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An Omnibus Test to Detect Time-Heterogeneity in Time Series

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Abstract

This paper focuses on a procedure to test for structural changes in the first two moments of a time series, when no information about the process driving the breaks is available. We model the series as a finite-order auto-regressive process plus an orthogonal Bernstein polynomial to capture heterogeneity. Testing for the null of time-invariance is then achieved by testing the order of the polynomial, using either an information criterion, or a restriction test. The procedure is an omnibus test in the sense that it covers both the pure discrete structural changes and some continuous changes models. To some extent, our paper can be seen as an extension of Heracleous, Koutris and Spanos (2008).

Keywords: Structural Changes - Time-homogeneity - Bernstein polynomial

1 Introduction

This paper deals with models of the form:

$$A(L)y_t = c_t \tag{1}$$

where: $A(L) = 1-\rho_1 L-\rho_2 L^2-...-\rho_p L^p$ and the roots $1-\rho_1 z-\rho_2 z^2-...-\rho_p z^p = 0$ lie all outside the unit circle,

c_t is either defined as $c_t = f(t) + \varepsilon_t$ or $c_t = \sqrt{f(t)}\varepsilon_t$, where in both cases $f(t)$ is an unknown, possibly time-varying signal, thus inducing heterogeneity in one of the two moments of the conditional distribution,

ε_t is an iid term.

For instance, assume the simple case where $f(t)$ is defined as a step-function for the mean:

$$f(t) = \begin{cases} c_1 \text{ if } t \leq t_0 \\ c_2, \text{ otherwise} \end{cases} \tag{2}$$

with $c_1 \neq c_2$ and $t_0 \in [t_1, t_2]$.

Perron (2005) reviews the huge literature dedicated to structural changes testing procedures. Clearly, testing for structural changes is a prior to modelling and testing. On the one hand, structural changes are a source of global non-stationarity (Granger and Starica 2005 and Guégan 2010), and on the other hand, they are likely to bias tests for stationarity (Perron 1989), and for long memory (Baek and Pipiras 2011, Charffedine and Guégan 2011, Berkes et al. 2006). Hence, in addition to causing parameter instability and spurious results, time-heterogeneity, or structural changes, may lead to erroneous statistical inference and thus to incorrect modelling. Main tests of structural changes include Nyblom (1989) , Andrews (1993), Andrews and Ploberger (1994), Bai (1999), Bai and Perron (1998, 2003) or Altissimo and Corradi (2003) among others.

In a recent contribution, Heracleous, Koutris and Spanos (2008) have pointed out that such procedures may not have power against continuous changes. They introduce a new test designed to track both discrete and continuous changes in moments. Their test consists in tracking heterogeneity in rolling moments of de-memorized series using an orthogonal Bernstein polynomial. With k the degree of the polynomial, the test amount to testing $k = 0$ against $k > 0$.

In this paper, we propose an extension of the Heracleous-Koutris-Spanos test. Like their procedure, we test for the null of time-homogeneity against a broad alternative including discrete and some continuous changes as deterministic and stochastic trends. Main differences are that i) We don't use de-memorized series, but rather the observed ones, ii) Tests are performed on the series itself, and not on rolling moments, thus avoiding the difficult choice of choosing a window, iii) To test for the null we consider two strategies, either based on an information criterion ($AICu$), or on a restriction test.

This paper is organized as follows. Section 2 presents the test, Section 3 implements the tests on two series, Section 4 runs Monte-Carlo simulations, and Section 5 concludes.

2 A test of no structural change

For the stationary series $\{y_t\}_{t=1}^{T}$ where y_t is real-valued, define the following Data Generating Process (DGP):

$$y_t = \sum_{i=1}^{p} \rho_i y_{t-i} + c_t \tag{3}$$

In (3), c_t is either defined as i) $c_t = f(t) + \varepsilon_t$ or ii) $c_t = \sqrt{f(t)}\varepsilon_t$, and ε_t is a white noise.

