Discussion Papers
Department of Economics
University of Copenhagen

No. 07-03

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ISSN: 1601-2461 (online)

Malthus Revisited: Fertility Decision Making based on Quasi-Linear Preferences*

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January 22, 2007

Abstract

Malthus' (1798) population hypothesis is inconsistent with the demographic transition and the concurrent massive expansion of incomes observed among industrialised countries. This study shows that eliminating the income-effect on the demand for children from the Malthusian model makes it harmonise well with industrial development.

JEL classification codes: J13, N30, O10
Keywords: Demographic Transition, Fertility, Malthus

*I gratefully acknowledge the feedback from seminar participants at University of Copenhagen and at the First Summer School of the Marie Curie Research Training Network 'Unifying the European Experience,' especially Tommy E. Murphy. *Contact*: `jacob.weisdorf@econ.ku.dk`.

1 Introduction

Malthus, in his *Essay on the Principle of Population*, identifies England as a *preventive check* society, meaning a society in which fertility respond to changes in economic conditions (Malthus, 1798). If income levels decrease and the price of provisions rises, then it becomes harder to rear a family; this, Malthus argues, results in fewer marriages, leading ultimately to lower birth rates.

In England, a u-shaped relationship between income and birth rates has been observed (Figure 1). Ironically, the initial positive relationship between income and birth rates, for which Malthus is a spokesman, breaks down shortly after the *Principle* is published. Subsequently, higher incomes accompany lower birth rates, an episode today recognised as the *demographic transition*.

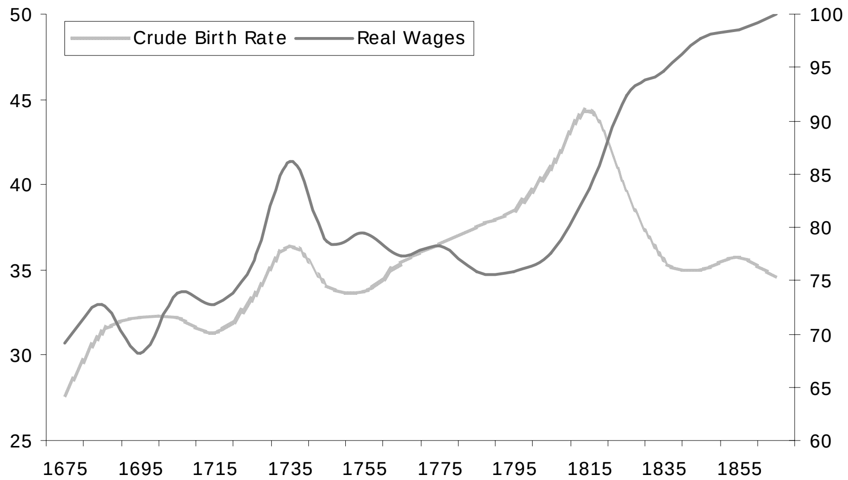

Figure 1
Crude Birth Rates and Real Wages, England 1675-1865
Sources: Clark (2001); Wrigley and Schofield (1981)

An incomplete list of studies that investigate the long-run relationship between income and birth rates from a theoretical viewpoint includes Boucekkine et al. (2002), Galor and Weil (2000), Lucas (2002), Strulik (2003) and Tamura (1996). The common story told is that human capital accumulation is key: on the one hand, it leads to income increase; on the other, it induces parents to trade off child-quantity for child-quality.

The current study differs from the related literature from two perspectives: First, human capital accumulation is not required to generate a demographic transition. Second, the present analysis is kept entirely within the boundaries of a Malthusian economy. Using a simply analytical framework, the current study shows that eliminating the income-effect on the demand for children from the Malthusian model makes it well in tune with industrial development.

More specifically, using so-called *quasi-linear* (‘*zero-income effect*’) preferences, the (indirect) effect on birth rates of income changes (which enter through the price of provisions) turns out to be sector-dependent: agricultural sector income growth leads to higher birth rates, industrial sector income growth to lower birth rates. According to the theory presented below, therefore, England’s u-shaped relationship between income and birth rates was caused by initial acceleration in agricultural productivity (an ‘agricultural revolution’), succeeded by acceleration in industrial productivity (an ‘industrial revolution’).

