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Capabilities and Equality of Health I

Hans Keiding

Studiestræde 6, DK-1455 Copenhagen K., Denmark
Tel. +45 35 32 30 82 - Fax +45 35 32 30 00
http://www.econ.ku.dk

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Hans Keiding

University of Copenhagen

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Abstract

The concept of capabilities, introduced originally by Sen with the aim to provide a better basis for the theory of inequality, has inspired many researchers but has not found any simple formal representation which might be instrumental in the construction of a comprehensive theory of equality.

In the present paper, we present a formalization of the concept of capabilities based on Lancasterian characteristics, whereby a functioning of an individual is a method for transforming an initial position to a final outcome. In this context, we investigate whether preferences over capabilities as sets of functionings can be rationalized by maximization of a suitable utility function over the set of functionings. Such a rationalization turns out to be possible only in cases which must be considered exceptional and which do not allow for interesting applications of the capability approach to questions of health or equality.

The conclusion which can be obtained from the predominantly negative results is that a formal description of capabilities much involve ideas which go beyond the simple representation as a family of choice sets.

Keywords: Capabilities, characteristics, equality of health.

JEL classification: D63, I10

1. Introduction

In recent years, the capability approach suggested by Sen (1980, 1985) has been applied in several different fields of economics, including research in poverty and inequality, but also individual health related quality of life. The capability of an individual is given as a set of functionings, each of which describes a way of transforming an intial given situation to a final outcome. A recent approach to the measurement of standard of living using functionings as a basic concept is Gaertner and Xu (2005).

Several recent studies (Herrero (1996), Herrero (1997), Herrero, Iturbe-Ormaetxe and Nieto (1998)) have applied the capability approach to problems of equality, whereby capabilities are taken as depending on the initial state, formalized as an allocation of goods. We shall expand slightly on this formulation, using the concept of characteristics introduced by Lancaster (see, e.g. Lancaster (1971)). We assume that individuals have access to a technology for transforming initial bundles of goods into characteristics. Each such transformation would then correspond to a functioning, but the set of functionings now depend in a rather simple way on the initial bundle, since they will be all the processes which are feasible and use the available endowment as inputs. This approach to capabilities is admittedly simplistic, in particular if we add standard assumptions on the overall technology, but this very simplicity has the advantage that it allows for a closer scrutiny of the nature of capabilities, and in particular, preferences over sets of capabilities, and their possible explanation.

In the present study, we shall be interested in situations where the individual or societal preferences over characteristics or characteristics profiles can be derived from utility maximization, so that one capability is better than another if the best possible outcome in the first is better than the best possible outcome in the second. If this is the case, we say that the preference relation over capability sets is rationalizable. It turns out that this will happen only in rather exceptional cases, at least when we consider setups which are realistic from the point of view of applications (in the first case, applications to health status measurement, and in the second case, to optimal choice of allocation in a socety). These results can be seen as a formal argument for the need of a more elaborate theory of capabilities, elaborating on the reason why one capability set is better than another and how much better it will be, since such questions cannot – as the present paper indicates – be answered based on the standard approach of economic theory, maximization of individual or societal welfare.

The paper is structured as follows. In the section 2, we consider individual preferences over capabilities, with a special view to application to health related quality of life, and in section 3, we discuss problems of allocation in society. Section 4 contains a short summary and discussion of the results obtained.

2. Rationalizing individual capability indices

As is well-known, measuring individual health is both conceptually and technically complicated; indeed the very nature of health is open to some debate, and in the by now very rich literature on this subject it has been suggested, that the capability approach outlined above and the measurement of health as QALYs are indeed closely related (Cookson, 2005).

In the following we consider a given family $\mathcal{C}$ of compact, convex, and comprehensive (meaning that if $x \in C$ and $y \in \mathbb{R}^L_+$ satisfies $y_h \leq x_h$ for $h = 1, \dots, L$, then $y \in C$) subsets of characteristics (interpreted as sets of functionings open to an individual in different health states). For our subsequent reasoning, it is convenient to assume that $\mathcal{C}$ is rich enough to contain some distinguished sets, in particular $\mathcal{C}$ contains $\{0\}$ and permits the operation of (Minkowski) weighted averages, i.e. if $C, C' \in \mathcal{C}$ and $\lambda \in [0, 1]$, then the set

$$\lambda C + (1 - \lambda)C' = \{y \in \mathbb{R}^L_+ \mid y = \lambda x + (1 - \lambda)x',\, x \in C,\, x' \in C'\}$$

belongs to $\mathcal{C}$ as well. We shall say that a family $\mathcal{C}$ with these properties is regular.