In this paper, we are interested in testing two kinds of assumptions: First-order time homogeneity, $H_0^1 : f(t) = c_1$ in i), and conditional on H_0^1 true, for second-order time-homogeneity, $H_0^2 : f(t) = c_2$ in ii). In such models if the process driving the changes is known, then it can be directly estimated. For instance, if one suspects discrete shifts in $f(t)$ in i), then one can use the Bai and Perron (1998) approach. Nevertheless, in most cases, $f(t)$ is generally unknown. To approximate it, i.e. to capture heterogeneity in the considered moment, we use an orthogonal Bernstein polynomial[1] of degree k. The Bernstein polynomial

[1] On the use of polynomials in structural changes models, see also MacNeil (1978) and

is given by:

$$B_{i,k}(t) = \sum_{i=0}^{k} \binom{k}{i} \left(\frac{t}{T}\right)^{i} \left(1 - \frac{t}{T}\right)^{k-i}, i = 0, 1, ..., k \tag{4}$$

Figure 1 plots few realizations of $B_{i,k}(t)$ for $k = 0, 1, 2$. Clearly, $k = 0$ corresponds to a constant signal, $k = 1$ to a linear trend in the moment, and $k > 1$ to a more complex signal[2].

Pease insert here Figure 1

The unconstrained model for the mean in then given by:

$$y_t = \sum_{i=1}^{p} \rho_i y_{t-i} + \sum_{i=0}^{k} \beta_i \binom{k}{i} \left(\frac{t}{T}\right)^{i} \left(1 - \frac{t}{T}\right)^{k-i} + \varepsilon_t \tag{5}$$

and for the variance under H_0^1 true by:

$$\varepsilon_t^2 = \sum_{i=0}^{k} \alpha_i \binom{k}{i} \left(\frac{t}{T}\right)^{i} \left(1 - \frac{t}{T}\right)^{k-i} + \nu_t \tag{6}$$

where: ε_t^2 are the squared residuals of model (5),
ν_t is an iid noise
the β_i, and α_i $i = 0, 1, .., k$ are estimated coefficients.

It is straightforward to see that in models (5) and (6), no structural change in the conditional distribution of y_t implies $k = 0$, corresponding to a constant signal. Thus, testing for the null amounts to testing: $H_0^i : k = 0$ against $k > 0$, $i = 1, 2$.

In this paper, two testing strategies are used. The first one consists in minimizing a Bayesian information criterion to jointly select the order p and the degree k in (5), and the degree k in (6). Since, we want to extract a signal, using a classical criterion as the AIC (Akaike 1974) will be inadequate, resulting

Perron (1991).

[2] To avoid any confusion, note that we test for second-order time-homogeneity conditional on H_0^1 true. The reason is that the Bernstein polynomial is used only as an approximation. Hence, especially in the discrete shift model, it is likely to produce non-spherical disturbances, thus possibly biaising tests for second-order time-homogeneity.

in overweighting the fit as showed by McQuarrie and Tsai (1988). This leads to use a more penalized AIC, i.e. the $AICu$ criterion. The $AICu$ is here given by:

$$AICu = \log(\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}(T-p-k)^{-1}) + 2(p+k+1)(p-k-2)^{-1} \tag{7}$$

for the mean, and:

$$AICu = \log(\boldsymbol{\nu}'\boldsymbol{\nu}(T-k)^{-1}) + 2(k+1)(k-2)^{-1} \tag{8}$$

for the variance under H_0^1 true.

One is then to accept the null if minimizing the $AICu$ lead to choose $k = 0$.

Alternatively, if the $AICu$ leads to select $k > 0$, one can use a classical restriction test in a non-nested environment (see Davidson and MacKinnon 2004), i.e. estimate (9)

$$y_t = \sum_{i=1}^{p} \rho_i y_{t-i} + \beta_0 + \sum_{i=0}^{k} \beta_i \binom{k}{i} \left(\frac{t}{T}\right)^i \left(1 - \frac{t}{T}\right)^{k-i} + \varepsilon_t \tag{9}$$

and test $H_0^1 : \beta_1 = \beta_2 ... = \beta_k$ using a standard $Ftest$[3].
For the variance, under H_0^1 true, we estimate (10):

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=0}^{k} \alpha_i \binom{k}{i} \left(\frac{t}{T}\right)^i \left(1 - \frac{t}{T}\right)^{k-i} + \nu_t \tag{10}$$

and test $H_0^2 : \alpha_1 = \alpha_2 ... = \alpha_k$.

We next turn to an application.

