K+C$). Thereafter, in the subsequent sensitivity and calibration experiments, we do not change this total number of firm agents anymore. Additionally, the total number of household agents (H) depends on the parameter ϖ. This constitutes the number of workers assigned to each (hash, bean or capital goods) firm; therefore the total number of household agents is given by $H = \varpi(J+K+C)$. However, the definition of the parameter ϖ is discussed within the subsection 'Main Parameters'.

Period length: We assume ad hoc that a period spans 52 weeks, i.e. one year; more detailed considerations as to the length of a period are made in section 2.1.3.

Working time per period: The working time per period is given by 52 weeks per period, times 40 hours per week. This results in 2,080 hours working time per period.

Initial wage w_1: The initial wage level in period $T = 1$ is set to 0.1.

Pre-initial inflation π_0: Pre-initial inflation, i.e. inflation in the period previous to the first period of the simulation, is set to 0.03. This is equal to the target inflation rate.

Target inflation π^*: The central bank on Agent Island aims for an inflation rate of 0.03 (i.e. 3%). It should be noted that the real–world inflation rate target lies above 0% due to several statistical biases in the determination of inflation rates.[4] The present model does not exhibit such statistical 'inflation biases'. Thus, the central bank is able to determine the inflation rate unbiased from the model output, and therefore it would be appropriate to target a level of 0%. However, in order to match the model to real world data, we fix the target inflation rate above zero, at 3%. This relatively small variation should not influence the model 'validation' notably.

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[3] In Germany, circa 3.5 million firms were existing in 2005, with 0.5 million corporate enterprises; see the internet database of www.destatis.de. Accordingly, 500 model firms could, for example, represent the 500,000 corporate enterprises in Germany, if they stood for large firms. Or, they could represent the total of 3.5 million firms. But this is an 'intellectual game'. Of course, they represent the *artificial* economy of Agent Island.

[4] For a discussion of 'inflation biases' see Bofinger, 2001.

Initial credit interest rate i_1: The initial credit interest rate, i.e. the nominal interest rate during the first simulation period, is set to 0.05. This takes place because of the fact that the central bank cannot set the nominal interest rate in the first period due to the lack of historical data.

Initial expected real interest rate $\mathbb{E}[r_1]$: Accordingly, the initial expected real interest rate for the first period is given by 0.02. Following the approximated 'Fisher equation', the expected real interest rate is given by the difference of the nominal interest rate and the expected inflation rate. In our setting we assume extrapolative expectations, i.e. $\mathbb{E}[\pi_1] = \pi_0$. This gives $\mathbb{E}[r_1] = i_1 - \mathbb{E}[\pi_1] = i_1 - \pi_0 = 0.05 - 0.03 = 0.02$.

Initial price for machines $\hat{p}_0^{machine}$: The average price for machines previous to the first period is set to 750 AD. This takes place due to the 'face validation' of the model in several runs. Unfortunately, the first–period's average machine price depends on several parameters, (for example) and in particular on ϖ; see the subsequent discussion in the following sections. According to this perspective, we are not able to adjust this initial value correctly with respect to its several determinants. Consequently, we should consider that the price of the machines, initially booked in the 'asset ledger' of firms can be too high (or too low) compared to the machine prices paid in the first periods; and the depreciation of the initial capital stock can be also too high (or too low).

Pre–initial mean per capita consumption expenditures $\hat{C}_0$: Due to 'face validation' runs this figure is set to 100 AD. This value is, however, only crucial for the consumption decision in the first period. Subsequently, it is adapted by current model data.

Firm Settings

Maximum machine run time: We assume ad hoc that one machine features a capacity to run 20 periods with 2,080 working hours per period. This results in 41,600 hours total machine run time. We do not apply a three–shift system.

Initial machines per firm: We assume ad hoc that a hash or bean firm features a capital stock of 35 machines pervious to the first period—in the 'homogenous case'. In the 'heterogenous case', the initial number of machines is drawn from a uniform distribution between 30 and 40. Consider, for example that initially 35 machines are booked into the 'asset ledger' of a firm. Each of the booked machines features three assigned entries in the 'asset ledger' ($(Maximum\ hours)$, $(Served\ hours_{z,j,T})$, $(P^{machine}_{z,\tilde{T}})$). The maximum run time is set to 41,600 hours, as derived above. The already served work time of each of these 35 machines is a random number drawn from a uniform distribution between 0 hours and the maximum machine run time of 41,600 hours. We maintain this design in the 'homogenous case'. Hence, consumer goods firms are *heterogenous* with respect to the age structure of their capital stock in the '*homogenous* case' as well. Finally, the machine purchase price ($P^{machine}_{z,\tilde{T}}$) is set to $\hat{p}^{machine}_0 = 750$.

Initial value for technology $A_{world,j,k,c,1}$: The initial value for the technology parameter in the production function of all firm agents (as well as the 'world') is set to 1.

Magnitude of the drift term in $A_{world,j,k,c}$, i.e. $\varrho_{world,j,k,c}$: In general, we assume that the 'drift term' within technical progress lies between 0.005 and 0.035. According to the 'random walk' specification, this figure is manipulated through a 'white noise error term' ϵ. The standard deviation of this error term defines the mean amplitude of supply shocks on the firm level. Therefore, it is treated as a 'main parameter', to be discussed in the next subsection. However, the mentioned magnitudes of the deterministic 'drift term' between 0.005 and 0.035 are verified by a cross country study of Sax, 2004. In this study the 'Solow residues'[5] for 17 OECD countries between 1960 and 2000 are calculated and compared. According to various countries and data sources, Sax determines 'Solow residues' between -0.001 and 0.042 (Sax, 2004). A similar study by Mathur, 2005, investigates 29 countries, consisting of some selected South Asian, East Asian and EU countries from 1966 to 2000. Mathur finds 'solow residues' between 0.0072 and 0.0626. The latter value is that for China, the former that for the Philippines. Comparable results are, among others, obtained by Senhadji, 1999. This study investigates

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[5] The starting point for calculating the 'Solow residual' (see e.g. Romer, 1996) is a common production function $Y = f(A, K, L)$, where Y defines output, A the technology parameter, K the capital stock, and L the magnitude of labor. Derivation after time delivers $\dot{Y} = f_A\dot{A} + f_K\dot{K} + f_L\dot{L}$, whereby f_x marks the marginal product for factor x. Rearrangement delivers $\frac{\dot{Y}}{Y} = g + \frac{f_K K}{Y}\frac{\dot{K}}{K} + \frac{f_L L}{Y}\frac{\dot{L}}{L}$, with $g \equiv \frac{f_A A}{Y}\frac{\dot{A}}{A}$ as the 'Solow residual'. When technology is 'Hicks–neutral', as in case of the production functions in the present model, i.e. $f(A, K, L) = Af(K, L)$, the 'Solow residual' is given by $g = \frac{\dot{A}}{A}$. Therefore, it illustrates the derivative of the technology parameter A with respect to time.

the sources of growth in 88 countries between 1960 and 1994. The results are that the mean $\Delta logA$ for several subperiods (7 to 13 years) lies between -0.004 and 0.0267 for industrial countries and -0.011 and 0.0301 for East Asian countries. It should be noted that these figures are average numbers for an aggregate production function. The growth rate for one single firm from year to year can be larger than the highest value or lower than smallest value. According to our specification of $A_{world,j,k,c}$ as 'random walk with drift', the (individual) deterministic drift parameter $\varrho_{world,j,k,c}$ is adopted each period by a stochastic 'white noise term' $\epsilon_{world,j,k,c}$. Insofar as we define drift terms between 0.05 and 0.035, the adjustment by $\epsilon_{world,j,k,c}$ results in growth rates for $A_{world,j,k,c}$ per year that are approximately located between 0% and 6%. To be precise, we define in the 'heterogenous case' that firms in the hash sector exhibit uniformly distributed 'drift terms' between 0.025 and 0.035, firms in the bean sector between 0.005 and 0.015, firms in the capital goods sector between 0.02 and 0.03, and the 'world drift term' is 0.025. In the 'homogenous case' all drift terms are fixed to 0.025.

Elasticity of output to labor for capital goods firm β_c: During the sensitivity and calibration experiments, we investigate the 'elasticity of output to labor' for consumer goods firms. Then again, as a simplification this elasticity for capital goods firms is not analyzed. Thus, we fix β_c for capital goods firms c (with $c \in [1, ... C]$) ad hoc to a deterministic level 0.5 in the 'homogenous case'. In the 'heterogenous case' individual β_c is a random variable drawn from a uniform distribution between 0.4 and 0.6. This specification accounts for large differences, up to a factor of 8, in the productivity of capital goods firms in the 'heterogenous case'.

Initial price mark–up $\mu_{c,1}$: The initial price 'mark–up' is defined as a random variable between 0.4 and 0.6 in the 'heterogenous case'. In the 'homogenous case', the 'mark–up' for all capital firms is fixed to 0.5. Several empirical studies investigate 'imperfect competition' and the dynamics of 'mark–ups'. Thereby, various theoretical and empirical approaches are used to derive 'mark–ups' of prices over marginal cost. The broad range of results spans from quite low 'mark–ups' of around 0%, up to 'mark–ups' of around 100%. For example, Martins and Scarpetta, 1999, review the literature and investigate 'mark–ups' in the U.S. manufacturing industries between 1970–1992. They state: "In broad terms, most of the sectoral mark–ups [...] are in the range of 30–60 per cent" (Martins and Scarpetta, 1999, p. 8). Other studies (among others Forsman et al., 1997, Marchetti, 2002, and Estrada and López-Salido, 2005) are in line with the results of Martins and Scarpetta, 1999.

Pay–out factor for (extra) profits of consumer firms $POF_{j,k}$: This parameter is fixed ad hoc to 0.5 in the 'homogenous case'. In the 'heterogenous case', consumer goods firms draw $POF_{j,k}$ as a random variable out of a uniform distribution between 0.4 and 0.6. These settings imply that consumer goods firms can use a part of their present extra profits to finance future investments (or future losses) by means of self financing, instead of credit financing.

Initial liabilities of firms $\underline{LL}_{j,k,c,1}$: We will discuss below that private households are endowed with initial financial wealth. The sum total of this wealth is $\sum_{H''} \underline{FA}_{h'',1} = \sum_{H''} 2000 = H''2000$, thereby H'' constitutes the number of agents in the wealthy subgroup of the household sector. This subgroup is per definition as large as the quantity of firms, i.e. $H'' \equiv (J+K+C)$. For the sake of simplicity, we distribute the initial debts evenly among firms, so that each firm exhibits 2,000 AD initial debt. We maintain this design in the 'homogenous' and the 'heterogenous case'.

The technical parameter of capital goods firms γ_c: This parameter does not feature any economic notion. It could be integrated in the initial level of A_c as well. We use this parameter in order to fine–tune the initial capacity utilizations of capital goods firms. After several 'face validation' runs we found out that a deterministic level of 0.03 for all capital goods firms delivers reasonable capacity utilizations in the capital goods industry. This setting is applied in the 'homogenous case' and in the 'heterogenous case'.

Household Settings

Consumer preference α_h: We do not model a general preference for hash or beans. Hence, in the 'homogenous case' the level of α is fixed to 0.5 for all household agents. If we switch to the 'heterogenous case', the mean of the distribution of the random variable is, of course, 0.5 too. Thereby, the uniform distribution has an upper limit of 0.65 and a lower limit of 0.35.

Initial basic financial savings rate $\tilde{s}_{h,1}$: In the 'homogenous case' the level of $\tilde{s}_{h,1}$ is fixed to 0.075 for all household agents. If we switch to the 'heterogenous case', the mean of the distribution of the random variable is also 0.075. Moreover, the uniform distribution has an upper limit of 0.15 and a lower limit of 0.

Initial financial assets $\underline{FA}_{h,1}$: Initial financial assets of household agent h ($\underline{FA}_{h,1}$) are determined by the subgroup agent h belongs to. There are two subgroups, each populated by H' or H'' members, whereby $H = H' + H''$. These subgroups represent the different wealth groups within the household sector. We assume that one subgroup, the pretty small subgroup (H''), possesses outstanding financial wealth compared to its complementary group (H'). We assume that each member of the H'–subgroup does not hold any financial assets at the beginning of period 1, i.e. $\underline{FA}_{h',1} = 0$ for all $h' \in [1 ... H']$. Then again, the members of the wealthy H''–group are initially endowed with 2,000 AD.[6] Besides this, we already explained that ϖ constitutes the number of workers (i.e. household agents) assigned to each firm. According to our design, the first of the ϖ agents assigned to each firm belongs to the wealthy H''–group, the other $(\varpi - 1)$ agents belong to the H'–group. Accordingly, the relative size of the wealthy group depends on ϖ. We will see that the final 'validation' is conducted with ϖ–levels of approximately 30 agents. In this case, the wealthy group includes 1/30 of the total household sector.

3.1.2 Domains of the Main Parameters

As explained in the introduction of this section, the following parameters are supposed to exhibit a significant impact on the business cycle of Agent Island. The 'domains' of these 'main parameters' constitute the 'input variable space' (or the 'experimental domain') of the subsequent sensitivity and calibration experiments. Thus, they are the starting base for the 'validation' of the model. In a first step, we have to highlight the difference between the 'homogenous case' and the 'heterogenous case'. Thereafter, we illustrate the parameters and their 'domains'.

The Implication of Homogenous vs. Heterogenous Sectors

The analysis within this chapter starts with a comparison of 'homogeneous' and 'heterogenous sectors'. As noted in the introduction of this section, the individual parameters in the 'homogenous case'

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[6] In 'face validation' we found out that the total initial wealth of $\sum_{H''} 2,000$ delivers a ratio of $\frac{\sum_{H''} 2,000}{GDP}$ in period 1 that corresponds approximately to German data at the beginning of the 1980s. See the preceding discussion in section 2.2.1.

are deterministic, while stochastic in the ‘heterogenous case’. What does that mean? The following overview delivers the ‘domain’ for each ‘main parameter’. Each (certain) level within the ‘domain’ of a ‘main parameter’ can be an element of a certain scenario. In addition, one level–combination (comprising all ‘main parameters’) defines one scenario that can be examined subsequently. Importantly, in the ‘homogenous case’, one certain level–combination (of the scenario) defines already the individual levels for the ‘main parameters’ of all agents in one sector. Hence, in the ‘homogenous case’, individual ‘main parameters’ are equal and deterministic for all agents in the sector in question. On the contrary, in the ‘heterogenous case’, one certain level–combination defines the distributions of the individual ‘main parameters’: It constitutes the means of the distributions, whereby each distribution spans with a band of ±20% around its mean. Accordingly, in the case comprising ‘heterogenous sectors’, the agents within one sector (e.g. each household agent in the household sector) exhibit different levels for each ‘main parameter’. Therefore, they are heterogenous. Lastly, each individual ‘main parameter’ is a continuous random variable, drawn from a uniform distribution[7], distributed around the mean.

We can make an example to highlight this design: For example, let us investigate a scenario where (next to all other main parameters) for consumer goods firms the level of θ, i.e the elasticity of output growth to marginal profitability, is set to 0.2, i.e. we investigate a $(\theta = 0.2)$–scenario. This implies that in the ‘homogenous case’ each consumer goods firm j (with $j \in [1, \dots J]$) exhibits a level $\theta_j = 0.2$. Then again, in the ‘heterogenous case’ each firm j draws its θ_j–level from a uniform distribution between 0.16 $(= 0.8 \times 0.2)$ and 0.24 $(= 1.2 \times 0.2)$, i.e. ±20% around the mean of 0.2. The individual level of a certain firm j could be 0.1847, for example. Thus, in the former case the individual θ_j is deterministic, and in the latter case it is stochastic. The reader should bear that in mind, when reading the following descriptions of the ‘main parameters’. It applies to all ‘main parameter’—except for one (to be mentioned below). By definition, global parameters feature only one deterministic specification, because they are defined on the global level. It should be noted that in the following the ‘main parameters’ are expressed without subscripts, which usually assign them to individual agents (in the ‘homogenous case’). This means that we omit the control variables h,

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[7] For the sake of simplicity we employ the uniform distribution. It is easy to handle and to adopt in our simulation model.

j, k, or c in the subscripts. This is due to simplification, because we mention those parameters pretty often in the following analysis in the remainder of this chapter. Finally, in the 'heterogenous case', these subscripts are not necessary, because the illustrated scenarios define only the means of the distributions of the individual levels of the parameters—but not the individual parameters themselves.

Global Parameter

The sensitivity of wage growth to output gaps λ: Our wage growth specification is similar to that of Fuhrer and Moore, 1995. Fuhrer and Moore, 1995, find very small values for λ, i.e. $\lambda = 0.00435$ in U.S. data between 1965 and 1993. Other results (delivered by Coenen and Wieland, 2000) define levels for λ between 0.006 and 0.0296 (for Germany, France, Italy and the Euro area). We can also interpret the slope of the 'New Keynesian Philips curve' (which connects the output gap to inflation) as an indicator for the λ: Usually, values between 0.025 and 0.39 are employed (see, for example, Woodford, 2003, and Mayer, 2005). To sum up, we investigate scenarios with λ varying between 0.04 to 0.4.

Central bank preference parameters in the interest rate rule (τ, ς and ρ): The study of Sauer and Sturm, 2007, reviews the empirical literature of 'Taylor rule' estimations for the Euro area. These estimations are based upon various sample periods and rule types (forward–looking or contemporaneous). The studies identify weights on the inflation gap (τ) between 1 and 4, weights on the output gap (ς) between 0.2 and 2, and smoothing parameters (ρ) between 0.2 and 0.9. We use these results as the domains for τ, ς, and ρ. It should noted that these studies are based on monthly or quarterly data. Insofar as the present simulation is based on yearly data, we expect that the smoothing parameter tends to be 0.2. Moreover, similar results are obtained by Judd and Rudebusch, 1998, who investigate the monetary policy of the FED between 1970 and 1997.

Number of workers per firm ϖ: This magnitude depends to a large extend on other settings of the model concerning the dimensions of sectors. We could set ϖ in loose analogy to real–world data: For example, German data suggest $10 \leq \varpi \leq 12$, given by a ratio of about 40 million households to about 3.5 million firms in Germany in 2005 (see again the internet database of www.destatis.de).[8] According to the analysis in the following section, we will see that ϖ represents an important regulator for the size and utilization of the capital goods industry. Hence, we use a rather broad domain for ϖ, with scenarios including ϖ between 5 and 30. Finally, one restriction is the computing power, i.e. the performance of the IT–system, needed to run the simulation. If, for example, ϖ is set to 2,000 and the number of firms is set to 500, the total number of agents goes to 1,000,500, which would be far too large to run the present rather complex simulation in the SeSAm environment. Thus, technical restrictions limit the overall size of the simulation, and therefore ϖ. According to this, we found out that levels for ϖ of approximately 50 (and, in addition, several 100 firm agents), define a technical upper bound.

Firm Parameter

The elasticity of output to labor of consumer firms β: In many studies, an aggregate 'Cobb–Douglas' production is estimated. The seminal paper of Cobb and Douglas, 1928, delivers a range for the 'factor' elasticity of labor (U.S. data for 1899–1922) between 0.8 and 0.9 (without technical progress).[9] Newer studies find values for β around 0.75 (Felipe and Adams, 2005), and from 0.35 to 0.91 (Fraiser, 2002). Thereby, the original data set of Cobb and Douglas is investigated, but with a different methodology, including, for example, technical progress. To some extend, the assumption of constants returns to scale have to be rejected (Fraiser, 2002)—whereby this is not verified by Felipe and Adams, 2005. Nevertheless, these studies investigate production functions on the *aggregate* level, so that we do not pursue this review—we rather switch to micro level specifications: Unfortunately, there are only few empirical estimations for firm–level data available. Biørn et al., 2004, use firm specific data to obtain aggregate production functions for the industry level. Their data comprise two Norwegian

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[8] We do, however, not incorporate 3.5 million firm agents into the model. If we assumed that the 500 firm agents represent 3.5 million firms in reality, each firm agent represents 7,000 real–world firms. According to the German data of 40 million households, this would imply that each firm agents has to employ 80,000 ($= 40,000,000/500$) households. If we, in addition, assume that one household agent represents 7,000 real–world households, we obtain a ratio of 11.4. This ratio stands for ϖ.

[9] In fact, the credit for already representing a hidden 'Cobb–Douglas' production function in the 18th century has to go to Thünen, 1930 (reprint). He modifies his original production function in such a way that labor is able to produce output even without any capital, which is also in contrast to the modern 'Cobb–Douglas' specification. Importantly, the modified approach of Thünen could be an appropriate alternative for the present model, because the capital stock of an individual firm could approach 0, in some rare cases. Besides this, an early formulation of an aggregate production function similar to the 'Cobb–Douglas' specification was also made by Knut Wicksell (Wicksell, 1923).

manufacturing industries (pulp & paper: 237 firms; basic metals: 166 firms) from 1972 to 1993. In this context a 'four–factor Cobb–Douglas' production function is used (in contrast to our 'two–factor' specification), which encompasses energy and materials besides labor and capital. Biørn et al., 2004, identify average labor elasticities of 0.1717 for pulp & paper and of 0.2749 for basic metals industries. Ichniowski, 1984, investigates firm–level data of eleven U.S. paper mills from 1976 to 1982. He employs a 'three–factor Cobb–Douglas' function (with labor, capital and energy), in which he identifies an elasticity of output to labor amounting to 0.77 (and a very small elasticity to capital of 0.04). The author identifies the paper mill industry as a highly capital intensive industry. Lastly, these results indicate a broad variety of reasonable values for β, so that we decide for a broad domain between 0.4 and 0.95.[10]

Capacity threshold (expansion investment) ι: This parameter cannot be determined from empirical observations in the literature. This stems from the fact that the variable does not possess any role in 'orthodox' models. Nevertheless, it is not impossible to find a reasonable scope for the parameter: Its natural upper bound is somewhat below 100%. A lower bound for ι can be around 70%. Note that if the capacity utilization of a firm lies below ι, it would not consider any expansion investments. Thus, we opt for a domain for ι between 0.7 and 0.95.

Magnitude of expansion investment ψ: The good descriptive study of Doms and Dunne, 1998, suggests plant–level values for the expansion investment parameter ψ between 0 and 0.4. We will allow values for ψ in a range between 0.05 and 0.3. In the 'heterogenous case', all, in principle possible, individual levels for ψ are drawn form a broader set than this 'domain', namely from 0.8×0.05 to 1.2×0.3. As a result, individual levels lie in a domain pretty close to the magnitudes identified by Doms and Dunne, 1998.

Variance in the random walk term ϵ: We apply values for the variance of the 'white noise term' in the 'random walk' specification between 0.01 and 0.0001. In 'face validation' we were testing the results of these defaults: The technical progress is pretty stable if $Var[\epsilon] = 0.0001$, so that the individual $\Delta A_{world,j,k,c}$ corresponds more or less to the 'drift term'. This defines a scenario

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[10] 'Face validation' runs verified the theoretical property of the production function, namely that in case of low β–levels, the marginal costs of firms are quite high. See table 3.5 in the following sensitivity analysis. Note that variable costs in the model are exclusively based upon wage costs. Marginal costs are, in turn, obtained by the derivation of marginal cost with respect to output. In addition, the level of β affects the amount of labor employed in the production process substantially. Consequently, a very low β–level enhances marginal costs substantially.

with almost no stochastic supply shocks. In case of $Var[\epsilon] = 0.01$, however, large variations in the growth rate of $A_{world,j,k,c}$ can occur. Consequently, stochastic supply shocks are quite large. Nevertheless, in most cases of $Var[\epsilon]$ between 0.0001 and 0.01, the growth rates of the individual technology parameter $A_{j,k,c}$ stay within a reasonable range, as stated above in the discussion of the magnitude of the individual 'drift terms' $\varrho_{world,j,k,c}$. Importantly and as a simplification, we do not distinguish between the 'homogenous' and the 'heterogenous case', so that in both cases the individual levels of $Var[\epsilon_{world,j,k,c}]$ are equal for all agents.

Sensitivity of mark–ups to output gaps ω^{Macro}: The influence of the business cycle (i.e. output gaps) on 'mark–up dynamics' is theoretically and empirically ambiguous. Many studies identify countercyclical behavior of 'mark–ups'.[11] Other empirical studies admit the presumption that 'mark–ups' behave procyclical.[12] For example, Whelan, 1991, calculates a positive coefficient for the output gap (0.658) in an estimation of the aggregate price 'mark–up' (U.S. data from 1960 to 1996). Estrada and López-Salido, 2005, find coefficients for sectoral 'mark-ups' (Spain, data from 1980 to 2002) between -0.687 (countercyclical) and 0.680 (procyclical). Thereby, the machinery sector (maybe similar to the capital goods industry of Agent Island) features a procyclical coefficient of 0.254. Moreover, in our model design firm agents change 'mark–ups' with respect to *last–period* business cycle conditions. After this decisions, it will be determined ex post as to how 'mark–ups' were procyclical or countercyclical against the output gap of the present model period. Finally, several 'face validation' runs produced pretty good results with non–negative levels for ω^{Macro}. As a result, we employ a domain for ω^{Macro} between 0.2 and 0.7.

Sensitivity of mark–ups to marginal earnings ω^{Micro}: There is no direct connection in the empirical literature between 'mark–ups' and the percentage deviation of marginal revenues from marginal costs. After several 'face validation' runs it became clear that it is a good approximation to employ the domain of ω^{Macro} also to ω^{Micro}. Note that both parameters are applied in the same rule. Thus, we allow values between 0.2 and 0.7.

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[11] See, among others, Rotemberg and Woodford, 1991; Martins and Scarpetta, 1999; Marchetti, 2002; Estrada and López-Salido, 2005.

[12] See, among others, Whelan, 1991; Forsman et al., 1997; Estrada and López-Salido, 2005; Geradi and Shapiro, 2007.

Sensitivity of output (of consumer goods firms) to marginal profitability θ: This figure is not investigated in the empirical literature. Thus, we have to employ ad doc upper and lower limits for the domain of θ. During 'face validation' runs it became clear that levels below 0.01 produce an approximately constant consumer goods output. The opposite takes place for large values, i.e. above 1, so that the fluctuation in real output would be extremely large in this case. But this is not desired. Consequently, we restrict the domain of θ by an upper limit of 1 and a lower limit of 0.01.

Household Parameters

Sensitivity of the savings rate to real interest rate gap η^{RR} & income growth rate η^{IC}: In section 2.2.1 we illustrate the results of several empirical studies concerning the impact of the real interest rate and income growth on aggregate savings rate (see table 2.2). However, we employ *financial* savings rates on the *individual* level. According to that, the results indicated by the empirical literature on savings rate estimations do not deliver the right connection. The interesting point for the present study is that the literature suggests (i) positive impacts of income growth on total savings rates, and (ii) ambiguous impacts of real interest rates on total savings rates. We use these insights and restrict individual financial savings rates in the following way: We (i) employ a non–negative domain for η^{IC}, (ii) the domain for η^{IC} is broader than that for η^{RR}, and (iii) we allow negative levels for η^{RR} as well. In anticipation of the results of the following sensitivity analysis (in section 3.3), we are able to state that the impact of both 'factors' on the chosen 'responses' is less significant, that what one can expect; in fact, it is insignificant. Thus, we define ad hoc both domains in reasonable ranges: This gives levels for η^{IC} between 0.1 and 1, and for η^{RR} between -0.1 and 0.5.

Subsistence & saturation levels of consumption expenditures given through ν: The subsistence and saturation levels of consumption are defined in relation to last–period average per capita consumption expenditures. This relation is represented through the behavioral parameters ν^{upper} and ν^{lower}. As explained, this design represents a 'quasi intertemporal budget constraint' as well. In a first step, we compress both variables (as a simplification), to one single variable ν, by defining $\nu \equiv \nu^{upper}$ and $(\nu)^{-1} \equiv \nu^{lower}$.[13]

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[13] This is necessary to reduce the number of 'factors' to 16, in order to reduce, in turn, the number of scenarios in

We know that consumption expenditures differ strongly with respect to several factors, such as income, age, education, etc. For example, German data for the distribution of consumption expenditure illustrates the following relations:[14] In 2005, the average consumption expenditures were 1,996 EUR. If we investigate the consumption expenditures for various income groups (given by the division of households by income into five groups)[15], the individuals in the lowest income group (i.e. with net income below 1,300 EUR) bought, on average, consumption goods amounting to 964 EUR, whereas goods amounting to 3,753 EUR were bought by individuals in the highest income group (i.e. with net income above 5,000 EUR). If we apply these figures to our design (represented by ν), this delivers $\nu = 2.1$, or $\nu = 1.9$.[16]

Different results are obtained, if we use the division of consumption expenditures with respect to age groups. The average consumption of the oldest group was 1,415 EUR in Germany in 2005; average consumption of the age group with the highest mean consumption expenditures (45 to 55 years) was 2,214 EUR. This gives figures for ν that amount for 1.4 or 1.1. We could continue such investigations endlessly, but it would never deliver the 'right' figures for the individual level, because those investigations deliver mean levels for certain groups. The respective figures for individual households (or subgroups) with the highest and the lowest consumption expenditures would definitely differ substantially from those values. Nevertheless, we have to define a reasonable domain for ν. We think that ν–levels between 1.1 and 2.5 should serve as a good proxy for the larger parts of the household sector as a whole in reality. Note that in the 'homogenous case' all household agents feature the same level for ν_h. In the 'heterogenous case', the individual levels are again stochastic. To give an example: If we investigate the extreme scenario with $\nu = 2.5$, the individual levels ν_h may alter between 2.0 ($= 0.8 \times 2.5$) and 3.0 ($= 1.2 \times 2.5$). In this extreme instance with $\nu_h = 3.0$, the individual saturation level of agent h is nine times as large as the subsistence level. [17]

Mental accounting parameter χ: This parameter is not investigated in any empirical study.

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the NOLH design substantially; 17 'factors' would require 129 scenarios, instead of only 65 scenarios in the case of 16 'factors'.

[14] The following figures are taken from Destatis, 2007.

[15] They are not divided into quintiles, because the groups are not equal–sized

[16] These figures are obtained when we divide the expenditures of the highest income group by the average per capita expenditures, i.e. $3.753/1.996 \approx 1.9$; or, by dividing the expenditures of the group with the smallest income by the average per capita expenditures, and thereafter inverting the result. This gives $(964/1.996)^{-1} \approx 2.1$.

[17] This is given by $\nu_h^{upper}/\nu_h^{lower} = \nu/(\nu)^{-1} = 3/(3)^{-1} = 9$.

Indeed, it emerges in the model due to the fact that we had to define higher savings rates for incomes from financial assets and equity compared to work income. This is one requirement for the control of hyperinflation outcomes due to exorbitantly rising interest incomes. However, we have to define the parameter ad hoc. The lower limit is given through a level near to 1. When χ is equal to 1, the savings rate of interest (and dividend) incomes are equal to the savings rate of the work income. In addition, the upper limit of χ is 2. In this case, the maximum savings rate for the interest income is 100%, which implies that the work income savings rate is reaching its upper limit of 50%; we have defined this boundary in chapter 2. Consequently, we apply a domain between 1.1 and 2.

3.2 Initial Analysis

This section analyzes the model with respect to some basic features. We are interested in the stability of the business cycle of Agent Island in the *reruns* of *specific scenarios*, and we are interested in the main determinants of this stability. Consequently, we investigate, at first, a scenario without any stochastic supply shocks. Thereafter, we compare the ‘homogenous case’ with the ‘heterogenous case’, as they were illustrated in the last section. We should note that the analysis of this section draws on the ‘baseline case’—the ‘Ponzi case’ is not investigated here.

3.2.1 Defaults

Before turning to the basic analysis of this section, the following table 3.1 summarizes the defaults we apply. Table 3.1 contains ‘peripheral settings’, as they were already fixed within the last section. Moreover, we had to select one level–combination for the ‘main parameters’. Insofar as a validated level–combination will not be available before the end of this chapter, we use ad hoc defined settings (selected by ‘face validation’ runs). The heading ‘Main settings’ in table 3.1 illustrates these settings. Finally, we expect that the mentioned stability of output primarily depends on the parameters grouped under the heading ‘Experimental domain’ below in table 3.1. Accordingly, we investigate in this section only the three ‘factors’ depicted there. Hence, the differences in the investigated scenarios are due to the variation of these three parameters. If the reader is not aware of the assignment of the individual letters and what they stand for, please see the description in the last section.

Parameter	Level[1] homogenous	Level[1] heterogenous	Parameter	Level[1] homogenous	Level[1] heterogenous
Peripheral settings					
Period length	52 weeks		Working time p.p.	2,080 h	
Machines per firm	35	[30–40] UD	Machine run time	41,600 h	
$A_{World,j,k,c,1}$	1		w_1	0.1	
π_0	0.03		i_1	0.05	
$\mathbb{E}r_1$	0.02		$\hat{p}_0^{machine}$	750	
ϱ_j	0.025	[0.025–0.035] UD	ϱ_k	0.025	[0.005–0.015] UD
ϱ_c	0.025	[0.02–0.03] UD	ϱ_{World}	0.025	
$\hat{C}_0$	100		$\frac{C}{J+K+C}$	0.4	
$\frac{J}{J+K+C}$	0.3		$\frac{K}{J+K+C}$	0.3	
$LL_{j,k,c,0}$	2,000		$\underline{FA}_{h',1}$	0	
$\underline{FA}_{h'',1}$	2,000		$\pi*$	0.03	
$POF_{j,k}$	0.5	[0.4–0.6]	α_h	0.5	[0.35–0.65] UD
γ_c	0.03		β_c	0.5	[0.4–0.6] UD
$\mu_{c,0}$	0.5	[0.4–0.6] UD	$\tilde{s}_{h,0}$	0.075	[0—0.15] UD
Main settings					
λ	0.2		ρ	0.2	
τ	0.5		ς	0.5	
$\beta_{j,k}$	0.75	[0.6–0.9] UD	$\theta_{j,k}$	0.26	[0.208–0.312] UD
$\psi_{j,k}$	0.2	[0.16–0.24] UD	$\iota_{j,k}$	0.8	[0.64–0.96] UD
ω_c^{Micro}	0.5	[0.35–0.65] UD	ω_c^{Macro}	0.5	[0.35–0.65] UD
η_h^{RR}	0.2	[0.16–0.24] UD	η_h^{IC}	0.3	[0.24–0.36] UD
$\chi_h^{I,D}$	1.5	[1.2–1.8] UD	ν_h^{upper}	1.5	[1.2–1.8]UD
ν_h^{lower}	0.5	[0.35–0.65] UD			
Experimental domain					
$(J+K+C)$	[10, 50, 100, 250, 500]		ϖ	[5, 10, 20, 30]	
$Var[\epsilon]$	[0.01, 0.001, 0.0001]				

Note: 1) The shortcut 'UD' stands for a 'uniform distribution'; the figures in the bracket indicate the statistical population of the distribution.

Table 3.1: Model settings within this section

3.2.2 Homogenous Case without Stochastic Supply Shocks

In the next step, we describe the results of a scenario where firms are not affected by stochastic supply shocks. This means we set the variance of the 'white noise' term $Var[\epsilon]$ in the exogenous technical progress of all relevant agents, i.e. hash, bean, and capital firms as well as the 'world' to 0. Hence, technical progress on Agent Island is no longer a 'random walk with drift', but a deterministic growth process. Stochastic supply shocks are therefore ruled out. Nevertheless, there are still stochastic elements that influence the supply decision of firms and the whole simulation. In particular, the capital market allocation is still dominated by a random search process. It is the aim of this subsection to investigate the remaining randomness of the model in a scenario without any stochastic supply shocks and with 'homogenous agents' within each sector. As a result, we eliminate all sources of randomness in the model, except for the inherent sources, which we cannot eliminate by parameter adjustments.

We call this setting with 'homogenous agents', and with a deterministic technical growth process the 'homogenous case without stochastic supply shocks'. Figure 3.1 shows the inflation and real output growth[18] time series in this case. We display the results for 10 simulation reruns; each run spans 20 periods. The illustration includes the following three scenarios:

1. $J + K + C = 500$, and $\varpi = 20$;
2. $J + K + C = 500$, and $\varpi = 10$;
3. $J + K + C = 500$, and $\varpi = 5$.

These three scenarios are taken from the full sketch of all examined scenarios in the 'homogenous case without stochastic supply shocks'. They are represented by the 'Experimental domain' in table 3.1, where we apply additionally the fact that $Var[epsilon] = 0$. Therefore, the full list of scenarios contains all combinations of $J + K + C$ (i.e. 10, 50, 100, 250, 500) and ϖ (i.e. 5, 10, 20, 30). This gives in sum 20 ($= 5 \times 4$) scenarios. The interested reader can review the time series of the full

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[18] In this section we prefer the output growth rate compared to the output gap. The former delivers better results with respect to the objective, i.e. the equivalence of time series becomes apparent through real output growth rates, but not so apparent through output gaps. See also footnote 23 below.

sketch of these 20 scenarios, with 47 reruns for each scenario, on the CD in appendix C.[19]

Although we eliminate most sources of randomness, the 10 displayed reruns illustrate some remaining variances in the data. Besides this, it is apparent that the various patterns in the 'responses' of these scenarios are due to the setting of the 'factor' ϖ. The global parameter ϖ defines the number of workers per firm. Firstly, it defines the total number of H household agents in the simulation via $H = \varpi(J + K + C)$. Therefore, it is not surprising that the model output is less stable in case of $\varpi = 5$, compared to larger levels of ϖ. Secondly, ϖ delivers the quantity of labor applicable to the production process. It determines indirectly the capacity limit of capital and consumer goods firms. However, a shift in labor resources, induced through a variation of ϖ, does not influence the consumer goods market: Consumer goods demand as well as supply are both reduced or enhanced proportionately through ϖ. That is, the market clearing price in the consumer good market is only marginally affected through ϖ. But this phenomenon does not apply to the capital goods markets. In this case, the amount of labor resources influence the supply side—but not immediately the demand side. This caused by the initial capital stock (i.e. the endowment of machines) of each consumer goods firm, which is fixed to 35 (see table 3.1) throughout the whole study. As mentioned, replacement and expansion investments depend on this initial capital stock. The demand for capital in the initial periods should therefore be within a narrow range in each of the scenario reruns illustrated in figure 3.1. At the same time, the capacity limit of a capital goods firm (i.e. its maximum supply) varies within these scenarios due to ϖ. The same applies to the marginal costs of producing a certain amount of machines; it varies with ϖ. Average machine supply prices vary therefore as well. As a result, the relation of machine supply conditions to machine demand conditions is changing with ϖ.[20] For example, in the bottom scenario ($\varpi = 5$) the capacity of the capital sector is 1/4 compared to the upper scenario ($\varpi = 20$), while capital demand is almost unchanged (at least in the initial periods). This phenomenon seems to affect the business cycle behavior, especially that of inflation and output growth rates, in the way illustrated in the charts of figure 3.1. Note that all parameters, except for the two varied 'factors', are held constant when switching from one scenario to another.

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[19] See the Excel files on the CD attached in appendix C.

[20] It should be mentioned that the ratio of capital goods firms to consumer goods firms is constant throughout this study. Hence, this cannot influence the capital market.

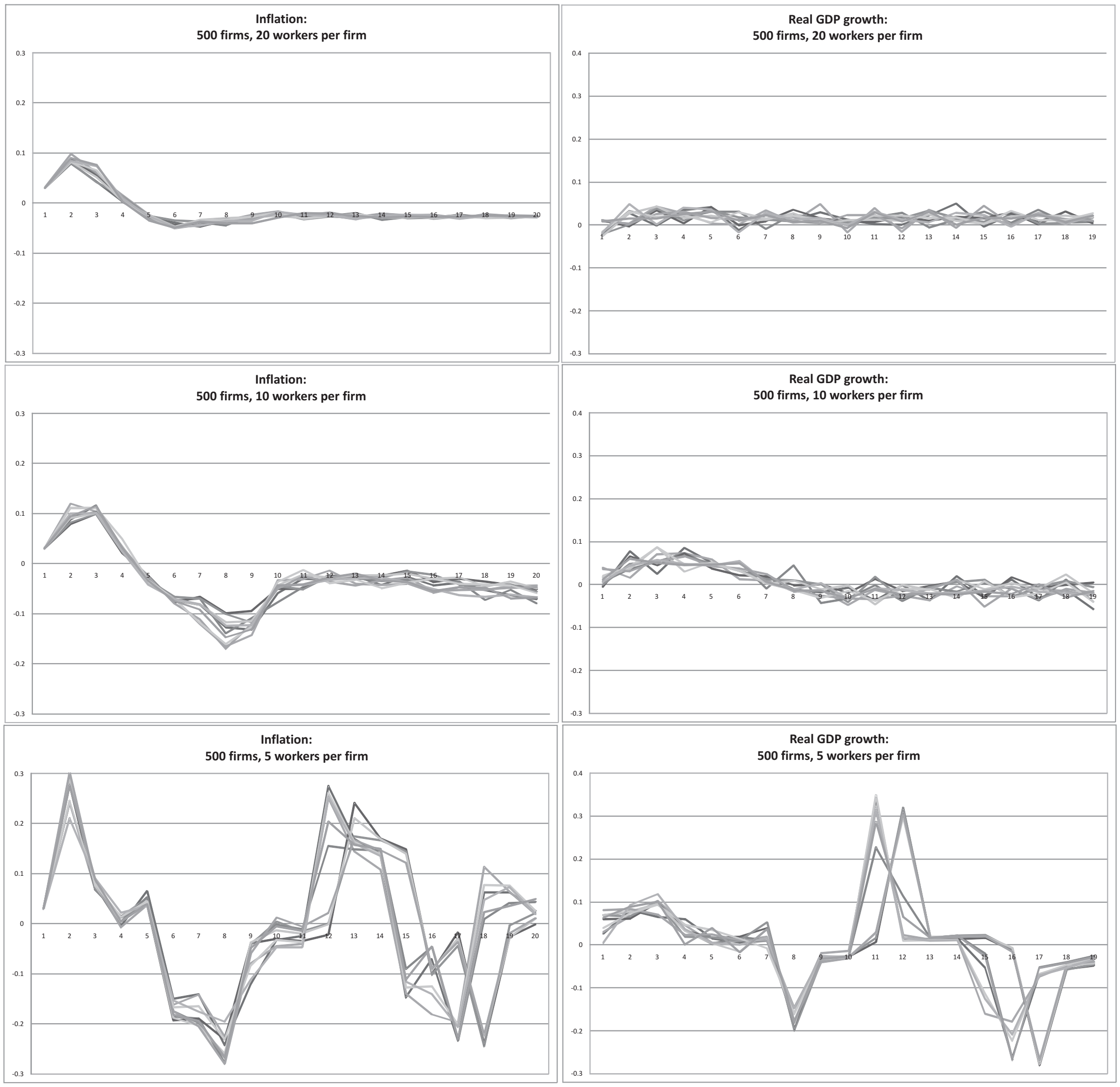

Figure 3.1: Homogenous case without stochastic supply shocks (10 reruns) – inflation time series (left panels) and output growth time series (right panels)

3.2.3 Homogenous vs. Heterogenous Sectors

In order to develop a reasonable validated macro model it is necessary to investigate all kinds of possible ‘factors’ that influence the ‘responses’ of interest. At this point it is, first of all, interesting to what extend the model output (i.e. the defined ‘response’) is stable during several reruns. Thereby, we apply a analysis where we observe and compare the ‘responses’ in the case where all agents in one sector behave identically, and in another case where they do not behave identically. As explained in the last section, we call the former a case with ‘homogenous sectors’, and the latter a case with ‘heterogenous sectors’. Hence, it is the objective of this subsection to compare the stability of simulation output with reference to the heterogeneity of agents in both sectors.

In this context it is important that the overall number of agents in an agent–based simulation is, first of all, a matter of time resources and computing power resources. Time resources constrain the number of agents to a certain upper bound; only below this bound a single simulation run is delivered in a reasonable time frame. In simulation experiments it often appears that several hundreds (or even thousands) single simulation runs have to be conducted. A single run should thus not exceed a time span of some minutes, in order to receive results within a few days. Obviously, the total number of agents affects this time span: For example, if we run the simulation in a scenario with only three firm agents (one hash, one bean and one capital firm) and with three private household agents (each working for one of the three firms), a single simulation run would be really short—probably some seconds long. The crucial aspect of such a scenario is that it would induce strong variances in the simulation output. One could assume that the stochastic elements of the model (i.e. supply shocks and stochastic individual parameters) will produce large variances in the simulation outputs (inflation, growth rates, etc.), if we rerun the simulation several times. In contrast, if there are many thousand firm and household agents, one can assume that many stochastic elements will cancel each other out.[21] This seems reasonable, due to the ‘law of large numbers’.[22] According to that,

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[21] This is especially true in case of the presented simulation. The cause of this phenomenon lies in our trivial specification of heterogeneity: Agents in one sector are different with respect to parameter(s), but not with respect to the rules types. Thus, parameters differ, but rules are equal. A more sophisticated approach to ‘heterogeneity’ would specify agents in one sector with different parameters *and* with different rules.

[22] The ‘law of large numbers’ is a theorem in probability theory that describes the long–term stability of a random variable. Given a sample of independent and identically distributed random variables with an expected value, the average of these observations will eventually approach and stay close to the expected value as the number of observations becomes large.

we assume that the sum total of agents on Agent Island influences the variance of simulation output.

As explained above, another source of randomness is the technical progress captured by an exogenous stochastic process. Technical progress does not only affect firms' capacity limits, but also marginal costs, which in turn influence supply decisions. Therefore, large variances of technical progress induces randomness in several decisions and therefore in the simulation 'responses' as well. This captures the idea of stochastic supply shocks. To sum up, the following 'experimental domain' has to be investigated:

1. **Number of firms ($J + K + C$):** 10, 50, 100, 250 and 500;
2. **Number of workers per firm (ϖ):** 5, 10, 20, and 30;
3. **Variance of the white–noise term ($Var[\epsilon]$):** 0.01, 0.001 and 0.0001.

Finally, it is necessary to identify reasonable 'responses' for our analysis. As mentioned, we investigate inflation and real output growth rates[23] (as displayed in figure 3.1) as the relevant simulation 'responses'. In order to compare the stability of the results in the 'homogenous' and 'heterogenous case' we conduct experiments with 47 simulation reruns. Thereby, the general framework of this section encompasses 60 possible scenarios, given through the possible level–combinations delivered by the enumeration above. Accordingly, we investigate in the present context a $5 \times 4 \times 3$ 'full factorial design' with 47 reruns. This gives in sum 2,820 (47×60) single simulations runs, which we have to evaluate. The interested reader can investigate the time series of those 2,820 simulation runs on the CD in the appendix.[24]

The question, as to how the 47 reruns for each scenario produce equal results or not, is fulfilled

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[23] We prefer output growth rates compared to output gaps, because the former delivers better results. As in reality, it is also difficult in agent–based data to define the correct output gap. This stems from the definition of the 'production potential', which is crucial for the determination of the output gap: According to Okun, the 'production potential' is defined as the economic output that occurs under 'full employment' (Okun, 1962). But full employment itself is a term that is not easy to define. Nowadays, one thinks of the 'production potential' as the output, which is generated without producing inflation, i.e. 'production potential' captures the notion of a 'neutral' growth path (SVR, 2003). In this study we follow the basic idea that the historical average capacity utilization delivers the 'production potential'. Consequently, output gaps depend on initial capacity utilizations, so that there can occur variances in output gaps due to variances in initial capacity utilizations. Moreover, a simulation starts without any historical data. As a result of these facts, we use the real output growth rates instead of output gaps within this section.

[24] See the Excel files on the CD in appendix C.

by an ‘analysis of variances’ (ANOVA):[25] The basis of an ANOVA–test are k groups of observations (in our case these groups are inflation or output growth rate time series) that are known to have been generated by k independent normal distributions with identical variances and respective means, $\mu_1, \mu_2, \ldots, \mu_k$. Hence, the ANOVA–test relies on two assumptions:

1. Normal distribution of the observations of each time series;
2. Homogeneity of the variances of these normal distributions.