In our present setup, the capabilities approach to QALY measurement would imply that there is complete preorder $\succsim$ on the sets $C \in \mathcal{C}$; we let $\succ$ and $\sim$ denote the associated strict order and indifference, respectively. For completeness of exposition, we state this as a first axiom.

AXIOM 1. *The preference relation $\succsim$ on the family $\mathcal{C}$ is a complete preorder, and it is continuous in the sense that $\{C' \mid C \succsim C'\}$ and $\{C' \mid C' \succsim C\}$ are closed (in the topology on $\mathcal{C}$ induced by the Hausdorff distance) for all $C \in \mathcal{C}$.*

We shall consider in some more details the properties of this preorder which seem reasonable if it is to be represented by an index with QALY-like properties. First of all, we assume that averages make sense and that the indifference relation is stable under such averages:

AXIOM 2. *Let (C_1, C_2) and (C_1', C_2') be pairs of elements of $\mathcal{C}$ with $C_1 \sim C_2$, $C_1' \sim C_2'$, and let $\lambda \in [0, 1]$. Then*

$$\lambda C_1 + (1 - \lambda)C_1' \sim \lambda C_2 + (1 - \lambda)C_2'.$$

The property stated as Axiom 2 is a strong one, inducing some linearity into the preferences (which indeed is what comes out of the present characterization). On the other hand it seems no more restrictive than what is usually assumed when considering preferences over health states, where indifference between suitable lotteries are instrumental for assessing the values of the utility indices.

For the intuitive interpretation of QALY score as size of the set of available functionings, we would like to have the following monotonicity axiom:

AXIOM 3. *If $C_1 \subset \operatorname{int} C_2$, then $C_2 \succ C_1$.*

In its present form, this axiom can hardly be controversial, stating that if there are strictly less functionings available, then the resulting smaller capability set is less desired than the large one. We shall later have to consider modifications of this axiom which are perhaps less immediately acceptable.

AXIOM 4. *For each $C \in \mathcal{C}$, there exists $x \in C$ such that $\{x\} - \mathbb{R}^L_+ \in \mathcal{C}$ and $\{x\} - \mathbb{R}^L_+ \sim C$.*

This axiom can be recognized as a version of the celebrated IIA principle (Independence of Irrelevant Alternatives). If preferences over capabilities can be rationalized by a utility function, then the utility-maximizing element of the availability set (extended by free disposal to satisfy comprehensiveness) should be exactly as good as the larger choice set containing options that will not be chosen anyway. Thus, an IIA axiom of some type (and we shall consider another type of IIA axiom later) seems to be a necessary ingredient in any system of axioms for preferences on availability sets which can be rationalized by utility maximization.

For ease of notation in the sequel, we introduce the notation $x^<$ for $\{x\} - \mathbb{R}^L_+$. We note that if Axiom 3 holds, then the vector x of Axiom 4 must belong to the boundary of C.

THEOREM 1. *Let $\mathcal{C}$ be a regular family of subsets of $\mathbb{R}^L_+$, and let $\succsim$ be relation on $\mathcal{C}$. Then the following are equivalent:*

(i) $(\mathcal{C}, \succsim)$ satisfies Axioms 1 – 4,

(ii) there is an linear map $u : \mathbb{R}_+ \to \mathbb{R}$ such that

$$C \succsim C' \Leftrightarrow \max_{x \in C} u(x) \geq \max_{x \in C'} u(x).$$

PROOF: The proof of the implication (ii)⇒(i) is straightforward and left to the reader. Define the set $L^1 = \{x \in \mathbb{R}^L_+ \mid x^< \in \mathcal{C},\, x^< \succsim C',\ \text{all}\ C' \in \mathcal{C}\}$. By Axiom 1 there are maximal elements for $\succsim$ on $\mathcal{C}$, and by Axiom 4, we get that L^1 is nonempty.