3 An empirical application

In this section, we implement the test on two series. To simplify, only the decision rule based on the $AICu$ is considered. The series, for the United States, are the inflation rate (1960Q1-2011Q2) and the growth rate of the real GDP (1970Q1-2011-Q2). Concerning the former (Figure 2), it clearly exhibits a stochastic trend in mean. Jointly selecting the order p and the degree k returns

[3] For the tests and estimations Heteroscedastic and Autocorrelation Consistent (HAC) matrices are used.

$p = 5$ and $k = 3$, supporting the rejection of the null, which is deeply coherent with the series. Concerning the latter, Figure 3 suggests a constant mean. Using the $AICu$ criterion leads to select $p = 2$ and $k = 0$ suggesting indeed a constant signal. Extracting the residuals of the regression, and estimating (10) for different orders k leads to select $k = 4$, thus indicating a change in the variance (Figure 4). Thus, clearly for the growth rate of the GDP, the second order time-homogeneity is rejected.

Please insert about here Figures 2,3 & 4

4 A small simulation study

We next turn to a small Monte-Carlo simulation study to estimate the size and power of the procedure. For the changes in mean, we also analyze its relative performance with regard to two competing tests: The CUSUM one (Brown, Durbin and Evans 1975) and the more recent Andrews and Ploberger (1994) approach, based on the $supF_n$ statistic. This latter consists in comparing the residuals sum of squares of two models using an $Ftest$: The model with no structural change, and the model with a structural change occurring at the period $t \in [t_1, t_2]$. Computing the $Fstat$ for each t and taking the supremum returns the $supF_n$. Following Hansen (2000), The pvalues for the $supF_n$ are computed using the fixed regressor bootstrap (using 1000 iterations). For breaks in variance, we also compare our procedure with the CUSUM one.

Our general DGP for the mean is given by:

$$y_t = 0.5y_{t-1} + f(t) + \varepsilon_t, \ \varepsilon_t \sim N(0, 1) \tag{11}$$

and the five considered cases for $f(t)$ are as follows (see Hansen 2000):

i) iid case: $f(t) = 0$,

ii) Mean break: $f(t) = 0$ for $t \leq t_0$ and $f(t) = 1$ otherwise and t_0 is randomly drawn in $[T/4, 3T/4]$ at each iteration,

iii) Deterministic trend in mean: $f(t) = (1 + 2t/T)$,

iv) Stochastic trend in mean: $f(t) = f(t-1) + \upsilon_t, \upsilon_t \sim N(0,1)$,

v) Stop-break model (Engle and Smith 1999): $y_t = f(t) + \varepsilon_t$, $f(t) = f(t-1) + \frac{\varepsilon_t^2}{\gamma + \varepsilon_t^2}$.

For each case, we run 10000 iterations. Table 1 returns the results of the simulations when the lag p and the order k are jointly choosen according to the $AICu$ criterion. The iid case returns the empirical size of the procedure, computed as $1 - P(k = 0) = P(k > 0)$, and the four other cases the power given by $P(k > 0)$. The size of the procedure does not exceed 0.187 for a small sample size ($T = 50$) and is of 0.118 for $T = 500$. Concerning the power, results are twofold. For the single discrete break in mean, the stop-break and the linear trend in mean models, the power is high ranging from 0.941 to 1.000 for T ranging from 100 to 500. For very small sample size ($T = 50$), the power remains high, indicating that the test performs well. For the stochastic trend in mean, the power is lower ranging from 0.748 to 0.772. It nevertheless stays within an acceptable range.

Please insert about here tables 1, 2, 3 & 4

We now turn to restriction tests, given by Table 2. Recall that we run the restrictions tests whenever the $AICu$ leads to select $k > 0$. Compared to the $AICu$ decision rule, at the 5% nominal size, the empirical size is lower: Slightly for the stop-break, the linear trend and the single discrete break models, and significantly for the stochastic trend in mean model.

