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Renewable Resources**

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# A New-Growth Perspective on Non-renewable Resources

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## Abstract

This article reviews issues related to the incorporation of non-renewable resources in the theory of economic growth and development. As an offshoot of the new growth theory of the last two decades a series of contributions have studied endogenous technical change in relation to resource scarcity. We discuss the main approaches within this literature and consider questions like: How is the new literature related to the wave of resource economics of the 1970s? What light is thrown on the limits-to-growth issue? Does the existence of non-renewable resources have implications for the controversies within new growth theory?

*Keywords:* Endogenous growth; innovation; non-renewable resources; knife-edge conditions; robustness; limits to growth.

*JEL Classification:* O4, Q3.

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# 1 Introduction

The aim of this article is to review issues related to the incorporation of scarce natural resources in the theory of economic growth and development. More specifically, we shall concentrate on the role of non-renewable resources. A *non-renewable resource* is a natural resource the amount of which on earth is finite and which has no natural regeneration process (at least not within a relevant time scale). Hence, the stock of a non-renewable resource is depletable. Fossil fuels and many non-energy minerals are examples. A *renewable resource* is also available only in limited supply, but its stock is replenished by a natural regeneration process. Hence, if the stock of a renewable resource is not over-exploited, it can be sustained in a more or less constant amount. Fertile soil, fish in the sea and environmental qualities (clean air etc.) would be examples. In this article the focus is on the specific features of *non-renewable* resources in relation to the feasibility of sustained economic growth.

The old Malthusian and Ricardian views were that scarce natural resources tend to cause diminishing returns to inputs of capital and labour taken together and thereby economic stagnation in the long run. Malthus and Ricardo had primarily land in mind. But what if also non-renewable, hence exhaustible, resources are essential inputs in production? Then the long-run prospect may be worse than stagnation according to the dire predictions of the Club of Rome set forth in the “Limits to growth” report by Meadows et al. (1972).<sup>1</sup> The world-wide oil crisis of the mid 1970s fuelled the interest in this topic.<sup>2</sup> Prominent economists like Solow (1974a, 1974b), Stiglitz (1974a, 1974b), Dasgupta and Heal (1974) and others took these challenges as an occasion for in-depth studies of the macroeconomics of non-renewable resources, including the big questions about sustainable development, defined as non-decreasing standard of living, or even sustained economic growth. Many issues were clarified, but since the big questions were essentially embedded in a framework with exogenous future technology (hence, unforeseeable), definitive answers could not be given. Although growth has not been hindered by resource shortages in the past, it is another thing whether this can continue in the future.

Beginning with the contributions by Paul Romer (1986, 1987, 1990) and Robert Lucas (1988) there has been, since the late 1980s, a surge of so-called new growth theory or endogenous growth theory. Characteristic traits of this theoretical development are: 1)

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<sup>1</sup>With a follow-up in Meadows et al. (1992).

<sup>2</sup>There are signs that the current renewed rise in oil prices may have a similar effect.

the focus on conditions that allow endogenous sustained productivity growth; and 2) the systematic incorporation of “ideas” (with their distinctive properties compared with other economic goods) into dynamic general equilibrium models with imperfect competition. In particular there have been great advances in the understanding of technological change. In this article we shall therefore ask:

What light does new growth theory throw on the limits-to-growth question?

Since there have been several controversies (for example about scale effects of different kinds or non-robustness due to knife-edge assumptions) within new growth theory, we add the additional question:

Does the existence of non-renewable resources have anything to say in relation to the controversies within new growth theory?

It turns out that a key distinction (which has not always received the requisite attention) is that between models where essential non-renewable resources are growth-essential and models where they are not. A non-renewable resource is called *growth-essential* if it is a necessary input to the growth-generating sector(s), the “growth engine”, in the economy. It can be so either directly or indirectly by being essential for the manufacturing sector which then delivers necessary input to the “growth engine”, usually an R&D or educational sector. Indeed, we shall see that whether non-renewable resources are growth-essential or not has non-trivial implications for the limits-to-growth question.

The remainder of the article discusses these issues within a unified framework. The next section gives an overview of new growth theory. Section 3 portrays the wave of natural resource economics of the 1970s. In Section 4 a simple one-sector growth model with endogenous technical change is introduced. Section 5 considers different approaches to two-sector models with non-renewable resources and endogenous technical change. The analysis lays bare the key role of the distinction between resources that are growth-essential and resources that are not. Section 6 debates the implications and briefly comments on other research directions, whereas Section 7 summarises.<sup>3</sup>

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<sup>3</sup>The focus on *endogenous* technical change in a world with essential non-renewable resources differentiates this brief (and selective) review from other reviews of the economics of non-renewable resources (Solow 1974b, Dixit 1976, Dasgupta and Heal 1979, Withagen 1990, Heal 1998 and Krautkraemer 1998).

## 2 New growth theory

Before considering the integration of non-renewable resources into new growth theory, let us recapitulate the key ingredients of new growth theory as such. The surge of new growth theory or endogenous growth theory began with Romer (1986, 1987, 1990) and Lucas (1988). The term *endogenous growth* refers to models where sustained positive growth in output per capita is driven by some internal mechanism (in contrast to exogenous technology growth).<sup>4</sup>

It is common to divide the endogenous growth literature into two broad classes: accumulation-based models and innovation-based models. The first class of models is based on the idea that the combination of physical and human capital accumulation may be enough to sustain long-run productivity growth. These contributions include the human capital model by Lucas (1988) and the “AK model” by Rebelo (1991). The second class of models, which is more central to our theme here, attempts to explain how technological change comes about and how it shapes economic growth. Technological progress is seen as evolving from purposeful decisions by firms in search for monopoly profits on innovations. An important ingredient in this approach is therefore an attempt at incorporating other market structures than perfect competition into a macroeconomic framework.

Within the class of innovation-based growth models we shall make a distinction between “first-generation” models and “second-generation” models. The first-generation models concentrated on *either* horizontal *or* vertical innovations. The second-generation models integrated these two one-sided lines of attack.

### 2.1 First-generation models

The first-generation innovation-based growth models have their origin in Romer (1987, 1990), where growth is driven by specialisation and increasing division of labour. That is, the focus is on *horizontal innovations*: the invention of new intermediate or final goods gives rise to new branches of trade. The invention of micro-processors is an example. Shortly after the Romer papers came out, Grossman and Helpman (1991, Chapter 4) and Aghion and Howitt (1992) proposed theories in which growth is driven by *vertical*

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<sup>4</sup>Whenever the term “growth” is used in this article, per capita growth is meant. For enlightening textbooks on new growth theory the reader is referred to Barro and Sala-i-Martin (2. ed., 2004), Aghion and Howitt (1998) and, at a more elementary level, Jones (2002b). It should also be mentioned that new growth theory has important forerunners such as Nordhaus (1969) and Shell (1973).

*innovations*. This strand of endogenous growth theory concentrates on the invention of better qualities of existing products and better production methods that make previous qualities and methods obsolete; improvement in the performance of microprocessors provides an example. The two kinds of models are often called *increasing variety* models versus *increasing quality* models (or *quality ladder* models), respectively.