2 The Model

Consider a two-sector, two-goods, closed economy. An agricultural sector produces foods, an industrial sector produces manufactured goods. The labour force is divided endogenously between the two sectors. Unless explicitly stated, all variables are considered in period t.

Output Food production is subject to constant returns to land and labour.[1] Land, measured by X, is fixed and set to unity. Total food output is thus

$$Y_A = \Omega_A L_A^{\alpha} X^{1-\alpha} \equiv \Omega_A L_A^{\alpha}, \quad \alpha \in (0,1)\,, \; X \equiv 1. \tag{1}$$

where Ω_A measures total factor productivity in agriculture and L_A is the number of farmers. The net rate of agricultural productivity growth between any two periods is denoted γ_A. Without property rights over land, the land rent is zero,[2] and a farmer's income is

$$w_A = \frac{Y_A}{L_A} = \frac{\Omega_A}{L_A^{1-\alpha}}. \tag{2}$$

Turning to industrial production, this is subject to constant returns to labour. Total industrial output is thus $Y_M = \Omega_M L_M$, where Ω_M measures industrial labour productivity (M for manufacturing) and L_M the number of manufacturers. The net rate of growth of industrial productivity between any two periods is denoted γ_M, and a manufacturer's income is

$$w_M = \frac{Y_M}{L_M} = \Omega_M. \tag{3}$$

Labour Force Dynamics Consider an overlapping-generations economy where people live for up to two periods: childhood and possibly adulthood. Income-generating activities take place only during adulthood. Development in the size of the adult population, i.e., the labour force, is affected by birth rates and death rates. That is, at the end of each period, the adult generation dies out and is replaced by a new generation consisting of children surviving childhood. The number of surviving children per adult equals the adult's (endogenous) birth rate b multiplied by the survival probability. With $d \in (0,1)$ denoting the (exogenous) risk of dying before adulthood, $1-d$ measures survival probability. With identical agents, the number of surviving children per adult is $n = (1-d)b$. Evolution in the size of the labour force from one period to the next is therefore $L_{t+1} = n_t L_t = (1-d)b_t L_t$, and the net rate of growth of the labour force between any two periods is thus

$$\gamma_L \equiv (1-d)\, b_t - 1. \tag{4}$$

Labour Market Equilibrium The total labour force L consists of farmers and manufacturers, i.e., $L = L_A + L_M$. Let $p \equiv p_A/p_M$ denote the relative price of foods and manufactured goods. Labour market equilibrium then requires that p adjusts, so that

$$p w_A = w_M. \tag{5}$$

[1] Throughout, we suppress the use of capital in production, an assumption commonly used in the related literature (see, e.g., Galor and Weil, 2000).

[2] In the related literature, this is also not an unusual assumption (e.g., Galor and Weil, 2000).

Food Market Equilibrium Suppose that, over the course of a lifetime, an individual consumes a fixed quantity of foods (or calories) measured by η. For simplicity, food is demanded only during childhood and some of it stored for adulthood.[3] Setting $\eta \equiv 1$, total food demand equals total food supply, given by (1), when $bL = \Omega_A L_A^{\alpha}$.

Preferences People, in a Darwinian sense, derive utility from the number of surviving offspring n and from the number of manufactured goods consumed m. The utility function of a representative adult is of a *quasi-linear* type whereby $u(n, m) = \ln n + m$. This implies that the price-effect on the demand for children is negative whereas the income-effect is zero (see further below).

The Budget Constraint Historically, the costs of bringing up children are largely equal to the costs of the foods that they consume.[4] The total costs of bringing up b children, measured in terms of manufactured goods, are thus pb. The budget constraint of a representative adult is therefore $w = pb + m$, where, following (5), $w \equiv pw_A = w_M$.

Utility Maximisation The solution to the utility maximisation problem: $\max_b u(n, m)$ subject to $w = pb + m$, is that an adult's demand for children is $b^* = 1/p$ and the demand for manufactured goods $m^* = w - 1$.

General Equilibrium Conditions Below, we first explore the static general equilibrium conditions. Then, we study the general equilibrium conditions along a balanced growth path.