Comparing the size and power of the test to the CUSUM (Table 3) and to the Andrews-Ploberger ones (Table 4), it appears that: i) Our procedure over-performs, in power, the CUSUM test, even in the simple discrete break in mean model. In fact, in our simulations, the CUSUM was able to recognize a single rupture only if it occured around the middle of the sample. A rupture located near the boundaries is unlikely to be detected by the CUSUM.. For the linear trend and the stochastic trend in mean the power is very low, and for the stop-break model the power is correct only for a large sample size. ii) The Andrews-Ploberger test is also over-performed by our test when the changes in means are continuous, especially for small sample sizes. It is equivalent to our test for discrete changes. This seems coherent, since the Andrews-Ploberger test is designed to track discrete changes. These results match those of Heracleous, Koutris and Spanos (2008).

Please insert here tables 5,6 & 7

We now analyze the size and power of the procedure to detect ruptures in variance. Our DGP is given by:

$$y_t = 0.5y_{t-1} + \sqrt{f(t)}\varepsilon_t,\ \varepsilon_t \sim N(0,1) \tag{12}$$

and the four cases considered are:

i) iid case: $\sqrt{f(t)} = 1$,

ii) Variance break: $\sqrt{f(t)} = 1$ for $t \leq t_0$ and $\sqrt{f(t)} = 2$ otherwise, and t_0 is randomly drawn in $[T/4, 3T/4]$ at each iteration,

iii) Deterministic trend in variance: $\sqrt{f(t)} = (1 + 2t/T)$,

iv) Stochastic trend in variance $\sqrt{f(t)} = \exp(h_t/2)$, $h_t = h_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0,1)$, $\varepsilon_t \sim N(0,1)$.

Table 5 presents the size and power of the procedure based on the $AICu$ decision rule. Clearly, the size is low and does not exceed 0.112. Unexpectedly the size doesn't decrease with the sample size. Considering the power, it is quite low for $T = 50$, especially when the variance moves according to a linear trend and generally for all considered models. It is nevertheless acceptable for sample sizes ranging from $T = 100$ to $T = 500$. Turning now to restriction tests, Table 6, it can be seen that the empirical size is less than the 5% nominal one. It also appears that the test has power against the three models exhibiting ruptures in variance only for large sample sizes, i.e. for $T \geq 150$. Table 7 returns results of the CUSUM test. Clearly the test has low size, but also low power when the variance moves according to a linear or stochastic trend. For discrete shifts, as in our procedure, the power is low for small sample sizes.

5 Conclusion

In this paper, we have introduced a procedure that tests for the null of time-homogeneity of the first two moments of a time series. The procedure uses an orthogonal Bernstein polynomial to extract the signal driving the time path of the moments. With k the order of the polynomial, the procedure amounts to testing $k = 0$ against $k > 0$ using either a Bayesian model selection criterion, here the $AICu$, or a restriction test. Running Monte-Carlo simulations, it appeared that: i) Concerning the structural changes in mean, it has power against both discrete and continuous changes, ii) It over-performs the CUSUM test and is equivalent to the Andrews-Ploberger one for discrete changes but over-performs it for continuous changes, iii) The test has good small sample properties, iv) The test is well suited to detect structural changes in the variance only for sample sizes more than 150 observations. In conclusion, the test could be used in empirical using either the $AICu$ decision rule, or a restriction test.

There is an avenue for further researches using orthogonal polynomials in this field. One possible extension would be altering the procedure to detect the breaking dates and/or the different regimes.

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Appendix A: Figures to be included in the paper