Next, choose any $x^* \in L^1$ and define for each $\lambda \in [0,1]$ the set $L^\lambda = \{x \in \mathbb{R}^L_+ \mid x^< \sim (\lambda x^*)^<$. By axiom 2, each set L^λ is convex, and we have that $\lambda' \lambda^{-1} L^\lambda \subseteq L^{\lambda'}$ whenever $\lambda' \leq \lambda$. Letting $\hat{L}^1 = \{x \mid \lambda x \in L^\lambda \text{ for some } \lambda \in [0,1]\}$ we get that $\hat{L}^1$ is convex and that for each $x \in \hat{L}^1$, the sets $\{x' \mid x' < x\}$ and $\hat{L}^1$ are disjoint. Consequently, by separation of convex sets there is $c \in \mathbf{R}^L_+$, $c \neq 0$, such that $L^1 \subset \{x' \mid c \cdot x' = 1\}$. It follows that $L^\lambda \subset \{x' \mid c \cdot x' = \lambda\}$ for each $\lambda \in [0,1]$.

Define the map $u : \mathbb{R}^L_+ \to \mathbb{R}$ by $u(x) = c \cdot x$ for each x. We show that u satisfies the conditions in (ii). Let C be arbitrary, and assume that $C \sim x^<$ for some $x \in L^\lambda$. Then there is $y \in C$ with $y^< \sim x^<$, and since $u(y) = u(x) = \lambda$, we have that $\max_{z \in C} u(z) \geq \lambda$. Suppose that $\max_{z \in C} u(z) = \lambda' > \lambda$; then C contains some vector $z \in L^{\lambda'}$, meaning that $z' = \lambda(\lambda')^{-1} z$ must belong to L^λ. But since $(z')^<$ is contained in the interior of C by Axiom 3, we have a contradiction. □

It is easily seen from the proof of the theorem that the linear function u is uniquely determined (up to a positive multiple) in the case where $\dim L^\lambda = L - 1$ for some $\lambda \in [0,1]$.

Also, if we do not insist on u being linear, we still need that $u(x) = \lambda$ for all $x \in L^\lambda$, each λ, meaning that the level sets of u will have large flat segments, possibly of dimension $L - 1$.

Since the assumption that $\mathcal{C}$ contains a sufficient large supply of sets of the form $x^{\leq}$ may be difficult to justify in the applications at hand, we consider below another axiom system which is tailored for our purpose. As before, we assume $\mathcal{C}$ to be compact (in topology induced by the Hausdorff metric), convex and to contain $\{0\}$. In our present setup we assume moreover that $\mathcal{C}$ is closed under the operation of taking (arbitrary) unions followed by convexification. Thus, if $\mathcal{D} \subset \mathcal{C}$, then the set

$$\text{conv}^*(\cup\mathcal{D}) = \text{cl}\left\{x \,\middle|\, x = \sum_{i=1}^{r} \mu_i x_i,\ \mu_i \in [0,1], \sum_{i=1}^{r} \mu_i = 1,\ x_i \in C_i \in \mathcal{D},\ i = 1, \ldots, r\right\}$$

belongs to $\mathcal{C}$. For ease of reference, a family satisfying all these conditions shall be called *rich.*

For the following result, we need to modify some of the axioms stated above; clearly Axiom 4 must be replaced by another one, but also Axiom 3 must me sharpened slightly.

AXIOM 3'. *Let $C, C' \in \mathcal{C}$. If $C \subseteq C'$ then $C' \succsim C$; if $C \sim C'$, then the set $bd\, C' \cap C$ is convex.*

In the context of regular families of convex sets, we replace Axiom 4 above by the following axiom which exploits the new structure.

AXIOM 4'. *Let $\mathcal{C}'$ be a family of sets from $\mathcal{C}$ such that $C \sim C'$ for all $C, C' \in \mathcal{C}'$. Then* conv $(\cup\mathcal{C}') \sim C$ *for each $C \in \mathcal{C}'$.*

It is easily seen that Axiom 4' captures the independence of irrelevant alternatives which we aspect to be satisfied by an ordering of sets which can be rationalized as utility maximization, but in another direction; instead of finding equivalent smaller sets we are now provided with a method for constructing equivalent supersets.