Provided that these conditions are satisfied, ANOVA will test the ‘null hypothesis’ $H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$, against the alternative hypothesis H_1, viz.: At least one of these means is different from the others. Within the analysis, a significance level α (typically 1%, 5% or 10%) is chosen, and ANOVA produces a p–value (i.e. a ‘probability value’): If the p–value is less than α, the ‘null hypothesis’ of the equal means will be rejected. Otherwise, if the p–value is larger than α, the conclusion will be that the data is not compatible (at the chosen significance level) with the rejection of the H_0 hypothesis that all means are equal. Note, that this is not equal to a confirmation that H_0 is true. One can only guess that H_0 is true.[26]

Appendix A shows the results for the ANOVA tests conducted for all 60 scenarios for both inflation and real output growth time series. In addition, the data in appendix A depicts also the ‘Bartlett’ and the ‘Levene test’[27] of the homogeneity of the variances. The homogeneity of the variances does not only reveal interesting properties concerning the equality of the time series, it constitutes also one of the two conditions of the ANOVA test, as stated above. Table 3.2 summarizes the best candidates with respect to equality of the time series of reruns in the ‘homogenous case’. These candidates are supposed to have equal means *and* variances (of inflation *and* output growth time series). In addition, all relevant time series should be distributed normally: If the last two columns show a low ratio (this ratio indicates the number of not normal distributed time series to the total number of time series), this assumption appears to be fulfilled. Table 3.2 illustrates 15 candidates. The investigation of the normal distributions shows, however, that many candidates do

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[25] The ANOVA f-test is conducted with a standard statistical software tool. We use the EViews 6.0 software package.

[26] This guess can be subject to an ‘error of second order’, namely that the ‘null hypothesis’ is maintained, even though it is not true.

[27] Again, both tests are conducted with EViews 6.0.

Agents[1]	Firms	Households per firm	Variance of random walk	ND inflation[2]	ND output growth[2]
15,500	500	30	0.0001	47/47	6/47
10,500	500	20	0.0001	47/47	3/47
7,750	250	30	0.001	47/47	3/47
7,750	250	30	0.0001	47/47	2/47
5,500	500	10	0.0001	16/47	0/47
5,250	250	20	0.0001	47/47	1/47
3,000	500	5	0.001	4/47	18/47
3,000	500	5	0.0001	0/47	21/47
1,500	250	5	0.001	1/47	18/47
1,500	250	5	0.0001	0/47	14/47
600	100	5	0.001	0/47	22/47
600	100	5	0.0001	0/47	23/47
300	50	5	0.0001	0/47	25/47
60	10	5	0.001	0/47	18/47
60	10	5	0.0001	1/47	22/47

Note: 1) Number of agents = $(1 + \varpi)(J + K + C)$. 2) Lower ratios indicate a better fit of the normal distribution assumption. The normal distribution of the time series is tested with ‘Jarque–Bera’. Again, tests are conducted with EViews 6.0. ‘Jarque–Bera’ is used to test the null hypothesis that the a time series is distributed normally. If the p-value is smaller than 0.1, the null hypothesis is rejected with 10% significance level. Column 5 depicts the ratio of not normal distributed series to total time series for the case of inflation rates, column 6 for the case of output growth rates.

Table 3.2: Scenarios that produce stable results in the homogenous case

not meet the normal distribution assumption. In case of ‘heterogenous agents’, displayed in table 3.3, similar results are obtained: 10 principle candidates are identified, 9 of these 10 candidates are likewise candidates on table 3.2. The larger quantity of stable scenarios in table 3.2 compared to table 3.3 is obviously due to the fact that the ‘homogenous case’ produces more stable results than the ‘heterogenous case’. As explained above, this result was expected.

Obviously, the dominant ‘factor’ in both tables is the variance of the ‘random walk term’, which defines the amplitude of stochastic supply side shocks. We find out that in scenarios where $Var[\epsilon]$ is set to 0.0001, many stable results are obtained. Consequently, this ‘factor’ marks an important determinant of the stability of output in simulation reruns: The smaller the amplitude of stochastic supply shocks, the more stable is the data generated by simulation reruns. This is a second (also expected) insight of the present analysis. What is striking is that a low value for ϖ generates many stable results as well: Surprisingly, some stable scenarios feature pretty low total numbers of agents (some below 1,000). This fact stems from the phenomenon that ϖ affects the relative size of capital supply to capital demand—as described above. The scenarios with $\varpi = 5$ exhibit a large unsatisfied 'excess demand' in the capital goods market, so that random rationing takes place (see also footnote 29). If the capital stock is not able to grow, due to the unsatisfied demand, this results in limited output expansion of the consumer goods industry, which in turn stabilizes inflation. In addition, if the capital goods supply rests at its limit, this, of course, leads to stable outputs and incomes generated in the capital goods sector. In sum, scenarios with $\varpi = 5$ produce undesired stabilizing effects, built upon capital rationing, so that it seems reasonable to fade out those scenarios. By doing this, only six (three) results remain in table 3.2 (3.3). Hence, the final result is that 5.250 (7.750) or more agents are necessary to deliver stable results in the case of homogenous (heterogenous) agents.

Agents[1]	Firms	Households per firm	Variance of random walk	ND inflation[2]	ND output growth[2]
15,500	500	30	0.0001	35/47	4/47
10,500	500	20	0.0001	24/47	3/47
7,750	250	30	0.0001	45/47	5/47
3,000	500	5	0.001	0/47	17/47
3,000	500	5	0.0001	0/47	13/47
1,500	250	5	0.001	0/47	6/47
1,500	250	5	0.0001	0/47	11/47
600	100	5	0.0001	0/47	8/47
300	50	5	0.001	0/47	16/47
300	50	5	0.0001	0/47	10/47

Note: 1) Number of agents = $(1 + \varpi)(J + K + C)$. 2) Lower ratios indicate a better fit of the normal distribution assumption. The normal distribution of the time series is tested with 'Jarque–Bera'. Again, tests are conducted with EViews 6.0. 'Jarque–Bera' is used to test the null hypothesis that the a time series is distributed normally. If the p-value is smaller than 0.1, the null hypothesis is rejected with 10% significance level. Column 5 depicts the ratio of not normal distributed series to total time series for the case of inflation rates, column 6 for the case of output growth rates.

Table 3.3: Scenarios that produce stable results in the heterogenous case

If we account for the normal distribution assumption as well, the best results are obtained with: (i) 5,500 agents, 500 firms, 10 households per firm and $Var[\epsilon] = 0.0001$ in case of 'homogenous agents'; and (ii) 10,500 agents, 500 firms, 20 households per firm and $Var[\epsilon] = 0.0001$ in case of 'heterogenous agents'. In most other cases the normal distribution assumption is strongly violated[28], or ϖ seems too low.

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[28] It is interesting to note to what extend inflation and output growth time series of real–world data are distributed normally: A test for the normal distribution of real GDP growth rates for 23 OECD countries between 1983 and 2007 delivers the result that in 13 cases the 'null hypothesis' (that the data are distributed normally) has to be rejected. The inflation time series of 47 countries (European and some other important Non–European countries, data between 1961 and 2007) indicate similar results, i.e. for 29 of 47 time series the 'null hypothesis' (that the data is distributed normally) must be rejected.

3.2.4 Summary of the Results

To sum up, we gain the following insights in this section:

- In the 'homogenous case without stochastic supply shocks' the randomness in the 'responses' does not entirely disappear.
- The global parameter ϖ exhibits a strong influence on business cycle patterns. This influence stems from the conditions on the capital goods market, which vary with the level of ϖ: If the level of ϖ is too low (e.g. $\varpi = 5$), undesired stabilizing effects on business cycle dynamics occur. This is caused by capital rationing and unsatisfied 'excess demand' in the capital goods market. Hence, the validated model should not be adjusted to very low ϖ–levels, even though they produce desirably stable outputs.[29]
- As expected, the 'homogenous case' generates more stable results than the 'heterogenous case'. But the difference is not extraordinary large.[30]
- More than 5.250 (7.750) agents are necessary to deliver stable results in case of homogenous (heterogenous) agents.
- Among the three investigated 'factors', the amplitude of stochastic supply side shocks (i.e. the level of $Var[\epsilon]$) appears to be the most important determinant of the stability of the business cycle with respect to simulation replications. Importantly, in the 'heterogenous case', the simulation output could be stabilized only by very low stochastic supply shocks (i.e. $Var[\epsilon] = 0.0001$). Provided that we exclude the undesired scenarios with $\varpi = 5$, all stable scenarios in the 'heterogenous case' include $Var[\epsilon] = 0.0001$. In fact, we do not restrict $Var[\epsilon]$ to such low levels in the remainder of this study. This would be misleading.

Due to our preference for a heterogenous design, the remainder of this study is based upon 'heterogenous sectors'. Moreover, we decide for a quantity of 500 firm agents, i.e. $(J+K+C) = 500$. This should guarantee at least some stability of the model outputs. Additionally, we will investigate

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[29] During early calibration steps we decide for such a scenario with $\varpi = 5$. Only after many 'face validation' runs it became clear that this setting produces undesired effects: We find out that the capacity utilization of capital goods firms were far too high—in fact firms were permanently at their capacity limits. Under such conditions the simulation does not generate reasonable results. The obtained stability of the simulation output was based upon a 'too small' capital goods industry.

[30] See the Excel files on the CD in appendix C.

the impact of stochastic supply shocks (measured by $Var[\epsilon]$) on the ‘responses’ during further sensitivity experiments in the next section. Insofar as we will allow large levels for $Var[\epsilon]$, the stability of the model output is not perfectly guaranteed. But this is an effect of the stochastic elements of the model. It would be misleading to fade them out. It is a philosophic question, as to how the reality would produce the same results, if we could ‘rerun’ it (the reality) several times. In short, it is fair to say that a model comprising stochastic elements is not able to produce perfect stable outputs, even though it would be desired from the perspective of simulation analysis and model ‘validation’.

3.3 Sensitivity Analysis

During the sensitivity analysis, the impacts of the ‘factors’ on several ‘responses’ are analyzed based upon computer experiments. As mentioned, we concentrate on inflation and output gap ‘responses’. Thereby, we pursue two objectives: (i) We inspect the connection between ‘responses’ and the ‘factors’. These connections should correspond to the micro–macro interrelations that are expected according to our model design. (ii) Equally important, we identify ‘factors’ without any significance for the ‘responses’. Those insignificant ‘factors’ drop out from the subsequent calibration process. In order to reach these objectives, it is first of all necessary to define reasonable ‘experimental points’ (concerning the ‘factors’). This is done in subsection 3.3.1. Thereafter, in subsection 3.3.2, we explain some ‘presentation tools’ necessary to present the results of the sensitivity analysis. The subsequent subsection discusses some ‘preliminary considerations’, before we turn to the discussion of the sensitivity analysis in the ‘baseline case’ (in subsection 3.3.4) and in the ‘Ponzi case’ (in subsection 3.3.5).

3.3.1 Defaults

The aim of the sensitivity analysis of this section is to identify and investigate the ‘factors’ of the model, which determine inflation and output gap ‘responses’. As usual in computer experiments the investigated parameters are called ‘factors’ or ‘input variables’. Thereby, we draw on the ‘main parameters’ determined in subsection 3.1.2—they give the experimental ‘factors’ or the ‘input variables’. Table 3.4 displays the total experimental domain (or ‘input variable space’) for the sensitivity analysis. Besides this, we use the settings of the ‘peripheral parameters’ represented in subsection

3.1.1 and in table 3.1. As already explained in subsection 3.1.2 and indicated by table 3.4, we decide for 16 ‘factors’. In principle, the ‘factors’ are continuous variables within the domains presented in table 3.4. Thus, there are infinite ‘factor–level–combinations’. The reduction of these infinite scenarios to a reasonable and manageable quantity is arranged by a ‘Nearly Orthogonal Latin Hypercube’ (NOLH).[31] The design of such a NOLH for 16 ‘factors’ produces a subset of only 65 scenarios. Hence, we employ a ‘fractional factorial design’ with a subset of 65 design points instead of the infinite design points in the ‘full factorial design’. The point is that the few scenarios of the NOLH–subset cover the full set of scenarios approximately. See appendix B for the concrete level combinations of the 65 investigated scenarios. Note that we run ten replications for each of the scenarios, so that the analysis comprises output data of 650 (= 65 × 10) single simulation runs. Therefore, we obtain a ‘factor matrix’ F with a dimension of 650×16.[32]

Factor[1]	Level	Factor[1]	Level
Experimental domain[3]			
β	[0.40, . . . 0.95]	ϖ	[5, . . . 30][2]
$Var[\epsilon]$	[0.0001, . . . 0.01]	θ	[0.01, . . . 1.00]
ψ	[0.05, . . . 0.30]	ι	[0.70, . . . 0.95]
ω^{Micro}	[0.20, . . . 0.70]	ω^{Macro}	[0.20, . . . 0.70]
η^{RR}	[−0.10, . . . 0.50]	η^{IC}	[0.10, . . . 1.00]
$\chi^{I,D}$	[1.10, . . . 2.00]	ν	[1.10, . . . 2.50]
ρ	[0.20, . . . 0.90]	τ	[1, . . . 4]
ς	[0.20, . . . 2.00]	λ	[0.04,. . . 0.40]

Note: 1) The term ‘factor’ contains all structural and behavioral parameters, initial values and endowments, which have to be determined before starting a simulation run. 2) The factor ϖ is a discrete integer variable. 3) The individual levels for each agent are drawn out of a uniform distribution ±20% around the investigated level of the parameter. This applies to all parameter except for $Var[\epsilon]$ (which is equal for all agents) and the global parameters ϖ, λ, τ, ς, and ρ.

Table 3.4: Experimental domain/input variable space

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[31] See chapter 1.3 for a description of the NOLH design.

[32] The 650 rows of the matrix are given by 65 single scenarios times 10 replications.

3.3.2 Presentation Tools

The sensitivity analysis of this section is conducted with the JMP 7.0 statistical software package. We use the 'Gaussian Process' platform for our analysis.[33] Within this platform several tools (mostly graphical) are available to display the results of the estimation. We use the following:

Actual vs. Predicted Plot: The 'Actual by Predicted Plot' illustrates the actual 'response' values on the y–axis and the predicted values on the x–axis. One measure of goodness–of–fit is how well the points lie along the 45 degree diagonal line. See, for example, the plots (with bad results) in figure 3.2 just below.

Model Report: The 'Model Report' shows a functional ANOVA table for the estimations. See, for example, the reports in figure 3.2 just below. The term 'Total Sensitivity' (third column) shows the integrated variability over the entire experimental space. For each covariate, we create a marginal prediction formula by averaging the overall prediction formula over the values of all the other 'factors'. The functional main effect (forth column) is the integrated total variation due to one 'factor' alone. Functional interaction effects, computed in a similar way, are also listed in the 'Model Report' table. Summing up the values of the main effect (forth column) and all interaction terms (starting with the fifth column) gives the 'Total Sensitivity' (third column), i.e. the amount of influence a 'factor' and all its two–way interactions have on the 'response' variable. The model reports the figure $-2 \times log$(Likelihood), that is, is minus two times the natural log of the likelihood function evaluated at the best–fit parameter estimates. This measure gives a quantitative goodness–of–fit, whereby smaller values are better fits.

Marginal Plots: These plots show the average value of each 'factor' across all other 'factors'. See, for example, the plots in figure 3.4 below. The deviations of actual data points from the marginal plot are produced by the variability due to all other 'factors'. The point is, only if the importance of one 'factor' was overwhelming, the actual data points would have been lying more or less around the marginal plot of the dominant 'factor'.

Surface Plots: See, for example, the plots in figure 3.5 below. These graphs show a three-dimensional surface plot of the 'response' surface, i.e. they depict the main and interaction

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[33] Again, chapter 1.3 gives a short review of the applied methodology. There, we explain the estimation technique of the 'Gaussian Kriging' method.

influence of two ‘factors’, given a setting of all other ‘factors’. Regularly, all other ‘factors’ are fixed to a level in the middle of their domain. For example, when investigating any interaction effect of θ and τ, the parameter β is always fixed to 0.675 (i.e. this represents the mean of the domain, given through $\frac{(0.4+0.95)}{2}$).

3.3.3 Preliminary Considerations

Before starting the sensitivity analysis, we have to tackle one problem. The estimation (of inflation ‘responses’, i.e. expected value μ^{π} and standard deviation σ^{π}) over the full data set, delivered by 650 simulation runs, indicate some non–satisfying results:

1. **Results not as predicted:** For instance, neither of the households behavioral parameters incorporated into the saving decision (η, ν or χ) exhibits any impact on μ^{π}. Besides this, the wage–setting parameter (λ) as well as the the weight on inflation gap in the ‘Taylor rule’ (τ) do not feature any influence. In this case, inflation mean is only influenced by β, θ, and ς. See the ‘Total Sensitivity’ in the (lower) model report in figure 3.2. The results do not appear very plausible. The reason for this anomaly brings us to the next point.

2. **Distortion due to outliers:** See the ‘Actual by Predicted Plots’ in figure 3.2. There, one can identify ranges for μ^{π} up to 10 (i.e. 1,000%) and σ^{π} up to 30 (3,000%). If we exclude magnitudes for μ^{π} and σ^{π} above 1 (i.e. 100%), 16 of the 650 ‘response’ figures will be canceled out. The named scenarios coincide for both ‘responses’, i.e., if μ^{π} is extremely large, σ^{π} is likewise very large. The interesting point is that these 16 cases belong to just two scenarios and their replications. These two scenarios obviously induce hyperinflation. By investigating both hyperinflation level combinations, we find out that (i) in one case θ and ν are set to their maximum values, i.e. $\theta = 1$ and $\nu = 2.5$, and (ii) in the second case θ is quite close to 1 (i.e. $\theta = 0.907$), and β is at the same time very low (i.e. $\beta = 0.495$). Hence, θ, ν and β seem to exhibit a large impact on inflation ‘responses’; we will discuss that topic below. See also the estimation results for those parameters in figure 3.2.

3. **Goodness–of–fit not optimal:** The maximum likelihood measure[34] seems to be oversized.

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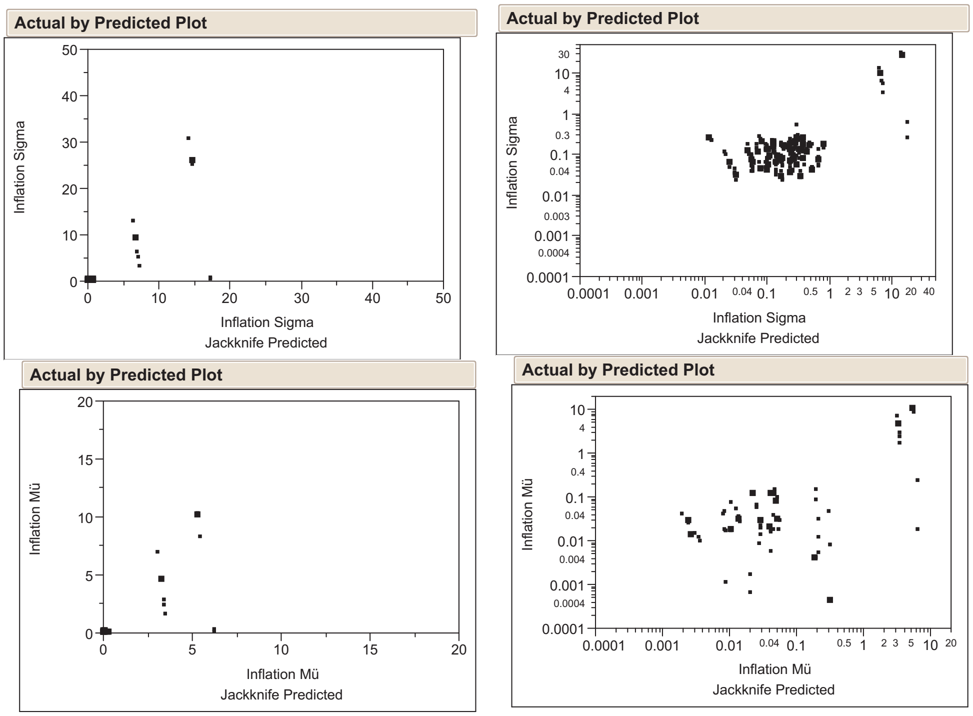
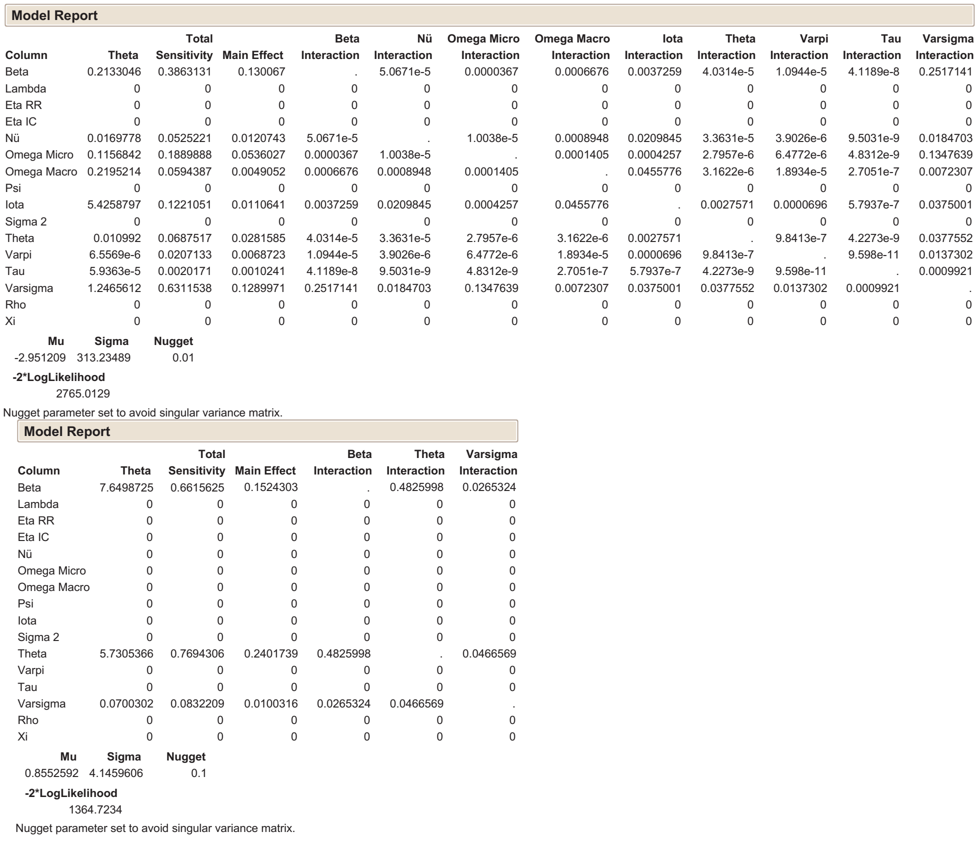
[34] The model reports the figure $-2*$LogLikelihood, that is, is minus two times the log of the likelihood function evaluated at the best–fit parameter estimates. Smaller values are better fits. Values can be positive or negative, and negative values describe a better fit.

In case of μ^{π}, the measure indicates 1,364.7 (in case of σ^{π}, 2,765). If we exclude the outliers, as described in the last point, the measure falls to -1,218.1 (-960,7), which indicates a substantial better goodness–of–fit. See, for example, the reports in figure 3.3 for the the case, where outliers are excluded. In addition, a good graphical measure of goodness–of–fit is how well the data points lie in the 'Actual by Predicted Plots' along the 45 degree diagonal. Especially the log–scaled plots highlight this point: In log-scaled graphs the densely populated area for low values of μ^{π} and σ^{π} is stretched. Thereby rather small deviations in the log–scaling stand for large absolute deviations. See the bad result illustrated by the plots in figure 3.2.

According to these discoveries, we exclude outliers with magnitudes of the inflation 'responses' above 1 (i.e. 100%): Level combinations that produce such hyperinflationary output data distort the result we are interested for. We think that within a limited set of scenarios, inflation figures stay within a narrow and reasonable band. If we employ scenarios that are beyond this limited set, the model falls into an area, where inflation is out of control. As a result, we identify and exclude these problematic scenarios, as it is one aim of this study (in the sense of model 'validation') to identify and exclude level–combinations that produce hyperinflation. To sum up, a scenario with a very high θ–level in combination with a low β–level is vulnerable to hyperinflation. The first interrelation seems obvious as θ defines the sensitivity of consumer goods firms' output adjustment to marginal profitability. If this sensitivity is high, output is pretty volatile in case of a volatile marginal profitability. Furthermore, β determines the output elasticity of labor in the production function, and it measures the responsiveness of output to a change in levels of labor used in production (c.p.). For example, when β is set to a low 0.45, a 1% increase in labor would lead approximately to a 0.45% increase in output. But the mechanism in the model works the other way around: A given, probably large, increase in output, according to a large θ–level in the output–adjustment rule of consumer goods firms, induces an extreme increase of labor demand due to a small β. This affects labor incomes and (c.p.) consumption expenditures substantially. Apparently, the combination of (i) a large increase in output, due to a high θ–figure, and (ii) a relatively large increase in labor demand, due to a small β–figure, leads to large and volatile inflation figures. Finally, the second hyperinflation scenario features a large level for ν. It is obvious that this influences consumption expenditures, insofar as ν defines the intertemporal budget constraint of household agents. If this





Model Report

Column	Theta	Total Sensitivity	Main Effect	Beta Interaction	Nü Interaction	Omega Micro Interaction	Omega Macro Interaction	Iota Interaction	Theta Interaction	Varpi Interaction	Tau Interaction	Varsigma Interaction
Beta	0.2133046	0.3863131	0.130067	.	5.0671e-5	0.0000367	0.0006676	0.0037259	4.0314e-5	1.0944e-5	4.1189e-8	0.2517141
Lambda	0	0	0	0	0	0	0	0	0	0	0	0
Eta RR	0	0	0	0	0	0	0	0	0	0	0	0
Eta IC	0	0	0	0	0	0	0	0	0	0	0	0
Nü	0.0169778	0.0525221	0.0120743	5.0671e-5	.	1.0038e-5	0.0008948	0.0209845	3.3631e-5	3.9026e-6	9.5031e-9	0.0184703
Omega Micro	0.1156842	0.1889888	0.0536027	0.0000367	1.0038e-5	.	0.0001405	0.0004257	2.7957e-6	6.4772e-6	4.8312e-9	0.1347639
Omega Macro	0.2195214	0.0594387	0.0049052	0.0006676	0.0008948	0.0001405	.	0.0455776	3.1622e-6	1.8934e-5	2.7051e-7	0.0072307
Psi	0	0	0	0	0	0	0	0	0	0	0	0
Iota	5.4258797	0.1221051	0.0110641	0.0037259	0.0209845	0.0004257	0.0455776	.	0.0027571	0.0000696	5.7937e-7	0.0375001
Sigma 2	0	0	0	0	0	0	0	0	0	0	0	0
Theta	0.010992	0.0687517	0.0281585	4.0314e-5	3.3631e-5	2.7957e-6	3.1622e-6	0.0027571	.	9.8413e-7	4.2273e-9	0.0377552
Varpi	6.5569e-6	0.0207133	0.0068723	1.0944e-5	3.9026e-6	6.4772e-6	1.8934e-5	0.0000696	9.8413e-7	.	9.598e-11	0.0137302
Tau	5.9363e-5	0.0020171	0.0010241	4.1189e-8	9.5031e-9	4.8312e-9	2.7051e-7	5.7937e-7	4.2273e-9	9.598e-11	.	0.0009921
Varsigma	1.2465612	0.6311538	0.1289971	0.2517141	0.0184703	0.1347639	0.0072307	0.0375001	0.0377552	0.0137302	0.0009921	.
Rho	0	0	0	0	0	0	0	0	0	0	0	0
Xi	0	0	0	0	0	0	0	0	0	0	0	0

Mu	Sigma	Nugget
-2.951209	313.23489	0.01

-2*LogLikelihood
2765.0129

Nugget parameter set to avoid singular variance matrix.

Model Report

Column	Theta	Total Sensitivity	Main Effect	Beta Interaction	Theta Interaction	Varsigma Interaction
Beta	7.6498725	0.6615625	0.1524303	.	0.4825998	0.0265324
Lambda	0	0	0	0	0	0
Eta RR	0	0	0	0	0	0
Eta IC	0	0	0	0	0	0
Nü	0	0	0	0	0	0
Omega Micro	0	0	0	0	0	0
Omega Macro	0	0	0	0	0	0
Psi	0	0	0	0	0	0
Iota	0	0	0	0	0	0
Sigma 2	0	0	0	0	0	0
Theta	5.7305366	0.7694306	0.2401739	0.4825998	.	0.0466569
Varpi	0	0	0	0	0	0
Tau	0	0	0	0	0	0
Varsigma	0.0700302	0.0832209	0.0100316	0.0265324	0.0466569	.
Rho	0	0	0	0	0	0
Xi	0	0	0	0	0	0

Mu	Sigma	Nugget
0.8552592	4.1459606	0.1

-2*LogLikelihood
1364.7234

Nugget parameter set to avoid singular variance matrix.

Note: The upper plots show the standard deviation response, the lower plots the expected value response (the left plots are normal-scaled and the right plots are log-scaled); the upper report table displays the results for standard deviation, the lower table that for expected value.

Figure 3.2: Actual by predicted plots and model reports for inflation responses (expected value and standard deviation) – case with outliers

budget constraint is eased, due to a higher ν, inflation is assumed to rise. In summary, there are good reasons to exclude these hyperinflation scenarios during the following sensitivity analysis. Thus, we exclude outliers producing outcomes above 1 in the inflation 'responses' (i.e. for $\mu^{\pi} > 1$, or $\sigma^{\pi} > 1$).

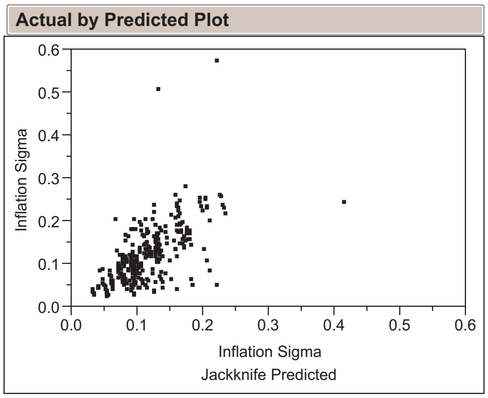
3.3.4 Analysis of the Baseline Model

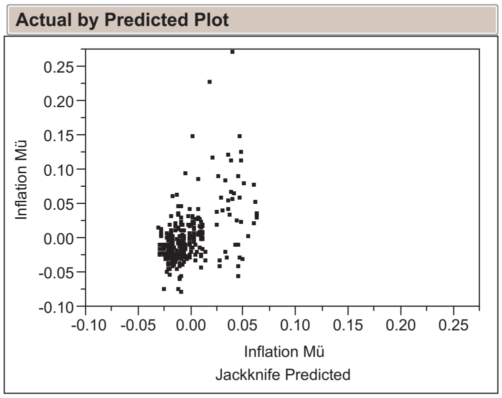
In this subsection, the results of the sensitivity analysis conducted in the 'baseline case' are discussed. First, we review the inflation 'responses', and thereafter the output gap 'responses'.

Inflation Responses (μ^{π} and σ^{π})

This 'response' is dominated by the main effect of ν, i.e. 63.5% of the variation in μ^{π} is due to ν. See the report in figure 3.3. If we add the interaction effects of ν with other 'factors', 81.5% of the the variation in μ^{π} is due to the main and interaction effects of ν. From the description in chapter 2, we know that ν defines upper and lower limits of consumption expenditures in relation to last period's average per capita consumption expenditure. It gives a intertemporal budget constraint for household agents; a lower level of ν defines a narrower budget, so that consumption expenditures are more constrained. Hence, it is not surprising that this mechanism affects inflation. In contrast, it is surprising that ν is dominating the expected value of inflation in such a strong way. It should be noted that very low magnitudes of ν do not result in the lowest 'responses'. See, for example, the marginal plot in figure 3.4: There, the lowest figure for μ^{π} results from ν–levels between 1.7 and 1.9. If ν falls below 1.5, μ^{π} is indeed rising but from a negative range to roughly 0%. The reason for this fact is straightforward: A very low ν (close to 1.1) reduces the budget of households in such a strong way that consumption expenditures are more or less stable. This implies stable prices, i.e. an inflation mean around 0%. To sum up, if ν is large, inflation mean is large too; if ν gets smaller, the inflation mean is getting smaller too, up to negative values. But, if ν approaches its minimum level of 1.1, inflation mean is rising and stabilizing around a mean of 0%.

In addition, the report in figure 3.3 identifies important interaction effects between ν and other 'factors'. First of all, interactions with ι, ϖ, and a rather small effect with θ are estimated. See the surface plots in figure 3.5. The interaction effect between ν and ι is straightforward: A large ι dampens the dominant effect of ν somewhat. Thereby, a large ι (slightly less than 1) indicates that





Model Report

Column	Theta	Total Sensitivity	Main Effect	Beta Interaction	Nü Interaction	Psi Interaction	Sigma 2 Interaction	Theta Interaction	Varpi Interaction	Tau Interaction	Rho Interaction	Xi Interaction
Beta	1.9622646	0.2421303	0.1471358	.	0.0568072	9.8675e-5	8.6121e-6	0.0006348	0.0224404	0.0001328	0.0148612	1.0948e-5
Lambda	0	0	0	0	0	0	0	0	0	0	0	0
Eta RR	0	0	0	0	0	0	0	0	0	0	0	0
Eta IC	0	0	0	0	0	0	0	0	0	0	0	0
Nü	1.6488036	0.683148	0.4728398	0.0568072	.	0.0019369	3.0625e-5	0.0326392	0.0227877	0.0181225	0.0776995	0.0002846
Omega Micro	0	0	0	0	0	0	0	0	0	0	0	0
Omega Macro	0	0	0	0	0	0	0	0	0	0	0	0
Psi	0.2647929	0.0118441	0.0097517	9.8675e-5	0.0019369	.	1.2927e-8	3.6126e-5	1.2758e-5	7.6741e-7	7.116e-6	3.7384e-8
Iota	0	0	0	0	0	0	0	0	0	0	0	0
Sigma 2	34.698827	0.002208	0.0021674	8.6121e-6	3.0625e-5	1.2927e-8	.	2.8115e-7	5.4219e-7	2.3979e-8	5.8496e-7	1.2052e-9
Theta	0.0647443	0.0344378	0.0008485	0.0006348	0.0326392	3.6126e-5	2.8115e-7	.	0.0001114	8.405e-5	8.3382e-5	1.1008e-7
Varpi	0.0001971	0.0789803	0.0324456	0.0224404	0.0227877	1.2758e-5	5.4219e-7	0.0001114	.	0.0001629	0.0010185	5.0979e-7
Tau	0.0044712	0.04164	0.022963	0.0001328	0.0181225	7.6741e-7	2.3979e-8	8.405e-5	0.0001629	.	0.0001738	9.6547e-8
Varsigma	0	0	0	0	0	0	0	0	0	0	0	0
Rho	0.3377993	0.1425373	0.0486889	0.0148612	0.0776995	7.116e-6	5.8496e-7	8.3382e-5	0.0010185	0.0001738	.	4.2858e-6
Xi	0.0069253	0.005039	0.0047385	1.0948e-5	0.0002846	3.7384e-8	1.2052e-9	1.1008e-7	5.0979e-7	9.6547e-8	4.2858e-6	.

Mu	Sigma	Nugget
0.1862367	0.0217274	0.1

-2*LogLikelihood

-960.7572

Nugget parameter set to avoid singular variance matrix.

Model Report

Column	Theta	Total Sensitivity	Main Effect	Nü Interaction	Psi Interaction	Iota Interaction	Sigma 2 Interaction	Theta Interaction	Varpi Interaction	Xi Interaction
Beta	0	0	0	0	0	0	0	0	0	0
Lambda	0	0	0	0	0	0	0	0	0	0
Eta RR	0	0	0	0	0	0	0	0	0	0
Eta IC	0	0	0	0	0	0	0	0	0	0
Nü	1.2481771	0.8146777	0.6348158	.	0.0001439	0.0562095	1.0836e-7	0.0122184	0.111041	0.0002489
Omega Micro	0	0	0	0	0	0	0	0	0	0
Omega Macro	0	0	0	0	0	0	0	0	0	0
Psi	0.0720861	0.0133695	0.0132214	0.0001439	.	3.2255e-7	1.048e-12	9.9036e-7	2.852e-6	5.6943e-9
Iota	0.6463725	0.0643737	0.0073715	0.0562095	3.2255e-7	.	6.835e-11	0.0000038	0.0007885	4.1526e-8
Sigma 2	0.6596907	6.5517e-6	6.4413e-6	1.0836e-7	1.048e-12	6.835e-11	.	1.467e-10	1.8498e-9	6.861e-12
Theta	0.0383724	0.0969367	0.0830467	0.0122184	9.9036e-7	0.0000038	1.467e-10	.	0.0016667	4.4261e-8
Varpi	0.0002825	0.1892925	0.0757928	0.111041	2.852e-6	0.0007885	1.8498e-9	0.0016667	.	5.6861e-7
Tau	0	0	0	0	0	0	0	0	0	0
Varsigma	0	0	0	0	0	0	0	0	0	0
Rho	0	0	0	0	0	0	0	0	0	0
Xi	0.0023255	0.0032427	0.0029931	0.0002489	5.6943e-9	4.1526e-8	6.861e-12	4.4261e-8	5.6861e-7	.

Mu	Sigma	Nugget
0.092032	0.0108868	0.1

-2*LogLikelihood

-1218.104

Nugget parameter set to avoid singular variance matrix.

Note: The upper report table displays the results for standard deviation, the lower table that for expected value.

Figure 3.3: Actual by predicted plots and model reports for inflation responses (expected value and standard deviation)

consumer goods firms postpone expansion investment until the capacity utilization is near to its absolute maximum of 100%. Accordingly, investment demand and incomes generated in the capital goods industry are lower, if ι gets large, which dampens the inflation mean. This effect is validated through the negative main effect of ι; see the respective plot in figure 3.4. The further interactions between ϖ and ν, or θ and ν, are explained below. Other interaction effects, where ν is not involved, appear unimportant in the 'baseline case'.

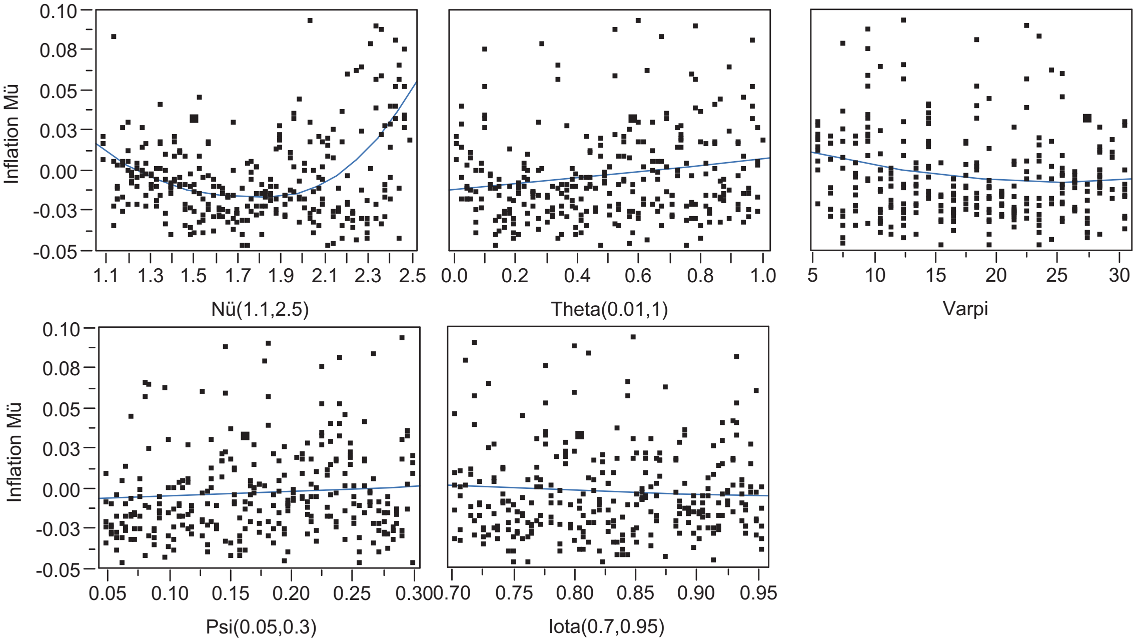
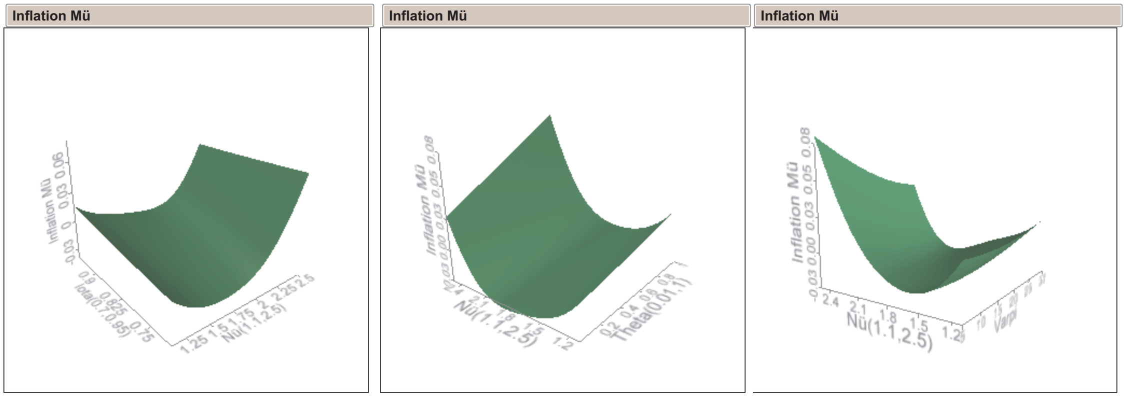


Figure 3.4: Marginal plots for expected value of inflation

Besides ν the 'factors' θ (8.3%), ϖ (7.6%), ψ (1.3%) and ι (0.7%) posses smaller main effects on μ^{π}. Together with interaction effects, ϖ accounts for 18.9% of the variation in μ^{π}; θ accounts for 9.7%, ψ for 1.3%, and ι for 6.4%. As explained above in section 3.2, ϖ affects the relative size of capital supply compared to capital demand. For the direction of the main effect of ϖ see figure 3.4. This effect seems to be driven by the capacity effects in the capital goods sector, viz: If ϖ is low, the capacity utilization in the capital goods sector is supposed to be high. Additionally, a high capacity utilization implies high incomes in this sector, and vice versa. On this account, the main effect of ϖ on the inflation mean is negative. Conversely, a low ϖ implies also that there is capital rationing and unsatisfied ‘excess demand’ in the capital goods market. According to this, the growth capabilities of Agent Island are restricted. One can assume that this would dampen inflation—but this is not the case. The effect of a high capacity utilization, due to a low ϖ, dominates. Moreover, the interaction effect between ϖ and ν is shown in figure 3.5. One can assert that a rising magnitude of ϖ dampens the effect of a large ν on μ^{π}.



Note: From the left panel to the right panel, interaction effects of (i) ν and ι, (ii) ν and θ, (iii) ν and ϖ are depicted.

Figure 3.5: Surface plots for expected value of inflation

In addition, the main effect of θ is positive: A larger θ (which defines the reaction of output to marginal profitability in the consumer goods industry) induces (c.p.) higher inflation mean. See figure 3.4. We already discussed this point in the last subsection, where we found out that in some cases a very high θ close to 1, produces extremely high inflation figures. The same results are identified through the interaction effects between θ and ν, depicted in figure 3.5. If θ becomes larger, the pressure on inflation due to ν is rising additionally. Besides this, the main effects of ψ and ι are negligible, but give the right direction, see again figure 3.4. It is interesting that none of the households behavioral parameters except of ν (and the negligible effect of χ) influence inflation mean, i.e. neither η^{IC} nor η^{RR} influence inflation mean. Obviously, the intertemporal budget constraint (defined via ν) dominates the savings decision of households, whereas other household parameters are negligible. Even more astonishing is that central bank preferences (τ, ς or ρ) do not exhibit any impact on the inflation mean. This seems to be a drawback of the ‘baseline case’. We will see below that these results change, when we switch to standard deviation of inflation, and later on, to the

'Ponzi case'.

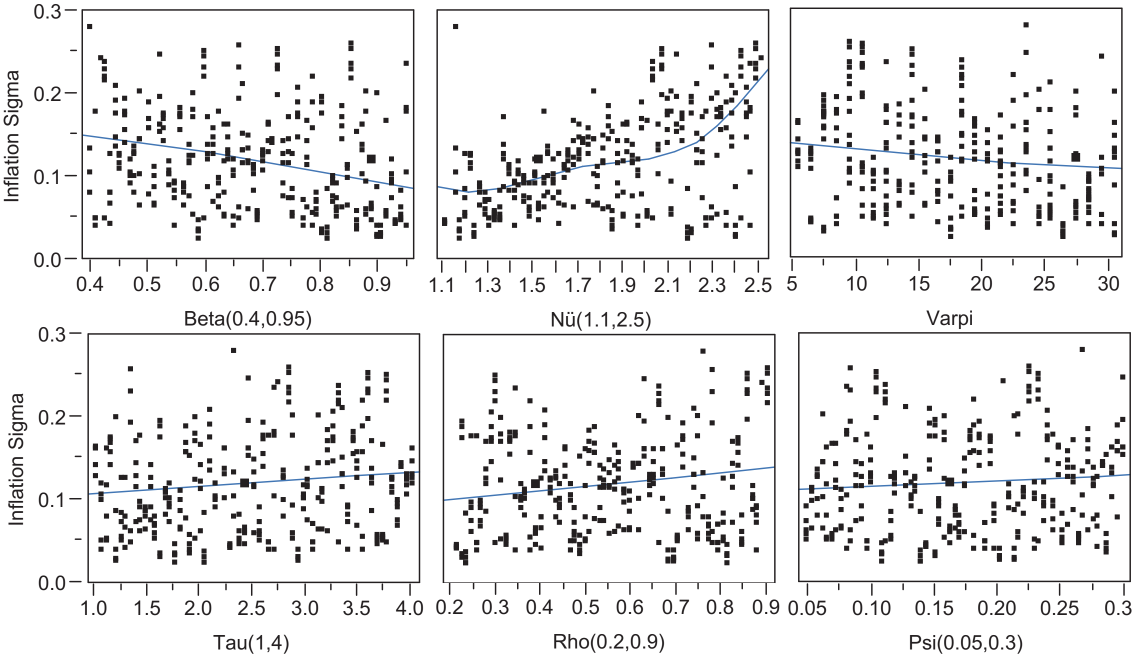


Figure 3.6: Marginal plots for standard deviation of inflation

The 'response' σ^{π} is dominated by main effects of ν (47.3%), and β (14.7%). The 'factors' ρ (4.9%), ϖ (3.2%), τ (2.3%), and ψ (1.0%) possess only a minor impact on this 'response'. See the report in figure 3.3 for the size of these main effects, and see the marginal plots in figure 3.6 for a graphical representation of the main effects. Let us start with the influence of ψ: It is straightforward that this effect is positive, insofar as ψ measures the size of investment expenditures of consumer goods firms. Investment expenditures in turn have an impact on the labor incomes of workers employed in that industry, and therefore (c.p.) on their consumption expenditures. Hence, a rising ψ enhances the volatility of investment demand and therefore influences indirectly σ^{π}.

The main effect for ν is as expected—for the same reasoning as in case of μ^{π}; see the explanation above. That is, the intertemporal budget constraint of the household agents is eased, if ν gets large. Therefore, the impact of ν on σ^{π} is positive. Together with interaction effects, ν accounts for 68.3%

in the variation of σ^π. For example, figure 3.7 shows the interaction of ν with τ. 'Factor' τ describes the weight on the inflation gap in the central bank rule. If this parameter is rising, the central bank reacts more volatile to inflation gaps with its policy instrument. To highlight the influence of the central bank rule on the standard deviation of inflation, figure 3.7 depicts the interactions of the most important 'factor' ν and the central bank 'factor' τ; this is examined under three different settings of the central bank 'factor' ρ. The parameter ρ describes the inertia of the interest rate set by the central bank due to the effect of 'interest rate smoothing'. One can identify that a rising ρ induces a stronger influence of ν on σ^π.[35] If, for example, ρ is set to 0.2, only large values of ν near to its maximum lead to a significant rise of σ^π. See the far left panel in figure 3.7. Moreover, in this case the influence of τ on σ^π is only partly as expected according to monetary transmission theory, viz.: The larger τ, the smaller the standard deviation of inflation. But this is only true for rather small levels of ν. If ν gets larger, the effect becomes neutral or even turns around, so that a higher τ does not reduce the standard deviation of inflation. In the other cases, i.e. in the right two panels in figure 3.7, the effect of τ on the standard deviation of inflation is not as expected, i.e. it is in contrast to the transmission theory. That is, a larger τ induces sometimes an even larger variation of inflation; but one would expect that a stronger reaction of interest rates on inflation gaps reduces standard deviation of inflation. The surprising opposite effect should be due to the disturbing effects of the zero lower bound restriction. According to this restriction, the central bank is able to counteract inflation, but it is not able to counteract deflation to the same extend. Consequently, this can produce ambiguous outcomes for the impact of τ on the standard deviation of inflation. However, the impact of the smoothing parameter ρ is as expected: If ρ becomes smaller, the effect of ν on σ^π is dampened.