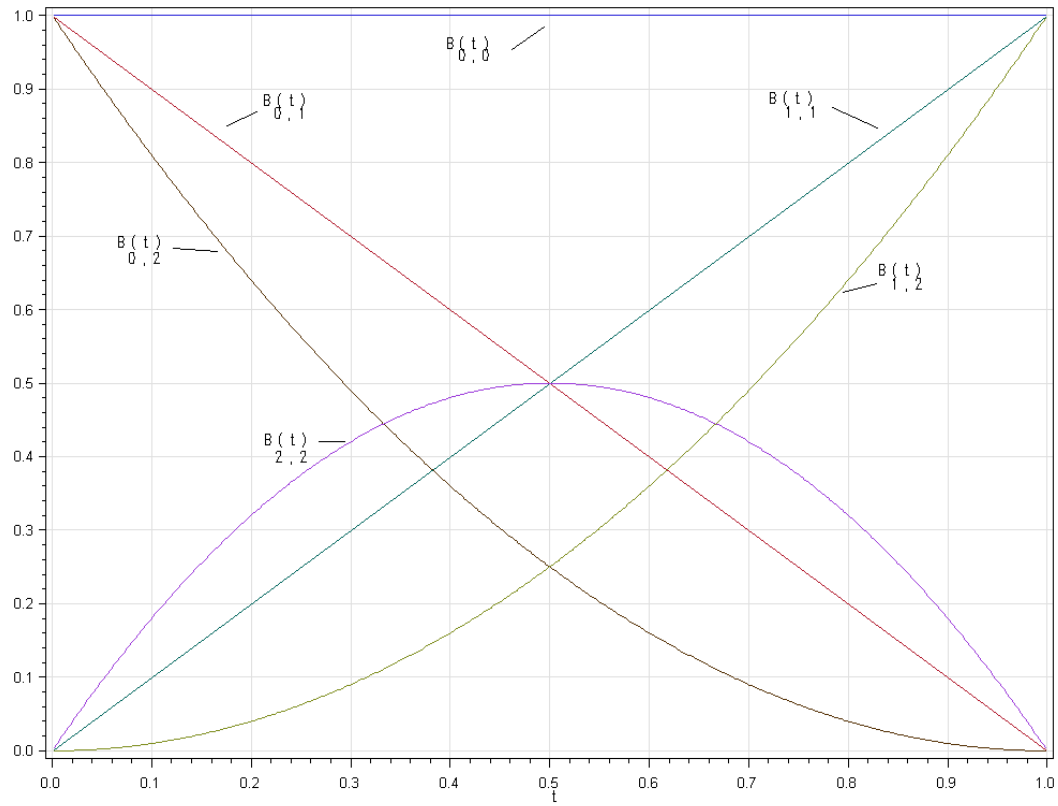

Figure 1: $B_{i,k}(t)$, $k = 0, 1, 2$.