Combining the food and labour market equilibrium conditions with the solution to the utility maximisation problem and the wage rates, the static general equilibrium birth rate becomes

$$b^{SGE} = \frac{\Omega_A}{\Omega_M^{\alpha} L^{1-\alpha}} \tag{6}$$

while the manufactured goods consumption level becomes

$$m^{SGE} = \Omega_M - 1. \tag{7}$$

Equation (6) says that birth rates (i) increase with agricultural productivity growth, but decrease (ii) with growth in the size of the labour force and (iii) with industrial productivity growth. Unlike the third effect, the first and the second effects are well-know elements in the traditional Malthusian framework: Starting from a constant population, an upward shift in agricultural productivity raises agricultural sector income. Through its negative effect on the price of provisions, this leads to higher birth rates and therefore to population growth. Due to diminishing returns to labour in agriculture, population growth causes agricultural income reduction. This reduces birth rates until population growth eventually peters out.

The third effect, however, which appears as a result of the quasi-linear preference function, provides a new facet to the Malthusian framework: Starting from a constant population, an upward shift in industrial productivity raises industrial sector income. Though its positive effect on the

[3] It will not affect the qualitative nature of the results, if, instead, individual food demand is divided over two periods. Such a construction, however, severely complicates matters.

[4] A construction whereby children, in addition to foods, require a certain amount of parents' time, or a given number of manufactured goods, will not change the quantitative nature of the results but severely complicates the analysis.

price of provisions, this causes birth rates to decline and therefore leads to negative population growth. Via diminishing returns in agriculture, a smaller population fosters higher agricultural income, which increases birth rates until the population growth rate is back to zero. As will become apparent below, the third effect plays a key role for the demographic transition.

Turning to the second static general equilibrium condition, equation (7), this shows that the number of manufactured goods produced and consumed depends solely on the level of industrial sector productivity. Unlike the traditional Malthusian model, consumption levels per capita in this modified population model are subject to sustained growth, restricted only by industrial productivity growth.

Dynamics and Stability To see how the static equilibriums evolve over time, we now characterize their steady-state balanced growth paths. Then, we explore the dynamics and stability properties of the two paths.

A balanced growth path is a path along which all variables grow at constant geometric rates (possibly zero). The left-hand-side of (6) is constant by definition, so the right-hand-side must also be constant. Using the growth rates of productivity and labour defined above, the equilibrium birth rate remains fixed over time when $\gamma_A = \alpha\gamma_M + (1-\alpha)\gamma_L$. Inserting (4), the steady state birth rate therefore becomes

$$b^{st.st.} = \frac{1}{1-d}\left(1 + \frac{\gamma_A - \alpha\gamma_M}{1-\alpha}\right).$$

Abstracting from variation in the (exogenous) death risk, changes in the steady state birth rate will be caused by a shift in the ratio of agricultural to industrial productivity growth. Defining $\gamma \equiv \gamma_A/\gamma_M$, an increase in γ increases the birth rate, and vice versa. Note that the effect of income variations on γ is sector-dependent: agricultural sector income growth leads to an increase in γ, whereas industrial sector income growth leads to a reduction in γ. Accordingly, controlling for changes in the death rate, England's u-shaped relationship between income and birth rates (Figure 1) would have been caused by initial acceleration in agricultural productivity growth (γ up), succeeded by acceleration in industrial productivity growth (γ down). In other words, as has also been observed in England (e.g., Overton, 1996), an 'agricultural revolution' is followed by an 'industrial revolution.'

Finally, the steady state rate of growth of manufactured goods consumption is $\gamma_m = \gamma_M$, where γ_M is the net rate of growth of industrial productivity and γ_m denotes the net rate of growth of manufactured goods consumption (or economic growth) between any two periods. Evidently, sustained economic growth is perfectly consistent with the current version of the Malthusian model.

3 Testable Implications

Testing the current theory means testing if ratio of agricultural to industrial productivity growth, i.e., γ, moves in the expected directions, keeping in mind that the theory considers a closed economy, and that, during industrialisation, countries typically become open up to foreign trade.

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