There are several further interaction effects concerning ν. Figure 3.8 shows the interaction between ν and ρ: This indicates the same results as just explained in the last paragraph, i.e. a low level of ρ dampens the effect of ν on σ^π due to faster interest rate reactions. In addition to that, ϖ has also some minor influence on the main effect of ν, but the direction is ambiguous. The interaction effect of ν and θ is also dominated by the main effect of ν; only in some areas one can identify an additional enhancing effect of θ on σ^π. Before we analyze the interaction effect of ν and β, we have to explain the mechanism of the main effect of β: This is due to the relevance of the output elasticity

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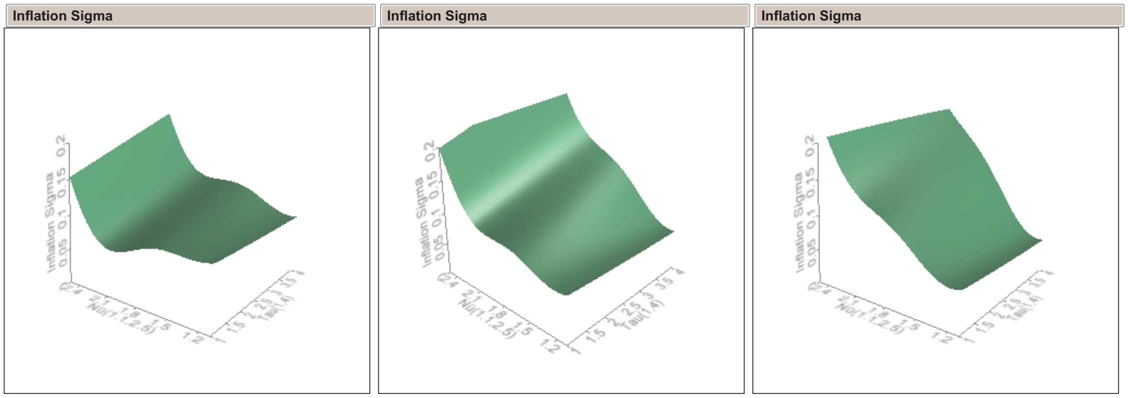
[35] The left panel in figure 3.7 shows interaction results for a very low ρ, and the right one for a very high ρ.



Note: The panels illustrate interaction effects of ν and τ. Thereby, the left panel displays results with $\rho = 0.2$, the middle that with $\rho = 0.55$, and the right that with $\rho = 0.9$.

Figure 3.7: Surface plots for standard deviation of inflation

of labor, which measures the responsiveness of output to a change in employed labor resources (c.p.). We have already mentioned this relationship. In case of a large figure, e.g. $\beta = 0.95$, a 1% increase in labor would lead to approximately a 0.95% increase in output. But the sequence of actions in the model works the other way around, viz.: According to the output–adjustment rule of consumer goods firms, a certain increase in output is given, and firms have to employ the quantity of labor necessary to produce the desired output (given the capital stock). Thus, a large β–figure implies a relative small enhancement of labor in order to increase the output as given. This effect influences also marginal costs, based upon the cost of labor, of consumer goods firms; see also the illustration in the next subsection. Consequently, the ‘factor’ β affects the standard deviation of inflation in such a way that inflation figures get more volatile, if β is rather small—and vice versa. This connection is verified by the marginal plot in figure 3.6.

The report in figure 3.3 shows that β accounts for 24.2% of the variation in σ^{π}, if interaction effects are included. For example, the interaction effects of β and ν are depicted in figure 3.8. There, one can identify that in the area of low ν–levels, a low level of β enhances standard deviation of inflation additionally—beyond the main effect of ν. If we start enhancing β while holding ν constant at its lowest level, σ^{π} is decreasing, exactly as explained above. However, for larger levels of ν, no additional influence of β on σ^{π} is identified. In addition, there exists an interesting interaction influence of ϖ and β on σ^{π}: If ϖ is very high (and the capital industry does not work at capacity limit, so that there is no capital goods rationing), the influence of β on standard deviation of inflation is as explained. That is, σ^{π} is falling with rising β. In contrast, if ϖ is small, the influence works the other way around so that a high β–level induces a relatively large σ^{π}. The disturbing income and rationing effects on the capital goods market are expected to account for this effect.

Last but not least, figure 3.6 depicts also some (small) main effects of ϖ, τ, as well as ρ on the 'response' σ^{π}: The parameter ϖ enlarges the quantity of consumer agents on Agent Island. If this number is rising, this should dampen the overall variability of model variables, such as inflation. In addition, the explained influence of ϖ on the capacity utilization in the capital goods industry also accounts for the direction of the main effect of ϖ. Moreover, we have already explained above the possible reasons for the surprisingly positive effect of τ on σ^{π}, i.e. the zero lower bound restriction.[36] The main effect of ρ is straightforward: If the central bank rule demands a strong inertia of interest rates, the central bank cannot react fast enough to inflation gaps, so that the standard deviation of inflation is higher compared to the case with little inertia represented through small ρ–levels. Finally, the interaction between β and ρ is also somewhat astonishing: If the central bank reacts immediately to inflation gaps, due to a small value for ρ, and if β is rising while fixing ρ, σ^{π} is also rising—instead of falling. This borderline case is difficult to explain intuitively. However, if ρ is enhanced a little bit, the effects are as expected, i.e. σ^{π} is rising in ρ and falling in β.

We summarize the main result of the sensitivity analysis with respect to inflation 'responses' in the 'baseline case': Most results are in accordance with our expectations. It is, however, striking that monetary policy does not work perfectly with respect to inflation control. In case of inflation mean, the central bank parameters do not feature any (significant) impact. In case of standard deviation of inflation, the influence of τ is partly in contrast to monetary theory. We verify in several 'face validation' runs that these controversial effects are due to the zero lower bound on nominal credit interest rates. Thus we see, if the model is well adjusted, in such a way that periods of large deflation

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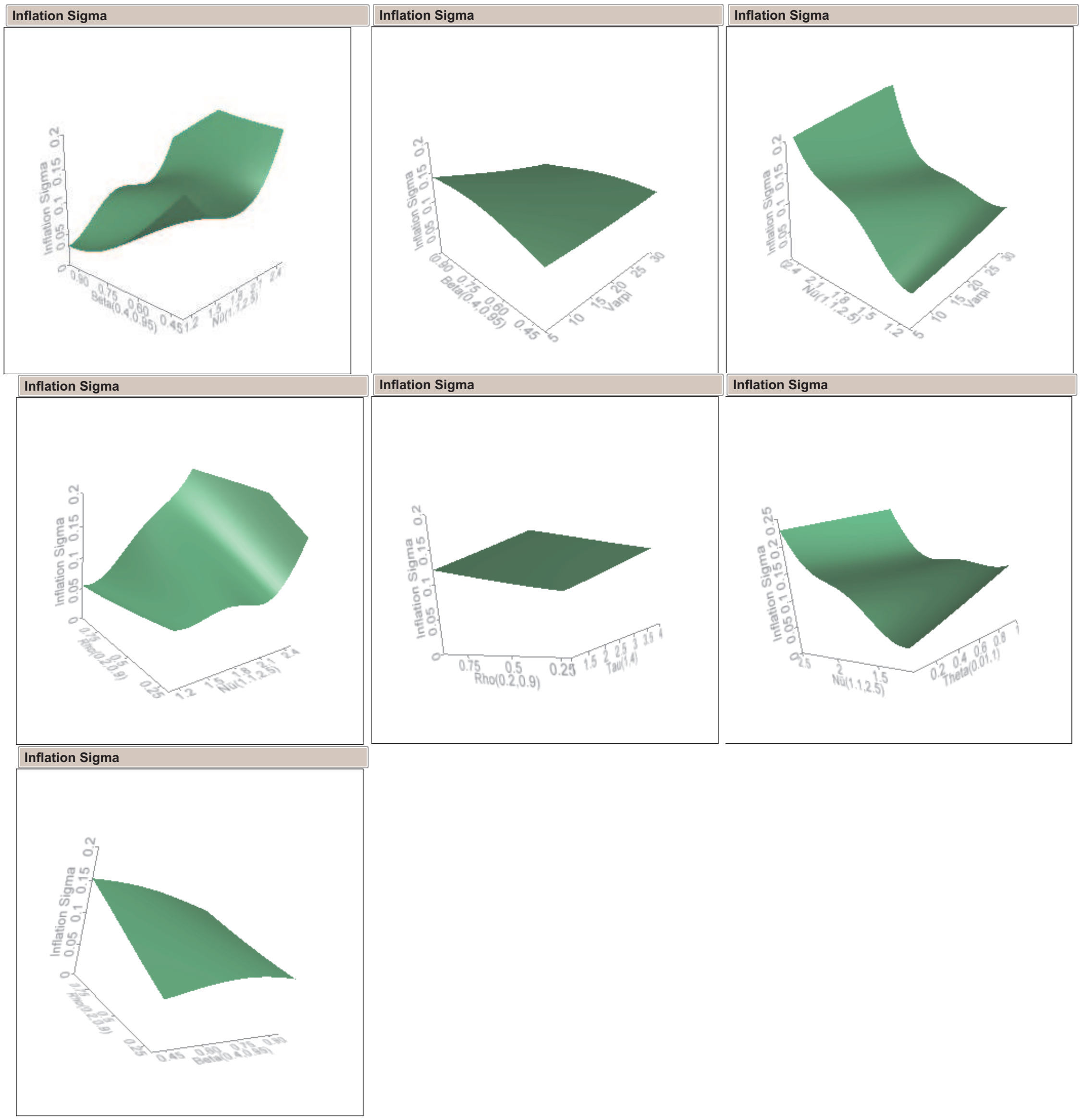
[36] Figure 3.8 depicts also a rather small interaction effect of ρ and τ on σ^{π}. One can identify that a strong central bank preference for inflation in the Taylor rule (i.e. a large τ) produces slightly higher standard deviation of inflation for all levels of ρ. Apparently, this unexpected effect is due to the zero lower bound restriction.



Note: From top left to bottom right, the panel depicts interaction effects of (i) β and ν, (ii) β and ϖ, (iii) ν and ϖ, (iv) ρ and ν, (v) ρ and τ, (vi) ν and θ, (vii) ρ and β.

Figure 3.8: Surface plots for standard deviation of inflation

(i.e. 'Great Depressions' on Agent Island) are ruled out, monetary transmission is functioning on Agent Island. We will illustrate this point within the 'statistical validation' and the final 'plausibility checks' of the calibrated model at the end of this chapter.

Output Gap Responses (μ^y and σ^y)

In the next step, we investigate the influence of the 16 'factors' on output gaps. Again, expected value and standard deviation of the time series are of interest. It should be noted that for most simulation outputs the expected value of the output gap time series is negative. Real–world evidence suggest a slightly negative magnitude for μ^y.[37] Consequently, optimal parameter settings should produce μ^y–figures approximately around 0% in the simulation data. According to the report in figure 3.9, the most dominant main effects on μ^y are determined by the 'factors' β (19.5%), ω^{Macro} (13.8%), and θ (11.8%). See also the panels in figure 3.10 for a graphical representation of the interrelations. Interestingly, the effect of the production function parameter β is positive, that is, the higher β, the higher is the output gap mean. The question is, why high output elasticities of labor induce higher means for output gap time series? This interrelation stems from the above mentioned fact that β affects the quantity of labor employed in the production process; this influence, in turn, determines the marginal costs of firms. We know from equation (2.2.13) that the marginal cost of producing output h_j through hash firm j is given by $MC_j = \frac{w}{\beta_j}(\frac{1}{A_j K_j})^{\frac{1}{\beta_j}}(h_j)^{\frac{1}{\beta_j}-1}$. The interesting point is, to what extend β drives the marginal cost of firm j, which in turn affects the output decision and the capacity utilization of the firm j. Table 3.5 illustrates the marginal costs for varied β and varied output figures—while all other variables in equation (2.2.13) are held constant and are fixed to 1 by a simplifying assumption, i.e. $A_j \equiv K_j \equiv w \equiv 1$.

According to the results of table 3.5, β affects marginal costs significantly. We know that the quantity of labor necessary to produce a given output rises with falling β. This effect produces rising marginal costs, if β decreases. For example, the results indicate for an output level of 1,000 a marginal cost amounting to 79,057 if $\beta = 0.4$, while this figure falls to 2.4 if $\beta = 0.9$. In case of $\beta = 0.9$, marginal costs are more than *30,000 times* lower compared to the case with $\beta = 0.4$. As explained, this effect is driven by labor demand: In this example (i.e. output amounting to 1,000

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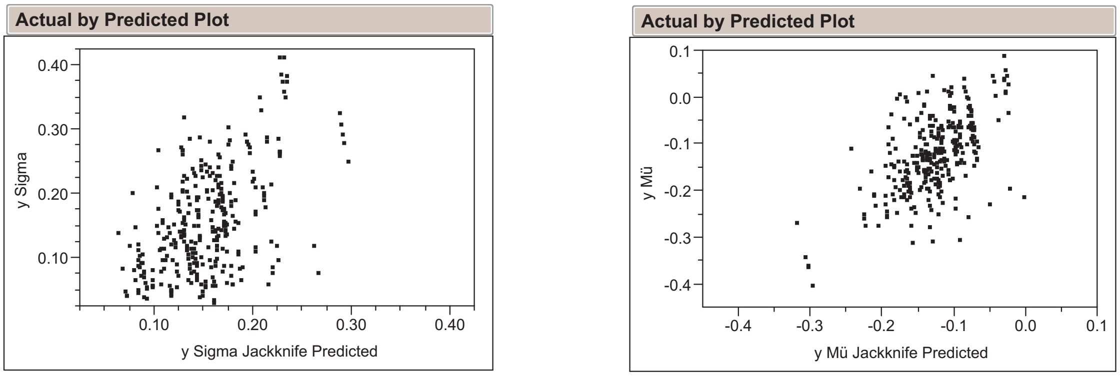
[37] See the description in section 3.4.



Model Report

Column	Theta	Total Sensitivity	Main Effect	Lambda Interaction	Eta RR Interaction	Nü Interaction	Omega Macro Interaction	Psi Interaction	Iota Interaction	Sigma 2 Interaction	Theta Interaction	Tau Interaction	Varsigma Interaction	Xi Interaction
Beta	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Lambda	0.1921045	0.0694155	0.0099133	.	3.1022e-8	4.2823e-7	0.0082012	0.0005482	1.3809e-7	0.0003529	0.0503841	2.7126e-6	0.0000112	1.3697e-6
Eta RR	0.0002372	0.0009426	0.0009417	3.1022e-8	.	1.904e-9	1.4855e-8	1.3518e-7	0	4.4999e-9	6.9095e-7	0	0	0
Eta IC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Nü	0.0075888	0.0862704	0.0700637	4.2823e-7	1.904e-9	.	0.0046881	0.0059973	5.9512e-8	5.3425e-5	0.0054553	7.6603e-7	1.0914e-5	3.0329e-7
Omega Micro	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Omega Macro	0.2123455	0.0963142	0.0348724	0.0082012	1.4855e-8	0.0046881	.	0.002646	1.902e-7	0.000412	0.0453418	1.424e-5	0.0001374	7.4553e-7
Psi	0.9151283	0.1740993	0.0597861	0.0005482	1.3518e-7	0.0059973	0.002646	.	4.759e-7	0.0021942	0.102858	1.6269e-6	6.4182e-5	3.1589e-6
Iota	0.0025632	0.0024464	0.0024381	1.3809e-7	0	5.9512e-8	1.902e-7	4.759e-7	.	6.8728e-9	7.4585e-6	0	0	0
Sigma 2	127.11149	0.1170388	0.0903685	0.0003529	4.4999e-9	5.3425e-5	0.000412	0.0021942	6.8728e-9	.	0.0236542	1.2066e-7	1.0724e-7	3.3848e-6
Theta	0.245698	0.5720041	0.3419955	0.0503841	6.9095e-7	0.0054553	0.0453418	0.102858	7.4585e-6	0.0236542	.	3.1854e-5	0.0022608	1.4337e-5
Varpi	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tau	0.0001065	0.034104	0.0340527	2.7126e-6	0	7.6603e-7	1.424e-5	1.6269e-6	0	1.2066e-7	3.1854e-5	.	2.7029e-9	4.4701e-9
Varsigma	0.0008463	0.0671202	0.0646356	0.0000112	0	1.0914e-5	0.0001374	6.4182e-5	0	1.0724e-7	0.0022608	2.7029e-9	.	4.5757e-8
Rho	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Xi	0.0012627	0.035299	0.0352756	1.3697e-6	0	3.0329e-7	7.4553e-7	3.1589e-6	0	3.3848e-6	1.4337e-5	4.4701e-9	4.5757e-8	.

Mu	Sigma	Nugget
0.4931949	0.4172203	0.01

-2*LogLikelihood
-787.8832

Nugget parameter set to avoid singular variance matrix.

Model Report

Column	Theta	Total Sensitivity	Main Effect	Beta Interaction	Eta RR Interaction	Nü Interaction	Omega Macro Interaction	Theta Interaction	Varpi Interaction	Tau Interaction	Varsigma Interaction	Xi Interaction
Beta	0.7221979	0.3479037	0.1950739	.	2.4646e-6	6.2739e-6	0.0120283	0.0037252	0.1364713	2.2742e-5	0.0002001	0.0003734
Lambda	0	0	0	0	0	0	0	0	0	0	0	0
Eta RR	0.0117293	0.0030899	0.0019862	2.4646e-6	.	2.9694e-9	0.0000654	1.1773e-5	0.0010236	6.0754e-8	2.4308e-7	1.5272e-7
Eta IC	0	0	0	0	0	0	0	0	0	0	0	0
Nü	0.0017389	0.0030942	0.0029524	6.2739e-6	2.9694e-9	.	8.4237e-5	2.9337e-6	4.7645e-5	5.2038e-9	7.1376e-7	3.3544e-8
Omega Micro	0	0	0	0	0	0	0	0	0	0	0	0
Omega Macro	3.3817076	0.2756746	0.1380349	0.0120283	0.0000654	8.4237e-5	.	0.040896	0.0777445	0.00134	0.0035823	0.001899
Psi	0	0	0	0	0	0	0	0	0	0	0	0
Iota	0	0	0	0	0	0	0	0	0	0	0	0
Sigma 2	0	0	0	0	0	0	0	0	0	0	0	0
Theta	0.3331106	0.1698545	0.1178864	0.0037252	1.1773e-5	2.9337e-6	0.040896	.	0.0016292	0.0000295	0.0051261	0.0005475
Varpi	0.0030169	0.3305782	0.0661871	0.1364713	0.0010236	4.7645e-5	0.0777445	0.0016292	.	0.0032046	0.039364	0.0049063
Tau	0.0016962	0.0262924	0.02169	2.2742e-5	6.0754e-8	5.2038e-9	0.00134	0.0000295	0.0032046	.	3.7689e-6	1.7794e-6
Varsigma	0.0176719	0.0588748	0.0105532	0.0002001	2.4308e-7	7.1376e-7	0.0035823	0.0051261	0.039364	3.7689e-6	.	4.4534e-5
Rho	0	0	0	0	0	0	0	0	0	0	0	0
Xi	0.0351244	0.0558193	0.0480466	0.0003734	1.5272e-7	3.3544e-8	0.001899	0.0005475	0.0049063	1.7794e-6	4.4534e-5	.

Mu	Sigma	Nugget
-0.171505	0.0378482	0.1

-2*LogLikelihood
-804.4828

Nugget parameter set to avoid singular variance matrix.

Note: The upper report table displays the results for standard deviation, the lower table that for expected value.

Figure 3.9: Actual by predicted plots and model reports for output gap responses (expected value and standard deviation)

Output↓ / β →	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
250	9,882.1	500	66.1	15.2	5.0	2.0
500	27,950.8	1,000	105.0	20.5	5.9	2.2
750	51,349.0	1,500	137.6	24.4	6.5	2.3
1,000	79,056.9	2,000	166.7	27.6	7.0	2.4
1,250	110,485.4	2,500	193.4	30.3	7.4	2.4
1,500	145,236.9	3,000	218.4	32.8	7.8	2.5
1,750	183,019.4	3,500	242.0	35.1	8.1	2.5
2,000	223,606.8	4,000	264.6	37.1	8.4	2.6
2,250	266,817.2	4,500	286.2	39.0	8.6	2.6
2,500	312,500.0	5,000	307.0	40.8	8.8	2.6
2,750	360,528.0	5,500	327.1	42.5	9.0	2.7
3,000	410,791.9	6,000	346.7	44.2	9.2	2.7
3,250	463,196.3	6,500	365.7	45.7	9.4	2.7
3,500	517,657.0	7,000	384.2	47.2	9.6	2.7
3,750	574,099.2	7,500	402.3	48.6	9.8	2.8
4,000	632,455.5	8,000	420.0	50.0	9.9	2.8
4,250	692,665.3	8,500	437.3	51.3	10.1	2.8
4,500	754,672.9	9,000	454.3	52.6	10.2	2.8
4,750	818,427.9	9,500	470.9	53.8	10.4	2.8
5,000	883,883.5	10,000	487.3	55.0	10.5	2.9

Note: In the calculation of marginal costs displayed in this table, all other determinants of marginal costs are hold constant and fixed to 1: We assume $A_j \equiv K_j \equiv w \equiv 1$.

Table 3.5: Marginal costs of consumer goods firm j

units), the labor demand is 31,622,776 hours if $\beta = 0.4$, while only 2,154 hours if $\beta = 0.9$. As a consequence, the marginal profitability of consumer firms is significantly higher, if β–figures are large. In addition, the capacity utilization of consumer goods firms is supposed to be higher under these circumstances. This supply side effect seems to determine the largely positive main effect of β on μ^y. See the panel in figure 3.10. Thereby, the opposite demand side effect via decreasing consumption expenditures due to decreasing labor incomes (if β is rising), seems to be exceeded by the previous supply side effect.

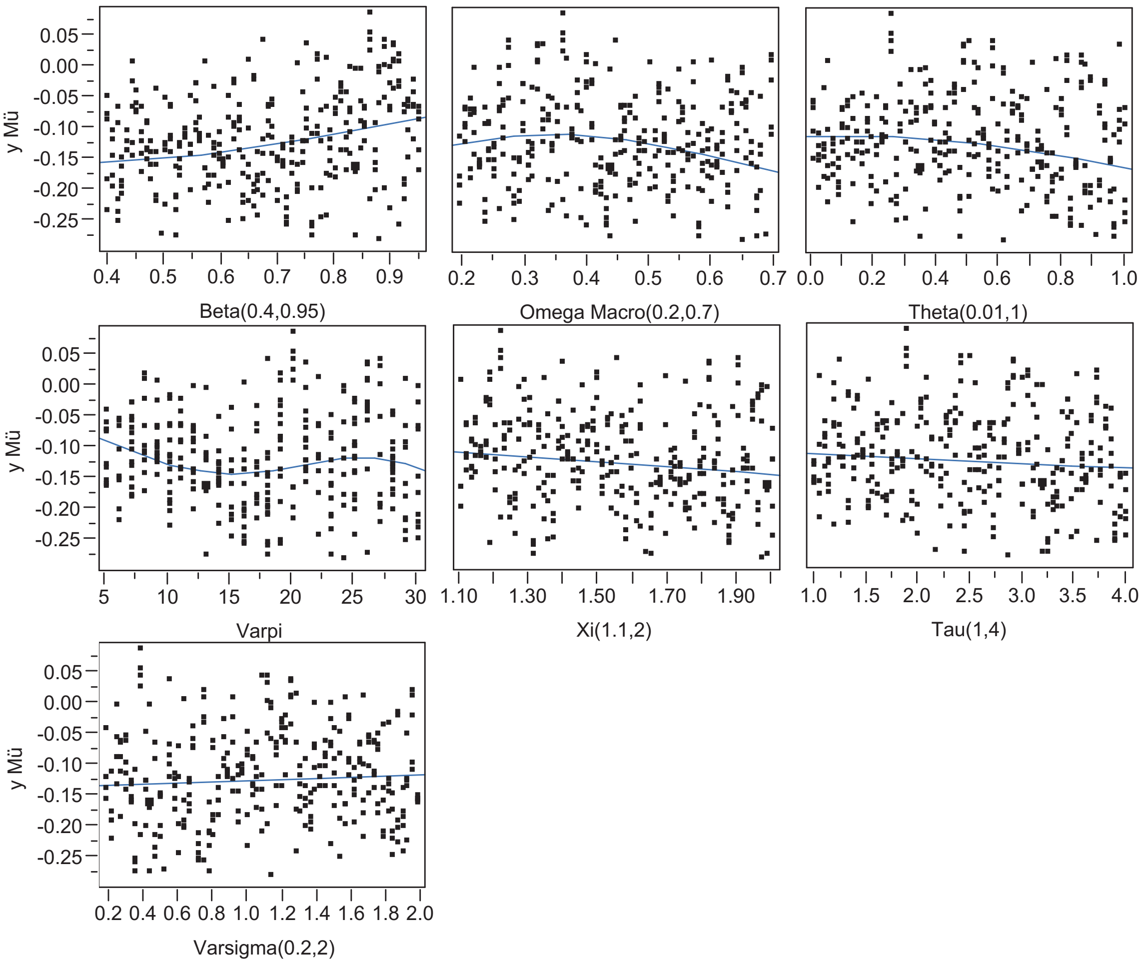
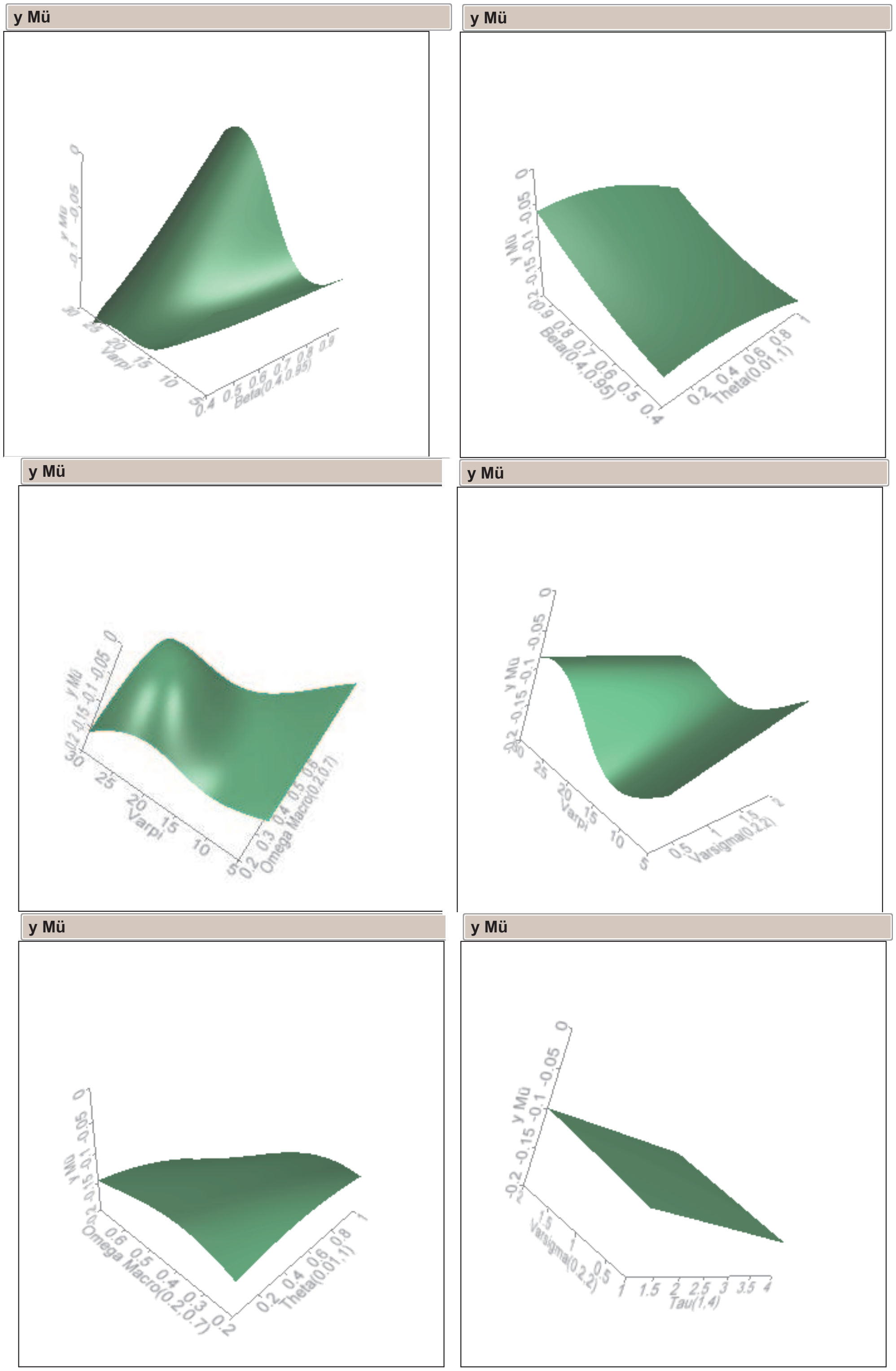


Figure 3.10: Marginal plots for expected value of the output gap

Besides this, the main effect of θ is negative: If θ is rising, the mean output gap is falling. Insofar as the capacity utilization in the consumer goods industry is stable, if θ is small, the results were as expected. Hence, a stabilizing policy of consumer goods firms stabilizes output gaps towards a mean of 0%. The interaction effect of both 'factors', i.e. β and θ, highlights the just explained interrelations: The best (i.e. highest) results for μ^y are obtained in case of high β–levels combined with low θ–levels. See the according panel in figure 3.11. If we add all interaction effects, β accounts for 34.8% of the variation in μ^y, and θ for about 17%. In the next step we investigate the main effect of ω^{Macro} on μ^y. According to this parameter, the firms in the capital goods industry adjust their price 'mark–up' in reaction to the macro–conditions of the capital goods industry. Thus, if the output gap of the industry is positive, 'mark–ups' are enlarged, and vice versa. The respective panel in figure 3.10 indicates that the sensitivity of 'mark–ups' should not be too high in order to improve business cycle conditions: If capital goods firms vary prices strongly, due to a large ω^{Macro}, the overall mean output gap decreases, i.e. μ^y falls further into negative areas. This must be due to the fact that capital goods trading declines, if prices are too high or too volatile. Consequently, the capacity utilization in the capital goods industry declines, and therefore output gaps are falling. The interaction effect of ω^{Macro} and θ underlines this phenomenon; see again the relevant panel in figure 3.11: If θ is close to 1, the expected value of output gap falls sharply with a rising ω^{Macro}. In this case consumer goods firms schedule large swings in production due to an extremely high θ–level. However, the supply conditions of consumer goods firms seem to be one–way limited, because their capital stock cannot expand optimally due to overly high or volatile capital goods prices (based upon the large ω^{Macro}). Then again, if θ is rather small, the effect of ω^{Macro} on μ^y is as explained. That is, initially μ^y increases with rising ω^{Macro} up to a peak; thereafter μ^y is falling, while increasing ω^{Macro} further on. As a result, we can assess that an optimal level of ω^{Macro} with respect to μ^y is located somewhere between 0.3 and 0.4.

Again, as in the analysis of the inflation mean, the parameter ϖ exhibits an important impact on the 'response', especially via interaction effects. The main effect of ϖ accounts for just 6.6% of the variation in μ^y; but with interaction effects this figure rises to 33.1%; see the report in figure 3.9. In addition, the main effect is ambiguous; see the relevant panel in figure 3.10. That is, if



Note: From the upper left to the lower right, interaction effects are depicted of (i) ϖ and β, (ii) β and θ, (iii) ϖ and ω^{Macro}, (iv) ϖ and ς, (v) ω^{Macro} and θ, (vi) ς and τ.

Figure 3.11: Surface plots for expected value of the output gap

we start from $\varpi = 5$ and enhance this figure, μ^y is falling and departing further from its optimum $\mu^y = 0$. If ϖ passes a level of about 15, μ^y is rising, and, thereafter, again slightly falling when ϖ approaches 30. The interpretation of this behavior seems difficult: As explained, the parameter features two opposite effects on the business cycle. First, if ϖ is low, the capacity utilization in the capital goods industry is high, and therefore incomes are high as well. Second, if ϖ is pretty low, there is a large unsatisfied excess demand in the capital goods markets, so that rationing takes place. We found out in 'face validation' runs that a ϖ below 10 does not deliver reasonable results, because the unsatisfied excess demand in the capital goods market is then overwhelming (see also footnote 29). In this case, virtually all capital goods firms are at capacity limit throughout the whole simulation run. As a result, we should not consider these cases henceforth. Except from these low levels of ϖ, it appears that an optimal level for ϖ with respect to the mean output gap is around 25. This is verified by the interaction effects between ϖ and other parameters; see the relevant panels in figure 3.11: Interestingly, the interaction between ϖ and β illustrates that in the area of β close to its maximum of 0.95 and ϖ at about 25, the expected value of output gap approaches its optimum level of slightly below 0%. When ϖ–levels are low, the results are again totally different. From this perspective it seems reasonable to target levels of ϖ close to 25 and β slightly below 1. The interaction effect of ϖ and ω^{Macro} verifies this assumption, i.e. for low ϖ–levels the outcomes appear to be distorted. However, around $\varpi = 25$, the 'response' approaches 0% from below, if ω^{Macro} is (in addition) located between 0.3 and 0.4.

Importantly, one can also identify main effects of the central bank parameters ς and τ: The main effect of ς (the weight on the output gap in the 'Taylor rule') is as expected. That is, μ^y tends to its optimum level of 0, as ς increases; see the relevant panel in figure 3.10. Accordingly, the central bank is able to close output gaps, but admittedly only to a small extend, and monetary transmission seems to work, at least, with respect to output gaps. In contrast to this, the negative effect of τ on μ^y is more difficult to explain: On the one hand, the correlation between inflation rates and output gaps is supposed to be positive, as stated through the notion of the 'Phillips curve'. Hence, if the central bank counteracts a negative inflation gap via τ through decreasing interest rates, this should dampen the negative output gap as well—and vice versa. In conclusion, (i) the larger τ, the smaller should be the gap between μ^y and 0; (ii) the correlation between τ and μ^y should be positive in the case of $\mu^y < 0$. Such a situation appears in case of demand shocks. On the other hand, the mechanism works the other way around in case of supply shocks. Consequently, a large marginal cost–increasing supply shock leads to lower output and lower capacity utilization, while consumer prices are rising (c.p.).[38] In such environments of supply shocks, a negative main effect of τ on μ^y can be explained when $\mu^y < 0$: If the central bank counteracts rising consumer goods prices via τ through higher interest rates, output gaps can fall additionally (i.e. in addition to the initial effect of the marginal cost–increasing supply shock). Thus, the direction of the main effect of τ depends on the relative importance of supply and demand shocks[39], whereby on Agent Island supply shocks appear to be slightly dominant.[40] This is verified by the interaction effect between τ and ς. Even though nearly not observable, it is true that the best outcome for μ^y is obtained for $\tau = 1$ and $\varsigma = 2$, i.e. for the lowest setting of τ and the highest for ς. This interaction effect can again be explained through dominating supply shocks. However, according to the total sensitivity, ς accounts for 5.9% and τ for 2.6% of the variation in μ^y; see the report in figure 3.9. Finally, the interaction effect between ϖ and ς is as expected: Up to a level of ϖ close to 20, the larger ς, the smaller are deviations of μ^y from 0%. In case of a large ϖ, this effect disappears.

In the last paragraphs of this subsection we investigate the 'factors', which influence the standard deviation of the output gap. If one looks at the data of the model report in figure 3.9, the 'factor' θ emerges as far more important than other 'factors', i.e. with interaction effects it accounts for 57.2% of the variation of σ^y. Moreover, the second, and third most important parameters are ψ and $Var[\epsilon]$, which account for 17.4% or 11.7% respectively. Figure 3.12 depicts the marginal plots for the relevant parameters. Thereby, θ influences σ^y as anticipated, i.e. if the variability of consumer goods output is high, due to large θ, σ^y is likewise high. Again as expected, the same positive correlation is identified between the amplitude of supply shocks given through $Var[\epsilon]$ and σ^y: If $Var[\epsilon]$ is large, the standard deviations of capacity utilizations, of marginal costs, and in turn of

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[38] If demand side conditions (consumption expenditures) are constant, a lower consumer goods output leads to a higher market clearing price in the model.

[39] This basic mechanism is in the line with monetary theory.

[40] However, the reader should bear in mind that monetary policy is restricted by the zero lower bound. This, in turn, distorts monetary transmission on Agent Island as explained, and this can also account for the negative impact of τ on μ^y.

output gaps are likewise large. The parameter ψ points in the same direction. That is, if its level is rather high, firms expand capacities in larger steps, which in turn enlarges standard deviation of capacity utilizations and therefore output gaps.

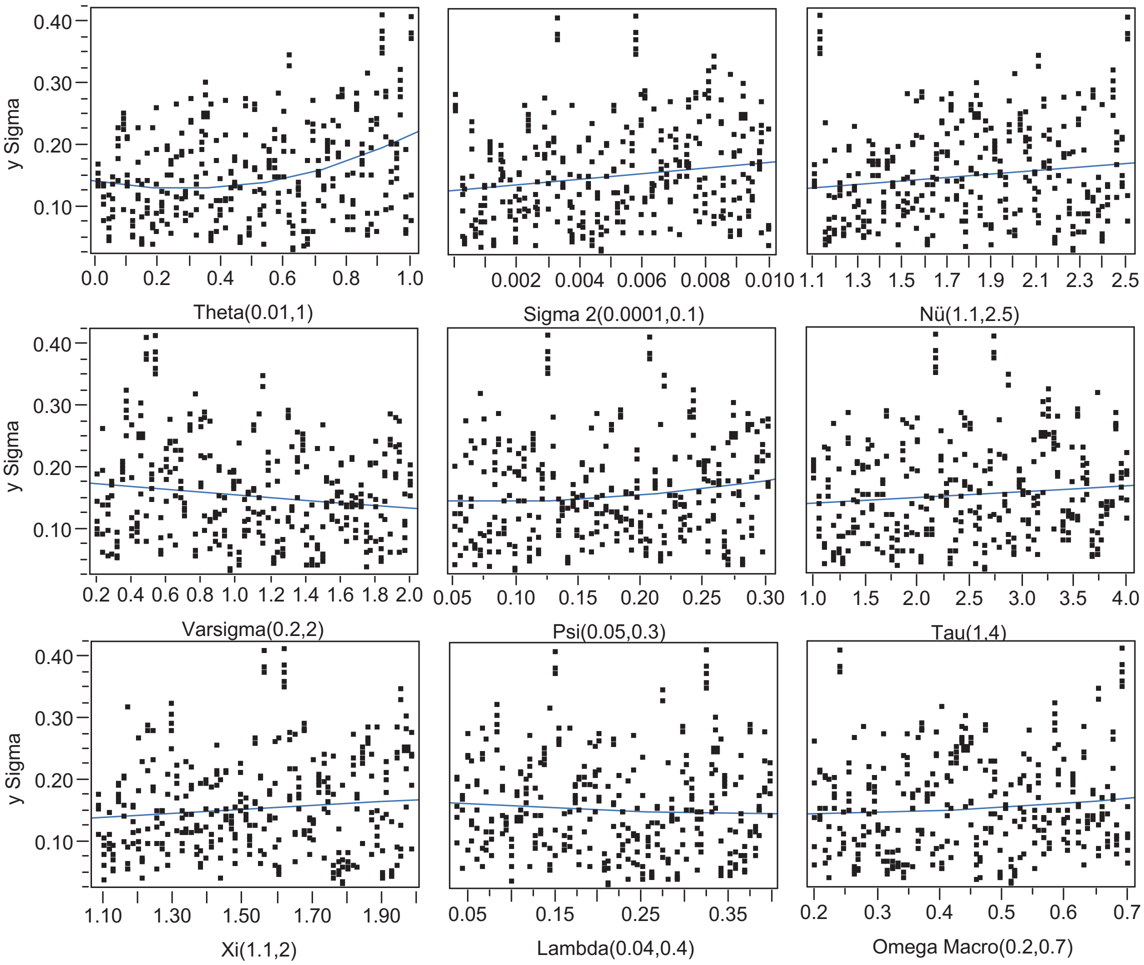
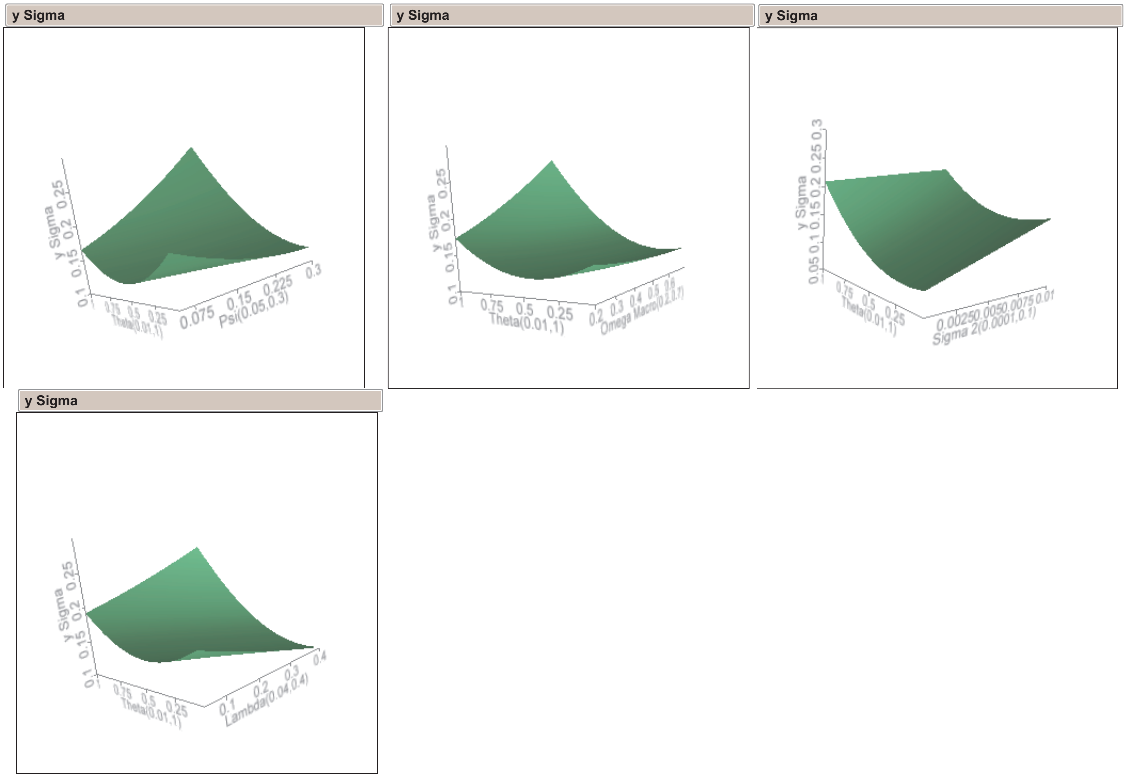


Figure 3.12: Marginal plots for standard deviation of the output gap

Next to this larger effects, some minor main effects on σ^y are at work (see the report in figure 3.9): The behavioral parameters ν and χ of households feature such effects amounting to 7% and 3.5% respectively on σ^y. The correlations of these 'factors' on σ^y are positive. For ν, this result is as expected, because it defines the intertemporal budget constraint of household agents. If this budget constraint is eased, due to higher ν, the variability of consumption expenditures rises, and therefore the standard deviation of output gaps will also rise. Conversely, we cannot explain the positive effect of χ on σ^y; it is, however, only of minor importance. Besides this, the wage setting parameter λ, and the 'mark–up' parameter ω^{Macro} possess also impact on the standard deviation of the output gap. The main effect of λ is 1.0%, that of ω^{Macro} is 3.5%. In here, the interrelation of λ and σ^y is a little surprising. If λ is relatively large (i.e. nominal wages react relatively strong to output gaps), the standard deviation of the output gap is decreasing—but this effect is pretty small. The correlation between ω^{Macro} and σ^y is positive, i.e. the output gap is more volatile, if ω^{Macro} is rather large (which is as expected). Finally, the influence of the 'Taylor rule' parameters τ and ς work similarly as in the context of μ^y. If ς is rising, the central bank is dampening output gaps and therefore μ^y; and if τ is rising, the standard deviation of the output gap enlarges. Again, both effects are in accordance to monetary theory, whereas the latter is due to dominant supply shocks. It should be noted that the identified main effects of these parameters are rather small, i.e. 6.5% for ς, and 3.4% for τ.

As a last step of this section we investigate interaction effects. As depicted in figure 3.13, we find only interaction effects between θ and other 'factors'. In case of effects between θ and ψ, ω^{Macro}, λ, or $Var[\epsilon]$, we find two dominant results: (i) The main effect of θ is overwhelming. It induces higher σ^y for rising θ. (ii) If θ is large, σ^y rises additionally with increasing 'factors' ψ, ω^{Macro} or λ. Both results are in line with the described main effects of θ, ψ, ω^{Macro}, or $Var[\epsilon]$. Apparently, the interaction between θ and $Var[\epsilon]$ is as expected. As a result, the highest standard deviation of the output gap is obtained with $\theta = 1$, and $Var[\epsilon] = 0.01$, i.e. for the largest possible levels of these parameters. Finally, we summarize again the main findings of the sensitivity analysis of output gaps in the 'baseline case': The parameters β, θ, and ω^{Macro} influence the mean of the output gap significantly, whereas θ, ψ, and $Var[\epsilon]$ influence its standard deviation. The influences of all of these behavioral or structural parameters are in accordance to the basic microeconomic and macroeconomic design of the model. Equally important, monetary policy is functioning with respect to output gaps (even though the identified effects are only small). Lastly, we identify that the model seems to be (more or less) dominated by supply shocks.



Note: From the upper left to the lower right, interaction effects are depicted of (i) θ and ψ, (ii) θ and ω^{Macro}, (iii) θ and $Var[\epsilon]$, (iv) θ and λ.