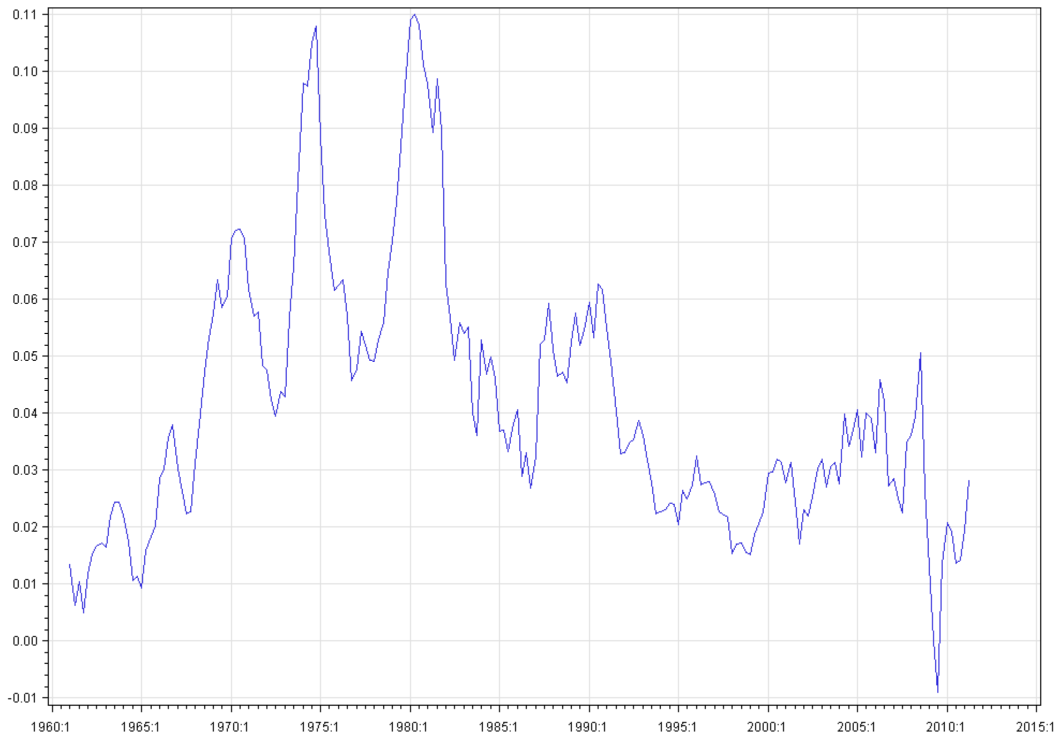

Figure 2: Inflation rate, USA, 1960:01-2011:02

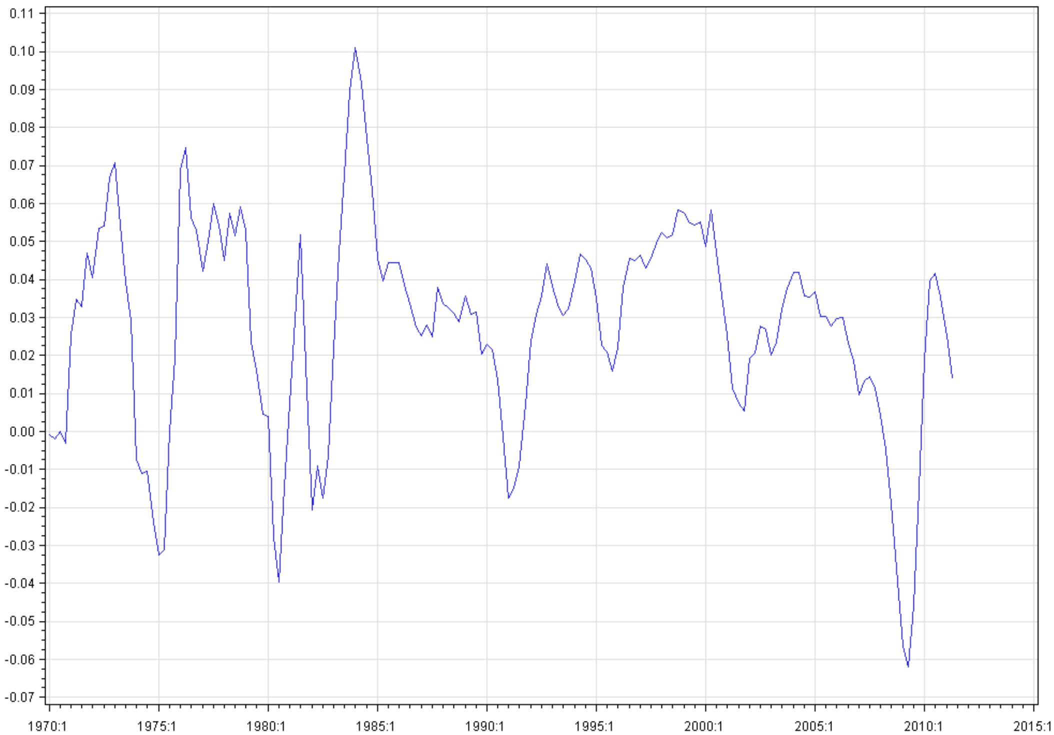

Figure 3: Real GDP, growth rate, USA, 1970:01-2011:02

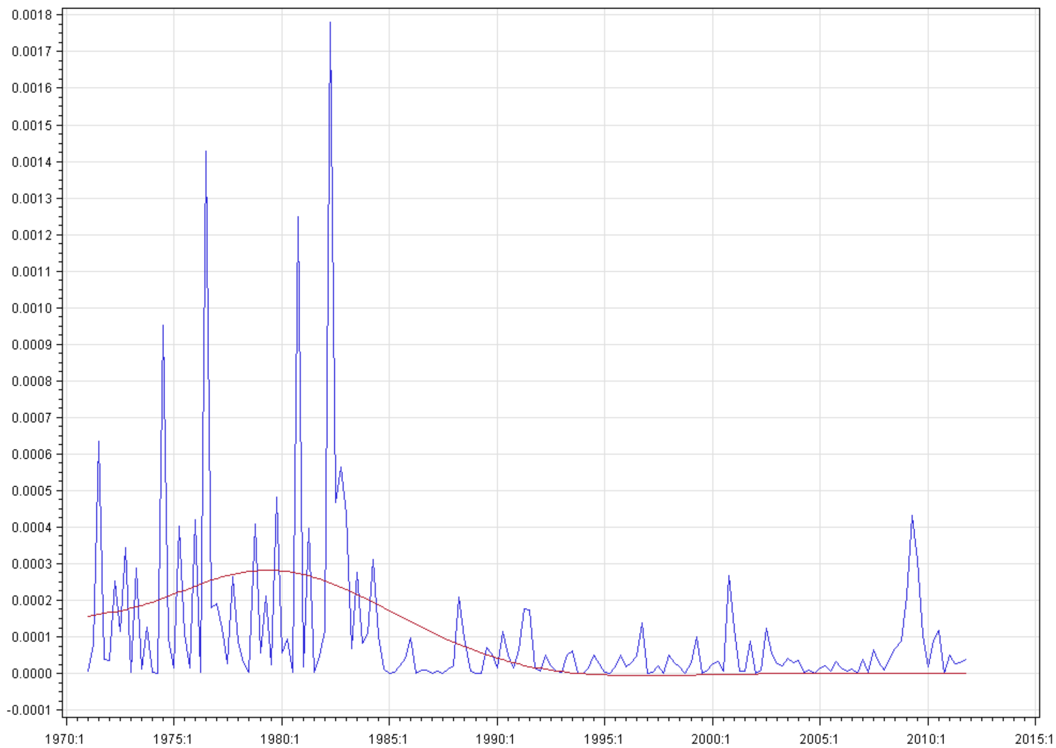

Figure 4: Squared residuals, together with a Bernstein polynomial ($k = 4$).

Appendix B: Tables to be included in the paper

Table 1: $AICu$ based criterion for five models, the last four ones exhibiting ruptures in mean

iid case: H_0 true					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.813	0.831	0.860	0.872	0.882
$P(k > 0)$	0.187	0.164	0.140	0.128	0.118
Single discrete break in mean: H_0 false					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.125	0.018	0.006	0.001	0.000
$P(k > 0)$	0.875	0.982	0.994	0.999	1.000
Linear trend in mean: H_0 false					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.047	0.000	0.000	0.000	0.000
$P(k > 0)$	0.953	1.000	1.000	1.000	1.000
Stochastic trend in mean: H_0 false					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.229	0.244	0.252	0.233	0.228
$P(k > 0)$	0.771	0.756	0.748	0.767	0.772
Stop-break model: H_0 false					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.156	0.059	0.049	0.064	0.000
$P(k > 0)$	0.844	0.941	0.951	0.936	1.000

Note 1: The iid case returns the size of the procedure, given by $1 - P(k = 0)$. Ideally it should be close to 0

Note 2: The other four cases return the power of the procedure, given by $P(k > 0)$. Ideally it should be close to 1