For both kinds of models the typical set-up is a two-sector framework. There is a manufacturing sector whose output is used for consumption as well as investment in capital of different varieties *or* new qualities (making the previous quality obsolete). The other sector is the “innovative sector”. In this sector two activities take place. Firstly, there is R&D activity leading to new capital-good varieties or new capital-good qualities. Secondly, once the technical design (blueprint) of a new variety or quality has been invented, the inventor starts supplying capital goods in the new form, protected by a patent or some kind of secrecy. The key feature behind the generation of sustained per capita growth in both the increasing variety models and the increasing quality models is the assumption of non-diminishing returns to the producible direct or indirect input(s) in the *growth-engine*, i.e., the sector or sectors that “drive growth”.<sup>5</sup> Usually the models are structured such that the innovative sector only uses (non-producible) labour as a direct input and therefore, by itself, constitutes the growth-engine. But the productivity of this labour input depends positively on society’s accumulated technical knowledge, hence this stock of knowledge can be seen as a produced indirect input.<sup>6</sup> Then non-diminishing returns to knowledge are needed to generate positive per capita growth. In practice exactly constant returns to knowledge (at least asymptotically) are assumed. This is because with increasing returns, growth would explode (see below).

Adding a description of the market structure and households’ preferences, the model can be solved. When certain parameter restrictions are satisfied two kinds of results stand out:

- Growth is *fully endogenous*<sup>7</sup> in the sense that the long-run growth rate in output per capita is positive without the support of growth in any exogenous factor; the key to this is the assumption of constant returns to the producible input(s) in the

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<sup>5</sup>Formally, the growth engine of an endogenous growth model is defined as the set of input-producing sectors or activities using their own output as an input.

<sup>6</sup>It is true that patents, concealment etc. can for a while exclude other firms from the commercial use of a specific innovation. Yet the general engineering principles behind the innovation are likely to diffuse rather quickly and add to the stock of common technical knowledge in society.

<sup>7</sup>Synonymous with this is the sometimes used term *strictly endogenous growth*.

growth engine.

- Via influencing incentives, *policy can affect growth* not only temporarily (i.e., during the transition to a new steady growth path), but also permanently (by affecting the slope of the steady growth path). This is in contrast to the traditional neoclassical growth models, like the Solow model or the Ramsey model, where economic policy (e.g., an investment subsidy) can have only a *level effect* in the long run.

An unwelcome implication of the models is the *scale effect on growth*. Indeed, the models imply the counterfactual predictions: (a) the larger the population is, *ceteris paribus*, the higher is the long-run per capita growth rate; and (b) sustained growth in population should be associated with a forever rising per capita growth rate. In fact, because of this scale effect the first-generation models simply ignore population growth and assume a constant labour force.

The scale effect is linked to the fact that technical knowledge, by which we mean a set of instructions or recipes about how to combine various inputs to obtain a specific output, is very different from ordinary economic goods in that it is a *non-rival* good. The use of knowledge by one agent does not in itself limit the simultaneous use of the same piece of knowledge by another agent or by many people. In this respect knowledge is dissimilar to human capital, which is embodied in an individual and therefore a rival good. The non-rival character of knowledge implies that output per capita depends on the *total* stock of ideas, not on the stock per person. A larger population breeds more ideas, leading to higher productivity. In the fully-endogenous growth models, due to the (knife-edge) assumption of constant returns to knowledge, this takes the extreme form of a scale effect not just on the level of output per capita, but on its growth rate.

The fact that technical knowledge is a non-rival good and only partially excludable (by patents, concealment etc.) makes it a very peculiar good which gives rise to market failures of many kinds. Thus, government intervention becomes an important ingredient in new growth theory.

## 2.2 The Jones critique and semi-endogenous growth

In two important papers, Charles Jones (1995a, 1995b) raised serious concerns about the predictions that not only levels, but also the long-run growth rate, are affected by economic policy and by scale. Jones claimed that: 1) both predictions are rejected by

time-series evidence for the industrialised world; 2) both predictions are theoretically non-robust (i.e., they are very sensitive to small changes in parameter values).

The empirical point is supported by, e.g., Evans (1996) and Romero-Avila (2006), although challenged by Li (2002b). As to the theoretical point, let us take Romer’s increasing variety model as an example.<sup>8</sup> Consider the aggregate invention production function:

$$\dot{A}(t) \equiv \frac{dA(t)}{dt} = \mu A(t)^\varphi L_A(t), \quad \mu > 0, \varphi \leq 1, \quad (1)$$

where  $A(t)$  is the number of existing different capital-good varieties at time  $t$  and  $L_A(t)$  is research labour, which leads to the invention of new capital-good varieties. The productivity of research labour depends, for  $\varphi \neq 0$ , on the stock of existing knowledge, which is assumed proportional to  $A(t)$ . The productivity of labour in manufacturing is similarly assumed proportional to  $A(t)$  so that manufacturing output is  $Y(t) = F(K(t), A(t)L_Y(t))$ , where  $K(t)$  and  $L_Y(t)$  are inputs of physical capital and labour, respectively, and the production function  $F$  is homogeneous of degree one. So far Romer and Jones agree. Their disagreement concerns the likely size of the parameter  $\varphi$ , i.e., the elasticity of research productivity with respect to the level of technical knowledge. In the Romer model, this parameter is (arbitrarily) made equal to one. It may be argued, however, that  $\varphi$  could easily be negative (the “fishing out” case, “the easiest ideas are found first”). Even if one assumes  $\varphi > 0$  (i.e., the case where the subsequent steps in knowledge accumulation requires less and less research labour), there is neither theoretical nor empirical reason to expect  $\varphi = 1$ . The standard “replication argument” for constant returns with respect to the complete set of *rival* inputs is not usable. Even worse,  $\varphi = 1$  is a *knife-edge* case. If  $\varphi$  is slightly above 1, then explosive growth arises - and does so in a very dramatic sense: infinite output in finite time. This simple mathematical point is made in Solow (1994). In the numerical example he calculates, the Big Bang - the end of scarcity - is only 200 years ahead! This seems too good to be true.<sup>9</sup>

On the other hand, with  $\varphi$  slightly less than 1, productivity growth peters out, unless assisted by growth in some exogenous factor, say population. To see this, let population (= labour force) be  $L(t) = L_Y(t) + L_A(t) = L_0 e^{nt}$ , where  $n \geq 0$  is a constant. For any positive variable  $x$ , let  $g_x \equiv \dot{x}/x$  (the growth rate of  $x$ ). Then, deriving from (1) an

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<sup>8</sup>An analogue argument goes through for the vertical innovations models.

<sup>9</sup>This knife-edge critique is equally relevant for the accumulation-based endogenous growth models (e.g., Lucas 1988 and Rebelo 1991), since they rely on a knife-edge condition similar to  $\varphi = 1$ .



expression for  $\dot{g}_A/g_A$ , we find that in a steady state (i.e., when  $\dot{g}_A = \dot{g}_K = \dot{g}_Y = 0$ ),

$$g_A = \frac{n}{1 - \varphi} = g_y, \quad (2)$$

where  $y$  is output per capita ( $\equiv Y/L$ ).<sup>10</sup> There are a number of observations to be made on this result. First, the unwelcome scale effect on growth has disappeared. Second, as indicated by (1), a positive scale effect on the *level* of  $y$  remains. This is also what we should expect. In view of the non-rival character of knowledge, the per capita cost of creating new knowledge is lower in a larger (closed) society than in a smaller one.<sup>11</sup> Empirically, the “very-long run” history of population and per capita income of different regions of the world gives evidence in favour of scale effects on levels (Kremer 1993). Econometric evidence is provided by, e.g., Alcalá and Ciccone (2004). Third, scale effects on levels also explain why the rate of productivity growth should be an increasing function of the rate of population growth, as implied by (2). In view of cross-border technology diffusion, this trait should not be seen as a prediction about individual countries in an internationalised world, but rather as pertaining to larger regions, perhaps the global economy. Finally, unless policy can affect  $\varphi$  or  $n$ ,<sup>12</sup> long-run growth is independent of policy, as in the old neoclassical story. Of course, “independence of policy” should not be interpreted as excluding that the general social, political and legal environments can be *barriers* to growth or that, via influencing incentives, policy can affect the long-run *level* of  $y$ .