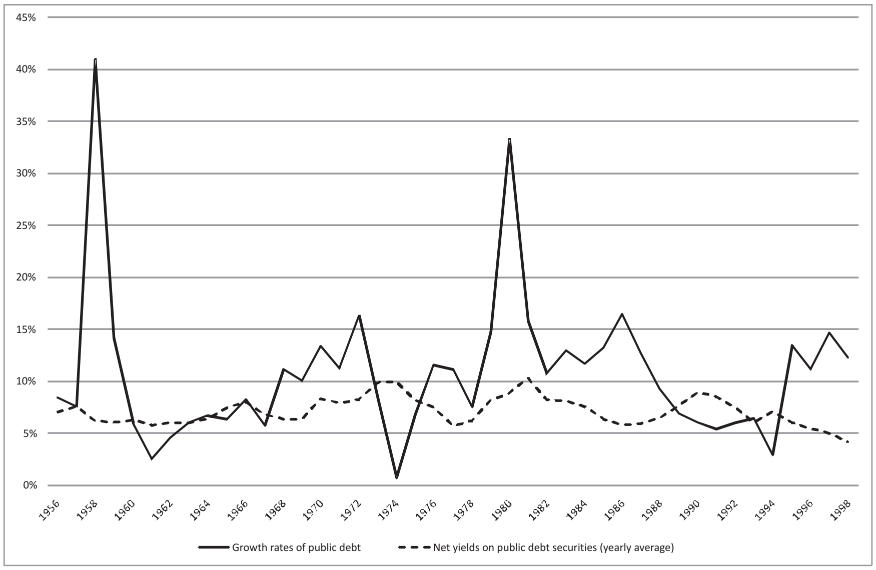
Figure 3.13: Surface plots for standard deviation of the output gap

3.3.5 Analysis of the Model with Ponzi Scheme lenders

In the last subsection we thoroughly investigated the sensitivity of inflation and output gap 'responses' compared to several parameters in a 'baseline setting' of the model. Within this subsection we apply a similar analysis, but with one modification: We alter the consumption and savings behavior of households out of the earned interest and capital incomes in such a way that these incomes are reinvested and (therefore) saved completely. The importance of this action could appear somewhat curious for 'orthodox' economist, but if we look at the flow–of–funds data, it becomes obvious that on the aggregate level, interest incomes of the private household sector are usually saved and not consumed. On the micro level such a behavior is correctly described by the logic of bank savings accounts, where the interest income is kept in the account. This leads to the effect of compounded interests: For example, the aggregate growth rates of financial assets of the household sector in Germany is depicted in figure 2.5 in chapter 2. It illustrates that in most years the growth rates of financial assets (from 1951 to 1998) were above—temporarily far above—the nominal government bond yield, which is a proxy for the average yield of financial assets. Figure 3.14 investigates the same phenomenon from the perspective of a large debtor, namely the German public sector. It depicts the growth rates of public debt in Germany from 1956 to 1998. However, the implication is closely related to that of figure 2.5 in chapter 2: In most years public debt builds up faster than the net yield on it is. Accordingly, the interest payments, due to government debts, are on average financed through new issued debts. This leads to the definition of 'Ponzi scheme borrowing', as described by Hyman Minsky:

> "For Ponzi financing units, the cash flows, from operations or from the way its assets perform, are not large enough to meet both the interest payments on their debts and all payments due on their maturing liabilities. Such units not only have to refinance or roll over maturing debts, but they have to borrow funds to pay interest." (Minsky, 1995, p. 200)

Hence, 'Ponzi borrowers' roll over debts and borrow funds in order to meet their interest payments. From this perspective combined with the data of figure 3.14, the German public sector tends



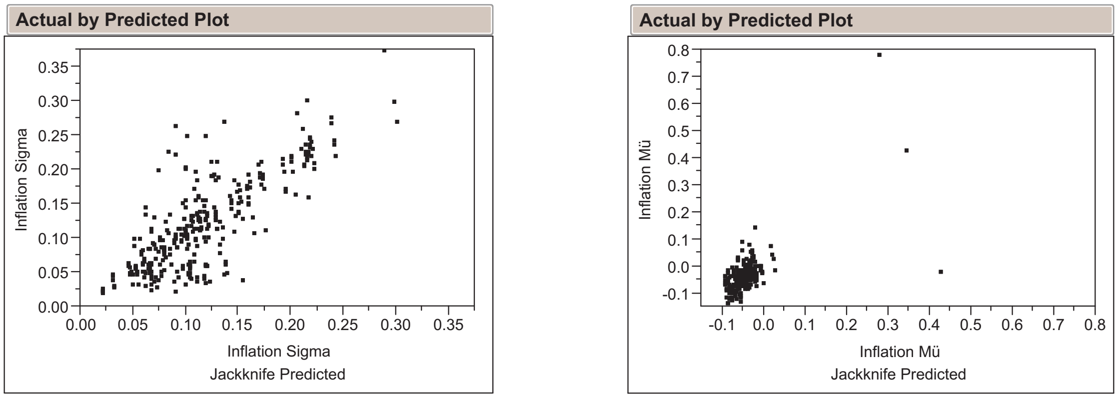
Note: In order to compare the displayed data of figure 3.14 to that of figure 2.5, the data set ends in 1998. In figure 2.5 in chapter 2 the time span ends 1998 because of a lack of suitable data: The methodology of the European national account system changed in 1996, and the 'old' data is only available till 1998. Especially the classification of the private household sector and the flow–of–fund data in the older European national account system fits better to our objectives.

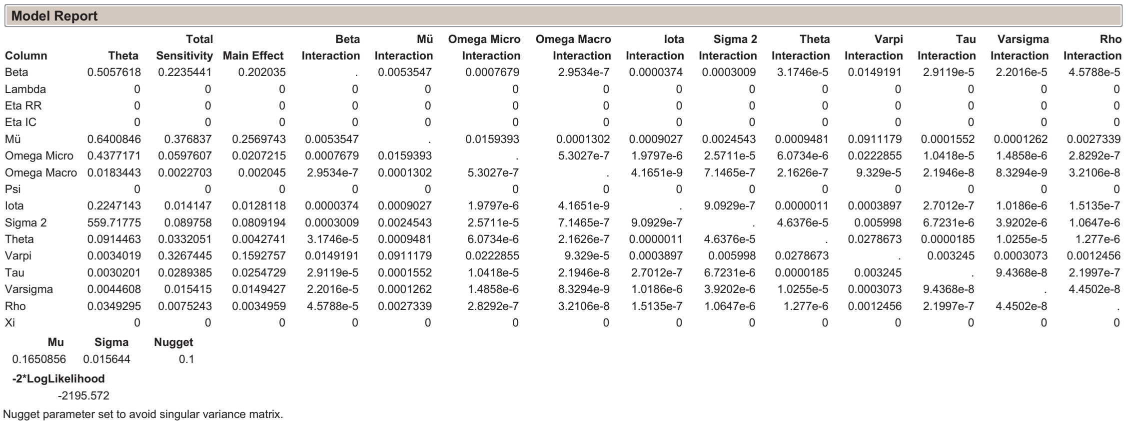
Figure 3.14: Growth rates and net yield of public debt; Germany, 1956 to 1998; Sources: Internet data bases of Deutsche Bundesbank and German Council of Economic Experts

to be a 'Ponzi borrower'—at least from the fact that it rolls over maturing debts and borrows funds in order to pay interest. In accordance to the definition of Hyman Minsky, we define 'Ponzi lenders' as household agents that roll over their financial assets and reinvest the received interest payments immediately. This is the starting point for the analysis in this subsection. From now on we investigate the results of the sensitivity analysis in the 'Ponzi case'. This takes place with reference to the results of the 'baseline case', as discussed in the previous subsection: We compare both cases and highlight similar and different results. In order to simplify this comparison, we show only the main effects of the 'factors'.

Inflation Responses (μ^{π} and σ^{π})

The results for the expected value of inflation change substantially, when we switch from the 'baseline case' to the 'Ponzi case'. In the 'baseline case', μ^{π} is dominated by the effect of ν (63.4% main effect), and minor effects of θ (8.3%), ϖ (7.6%) and ι (0.7%). In contrast, in the 'Ponzi case' several new 'factors' become important. Next to the parameters known from the baseline case, i.e.





Model Report

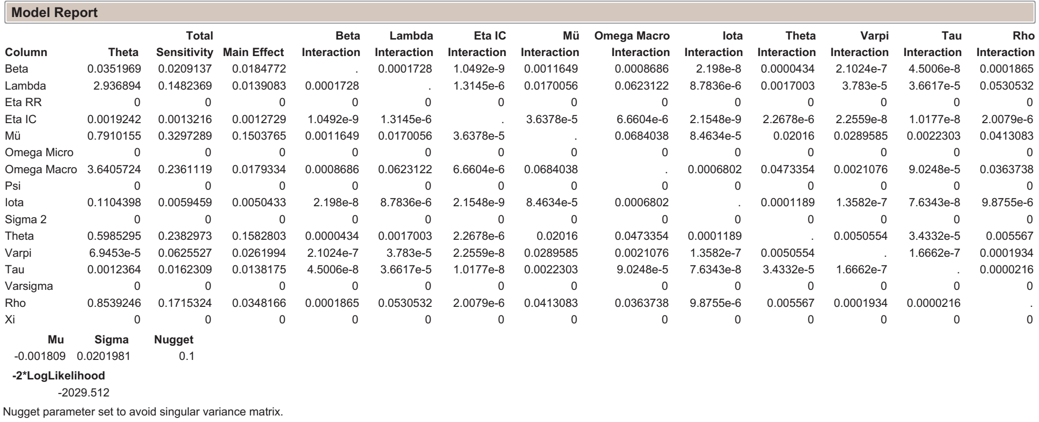
Column	Theta	Total Sensitivity	Main Effect	Beta Interaction	Mü Interaction	Omega Micro Interaction	Omega Macro Interaction	Iota Interaction	Sigma 2 Interaction	Theta Interaction	Varpi Interaction	Tau Interaction	Varsigma Interaction	Rho Interaction
Beta	0.5057618	0.2235441	0.202035	.	0.0053547	0.0007679	2.9534e-7	0.0000374	0.0003009	3.1746e-5	0.0149191	2.9119e-5	2.2016e-5	4.5788e-5
Lambda	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Eta RR	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Eta IC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mü	0.6400846	0.376837	0.2569743	0.0053547	.	0.0159393	0.0001302	0.0009027	0.0024543	0.0009481	0.0911179	0.0001552	0.0001262	0.0027339
Omega Micro	0.4377171	0.0597607	0.0207215	0.0007679	0.0159393	.	5.3027e-7	1.9797e-6	2.5711e-5	6.0734e-6	0.0222855	1.0418e-5	1.4858e-6	2.8292e-7
Omega Macro	0.0183443	0.0022703	0.002045	2.9534e-7	0.0001302	5.3027e-7	.	4.1651e-9	7.1465e-7	2.1626e-7	9.329e-5	2.1946e-8	8.3294e-9	3.2106e-8
Psi	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Iota	0.2247143	0.014147	0.0128118	0.0000374	0.0009027	1.9797e-6	4.1651e-9	.	9.0929e-7	0.0000011	0.0003897	2.7012e-7	1.0186e-6	1.5135e-7
Sigma 2	559.71775	0.089758	0.0809194	0.0003009	0.0024543	2.5711e-5	7.1465e-7	9.0929e-7	.	4.6376e-5	0.005998	6.7231e-6	3.9202e-6	1.0647e-6
Theta	0.0914463	0.0332051	0.0042741	3.1746e-5	0.0009481	6.0734e-6	2.1626e-7	0.0000011	4.6376e-5	.	0.0278673	0.0000185	1.0255e-5	1.277e-6
Varpi	0.0034019	0.3267445	0.1592757	0.0149191	0.0911179	0.0222855	9.329e-5	0.0003897	0.005998	0.0278673	.	0.003245	0.0003073	0.0012456
Tau	0.0030201	0.0289385	0.0254729	2.9119e-5	0.0001552	1.0418e-5	2.1946e-8	2.7012e-7	6.7231e-6	0.0000185	0.003245	.	9.4368e-8	2.1997e-7
Varsigma	0.0044608	0.015415	0.0149427	2.2016e-5	0.0001262	1.4858e-6	8.3294e-9	1.0186e-6	3.9202e-6	1.0255e-5	0.0003073	9.4368e-8	.	4.4502e-8
Rho	0.0349295	0.0075243	0.0034959	4.5788e-5	0.0027339	2.8292e-7	3.2106e-8	1.5135e-7	1.0647e-6	1.277e-6	0.0012456	2.1997e-7	4.4502e-8	.
Xi	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Mu	Sigma	Nugget
0.1650856	0.015644	0.1

-2*LogLikelihood

-2195.572

Nugget parameter set to avoid singular variance matrix.



Model Report

Column	Theta	Total Sensitivity	Main Effect	Beta Interaction	Lambda Interaction	Eta IC Interaction	Mü Interaction	Omega Macro Interaction	Iota Interaction	Theta Interaction	Varpi Interaction	Tau Interaction	Rho Interaction
Beta	0.0351969	0.0209137	0.0184772	.	0.0001728	1.0492e-9	0.0011649	0.0008686	2.198e-8	0.0000434	2.1024e-7	4.5006e-8	0.0001865
Lambda	2.936894	0.1482369	0.0139083	0.0001728	.	1.3145e-6	0.0170056	0.0623122	8.7836e-6	0.0017003	3.783e-5	3.6617e-5	0.0530532
Eta RR	0	0	0	0	0	0	0	0	0	0	0	0	0
Eta IC	0.0019242	0.0013216	0.0012729	1.0492e-9	1.3145e-6	.	3.6378e-5	6.6604e-6	2.1548e-9	2.2678e-6	2.2559e-8	1.0177e-8	2.0079e-6
Mü	0.7910155	0.3297289	0.1503765	0.0011649	0.0170056	3.6378e-5	.	0.0684038	8.4634e-5	0.02016	0.0289585	0.0022303	0.0413083
Omega Micro	0	0	0	0	0	0	0	0	0	0	0	0	0
Omega Macro	3.6405724	0.2361119	0.0179334	0.0008686	0.0623122	6.6604e-6	0.0684038	.	0.0006802	0.0473354	0.0021076	9.0248e-5	0.0363738
Psi	0	0	0	0	0	0	0	0	0	0	0	0	0
Iota	0.1104398	0.0059459	0.0050433	2.198e-8	8.7836e-6	2.1548e-9	8.4634e-5	0.0006802	.	0.0001189	1.3582e-7	7.6343e-8	9.8755e-6
Sigma 2	0	0	0	0	0	0	0	0	0	0	0	0	0
Theta	0.5985295	0.2382973	0.1582803	0.0000434	0.0017003	2.2678e-6	0.02016	0.0473354	0.0001189	.	0.0050554	3.4332e-5	0.005567
Varpi	6.9453e-5	0.0625527	0.0261994	2.1024e-7	3.783e-5	2.2559e-8	0.0289585	0.0021076	1.3582e-7	0.0050554	.	1.6662e-7	0.0001934
Tau	0.0012364	0.0162309	0.0138175	4.5006e-8	3.6617e-5	1.0177e-8	0.0022303	9.0248e-5	7.6343e-8	3.4332e-5	1.6662e-7	.	0.0000216
Varsigma	0	0	0	0	0	0	0	0	0	0	0	0	0
Rho	0.8539246	0.1715324	0.0348166	0.0001865	0.0530532	2.0079e-6	0.0413083	0.0363738	9.8755e-6	0.005567	0.0001934	0.0000216	.
Xi	0	0	0	0	0	0	0	0	0	0	0	0	0

Mu	Sigma	Nugget
-0.001809	0.0201981	0.1

-2*LogLikelihood

-2029.512

Nugget parameter set to avoid singular variance matrix.

Note: The upper report table displays the results for standard deviation, the lower table that for expected value.

Figure 3.15: Actual by predicted plots and model reports for inflation responses (expected value and standard deviation)

ν (15.0%), θ (15.8% main effect in the 'Ponzi case'), and ϖ (2.6%), the new 'factors' are ω^{Macro} (1.8%), λ (1.4%), ρ (3.5%), β (1.8%), and τ (1.4%). See the report in figure 3.15. In this case, several interesting facts become apparent: (i) In the 'Ponzi case' the importance of household parameter ν declines to a rather normal stance. (ii) The parameter θ, which describes the output adjustment of consumer goods firms over time, subsequently becomes the most dominant 'factor'. (iii) The importance of the 'factors' ϖ and ι decreases somewhat as compared to the 'baseline case'. This can be due to the emergence of new 'factors' in the estimation of μ^{π}. (iv) The wage setting parameter λ becomes a quite important 'factor'—at least if we consider the total sensitivity of λ, i.e. main and interaction effects. (v) Lastly, the central bank parameters ρ and τ exhibit influence in the 'Ponzi case' as opposed to the 'baseline case'.

By investigating the directions of the main effects in question, we work out several insights: Figure 3.16 depicts the concerning marginal plots. First of all, in the 'Ponzi case' μ^{π} holds more often negative values than in the 'baseline case'. According to the immediate reinvestment, interest (and dividend) payments do not enter in the 'income circuit', and therefore, they cannot induce inflation. As a result, the economy of Agent Island tends to be deflationary. Deflation becomes the crucial issue of the Agent Island economy in the 'Ponzi case'. In the following we discuss the main effects of several 'factors': The direction of the main effect of θ stays unchanged, i.e. μ^{π} is rising in θ. In case of θ close to 1, μ^{π} approaches 0%. The connection between ν and μ^{π} does not change either. But, as explained above, the effect becomes less extensive than in the baseline case. For example, if ν becomes large, i.e. between 1.7 and 2.5, there is only a minor positive influence of ν on μ^{π}. As a consequence, the main effect of ϖ changes: In contrast to the baseline case, the connection between ϖ and μ^{π} is now positive in all areas of the domain of ϖ. However, because we are only interested for levels above 10 (below the results are absolutely misleading), both cases deliver quite similar results.

In the 'baseline case' β does not exhibit any influence on μ^{π}. This situation changes in the 'Ponzi case': The inflation mean increases and approaches 0% with rising β. In principle, the dominant supply side effect of β was already explained in the last subsection. We expect that this effect is

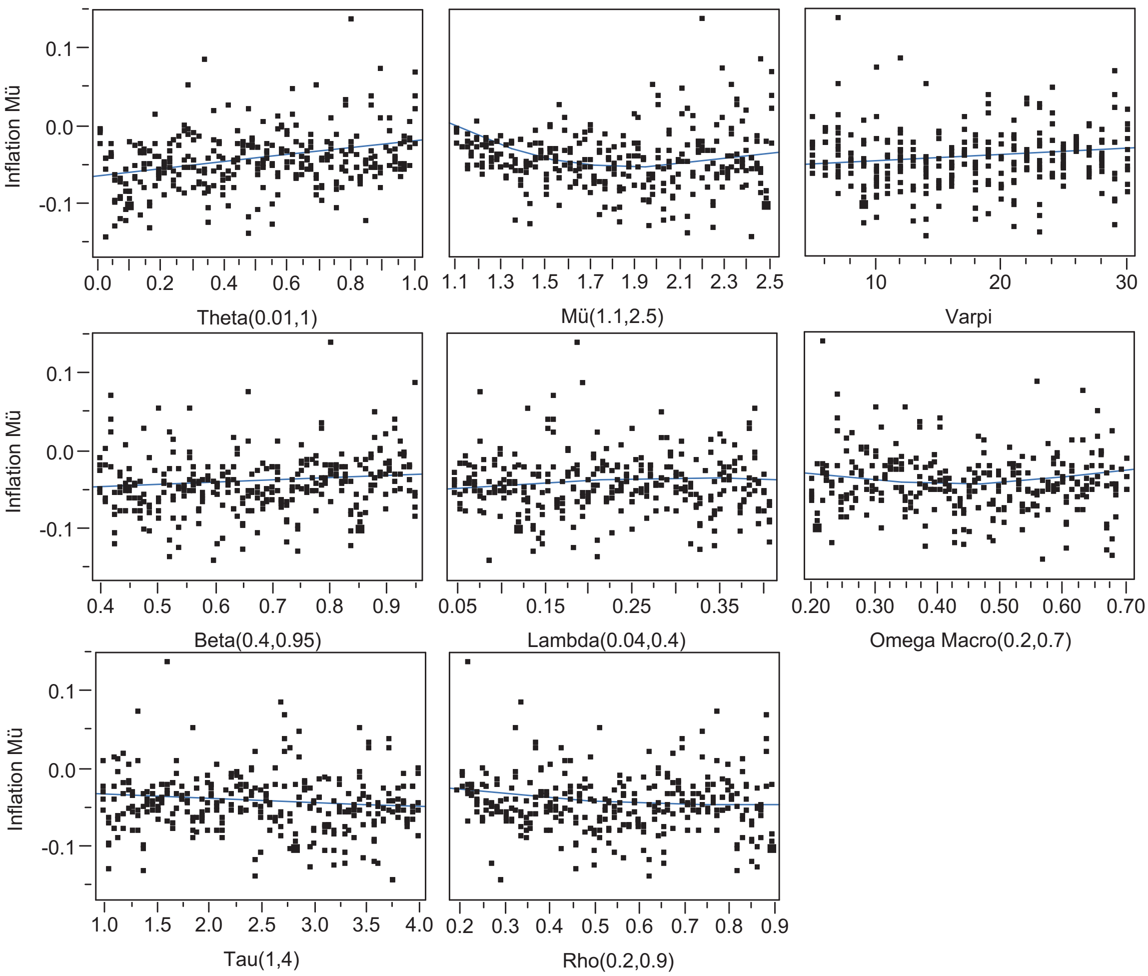


Figure 3.16: Marginal plots for the expected value of inflation

counteracted by an opposite demand side effect. Apparently, the supply side effect of β seems to dominate its influence on μ^{π}, so that a positive impact can be identified. Besides this, λ emerges as a new determinant of μ^{π}: The parameter λ generally determines the sensitivity of the wage growth rate to the output gap. In case of a growing economy where output increases, it is expected that a larger level of λ tends to result in higher income growth and (c.p.) higher growth of consumption expenditures. This results in higher inflation. In addition, decreasing (nominal) wages are ruled out by definition. Hence, the influence of the output gap on income growth is one–sided. This can account for the slightly positive interrelation between λ and μ^{π}—see the marginal plot for λ in figure 3.16. In the 'baseline case' such an effect was not identified. The new results stem from the more important role of labor incomes in the 'Ponzi case'.[41]

The connection between ω^{Macro} and μ^{π} is ambiguous and therefore difficult to explain. Because the overall main effect of ω^{Macro} is rather small, we will leave out the description of this parameter. Finally, the central bank parameters τ and ρ exhibit effects on μ^{π} in the 'Ponzi case' (in contrast to the 'baseline case'): The effect of ρ is straightforward, i.e. if ρ is large, the inertia of credit interest rates is likewise large so that the central bank cannot react immediately to inflation or output gaps. So it is evident that a larger ρ induces a larger deviation of the expected value of inflation from its target value, i.e. from 3%. From this perspective, it seems reasonable that μ^{π} is falling further into negative values, i.e. away form its optimum level at 3%, in the case of a rising ρ. On the contrary, the negative correlation between τ and μ^{π} must be due to the explained zero lower bound on nominal credit interest rates: Because the 'Ponzi case' is dominated by negative inflation rates, monetary policy looses its power almost completely. Provided that inflation is negative, nominal interest rates cannot react perfectly; they cannot fall below the zero boundary. On the other hand, if inflation is above its target value of 3%, the central bank reacts regularly and dampens upward pressure on inflation. Consequently, the overall effect of τ on μ^{π} tends to be one–sided (i.e. reduced to the negative influence of increasing interest rates on inflation); it is therefore negative.

In the retrospective of the 'baseline casse', this insight enables us to derive the cause of the

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[41] Note that in the 'Ponzi case' interest and dividend earning are reinvested each period, so that they are withdrawn from the 'income circuit'. As a result, the relative importance of labor incomes for the determination of inflation rates is expected to rise.

not expected absence of the parameter τ in the estimation of μ^{π}: In 'normal times' (with positive inflation) one expects that inflation mean approaches its target value of 3%, if τ is rising; we call this the 'regular effect' of τ. However, in the presented 'Ponzi case', with a strong tendency to deflation, the correlation between τ and μ^{π} is negative, so that μ^{π} drifts further apart from 3% when τ is rising. The zero lower bound restriction accounts for this effect; we call this the 'effect of powerless monetary policy'. In the 'baseline case', however, where phases of inflation and deflation are quite balanced, we assume in contrast that both effects (the 'regular effect' and the 'effect of powerless monetary policy') cancel each other out. This leads to the absence of *any* identified impact of τ on μ^{π} in the 'baseline case'. It constitutes one major insight of the sensitivity analysis with respect to the 'validation': It is our task to adjust the model in such a way that overwhelming deflation is absent in simulation runs. Provided that this is guaranteed, we expect that monetary policy on Agent Island is functioning, and that the influence of τ on μ^{π} is as expected. All in all, the 'baseline case' seems to be a better candidate for a reasonable validated model, because it does not generate as much deflationary pressure as the 'Ponzi case' does.

Let us return to the analysis of the 'Ponzi case'. The results for the standard deviation 'response' (σ^{π}) are related to the results for the expected value (μ^{π}). The model of the 'Ponzi case' exhibits obviously a decreasing importance of the parameter ν. In the 'baseline case', the variations in the 'response' σ^{π} are mainly based upon the parameter ν, i.e. its main effect is responsible for 47.3% in the variations of σ^{π}; in the 'Ponzi case' this value decreases to 25.7%. Next to ν, the importance of the parameters β, θ and τ do not change substantially, if we switch from one case to the other: In the 'Ponzi case', β exhibits a main effect of 20.2%, θ a main effect of 0.4%, and τ a main effect of 2.5%. These figures correspond roughly to those estimated in the 'baseline case'. Conversely, the parameters $Var[\epsilon]$ (8.1%), ϖ (15.9%) and ω^{Micro} (2.1%) appear to be newly significant in the 'Ponzi case'. It is apparent that the parameter ρ looses much of its impact compared to the 'baseline case', i.e. the main effect falls from 4.9% to 0.3%. In contrast, the central bank parameter ς gains some influence; unlike the 'baseline case', it exhibits a small main effect of 1.5% in the 'Ponzi case'.

The directions of the main effects are illustrated in figure 3.17. We leave out the discussion of ν,

β and ϖ, because they correspond roughly to the results of the 'baseline case'. In addition, the main effects of the central bank parameters τ and ρ do not change qualitatively: That is, if ρ (or τ) is rising, σ^{π} rises as suggested (or as not suggested) by transmission theory. The explanations given in the baseline case can be applied likewise, such as the role of the zero lower bound restriction. Then again, the direction of the newly significant main effect of $Var[\epsilon]$ is straightforward: The higher its value, the higher is the amplitude of stochastic supply shocks and the higher is therefore σ^{π}. This is due to movements in the marginal costs of consumer goods firms, which in turn affect the supply decision of consumer firms and therefore consumer goods prices immediately. Besides this, the main effect of θ is slightly negative. This seems at first to be in contrary to intuition. One could assume that a higher sensitivity of output to marginal profitability enhances standard deviation of inflation. However, the determination of consumer goods prices can account for the unexpected effect: If consumer prices and marginal profitabilities were high in the previous period and θ is large, consumer goods firms enhance present output strongly. In this respect, both consumer goods supply and consumption expenditures are enhanced, and the overall effect could stabilize consumer goods prices. This takes place, if aggregate consumer goods supply grows as strong as aggregate consumption expenditures. Apparently, this effect should depend on other parameters, such as λ. However, the estimated negative effect illustrated in figure 3.17 is rather small.

Finally, the main effect of ω^{Micro} is straightforward: Even though machine prices are not part of the consumer price index, and movements in the machine prices cannot influence consumer price inflation directly, it is not a surprise that there must be an indirect channel. For example, provided that the variation of machine prices is relatively high due to a large level of ω^{Micro}, the variation of the capacity utilization in the capital goods industry is likewise large. As a result, the labor incomes generated in the capital goods industry are also volatile, which in turn influence consumption expenditures and inflation. We should note one point: The parameters ω^{Macro} and ω^{Micro} determine the price–setting behavior of capital goods firms. But via the macro parameter the prices of all firms are influenced in the same way. That is, if ω^{Macro} is large and the output gap of the capital goods industry is likewise positive, *all* capital goods firms *enlarge* prices in large steps. Then again, a large level of the parameter ω^{Micro} implies that *some* manufacturers enlarge their supply prices in

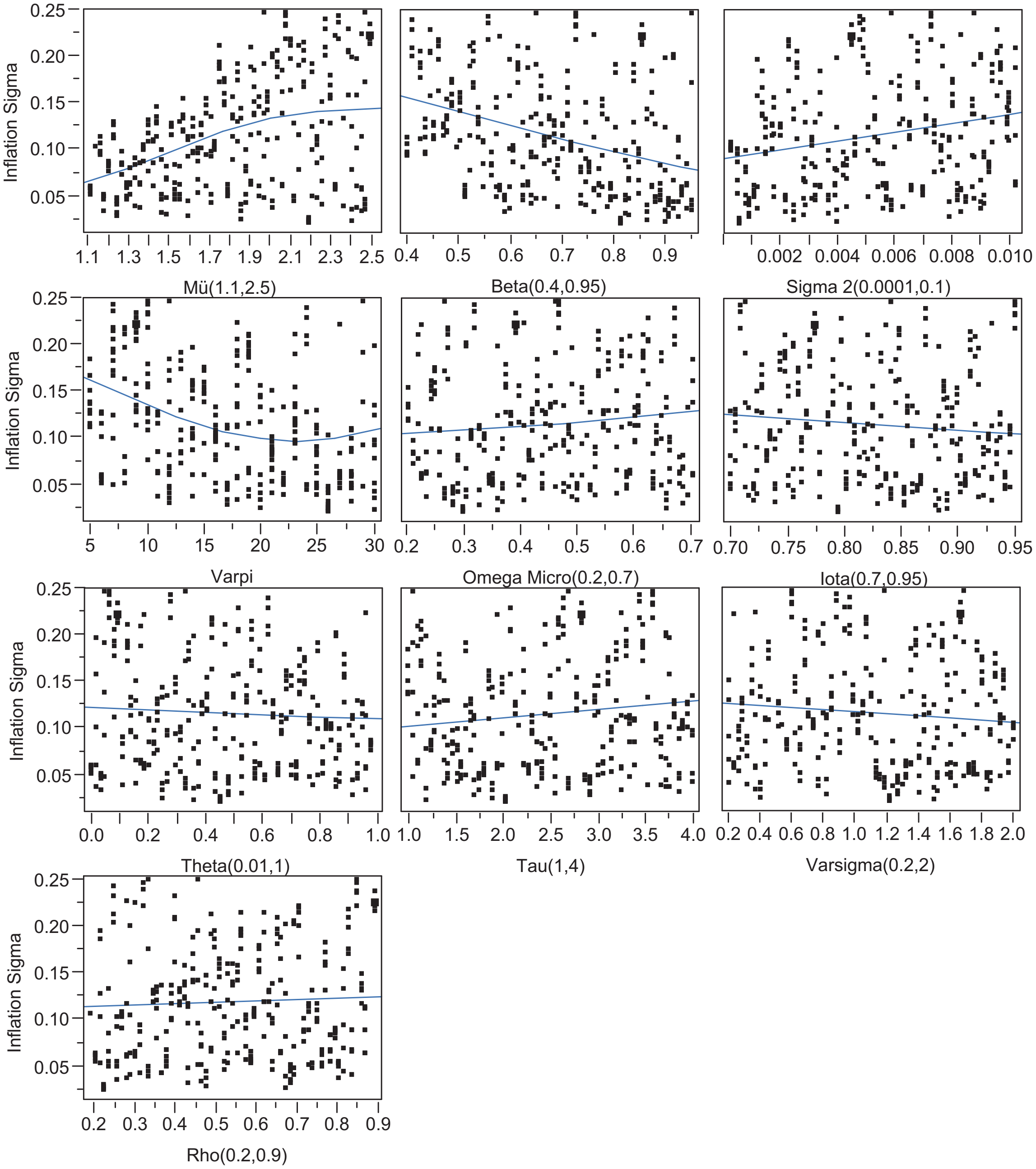


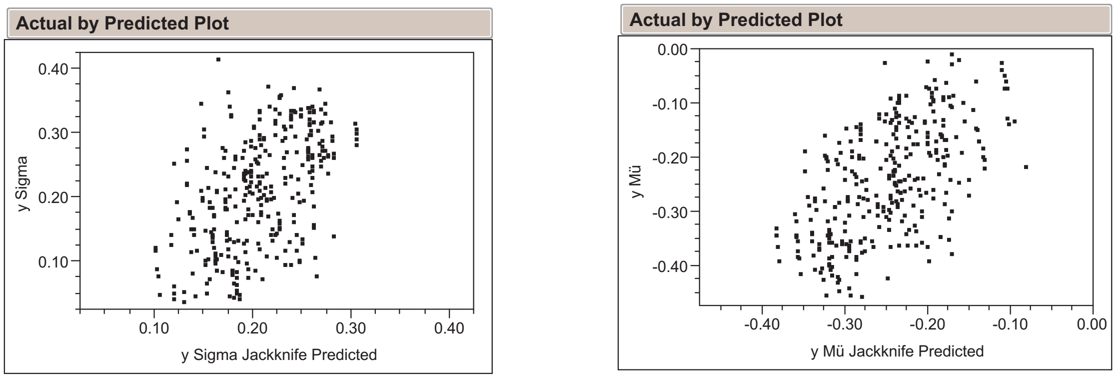
Figure 3.17: Marginal plots for standard deviation of inflation

large steps (namely those firms with positive marginal profits), while other firms lower their prices in large steps (due to negative marginal profits). Whether this produces a positive aggregate effect or a negative in the capital goods market depends on relative size of the group of price–lowering to the group of price–enhancing firms.

Output Gap Responses (μ^y and σ^y)

Similar to the inflation 'response' the mean output gap tends to be negative in more simulation runs in the 'Ponzi case' as compared to the 'baseline case'. Apparently, this is a result of the demand–reducing (i.e. deflationary) effect of the savings behavior of 'Ponzi lenders'. This dampens consumption expenditures, consumer goods prices and real economic activity. However, in the following we compare the results of the 'Ponzi case' with that of the 'baseline case'. In the 'Ponzi case', the variation of μ^y is dominated by the effects of ϖ (30.9% main effect), θ (24.3%), ν (23.4%) and β (11.2%). In here, the parameters ϖ, θ, and β play a significant role in the 'baseline case' as well. The parameter ν is not significant in the 'baseline case'; and the ω^{Macro} is only in the 'baseline case' significant. In addition, the parameters λ (3%) and ς (1.9%) are of minor importance in the 'Ponzi case'. The report in figure 3.18 displays the results of the estimation.

By investigating the direction of the main effects, the panels in figure 3.19 indicate that the pattern of the effects of β, θ and ς do not change qualitatively. In contrast to that, the direction of the ϖ–effect switches from a (downward and upward) swinging curve to an approximately linear upward trend. Because we are not interested in small ϖ–levels (as suggested above), the positive effect is in accordance to the results of the 'baseline case', if we consider $\varpi > 10$. Besides this, the main effects of λ and ν have to be explained, because they do not arise in the 'baseline case': The negative correlation of λ is straightforward and in correspondence to the standard economic theory. This means that a larger λ enhances firms' marginal costs, and this in turn (i) decreases supplied output in the consumer goods industry (c.p.), and (ii) increases supply prices in the capital goods industry (c.p.), and therefore the possibilities to sell machines. Consequently, the mean output gap is decreasing, if λ is rising. Finally, μ^y is falling if ν is rising, and then stabilizing if ν approaches its maximum of 2.5. The effect that μ^y increases, if ν is decreasing (and approaching 1.1), is due


Actual by Predicted Plot

Actual by Predicted Plot

Model Report

Column	Theta	Total Sensitivity	Main Effect	Lambda Interaction	Mü Interaction	Omega Macro Interaction	Iota Interaction	Sigma 2 Interaction	Theta Interaction	Varpi Interaction	Varsigma Interaction	Xi Interaction
Beta	0	0	0	0	0	0	0	0	0	0	0	0
Lambda	0.0796009	0.0397502	0.0395905	.	0.000118	4.2327e-8	2.2247e-8	1.0322e-7	2.7753e-5	0.0000132	5.2871e-7	9.032e-11
Eta RR	0	0	0	0	0	0	0	0	0	0	0	0
Eta IC	0	0	0	0	0	0	0	0	0	0	0	0
Mü	0.2209407	0.2777119	0.2542961	0.000118	.	0.0007109	7.6654e-5	0.0003438	0.0015086	0.0021419	0.0185158	1.2456e-8
Omega Micro	0	0	0	0	0	0	0	0	0	0	0	0
Omega Macro	0.0197543	0.0077902	0.0070564	4.2327e-8	0.0007109	.	7.168e-10	4.1263e-8	1.0933e-5	0.0000114	4.8005e-7	4.847e-12
Psi	0	0	0	0	0	0	0	0	0	0	0	0
Iota	0.0322794	0.0034672	0.0033874	2.2247e-8	7.6654e-5	7.168e-10	.	3.0195e-8	1.638e-6	1.3317e-6	6.9371e-8	0
Sigma 2	145.23464	0.0325664	0.0320117	1.0322e-7	0.0003438	4.1263e-8	3.0195e-8	.	6.0959e-5	6.5362e-5	8.4479e-5	1.149e-10
Theta	0.0589641	0.2526477	0.2509899	2.7753e-5	0.0015086	1.0933e-5	1.638e-6	6.0959e-5	.	1.6027e-5	3.1837e-5	7.0374e-9
Varpi	9.0074e-5	0.346813	0.3442961	0.0000132	0.0021419	0.0000114	1.3317e-6	6.5362e-5	1.6027e-5	.	0.0002677	2.336e-10
Tau	0	0	0	0	0	0	0	0	0	0	0	0
Varsigma	0.015186	0.0632403	0.0443395	5.2871e-7	0.0185158	4.8005e-7	6.9371e-8	8.4479e-5	3.1837e-5	0.0002677	.	2.382e-10
Rho	0	0	0	0	0	0	0	0	0	0	0	0
Xi	9.172e-5	0.0000108	1.0772e-5	9.032e-11	1.2456e-8	4.847e-12	0	1.149e-10	7.0374e-9	2.336e-10	2.382e-10	.

Mu	Sigma	Nugget
0.0044333	0.0528102	0.1

-2*LogLikelihood

-744.8954

Nugget parameter set to avoid singular variance matrix.

Model Report

Column	Theta	Total Sensitivity	Main Effect	Beta Interaction	Lambda Interaction	Mü Interaction	Iota Interaction	Sigma 2 Interaction	Theta Interaction	Varpi Interaction	Tau Interaction	Varsigma Interaction
Beta	1.2395432	0.1276391	0.1124989	.	2.3956e-5	0.0125067	4.2838e-6	1.9829e-5	0.0022621	0.0002028	2.5366e-5	9.514e-5
Lambda	0.0988711	0.0303292	0.0298785	2.3956e-5	.	4.7647e-5	5.0598e-9	3.7285e-8	0.0002625	0.0001163	5.5682e-8	2.5412e-7
Eta RR	0	0	0	0	0	0	0	0	0	0	0	0
Eta IC	0	0	0	0	0	0	0	0	0	0	0	0
Mü	0.1844827	0.2509664	0.2342939	0.0125067	4.7647e-5	.	1.3149e-5	2.2118e-5	0.002103	0.0006782	3.4567e-5	0.0012671
Omega Micro	0	0	0	0	0	0	0	0	0	0	0	0
Omega Macro	0	0	0	0	0	0	0	0	0	0	0	0
Psi	0	0	0	0	0	0	0	0	0	0	0	0
Iota	0.043954	0.0041467	0.0041015	4.2838e-6	5.0598e-9	1.3149e-5	.	1.0465e-9	2.1759e-5	6.0183e-6	8.3752e-9	4.2179e-9
Sigma 2	50.015887	0.009596	0.0095435	1.9829e-5	3.7285e-8	2.2118e-5	1.0465e-9	.	5.8251e-6	4.5401e-6	7.4121e-8	4.5647e-8
Theta	0.2684601	0.2526742	0.2433749	0.0022621	0.0002625	0.002103	2.1759e-5	5.8251e-6	.	0.004539	6.3179e-5	4.1928e-5
Varpi	0.0001474	0.3148709	0.3093186	0.0002028	0.0001163	0.0006782	6.0183e-6	4.5401e-6	0.004539	.	8.4836e-7	0.0000046
Tau	0.0006609	0.0124199	0.0122958	2.5366e-5	5.5682e-8	3.4567e-5	8.3752e-9	7.4121e-8	6.3179e-5	8.4836e-7	.	1.8171e-8
Varsigma	0.0037588	0.0209208	0.0195117	9.514e-5	2.5412e-7	0.0012671	4.2179e-9	4.5647e-8	4.1928e-5	0.0000046	1.8171e-8	.
Rho	0	0	0	0	0	0	0	0	0	0	0	0
Xi	0	0	0	0	0	0	0	0	0	0	0	0

Mu	Sigma	Nugget
0.0746432	0.0656785	0.1

-2*LogLikelihood

-658.7494

Nugget parameter set to avoid singular variance matrix.

Note: The upper report table displays the results for standard deviation, the lower table that for expected value.

Figure 3.18: Actual by predicted plots and model reports for output gap responses (expected value and standard deviation)

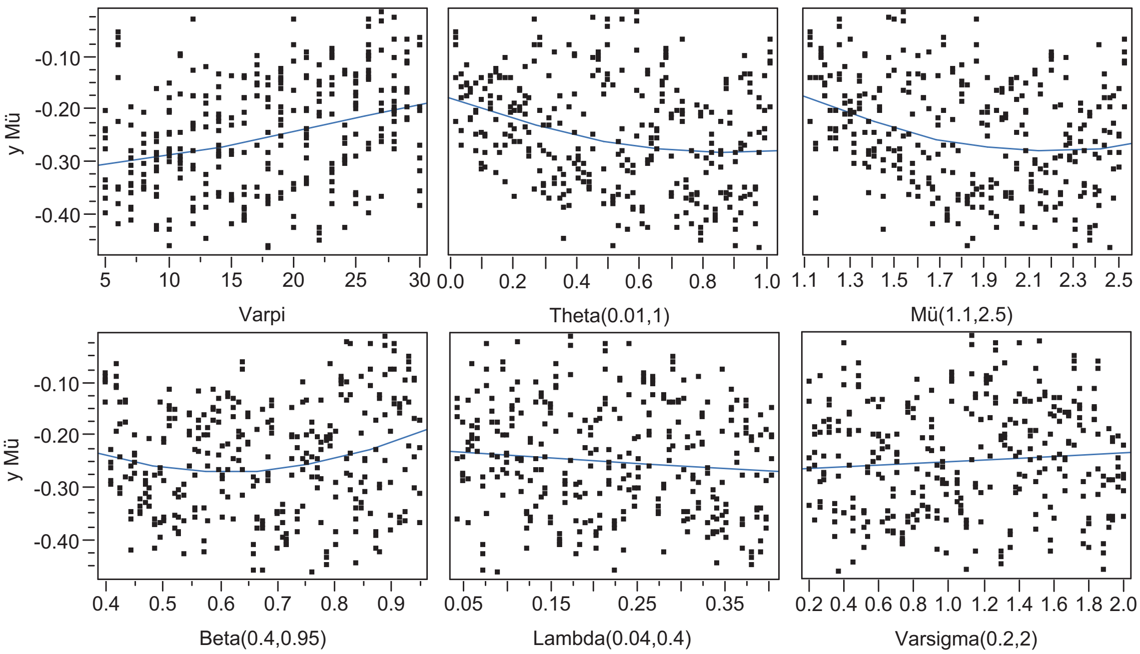


Figure 3.19: Marginal plots for expected value of the output gap

to the overall stabilizing effect of small ν–levels. Note that overall mean output gap is negative in the data of the 'Ponzi case', which implies that the named stabilizing effect increases the output gap. It can be assumed that the negative of effect of ν between 1.1 and 1.8 will reverse, if ν would exceed the depicted maximum of 2.5. It is, however, necessary to restrict ν to an upper boundary of 2.5, because otherwise inflation rates would rise exponentially in the 'baseline case', which is not desired. Consequently, we generally restrict ν in the described way.

Finally, we discuss the results for the standard deviation of the output gap: In the 'baseline case' the parameter θ constitutes the dominant main effect; thereafter several parameters ($Var[\epsilon]$, ν, ς, ψ, χ, ω^{Macro} and τ) feature minor effects on σ^y. This result changes somewhat in the 'Ponzi case'. In here, the dominant 'factors' are ϖ (34.4% main effect), ν (25.4%), and θ (25.1%). It is interesting that ϖ does not possess any impact on σ^y (but on μ^y) in the 'baseline case'. In the 'Ponzi case', ϖ dominates both output gap 'responses'. The influence of ν improves somewhat as compared to the 'baseline case'. Besides this, ς (4.4%), λ (4.0%), and $Var[\epsilon]$ (3.2%) have minor impact on σ^y. To summarize, the estimations for σ^y do not change substantially as compared to the 'Ponzi case'.

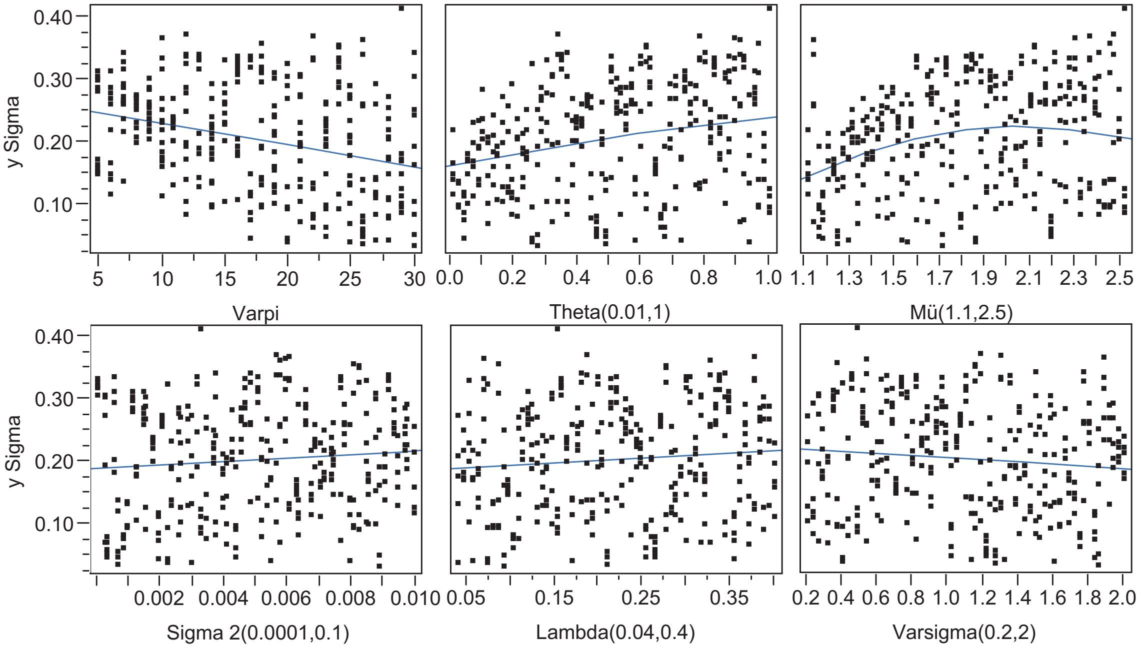


Figure 3.20: Marginal plots for standard deviation of the output gap

The same conclusion can be drawn from the marginal plots, as shown in the panels of figure 3.20. According to these plots, the direction of the effects of θ, ν, $Var[\epsilon]$ and ς do not change between the 'baseline' and 'Ponzi case'. In contrast, the parameter ϖ emerges as an important determinant of σ^y in the 'Ponzi case'; therefore its influence has to be explained: A rising ϖ enhances the number of household agents on Agent Island. In this case the variability of aggregate variables, such as inflation or output gaps, is assumed to decrease. Obviously, the effect appears that heterogenous actions cancel each other out due to the larger quantity of household agents. Finally, the direction of the parameter λ is positive in the 'Ponzi case'. Apparently, this results from the effect that the wage growth rate depends on λ. If λ parameter is large, the variability of labor incomes is likewise large; this affects (c.p.) the fluctuations of consumption expenditures, firms' capacity utilizations, and therefore output gaps.

3.3.6 Summary of the Results

The following table 3.6 summarizes the 'factors', which possess significant effect (i.e. at least 2% total sensitivity) in the 'baseline case' or in the 'Ponzi case', ranked by decreasing order of importance. The summary defines the minimal model(s) to be investigated within the calibration procedure of the next section. The parameters without any impact drop out from further investigations, and are fixed somewhere in the middle of their domains during the final 'validation'. The results illustrated in table 3.6 are particularly interesting for the parameters determining savings rates, i.e. η^{RR} as well as η^{IC}. In neither case these parameters exhibit any significant influence on one of the 'responses'. In this context the exclusion of η^{RR} fits to the analytical as well as empirical ambiguous effect of real interest rates on savings rates; see the discussion in subsection 2.2.1. Hence, the initially assumed 'savings–channel' seems to be ineffective on Agent Island.

Factors[1]	Agents[2]	Baseline case				Ponzi case			
		μ^{π}	σ^{π}	μ^{y}	σ^{y}	μ^{π}	σ^{π}	μ^{y}	σ^{y}
θ	Consumer goods firms	X	X	X	X	X	X	X	X
ν^{lower}	Households	X	X		X	X	X	X	X
ν^{upper}	Households	X	X		X	X	X	X	X
ϖ	All agents	X	X	X		X	X	X	X
β	Consumer goods firms		X	X		X	X	X	
λ	All agents				X	X		X	X
τ	Central bank		X	X	X		X		
ς	Central bank			X	X			X	X
ω^{Macro}	Capital goods firms			X	X	X			
$Var[\epsilon]$	All firms				X		X		X
ρ	Central bank		X			X			
χ	Households			X	X				
ι	Consumer goods firms	X							
ψ	Consumer goods firms				X				
ω^{Micro}	Capital goods firms						X		
η^{RR}	Households								
η^{IC}	Households								

Note: 1) The column 'Factors' contains all structural and behavioral parameters, which are investigated during the sensitivity analysis of this section. In the table, the factors are sorted for their relevance identified through the sensitivity analysis, i.e. the table starts in the first row with the most important factor θ, and so on. 2) The entries in the column 'Agents' constitute the sector(s) the 'factor' in question belongs to.