Table 2: Size and power of restriction tests at 4 nominal sizes for five models, the last four ones exhibiting ruptures in mean.

iid case: H_0 true					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.045	0.039	0.024	0.020	0.011
0.05	0.105	0.086	0.063	0.051	0.045
0.10	0.139	0.120	0.093	0.088	0.070
0.15	0.166	0.148	0.111	0.105	0.092
Single discrete break in mean: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.343	0.716	0.910	0.944	1.000
0.05	0.671	0.925	0.970	0.982	1.000
0.10	0.810	0.967	0.989	0.996	1.000
0.15	0.848	0.978	0.992	0.998	1.000
Linear trend in mean: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.489	0.918	0.995	1.000	1.000
0.05	0.788	0.986	1.000	1.000	1.000
0.10	0.897	0.994	1.000	1.000	1.000
0.15	0.938	0.998	1.000	1.000	1.000
Stochastic trend in mean: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.512	0.467	0.442	0.442	0.432
0.05	0.689	0.652	0.639	0.638	0.654
0.10	0.739	0.718	0.695	0.717	0.729
0.15	0.763	0.741	0.736	0.751	0.749
Stop-break model: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.670	0.871	0.880	0.860	0.999
0.05	0.792	0.913	0.936	0.914	1.000
0.10	0.817	0.933	0.946	0.925	1.000
0.15	0.831	0.936	0.947	0.932	1.000

Note 1: The iid case returns the size of the procedure. Ideally it should be close to the nominal one

Note 2: The three other cases return the power of the procedure. Ideally it should be close to 1