THEOREM 2. *Let $\mathcal{C}$ be a rich family of closed, convex, and comprehensive subsets of $\mathbb{R}^L_+$, and let $\succsim$ be a complete and continuous preorder on $\mathcal{C}$. Then the following are equivalent:*

(i) $(\mathcal{C}, \succsim)$ satisfies Axioms 1,2,3', 4' and 5,

(ii) there is an affine function $u : \mathbb{R}^L_+ \to \mathbb{R}$ such that

$$C \succsim C' \Leftrightarrow \max_{x \in C} u(x) \geq \max_{x \in C'} u(x).$$

PROOF. As before we leave the implication (i)⇒(ii) to the reader. For the converse implication, we follow the steps in the proof of Theorem 1. Let $\mathcal{D}^1$ be the set of elements of $\mathcal{C}$ which are maximal for $\succsim$, and let $C^1 = \text{conv}^*(\cup\mathcal{L}^1)$. Then C^1 belongs to $\mathcal{D}^1$ by Axiom 4'. For each $\lambda \in [0,1]$, we define $\mathcal{L}^\lambda$ as the set of all C such that $C \sim \lambda C^1$, and let $C^\lambda = \text{conv}^*(\cup\mathcal{D}^\lambda)$; again $C^\lambda \in \mathcal{D}^\lambda$ by Axiom 4'. If for all λ, the set $\mathcal{D}^\lambda$ consists of the single element C^λ, then $C^\lambda = \lambda C^1$ for each λ. Choosing any $x \in \text{bd}\, C^1$ and any p which supports C^1 at x (i.e. $p \cdot c \leq p \cdot x$ for all $c \in C^1$, we have that the function u defined by $u(x) = p \cdot x$ satisfies the conditions stated in (ii).

If there is some λ such that $\mathcal{D}^\lambda$ contains more than one element, then we define the set

$$E = \{x \in \mathbb{R}^L_+ \mid \exists \lambda \in [0,1], C \in \mathcal{D}\lambda,\, C \neq C^\lambda : \lambda x \in \text{bd}\, C^\lambda \cap C\}.$$

We claim that E is convex. Indeed, let $x_1, x_2 \in E$ and let $\mu \in [0,1]$ be arbitrary. Then there is some λ (chosen small enough), together with sets $C_1, C_2 \in \mathcal{D}^\lambda$, $C_1, C_2 \neq C^\lambda$, such that $\lambda x_i \in \text{bd}\, C^\lambda \cap C_i$, $i = 1, 2$. It follows that $\lambda x_1, \lambda x_2 \in \text{bd}\, C^\lambda \cap \text{conv}\,(C_1 \cup C_2)$, and by Axiom 3', we have that

$$\mu\lambda x_1 + (1-\mu)\lambda x_2 \in \text{bd}\, C^\lambda \cap \text{conv}\,(C_1 \cup C_2)$$

for any $\mu \in [0,1]$. If $\text{conv}\,(C_1 \cup C_2) \neq C^\lambda$, then $\mu x_1 + (1-\mu)x_2 \in E$; if not, we repeat the argument above with C_1 replaced by $\nu C_1 + (1-\nu)C_2$ for ν small enough so that $\mu x_1 + (1-\mu)x_2$ can be written as a convex combination of $\nu x_1 + (1-\nu)x_2$ and x_2.

Next, consider the family of closed convex sets $(\lambda^{-1}C^\lambda)_{0<\lambda\leq 1}$. We have that $\lambda^{-1}C^\lambda \subseteq \hat{\lambda}^{-1}C^{\hat{\lambda}}$ for $\hat{\lambda} \leq \lambda$, so the set $F = \cup_{0<\lambda\leq 1}\lambda^{-1}C^\lambda$ is convex. Moreover, E does not intersect the interior of E, since in that case E would intersect the interior of C^λ for some $\lambda > 0$, a contradiction. It follows that E can be separated from the set $\text{int}\, F$, so that there is a linear form p such that $p \cdot x = 1$ for $x \in E$ and $p \cdot x \leq 1$ for $x \in F$.