The case  $\varphi < 1$  constitutes an example of *semi-endogenous growth*. We say there is semi-endogenous growth when 1) per capita growth is driven by some internal mechanism (as distinct from exogenous technology growth), but 2) sustained per capita growth requires support in the form of growth in some exogenous factor. In innovation-based growth theory, this factor is typically population size. In Jones (1995b), (1) takes the extended form,  $\dot{A} = \mu A^\varphi L_A^\lambda$ ,  $0 < \lambda \leq 1$ , where  $1 - \lambda$  represents a likely congestion externality of simultaneous research (duplication of effort); but this externality is not crucial for the discussion here.<sup>13</sup> As we have defined the first-generation models of endogenous

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<sup>10</sup>The result that  $g_y = g_A$  in a steady state follows, as a special case, by the method applied to balanced growth analysis in Section 3.3.

<sup>11</sup>Emphasis on the non-rival character of technical knowledge is not specific to new growth theory, but can be found already in, e.g., Arrow (1962a) and Nordhaus (1969). What is new is rather the elaborate integration of this facet into dynamic general equilibrium models with imperfect competition.

<sup>12</sup>This is usually ruled out by assumption. But not always. Indeed, one may allow for endogenous fertility, thereby endogenizing  $n$  (as in Jones 2003). And Cozzi (1997) develops a model where even  $\phi$  is endogenous.

<sup>13</sup>For more elaborate variants of the semi-endogenous approach, with detailed accounts of R&D and

growth, the Jones (1995b) model also belongs to this group, being a modified Romer-style increasing-variety growth model. Indeed, whether an analysis concentrates on the robust case  $\varphi < 1$  or the non-robust (but analytically much simpler) case  $\varphi = 1$ , is in our terminology not decisive for what generation the applied model framework belongs to. A further terminological remark is perhaps warranted. Speaking of “fully endogenous” vs. “semi-endogenous” growth may give the impression that the first term refers to something going deeper than the second; nothing of that sort should be implied.

## 2.3 Second-generation models

The Jones-critique provoked numerous answers and fruitful new developments. These include different ways of combining the horizontal and the vertical innovation approach (Young 1998, Peretto 1998, Aghion and Howitt 1998, Ch. 12, Dinopoulos and Thompson 1998, Howitt 1999 and Peretto and Smulders 2002).<sup>14</sup> On the one hand these models succeeded in reconciling policy-dependent long-run growth with the absence of a scale-effect on growth and thereby the absence of accelerating growth as soon as population growth is present. On the other hand, as maintained by Jones (1999), Li (2000) and Li (2002a), this reconciliation relies on *several* questionable knife-edge conditions; a generic model with innovations along two dimensions tends to have policy-invariant long-run growth, as long as population growth is exogenous, and tends to feature semi-endogenous growth, not fully endogenous growth.<sup>15</sup>

What do these developments within growth theory have to say about the role of natural resources for sustainable development and the role of technological change for overcoming the finiteness of natural resources? In the wake of the first-generation endogenous growth models appeared a series of papers considering the relationship between growth and environmental problems (Brock and Taylor 2005 and Fullerton and Kim 2006 depict the state of the art). Much of this literature does not take the specifics of *non-renewable* resources

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market structure, see Kortum (1997) and Segerstrom (1998). An early example is Arrow (1962b). A somewhat different way to alleviate or eliminate scale effects on growth is based on *adoption costs* (Jovanovic 1997).

<sup>14</sup>Another strand is the new theories about how the market mechanism and profit incentives affect not only the *rate* of technical change, but also its *direction* (Acemoglu 2003). Yet a new strand, perhaps deserving to be categorized as *third-generation* models, is the integration of industrial organisation theory and growth theory in an endeavour to achieve a nuanced understanding of the relationship between market structure and innovation (see, e.g., Aghion and Grifitt, 2005).

<sup>15</sup>At least within the second-generation framework this is so. To my knowledge there exists, so far, no compelling demonstration of fully endogenous growth arising generically from a more in-depth framework. Yet, Weitzman (1998a) is an attempt in this direction.

into account. There has also, however, been done some work on the relationship between endogenous growth and non-renewable resources (Jones and Manuelli 1997,<sup>16</sup> Aghion and Howitt 1998, Chapter 5, Scholz and Ziemes 1999, Schou 2000, Schou 2002, Groth and Schou 2002, Grimaud and Rougé 2003). These contributions link new growth theory to the resource economics of the 1970s and the limits-to-growth debate. Since the resource economics of the 1970s is still of central importance, the next section is devoted to a summary before the new literature is taken up.

### 3 The wave of resource economics in the 1970s

From the literature of the 1970s on non-renewable resources in a macroeconomic framework four contributions published in a symposium issue of *Review of Economic Studies* in 1974 stand out: Dasgupta and Heal (1974), Solow (1974a) and Stiglitz (1974a and 1974b). For the purpose at hand we group these contributions together, notwithstanding they concentrated on partly different aspects and contain far more insight than is visible in this brief account.

#### 3.1 The Dasgupta-Heal-Solow-Stiglitz model

What we may call the Dasgupta-Heal-Solow-Stiglitz model, or D-H-S-S model for short, is a one-sector model with technology and resource constraints described by:

$$Y(t) = F(K(t), L(t), R(t), t), \quad \partial F/\partial t \geq 0, \quad (3)$$

$$\dot{K}(t) = Y(t) - C(t) - \delta K(t), \quad \delta \geq 0, \quad (4)$$

$$\dot{S}(t) = -R(t) \equiv -u(t)S(t), \quad (5)$$

$$L(t) = L_0 e^{nt}, \quad n \geq 0, \quad (6)$$

where  $Y(t)$  is aggregate output and  $K(t)$ ,  $L(t)$  and  $R(t)$  are inputs of capital, labour and a non-renewable resource (say oil), respectively, at time  $t$ . Input of *renewable* natural resources is ignored. The aggregate production function  $F$  is neoclassical<sup>17</sup> and has constant returns to scale w.r.t.  $K$ ,  $L$  and  $R$ . The assumption  $\partial F/\partial t \geq 0$  represents exogenous technical progress. Further,  $C(t)$  is aggregate consumption ( $\equiv c(t)L(t)$ , where  $c(t)$  is per

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<sup>16</sup>Not Charles I. Jones, but Larry E. Jones.

<sup>17</sup>That is, marginal productivities are positive, but diminishing in own factor.

capita consumption),  $\delta$  denotes a constant rate of capital depreciation (decay),<sup>18</sup>  $S(t)$  is the *stock* of the non-renewable resource (e.g., oil reserves) and  $u(t)$  is the rate of depletion. Since we must have  $S(t) \geq 0$  for all  $t$ , there is a finite upper bound on cumulative resource extraction:

$$\int_0^{\infty} R(t)dt \leq S(0). \quad (7)$$

Uncertainty and costs of extraction are ignored.<sup>19</sup> There is no distinction between employment  $L(t)$  and population. The population growth rate  $n$  is assumed constant.

Adding households' preferences and a description of the institutional skeleton (for example competitive markets), the model can be solved. The standard neoclassical (or Solow-Ramsey) growth model (see Barro and Sala-i-Martin 2004) corresponds to the case where neither the production function nor the utility function depends on  $R$  or  $S$ . This amounts to considering the finiteness of natural resources as economically irrelevant, at least in a growth context. One of the pertinent issues is whether this traditional approach is tenable.