Table 3: Size and power of the CUSUM test at the 5% nominal size, for five models, the last four ones exhibiting ruptures in mean

iid case: H_0 true					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.079	0.041	0.053	0.045	0.044
Single discrete break in mean: H_0 false					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.04	0.402	0.426	0.400	0.405
Linear trend in mean: H_0 false					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.102	0.194	0.199	0.227	0.238
Stochastic trend in mean: H_0 false					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.087	0.204	0.228	0.263	0.252
Stop-break model: H_0 false					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.107	0.294	0.520	0.663	0.987

Note 1: The iid case returns the size of the procedure. Ideally it should be close to the nominal one

Note 2: The three other cases return the power of the procedure. Ideally it should be close to 1

Table 4: Size and power of the Andrews-Ploberger test at the 5% nominal size, for five models, the last four ones exhibiting ruptures in mean

iid case: H_0 true					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.160	0.052	0.020	0.017	0.000
Single discrete break in mean: H_0 false					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.849	0.951	0.979	0.988	1.000
Linear trend in mean: H_0 false					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.234	0.450	0.619	0.720	0.955
Stochastic trend in mean: H_0 false					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.087	0.204	0.228	0.263	0.252
Stop-break model: H_0 false					
Size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.727	0.752	0.712	0.705	0.749

Note 1: The iid case returns the size of the procedure. Ideally it should be close to the nominal one

Note 2: The three other cases return the power of the procedure. Ideally it should be close to 1

Table 5: $AICu$ based criterion for four models, the last three ones exhibiting ruptures in variance

iid case: H_0 true					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.932	0.898	0.900	0.897	0.888
$P(k > 0)$	0.066	0.052	0.100	0.060	0.112
Single discrete break in variance: H_0 false					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.391	0.085	0.001	0.001	0.000
$P(k > 0)$	0.609	0.915	0.999	0.999	1.000
Linear trend in variance: H_0 false					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.652	0.186	0.111	0.048	0.000
$P(k > 0)$	0.348	0.720	0.829	0.952	1.000
Stochastic trend in variance: H_0 false					
	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
$P(k = 0)$	0.380	0.186	0.160	0.110	0.107
$P(k > 0)$	0.620	0.814	0.840	0.890	0.893

Note 1: The iid case returns the size of the procedure, given by $1 - P(k = 0)$. Ideally it should be close to 0

Note 2: The other four cases return the power of the procedure, given by $P(k > 0)$. Ideally it should be close to 1

Table 6: Size and power of restriction tests at 4 nominal sizes for four models, the last three ones exhibiting ruptures in variance

iid case: H_0 true					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.007	0.007	0.013	0.009	0.012
0.05	0.024	0.026	0.030	0.031	0.038
0.10	0.043	0.052	0.055	0.060	0.061
0.15	0.054	0.070	0.076	0.080	0.082
Single discrete break in variance: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.109	0.459	0.768	0.913	1.000
0.05	0.305	0.729	0.947	0.985	1.000
0.10	0.447	0.839	0.980	0.995	1.000
0.15	0.538	0.880	0.988	0.998	1.000
Linear trend in variance: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.048	0.194	0.399	0.534	0.994
0.05	0.148	0.415	0.659	0.807	1.000
0.10	0.212	0.536	0.786	0.887	1.000
0.15	0.281	0.608	0.835	0.917	1.000
Stochastic trend in variance: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.01	0.226	0.577	0.666	0.761	0.849
0.05	0.407	0.707	0.765	0.825	0.864
0.10	0.502	0.776	0.795	0.853	0.873
0.15	0.577	0.792	0.818	0.871	0.882

Note 1: The iid case returns the size of the procedure. Ideally it should be close to the nominal one

Note 2: The three other cases return the power of the procedure. Ideally it should be close to 1

Table 7: Size and power of the CUSUM test at the 5% nominal size for four models, the last three ones exhibiting ruptures in variance

iid case: H_0 true					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.092	0.050	0.043	0.041	0.040
Single discrete break in variance: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.332	0.736	0.885	0.957	1.000
Linear trend in variance: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.222	0.224	0.326	0.474	0.957
Stochastic trend in variance: H_0 false					
size	$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 500$
0.05	0.171	0.404	0.463	0.481	0.504

Note 1: The iid case returns the size of the procedure. Ideally it should be close to the nominal one

Note 2: The three other cases return the power of the procedure. Ideally it should be close to 1