Table 3.6: Significance of factors

The exclusion of η^{RR} and the according absence of the 'savings–channel' seems to be due to the misleading effects of monetary policy, identified in this section. These misleading effects are one result of fact that the model tends to produce negative inflation and output gap figures. Note that during early 'face validation' runs, the model tended to produce hyperinflationary outcomes. We manage this problem through the integration of several restrictions into the savings decision of household agents. Thereafter, within the later 'validation' of the model, the simulation outcomes feature overall deflationary trends. Such downward pressure on prices distort credit interest rates through the zero lower bound restriction: On condition that both inputs of the 'Taylor rule' (i.e. inflation and output gaps) are negative, the influence of monetary policy is distorted and reduced. For example, real interest rates are rising (instead of falling), if the nominal interest rate rests at 0%. As a consequence, individual savings rates are rising (instead of falling), which indicates a procyclical effect. Finally, the mixtures of such procyclical effects of monetary policy in case of deflation and anticyclical effects in case of inflation can account for the unidentified (or wrong signed) influences of the central bank parameters ς and τ on the 'responses'. This can also account for the result that the parameters η^{RR} and η^{IC} do not affect inflation or the output gap, as it is expected.

We have to bear these problems in mind during the statistical 'validation' in the next section. There, we will investigate to what extend a negative 'interest rate shock' (i.e. an one percentage point increase of the credit interest rate by the central bank) influences inflation and output growth. We will compare the results with real–world evidence. Apparently, such an investigation is only possible, if credit interest rates are moving up during a simulation run. In the presented simulation—in particular in the 'Ponzi case'—this occurs in many runs only during the first ten periods of one single simulation run. When the agent economy remains in a deflationary state, which the central bank cannot counteract with its policy instrument, the credit interest rate falls to 0% and remains there. It is therefore of interest to adjust the model in such a way that it produces regular cyclical trends for the business cycle. To sum up, it is one necessary condition of the validated model that it produces downswings and upswings of economic activity, accompanied by decreasing and increasing credit interest rates.

Before we turn to the last step in the validating process in the next section, we want to highlight an important insight generated during the analysis until now: It is an interesting feature of an agent-based model that the researcher is able to investigate the model on various time scales. Consequently, we are in principle able to analyze the model in short, medium, and long term. Economic analysis is normally divided in the business cycle perspective (short or medium term perspective), and the growth perspective (long term perspective). It is assumed that monetary policy is only effective in the short or medium term, and therefore monetary issues do not affect the long term growth prospects of an economy.[42] This is based upon the 'neoclassical' idea that monetary policy is not able to influence real magnitudes in the long run. As a consequence of this view, 'orthodox' models do often not consider any financial variables; even credit transactions are expressed in real terms. However, the fact that the 'orthodox' approach of 'New Keynesian' macroeconomics identifies effects of monetary policy on real output is due to the existence of price rigidities. In the sensitivity analysis of this section (encompassing a time frame of 20 periods/years), we find out that monetary policy, represented by ς, τ and ρ, is able to affect real magnitudes in the medium term. Indeed, the estimated influences are quite small (i.e. at most about 7% in the ANOVA reports, see the figures 3.9 and 3.18), but they do exist. It could be the task of a further analysis to investigate the time perspective of monetary policy in an agent–based environment, and to review the 'orthodox' perspective that issues of macroeconomic growth and monetary policy have to be analyzed in different (time) frameworks.

3.4 Final Validation

Within this section we take the final steps towards a reasonable validated model of Agent Island. Thereby, we employ the minimal models (for both cases) as described in the end of the last section. The crucial point of the final 'validation' step involves the application of a multi–level calibration procedure. In the first step (in subsection 3.4.1), we calibrate the model towards stylized facts of the business cycle, viz. the expected value and the standard deviation of inflation and output gap time series. The results of this calibration (i.e. a specific scenario for each case) is thereafter verified

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[42] See, for example, Bofinger, 2001, for the time–perspective of monetary policy.

through a statistical 'validation' of the 'Phillips curve' relationship: If we merely adjust the model of Agent Island to the stylized facts of the business cycle represented by the expected value and the standard deviation of inflation and output gap time series, nothing is said about the interrelation between both time series. In reality, the 'Phillips curve' describes this interrelation. That is, if the output gap is positive, upside pressure on inflation is expected. Hence, we investigate in the statistical 'validation' the cross–correlation of both time series, and we compare them to the data of the 'original system'. During many 'face validation' runs we found that the model of Agent Island usually reproduces the 'Phillips curve' relationship.[43]

Subsequently, in the calibration at the second level, we adjust the model to the 'Keynesian business cycle equilibrium' (see subsection 3.4.2). Importantly, we use the results of the calibration process at level I as the *initial values* of the search process at level II. The aim of level II is to guarantee that the business cycle is neither dominated by economic downswings nor by upswings. Both directions should be balanced over the simulation span, so that overall deflationary trends (as documented in the sensitivity analysis) are avoided. After this calibration procedure, we again employ a statistical 'validation': Thereby, we investigate the relevance of the 'circuit equilibrium' for upswings and downswings of the business cycle. It is expected that a negative 'circuit equilibrium indicator' (CEI) appears in economic upswings, and that a positive one appears in economic downswings. This connection was already verified in some 'face validation' runs.

In order to combine both perspectives (i.e. level I and level II), the multi–level calibration procedure concludes with an iteration of level I and level II (in subsection 3.4.3). This iteration implies that we use the previous results of level II as the initial values for a renewed level I calibration. Thereafter, the results of this renewed calibration procedure at level I are compared to the results of the previous one at level II. If they do not differ significantly, the iteration process is finished, and we treat the results as the final adjustment of the model. In this case, the results represent the validated model. If they do differ, we in turn use them as the initial values of a renewed level II calibration, and so on. The iteration will continue until two consecutive calibration experiments show comparable

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[43] The core problem of these early 'face validation' runs was that we were not able to investigate *calibrated* models as opposed to this section.

results. Such an iteration procedure guarantees that the model is adjusted reasonably towards the minimization of two ‘objective functions’: (i) The first of them is incorporating stylized facts of the business cycle (at level I), (ii) and the second one is incorporating the ‘circuit equilibrium indicator’ (at level II). The result of this iteration procedure constitutes the calibrated model. Subsequently, the calibrated model has to endure (i) a final statistical ‘validation’ (at the end of subsection 3.4.3) and (ii) several ‘plausibility checks’ (in subsection 3.4.4): Firstly, in the final statistical ‘validation’, we investigate the influence of negative ‘monetary policy shocks’ on inflation and output growth, and we compare the derived results with real–world evidence. Secondly, we finish the ‘validation’ task of this chapter with a ‘plausibility check’, in which we analyze qualitatively the aggregate data of several ‘face validation’ runs. Beyond the discussed topics in this section,[44] we investigate as to how aggregate firm profits, aggregate debt, and aggregate financial wealth affect the business cycle. As a consequence, we illustrate the time series in question and explain them qualitatively in the ‘plausibility checks’.

3.4.1 Level I Calibration: Stylized Facts of the Business Cycle

As explained, we start with a calibration procedure. The results of the calibration are verified by a subsequent statistical analysis. Finally, the results of this subsection (i.e. specific scenarios for the ‘baseline case’ and the ‘Ponzi case’) are used as initial values in the following level II calibration in subsection 3.4.2.

Calibration Experiments

Parameter	Level[1]	Parameter	Level[1]
η_h^{RR}	[0.168–0.252] UD	η_h^{IC}	[0.088–0.132] UD
$\psi_{j,k}$	[0.2–0.3] UD	$\iota_{j,k}$	[0.73–0.99] UD
$\chi_h^{I,D}$	[1.09–1.63] UD	ω_c^{Micro}	[0.36–0.54] UD

Note: 1) The shortcut ‘UD’ describes that the individual parameter is a ‘uniformly distributed’ random variable. The figures in the bracket indicate the statistical population of the distribution.

Table 3.7: Settings of the not investigated main parameters

The sensitivity analysis of the previous section delivers the ‘minimal models’ (for the ‘baseline

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[44] That is, the ‘Phillips curve’ relationship, the role of the ‘circuit equilibrium indicator’, and the impact of negative ‘monetary policy shocks’.

case' and the 'Ponzi case'). Besides this, we use again the 'peripheral settings' as specified in table 3.1 in section 3.2. Note that some main parameters are eliminated from the calibration procedure, due to the minimization of the model in the last section. So it is clear that we have to fix them to reasonable levels: See the data in table 3.7. The calibration of the model is arranged in order to adjust the model with respect to stylized macro facts. The reference data are inflation and output gap time series of major industrial economies (Germany, U.S., U.K., and Japan) from 1980 to 2007. We treat these countries as the 'original system'. See the illustration of the named time series in the panels in figure 3.21.

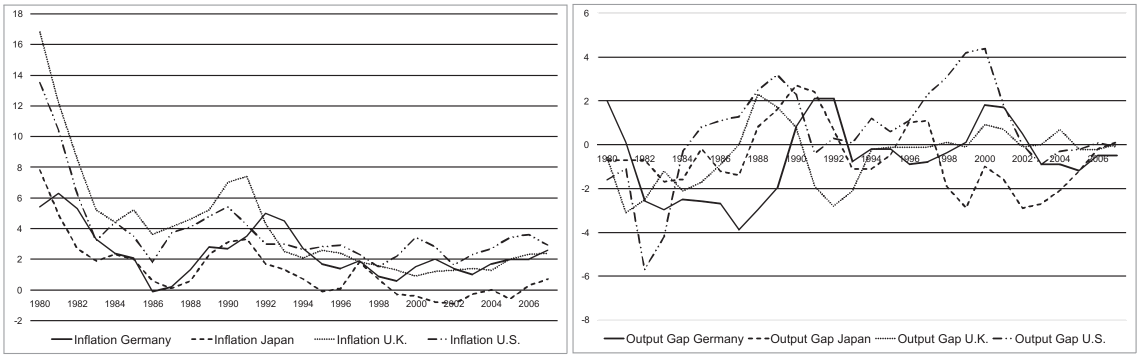

Figure 3.21: Inflation (left panel) and output gap (right panel) time series; Germany, United States, United Kingdom and Japan, data from 1980 to 2007; Source: IMF, World Economic Outlook

We need to compress the data to single figures; again we use the expected value and the standard deviation, i.e. the first and the second central moments. Table 3.8 shows these figures. In the calibration approach, we use average values over the four countries mentioned above as reference values of the 'original system'; see last row in table 3.8. In case of inflation time series, this results in an expected value of 0.0396 (i.e. 3.96%) and a standard deviation of 0.0369. For output gap time series, the average value over all four countries is -0.003 (-0.3%) for the expected value, and 0.017 for the standard deviation. In the sensitivity analysis we already referred to the fact that the mean output gap in many simulation runs tended to be negative. In real world data only the U.S. data shows a positive figure (0.6%), while the other three countries feature negative values.[45] Indeed, it

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[45] We use IMF data from 1980 to 2007. The alternative OECD output gap data is pretty heterogenous with respect to the depicted period, e.g. the German data set starts after reunification in 1990, while other data sets start in the

would be expected that the mean output gap data is close to 0%.

Country	μ^{π}_{ref}	σ^{π}_{ref}	μ^{y}_{ref}	σ^{y}_{ref}
Germany	0.0262	0.0225	-0.007	0.017
United States	0.0381	0.0296	0.006	0.022
United Kingdom	0.0573	0.0484	-0.005	0.013
Japan	0.0368	0.0470	-0.006	0.015
∅ (Reference values)	0.0396	0.0369	-0.003	0.017

Table 3.8: Reference data represented by the central moments (of first and second order) of inflation and output gap time series; Germany, United States, United Kingdom and Japan, data from 1980 to 2007; Source: IMF, World Economic Outlook

The results of table 3.8 are the basis for the calibration experiments at level I. Thereby, we minimize a 'goal objective function' (*GOF I*) through variations of the calibration parameters (i.e. the investigated main parameters). This search process is executed by a 'calibration plug–in' within the SeSAm programing environment. As explained in section 1.3, the 'plug–in' employs the 'simulated annealing' optimization algorithm. According to our aim to fit the model to the stylized facts, delivered by the figures in table 3.8, we construct the following 'goal objective function':

$$\begin{aligned} GOF\,I \quad &= \quad [100\mu^{\pi}_{sim} - 100\mu^{\pi}_{ref}]^2 + [100\sigma^{\pi}_{sim} - 100\sigma^{\pi}_{ref}]^2 + \\ &\quad\quad [100\mu^{y}_{sim} - 100\mu^{y}_{ref}]^2 + [100\sigma^{y}_{sim} - 100\sigma^{y}_{ref}]^2. \end{aligned} \tag{3.4.1}$$

Equation (3.4.1) contains the expected value and the standard deviation of inflation and output gap time series for (i) the reference data (μ^{π}_{ref}, σ^{π}_{ref}, μ^{y}_{ref} and σ^{y}_{ref}) delivered by table 3.8, and (ii) for the data delivered by the calibration experiments (μ^{π}_{sim}, σ^{π}_{sim}, μ^{y}_{sim} and σ^{y}_{sim}). All figures in equation (3.4.1) are multiplied by 100 in order to produce %–figures above 1 (as opposed to the rest of the study). The squaring of the deviations in the 'goal objective function' guarantees that positive *and* negative deviations from the reference figures produce in each case a loss. Therefore, the minimal loss would be 0, if the simulated data matched the reference data perfectly for each

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1960s or 1970s. As a result, we use the IMF data, where the time series for all countries start 1980. Nevertheless, the OECD data also indicates that the mean output gap is slightly negative (for many countries).

target figure. As stated, the 'simulated annealing' algorithm searches for that scenario which minimizes the 'goal objective function' (3.4.1).[46] It is therefore necessary to define a suitable framework for the parameter search: This contains a starting value, a minimum and maximum value for each parameter, as well as the start, minimum and maximum step sizes applied in the search process. The following table 3.9 summarizes the settings in question.

Parameter[1,2,3]	Value start	Value min.	Value max.	Step size start	Step size min.	Step size max.
Parameters used in the calibration of both cases						
β	0.98	0.4	0.98	0.05	0.01	0.1
ϖ	25	5	35	3	1	5
$Var[\epsilon]$	0.001	0.0001	0.01	0.0025	0.0005	0.005
θ	0.1	0.01	1	0.05	0.01	0.2
ω^{Macro}	0.1	0.2	0.7	0.05	0.01	0.1
λ	0.1	0.04	0.4	0.05	0.01	0.1
ν^{lower}	0.5	0.3	0.9	0.1	0.05	0.2
ν^{upper}	1.5	1.1	3.0	0.1	0.05	0.4
τ	2.0	1.0	4.0	0.5	0.1	1.0
ς	2.0	0.2	2.0	0.2	0.1	0.5
ρ	0.3	0.2	0.9	0.1	0.02	0.2
Parameters exclusively used in the calibration of the baseline case						
ψ	0.075	0.05	0.3	0.05	0.01	0.1
ι	0.875	0.7	0.95	0.05	0.01	0.1
$\chi^{I,D}$	1.5	1.1	2	0.05	0.01	0.2
Parameters exclusively used in the calibration of the Ponzi case						
ω^{Micro}	0.2	0.2	0.7	0.05	0.01	0.1

Note: 1) The individual levels of the parameters are drawn from a 'uniform distribution' ± 20% around the investigated level. The exceptions are the parameters ν^{lower} and ν^{upper}; see also the following note. 2) The parameters ν^{lower} and ν^{upper} are drawn form a 'uniform distribution' ± 40% around the investigated level. 3) The factor ϖ is a discrete integer variable.

Table 3.9: Parameter information concerning the calibration experiments

It is worth noting that we apply some small modifications compared to the sensitivity analysis of the last section: (i) We lift the upper limit of β from 0.95 to 0.98. (ii) The upper limit of the domain of ϖ is increased from 30 to 35. (iii) Most importantly, we change the heterogeneity of consumer agents with respect to the parameters ν^{lower} and ν^{upper}, viz.: (i) The heterogeneity of the individual parameters ν_h^{lower} and ν_h^{upper} of any agent h is broadened from ± 20% to ± 40 % (around the investigated levels ν^{lower} and ν^{upper}). See also the notes (1) and (2) in table 3.9. We notice that

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[46] See chapter 1.3 for a short discussion of the 'simulated annealing' optimization method.

this modification fits somewhat better to the presumed heterogeneity of real world consumption and savings behavior. For example, if we investigate a scenario where ν^{upper} is fixed to 1.6, household agent h can now draw ν_h^{upper} from a statistical population between 1.1 and 2.24.[47,48] (ii) In addition, we broaden the domain of ν^{lower} and ν^{upper}. In the new framework we apply: $0.3 < \nu^{lower} < 0.9$ and $1.1 < \nu^{upper} < 3.0$.[49] To sum up, the largest ratio of (individual) ν_h^{upper} to ν_h^{lower} is constituted by $\frac{4.2}{0.18} \approx 23.33$.[50] In this extreme case, the saturation level of consumption expenditures is 23.33 times the subsistence level of consumption expenditures. However, the average ratio over all agents is given by $\frac{3.0}{0.3} = 10$, i.e. the saturation level is 10 times the subsistence level. Then again, in the other extreme case the average ratio is only $\frac{1.1}{0.9} = 1.22$. The task of the calibration procedure is to find levels for ν^{lower} and ν^{upper} within these boundaries that guarantee reasonable results for inflation and output gap time series.

Finally, the results calculated by the calibration tool are presented in table 3.10 for the 'baseline case', and in table 3.11 for the 'Ponzi case'. During each calibration procedure more than 400 simulations runs are conducted, plus several 100 refinement runs. Both tables show the 20 best results delivered by this procedure.[51] In the 'baseline case', the best result for the *GOF I* (see the figures in the column 'target value') is represented by 9,734. According to this, the average deviation for each of the four target values is approximately 50 percentage points, which is a bad result. [52] In the 'Ponzi case', the best 'target value' is 12,045, which gives an average deviation of about 55 percentage points.[53] It should be noted that these are not percentage deviations—they are in fact percentage points. According to these bad results, it becomes clear that another calibration step (at

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[47] Note that individual parameters are (uniformly distributed) random variables.

[48] Determination of the lower boundary of the population: In fact, we obtain $0.96 = 0.6 \times 1.6$. By definition ν^{upper} is, however, not allowed to be smaller than 1. Moreover, we restrict ν^{upper} to $\nu^{upper} \geq 1.1$, i.e. levels below 1.1 are not allowed. This delivers the stated value of 1.1. Determination of the upper boundary of the population: This is given through $2.24 = 1.6 \times 1.6$.

[49] We will see below that this new assumption is not effective, because the calibration procedure delivers levels for ν^{lower} and ν^{upper}, which lie within the old domain.

[50] In the extremest case of spreads between ν_h^{lower} and ν_h^{upper}, we obtain an individual $\nu_h^{lower} = 0.6 \times 0.3 = 0.18$ (i.e. 0.3 − 40%), and an individual $\nu_h^{upper} = 1.4 \times 3.0 = 4.2$ (i.e. 3.0 + 40%). This gives the ratio $\frac{4.2}{0.18} \approx 23.33$.

[51] In each of the following calibration experiments at level II (see section 3.4.2) and in the subsequent iteration (see section 3.4.3) we represent only the best single results. The overviews in figures 3.10 and 3.11 should give an idea of how the results are derived by the 'simulated annealing' search algorithm.

[52] See equation (3.4.1) for the construction of the 'goal objective function'. The average deviation is delivered by dividing the 'target value' by four, due to the fact that the 'goal objective function' consists of four sub–target values; and by extracting the root of the result. This gives $50 \approx \sqrt{\frac{9{,}734}{4}}$.

[53] This figure is given by $55 \approx \sqrt{\frac{12{,}045}{4}}$.

β	λ	ν^{lower}	ν^{upper}	ω^{Macro}	χ	$Var[\epsilon]$	ϖ	θ	τ	ς	ρ	ι	ψ	Target Value
0.97	0.13	0.55	1.75	0.21	1.55	0.003250	28	0.09	1.68	2	0.32	0.88	0.06	9734.32
0.96	0.14	0.5	1.8	0.2	1.5	0.002750	27	0.1	1.58	2	0.3	0.87	0.05	10728.31
0.97	0.15	0.45	1.85	0.21	1.55	0.002250	26	0.09	1.68	1.9	0.32	0.86	0.05	12099.18
0.97	0.15	0.45	1.75	0.21	1.5	0.002250	28	0.11	1.48	1.9	0.28	0.86	0.06	12099.45
0.98	0.19	0.6	1.7	0.2	1.5	0.000100	24	0.15	1.08	2	0.39	0.82	0.05	13749.28
0.95	0.15	0.45	1.75	0.21	1.55	0.002250	26	0.11	1.68	2	0.28	0.86	0.05	17261.02
0.97	0.15	0.55	1.85	0.21	1.6	0.002250	26	0.11	1.48	2	0.32	0.86	0.06	21183.06
0.95	0.13	0.45	1.85	0.2	1.55	0.003375	28	0.11	1.68	2	0.28	0.86	0.05	21856.68
0.98	0.14	0.6	1.8	0.2	1.55	0.002750	29	0.08	1.78	2	0.3	0.87	0.05	22344.65
0.95	0.13	0.55	1.85	0.21	1.55	0.003250	26	0.11	1.68	1.9	0.28	0.88	0.06	24849.51
0.95	0.13	0.55	1.85	0.2	1.55	0.003250	28	0.11	1.68	2	0.32	0.86	0.05	25002.29
0.98	0.09	0.6	1.7	0.2	1.55	0.000100	30	0.15	2.08	2	0.2	0.82	0.05	32034.02
0.97	0.13	0.45	1.85	0.2	1.6	0.003250	28	0.11	1.48	2	0.28	0.86	0.05	34850.07
0.97	0.15	0.45	1.75	0.2	1.52	0.003250	28	0.11	1.48	2	0.28	0.86	0.06	36248.98
0.95	0.15	0.55	1.85	0.2	1.50	0.002250	26	0.11	1.48	2	0.28	0.88	0.05	37449.43
0.95	0.15	0.45	1.85	0.2	1.5	0.003250	28	0.09	1.68	2	0.32	0.86	0.05	39956.86
0.95	0.15	0.55	1.85	0.2	1.48	0.003250	26	0.09	1.48	2	0.32	0.86	0.05	43317.16
0.95	0.15	0.55	1.75	0.21	1.4	0.002125	28	0.11	1.48	2	0.32	0.88	0.05	44804.82
0.95	0.13	0.55	1.85	0.21	1.5	0.002250	28	0.09	1.68	1.9	0.28	0.86	0.05	44853.23
0.98	0.11	0.55	1.75	0.2	1.6	0.000250	28.5	0.13	1.33	2	0.25	0.9	0.08	47132.69

Table 3.10: 20 best calibration results out of 724 investigated simulation runs in the baseline case

β	λ	ν^{lower}	ν^{upper}	ω^{Macro}	ω^{Micro}	$Var[\epsilon]$	ϖ	θ	τ	ς	ρ	Target Value
0.97	0.13	0.45	1.45	0.21	0.21	0.0016250	26	0.11	1.9	2	0.36	12045.21
0.96	0.12	0.5	1.5	0.22	0.22	0.0021250	25	0.12	2	2	0.34	12885.4
0.97	0.13	0.45	1.55	0.21	0.21	0.0026250	24	0.11	1.9	2	0.32	13600.48
0.96	0.12	0.5	1.5	0.22	0.22	0.0038750	25	0.1	2.03	1.9	0.3	17968.83
0.98	0.14	0.4	1.5	0.22	0.2	0.0021250	27	0.1	2	1.9	0.38	20949.31
0.98	0.14	0.5	1.4	0.22	0.2	0.0021250	27	0.12	2	2	0.34	22393.06
0.96	0.14	0.4	1.5	0.2	0.22	0.0011250	25	0.12	1.8	2	0.34	23080.61
0.98	0.14	0.5	1.5	0.2	0.22	0.0020000	23	0.12	1.8	1.9	0.34	24545.74
0.96	0.14	0.5	1.5	0.22	0.2	0.0031250	25	0.12	2	2	0.34	32937.2
0.98	0.14	0.4	1.5	0.22	0.2	0.0021250	25	0.1	1.8	2	0.34	35445.59
0.98	0.12	0.5	1.4	0.22	0.22	0.0021250	25	0.1	1.8	2	0.34	44653.88
0.98	0.14	0.4	1.5	0.22	0.22	0.0021250	25	0.12	2	2	0.38	52683.65
0.98	0.12	0.4	1.5	0.22	0.22	0.0021250	25	0.12	2	2	0.38	52752.25
0.98	0.12	0.4	1.4	0.2	0.22	0.0021250	25	0.12	1.8	2	0.34	55205.53
0.96	0.12	0.5	1.4	0.2	0.2	0.0011250	27	0.1	1.8	2	0.34	72833.22
0.96	0.12	0.4	1.5	0.2	0.2	0.0021250	23	0.1	1.8	2	0.34	75379.31
0.98	0.14	0.5	1.4	0.2	0.2	0.0011250	27	0.12	1.8	2	0.34	90140.68
0.96	0.14	0.5	1.5	0.22	0.2	0.0011250	27	0.1	2	2	0.38	96457.03
0.96	0.12	0.5	1.5	0.2	0.2	0.0011250	27	0.1	1.8	2	0.34	100091.97
0.98	0.12	0.4	1.6	0.22	0.22	0.0031250	23	0.12	1.8	2	0.3	107350.7

Table 3.11: 20 best calibration results out of 867 investigated simulation runs in the Ponzi case

level II) will be necessary. Moreover, the results illustrated in tables 3.10 and 3.11 determine the basis for the next step in the 'validation' process—the statistical 'validation' at level I.

Statistical Validation: The Phillips Curve Relationship

In the following paragraphs we verify the simulation output through some descriptive statistics. Again, we compare the 'baseline case' and the 'Ponzi case'. The investigated scenario of the 'baseline case' is specified by the settings represented in the first row in table 3.10; the scenario of the 'Ponzi case' is specified by the first row in table 3.11. Besides this, the 'peripheral settings' represented in the tables 3.1 and 3.7 still apply. We will investigate the 'Phillips curve' relationship by comparing the simulation output with evidence from large industrial economies. The reference data is given by the reports in figure 3.22: This shows the cross–correlation of inflation and output gap time series (from 1980 to 2007) for Germany, United States, United Kingdom and Japan. As suggested by the theory of the 'New Keynesian Phillips curve', we expect that the cross–correlation is positive (see, for example, Bofinger, 2001) and that the output gap causes inflation (see, for example, Woodford, 2003). The results displayed in figure 3.22 indicate that the expected positive cross–correlation is not identified for all countries in the investigated period. The data for Germany and Japan indicates a positive cross–correlation, when the lag or lead is 0, or when there is a 1– or 2–period(s) lead of inflation. In addition, the 'Granger causality tests'[54] for Germany and Japan indicate that output gaps cause inflation.[55] The results for U.S. and U.K. data, however, show negative cross–correlations between output gap and inflation, when the lag or lead is 0. This is also true, if lags or leads are added to the investigation. In this respect, the results contradict partially the notion of the 'New Keynesian Phillips curve'. This can be due to the fact that the investigation is based upon yearly data, and one could obtain better results with quarterly data.

Figure 3.23 displays the time series for countries with the best results, i.e. for Germany and Japan. For both countries the positive correlation between output gap and inflation is evident. The

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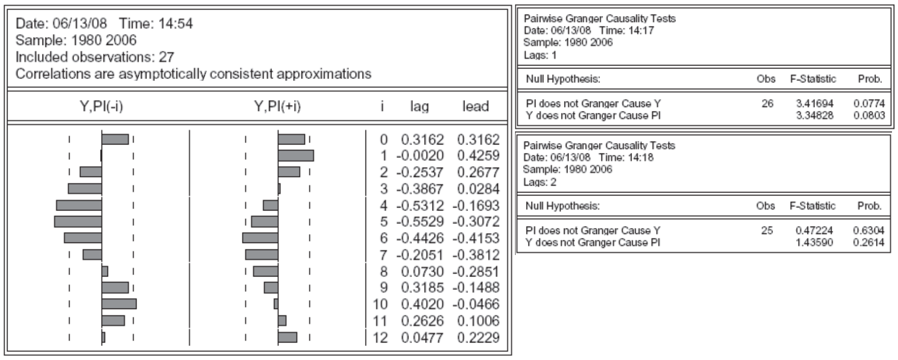
[54] A time series Y is said to 'Granger-cause' X, if it can be shown, usually through a series of F–tests on lagged values of Y (and with lagged values of X also known), that those Y values provide statistically significant information about future values of X.

[55] In case of a 1-period lag (see the reports in figure 3.22), the 'Granger causality test' delivers the result that the 'null hypothesis' (i.e. 'y does not Granger cause π') should be refused. Unfortunately, the test exhibits some inaccuracy, because for the German data the opposite 'null hypothesis' (i.e. 'π does not Granger cause y') has to be refused as well. The phenomenon is also found in the U.K. data set. Hence, only in the Japanese data set the expected causality is perfectly identified.

Germany: Cross Correlogram of Y and PI

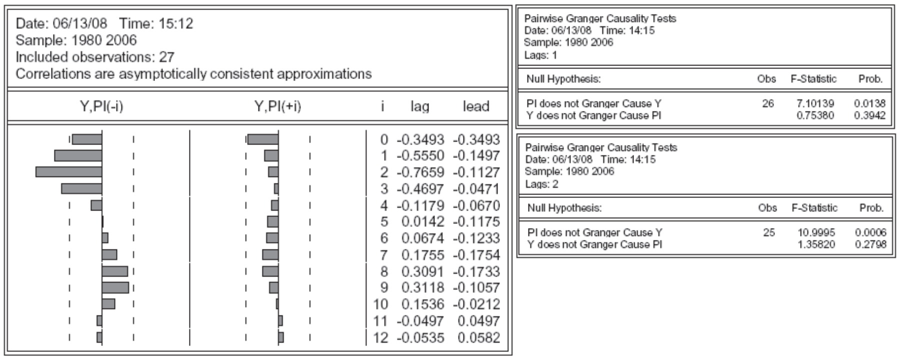


```
Date: 06/13/08   Time: 14:54
Sample: 1980 2006
Included observations: 27
Correlations are asymptotically consistent approximations

        Y,PI(-i)              Y,PI(+i)          i    lag      lead

                                                0   0.3162   0.3162
                                                1  -0.0020   0.4259
                                                2  -0.2537   0.2677
                                                3  -0.3867   0.0284
                                                4  -0.5312  -0.1693
                                                5  -0.5529  -0.3072
                                                6  -0.4426  -0.4153
                                                7  -0.2051  -0.3812
                                                8   0.0730  -0.2851
                                                9   0.3185  -0.1488
                                               10   0.4020  -0.0466
                                               11   0.2626   0.1006
                                               12   0.0477   0.2229
```

```
Pairwise Granger Causality Tests
Date: 06/13/08   Time: 14:17
Sample: 1980 2006
Lags: 1

Null Hypothesis:                        Obs   F-Statistic   Prob.

PI does not Granger Cause Y              26     3.41694     0.0774
Y does not Granger Cause PI                     3.34828     0.0803
```

```
Pairwise Granger Causality Tests
Date: 06/13/08   Time: 14:18
Sample: 1980 2006
Lags: 2

Null Hypothesis:                        Obs   F-Statistic   Prob.

PI does not Granger Cause Y              25     0.47224     0.6304
Y does not Granger Cause PI                     1.43590     0.2614
```

United States: Cross Correlogram of Y and PI

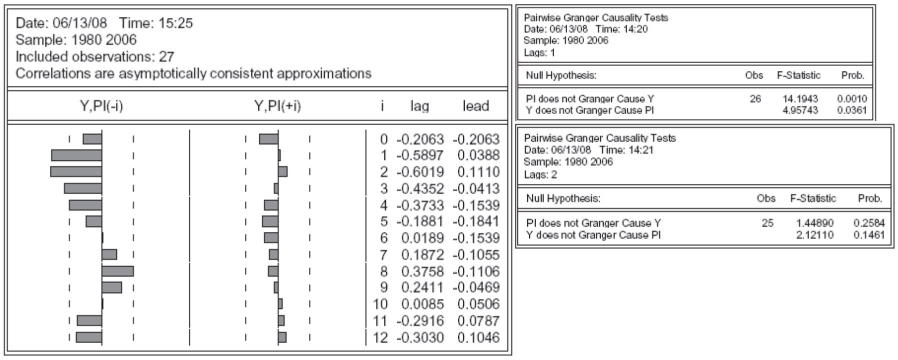


```
Date: 06/13/08   Time: 15:12
Sample: 1980 2006
Included observations: 27
Correlations are asymptotically consistent approximations

        Y,PI(-i)              Y,PI(+i)          i    lag      lead

                                                0  -0.3493  -0.3493
                                                1  -0.5550  -0.1497
                                                2  -0.7659  -0.1127
                                                3  -0.4697  -0.0471
                                                4  -0.1179  -0.0670
                                                5   0.0142  -0.1175
                                                6   0.0674  -0.1233
                                                7   0.1755  -0.1754
                                                8   0.3091  -0.1733
                                                9   0.3118  -0.1057
                                               10   0.1536  -0.0212
                                               11  -0.0497   0.0497
                                               12  -0.0535   0.0582
```

```
Pairwise Granger Causality Tests
Date: 06/13/08   Time: 14:15
Sample: 1980 2006
Lags: 1

Null Hypothesis:                        Obs   F-Statistic   Prob.

PI does not Granger Cause Y              26     7.10139     0.0138
Y does not Granger Cause PI                     0.75380     0.3942
```

```
Pairwise Granger Causality Tests
Date: 06/13/08   Time: 14:15
Sample: 1980 2006
Lags: 2

Null Hypothesis:                        Obs   F-Statistic   Prob.

PI does not Granger Cause Y              25     10.9995     0.0006
Y does not Granger Cause PI                     1.35820     0.2798
```

United Kingdom: Cross Correlogram of Y and PI

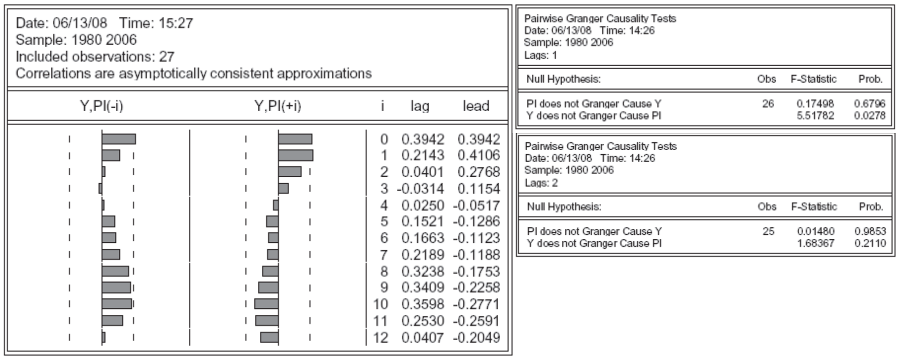


```
Date: 06/13/08   Time: 15:25
Sample: 1980 2006
Included observations: 27
Correlations are asymptotically consistent approximations

        Y,PI(-i)              Y,PI(+i)          i    lag      lead

                                                0  -0.2063  -0.2063
                                                1  -0.5897   0.0388
                                                2  -0.6019   0.1110
                                                3  -0.4352  -0.0413
                                                4  -0.3733  -0.1539
                                                5  -0.1881  -0.1841
                                                6   0.0189  -0.1539
                                                7   0.1872  -0.1055
                                                8   0.3758  -0.1106
                                                9   0.2411  -0.0469
                                               10   0.0085   0.0506
                                               11  -0.2916   0.0787
                                               12  -0.3030   0.1046
```

```
Pairwise Granger Causality Tests
Date: 06/13/08   Time: 14:20
Sample: 1980 2006
Lags: 1

Null Hypothesis:                        Obs   F-Statistic   Prob.

PI does not Granger Cause Y              26     14.1943     0.0010
Y does not Granger Cause PI                     4.95743     0.0361
```

```
Pairwise Granger Causality Tests
Date: 06/13/08   Time: 14:21
Sample: 1980 2006
Lags: 2

Null Hypothesis:                        Obs   F-Statistic   Prob.

PI does not Granger Cause Y              25     1.44890     0.2584
Y does not Granger Cause PI                     2.12110     0.1461
```

Japan: Cross Correlogram of Y and PI



```
Date: 06/13/08   Time: 15:27
Sample: 1980 2006
Included observations: 27
Correlations are asymptotically consistent approximations

        Y,PI(-i)              Y,PI(+i)          i    lag      lead

                                                0   0.3942   0.3942
                                                1   0.2143   0.4106
                                                2   0.0401   0.2768
                                                3  -0.0314   0.1154
                                                4   0.0250  -0.0517
                                                5   0.1521  -0.1286
                                                6   0.1663  -0.1123
                                                7   0.2189  -0.1188
                                                8   0.3238  -0.1753
                                                9   0.3409  -0.2258
                                               10   0.3598  -0.2771
                                               11   0.2530  -0.2591
                                               12   0.0407  -0.2049
```

```
Pairwise Granger Causality Tests
Date: 06/13/08   Time: 14:26
Sample: 1980 2006
Lags: 1

Null Hypothesis:                        Obs   F-Statistic   Prob.

PI does not Granger Cause Y              26     0.17498     0.6796
Y does not Granger Cause PI                     5.51782     0.0278
```

```
Pairwise Granger Causality Tests
Date: 06/13/08   Time: 14:26
Sample: 1980 2006
Lags: 2

Null Hypothesis:                        Obs   F-Statistic   Prob.

PI does not Granger Cause Y              25     0.01480     0.9853
Y does not Granger Cause PI                     1.68367     0.2110
```

Figure 3.22: Reference results for the cross–correlation and the Granger causality test of output gap and inflation time series; Germany, United States, United Kingdom and Japan, data from 1980 to 2006; Source: IMF, World Economic Outlook

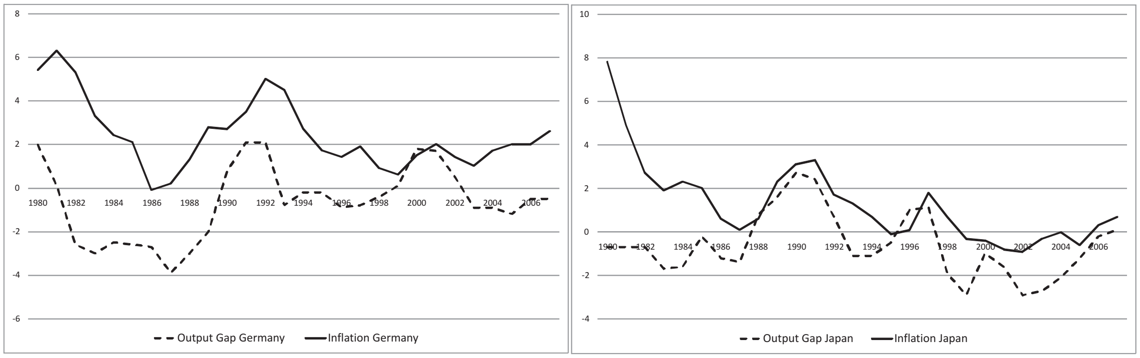


Figure 3.23: Output gap and inflation time series for Germany (left panel) and Japan (right panel); data from 1980 to 2006; Source: IMF, World Economic Outlook

panels in figure 3.24 illustrate the simulation data of the time series in question, i.e. we show the results of four arbitrarily chosen single simulation runs. It is interesting to note that we investigate only simulation runs lasting 10 periods. This is due to the phenomenon that before calibration at level II, deflation is the crucial problem of business cycle on Agent Island—especially if we consider more than 10 periods. From calibration at level II onwards, we investigate simulation runs including 20 periods. However, the displayed results are convincing—all panels indicate a positive cross–correlation. In addition, the graphs illustrate the already mentioned problem that the overall results (especially in the ‘Ponzi case’, see the right panels in figure 3.24) exhibit a deflationary trend. In some cases, the economy is able to escape deflation, but regularly this does not happen. The zero lower bound restriction can account for this fact. The result indicated by the panels in figure 3.24 are verified by the cross–correlation of output gap and inflation time series in the simulation data: Figure 3.25 displays the cross–correlograms for the ‘baseline case’, and figure 3.26 those for the ‘Ponzi case’. In the ‘baseline case’ the cross–correlation is positive in all four simulation runs, when no lags or leads are considered. The ‘Ponzi case’ data produces quite similar results, i.e. when the lag or lead is 0, the cross–correlation is positive in the selected simulation runs. The situation changes somewhat, if we investigate a lead of 1 or 2 period(s) for the inflation time series. Consequently, in the ‘baseline case’, the results for a 1–period lead of inflation are positive, even though one results is close to zero. In the ‘Ponzi case’ one result is close to zero but positive, and one is negative. Basically, the identified ‘Phillips curve’ relationship is more significant as compared to the

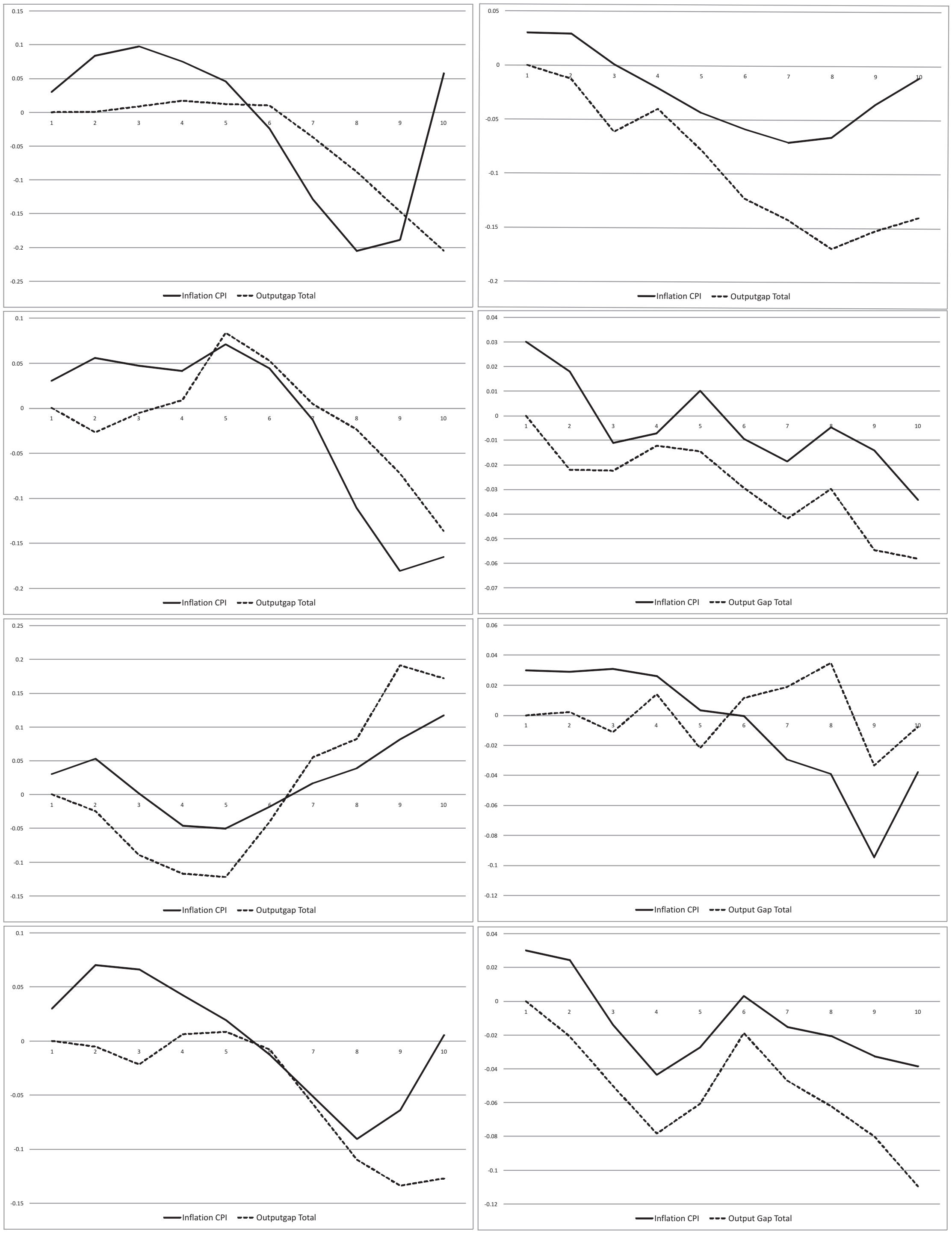
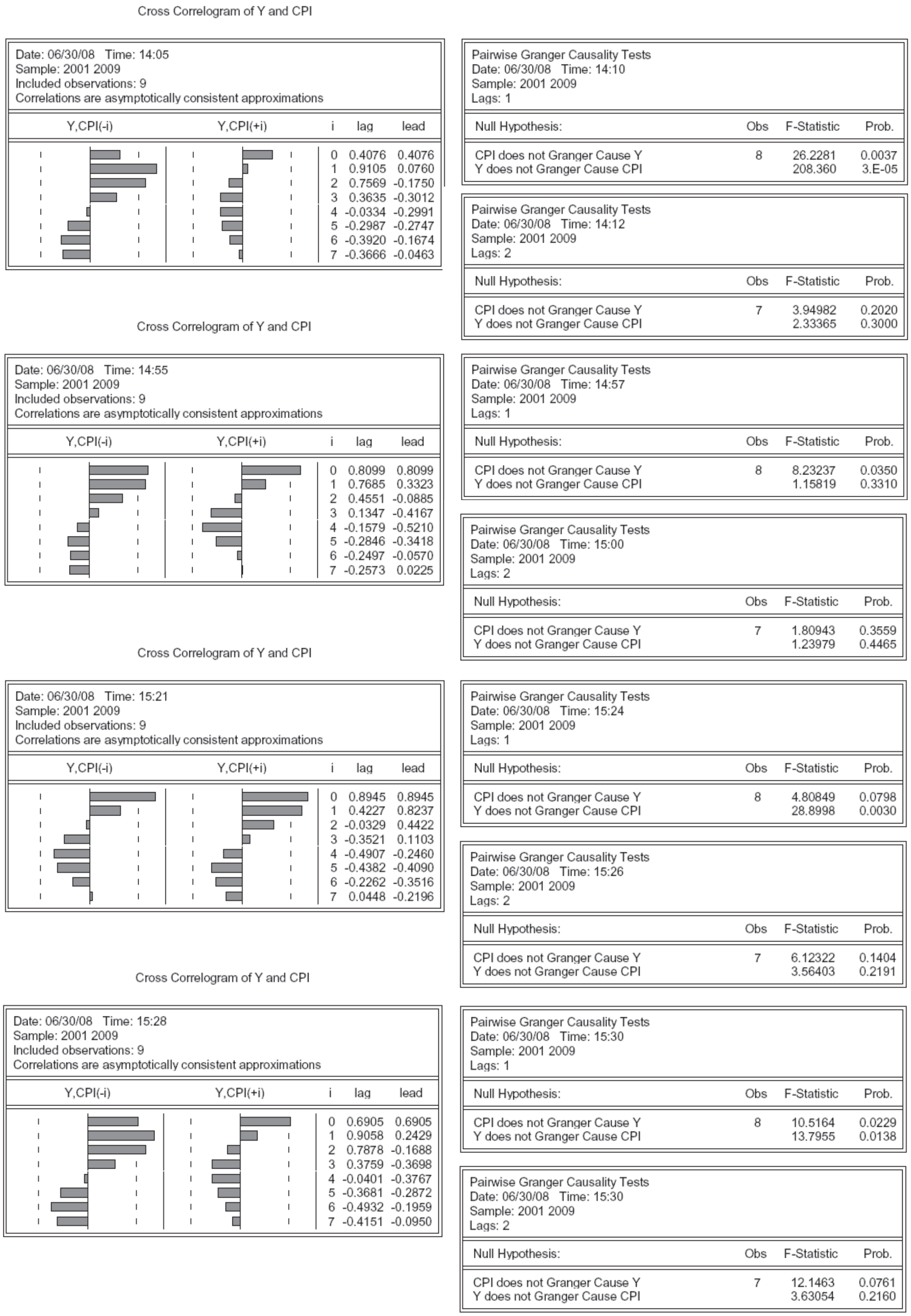


Figure 3.24: Output gap and inflation time series of eight exemplary simulations runs; four runs in the baseline case (left panels) and four runs in the Ponzi case (right panels)

‘original system’.