Define u by $u(x) = p \cdot x$; we check that u has the desired properties by showing that $\max_{x \in C} = \lambda$ for $C \in \mathcal{D}^\lambda$, $\lambda \in [0,1]$. Thus, let $C \in \mathcal{D}^\lambda$. Then $\text{bd}\, C^\lambda \cap C \neq \emptyset$, since otherwise $C \prec C^\lambda$ by Axiom 3. Since $u(\hat{x}) = \lambda$ for $\hat{x} \in \text{bd}C^\lambda \cap C$, we have that $\max_{x \in C} u(x) \geq \lambda$. The fact that $\max_{x \in C} u(x) \geq \lambda$ follows from the separation property, since $u(x) \leq 1$ on F implies $u(x) \leq \lambda$ for $x \in C^\lambda$. □

We conclude this section with some considerations of the conditions for a family $\mathcal{C}$ to be rich. In the application that we have had in mind, sets $C \in \mathcal{C}$ arise as sets of feasible (characteristics) outputs in a technology with commodity bundles as inputs. Let $T \subset \mathbb{R}^l_+ \times \mathbb{R}^L_+$ be a convex set, interpreted as a production set for the household transforming commodity bundles $x \in \mathbb{R}^l_+$ to characteristics bundles $\xi \in \mathbb{R}^L_+$. Then any set of the form

$$T(x) = \{\xi \mid (x, \xi) \in T\}$$

would be a feasible availability set. The set $\{0\}$ will appear as $T(0)$ provided that T satisfies the standard assumption that no output is obtainable without input. The remaining properties of regular or rich families do not however follow if we restrict ourselves to sets of the type $T(x)$ or even sets $\cup_{x\in A}T(x)$ for suitable subsets A of $\mathbb{R}^l_+$, the standard example of A being bundles obtainable in a market from a given bundle of ressources. Thus, for our results above to make sense in this context we might have to assume that individuals can order also availability sets that do not arise in any natural way.

Needless to say, any rich family $\mathcal{C}$ may arise from *some* technology T, at least if we allow for infinite-dimensional commodity spaces; as the input bundle giving rise to C, we may then choose the support function of C, giving us a suitable subspace of C^0 as the commodity space; the operatios of averaging and of convex combinations carry over to support functions. Since this construction has only limited interest unless combined with some method of reduction to finite dimensions, we shall not pursue this matter any further.

3. Nonrationalizable capability indices and choices of allocation

In the previous section, we considered the case where the individual ordering of availability sets could be rationalized as utility maximization. The results showed that when this did happen, the utility function involved would typically be uniquely determined (up to an ordinal transformation).

In the following, we shall develop this line of thought further, with a special view to policies striving at equality in health. For this, we need a model with several individuals, and we need that they are exposed to different health conditions, which here are specified using the concepts of capabilities in the form introduced above. We assume in the sequel that the household technologies differ only according to health state, so we have a given family of technologies $T^s \subset \mathbb{R}^l_+ \times \mathbb{R}^L_+$, describing all the ways of transforming goods to characteristics which are open to persons in health condition s, for each $s \in S$.

We consider in the present section a simple model where agents receive commodity bundles to be transformed according to a known and common technology. If agent i inserts the bundle x_i into the common technology T, she obtains the option of choosing a characteristics bundle from $T(x_i)$. As in the previous section, we may consider orderings of the availability sets which either can or cannot be rationalized by utility maximization. In the following, we assume that agents may have different utility functions u_i on characteristics bundles $\xi_i \in \mathbb{R}^L_+$.

A *policy* in this model is a redistribution of initial commodity bundles, i.e. an array $(z_1,\dots,z_n) \in (\mathbb{R}^l)^n$ with $\sum_{i\in N} z_i = 0$. The policy is feasible if $\omega_i + z_i \in \mathbb{R}^l_+$ for each i. Alternatively, we may consider a policy as given by the array of final commodity bundles $x = (x_1,\dots,x_n) \in (\mathbb{R}^l_+)^n$; we shall use this way of describing policies in the present section.

We shall need some properties of the techonology T for the following result. We say that T satisfies monotonicity if $x_h > x'_h$ for $h = 1, \dots, l$ implies that $T(x') \subset \text{int}\, T(x)$. This proporty seems reasonable enough and will be fulfilled in standard situations. The next property is somewhat more restrictive and deals with situations where the input combinations are in some sense on the effective boundary – meaning that certain changes of input will give no improvement of capabilities. A vector $z \in \mathbb{R}^l$ is said to be a *feasible input change* at $x \in \mathbb{R}^l_+$ if $x + z \in \mathbb{R}^l_+$ and $T(x + z)$ is not contained in $T(x)$. The technology is said to be strictly convex if the set $H_T(x)$ of feasible input changes at x is convex, each $x \in \mathbb{R}^l_+$.