Dasgupta-Heal-Solow-Stiglitz responded to the pessimistic Malthusian views of the Club of Rome (Meadows et al., 1972) by emphasizing that feedback from relative price changes should be taken into account. More specifically they asked the question: what are the conditions needed to avoid a falling level of per capita consumption in the long run in spite of the inevitable decline in resource use? The answer is that there are three ways in which this decline in resource use may be counterbalanced: substitution, resource-augmenting technical progress and increasing returns to scale. Let us consider each of them in turn (although in practice the three mechanisms tend to be intertwined).

## 3.2 Substitution

By substitution is meant the gradual replacement of the input of the exhaustible natural resource by man-made input, capital. An example might be the substitution of fossil fuel energy by solar, wind, tidal and wave energy resources; more abundant lower-grade non-renewable resources can be substituted for scarce higher-grade non-renewable resources - and this *will* happen when the scarcity price of these has become sufficiently high; a rise

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<sup>18</sup>D-H-S-S had  $\delta = 0$ , thereby ignoring capital depreciation, because they considered exponential decay unrealistic and other depreciation formulas too cumbersome. Here, we allow  $\delta > 0$ , because exponential decay is a normal simplifying assumption in growth theory.

<sup>19</sup>Thus the model's description of resource extraction is trivial. That is why it is natural to classify the model as a *one-sector* model notwithstanding there are two activities in the economy, manufacturing and resource extraction.

in the price of a mineral may make a synthetic substitute cost-efficient or lead to increased recycling of the mineral; finally, the composition of final output can change toward goods with less material content. The conception is that capital accumulation is at the heart of such processes (though also the arrival of new technical knowledge may be involved - we come back to this).

Whether capital accumulation can do the job depends critically on the degree of substitutability between  $K$  and  $R$ . To see this, let the production function  $F$  be a Constant-Elasticity-of-Substitution (CES) function with no technical change. That is, suppressing the explicit dating of the variables when not needed for clarity, we have.

$$Y = (\alpha K^\psi + \beta L^\psi + \gamma R^\psi)^{1/\psi}, \quad \alpha, \beta, \gamma > 0, \alpha + \beta + \gamma = 1, \psi < 1, \psi \neq 0. \quad (8)$$

The important parameter is  $\psi$ , the substitution parameter. Let  $p_R$  denote the cost to the firm per unit of the resource flow and let  $\tilde{r}$  be the cost per unit of capital (generally,  $\tilde{r} = r + \delta$ , where  $r$  is the real rate of interest). Then  $p_R/\tilde{r}$  is the relative factor price, which may be expected to increase as the resource becomes more scarce. The *elasticity of substitution* between  $K$  and  $R$  is  $[d(K/R)/d(p_R/\tilde{r})] (p_R/\tilde{r})/(K/R)$  along an isoquant curve, i.e., the percentage rise in the  $K$ - $R$  ratio that a cost-minimizing firm will choose in response to a one-percent rise in the relative factor price,  $p_R/\tilde{r}$ . For the CES production function this elasticity is a constant  $\sigma = 1/(1-\psi) > 0$ . Moreover, (8) depicts the standard case where the elasticity of substitution between all pairs of production factors is the same.<sup>20</sup>

First, suppose  $\sigma > 1$ , i.e.,  $0 < \psi < 1$ . Then, for fixed  $K$  and  $L$ ,  $Y \rightarrow (\alpha K^\psi + \beta L^\psi)^{1/\psi} > 0$  when  $R \rightarrow 0$ . In this case of high substitutability the resource is seen to be *inessential* in the sense that it is not necessary for a positive output. That is, from an economic perspective, conservation of the resource is not vital. Instead suppose  $\sigma < 1$ , i.e.,  $\psi < 0$ . Then output per unit of the resource flow, though increasing when  $R$  decreases, is bounded from above. Consequently, the finiteness of the resource inevitably implies doomsday sooner or later (unless, of course, one of the other two salvage mechanisms can prevent it). To

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<sup>20</sup>A more general case is  $Y = [(1-\gamma)\tilde{F}(K,L)^\psi + \gamma R^\psi]^{1/\psi}$ , where  $\tilde{F}(K,L)$  has constant returns to scale. Here the elasticity of substitution between  $R$  and the “composite input”  $\tilde{F}(K,L)$  is  $1/(1-\psi)$ , whereas that between  $K$  and  $L$  can be different (and may be variable). This makes it easier to obtain compliance with the empirical time trends in factor shares. A further generalisation allows  $\sigma$  to depend on the input ratio  $R/\tilde{F}(K,L)$ . In fact, what really matters is whether  $\sigma(R/\tilde{F}(K,L))$  remains low (below 1) for  $R/\tilde{F}(K,L)$  approaching 0. Cass and Mitra (1991) generalise the D-H-S-S analysis by providing necessary and sufficient conditions for non-decreasing consumption in a capital-resource model with minimal technological restrictions, including allowance for extraction costs of many different kinds.

see this, keeping  $K$  and  $L$  fixed, we get

$$\frac{Y}{R} = Y(R^{-\psi})^{1/\psi} = \left[ \alpha \left( \frac{K}{R} \right)^\psi + \beta \left( \frac{L}{R} \right)^\psi + \gamma \right]^{1/\psi} \rightarrow \gamma^{1/\psi} \text{ for } R \rightarrow 0, \quad (9)$$

since  $\psi < 0$ . In fact, even if  $K$  and  $L$  are increasing,  $\lim_{R \rightarrow 0} Y = \lim_{R \rightarrow 0} (Y/R)R = \gamma^{1/\psi} \cdot 0 = 0$ . Thus, when substitutability is low, the resource is *essential* in the sense that output is nil in its absence.

What about the intermediate case  $\sigma = 1$ ? Although (8) is not defined for  $\psi = 0$ , it can be shown (using L'Hôpital's rule) that  $(\alpha K^\psi + \beta L^\psi + \gamma R^\psi)^{1/\psi} \rightarrow K^\alpha L^\beta R^\gamma$  for  $\psi \rightarrow 0$ . This limiting function, a Cobb-Douglas function, has  $\sigma = 1$  (corresponding to  $\psi = 0$ ). The interesting aspect of the Cobb-Douglas case is that it is the only case where the resource is essential and at the same time output per unit of the resource is not bounded from above (since  $Y/R = K^\alpha L^\beta R^{\gamma-1} \rightarrow \infty$  for  $R \rightarrow 0$ ).<sup>21</sup> Under these circumstances it was an open question whether non-decreasing per capita consumption can be sustained. Therefore the Cobb-Douglas case was studied intensively. For example, Solow (1974a) showed the key result that if  $n = \delta = 0$ , then a necessary and sufficient condition that a constant positive level of consumption can be sustained is that  $\alpha > \gamma$ . Moreover, this condition seems fairly realistic, since empirically  $\alpha$  is several times the size of  $\gamma$  (Nordhaus and Tobin, 1972, Neumayer 2000).<sup>22</sup> Solow added the observation that under competitive conditions, the *highest* sustainable level of consumption is obtained when investment in capital exactly equals the resource rent,  $R \cdot \partial Y / \partial R$ . This result was generalized in Hartwick (1977) and became known as *Hartwick's rule*.

Neumayer (2000) reports that the empirical evidence on the elasticity of substitution between capital and energy is inconclusive. In any case, ecological economists claim the poor substitution case to be much more realistic than the optimistic Cobb-Douglas case, not to speak of the case  $\sigma > 1$ . This invites considering the role of technical progress.