It is remarkable that the positive cross–correlation prevails, if we add a 1– or 2–period(s) lag for *inflation*. This disagrees with the ‘New Keynesian’ assumption that the output gap causes inflation. Indeed, the ‘Granger causality’ tests for the four time series (illustrated in the right panels of figure 3.25) verify these results: For lags equal to 1 period, the ‘null hypothesis’ that ‘CPI does not Granger cause y’ is rejected in all four examples. This indicates that π can ‘Granger cause’ y, which is in contradiction to the common notion that y causes π. The mechanism behind these results is, however, straightforward: Provided that in a certain period the capacity utilization of the firm sector is relatively high, the output gap is by definition positive. In this case the income of the household sector (generated in the firm sector) is likewise high, which in turn implies (c.p.) high consumption expenditures and high inflation. This is the first result identified in the panels of figure 3.25, namely that the cross correlation with lag or lead 0 is entirely positive. As a result of this explanation, we can assume that *within* one period the output gap *causes* inflation. In a next step, however, inflation propagates into the subsequent periods through wage agreements, i.e. via the wage setting rule. We denote this mechanism as the ‘wage–price spiral’ or the ‘expectations–channel’ of monetary transmission. In the following period wages are therefore rising at least with the inflation rate of the last period. In turn, this results in rising consumer incomes and (c.p.) in higher consumption expenditures. Finally, the capacity utilization of the firm sector, and therefore the output gap tend to rise in the following period as well. This connection accounts for the identified causality between inflation and output gaps between two consecutive periods.

Finally, the described effect of the ‘wage–price spiral’ propagates the inflation into the future. We verify these results by investigating the auto–correlation of inflation time series. See the panels in figure 3.27: The auto-correlation of inflation time series in the simulation data clearly corresponds to the real world data. In case of lags equal to 1, the auto–correlation in the simulation data is entirely positive (see right panels of figure 3.27), featuring values between 0.522 to 0.755. In the real–world data (left panels of figure 3.27) the values lie between 0.579 and 0.750. For lags equal to 2, the simulation data applies as well to the real world. In summary we can state that the ‘Phillips curve’



Cross Correlogram of Y and CPI

```
Date: 06/30/08   Time: 14:05
Sample: 2001 2009
Included observations: 9
Correlations are asymptotically consistent approximations

      Y,CPI(-i)          Y,CPI(+i)       i    lag     lead

                                         0  0.4076   0.4076
                                         1  0.9105   0.0760
                                         2  0.7569  -0.1750
                                         3  0.3635  -0.3012
                                         4 -0.0334  -0.2991
                                         5 -0.2987  -0.2747
                                         6 -0.3920  -0.1674
                                         7 -0.3666  -0.0463
```

```
Pairwise Granger Causality Tests
Date: 06/30/08   Time: 14:10
Sample: 2001 2009
Lags: 1

 Null Hypothesis:                  Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y       8     26.2281      0.0037
 Y does not Granger Cause CPI             208.360      3.E-05
```

```
Pairwise Granger Causality Tests
Date: 06/30/08   Time: 14:12
Sample: 2001 2009
Lags: 2

 Null Hypothesis:                  Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y       7     3.94982      0.2020
 Y does not Granger Cause CPI             2.33365      0.3000
```

Cross Correlogram of Y and CPI

```
Date: 06/30/08   Time: 14:55
Sample: 2001 2009
Included observations: 9
Correlations are asymptotically consistent approximations

      Y,CPI(-i)          Y,CPI(+i)       i    lag     lead

                                         0  0.8099   0.8099
                                         1  0.7685   0.3323
                                         2  0.4551  -0.0885
                                         3  0.1347  -0.4167
                                         4 -0.1579  -0.5210
                                         5 -0.2846  -0.3418
                                         6 -0.2497  -0.0570
                                         7 -0.2573   0.0225
```

```
Pairwise Granger Causality Tests
Date: 06/30/08   Time: 14:57
Sample: 2001 2009
Lags: 1

 Null Hypothesis:                  Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y       8     8.23237      0.0350
 Y does not Granger Cause CPI             1.15819      0.3310
```

```
Pairwise Granger Causality Tests
Date: 06/30/08   Time: 15:00
Sample: 2001 2009
Lags: 2

 Null Hypothesis:                  Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y       7     1.80943      0.3559
 Y does not Granger Cause CPI             1.23979      0.4465
```

Cross Correlogram of Y and CPI

```
Date: 06/30/08   Time: 15:21
Sample: 2001 2009
Included observations: 9
Correlations are asymptotically consistent approximations

      Y,CPI(-i)          Y,CPI(+i)       i    lag     lead

                                         0  0.8945   0.8945
                                         1  0.4227   0.8237
                                         2 -0.0329   0.4422
                                         3 -0.3521   0.1103
                                         4 -0.4907  -0.2460
                                         5 -0.4382  -0.4090
                                         6 -0.2262  -0.3516
                                         7  0.0448  -0.2196
```

```
Pairwise Granger Causality Tests
Date: 06/30/08   Time: 15:24
Sample: 2001 2009
Lags: 1

 Null Hypothesis:                  Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y       8     4.80849      0.0798
 Y does not Granger Cause CPI             28.8998      0.0030
```

```
Pairwise Granger Causality Tests
Date: 06/30/08   Time: 15:26
Sample: 2001 2009
Lags: 2

 Null Hypothesis:                  Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y       7     6.12322      0.1404
 Y does not Granger Cause CPI             3.56403      0.2191
```

Cross Correlogram of Y and CPI

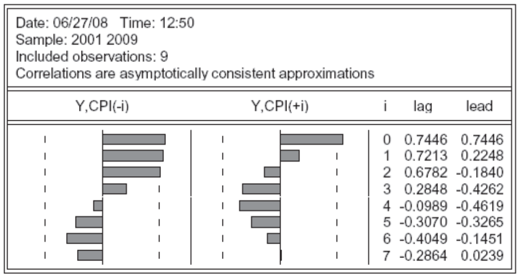
```
Date: 06/30/08   Time: 15:28
Sample: 2001 2009
Included observations: 9
Correlations are asymptotically consistent approximations

      Y,CPI(-i)          Y,CPI(+i)       i    lag     lead

                                         0  0.6905   0.6905
                                         1  0.9058   0.2429
                                         2  0.7878  -0.1688
                                         3  0.3759  -0.3698
                                         4 -0.0401  -0.3767
                                         5 -0.3681  -0.2872
                                         6 -0.4932  -0.1959
                                         7 -0.4151  -0.0950
```

```
Pairwise Granger Causality Tests
Date: 06/30/08   Time: 15:30
Sample: 2001 2009
Lags: 1

 Null Hypothesis:                  Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y       8     10.5164      0.0229
 Y does not Granger Cause CPI             13.7955      0.0138
```

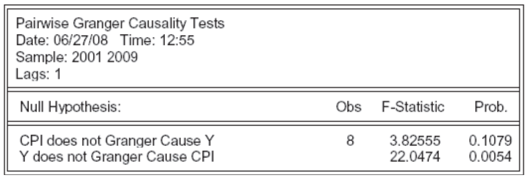
```
Pairwise Granger Causality Tests
Date: 06/30/08   Time: 15:30
Sample: 2001 2009
Lags: 2

 Null Hypothesis:                  Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y       7     12.1463      0.0761
 Y does not Granger Cause CPI             3.63054      0.2160
```

Figure 3.25: Baseline case results for the cross correlation and the Granger causality test of output gap and inflation time series in the data of four exemplary simulations runs

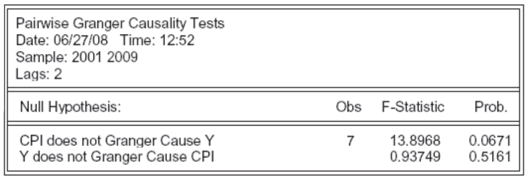
Cross Correlogram of Y and CPI



```
Date: 06/27/08   Time: 12:50
Sample: 2001 2009
Included observations: 9
Correlations are asymptotically consistent approximations

      Y,CPI(-i)              Y,CPI(+i)          i    lag      lead

                                                0   0.7446   0.7446
                                                1   0.7213   0.2248
                                                2   0.6782  -0.1840
                                                3   0.2848  -0.4262
                                                4  -0.0989  -0.4619
                                                5  -0.3070  -0.3265
                                                6  -0.4049  -0.1451
                                                7  -0.2864   0.0239
```

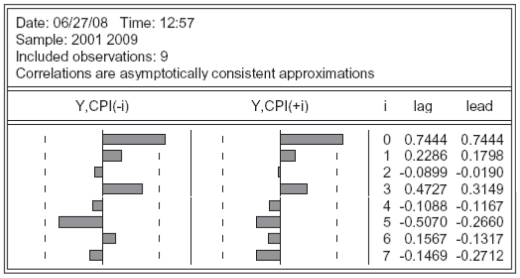


```
Pairwise Granger Causality Tests
Date: 06/27/08   Time: 12:55
Sample: 2001 2009
Lags: 1

 Null Hypothesis:                      Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y            8     3.82555     0.1079
 Y does not Granger Cause CPI                 22.0474      0.0054
```

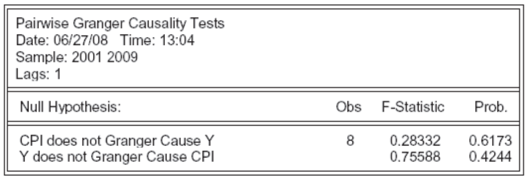


```
Pairwise Granger Causality Tests
Date: 06/27/08   Time: 12:52
Sample: 2001 2009
Lags: 2

 Null Hypothesis:                      Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y            7    13.8968      0.0671
 Y does not Granger Cause CPI                  0.93749     0.5161
```

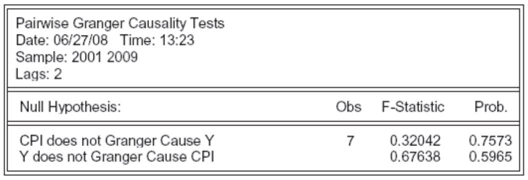
Cross Correlogram of Y and CPI



```
Date: 06/27/08   Time: 12:57
Sample: 2001 2009
Included observations: 9
Correlations are asymptotically consistent approximations

      Y,CPI(-i)              Y,CPI(+i)          i    lag      lead

                                                0   0.7444   0.7444
                                                1   0.2286   0.1798
                                                2  -0.0899  -0.0190
                                                3   0.4727   0.3149
                                                4  -0.1088  -0.1167
                                                5  -0.5070  -0.2660
                                                6   0.1567  -0.1317
                                                7  -0.1469  -0.2712
```

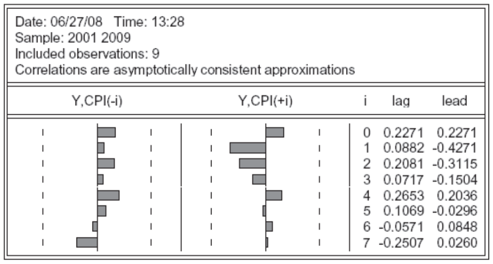


```
Pairwise Granger Causality Tests
Date: 06/27/08   Time: 13:04
Sample: 2001 2009
Lags: 1

 Null Hypothesis:                      Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y            8     0.28332     0.6173
 Y does not Granger Cause CPI                  0.75588     0.4244
```

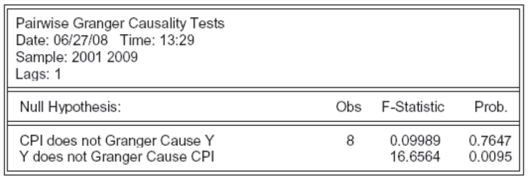


```
Pairwise Granger Causality Tests
Date: 06/27/08   Time: 13:23
Sample: 2001 2009
Lags: 2

 Null Hypothesis:                      Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y            7     0.32042     0.7573
 Y does not Granger Cause CPI                  0.67638     0.5965
```

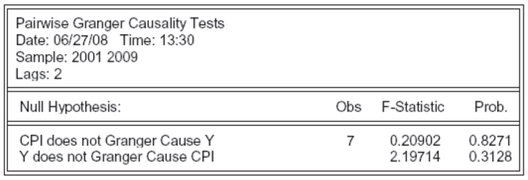
Cross Correlogram of Y and CPI



```
Date: 06/27/08   Time: 13:28
Sample: 2001 2009
Included observations: 9
Correlations are asymptotically consistent approximations

      Y,CPI(-i)              Y,CPI(+i)          i    lag      lead

                                                0   0.2271   0.2271
                                                1   0.0882  -0.4271
                                                2   0.2081  -0.3115
                                                3   0.0717  -0.1504
                                                4   0.2653   0.2036
                                                5   0.1069  -0.0296
                                                6  -0.0571   0.0848
                                                7  -0.2507   0.0260
```

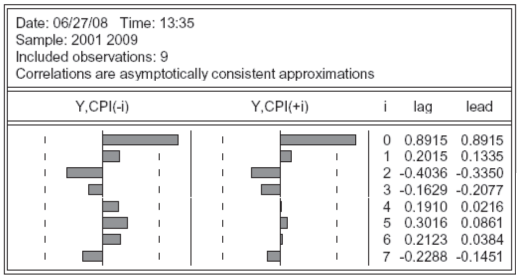


```
Pairwise Granger Causality Tests
Date: 06/27/08   Time: 13:29
Sample: 2001 2009
Lags: 1

 Null Hypothesis:                      Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y            8     0.09989     0.7647
 Y does not Granger Cause CPI                 16.6564      0.0095
```

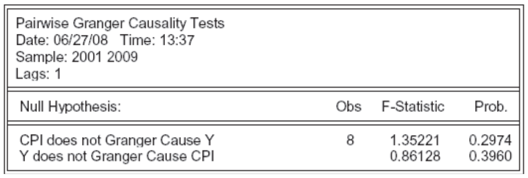


```
Pairwise Granger Causality Tests
Date: 06/27/08   Time: 13:30
Sample: 2001 2009
Lags: 2

 Null Hypothesis:                      Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y            7     0.20902     0.8271
 Y does not Granger Cause CPI                  2.19714     0.3128
```

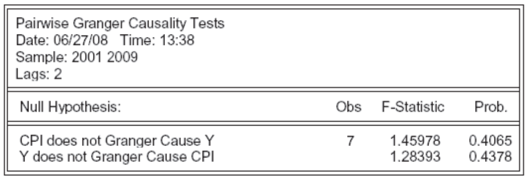
Cross Correlogram of Y and CPI



```
Date: 06/27/08   Time: 13:35
Sample: 2001 2009
Included observations: 9
Correlations are asymptotically consistent approximations

      Y,CPI(-i)              Y,CPI(+i)          i    lag      lead

                                                0   0.8915   0.8915
                                                1   0.2015   0.1335
                                                2  -0.4036  -0.3350
                                                3  -0.1629  -0.2077
                                                4   0.1910   0.0216
                                                5   0.3016   0.0861
                                                6   0.2123   0.0384
                                                7  -0.2288  -0.1451
```



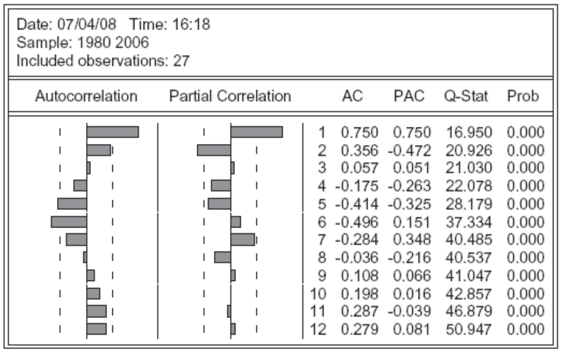
```
Pairwise Granger Causality Tests
Date: 06/27/08   Time: 13:37
Sample: 2001 2009
Lags: 1

 Null Hypothesis:                      Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y            8     1.35221     0.2974
 Y does not Granger Cause CPI                  0.86128     0.3960
```



```
Pairwise Granger Causality Tests
Date: 06/27/08   Time: 13:38
Sample: 2001 2009
Lags: 2

 Null Hypothesis:                      Obs   F-Statistic    Prob.

 CPI does not Granger Cause Y            7     1.45978     0.4065
 Y does not Granger Cause CPI                  1.28393     0.4378
```

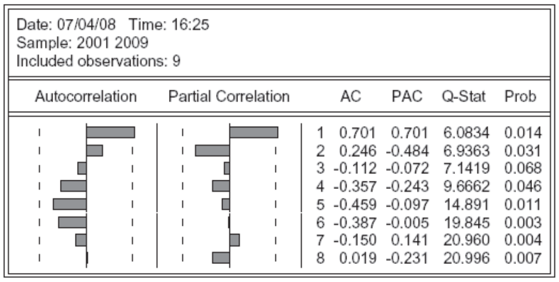
Figure 3.26: Ponzi case results for the cross correlation and the Granger causality test of output gap and inflation time series in the data of four exemplary simulations runs

Germany: Correlogram of PI



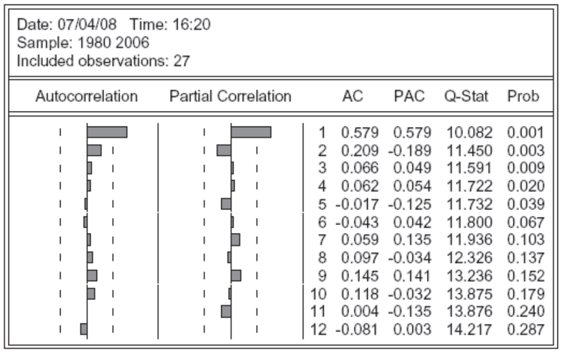
```
Date: 07/04/08   Time: 16:18
Sample: 1980 2006
Included observations: 27

Autocorrelation   Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.750   0.750  16.950  0.000
                                        2   0.356  -0.472  20.926  0.000
                                        3   0.057   0.051  21.030  0.000
                                        4  -0.175  -0.263  22.078  0.000
                                        5  -0.414  -0.325  28.179  0.000
                                        6  -0.496   0.151  37.334  0.000
                                        7  -0.284   0.348  40.485  0.000
                                        8  -0.036  -0.216  40.537  0.000
                                        9   0.108   0.066  41.047  0.000
                                       10   0.198   0.016  42.857  0.000
                                       11   0.287  -0.039  46.879  0.000
                                       12   0.279   0.081  50.947  0.000
```

Correlogram of CPI



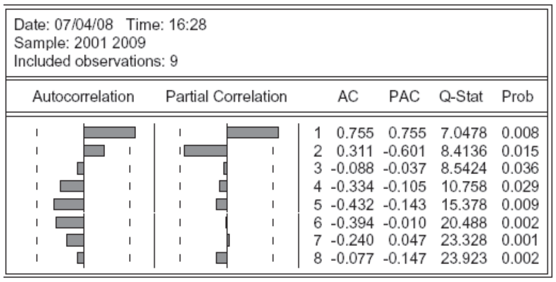
```
Date: 07/04/08   Time: 16:25
Sample: 2001 2009
Included observations: 9

Autocorrelation   Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.701   0.701  6.0834  0.014
                                        2   0.246  -0.484  6.9363  0.031
                                        3  -0.112  -0.072  7.1419  0.068
                                        4  -0.357  -0.243  9.6662  0.046
                                        5  -0.459  -0.097  14.891  0.011
                                        6  -0.387  -0.005  19.845  0.003
                                        7  -0.150   0.141  20.960  0.004
                                        8   0.019  -0.231  20.996  0.007
```

U.S.: Correlogram of PI



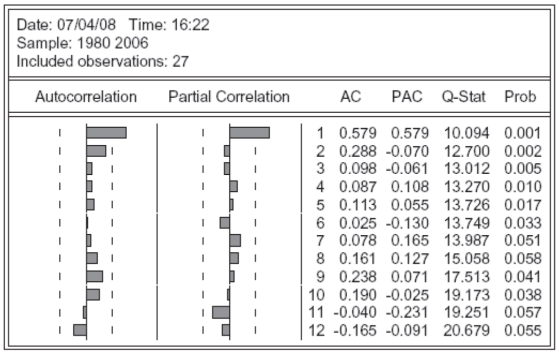
```
Date: 07/04/08   Time: 16:20
Sample: 1980 2006
Included observations: 27

Autocorrelation   Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.579   0.579  10.082  0.001
                                        2   0.209  -0.189  11.450  0.003
                                        3   0.066   0.049  11.591  0.009
                                        4   0.062   0.054  11.722  0.020
                                        5  -0.017  -0.125  11.732  0.039
                                        6  -0.043   0.042  11.800  0.067
                                        7   0.059   0.135  11.936  0.103
                                        8   0.097  -0.034  12.326  0.137
                                        9   0.145   0.141  13.236  0.152
                                       10   0.118  -0.032  13.875  0.179
                                       11   0.004  -0.135  13.876  0.240
                                       12  -0.081   0.003  14.217  0.287
```

Correlogram of CPI



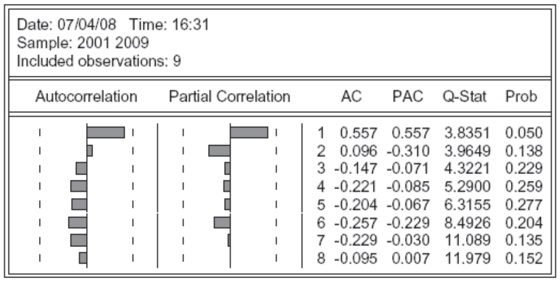
```
Date: 07/04/08   Time: 16:28
Sample: 2001 2009
Included observations: 9

Autocorrelation   Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.755   0.755  7.0478  0.008
                                        2   0.311  -0.601  8.4136  0.015
                                        3  -0.088  -0.037  8.5424  0.036
                                        4  -0.334  -0.105  10.758  0.029
                                        5  -0.432  -0.143  15.378  0.009
                                        6  -0.394  -0.010  20.488  0.002
                                        7  -0.240   0.047  23.328  0.001
                                        8  -0.077  -0.147  23.923  0.002
```

U.K.: Correlogram of PI



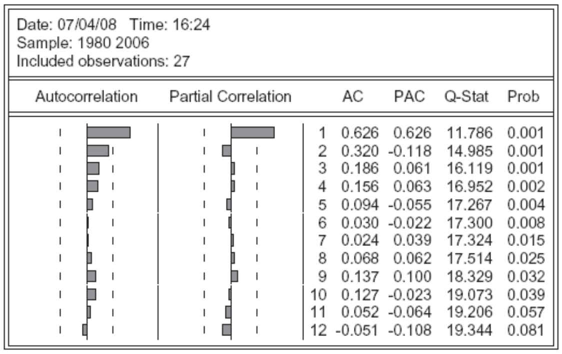
```
Date: 07/04/08   Time: 16:22
Sample: 1980 2006
Included observations: 27

Autocorrelation   Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.579   0.579  10.094  0.001
                                        2   0.288  -0.070  12.700  0.002
                                        3   0.098  -0.061  13.012  0.005
                                        4   0.087   0.108  13.270  0.010
                                        5   0.113   0.055  13.726  0.017
                                        6   0.025  -0.130  13.749  0.033
                                        7   0.078   0.165  13.987  0.051
                                        8   0.161   0.127  15.058  0.058
                                        9   0.238   0.071  17.513  0.041
                                       10   0.190  -0.025  19.173  0.038
                                       11  -0.040  -0.231  19.251  0.057
                                       12  -0.165  -0.091  20.679  0.055
```

Correlogram of CPI



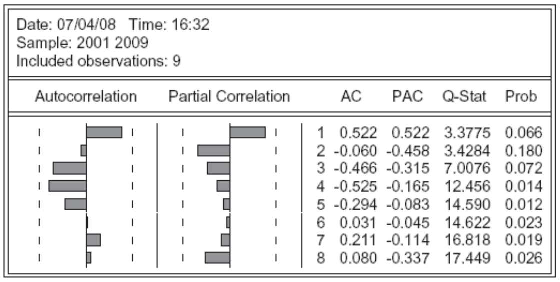
```
Date: 07/04/08   Time: 16:31
Sample: 2001 2009
Included observations: 9

Autocorrelation   Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.557   0.557  3.8351  0.050
                                        2   0.096  -0.310  3.9649  0.138
                                        3  -0.147  -0.071  4.3221  0.229
                                        4  -0.221  -0.085  5.2900  0.259
                                        5  -0.204  -0.067  6.3155  0.277
                                        6  -0.257  -0.229  8.4926  0.204
                                        7  -0.229  -0.030  11.089  0.135
                                        8  -0.095   0.007  11.979  0.152
```

Japan: Correlogram of PI



```
Date: 07/04/08   Time: 16:24
Sample: 1980 2006
Included observations: 27

Autocorrelation   Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.626   0.626  11.786  0.001
                                        2   0.320  -0.118  14.985  0.001
                                        3   0.186   0.061  16.119  0.001
                                        4   0.156   0.063  16.952  0.002
                                        5   0.094  -0.055  17.267  0.004
                                        6   0.030  -0.022  17.300  0.008
                                        7   0.024   0.039  17.324  0.015
                                        8   0.068   0.062  17.514  0.025
                                        9   0.137   0.100  18.329  0.032
                                       10   0.127  -0.023  19.073  0.039
                                       11   0.052  -0.064  19.206  0.057
                                       12  -0.051  -0.108  19.344  0.081
```

Correlogram of CPI



```
Date: 07/04/08   Time: 16:32
Sample: 2001 2009
Included observations: 9

Autocorrelation   Partial Correlation       AC     PAC    Q-Stat   Prob

                                        1   0.522   0.522  3.3775  0.066
                                        2  -0.060  -0.458  3.4284  0.180
                                        3  -0.466  -0.315  7.0076  0.072
                                        4  -0.525  -0.165  12.456  0.014
                                        5  -0.294  -0.083  14.590  0.012
                                        6   0.031  -0.045  14.622  0.023
                                        7   0.211  -0.114  16.818  0.019
                                        8   0.080  -0.337  17.449  0.026
```

Figure 3.27: Auto–correlation of inflation time series – reference results (left panels); Germany, United States, United Kingdom and Japan, data from 1980 to 2007; Source: IMF, World Economic Outlook internet database; and simulation results for the baseline case (right panels)

relationship and the ‘expectations–channel’ of monetary transmission are explicit macroeconomic features of the artificial economy of Agent Island. Inflation and output gap time series are positively correlated, and due to the mechanism of the ‘wage–price spiral’ past inflation propagates into the future.

3.4.2 Level II Calibration: Keynesian Business Cycle Equilibrium

As explained in the introduction of this section, we restart a calibration experiment at level II. Again, the results of this process are verified by a subsequent statistical analysis. Furthermore, the results are used as initial values in the iteration procedure of the following subsection 3.4.3.

Calibration

In short, the representation of the calibration experiments at level II builds upon the description of level I in the last subsection. Hence, the ‘peripheral settings’ of tables 3.1 and 3.7 still apply, as well as the experimental settings of table 3.9. We merely implement the results derived at level I as the starting values in level II: In the ‘baseline case’, the figures in the first column of table 3.9 (i.e. the *starting values* of the calibration experiments at level) are substituted by the figures displayed in the first row of table 3.10 (i.e. the *results* of the calibration at level I). In the ‘Ponzi case’, we apply the figures displayed in the first row of table 3.11 as the new starting values. Finally, we make two modifications to ‘peripheral settings’ of the model, insofar as we change two initial values: (i) The initial values of the individual savings rates $\tilde{s}_{h,1}$ are changed, i.e. we change the statistical population of the ‘uniform distribution’ from [0–0.15] to the new population [0–0.025]. This reduces the mean of $\tilde{s}_{h,1}$ substantially from 7.5% to 1.125%, which takes place in order to reduce the deflationary pressure in the Agent Island economy. (ii) In addition, we change the initial values of individual price ‘mark–ups’ for machines $\mu_{c,1}$, in order to improve the trading of capital goods in the initial periods. We switch from the population [0.4–0.6] to a new one [0.05–0.15], which again should reduce deflation on Agent Island. Both modifications were identified in several ‘face validation’ runs of the scenarios delivered by level I calibration.

In the next step we have to define the ‘goal objective function’ of level II, which is again minimized by the ‘simulated annealing’ method:

$$GOF\ II = |\sum_{T=1}^{\bar{T}} CEI_T|. \tag{3.4.2}$$

This ‘objective function’ draws on the ‘circuit equilibrium indicator’ CEI, defined in subsection 2.2.5. It implies that the indicator characterizes ‘inflationary’ and ‘deflationary gaps’ of the business cycle on Agent Island. As explained in detail in section 2.2.5, the indicator is expected to be less than 0 in case of an ‘inflationary gap’, and above 0 in case of a ‘deflationary gap’. This idea is incorporated into equation (3.4.2): The ‘goal objective function’ at level II is defined as the sum of CEI_T over all simulation periods $T \in [1, ... \bar{T}]$. Because we minimize $GOF\ II$, we use the absolute value of this sum, i.e. a negative sum should be maximized. The closer the absolute value of $GOF II$ to 0, the more balanced are economic upswings and downswings on Agent Island.[56]

	Scenario	
Parameter	Baseline case	Ponzi case
β	0.87	0.98
ϖ	28	32
$Var[\epsilon]$	0.001375	0.0001
θ	0.1	0.01
ω^{Macro}	0.31	0.21
λ	0.04	0.22
ν^{lower}	0.65	0.86
ν^{upper}	1.65	1.5
τ	1.1	1.65
ς	2.0	1.9
ρ	0.32	0.22
ψ	0.16	—
ι	0.88	—
$\chi^{I,D}$	1.42	—
ω^{Micro}	—	0.23

Table 3.12: Resulting scenarios of the level II calibration

As stated, the function (3.4.2) is minimized by the ‘simulated annealing’ heuristic optimization method, applied in the calibration ‘plug–in’ in SeSAm. By this means we derive a scenario (in each of both ‘baseline case’ and ‘Ponzi case’), which is supposed to constitute an optimal adjustment of

---

[56] This is an implication of the ‘circuit equilibrium indicator’, which will be verified in the subsequent statistical analysis at the end of this subsection.

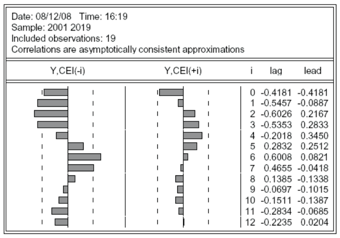
the model with respect to the minimization of function (3.4.2). The table 3.12 illustrates these two scenarios in question. We will use them in the subsequent statistical 'validation'.

Statistical Validation: CEI and the State of the Business Cycle

In the following we analyze the business cycle of Agent Island with respect to the 'circuit equilibrium indicator' (CEI). As explained, when the indicator is below 0, planned *excess expenditures* occur, which leads to an 'inflationary gap'; when the figure is located above 0, planned *excess receipts* occur, which leads to a 'deflationary gap'. Finally, when CEI is close to 0, the situation is balanced; neither planned excess expenditures nor excess receipts occur. In fact, the present analysis cannot be build upon reference data of the 'original system'; rather, we have to investigate the simulation data, and compare it to the stated theoretical considerations. This is fulfilled through a statistical comparison of inflation and output gap time series with the CEI time series.

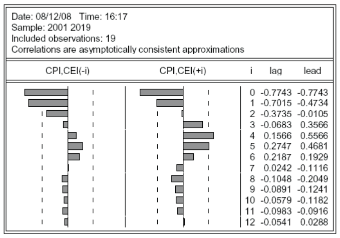
The right panels in figure 3.28 illustrate the named time series data of three arbitrary reruns in the 'baseline case'. Thereby, the expected anticyclical course of CEI is apparent. This intuitive result is verified by an analysis of the cross–correlation of CEI and inflation or output gap time series, illustrated in the left panels in figure 3.28: In here, the cross–correlations between inflation and CEI are throughout significantly negative for lags of CEI equal to 0, 1 or 2. If we consider the cross–correlations between CEI and the output gap for a 0–period lag, the correlations are somewhat less significant, and in one case they are even slightly positive. Moreover, the panels representing the 'Ponzi case' in figure 3.29 indicate similar results. In fact, these results are even more significant than those of the 'baseline case'. The cross–correlation between CEI and inflation is below -0.9 (for a lag/lead equal to 0) in two cases, i.e. both time series are nearly perfect negative correlated. This negative correlation prevails, if we add 1– or 2–, in some case even 3–, period(s) lags for CEI. For the cross–correlation between CEI and output gap time series we again obtain less significant results. In sum, these results stick closely to our expectations. We can record that the 'circuit equilibrium indicator' is a good proxy for the stance of the business cycle. In addition, our investigation verifies an inertia of the business cycle course. All in all, this implies that the calibration procedure applied above, i.e. the minimization of $GOF\ II$, is appropriate. Finally, we can state that (i) we are able to validate the present model with respect to the 'Keynesian' notion

Cross Correlogram of Y and CEI

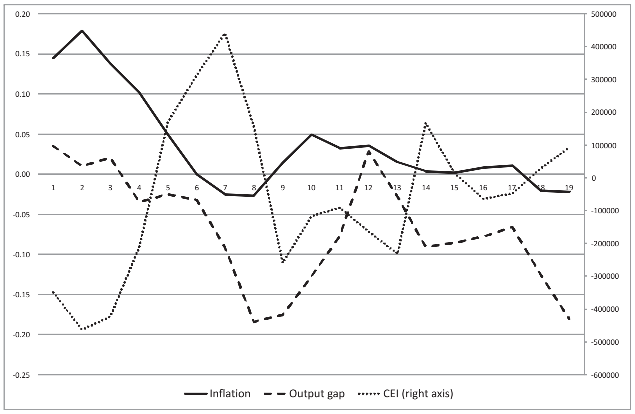


```
Date: 08/12/08   Time: 16:19
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      Y,CEI(-i)            Y,CEI(+i)        i    lag      lead

                                            0  -0.4181  -0.4181
                                            1  -0.5457  -0.0887
                                            2  -0.6026   0.2167
                                            3  -0.5353   0.2833
                                            4  -0.2018   0.3450
                                            5   0.2832   0.2512
                                            6   0.6008   0.0821
                                            7   0.4655  -0.0418
                                            8   0.1385  -0.1338
                                            9  -0.0697  -0.1015
                                           10  -0.1511  -0.1387
                                           11  -0.2834  -0.0685
                                           12  -0.2235   0.0204
```

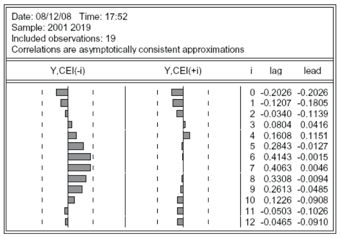
Cross Correlogram of CPI and CEI



```
Date: 08/12/08   Time: 16:17
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      CPI,CEI(-i)          CPI,CEI(+i)      i    lag      lead

                                            0  -0.7743  -0.7743
                                            1  -0.7015  -0.4734
                                            2  -0.3735  -0.0105
                                            3  -0.0683   0.3566
                                            4   0.1566   0.5566
                                            5   0.2747   0.4681
                                            6   0.2187   0.1929
                                            7   0.0242  -0.1116
                                            8  -0.1048  -0.2049
                                            9  -0.0891  -0.1241
                                           10  -0.0579  -0.1182
                                           11  -0.0983  -0.0916
                                           12  -0.0541   0.0288
```

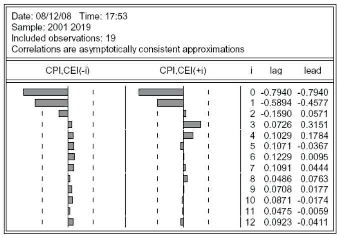


Cross Correlogram of Y and CEI

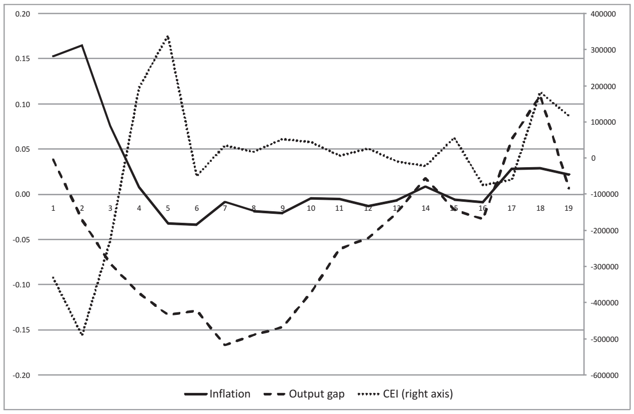


```
Date: 08/12/08   Time: 17:52
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      Y,CEI(-i)            Y,CEI(+i)        i    lag      lead

                                            0  -0.2026  -0.2026
                                            1  -0.1207  -0.1805
                                            2  -0.0340  -0.1139
                                            3   0.0804   0.0416
                                            4   0.1608   0.1151
                                            5   0.2843  -0.0127
                                            6   0.4143  -0.0015
                                            7   0.4063   0.0046
                                            8   0.3308  -0.0094
                                            9   0.2613  -0.0485
                                           10   0.1226  -0.0908
                                           11  -0.0503  -0.1026
                                           12  -0.0465  -0.0910
```

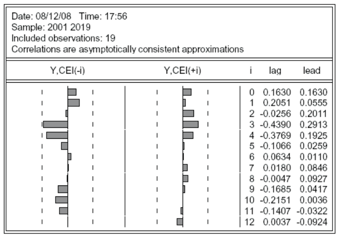
Cross Correlogram of CPI and CEI



```
Date: 08/12/08   Time: 17:53
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      CPI,CEI(-i)          CPI,CEI(+i)      i    lag      lead

                                            0  -0.7940  -0.7940
                                            1  -0.5894  -0.4577
                                            2  -0.1590   0.0571
                                            3   0.0726   0.3151
                                            4   0.1029   0.1784
                                            5   0.1071  -0.0367
                                            6   0.1229   0.0095
                                            7   0.1091   0.0444
                                            8   0.0486   0.0763
                                            9   0.0708   0.0177
                                           10   0.0871  -0.0174
                                           11   0.0475  -0.0059
                                           12   0.0923  -0.0411
```

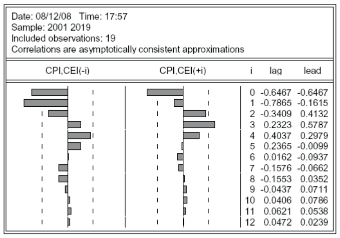


Cross Correlogram of Y and CEI



```
Date: 08/12/08   Time: 17:56
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      Y,CEI(-i)            Y,CEI(+i)        i    lag      lead

                                            0   0.1630   0.1630
                                            1   0.2051   0.0555
                                            2  -0.0256   0.2011
                                            3  -0.4390   0.2913
                                            4  -0.3769   0.1925
                                            5  -0.1066   0.0259
                                            6   0.0634   0.0110
                                            7   0.0180   0.0846
                                            8  -0.0047   0.0927
                                            9  -0.1685   0.0417
                                           10  -0.2151   0.0036
                                           11  -0.1407  -0.0322
                                           12   0.0037  -0.0924
```

Cross Correlogram of CPI and CEI



```
Date: 08/12/08   Time: 17:57
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      CPI,CEI(-i)          CPI,CEI(+i)      i    lag      lead

                                            0  -0.6467  -0.6467
                                            1  -0.7865  -0.1615
                                            2  -0.3409   0.4132
                                            3   0.2323   0.5787
                                            4   0.4037   0.2979
                                            5   0.2365  -0.0099
                                            6   0.0162  -0.0937
                                            7  -0.1576  -0.0662
                                            8  -0.1553   0.0352
                                            9  -0.0437   0.0711
                                           10   0.0406   0.0786
                                           11   0.0621   0.0538
                                           12   0.0472   0.0239
```

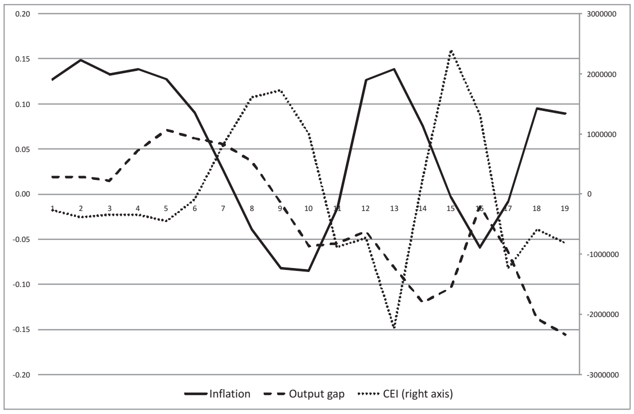
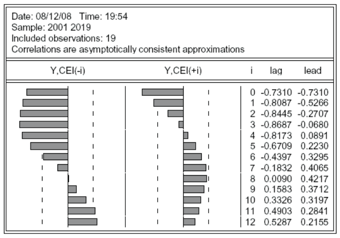


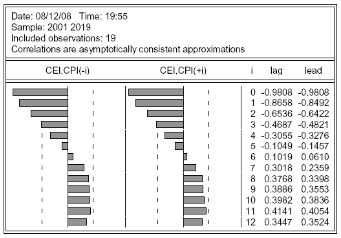
Figure 3.28: Cross–correlation between inflation/output gap and CEI time series in three exemplary simulations runs in the baseline case (left panels); the named time series (right panels)

Cross Correlogram of Y and CEI

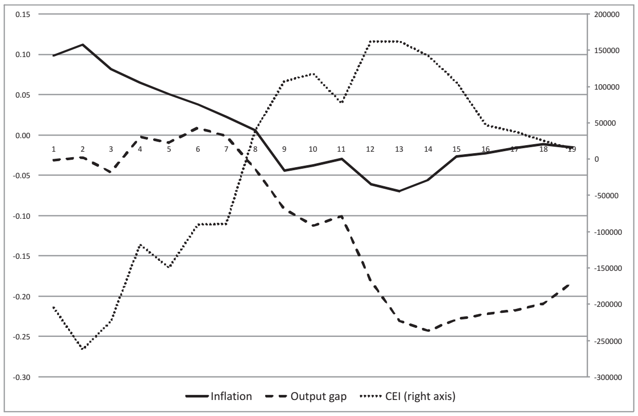


```
Date: 08/12/08   Time: 19:54
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      Y,CEI(-i)          Y,CEI(+i)        i   lag     lead

                                          0  -0.7310 -0.7310
                                          1  -0.8087 -0.5266
                                          2  -0.8445 -0.2707
                                          3  -0.8687 -0.0680
                                          4  -0.8173  0.0891
                                          5  -0.6709  0.2230
                                          6  -0.4397  0.3295
                                          7  -0.1832  0.4065
                                          8   0.0090  0.4217
                                          9   0.1583  0.3712
                                         10   0.3326  0.3197
                                         11   0.4903  0.2841
                                         12   0.5287  0.2155
```

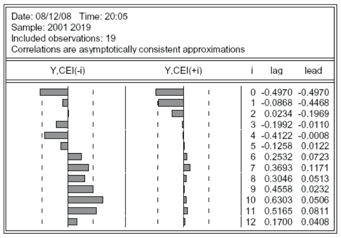
Cross Correlogram of CEI and CPI



```
Date: 08/12/08   Time: 19:55
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      CEI,CPI(-i)        CEI,CPI(+i)      i   lag     lead

                                          0  -0.9808 -0.9808
                                          1  -0.8658 -0.8492
                                          2  -0.6536 -0.6422
                                          3  -0.4687 -0.4821
                                          4  -0.3055 -0.3276
                                          5  -0.1049 -0.1457
                                          6   0.1019  0.0610
                                          7   0.3018  0.2359
                                          8   0.3768  0.3398
                                          9   0.3886  0.3553
                                         10   0.3982  0.3836
                                         11   0.4141  0.4054
                                         12   0.3447  0.3524
```

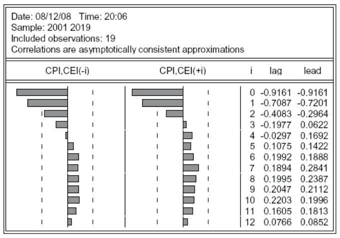


Cross Correlogram of Y and CEI

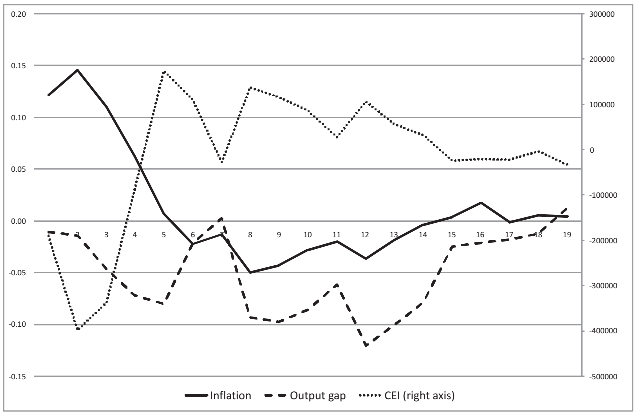


```
Date: 08/12/08   Time: 20:05
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      Y,CEI(-i)          Y,CEI(+i)        i   lag     lead

                                          0  -0.4970 -0.4970
                                          1  -0.0868 -0.4468
                                          2   0.0234 -0.1969
                                          3  -0.1992 -0.0110
                                          4  -0.4122 -0.0008
                                          5  -0.1258  0.0122
                                          6   0.2532  0.0723
                                          7   0.3693  0.1171
                                          8   0.3046  0.0513
                                          9   0.4558  0.0232
                                         10   0.6303  0.0506
                                         11   0.5165  0.0811
                                         12   0.1700  0.0408
```

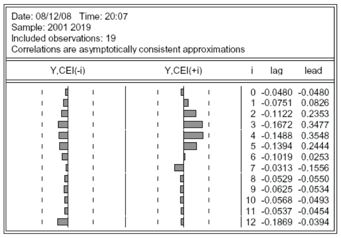
Cross Correlogram of CPI and CEI



```
Date: 08/12/08   Time: 20:06
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      CPI,CEI(-i)        CPI,CEI(+i)      i   lag     lead

                                          0  -0.9161 -0.9161
                                          1  -0.7087 -0.7201
                                          2  -0.4083 -0.2964
                                          3  -0.1977  0.0622
                                          4  -0.0297  0.1692
                                          5   0.1075  0.1422
                                          6   0.1992  0.1888
                                          7   0.1894  0.2841
                                          8   0.1995  0.2387
                                          9   0.2047  0.2112
                                         10   0.2203  0.1996
                                         11   0.1605  0.1813
                                         12   0.0766  0.0852
```

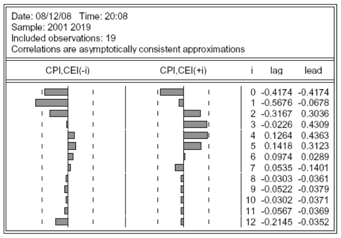


Cross Correlogram of Y and CEI



```
Date: 08/12/08   Time: 20:07
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      Y,CEI(-i)          Y,CEI(+i)        i   lag     lead

                                          0  -0.0480 -0.0480
                                          1  -0.0751  0.0826
                                          2  -0.1122  0.2353
                                          3  -0.1672  0.3477
                                          4  -0.1488  0.3548
                                          5  -0.1394  0.2444
                                          6  -0.1019  0.0253
                                          7  -0.0313 -0.1556
                                          8  -0.0529 -0.0550
                                          9  -0.0625 -0.0534
                                         10  -0.0568 -0.0493
                                         11  -0.0537 -0.0454
                                         12  -0.1869 -0.0394
```

Cross Correlogram of CPI and CEI



```
Date: 08/12/08   Time: 20:08
Sample: 2001 2019
Included observations: 19
Correlations are asymptotically consistent approximations

      CPI,CEI(-i)        CPI,CEI(+i)      i   lag     lead

                                          0  -0.4174 -0.4174
                                          1  -0.5676 -0.0678
                                          2  -0.3167  0.3036
                                          3  -0.0226  0.4309
                                          4   0.1264  0.4363
                                          5   0.1418  0.3123
                                          6   0.0974  0.0289
                                          7   0.0535 -0.1401
                                          8  -0.0303 -0.0361
                                          9  -0.0522 -0.0379
                                         10  -0.0302 -0.0371
                                         11  -0.0567 -0.0369
                                         12  -0.2145 -0.0352
```

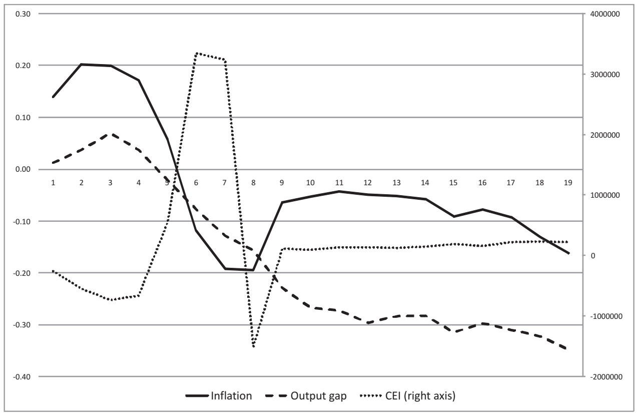


Figure 3.29: Cross–correlation between inflation/output gap and CEI time series in three exemplary simulations runs in the Ponzi case (left panels); the named time series (right panels)

of a ‘business cycle equilibrium’. (ii) At the same time, we are not able to ‘falsify’ (in the sense of Karl Popper’s view of philosophy of science) the ‘Keynesian business cycle equilibrium’ within our agent–based environment. See the explanations in subsection 2.2.5 for that point.