We shall be interested in allocations x giving rise to capabilities $(T(x_1), \dots, T(x_n))$ which are maximal for some preference relation $\mathcal{R}$ over capability profiles, considered to be an expression of the values that society assigns to equality (in health). We assume that society's preferences $\mathcal{R}$ over profiles of capabilities is derived from individual preference relations $\mathcal{R}_i$ over capabilities; let $\mathcal{P}_i$ be the associated strict preference. The individual preference relations $\mathcal{R}_i$ are assumed to be monotonic in the sense that if $C \subseteq C'$ for two capability sets, then $C' \; \mathcal{R}_i \; C$.

THEOREM 3. *Assume that the technology T satisfies monotonicity and strict convexity. Let $x^* = (x_1^*, \dots, x_n^*)$ be a commodity allocation with associated capability profile $(T(x_1^*), \dots, T(x_n^*))$. Then one of the following holds:*

(i) there is a utility profile $u = (u_1, \dots, u_n)$ and a commodity allocation $\hat{x} = (\hat{x}_1, \dots, \hat{x})$ such that if

$$\hat{\xi} = (\hat{\xi}_1, \dots, \hat{\xi}_n),\ \hat{\xi}_i \in \text{Argmax}_{\xi \in T(\hat{x}_i)} u_i(\xi),\ i \in N,$$
$$\xi^* = (\xi_1^*, \dots, \xi_n^*),\ \xi_i^* \in \text{Argmax}_{\xi \in T(x_i^*)} u_i(\xi),\ i \in N.$$

then $u_i(\hat{\xi}_i) > u_i(\xi_i^*)$ for each i,

(ii) there is a price vector $p \in \mathbf{R}^l_+$ such that $(x_1^*, \dots, x_n^*, p)$ is an equilibrium in the sense that if for each $i \in N$, if x'_i is such that $T(x'_i) \; \mathcal{P}_i \; T(x_i^*)$, then $p \cdot x'_i > p \cdot x_i^*$.

PROOF: For each $i \in N$, let $\delta : \mathbb{R}^L_+ \times \mathbb{R}^l_+ \to \mathbb{R}$ be the be defined by

$$\delta_i(\pi, x) = \text{max}_{\xi \in T(x)} \pi \cdot \xi;$$

thus, $\delta(\cdot, x)$ is the support function of $T(x)$. For each array $z = (z_1, \dots, z_n) \in (\mathbb{R}^l_+)^n)$ with $\sum_{i \in N} z_i = 0$ (that is, each redistribution of commodities) and for each $i \in N$,

$$\varepsilon_i(z_i) = \text{max}_{\pi \in \triangle_L} [\delta(\pi, x_i^* + z_i) - \delta(\pi, x_i^*)].$$

Suppose that there is a redistribution $\bar{z}$ such that $\varepsilon_i(\bar{z}_i) \geq 0$ for all i and $\varepsilon_{i^0}(\bar{z}_{i^0}) > 0$ for some $i^0 \in N$. For each i, let π_i be such that

$$\delta_i(\pi_i, x_i^*) = \text{max}_{\pi \in \triangle_L} \delta(\pi, x_i^* + \bar{z}_i)$$

and define utility functions u_i by $u_i(\xi_i) = \pi_i \cdot \xi_i$. Then we have for the utility profile $(u_1, \dots, u_n)$ that the characteristics allocation $(\bar{\xi}_1, \dots, \bar{\xi}_n)$, where

$$\bar{\xi} \in \text{Argmax}_{\xi \in T(x_i^* + \bar{z}_i)} u_i(\xi), \text{ all } i \in N,$$

Pareto dominates the characteristics allocation ξ^*. Using monotonicity of T we then have that by transferring a slight amount of commodities from i_0 to the other individuals we may obtain that every individual becomes better off, giving an allocation ξ such that $u_i(\xi_i) > u_i(xi^*)$ for all i.