### 3.3 Technical progress

Solow (1974a) and Stiglitz (1974a,b) analysed the theoretical possibility that resource-saving technological change can overcome the declining resource use that must be expected in the future. In this context the focus is not only on whether a non-decreasing

<sup>21</sup>To avoid misunderstanding: by ‘‘Cobb-Douglas case’’ we refer to any function where  $R$  enters in a ‘‘Cobb-Douglas fashion’’, i.e., any function like  $Y = \tilde{F}(K, L)^{1-\gamma} R^\gamma$ .

<sup>22</sup>Also the assumption  $n = 0$  seems acceptable for the very long run on this finite planet. It appears harder to swallow  $\delta = 0$ , but a generalisation of Solow's result is possible for certain patterns of non-exponential depreciation (Dasgupta and Heal, 1979, p. 226).

consumption level can be maintained, but also on the possibility of sustained per capita growth in consumption.

New production techniques may raise the efficiency of resource use. For example, Dasgupta (1993) reports that during the period 1900 to the 1960s, the quantity of coal required to generate a kilowatt-hour of electricity fell from nearly seven pounds to less than one pound.<sup>23</sup> Further, technological developments make extraction of lower quality ores cost-effective and make more durable forms of energy economical. Incorporating resource-saving technical progress at the (exogenous) rate  $\lambda > 0$ , the CES production function reads

$$Y = (\alpha K^\psi + \beta L^\psi + \gamma (A_3 R)^\psi)^{1/\psi}, \quad (10)$$

where  $A_3 = e^{\lambda t}$ , assuming, for simplicity,  $\lambda$  to be constant. If the (proportionate) rate of decline of  $R$  is kept smaller than  $\lambda$ , then the “effective” resource input is no longer decreasing over time. As a consequence, even if  $\sigma < 1$  (the poor substitution case), the finiteness of nature need not be an insurmountable obstacle within any time scale of practical relevance.

Actually, a technology with  $\sigma < 1$  *needs* a considerable amount of resource-saving technical progress to obtain compliance with the empirical fact that the income share of natural resources has not been rising (Jones, 2002b). When  $\sigma < 1$ , market forces tend to increase the income share of the factor that is becoming relatively more scarce. Empirically,  $K/R$  and  $Y/R$  have increased systematically. However, with a sufficiently increasing  $A_3$ , the income share  $p_R R/Y$  need not increase in spite of  $\sigma < 1$ . Similarly, for the model to comply with Kaldor’s “stylized facts” (more or less constant growth rates of  $K/L$  and  $Y/L$  and stationarity of the output-capital ratio, the income share of labour and the rate of return on capital), we should replace  $L$  in (10) by  $A_2 L$ , where  $A_2$  is growing over time. In view of the absence of trend in the rate of return to capital, however, we assume technical progress is on average neither capital-saving nor capital-using, i.e., we do not replace  $K$  by  $A_1 K$ , but leave it as it is.

A concept which has proved extremely useful in the theory of economic growth is the concept of balanced growth. A *balanced growth path* (BGP for short) is defined as a path along which the quantities  $Y$ ,  $C$  and  $K$  change at constant proportionate rates (some or all of which may be negative). It is well-known, first, that compliance with Kaldor’s “stylized facts” is generally equivalent with existence of a balanced growth path; second,

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<sup>23</sup>For a historical account of energy technology, see Smil (1994).

that existence of a balanced growth path requires  $A_1$  to be stationary in the long run, when  $\sigma \neq 1$ .<sup>24</sup> Of course, one thing is that such a framework may allow for constant growth in per capita consumption - which is more or less what we have seen since the industrial revolution. Another thing is whether such a development will be sustainable for a long time in the future. To come nearer an answer to that question, we need theory about the relation between *endogenous* technical change and non-renewable resources.

Before entering that area, note that the Cobb-Douglas production function is again a convenient intermediate case, in that capital-saving, labour-saving and resource-saving technical progress are indistinguishable. Hence technical progress can simply be represented by

$$Y = AK^\alpha L^\beta R^\gamma, \quad (11)$$

where “total factor productivity”,  $A$ , is growing at some constant rate  $\tau > 0$ . Log-differentiating w.r.t. time yields the “growth-accounting relation”

$$g_Y = \tau + \alpha g_K + \beta n + \gamma g_R, \quad (12)$$

where, for any positive variable  $x$ ,  $g_x$  denotes the growth rate  $\dot{x}/x$ . It is easily shown that along a BGP  $g_K = g_Y = g_C \equiv g_c + n$  and, if nothing of the resource is left unutilised forever,  $g_R = g_S = -R/S \equiv -u = \text{constant}$ , so that (12) gives

$$g_c = \frac{1}{1 - \alpha}(\tau - \gamma n - \gamma u), \quad (13)$$

since  $\alpha + \beta - 1 = \gamma$ . Consequently, as observed by Stiglitz (1974a), a positive constant growth rate of  $c$  is technologically feasible, if and only if  $\tau > \gamma n$ . It is also visible from (13) that in spite of technical progress being exogenous, there is scope for policy affecting long-run growth to the extent that policy can affect the rate of depletion  $u$  in the opposite direction (a property about which we shall have more to say later).

Of course, when speaking of “sustained growth” in  $K$  and  $c$ , it should not be understood in a narrow physical sense. We have to understand  $K$  broadly as “produced means of production” of rising quality and falling material intensity; similarly,  $c$  must be seen as a composite of consumer “goods” with declining material intensity over time. This accords with the empirical fact that as income rises, the share of consumption expenditures devoted to agricultural and industrial products declines and the share devoted to services, hobbies and amusement increases. Although “economic development” is perhaps a more appropriate term, we shall retain standard terminology and speak of “economic growth”.

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<sup>24</sup>For a lucid account of this theorem by Uzawa (1961), see Jones and Scrimgeour (2005).



In any event, simple aggregate models like this should be seen as no more than a frame of reference, a tool for thought experiments. At best such models might have some validity as an approximate summary description of a certain period of time. One should be aware that an economy in which the ratio of capital to resource input grows without limit might well enter a phase where technological relations (including the elasticity of substitution) are very different from now.<sup>25</sup>

Dasgupta and Heal (1974) typify a different approach to resource-saving technical change, considering it not as a smooth gradual process, but as something arriving in a discrete once-for-all manner. They envision a future major discovery of, say, how to harness a lasting energy source such that a hitherto essential resource like fossil fuel becomes inessential. The contour of such a “backstop technology” might be currently known, but its practical applicability still awaits a technological breakthrough. The time until the arrival of this breakthrough is uncertain and may well be long. In Dasgupta, Heal and Majumdar (1977) and Dasgupta, Heal and Pand (1980) the idea is pursued further, by incorporating costly R&D. The likelihood of the technological breakthrough to appear in a given time interval depends positively on the accumulated R&D as well as the current R&D. It is shown that under certain conditions an index reflecting the probability that the resource becomes unimportant acts like an addition to the utility discount rate and that R&D expenditure begins to decline after some time. This is an interesting example of an early study of *endogenous* technological change. A similar problem has been investigated by Kamien and Schwartz (1978) and Just et al. (2005), using somewhat different approaches.