3.4.3 Iteration of Level I and Level II

Calibration

As the final step of the calibration procedure, we conduct an iteration of level I and level II calibration experiments. The descriptions of the previous subsections 3.4.1 and 3.4.2 still apply here. As in level II, we use the results of the previous iterations as starting values. All in all, the complete iteration was finished after one renewed level I calibration and one renewed level II calibration. According to this, four total calibration experiments (each of them comprising more than 600 simulation runs) were necessary to finish the iteration (in each of both ‘baseline case’ and ‘Ponzi case’). The outcomes of the last two steps of the iteration are displayed in table 3.13. In here, one has to compare level I and level II values for each of the two cases. Even though the levels of some parameters differ noticeably (e.g. $Var[\epsilon]$ in the ‘baseline case’), we assume that the results overlap very well. As a consequence, we stop the calibration iteration and use the scenarios displayed in level–II columns in table 3.13 as the final validated model. These scenarios are investigated in the subsequent statistical ‘validation’.

Statistical Validation: A Negative Monetary Policy Shock

In the following statistical ‘validation’, we investigate the effects of a negative ‘monetary policy shock’ on inflation rates, on real output, and on investment demand. Our investigation is referring to the analysis of De Grauwe and Storti, 2007. They conduct an empirical meta–analysis, in which they explore the short and long term effects of a negative interest rate shock on inflation and output identified in 83 empirical studies from 1990 onwards. In the descriptive part of their study, they illustrate the short and long term effects, i.e. the effects one and five year after the interest rate shock. In our analysis, we concentrate on the period where the shock occurs, and on the following period. We therefore discard the long term effect. It should be noted that in our model an interest

	The resulting scenario			
	Baseline case		Ponzi case	
Parameter	Level I	Level II	Level I	Level II
β	0.85	0.79	0.97	0.98
ϖ	28	32	32	32
$Var[\epsilon]$	0.0011	0.0016	0.0046	0.0051
θ	0.17	0.13	0.1	0.01
ω^{Macro}	0.21	0.21	0.29	0.37
λ	0.15	0.21	0.26	0.16
ν^{lower}	0.5	0.65	0.71	0.86
ν^{upper}	1.4	1.25	1.65	1.75
τ	1.75	1.9	1.1	1.25
ς	1.8	1.5	1.4	1.2
ρ	0.65	0.76	0.34	0.58
ψ	0.28	0.25	—	—
ι	0.86	0.9175	—	—
$\chi^{I,D}$	1.34	1.36	—-	—
ω^{Micro}	—	—	0.24	0.22

Table 3.13: Resulting scenarios of the iteration of level I and II

rate shock affects investment demand immediately, i.e. in the period of the shock.[57] In addition, we decide to analyze the effects in the period after the shock. Both effects, i.e. in the present and in the following period, can be treated as short or medium term effects of the monetary policy on Agent Island.

In here, we perform 400 reruns in both scenarios. In the data of these 400 reruns we search for periods where the credit interest rate rises between 0.5 and 1.5 percentage points compared to the previous period. Thereafter, we normalize the policy shock to 1 percentage point by assuming that between 0.5 and 1.5 percentage points the effect on the model output is linear. This implies that the connection between the interest rate increase, on the one hand, and its impact on inflation and output, on the other hand, is linear. For example, if the credit interest rate rises 1.35 percentage points, we divide the inflation rate and the output growth rate by 1.35. The according result represents the normalized effect of a 1 percentage point increase of the credit interest rate.[58] In

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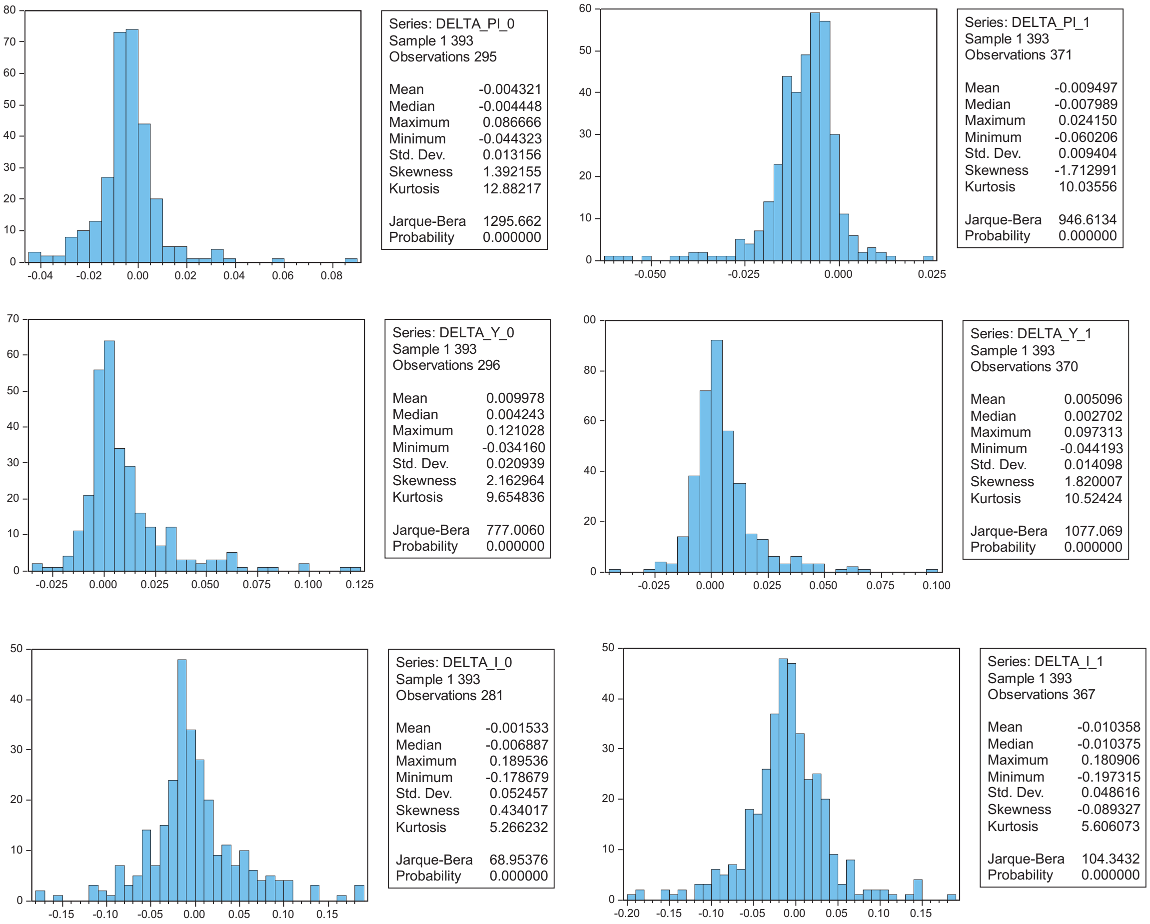
[57] Note that the sequence of decisions produces the phenomenon that at the beginning of a period the central bank sets the interest rate, and thereafter the investment decisions are affected by the interest rate policy. Consequently, the monetary policy affects investment decisions in the *present* period. See the discussion in subsection 2.1.3.

[58] Such a data normalization is also conducted by De Grauwe and Storti, 2007.



Series: DELTA_PI_0	
Sample 1 393	
Observations 295	
Mean	-0.004321
Median	-0.004448
Maximum	0.086666
Minimum	-0.044323
Std. Dev.	0.013156
Skewness	1.392155
Kurtosis	12.88217
Jarque-Bera	1295.662
Probability	0.000000

Series: DELTA_PI_1	
Sample 1 393	
Observations 371	
Mean	-0.009497
Median	-0.007989
Maximum	0.024150
Minimum	-0.060206
Std. Dev.	0.009404
Skewness	-1.712991
Kurtosis	10.03556
Jarque-Bera	946.6134
Probability	0.000000

Series: DELTA_Y_0	
Sample 1 393	
Observations 296	
Mean	0.009978
Median	0.004243
Maximum	0.121028
Minimum	-0.034160
Std. Dev.	0.020939
Skewness	2.162964
Kurtosis	9.654836
Jarque-Bera	777.0060
Probability	0.000000

Series: DELTA_Y_1	
Sample 1 393	
Observations 370	
Mean	0.005096
Median	0.002702
Maximum	0.097313
Minimum	-0.044193
Std. Dev.	0.014098
Skewness	1.820007
Kurtosis	10.52424
Jarque-Bera	1077.069
Probability	0.000000

Series: DELTA_I_0	
Sample 1 393	
Observations 281	
Mean	-0.001533
Median	-0.006887
Maximum	0.189536
Minimum	-0.178679
Std. Dev.	0.052457
Skewness	0.434017
Kurtosis	5.266232
Jarque-Bera	68.95376
Probability	0.000000

Series: DELTA_I_1	
Sample 1 393	
Observations 367	
Mean	-0.010358
Median	-0.010375
Maximum	0.180906
Minimum	-0.197315
Std. Dev.	0.048616
Skewness	-0.089327
Kurtosis	5.606073
Jarque-Bera	104.3432
Probability	0.000000

Note: Left panels illustrate the effect in the period of the shock, right panels illustrate the effect one period after the interest rate shock.

Figure 3.30: Baseline case – effects of an increase of credit interest rates by 1 percentage point in the simulation data depicted by histograms for inflation rates (upper panels), real output (central panels), and investment demand (lower panels)

the following analysis we exclude outliers, where the effects lie above (below) +20% (-20%).[59] The reference analysis of De Grauwe and Storti, 2007, identifies on average short term effects on inflation of -0.04 percentage points and long term effects of -0.3 percentage points. The phenomenon that the long term effect is larger than the short term effect is due to price rigidities (De Grauwe and Storti, 2007). In addition, the mean effect on real output in the short term is -0.23%, and -0.15% in the long term.

The histograms in figure 3.30 illustrate the results for the 'baseline case': We find out that the mean of the inflation effect is -0.43 percentage points in the present period, and -0.95 percentage points in the following period. From this perspective the monetary transmission on Agent Island seems to be more pronounced than in reality. At first sight, the effects on real output indicate a different implication. Accordingly, in the period of the interest rate shock, the mean of the effect is +1.0%, and +0.51% in the subsequent period. This is in contradiction to the results delivered by De Grauwe and Storti, 2007. In the simulation, a negative policy shock produces on average *positive* output growth rates. Unfortunately, we cannot answer how large the growth rates would be, if monetary policy was neutral and the central bank did not change the nominal interest rate. As indicated by the 'Taylor rule', the central bank on Agent Island enlarges interest rates in economic upswings, where output growth rates are usually positive. As a consequence, there are two possibilities: (i) Monetary policy on Agent Island is completely ineffective, i.e. growth rates are not dampened by the interest rate policy; or (ii) it can be the case that monetary policy dampens growth rates, but only to a small extend. In the latter case, the central bank cannot reverse output growth, i.e. growth rates are effected, but they remain positive.

Therefore, we are particularly interested in the question, to what extend the interest rate policy is effective and leads to real effects? This is shown by an investigation of its influence on investment demand. According to the structure of firms' investment decisions, credit interest rates should affect individual and aggregate investment demand. This is the aim of the 'Wicksellian' transmission channel in the model: The results represented in figure 3.30 indicate that a negative monetary policy

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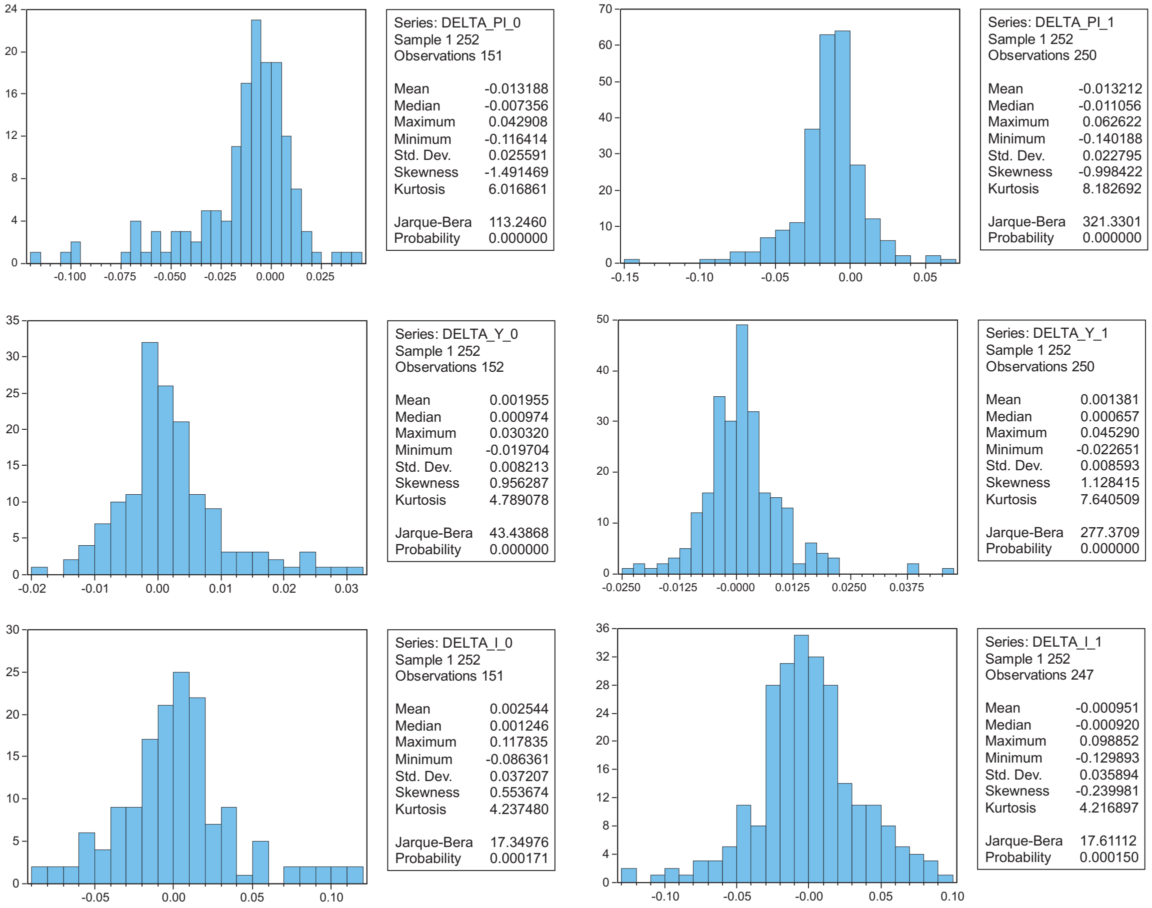
[59] De Grauwe and Storti, 2007, exclude outliers above +1% and below -1%. The according difference is due to the substantially higher variances in our data. See the explanation below.

shock reduces investment demand. In the period where the interest rate shock appears, the mean of the effect is -0.15% (the median is -0.69%); the average effect in the subsequent period is -1.04%. This implies that investment demand is on average decreasing by 0.15% in the period of the shock, and decreasing by 1.04% in the period after the increase of the interest rate. Therefore, we can conclude that monetary transmission is effective on Agent Island, and that the 'investment–channel' is functioning under the settings of the 'baseline case'. Nevertheless, the real effects are not sufficient to reduce real output. We can, however, expect that (c.p.) the output growth is smaller in case of a negative 'monetary policy shock', compared to the situation with constant credit interest rates.

Similar results are obtained in the 'Ponzi case': In the period of the negative shock inflation is on average 1.31 percentage points lower than in the period before. In the following period the mean of the effect is -1.32 percentage points. In addition, the impact on output growth is again (in contrast to the evidence delivered by De Grauwe and Storti, 2007) positive, viz.: In the present period we identify an effect of +0.2% (whereby the median is +0.1%), and +0.14% in the following period (median is +0.07%). The influence on investment demand is +0.25% in the present period, and -0.1% in the following period. Hence, in the 'Ponzi case' monetary transmission with respect to investment demand is not as effective as in the 'baseline case'. We finish this analysis with some general remarks: (i) The standard deviation of the output effects identified be De Grauwe and Storti, 2007, lies between 0.0034 and 0.0030, that of inflation effects between 0.0043 and 0.0093. According to De Grauwe and Storti, 2007, this seems to be quite high. In fact, the standard deviation of output effects on Agent Island lies between 0.0082 ('Ponzi case') and 0.0209 ('baseline case'), which is substantially higher than in the 'original system'. Similar results are obtained for the standard deviation of the inflation effects in the simulation data; the named figures lie between 0.0228 and 0.0256. For that reason, the effects in the presented simulation are more volatile than in the 'original system'. (ii) The standard deviation of growth rates of investment demand in the simulation data lies between 0.0359 ('Ponzi case') and 0.0525 ('baseline case'). If we investigate the growth rates for gross investment in Germany from 1970 onwards, we derive a mean of the growth rates amounting to 0.0411, and a standard deviation of 0.0620. It should be noted that the German data is not selected with respect to a negative 'monetary policy shock' as opposed to the presented simulation



Note: Left panels illustrate the effect in the period of the shock, right panels illustrate the effect one period after the interest rate shock.

Figure 3.31: Ponzi case – effects of an increase of credit interest rates by 1 percentage point in the simulation data depicted by histograms for inflation rates (upper panels), real output (central panels), and investment demand (lower panels)

data. However, the identified standard deviations of the growth rates of investment demand on Agent Island fits quite well to the German data.

3.4.4 Plausibility Check

As explained in chapter 1, the focus of the model development lies on the potential to provide a computational demonstration that a given microspecification is able to generate the macrostructure of interest. Calibration, adjustments and tests done till now verify the correspondence between the macro dynamics of the presented simulation and reality. This approach is based upon the concept of ‘generative sufficiency’—as illustrated in section 1.3. In this final subsection we check the model output on the macro level for ‘plausibility’: We illustrate and investigate aggregate data of the artificial economy of Agent Island produced by several ‘face validation’ runs. In here, we use the final scenarios delivered by the iteration procedure of the last subsection. In the following analysis, we conduct a qualitative analysis of time series data representing the business cycle of Agent Island.

Baseline Case

The following figures depict several time series data illustrating the dynamics of the business cycle over a period of 20 years. Thereby, we select and illustrate the data of four simulation reruns. This selection takes place with respect to some specific phenomena, such as alternating downswings and upswings or strong recession. The subsequent descriptions explain the results of each of these simulation reruns qualitatively:

First rerun (panels in figure 3.32): The situation in the first three reruns is characterized by economic upswings and downswings. Thereby, each of the reruns features three upswings within the period of 20 years. In the situation of the upper panel in figure 3.32 one can identify that inflation rises initially up to about +10% in period 3. In reaction to this upside pressure the central bank boost the nominal credit interest rate to +20% in order to control inflation (the real interest rate rises up to 13%). Such a strong reaction, while the output gap is quite low, is due to two reasons: (i) In the final scenario, the weight on the inflation gap τ is set to a rather high level of 1.75 (see table 3.13). According to that, the ‘Taylor rule’ implies that the central bank lifts the

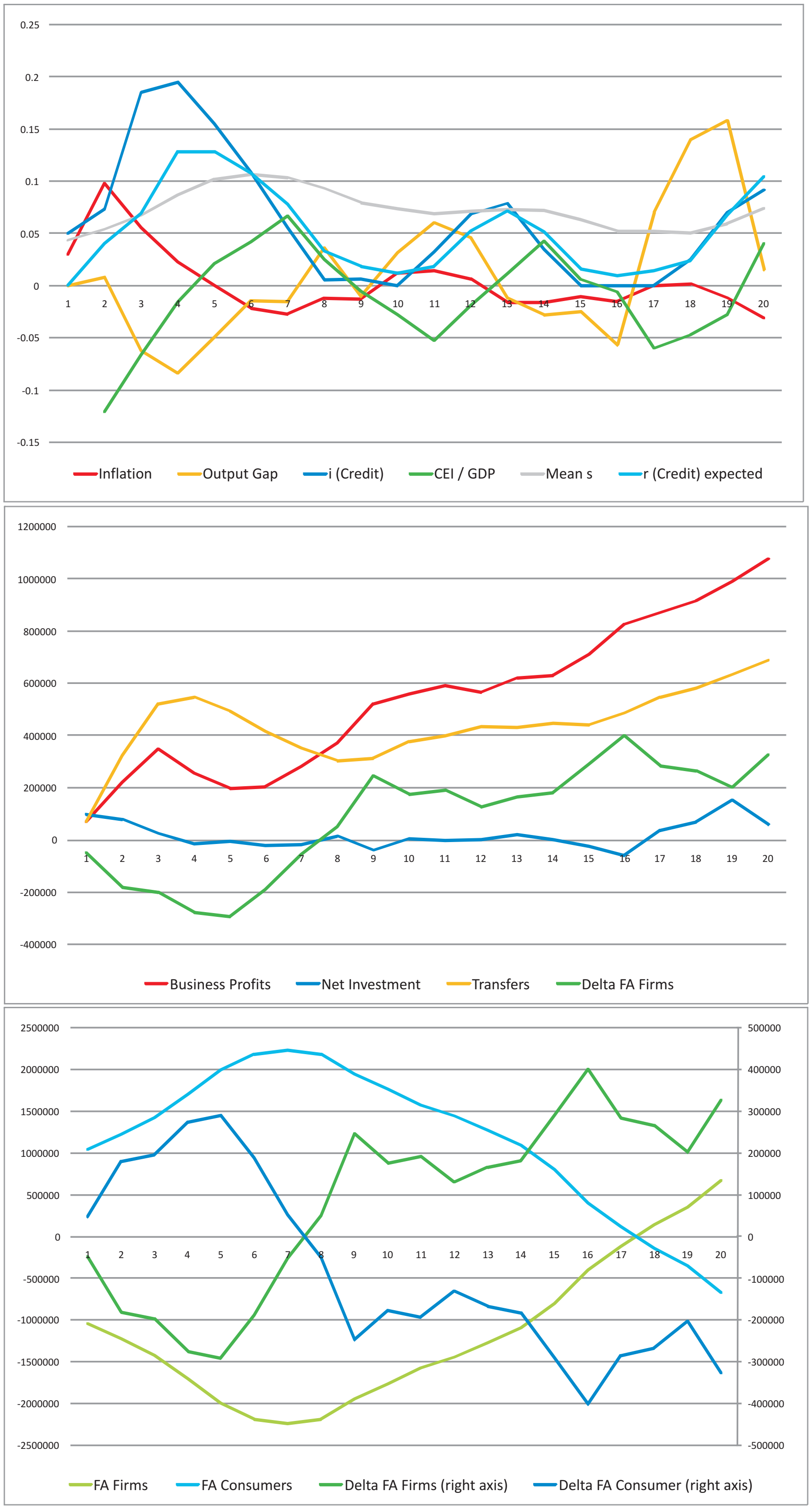


Figure 3.32: Stylized facts of the first rerun in the baseline case

nominal credit interest rate approximately 12 percentage points[60] above the average real interest rate plus inflation. (ii) The second reason for this high interest rate stems from the computation of the average real interest rate. In early stages of a simulation run, historical data are hardly available. According to this phenomenon, the historical average real interest rate up to the 4th period builds upon the average real interest rate of the first three periods. In case of the present simulation run, this produces an historical average real interest rate of about +5%. Moreover, the narrow definition of the intertemporal budget constraint of household agents (represented by $\nu^{lower} = 0.65$, and $\nu^{upper} = 1.25$) accounts for the illustrated small variations of inflation rates in the following periods. According to this specification, the growth rates of individual consumption expenditures are limited considerably. In addition, an apparently positive cross–correlation between inflation and output gap time series can be identify in economic upswings (i.e. in periods 1 and 2, 9 to 11, and 16 to 18). The business cycle indicator (CEI) is furthermore negative throughout these periods, which is as expected.

The center panel illustrates the aggregate business profit equation. According to this equation, aggregate profits of the firm sector are composed of (i) net investments of the firm sector, (ii) transfers of the firm sector to the household sector (i.e. dividend payments), and (iii) net changes of financial assets of the firm sector. It is remarkable that net investments in figure 3.32 are circumstantial within the business profit equation. In addition, transfers of the firm sector to the household sector are given through the aggregate dividend income distributed in the last period, which is in turn determined by last–period's profits. Consequently, the key to changes of firm profits is delivered by the change of financial assets of the firm sector (ΔFA_{firm}), which is in turn represented by the financial savings (or borrowings) of the household sector, i.e. $\Delta FA_{firm} = -\Delta FA_{house}$. According to the lower panel, household agents as a whole start lowering financial savings after period 5. Additionally, after period 7, these financial 'savings' become even negative. At the same time aggregate firm profits start rising significantly. The named financial dissavings of the household sector and the high aggregate business profits of firms generate an upswing in economic activity, which leads to an output gap of about +6% in period 11 (see the upper panel). Thereafter, from period 13 onwards, a recession appears. It strikes that the central bank is able to recover growth subsequently by lowering

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[60] Thereby the following calculation applies: $\tau(\pi - \pi^*) = 1.75(0.1 - 0.03) \approx 0.12$.

nominal credit interest rates to 0%; see periods 15 to 17. This generates an increase in aggregate net investment from period 16 onwards (see the center panel), which is accompanied by an upswing of aggregate firm profits (see again the center panel). These dynamics generate in sum an economic boom on Agent Island, with a peak output gap of about +16% in period 19. Importantly, while net investments are rising (which implies an increase of the aggregate *real* savings), the household sector in sum becomes a net borrower after period 17. While households are financial dissaving, the economy of Agent Island is in sum saving more in real capital. This represents an illustration of the 'Keynesian paradox of thrift'. However, inflation is close to 0% in this last stage of the simulation run. This can be due to the large growth of consumer goods supply, which dampens consumer prices, or due to the strong restriction of consumption expenditures by the narrow definition of the intertemporal budget constraint of household agents.

Second rerun (panels in figure 3.33): If we compare the business cycle of the first and the second simulation rerun, it stands out that the courses of output gap time series are quite different. For example, in the first rerun the output gap peaks are about 0%, 6%, and 16%, while in the second rerun the peaks are 0%, 12%, and 10%. This heterogeneity in the output gap data is presumably due to the stochastic elements in individual production functions. Hence, various supply shocks affect the production capacities of firms, so that output gaps are likewise different in both reruns. Besides this, it is apparent that the financial savings behavior of the household sector is pretty different compared to the first rerun (see the lower panels): In the second rerun, financial wealth of the household sector is only decreasing slightly between periods 8 and 11 as well as between periods 16 and 18. On the contrary, in the first rerun the financial assets of the household sector are decreasing substantially from period 8 onwards, until falling into a negative area at the end of the simulation run. In figure 3.33 we can identify that declining financial savings, or even rising financial borrowings (i.e. financial dissavings), of the household sector (see periods 5 to 11, and 13 to 17) initiate subsequent periods of economic upswings (periods 9 to 11, and 15 to 18), in which both inflation rates and output gaps are rising simultaneously. Those phases are again (like in the first rerun) accompanied by rising net investments. Thus, the 'Keynesian paradox of thrift' works perfectly in this simulation run. It strikes that the phases of a rising real capital stock (periods 10

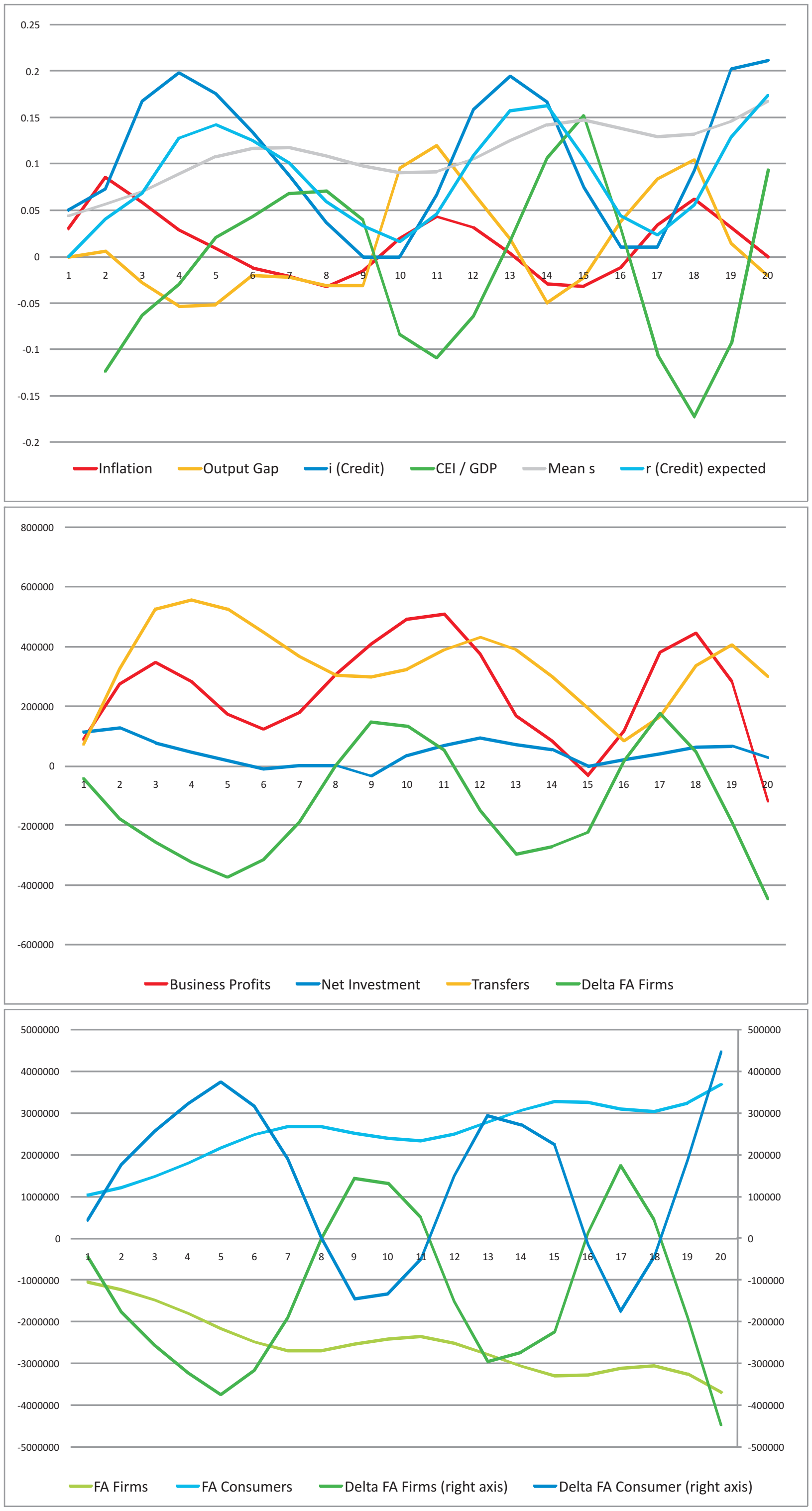

Figure 3.33: Stylized facts of the second rerun in the baseline case

to 14 and 16 to 20, see the center panel) are initiated by a financial dissaving of the household sector (periods 8 to 11 and 16 to 18, see the lower panel).

Again, as in the first rerun, the central bank of Agent Island is able to stimulate economic activity through a zero–interest–rate policy in periods 9 and 10, and by interest rates close to zero in periods 16 and 17. Real interest rates are in both periods likewise low, and close to 0%. This enhances net investments from periods 10 and 16 onward. In fact, a look into the data (not depicted in the panels here) reveals that gross investments (in present prices) are rising by more than 18% in period 10, and about 6% in period 16.[61] As explained above, these episodes of an increasing real capital stock (i.e. an increase of real savings of the Agent Island economy as a whole) are activated by decreasing financial savings of the household sector. See also the downswings of the average savings rate between periods 7 and 10 as well as between periods 15 and 17, depicted in the upper panel. Note that the jump in investment expenditures lifts incomes generated in the capital goods sector, which in turn enlarges overall economic activity. Again, the phases of economic upswings (periods 9 to 11, and 15 to 18) are accompanied by large upswings of firm profits; see the center panel. It is important to remember that positive profits of consumer goods firms are one condition of net investments. Besides this, rising profits are in turn one consequence of the willingness of the household sector to dissave. See decreasing financial wealth of the household sector within periods 8 to 11, and 16 to 18. This enhances—as already mentioned—firm profits and overall economic activity as well.

Third rerun (panels in figure 3.34): The third rerun features somewhat different patterns compared to the previous cases. For example, the output gap is negative until period 17. However, there are again three periods of inflation upswings; and initially the nominal credit interest rate rises up to 25% (the real interest rate peaks close to 20%). Importantly, the third rerun exhibits a recession in the periods 7 to 9, and subsequently a deep recession in periods 11 to 16. See for example, the outstanding 'deflationary gap' (CEI at about +30% of the GDP) accompanied by a -7% deflation and -4% output gap between periods 7 and 9. The central bank of Agent Island tries to counteract this recession by reducing credit interest rates from 22% in the 7th period to

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[61] When we use real figures, i.e. deflated gross investments, aggregate net investment enlarge by 26% and 11%.

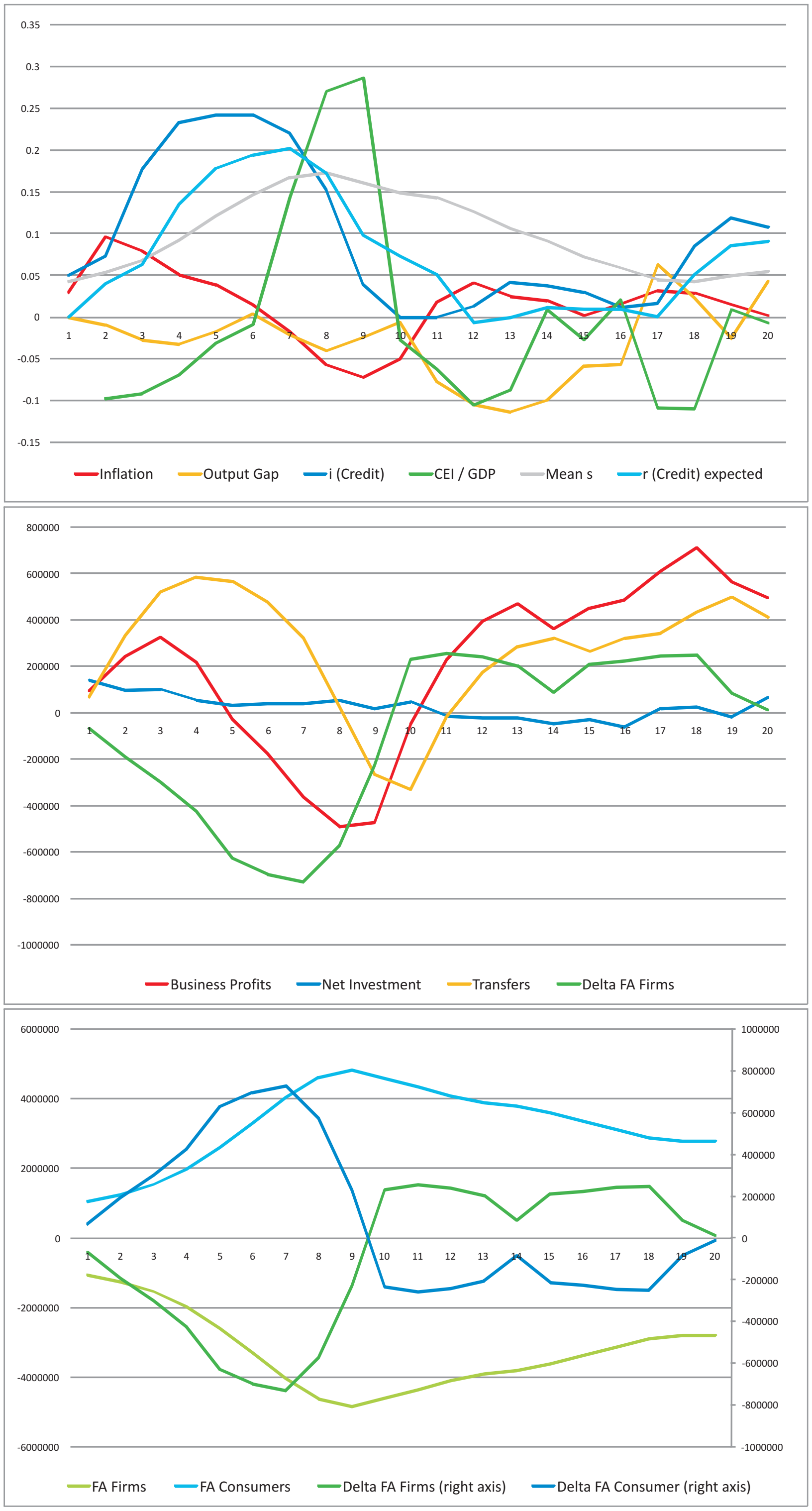

Figure 3.34: Stylized facts of the third rerun in the baseline case

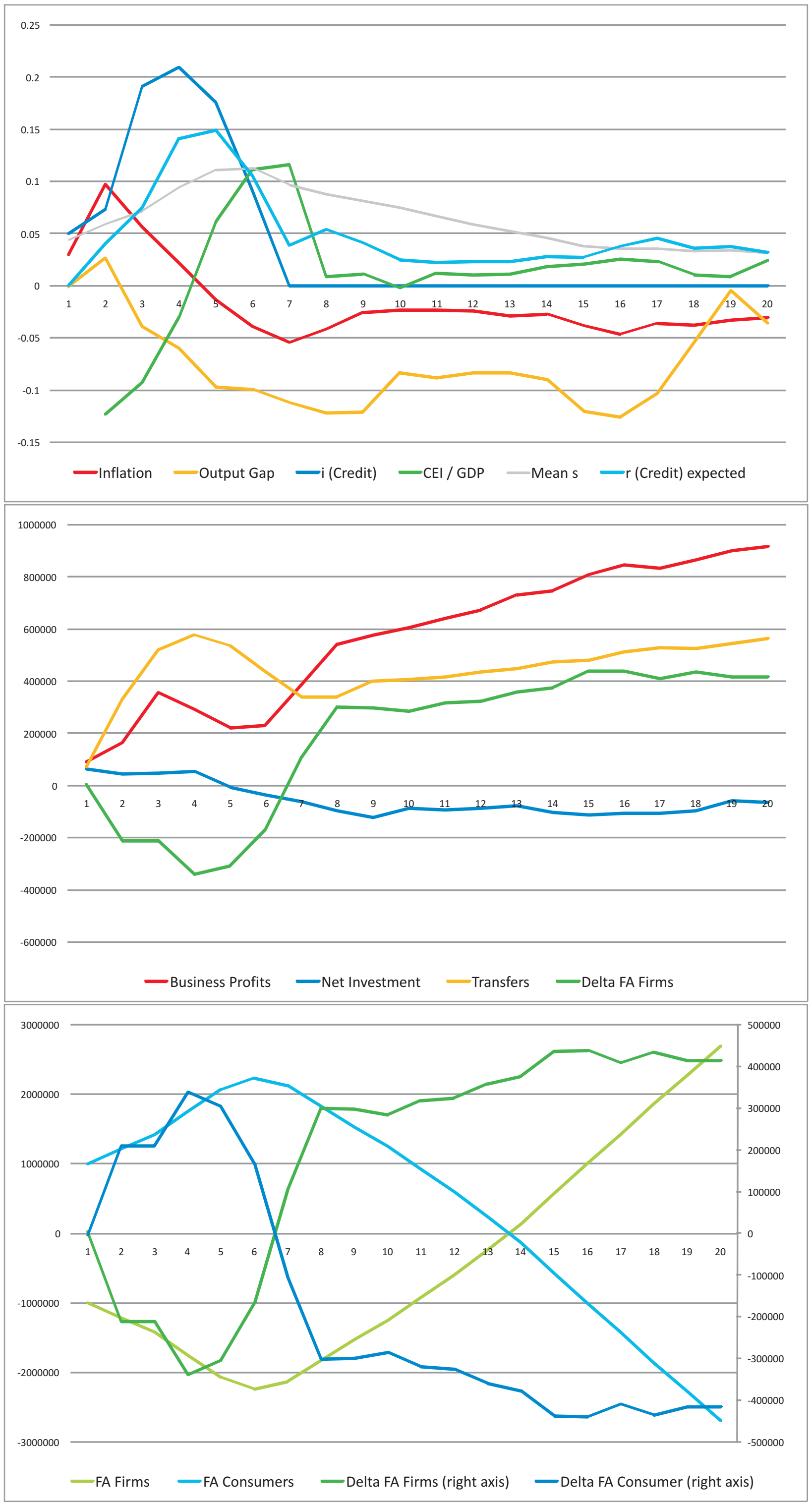

Figure 3.35: Stylized facts of the fourth rerun in the baseline case

0% in the 10th period. Real interest rates are falling to approximately 0% after period 11. The crucial point of this rerun is the fact that the central bank cannot not lift net investments; see the center panel, and monetary policy is therefore ineffective. This is obviously due to the phenomenon that business profits are negative between periods 5 and 10; see the center panel. One can assume that the large financial savings of the household sector in the first nine periods account for this effect. From period 10 onwards this situation changes, so that firm profits become positive—and the financial assets of the household sector are decreasing. Thereafter, the low interest rates in periods 16 and 17 combined with the positive firm profits activate gross investments substantially. In the simulation data (not depicted in the panels here), we find a growth rate of (nominal) gross investments of more than 20% in period 17.[62] Consequently, the output gap and inflation recover in the periods 17 and 18. It stands out that households' savings rates are in average very low at the end of the simulation run. The household sector features excess expenditures in this final stage.[63]

Fourth rerun (panels in figure 3.35): The fourth rerun represents a deep recession from the initial periods onwards. Thereby, the output gap falls to about -10%, while deflation stabilizes between -2% and -5%. The Agent Island central bank is thereby not able to activate economic activity: In period 7 the credit interest rate falls to the zero lower bound, but aggregate net investment does not recover, and the capital stock decreases continuously. The reason for the 'powerless monetary policy' is not obvious, because firm profits are positive[64] and financial savings of the household sector are falling from period 4 onwards. They become even negative in period 7; from period 14 onwards the household sector becomes a net debtor. Nevertheless, the business cycle does not recover. It should be noted that real output in the consumer goods industry is still growing by 2% to 4% (not depicted in figure 3.35). However, output gaps are negative, because capacity limits in the industry are rising more compared to real output. The latter effect is due to the technical progress on Agent

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[62] Real gross investment is rising by more than 27% in period 17.

[63] The reader could be surprised, as to how a positive average savings rate could produce the mentioned dissaving of the household sector. This phenomenon should be due to the narrow intertemporal budget constraints of household agents: The intertemporal budget dominates the savings behavior of a household agent; it generates negative savings, if the subsistence level is reached even though the savings rate is still positive.

[64] The interested reader could be surprised why net transfers in the initial periods are larger than firm profits, while firms distribute only a *part* of their profits. This phenomenon is caused by heterogeneity in the firm sector: Some firms exhibit large profits, which lead to large dividend payments, while other firms feature even larger losses. In sum, aggregate profits can be smaller than aggregate dividends in this rare case.

Island. The effect of an overall decreasing capacity utilization in the consumer goods industry accounts for the ineffectiveness of monetary policy: When consumer goods firms are far below their capacity limits, they do not expand the capital stock. This phenomenon is independent from the level of the credit interest rate. As a result, in a deep recession, accompanied by plunging capacity utilizations, the central bank of Agent Island cannot recover growth through zero interest rates. In this case, it would be necessary to stabilize the business cycle through fiscal policy. Unfortunately, this policy instrument is not integrated into the model.

Ponzi Case

The discussion of a 'plausibility check' in the framework of the 'Ponzi case' is problematic. This is due to the final settings of the scenario obtained by the calibration procedure. See the scenario in table 3.13. There, we illustrate that the parameter θ, which defines the output adjustment of consumer goods firms, is fixed to its minimum level of 0.01. This leads to the problem that overall output gaps are mostly negative in all periods of all simulation reruns. Note that technical progress on Agent Island causes growing output capacities of firms: According to the specification of the exogenous technical progress, the deterministic productivity growth (represented by $\varrho_{world,j,k,c}$) lies between 0.5% and 3.5%. The real output growth is, however, pretty small (due to the small θ–levels); it is in fact smaller than capacity growth, due to the relatively large technical progress. Consequently, output gaps are mostly negative.[65] We try to 'solve' this problem by setting the 'drift term' in the technical progress to 0, i.e. we define $\varrho_{world,j,k,c} \equiv 0$. Otherwise, the simulation output could not match the real–world data. The following descriptions, however, highlight several paradox outcomes occurring in this scenario.

First rerun (panels in figure 3.36): The best result of this rerun is the simultaneous course of inflation and output gap time series; see the upper panel. There are, however, several undesired results: (i) Aggregate net investment peaks in period 6, while output gap is falling strongly between period 5 and 7. (ii) At the end of the simulation run (from period 14 onwards), firm profits are

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[65] This problem is rooted in the deviation of the output gap via capacity utilization.