Assume now that for each $z = (z_1, \dots, z_n)$ with $\sum_{i=1}^n z_i = 0$, there is $i \in N$ such that $\varepsilon_i(x_i^* + z_i) \leq 0$ for all $i \in N$. Then we must have that if $y = \sum_{i \in N} h_i$ with $h_i \in H(x_i^*)$, each i, then $y \notin \mathbb{R}^l_-$, or

$$\Big[\sum_{i \in N} H(x_i^*)\Big] \cap \mathbb{R}^l_-.$$

By separation of convex sets, there is a positive linear form p on $\mathbb{R}^l_+$ such that $p \cdot h > 0$ for $h \in \sum_{i \in N} H(x_i^*)$; by convexity of T, each $H(x_i^*)$ contains elements arbitrarily close to 0, so we conclude that $p \cdot h_i > 0$ for all $h_i \in H(x_i^*)$, $i \in N$.

We check that $(x_1^*, \dots, x_n^*, p)$ is an equilibrium; indeed, if $i \in N$ and x_i' is such that $T(x_i') \; \mathcal{R}_i \; T(x_i^*)$, then $x_i' - x_i^*$ must belong to $H(x_i^*)$ (since otherwise $T(x_i') \subset T(x_i^*)$ contradicting monotonicity of $\mathcal{R}_i$); consequently $p \cdot (x_i' - x_i^*) > 0$ or $p \cdot x_i' > p \cdot x_i^*$ which shows that the equilibrium condition is satisfied. □

In the formulation of Theorem 3, the allocation $(x_1^*, \dots, x_n^*)$ was chosen arbitrarily. In particular, it may be such that the associated capability profile is maximal for society's preference relation $\mathcal{R}$ over all profiles of the type $(T(x_1), \dots, T(x_n))$. For this particular choice of commodity allocation, the theorem tells us that either the resulting characteristics allocation is potentially inefficient, in the sense that there are utility assignments such that every individual could obtain something better after redistribution of the commodity endowment, or the $\mathcal{R}$-maximal capability profile is sustained by a price equilibrium in the commodity market, so that no individual can obtain an $\mathcal{R}_i$-better capability set by trading commodities at the market price.

As the reader will have by now realized, Theorem 3 is actually a somewhat unusual version of the second fundamental theorem of welfare economics; we actually need stronger assumptions (in particular, our assumption of strict convexity) than what is usual, since we let the utility functions of the individuals be part of the problem rather than being given at the start. The situation described in part (ii) is so special that it can hardly be expected to obtain unless the technology has a very special form (for example if all capability sets are blown-up versions of the same set), meaning that case (i) would be the rule and (ii) the exception. In our interpretation, this enforces our results from the previous section about rationalizing preorders on families of capability sets by utility maximization. In most cases, such a rationalization is beyond reach.

Since our strict convexity assumption is somewhat dubious, we present as a conclusion of this section a version of Theorem 3 which can be proved without this assumption. For this, need to consider economies with so may agents that a set of l agents (where l is the number of commodities) may be considered as a small or exceptional set.

COROLLARY. *Assume that the technology T satisfies monotonicity and strict convexity. Let $x^* = (x_1^*, \ldots, x_n^*)$ be a commodity allocation with associated capability profile $(T(x_1^*), \ldots, T(x_n^*))$. Then one of the following holds:*

(i) there is a utility profile $u = (u_1, \ldots, u_n)$ and a commodity allocation $\hat{x} = (\hat{x}_1, \ldots, \hat{x})$ such that if

$$\hat{\xi} = (\hat{\xi}_1, \ldots, \hat{\xi}_n),\ \hat{\xi}_i \in \text{Argmax}_{\xi \in T(\hat{x}_i)} u_i(\xi),\ i \in N,$$
$$\xi^* = (\xi_1^*, \ldots, \xi_n^*),\ \xi_i^* \in \text{Argmax}_{\xi \in T(x_i^*)} u_i(\xi),\ i \in N.$$

then $u_i(\hat{\xi}_i) > u_i(\xi_i^*)$ for each i,

(ii) there is a price vector $p \in \mathbf{R}_+^l$ and for each $\delta > 0$ an exceptional subset N_δ of N (depending only on δ) such that $(x_1^*, \ldots, x_n^*, p)$ is an δ-approximate equilibrium in the sense that if for each $i \in N \backslash N_0$, if x_i' is such that $T(x_i')\, \mathcal{P}_i\, T(x_i^*)$, and $\max_h |x_{ih}' - x_{ih}^*| \leq 1$, then $p \cdot x_i' > p \cdot x_i^* - \delta$.