### 3.4 Increasing returns to scale

The third circumstance that might help overcoming the finiteness of nature is increasing returns to scale. For the CES function with poor substitution ( $\sigma < 1$ ), however, increasing returns to scale, though helping, are not by themselves sufficient to avoid doomsday. To see this, let  $Y = (\alpha K^\psi + \beta L^\psi + \gamma R^\psi)^{\eta/\psi}$ ,  $\eta > 1$ . Then

$$\frac{Y}{R^\eta} = \left[ \alpha \left(\frac{K}{R}\right)^\psi + \beta \left(\frac{L}{R}\right)^\psi + \gamma \right]^{\eta/\psi} \rightarrow \gamma^{\eta/\psi} \text{ for } R \rightarrow 0,$$

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<sup>25</sup>For example, along *any* economic development path, the input of the non-renewable resource must in the long run asymptotically approach zero. From a physical point of view, however, there must be some minimum amount of the resource below which it can not fulfil its role as a productive input. Thus, strictly speaking, sustainability requires that in the very long run non-renewable resources become inessential.

since  $\psi < 0$ , when  $\sigma < 1$ . Hence, even if  $K$  and  $L$  are increasing,  $\lim_{R \rightarrow 0} Y = \lim_{R \rightarrow 0} (Y/R^n) R^n = \gamma^{n/\psi} \cdot 0 = 0$ . In contrast, in the Cobb-Douglas case (11) with  $\alpha + \beta + \gamma > 1$ , sustained positive per capita growth may be possible. Indeed, as Stiglitz (1974a) noted in a short remark, with increasing returns to scale it is enough that  $\tau > (1 - \alpha - \beta)n$ , which can be true even if  $\tau = 0$ .

### 3.5 Summary of D-H-S-S

Apart from the just mentioned observation by Stiglitz, the focus of D-H-S-S was on constant returns to scale; and, as in the original Solow-Ramsey growth model, only *exogenous* technical progress was considered. For our purposes we may summarize the D-H-S-S results in the following way. Non-renewable resources do not really matter if the elasticity of substitution between them and man-made inputs is above one. If not, then:

- (a) absent technical progress, if  $\sigma = 1$ , sustainable per capita consumption requires  $\alpha > \gamma$  and  $n = 0 = \delta$ ; otherwise, declining per capita consumption is inevitable and this is definitely the prospect, if  $\sigma < 1$ ;
- (b) on the other hand, if there is enough resource-saving technical progress, non-decreasing per capita consumption and even growing per capita consumption may be sustained;
- (c) population growth (more mouths to feed) exacerbates the drag on growth implied by a declining resource input; indeed, as seen from (13), the drag on growth is  $\gamma(n + u)/(1 - \alpha)$  along a BGP.

The next sections examine how *endogenising* technical change may throw new light on the issues, in particular the visions (b) and (c). We shall derive some basic conditions needed for vision (b) to show up. As to point (c), we shall see that the relationship between population growth and economic growth tends to be circumvented when endogenous creation of ideas (generating increasing returns to scale) is considered.

## 4 Endogenous growth theory with non-renewable resources

It is not always recognised that the research of the 1970s on macro implications of essential non-renewable natural resources already laid the groundwork for a theory of endogenous

and policy-dependent growth with natural resources. Actually, by extending the D-H-S-S model, Suzuki (1976), Chiarella (1980), Robson (1980) and Takayama (1980) studied how endogenous innovation may affect the prospect of overcoming the finiteness of natural resources. The one-sector model by Suzuki (1976) constitutes an expedient benchmark case.

#### 4.1 An extended D-H-S-S model

Suzuki (1976) added *endogenous* technical change to the D-H-S-S model. He insisted that technical innovations are the costly result of intentional R&D. A part of aggregate output is used as R&D investment and results in additional technical knowledge and thereby higher productivity. Aggregate output is

$$Y = A^\varepsilon K^\alpha L^\beta R^\gamma, \quad \varepsilon, \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma = 1, \quad (14)$$

where  $A$  is proportional to the “stock of knowledge”. Due to this proportionality we can simply identify  $A$  with the stock of knowledge, which increases through R&D investment  $I_A$ :

$$\dot{A} = I_A - \delta_A A, \quad \delta_A \geq 0. \quad (15)$$

The interpretation is that the technology for creating new knowledge uses the same inputs as manufacturing, in the same proportions. The parameter  $\delta_A$  is the (exogenous) rate of depreciation (obsolescence) of knowledge. After consumption and R&D investment, the remainder of output is invested in physical capital:

$$\dot{K} = Y - cL - I_A - \delta_K K, \quad \delta_K \geq 0, \quad (16)$$

where  $\delta_K$  is the (exogenous) rate of depreciation (decay) of capital. Finally, resource extraction and population growth are described as in (5) and (6), respectively. Uncertainty is ignored.

We shall limit our attention to *efficient paths*, i.e., paths such that consumption can not be increased in some time interval without being decreased in another time interval. Assuming, for simplicity, that  $\delta_A = \delta_K = \delta$ ,<sup>26</sup> the net marginal productivities of  $A$  and  $K$  are equal if and only if  $\varepsilon Y/A - \delta = \alpha Y/K - \delta$ , i.e.,

$$A/K = \varepsilon/\alpha.$$

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<sup>26</sup>Suzuki (1976) has  $\delta_A = \delta_K = 0$ . But in order to comply with the general framework in this article, we allow  $\delta_K > 0$ , hence  $\delta \geq 0$ . Chiarella (1980) modifies (15) into  $\dot{A} = I_A^\xi$ ,  $\xi > 0$ , and focuses on the resulting quite complicated transitional dynamics.

Initial stocks,  $A_0$  and  $K_0$  are historically given. Suppose  $A_0/K_0 > \varepsilon/\alpha$ . Then, initially, the net marginal product of capital is larger than that of knowledge, i.e., capital is relatively scarce. An investing efficient economy will therefore for a while invest only in capital, i.e., there will be a phase where  $I_A = 0$ . This phase of complete specialisation lasts until  $A/K = \varepsilon/\alpha$ , a state reached in finite time, say at time  $\bar{t}$ . Hereafter, there is investment in both assets so that their ratio remains equal to the efficient ratio  $\varepsilon/\alpha$  forever. Similarly, if initially  $A_0/K_0 < \varepsilon/\alpha$ , then there will be a phase of complete specialisation in R&D, and after a finite time interval the efficient ratio  $A/K = \varepsilon/\alpha$  is achieved and maintained forever. Thus, for  $t > \bar{t}$  it is as if there were only one kind of capital, which we may call “broad capital” and define as  $\tilde{K} = K + A = (\alpha + \varepsilon)K/\alpha$ . Indeed, substitution of  $A = \varepsilon K/\alpha$  and  $K = \alpha \tilde{K}/(\varepsilon + \alpha)$  into (14) gives

$$Y = \frac{\varepsilon^\varepsilon \alpha^\alpha}{(\varepsilon + \alpha)^{\varepsilon + \alpha}} \tilde{K}^{\varepsilon + \alpha} L^\beta R^\gamma \equiv B \tilde{K}^{\tilde{\alpha}} L^\beta R^\gamma, \quad \tilde{\alpha} \equiv \alpha + \varepsilon, \quad (17)$$

so that  $\tilde{\alpha} + \beta + \gamma > 1$ . Further, adding (15) and (16) gives

$$\dot{\tilde{K}} = \dot{A} + \dot{K} = Y - cL - \delta \tilde{K}. \quad (18)$$

Thus, we can proceed with a model based on broad capital, using (17), (18) and the usual resource depletion equation (5). Essentially, this model provides a theoretical basis for extending the D-H-S-S model to include increasing returns to scale, thereby offering a simple framework for studying *endogenous* growth with essential non-renewable resources. Groth and Schou (2006) study a similar configuration where the source of increasing returns to scale is not intentional creation of knowledge, but learning as a by-product of investing as in Arrow (1962a) and Romer (1986). Empirically, the evidence furnished by, e.g., Hall (1990) and Caballero and Lyons (1992) suggests that there are quantitatively significant increasing returns to scale w.r.t. capital and labour or external effects in US and European manufacturing. Similarly, Antweiler and Treffer (2002) examine trade data for goods-producing sectors and find evidence for increasing returns to scale. Whatever the source of increasing returns to scale we shall call a D-H-S-S framework with  $\tilde{\alpha} + \beta + \gamma > 1$  an *extended D-H-S-S model*.