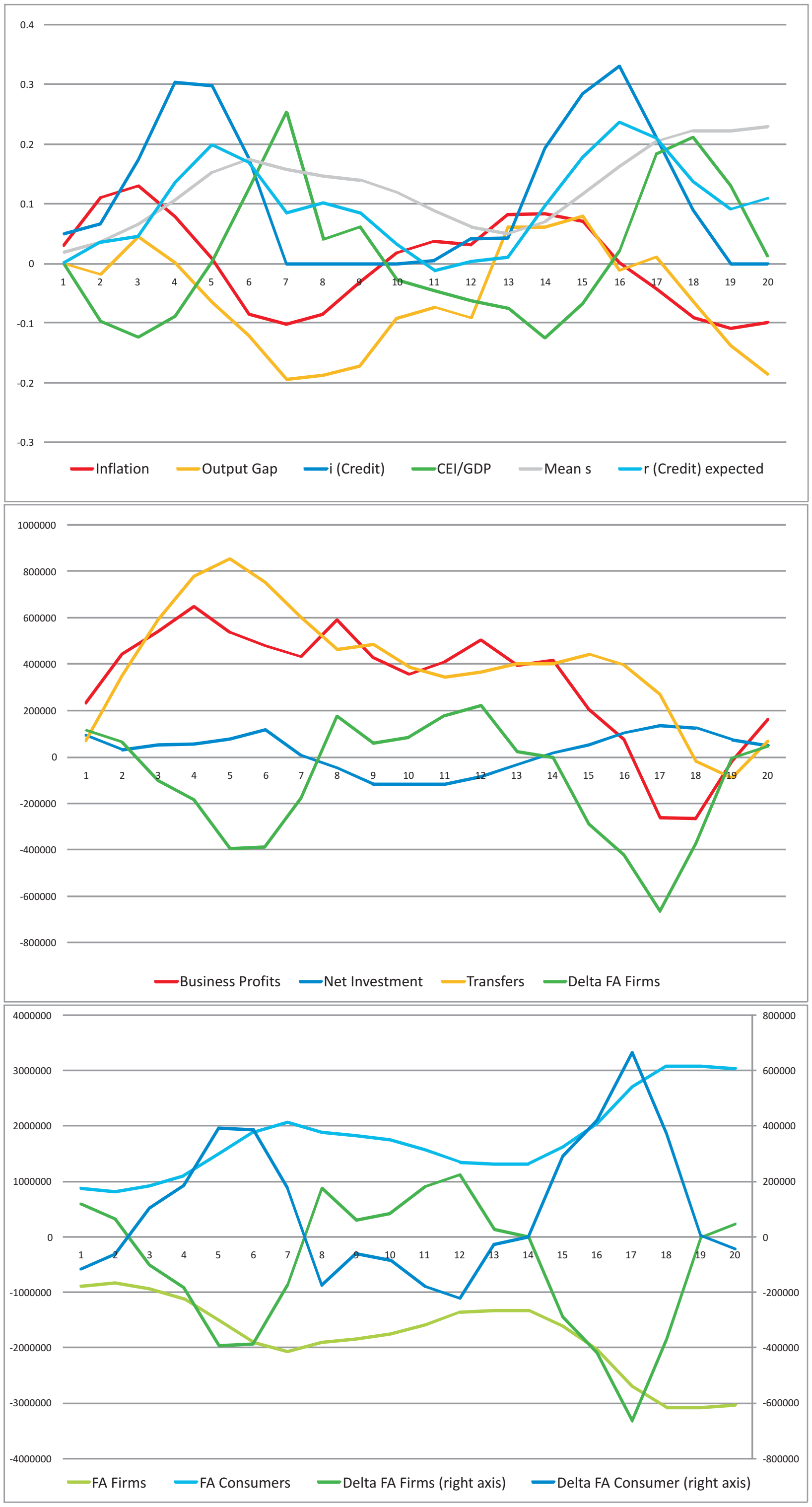

Figure 3.36: Stylized facts of the first rerun in the Ponzi case

falling substantially. In period 17 firm profits become even overall negative. Nevertheless, net investment is rising from period 15 onwards. (iii) This episode is accompanied by a boost in credit interest rates—up to 32% in period 16 (real interest rates are also above 20%). At the same time, net investment is rising unexpectedly, while credit interest rates are high and firm profits are falling. (iv) The final phase of rising net investment is accompanied by deeply negative output gaps (from period 18 onwards) as well as strongly rising financial savings of the household sector (from period 14 onwards). These astonishing results can be due to the small variation of real output of consumer goods firms and the dominant role of *stochastic* supply shocks. The latter is based upon the fact that we set the 'drift term' to 0; technical progress is therefore a 'random walk'. Finally, the central bank is not able to activate economic activity. See for example period 7, where the credit interest rate falls to 0%, while net investment becomes even negative. These strange results are verified by several other runs, which we do not present here.

Second rerun (panels in figure 3.37): We finish the discussion of the 'Ponzi case' with a simulation run featuring deflationary results. From period 6 onwards inflation is negative, while recovering a little bit after period 16. The output gap times series is even more negative, and does not recover. Between periods 6 and 9 the central bank lowers the interest rate to 0%. This actually stimulates net investments in periods 6 to 8—even though overall firm profits become negative after period 6. However, it is possible that some firms are profitable, while overall profits are negative. The investment peak in period 8 generates a slightly positive output gap in period 8; but, thereafter the output gap is falling subsequently.

We finish this investigation here. This is due to the strange results identified in the 'Ponzi case'. We must admit that it was not successful to modify the scenario and fix the 'drift term' of technological progress to 0. This is obviously due to the fact that the calibration procedure was conducted under a different setting. The generated strange results highlight the fact that the model contains complex, non–trivial and highly non–linear interaction effects: As stated in chapter 1, 'everything seems to depend on everything else'. The modification of just one 'peripheral parameter', after the calibration and 'validation' procedure, leads to undesired effects. It is important for us to

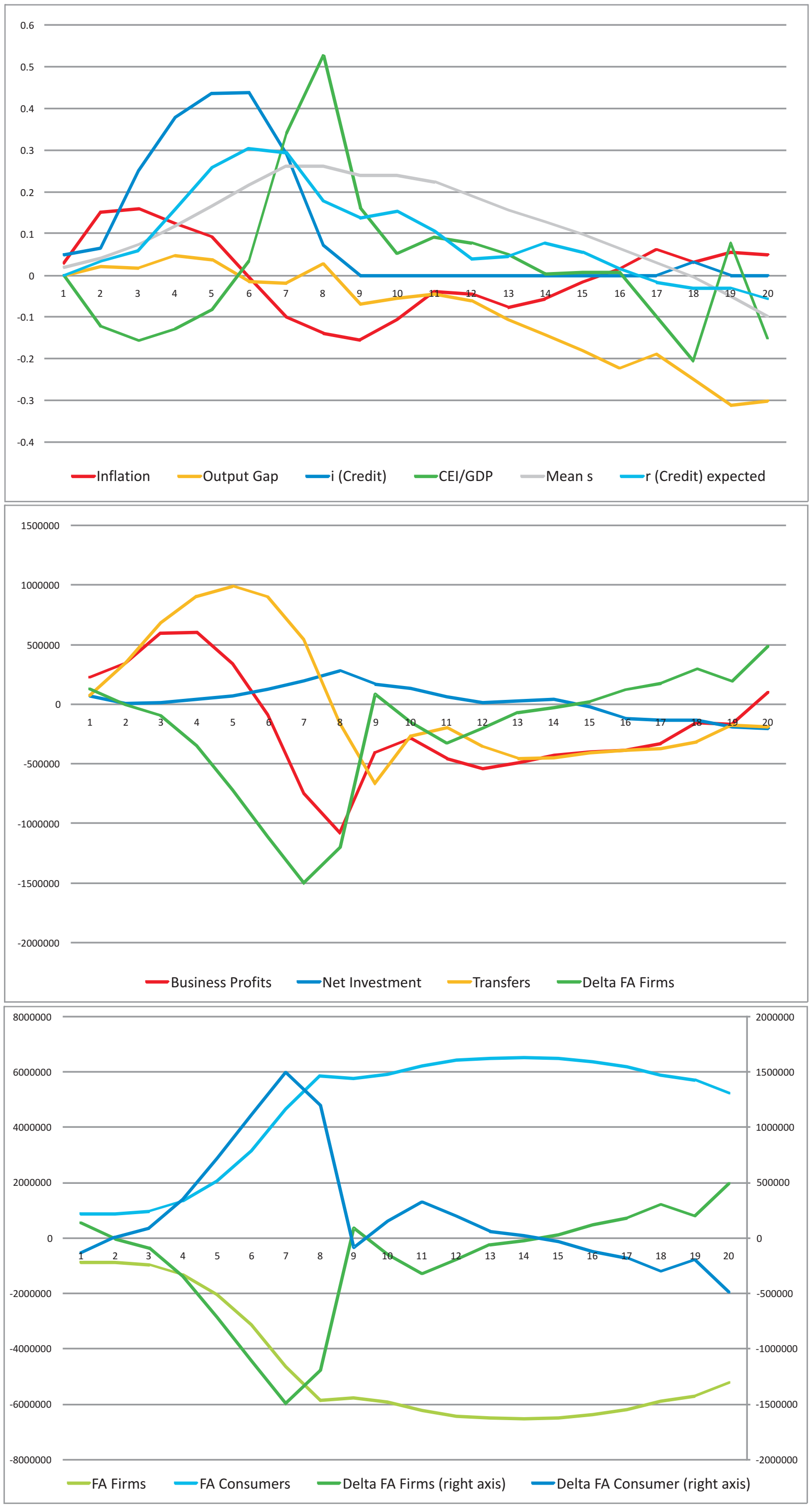

Figure 3.37: Stylized facts of the second rerun in the Ponzi case

remember that the original results of the ‘Ponzi case’ delivered by the calibration procedure did not match the real world data. The output gaps were too low.

3.5 Conclusion

In this chapter we execute a comprehensive ‘validation’ of the agent–based computational model of Agent Island. In fact, the model had to pass several steps including an initial analysis, a sensitivity analysis, several calibration experiments, several statistical ‘validations’ and final ‘plausibility checks’. We investigate in each step the ‘baseline case’ and the ‘Ponzi case’. In the sensitivity analysis, we find out that both frameworks contain some weakness: The ‘baseline case’ generates an overstated importance of the parameter ν. In our view, it appears hardly plausible that ν dominates the effects on μ^y in such a manner as stated in the sensitivity analysis. Moreover, it strikes that none of the households behavioral parameters, except ν, influence μ^π. Besides this, the wage setting parameter λ does not exhibit any impact. Even more astonishing is that the preference parameters of the central bank (τ, ς or ρ) do not carry out any impact on inflation mean. On the contrary, the ‘Ponzi case’ generates undesired deflation. This latter point is an insight identified in every stage of the ‘validation’ steps. It becomes especially apparent in the ‘plausibility check’ of the final step. A modification of the chosen scenario does not solve this problem. Therefore, we decide for the ‘baseline case’ with settings depicted in table 3.13. Especially the final ‘plausibility checks’ verify that the ‘baseline case’ generates good results: It reproduces the macro dynamics of the ‘original system’ very well, and it features a central bank, which is able to control the business cycle—at least to some extend. Equally important, the settings of the ‘baseline case’ prevent that deflationary outcomes are generated permanently. This accounts for the effectiveness of monetary policy in this case.

Concluding Remarks

The present study delivers a reasonable validated agent–based model that can be placed into the field of monetary macroeconomics. The comprehensive ‘validation’ approach applied in chapter 3 guarantees that the model of Agent Island, presented in chapter 2, is well–adjusted with respect to several macro phenomena. These phenomena are basic issues of monetary policy and theory. They include the following aspects:

1. **Stylized facts of the business cycle:** The model is adjusted with respect to inflation and output gap time series. In here, we use the first and second moment of both times series as reference values. The expected value and the standard deviation of both time series, delivered by simulation runs, match the real world data approximately.
2. **Phillips curve relationship:** In addition, both times series are significantly positive correlated. Therefore, one could expect that if the central bank on Agent Island is able to control the output gap, inflation is controlled indirectly.
3. **Regular course of the business cycle:** The calibration procedure guarantees that within a simulation run several upswings and downswings of economic activity are generated. According to this, it is one achievement of the ‘validation’ procedure that we were able to overcome the problems of overwhelming inflation or deflation.
4. **Keynesian business cycle equilibrium:** We identify that the ‘Keynesian’ equilibrium concept is functioning in the simulation. Insofar we are able to calibrate the model with respect to this equilibrium concept. This guarantees the regular course of the business cycle, as mentioned in the last point. It strikes that the equilibrium concept emerges bottom–up, out of

the individual behaviors, rather than being imposed to the model exogenously on the aggregate level. All in all, we are convinced that the business cycle of Agent Island behaves in a 'Keynesian' manner.

5. **Monetary Transmission** We investigate and verify that the central bank of Agent Island is able to affect overall economic activity via a 'Wicksellian investment–channel'. In the statistical 'validation' and the final 'plausibility checks', we work out that this channel is effective. However, the interrelationships are complex, and the conditions for net investments are manifold. So it is clear that monetary transmission does not work mechanical, if the central bank lowers the credit interest rate. However, it is effective in many circumstances. Moreover, we identify that the zero lower bound restriction influences monetary policy significantly in the case of enduring deflation. This implies a 'powerless monetary policy' in case of deflation.

6. **Plausibility Check** Finally, we investigate several 'face validation' runs with respect to the 'plausibility' of the previous aspects. Besides this, the role of aggregate business profits is checked and verified. Last but not least, we find out that the 'Keynesian paradox of thrift' is an important phenomenon of the business cycle of Agent Island.

In sum, the presented model is reasonable validated to several phenomena, which are assumed to be important building blocks for monetary theory and policy. However, the model could be tackled with advice to some probably unrealistic assumptions concerning the individual behavior of agents. Especially the design of households' savings behaviors could be considered as problematic.[66] Even though this potential source of criticism is not beside the point, we can counter it in advance: The presented 'validation' framework is focused on macroeconomic dynamics exclusively. This implies that we were searching for a microeconomic design that matches the *aggregate* data of the 'original system'. If this is fulfilled, 'validation' is successful in the sense of 'generative sufficiency' (as developed by Joshua M. Epstein). Remember, this happens without any 'validation' on the micro level. It should be the task of further research to refine the obtained results: Based upon the present study it could be possible to develop a 'better' model, which implies that new microeconomic assumptions have to be defined. Such a new (or further developed) 'microfoundation' could be more realistic

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[66] For example, it would be a straightforward alternative to model the individual savings behavior of household agents based upon a partly random behavior. The according randomness would represent stochastic demand shocks. This could deliver a less complex set of savings routines.

and easier to operate at once. If such a new model is also able to reproduce macro dynamics, as in the present study, this could enable the ‘falsification’ of the present model. At the end of the day, a reasonable validated model could be obtained. All in all, this would be the basis for a subsequent normative analysis of monetary topics, such as the search for optimal interest rate rules. It is beyond the scope of the present study to employ such a normative analysis. In conclusion, the present study delivers a first agent–based computer simulation for purposes of monetary policy. Its ‘Keynesian’ elements, the integrated ‘monetary circuit’, the ‘Wicksellian’ transmission framework, and the role of the central bank constitute a suitable framework that guarantees this perspective. Equally important, the model of Agent Island constitutes a business cycle framework, in which complex and non–linear micro–macro interactions appear. Hence, we are able to satisfy the basic notion of ‘generative sufficiency’, as coined by Joshua M. Epstein: “If you didn’t grow it [author’s note: the aggregate model], you didn’t explain its emergence” (Epstein, 2006a, p. 9). Indeed, we are able to declare that we grew the economy of Agent Island, and therefore we are able to explain its emergence. In this sense the model closes a gap in monetary theory.

Appendix A

Test of Equality of Means and Variances (Section 3.3)

Table A.1: Test of equality of means and variances of inflation time series in 47 simulation reruns with homogenous agents

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
15.500	500	30	0.01	Mean	0.1224
				Variance a	0.0000***
				Variance b	0.0000***
15.500	500	30	0.001	Mean	0.3056
				Variance a	0.00001***
				Variance b	0.0008***
15.500	500	30	0.0001	Mean	1.000
				Variance a	1.000
				Variance b	1.000
10.500	500	20	0.01	Mean	0.0090***
				Variance a	0.0000***
				Variance b	0.0000***
10.500	500	20	0.001	Mean	0.9260
				Variance a	0.0000***
				Variance b	0.0000***
10.500	500	20	0.0001	Mean	1.0000

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance a	1.0000
				Variance b	1.0000
7.750	250	30	0.01	Mean	0.8786
				Variance a	0.0000***
				Variance b	0.0000***
7.750	250	30	0.001	Mean	0.9835
				Variance a	0.9445
				Variance b	0.9998
7.750	250	30	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	1.0000
5.500	500	10	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
5.500	500	10	0.001	Mean	0.9910
				Variance a	0.0000***
				Variance b	0.0000***
5.500	500	10	0.0001	Mean	1.0000
				Variance a	0.8543
				Variance b	1.0000
5.250	250	20	0.01	Mean	0.0068***
				Variance a	0.0000***
				Variance b	0.0000***
5.250	250	20	0.001	Mean	0.9623
				Variance a	0.0000***
				Variance b	0.0054***
5.250	250	20	0.0001	Mean	1.000
				Variance a	0.9840
				Variance b	1.0000
3.100	100	30	0.01	Mean	0.7277
				Variance a	0.0000***
				Variance b	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
3.100	100	30	0.001	Mean	0.1359
				Variance a	0.0000***
				Variance b	0.0000***
3.100	100	30	0.0001	Mean	1.0000
				Variance a	0.9972
				Variance b	1.0000
3.000	500	5	0.01	Mean	0.1655
				Variance a	0.0325**
				Variance b	0.0000***
3.000	500	5	0.001	Mean	1.0000
				Variance a	0.7723
				Variance b	0.2611
3.000	500	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	1.0000
2.750	250	10	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
2.750	250	10	0.001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.0000***
2.750	250	10	0.0001	Mean	1.0000
				Variance a	0.4154
				Variance b	0.9422
2.100	100	20	0.01	Mean	0.5938
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.001	Mean	0.9565
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.0001	Mean	1.0000
				Variance a	0.3683

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance b	0.9879
1.550	50	30	0.01	Mean	0.6637
				Variance a	0.0000***
				Variance b	0.0000***
1.550	50	30	0.001	Mean	0.6653
				Variance a	0.0000***
				Variance b	0.0386**
1.550	50	30	0.0001	Mean	1.0000
				Variance a	0.9929
				Variance b	1.0000
1.500	250	5	0.01	Mean	0.0238**
				Variance a	0.2307
				Variance b	0.0066***
1.500	250	5	0.001	Mean	1.0000
				Variance a	0.1595 †
				Variance b	0.0600*
1.500	250	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	1.0000
1.100	100	10	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
1.100	100	10	0.001	Mean	0.9984
				Variance a	0.0000***
				Variance b	0.0000***
1.100	100	10	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.01	Mean	0.0024***
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.001	Mean	0.9473

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.0001	Mean	0.9990
				Variance a	0.0000***
				Variance b	0.0000***
600	100	5	0.01	Mean	0.0068***
				Variance a	0.0164**
				Variance b	0.0000***
600	100	5	0.001	Mean	1.0000
				Variance a	0.5259
				Variance b	0.2050
600	100	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	1.000
550	50	10	0.01	Mean	0.0001***
				Variance a	0.0000***
				Variance b	0.0000***
550	50	10	0.001	Mean	0.9879
				Variance a	0.0000***
				Variance b	0.0000***
550	50	10	0.0001	Mean	0.9999
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.01	Mean	0.7748
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.001	Mean	0.1610
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.0001	Mean	0.9999
				Variance a	0.0000***
				Variance b	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
300	50	5	0.01	Mean	0.1612
				Variance a	0.2311
				Variance b	0.0052***
300	50	5	0.001	Mean	1.0000
				Variance a	0.5473
				Variance b	0.2906
300	50	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	1.0000
210	10	20	0.01	Mean	0.4717
				Variance a	0.0000***
				Variance b	0.0000***
210	10	20	0.001	Mean	0.8588
				Variance a	0.0000***
				Variance b	0.0000***
210	10	20	0.0001	Mean	0.0710*
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.01	Mean	0.0006***
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.001	Mean	0.9842
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.0001	Mean	0.9424
				Variance a	0.0000***
				Variance b	0.0000***
60	10	5	0.01	Mean	0.1701
				Variance a	0.0001***
				Variance b	0.0001***
60	10	5	0.001	Mean	1.0000
				Variance a	0.4878

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance b	0.6863
60	10	5	0.0001	Mean	1.0000
				Variance a	0.3535
				Variance b	0.5284

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Table A.2: Test of equality of means and variances of inflation time series in 47 simulation reruns with heterogenous agents

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
15.500	500	30	0.01	Mean	0.8066
				Variance a	0.0000***
				Variance b	0.0000***
15.500	500	30	0.001	Mean	0.9831
				Variance a	0.0000***
				Variance b	0.0000***
15.500	500	30	0.0001	Mean	1.000
				Variance a	1.000
				Variance b	1.000
10.500	500	20	0.01	Mean	0.1107
				Variance a	0.0000***
				Variance b	0.0000***
10.500	500	20	0.001	Mean	0.9982
				Variance a	0.0000***
				Variance b	0.0018***
10.500	500	20	0.0001	Mean	1.0000
				Variance a	0.6712
				Variance b	0.9973
7.750	250	30	0.01	Mean	0.0275**
				Variance a	0.0000***
				Variance b	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents^1	Firms	Households per firm	Variance of Random Walk	Test^2	Probability^{2,3,4}
7.750	250	30	0.001	Mean	0.8338
				Variance a	0.0001***
				Variance b	0.0011***
7.750	250	30	0.0001	Mean	1.0000
				Variance a	0.9751
				Variance b	1.0000
5.500	500	10	0.01	Mean	0.0002***
				Variance a	0.0000***
				Variance b	0.0000***
5.500	500	10	0.001	Mean	0.9998
				Variance a	0.0000***
				Variance b	0.0000***
5.500	500	10	0.0001	Mean	1.0000
				Variance a	0.0582*
				Variance b	0.0044***
5.250	250	20	0.01	Mean	0.0380**
				Variance a	0.0000***
				Variance b	0.0000***
5.250	250	20	0.001	Mean	0.9145
				Variance a	0.0001***
				Variance b	0.0570**
5.250	250	20	0.0001	Mean	1.000
				Variance a	0.1259
				Variance b	0.9053
3.100	100	30	0.01	Mean	0.8778
				Variance a	0.0000***
				Variance b	0.0000***
3.100	100	30	0.001	Mean	0.3469
				Variance a	0.0000***
				Variance b	0.0000***
3.100	100	30	0.0001	Mean	0.9957
				Variance a	0.1171

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance b	0.7639
3.000	500	5	0.01	Mean	0.2235
				Variance a	0.0000***
				Variance b	0.0000***
3.000	500	5	0.001	Mean	1.0000
				Variance a	1.0000
				Variance b	0.9551
3.000	500	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	1.0000
2.750	250	10	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
2.750	250	10	0.001	Mean	0.9937
				Variance a	0.0000***
				Variance b	0.0000***
2.750	250	10	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.01	Mean	0.0005***
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.001	Mean	0.9423
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.0001	Mean	0.5607
				Variance a	0.0001***
				Variance b	0.0111**
1.550	50	30	0.01	Mean	0.1097
				Variance a	0.0000***
				Variance b	0.0000***
1.550	50	30	0.001	Mean	0.0539*

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance a	0.0000***
				Variance b	0.0000***
1.550	50	30	0.0001	Mean	0.2377
				Variance a	0.0000***
				Variance b	0.0000***
1.500	250	5	0.01	Mean	0.7694
				Variance a	0.0026***
				Variance b	0.0000***
1.500	250	5	0.001	Mean	1.000
				Variance a	1.000
				Variance b	0.9997
1.500	250	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	1.0000
1.100	100	10	0.01	Mean	0.1307
				Variance a	0.0000***
				Variance b	0.0000***
1.100	100	10	0.001	Mean	0.9914
				Variance a	0.0000***
				Variance b	0.0000***
1.100	100	10	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.01	Mean	0.1504
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.001	Mean	0.9451
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.0001	Mean	0.0815*
				Variance a	0.0000***
				Variance b	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
600	100	5	0.01	Mean	0.4134
				Variance a	0.2163 †
				Variance b	0.0267**
600	100	5	0.001	Mean	1.0000
				Variance a	0.9733
				Variance b	0.6314
600	100	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	0.9934
550	50	10	0.01	Mean	0.0002***
				Variance a	0.0000***
				Variance b	0.0000***
550	50	10	0.001	Mean	0.9662
				Variance a	0.0000***
				Variance b	0.0000***
550	50	10	0.0001	Mean	0.9994
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.01	Mean	0.0063***
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.001	Mean	0.9865
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.0001	Mean	0.1356
				Variance a	0.0000***
				Variance b	0.0000***
300	50	5	0.01	Mean	0.6161
				Variance a	0.0446**
				Variance b	0.0026***
300	50	5	0.001	Mean	1.0000
				Variance a	0.2304

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance b	0.1177
300	50	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	0.9919
210	10	20	0.01	Mean	0.9181
				Variance a	0.0000***
				Variance b	0.0000***
210	10	20	0.001	Mean	0.8520
				Variance a	0.0000***
				Variance b	0.0000***
210	10	20	0.0001	Mean	0.6949
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.01	Mean	0.0027***
				Variance a	0.0005***
				Variance b	0.0006***
110	10	10	0.001	Mean	0.7954
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.0001	Mean	0.5443
				Variance a	0.0000***
				Variance b	0.0000***
60	10	5	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
60	10	5	0.001	Mean	0.9880
				Variance a	0.0000***
				Variance b	0.0001***
60	10	5	0.0001	Mean	0.9963
				Variance a	0.0001***
				Variance b	0.0021***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Table A.3: Test of equality of means and variances of output gap time series in 47 simulation reruns with homogenous agents

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
15.500	500	30	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
15.500	500	30	0.001	Mean	0.0000***
				Variance a	0.9137
				Variance b	0.9611
15.500	500	30	0.0001	Mean	1.0000
				Variance a	0.8339
				Variance b	0.9649
10.500	500	20	0.01	Mean	0.4683
				Variance a	0.0000***
				Variance b	0.0000***
10.500	500	20	0.001	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
10.500	500	20	0.0001	Mean	0.9950
				Variance a	0.5277
				Variance b	0.2547
7.750	250	30	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
7.750	250	30	0.001	Mean	0.9947
				Variance a	0.7455
				Variance b	0.3191
7.750	250	30	0.0001	Mean	1.0000
				Variance a	0.7562
				Variance b	0.6359
5.500	500	10	0.01	Mean	0.0000***
				Variance a	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance b	0.0000***
5.500	500	10	0.001	Mean	0.9461
				Variance a	0.0000***
				Variance b	0.0000***
5.500	500	10	0.0001	Mean	0.9950
				Variance a	0.8952
				Variance b	0.3768
5.250	250	20	0.01	Mean	0.0003***
				Variance a	0.0000***
				Variance b	0.0000***
5.250	250	20	0.001	Mean	0.0000***
				Variance a	0.0053***
				Variance b	0.0005***
5.250	250	20	0.0001	Mean	0.9996
				Variance a	0.2927
				Variance b	0.3794
3.100	100	30	0.01	Mean	0.0003***
				Variance a	0.0000***
				Variance b	0.0000***
3.100	100	30	0.001	Mean	0.6096
				Variance a	0.0000***
				Variance b	0.0000***
3.100	100	30	0.0001	Mean	1.0000
				Variance a	0.0032***
				Variance b	0.0147**
3.000	500	5	0.01	Mean	0.7766
				Variance a	0.0000***
				Variance b	0.0001***
3.000	500	5	0.001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.3001 †
3.000	500	5	0.0001	Mean	1.0000

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance a	0.9956
				Variance b	1.0000
2.750	250	10	0.01	Mean	0.0026***
				Variance a	0.0000***
				Variance b	0.0001***
2.750	250	10	0.001	Mean	0.7168
				Variance a	0.0000***
				Variance b	0.0000***
2.750	250	10	0.0001	Mean	0.9554
				Variance a	0.0000***
				Variance b	0.0001***
2.100	100	20	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.001	Mean	0.54011
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.0001	Mean	1.0000
				Variance a	0.0210**
				Variance b	0.0007***
1.550	50	30	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
1.550	50	30	0.001	Mean	0.9992
				Variance a	0.1196
				Variance b	0.2481
1.550	50	30	0.0001	Mean	1.0000
				Variance a	0.0076***
				Variance b	0.0310**
1.500	250	5	0.01	Mean	0.5919
				Variance a	0.0000***
				Variance b	0.0191**

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
1.500	250	5	0.001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.7469 †
1.500	250	5	0.0001	Mean	1.0000
				Variance a	0.9971
				Variance b	1.0000
1.100	100	10	0.01	Mean	0.0295**
				Variance a	0.0000***
				Variance b	0.0000***
1.100	100	10	0.001	Mean	0.9181
				Variance a	0.0000***
				Variance b	0.0000***
1.100	100	10	0.0001	Mean	0.9937
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.01	Mean	0.0334**
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.001	Mean	0.8907
				Variance a	0.0001***
				Variance b	0.0080***
1.050	50	20	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.0000***
600	100	5	0.01	Mean	0.2661
				Variance a	0.0000***
				Variance b	0.0000***
600	100	5	0.001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.6457 †
600	100	5	0.0001	Mean	1.0000
				Variance a	0.9904

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance b	1.0000
550	50	10	0.01	Mean	0.4759
				Variance a	0.0000***
				Variance b	0.0000***
550	50	10	0.001	Mean	0.8538
				Variance a	0.0000***
				Variance b	0.0000***
550	50	10	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.01	Mean	0.7112
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.001	Mean	0.9999
				Variance a	0.0317**
				Variance b	0.0541*
310	10	30	0.0001	Mean	1.0000
				Variance a	0.0075***
				Variance b	0.0051***
300	50	5	0.01	Mean	0.3972
				Variance a	0.0000***
				Variance b	0.0000***
300	50	5	0.001	Mean	0.9999
				Variance a	0.0000***
				Variance b	0.0771*
300	50	5	0.0001	Mean	1.0000
				Variance a	0.7521
				Variance b	1.0000
210	10	20	0.01	Mean	0.4810
				Variance a	0.0000***
				Variance b	0.0000***
210	10	20	0.001	Mean	0.9997

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance a	0.0000***
				Variance b	0.0001***
210	10	20	0.0001	Mean	0.9979
				Variance a	0.0001***
				Variance b	0.0462*
110	10	10	0.01	Mean	0.9213
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.001	Mean	0.9924
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.0001	Mean	0.9993
				Variance a	0.0000***
				Variance b	0.0000***
60	10	5	0.01	Mean	0.9059
				Variance a	0.0000***
				Variance b	0.0001***
60	10	5	0.001	Mean	0.9998
				Variance a	0.0000***
				Variance b	0.3693 †
60	10	5	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.9038 †

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Table A.4: Test of equality of means and variances of output gap time series in 47 simulation reruns with heterogenous agents

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
15.500	500	30	0.01	Mean	0.0336**
				Variance a	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance b	0.0000***
15.500	500	30	0.001	Mean	0.0000***
				Variance a	0.8065
				Variance b	0.8761
15.500	500	30	0.0001	Mean	1.0000
				Variance a	0.8207
				Variance b	0.7789
10.500	500	20	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
10.500	500	20	0.001	Mean	0.0000***
				Variance a	0.3224
				Variance b	0.1739
10.500	500	20	0.0001	Mean	0.1098
				Variance a	0.4336
				Variance b	0.6783
7.750	250	30	0.01	Mean	0.2178
				Variance a	0.0000***
				Variance b	0.0000***
7.750	250	30	0.001	Mean	0.0000***
				Variance a	0.0357**
				Variance b	0.0666*
7.750	250	30	0.0001	Mean	1.0000
				Variance a	0.7296
				Variance b	0.7664
5.500	500	10	0.01	Mean	0.4333
				Variance a	0.0000***
				Variance b	0.0000***
5.500	500	10	0.001	Mean	0.9999
				Variance a	0.0000***
				Variance b	0.0000***
5.500	500	10	0.0001	Mean	0.9992

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance a	0.0000***
				Variance b	0.0000***
5.250	250	20	0.01	Mean	0.2356
				Variance a	0.0000***
				Variance b	0.0000***
5.250	250	20	0.001	Mean	0.0000***
				Variance a	0.0001***
				Variance b	0.0004***
5.250	250	20	0.0001	Mean	0.9886
				Variance a	0.0932* †
				Variance b	0.1328
3.100	100	30	0.01	Mean	0.0000***
				Variance a	0.0000***
				Variance b	0.0000***
3.100	100	30	0.001	Mean	0.8259
				Variance a	0.1163
				Variance b	0.3059
3.100	100	30	0.0001	Mean	1.0000
				Variance a	0.0848*
				Variance b	0.0749*
3.000	500	5	0.01	Mean	0.1952
				Variance a	0.0000***
				Variance b	0.0000***
3.000	500	5	0.001	Mean	1.0000
				Variance a	0.0001***
				Variance b	0.9923 †
3.000	500	5	0.0001	Mean	1.0000
				Variance a	1.0000
				Variance b	1.0000
2.750	250	10	0.01	Mean	0.0573*
				Variance a	0.0000***
				Variance b	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
2.750	250	10	0.001	Mean	0.9943
				Variance a	0.0000***
				Variance b	0.0000***
2.750	250	10	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.00001***
2.100	100	20	0.01	Mean	0.0118**
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.001	Mean	0.8497
				Variance a	0.0000***
				Variance b	0.0000***
2.100	100	20	0.0001	Mean	0.4130
				Variance a	0.0250**
				Variance b	0.0178**
1.550	50	30	0.01	Mean	0.1710
				Variance a	0.0000***
				Variance b	0.0000***
1.550	50	30	0.001	Mean	0.9300
				Variance a	0.0000***
				Variance b	0.0001***
1.550	50	30	0.0001	Mean	0.9992
				Variance a	0.0000***
				Variance b	0.0001***
1.500	250	5	0.01	Mean	0.7062
				Variance a	0.0000***
				Variance b	0.0088***
1.500	250	5	0.001	Mean	1.0000
				Variance a	0.6188
				Variance b	0.9999
1.500	250	5	0.0001	Mean	1.0000
				Variance a	0.9735

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance b	1.0000
1.100	100	10	0.01	Mean	0.2204
				Variance a	0.0000***
				Variance b	0.0000***
1.100	100	10	0.001	Mean	0.9645
				Variance a	0.0000***
				Variance b	0.0000***
1.100	100	10	0.0001	Mean	0.9999
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.01	Mean	0.4523
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.001	Mean	0.9864
				Variance a	0.0000***
				Variance b	0.0000***
1.050	50	20	0.0001	Mean	0.3644
				Variance a	0.0000***
				Variance b	0.0001***
600	100	5	0.01	Mean	0.6706
				Variance a	0.0000***
				Variance b	0.0002***
600	100	5	0.001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.7273 †
600	100	5	0.0001	Mean	1.0000
				Variance a	0.2556
				Variance b	0.9997
600	50	10	0.01	Mean	0.0384**
				Variance a	0.0000***
				Variance b	0.0000***
550	50	10	0.001	Mean	0.9970

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
				Variance a	0.0000***
				Variance b	0.0000***
550	50	10	0.0001	Mean	0.9993
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.01	Mean	0.5872
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.001	Mean	0.9683
				Variance a	0.0000***
				Variance b	0.0000***
310	10	30	0.0001	Mean	0.9763
				Variance a	0.0000
				Variance b	0.0030
300	50	5	0.01	Mean	0.4346
				Variance a	0.0000***
				Variance b	0.0089***
300	50	5	0.001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.8514 †
300	50	5	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.5570 †
210	10	20	0.01	Mean	0.9658
				Variance a	0.0000***
				Variance b	0.0000***
210	10	20	0.001	Mean	0.9853
				Variance a	0.0000***
				Variance b	0.0000***
210	10	20	0.0001	Mean	1.0000
				Variance a	0.0000***
				Variance b	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Agents[1]	Firms	Households per firm	Variance of Random Walk	Test[2]	Probability[2,3,4]
110	10	10	0.01	Mean	0.8329
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.001	Mean	0.9979
				Variance a	0.0000***
				Variance b	0.0000***
110	10	10	0.0001	Mean	0.9921
				Variance a	0.0000***
				Variance b	0.0000***
60	10	5	0.01	Mean	0.0956*
				Variance a	0.0000***
				Variance b	0.0000***
60	10	5	0.001	Mean	0.9501
				Variance a	0.0000***
				Variance b	0.0000***
60	10	5	0.0001	Mean	0.9968
				Variance a	0.0000***
				Variance b	0.0000***

Note: 1) Number of agents = $(J + K + C)(1 + \varpi)$. 2) Test statistics for the test of equality of (i) means = Anova F-statistic; (ii) variance a = Bartlett test statistic; (iii) variance b = Levene test statistic. 3) *** significant at 1% level; ** significant at 5% level; * significant at 10% level. 4) †= ambiguous cases.

Appendix B

Nearly Orthogonal Latin Hypercube (Section 3.4)

The source file for the presented NOLH design is available online. You can download the file via http://diana.cs.nps.navy.mil/SeedLab/.

β	λ	η^{RR}	η^{IC}	ν	ω^{Micro}	ω^{Macro}	ψ	ι	θ	$Var[\epsilon]$	ϖ	τ	ς	ρ	χ
0.795	0.057	0.116	0.395	1.275	0.583	0.598	0.171	0.942	0.722	0.00547	28	1.563	1.325	0.769	1.719
0.924	0.299	-0.034	0.48	1.581	0.325	0.473	0.238	0.88	0.923	0.00766	17	1.703	0.819	0.441	1.241
0.89	0.169	0.472	0.297	1.516	0.63	0.278	0.163	0.802	0.582	0.00797	27	2.406	1.128	0.386	1.902
0.752	0.361	0.331	0.508	1.188	0.411	0.333	0.116	0.731	0.985	0.00906	21	1.234	0.566	0.802	1.395
0.907	0.209	0.013	0.114	1.231	0.278	0.325	0.202	0.852	0.49	0.00063	26	2.922	1.269	0.692	1.817
0.692	0.372	0.041	0.536	1.319	0.528	0.27	0.265	0.923	0.118	0.00407	25	3.672	0.313	0.32	1.142
0.821	0.108	0.219	0.128	1.45	0.364	0.645	0.155	0.739	0.443	0.00344	28	3.859	1.522	0.266	1.592
0.847	0.316	0.453	0.381	1.625	0.661	0.552	0.054	0.763	0.319	0.00017	21	3.156	0.256	0.583	1.269
0.778	0.051	-0.091	0.831	1.669	0.505	0.395	0.073	0.899	0.304	0.00844	15	1.984	0.622	0.561	2
0.933	0.288	0.191	0.789	1.1	0.341	0.661	0.144	0.868	0.01	0.00672	6	2.266	1.719	0.211	1.522
0.684	0.046	0.481	0.578	1.384	0.419	0.356	0.222	0.704	0.242	0.00719	10	1.656	0.284	0.364	1.845
0.941	0.226	0.313	0.93	1.341	0.231	0.388	0.253	0.778	0.381	0.00532	11	2.359	1.241	0.845	1.297
0.701	0.119	0.097	0.677	1.647	0.442	0.294	0.12	0.938	0.954	0.00141	6	3.953	0.509	0.638	1.761
0.83	0.237	0.153	0.902	1.472	0.684	0.223	0.05	0.829	0.536	0.00251	12	2.547	1.409	0.473	1.452
0.718	0.158	0.369	0.958	1.713	0.309	0.434	0.284	0.735	0.83	0.0001	16	3.906	0.734	0.605	1.873
0.77	0.248	0.228	0.747	1.297	0.677	0.7	0.191	0.813	0.814	0.0036	8	3.063	1.972	0.791	1.494
0.873	0.198	0.134	0.213	2.041	0.7	0.591	0.292	0.845	0.598	0.00313	12	2.219	1.353	0.67	1.438
0.709	0.349	0.059	0.409	1.909	0.427	0.614	0.249	0.79	0.737	0.0011	5	1	0.341	0.452	1.733
0.804	0.181	0.238	0.367	2.194	0.645	0.216	0.152	0.833	0.799	0.00485	7	1.609	1.775	0.222	1.564
0.735	0.304	0.416	0.156	1.866	0.286	0.419	0.093	0.903	0.876	0.00058	16	1.516	0.65	0.528	1.747
0.855	0.113	0.144	0.184	2.478	0.388	0.208	0.226	0.774	0.103	0.00422	9	2.828	1.663	0.9	1.466
0.838	0.338	-0.1	0.241	1.822	0.552	0.442	0.273	0.743	0.35	0.00922	13	3.203	0.453	0.506	1.986
0.95	0.186	0.378	0.325	2.456	0.263	0.559	0.081	0.841	0.335	0.00563	12	2.688	1.184	0.342	1.325
0.727	0.4	0.397	0.438	2.063	0.466	0.489	0.105	0.95	0.072	0.00703	10	3.578	0.594	0.856	1.93
0.744	0.13	-0.044	0.634	2.238	0.567	0.669	0.089	0.723	0.165	0.00266	21	1.375	0.2	0.823	1.423

Table B.1: Nearly Orthogonal Latin Hypercube (Part I)

β	λ	η^{RR}	η^{IC}	ν	ω^{Micro}	ω^{Macro}	ψ	ι	θ	$Var[\epsilon]$	ϖ	τ	ς	ρ	χ
0.813	0.265	-0.006	1	2.172	0.294	0.52	0.14	0.794	0.459	0.00032	26	2.031	1.213	0.233	1.944
0.864	0.074	0.35	0.817	1.844	0.653	0.364	0.288	0.891	0.258	0.0022	20	1.891	0.397	0.484	1.213
0.898	0.344	0.275	0.845	2.391	0.356	0.263	0.214	0.884	0.366	0.00376	30	1.188	1.494	0.681	1.62
0.916	0.147	-0.025	0.648	1.997	0.302	0.403	0.07	0.755	0.861	0.00875	19	3.719	0.763	0.703	1.184
0.787	0.378	0.106	0.761	2.347	0.692	0.372	0.183	0.716	0.783	0.0061	22	3.531	1.297	0.375	1.691
0.761	0.085	0.322	0.606	2.434	0.403	0.583	0.241	0.93	0.969	0.00813	18	3.25	0.369	0.288	1.311
0.881	0.276	0.463	0.873	2.106	0.52	0.653	0.218	0.864	0.613	0.00828	24	2.875	1.156	0.747	1.972
0.675	0.22	0.2	0.55	1.8	0.45	0.45	0.175	0.825	0.505	0.0081	18	2.5	1.1	0.55	1.55
0.555	0.383	0.284	0.705	2.325	0.317	0.302	0.179	0.708	0.288	0.00454	7	3.438	0.875	0.331	1.381
0.426	0.141	0.434	0.62	2.019	0.575	0.427	0.113	0.77	0.087	0.00235	18	3.297	1.381	0.659	1.859
0.46	0.271	-0.072	0.803	2.084	0.27	0.622	0.187	0.848	0.428	0.00204	8	2.594	1.072	0.714	1.198
0.598	0.079	0.069	0.592	2.413	0.489	0.567	0.234	0.919	0.025	0.00095	14	3.766	1.634	0.298	1.705
0.443	0.231	0.388	0.986	2.369	0.622	0.575	0.148	0.798	0.52	0.00938	9	2.078	0.931	0.408	1.283
0.658	0.068	0.359	0.564	2.281	0.372	0.63	0.085	0.727	0.892	0.00594	10	1.328	1.888	0.78	1.958
0.529	0.333	0.181	0.972	2.15	0.536	0.255	0.195	0.911	0.567	0.00657	7	1.141	0.678	0.834	1.508
0.503	0.124	-0.053	0.719	1.975	0.239	0.348	0.296	0.888	0.691	0.00984	14	1.844	1.944	0.517	1.831
0.572	0.389	0.491	0.269	1.931	0.395	0.505	0.277	0.751	0.706	0.00157	20	3.016	1.578	0.539	1.1
0.417	0.153	0.209	0.311	2.5	0.559	0.239	0.206	0.782	1	0.00329	29	2.734	0.481	0.889	1.578
0.666	0.394	-0.081	0.522	2.216	0.481	0.544	0.128	0.946	0.768	0.00282	25	3.344	1.916	0.736	1.255
0.409	0.214	0.088	0.17	2.259	0.669	0.513	0.097	0.872	0.629	0.00469	24	2.641	0.959	0.255	1.803
0.649	0.321	0.303	0.423	1.953	0.458	0.606	0.23	0.712	0.056	0.0086	29	1.047	1.691	0.463	1.339
0.52	0.203	0.247	0.198	2.128	0.216	0.677	0.3	0.821	0.474	0.0075	23	2.453	0.791	0.627	1.648
0.632	0.282	0.031	0.142	1.888	0.591	0.466	0.066	0.915	0.18	0.01	19	1.094	1.466	0.495	1.227
0.58	0.192	0.172	0.353	2.303	0.223	0.2	0.159	0.837	0.196	0.00641	27	1.938	0.228	0.309	1.606
0.477	0.243	0.266	0.888	1.559	0.2	0.309	0.058	0.805	0.412	0.00688	23	2.781	0.847	0.43	1.663

Table B.2: Nearly Orthogonal Latin Hypercube (Part II)

β	λ	η^{RR}	η^{IC}	ν	ω^{Micro}	ω^{Macro}	ψ	ι	θ	$Var[\epsilon]$	ϖ	τ	ς	ρ	χ
0.641	0.091	0.341	0.691	1.691	0.473	0.286	0.101	0.86	0.273	0.00891	30	4	1.859	0.648	1.367
0.546	0.259	0.163	0.733	1.406	0.255	0.684	0.198	0.817	0.211	0.00516	28	3.391	0.425	0.878	1.536
0.615	0.136	-0.016	0.944	1.734	0.614	0.481	0.257	0.747	0.134	0.00953	19	3.484	1.55	0.572	1.353
0.495	0.327	0.256	0.916	1.122	0.513	0.692	0.124	0.876	0.907	0.00579	26	2.172	0.538	0.2	1.634
0.512	0.102	0.5	0.859	1.778	0.348	0.458	0.077	0.907	0.66	0.0009	22	1.797	1.747	0.594	1.114
0.4	0.254	0.022	0.775	1.144	0.638	0.341	0.269	0.809	0.675	0.00438	23	2.313	1.016	0.758	1.775
0.623	0.04	0.003	0.663	1.538	0.434	0.411	0.245	0.7	0.938	0.00298	25	1.422	1.606	0.244	1.17
0.606	0.31	0.444	0.466	1.363	0.333	0.231	0.261	0.927	0.845	0.00735	14	3.625	2	0.277	1.677
0.538	0.175	0.406	0.1	1.428	0.606	0.38	0.21	0.856	0.551	0.00969	9	2.969	0.988	0.867	1.156
0.486	0.366	0.05	0.283	1.756	0.247	0.536	0.062	0.759	0.753	0.00781	15	3.109	1.803	0.616	1.888
0.452	0.096	0.125	0.255	1.209	0.544	0.638	0.136	0.766	0.644	0.00625	5	3.813	0.706	0.419	1.48
0.434	0.293	0.425	0.452	1.603	0.598	0.497	0.28	0.895	0.149	0.00126	16	1.281	1.438	0.397	1.916
0.563	0.063	0.294	0.339	1.253	0.208	0.528	0.167	0.934	0.227	0.00391	13	1.469	0.903	0.725	1.409
0.589	0.355	0.078	0.494	1.166	0.497	0.317	0.109	0.72	0.041	0.00188	17	1.75	1.831	0.813	1.789
0.469	0.164	-0.063	0.227	1.494	0.38	0.247	0.132	0.786	0.397	0.00173	11	2.125	1.044	0.353	1.128

Table B.3: Nearly Orthogonal Latin Hypercube (Part III)

Appendix C

CD

The attached CD provides the SeSAm installer. When one starts the installer, both the SeSAm programing environment and the model library including the model of Agent Island installs on your computer. If one runs SeSAm, a model library opens. By starting the model of Agent Island in the library, a 'macroeconomic cockpit' opens. Through this 'cockpit' the researcher is able to run the model via the 'Run' button and to investigate several model charts. The following four panels are depicted containing several time series data:

1. **Inflation & Output gap:** This panel contains eight time series ['Inflation CPI' = consumer price inflation; 'Inflation IPI' = capital goods inflation; 'Output Gap Total' = overall output gap of all business sectors; 'Output Gap Hash & Beans' = output gap of the consumer goods sector; 'Output Gap Capital' = overall output gap of the capital goods sector'; 'Delta GDP Real' = growth rate of real GDP; 'CEI / GDP' = circuit equlibrium indicator per GDP; 'i(Credit)' = nominal credit interest rate].
2. **Interest Rates:** This panel contains five time series ['i(Credit)' = nominal credit interest rate; 'r(Credit) ex post' = ex post real credit interest rate; 'r(Credit) expected' = ex ante real credit interest rate; 'i(Assets)' = average nominal return on assets of all consumer goods firms; 'r(Assets) ex post' = average ex post real return on assets of all consumer goods firms].
3. **Delta GDP real:** This panel contains three time series ['Delta GDP Real' = growth rate of real GDP; 'Delta C Real' = growth rate of real consumption; 'Delta I Real' = growth rate of real (gross) investment; 'Delta A(t) World' = growth rate of the technology parameter on the

'world' level].

4. **Firm profits:** This panel contains four time series ['Business Profit' = aggregate profits of the firm sector; 'net Investment' = net investment of the firm sector; 'Transfers' = transfers, i.e. dividend payments, of the firm sector to the household sector; 'Delta FA Firms' = net non–financial sources or borrowings of funds of the firm sector].

Next to the SeSAM installer, the CD provides three Excel files. These files belong to the discussion of section 3.2. The CD contains the following files:

1. **Homogenous case without stochastic supply shocks**: In here, each Excel folder shows the time series of one scenario. The folders are labeled by the description of the contained scenario, i.e. $((J + K + C)\_\varpi)$,
2. **Homogenous sectors**: In here, each Excel folder shows the time series of one scenario. The folders are labeled by the description of the contained scenario, i.e. $((J + K + C)\_\varpi\_Var[\epsilon])$,
3. **Heterogenous sectors**: In here, each Excel folder shows the time series of one scenario. The folders are labeled by the description of the contained scenario, i.e. $((J + K + C)\_\varpi\_Var[\epsilon])$.

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