PROOF: In the proof of Theorem 3, replace for each $i \in N$ $H(x_i^*)$ by

$$\hat{H}(x_i^*) = \text{conv}(H(x_i^*)) \cap \{z \in \mathbb{R}^l \mid |z_h| \leq 1,\ h = 1, \ldots, l\}.$$

If alternative (i) does not hold, then

$$\Big[\sum_{i \in N} \hat{H}(x_i^*)\Big] \cap \{-le\} - \mathbb{R}_+^l = \emptyset,$$

where $e = (1, \ldots, 1)$ is the unit diagonal vector in $\mathbb{R}^l$. Indeed, if $u \in \{-lKe\} - \mathbb{R}_+^l$ belongs to $\sum_{i \in N} \hat{H}(x_i^*)$, then by the Shapley-Folkman theorem (see e.g. Hildenbrand (1974)), u has a representation $u = \sum_{i \in N} u_i$, where $u_i \in \hat{H}(x_i^*)$ for all i and $u_i \in H(x_i^*) \cap \{z \mid |z_h| \leq 1,\ \text{all } h\}$ for all $i \in N \backslash N_0$ where N_0 is a set of cardinality at most l. It follows that $\sum_{i \in N \backslash N_0} u_i \in \mathbb{R}_-^l$, a contradiction.

By separation of convex sets, there is $p \in \mathbb{R}_+^l$, $\sum_{h=1}^l p_h = 1$, such that

$$p \cdot z \geq p \cdot le = l,\ z \in \sum_{i \in N} \hat{H}(x_i^*).$$

It follows that for given $\delta > 0$, the cardinality of the set N_δ such that $\min_{z \in H(x_i^*)} p \cdot z \leq \delta$ cannot exceed $\lfloor \delta^{-1} \rfloor + 1$ (where $\lfloor r \rfloor$ is the integer part of the real number r). This proves the statement in (ii). $\square$

4. Concluding remarks

In this work, we have been concerned with the foundations of the capability approach and its application to problems of equality, with special regard to equality in health. The basic question posed was whether the capability approach is compatible with the traditional view of the individual as utility maximizer, and we have found that the such a compatibility of approaches, at least in its simplest possible form, is not to be expected.

Looking at the single individual and considering the capability, considered as a set of functioning, as an expression of the individual to cope with her situation and thus as a formalization of the health-related quality of life, we found that an assignment of indiced so capabilities would be compatible with utility maximization only for a small family of utility functions – in the generic case a uniquely determined utility function. Classical economics would typically reject a theory which holds only for one particular utility function, meaning that preferences over capabilities must be founded on something which is different from preferences over final outcomes; even if it is assumed, as in the widely accepted QALY approach, that all individuals have identical orderings of health states, we still need to explain why these preferences should be connected intimately with the technological conditions for attaining these states.

In the context of preferences over capability profiles in a society, we saw that these preferences can be explained by utility maximization only in very special cases, namely such cases where the socially optimal allocation could be achieved as an equilibrium in the market, that is by assigning suitable incomes to individuals and allowing them to buy commodities at given prices. Thus, we have that either the social ordering of capability profiles cannot be explained by utility maximization, or the social optima considered are of a very simple type.

It should be stressed that the results obtained are not to be considered as in any way reducing the potential usefulness of the capability approach. Rather, they should be seen as an additional argument for intensifying research in the nature of capabilities; since preferences on capabilities cannot be trivially deduced from standard utility maximization, we need an explanation of how preferences on sets of functionings are formed and whether they can at all be reduced to simpler structures.

Thus, the main insights to be gained from the present analysis is that a satisfactory theory of capabilities must go beyond the traditional approaches; capabilities must involve something not captured by the idea of choosing from a given family of objects according to a known method of evaluation. In particular, time and uncertainty both of which were spectacularly absent in this study, should enter in a meaningful way. This is a matter of future research.

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