Log-differentiating (17) w.r.t.  $t$  gives the “growth-accounting equation”

$$g_Y = \tilde{\alpha} g_{\tilde{K}} + \beta n + \gamma g_R. \quad (19)$$

Hence, along a BGP we get, instead of (13),

$$(1 - \tilde{\alpha})g_c + \gamma u = (\tilde{\alpha} + \beta - 1)n. \quad (20)$$

Since  $u > 0$ , it follows immediately that:

**Result (i)** A BGP with  $g_c > 0$  is technologically feasible only if

$$(\tilde{\alpha} + \beta - 1)n > 0 \quad \text{or} \quad \tilde{\alpha} > 1. \quad (21)$$

This result warrants some remarks from the perspective of new growth theory. In Section 2 we defined *endogenous growth* to be present if sustained positive per capita growth ( $g_c > 0$ ) is driven by some internal mechanism (in contrast to exogenous technology growth). Hence, Result (i) tells us that endogenous growth is theoretically possible, if there are either increasing returns to the capital-*cum*-labour input combined with population growth *or* increasing returns to capital (broad capital) itself. At least one of these conditions is required in order for capital accumulation to offset the effects of the inescapable waning of resource use over time. The reasoning of Mankiw (1995) suggests  $\beta$  to be in the neighbourhood of 0.25. And Barro and Sala-i-Martin (2004, p. 110) argue that, given the “broad capital” interpretation of capital,  $\tilde{\alpha}$  being around 0.75 accords with the empirical evidence. In view of this,  $\tilde{\alpha}$  and  $\beta$  summing to a value above 1 cannot be excluded (but it is, on the other hand, not assured). Hence,  $(\tilde{\alpha} + \beta - 1)n > 0$  seems possible when  $n > 0$ .

We have defined *fully endogenous growth* to be present if the long-run growth rate in per capita output is positive without the support of growth in any exogenous factor. Result (i) shows that only if  $\tilde{\alpha} > 1$ , is *fully* endogenous growth possible. Although the case  $\tilde{\alpha} > 1$  has potentially explosive effects on the economy, if  $\tilde{\alpha}$  is not too much above 1, these effects can be held back by the strain on the economy imposed by the declining resource input.<sup>27</sup>

In some sense this is “good news”: fully endogenous steady growth is theoretically possible and no knife-edge assumption is needed. As we saw in Section 2, in the conventional framework, without non-renewable resources, fully endogenous growth requires constant returns to the producible input(s) in the growth engine. In our one-sector model the growth engine is the manufacturing sector itself, and without the essential non-renewable resource, fully endogenous growth would require the knife-edge condition  $\tilde{\alpha} = 1$  ( $\tilde{\alpha}$  being

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<sup>27</sup>It is shown in Groth (2004) that “only if” in Result (i) can be replaced by the stronger “if and only if”. Note also that if some irreducibly exogenous element in the technological development is allowed in the model by replacing the constant  $B$  in (17) by  $e^{\tau t}$ , where  $\tau \geq 0$ , then (21) is replaced by  $\tau + (\tilde{\alpha} + \beta - 1)n > 0$  or  $\tilde{\alpha} > 1$ . Both Stiglitz (1974a, p. 131) and Withagen (1990, p. 391) ignore implicitly the possibility  $\tilde{\alpha} > 1$ . Hence, from the outset they preclude fully endogenous growth.

above 1 is excluded in this case, because it would lead to explosive growth in a setting without some countervailing factor). When non-renewable resources are an essential input in the growth engine, they entail a drag on the growth potential. In order to offset this drag, fully endogenous growth requires *increasing* returns to capital.

However, the “bad news” is that even in combination with essential non-renewable resources, an assumption of increasing returns to capital seems too strong and too optimistic. A technology having  $\tilde{\alpha}$  just slightly above 1 can sustain *any* per capita growth rate - there is no upper bound on  $g_c$ .<sup>28</sup> This appears overly optimistic.

This leaves us with *semi-endogenous* growth as the only plausible form of endogenous growth (as long as  $n$  is not endogenous). Indeed, Result (i) indicates that semi-endogenous growth corresponds to the case  $1 - \beta < \tilde{\alpha} \leq 1$ . In this case sustained positive per capita growth driven by some internal mechanism is possible, but only if supported by  $n > 0$ , that is, by growth in an exogenous factor, here population size.

## 4.2 Growth policy and conservation

Result (i) is about as far as Suzuki’s analysis takes us, since his focus is only on whether the technology as such allows the growth rate to be positive or not.<sup>29</sup> That is, he does not study the *size* of the growth rate. A key issue in new growth theory is to explain the size of the growth rate and how it can temporarily or perhaps permanently be affected by economic policy. The simple growth-accounting relation (20) immediately shows:

**Result (ii)** Along a BGP, policies that decrease (increase) the depletion rate  $u$  (and only such policies) will increase (decrease) the per capita growth rate (here we presuppose  $\tilde{a} < 1$ , the plausible case).

This observation is of particular interest in view of the fact that changing the perspective from exogenous to endogenous technical progress implies bringing a source of numerous market failures to light. On the face of it, the result seems to run against common sense. Does high growth not imply *fast* depletion (high  $u$ )? Indeed, the answer is affirmative, but with the addition that exactly because of the fast depletion such high growth will only be temporary - it carries the seeds to its own obliteration. For faster

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<sup>28</sup>See Groth (2004).

<sup>29</sup>Suzuki’s (1976) article also contains another model, with a resource externality. We touch upon this model in Section 6.

sustained growth there must be sustained slower depletion. The reason for this is that with protracted depletion, the rate of decline in resource input becomes smaller; hence, so does the drag on growth caused by this decline.

As a statement about policy and long-run growth, (ii) is a surprisingly succinct conclusion. It can be clarified in the following way. For policy to affect long-run growth, it must affect a linear differential equation linked to the basic goods sector in the model (Romer 1995). In the present framework the resource depletion relation,

$$\dot{S} = -uS,$$

is such an equation. In balanced growth  $g_S = -R/S \equiv -u$  is constant so that the proportionate rate of decline in  $R$  must comply with, indeed be equal to, that of  $S$ . Through the growth accounting relation (19), given  $u$ , this fixes  $g_Y$  and  $g_{\bar{K}}$  (equal in balanced growth), hence also  $g_c = g_Y - n$ . The conventional wisdom in the endogenous growth literature is that interest income taxes impede economic growth and investment subsidies promote economic growth. Interestingly, this is not so when non-renewable resources are an essential input in the growth engine (which is here the manufacturing sector itself). Then, generally, only those policies that interfere with the depletion rate  $u$  in the long run (like a profits tax on resource-extracting companies or a time-dependent tax on resource use) can affect long-run growth. This is further explored in Groth and Schou (2006). It is noteworthy that this long-run policy result holds whether  $g_c > 0$  or not and whether growth is exogenous, semi-endogenous or fully endogenous.<sup>30</sup> The general conclusion is that with non-renewable resources entering the growth-generating sector in an essential way, conventional policy tools receive a different role and there is a role for new tools (affecting long-run growth through affecting the depletion rate).

### 4.3 Further implications

In order to be more specific we introduce household preferences and a “social planner”. The resulting resource allocation will coincide with that of a decentralized economy with appropriate subsidies and taxes. As in Stiglitz (1974a), let the utilitarian social planner optimise

$$U_0 = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} L(t) e^{-\rho t} dt, \quad \theta > 0, \rho \geq n \geq 0, \quad (22)$$

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<sup>30</sup>This is a reminder that the distinction between fully endogenous growth and semi-endogenous growth is not the same as the distinction between policy-dependent and policy-invariant growth.

subject to the constraints given by technology ((17), (18) and (5)) and initial conditions. Here,  $\theta$  is the (numerical) elasticity of marginal utility (desire for consumption smoothing) and  $\rho$  is a constant rate of time preference (impatience).<sup>31</sup>

Using the Pontryagin Maximum Principle, the first order conditions for this problem lead to, first, the *Ramsey rule*,<sup>32</sup>

$$g_c = \frac{1}{\theta} \left( \frac{\partial Y}{\partial \tilde{K}} - \delta - \rho \right) = \frac{1}{\theta} (\tilde{\alpha} \frac{Y}{\tilde{K}} - \delta - \rho), \quad (23)$$

second, the *Hotelling rule*,<sup>33</sup>

$$\frac{d(\partial Y / \partial R)}{dt} = \frac{\partial Y}{\partial R} \left( \frac{\partial Y}{\partial \tilde{K}} - \delta \right) = \gamma \frac{Y}{R} (\tilde{\alpha} \frac{Y}{\tilde{K}} - \delta). \quad (24)$$

The first rule says: as long as the net return on investment in capital is higher than the rate of time preference, one should let current  $c$  be low enough to allow positive net saving (investment) and thereby higher consumption in the future. The second rule is a no-arbitrage condition saying that the return (“capital gain”) on leaving the marginal unit of the resource in the ground must equal the return on extracting and using it in production and then investing the proceeds in the alternative asset (reproducible capital).<sup>34</sup>

Using the Cobb-Douglas specification, we may rewrite the Hotelling rule as  $g_Y - g_R = \tilde{\alpha} Y / \tilde{K} - \delta$ . Along a BGP  $g_Y = g_C = g_c + n$  and  $g_R = -u$ , so that the Hotelling rule combined with the Ramsey rule gives

$$(\theta - 1)g_c - u = n - \rho. \quad (25)$$

This linear equation in  $g_c$  and  $u$  combined with the growth-accounting relationship (20) constitutes a linear two-equation system in the growth rate and the depletion rate. The

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<sup>31</sup>If  $\rho = n$ , the improper integral  $U_0$  tends to be unbounded and then the optimization criterion is not maximization, but “overtaking” or “catching-up” (see Seierstad and Sydsaeter, 1987). For simplicity we have here ignored (as does Stiglitz) that also environmental quality should enter the utility function.

<sup>32</sup>After Ramsey (1928).

<sup>33</sup>After Hotelling (1931). Assuming perfect competition, the real resource price becomes  $p_R = \partial Y / \partial R$  and the real rate of interest is  $r = \partial Y / \partial K - \delta$ . Then the rule takes the more familiar form  $\dot{p}_R / p_R = r$ . If there are extraction costs at rate  $C(R, S, t)$ , then the rule takes the form  $\dot{p}_S - \partial C / \partial S = r p_S$ , where  $p_S$  is the price of the unextracted resource (whereas  $p_R = p_S + \partial C / \partial R$ ).

It is another thing that the rise in resource prices and the predicted decline in resource use have not yet shown up in the data (Krautkraemer 1998, Smil 2003); this may be due to better extraction technology and discovery of new deposits. But in the long run, if non-renewable resources *are* essential, this tendency inevitably will be reversed.

<sup>34</sup>After the initial phase of complete specialization described in Section 4.1, we have, due to the proportionality between  $K$ ,  $A$  and  $\tilde{K}$ , that  $\partial Y / \partial K = \partial Y / \partial A = \partial Y / \partial \tilde{K} = \tilde{\alpha} Y / \tilde{K}$ . Notice that the Hotelling rule is independent of preferences; *any* path that is *efficient* must satisfy the Hotelling rule (as well as the exhaustion condition  $\lim_{t \rightarrow \infty} S(t) = 0$ ).



determinant of this system is  $D \equiv 1 - \tilde{\alpha} - \gamma + \theta\gamma$ . We assume  $D > 0$ , which seems realistic and is in any case necessary (and sufficient) for stability.<sup>35</sup> Then

$$g_c = \frac{(\tilde{\alpha} + \beta + \gamma - 1)n - \gamma\rho}{D}, \quad \text{and} \quad (26)$$

$$u = \frac{[(\tilde{\alpha} + \beta - 1)\theta - \beta]n + (1 - \tilde{\alpha})\rho}{D}. \quad (27)$$

Interesting implications are:

**Result (iii)** If there is impatience ( $\rho > 0$ ), then even when a non-negative  $g_c$  is technologically feasible ((21) satisfied), a negative  $g_c$  can be optimal and stable.

**Result (iv)** Population growth is *good* for economic growth. In its absence, when  $\rho > 0$ , we get  $g_c < 0$  along an optimal BGP; if  $\rho = 0$ ,  $g_c = 0$  when  $n = 0$ .

**Result (v)** There is never a scale effect on the growth rate.

Result (iii) reflects that utility discounting and consumption smoothing weaken the “growth incentive”. Result (iv) is completely contrary to the conventional (Malthusian) view and the learning from the D-H-S-S model. The point is that two offsetting forces are in play. On the one hand, higher  $n$  means more mouths to feed and thus implies a drag on per capita growth (Malthus). On the other hand, a growing labour force is exactly what is needed in order to exploit the benefits of increasing returns to scale (anti-Malthus).<sup>36</sup> And in the present framework this dominates the first effect.<sup>37</sup> This feature might seem to be contradicted by the empirical finding that there is no robust correlation between  $g_c$  and population growth in cross-country regressions (Barro and Sala-i-Martin 2004, Ch. 12). However, the proper unit of observation in this context is not the individual country. Indeed, as argued in Section 2.2, in an internationalized world with technology diffusion a positive association between  $n$  and  $g_c$  as in (26) should not be seen as a prediction about individual countries, but rather as pertaining to larger regions, perhaps the global economy. In any event, the second part of Result (iv) is a dismal part - in view of the projected long-run stationarity of world population (United Nations 2005).

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<sup>35</sup>As argued above,  $\tilde{\alpha} < 1$  seems plausible. Generally,  $\theta$  is estimated to be greater than one (see, e.g., Attanasio and Weber 1995); hence  $D > 0$ . The stability result as well as other findings reported here are documented in Groth and Schou (2002).

<sup>36</sup>This aspect will become more lucid in the two-sector models of the next section, where the non-rival character of technical knowledge is more transparent.

<sup>37</sup>This as well as the other results go through if a fixed resource like land is included as a necessary production factor. Indeed, letting  $J$  denote a fixed amount of land and replacing (14) by  $Y = A^\varepsilon K^\alpha L^\beta R^\gamma J^{1-\alpha-\beta-\gamma}$ , where now  $\alpha + \beta + \gamma < 1$ , leave (19)-(21), (26) and (27) unchanged.

A somewhat surprising result appears if we imagine (unrealistically) that  $\tilde{\alpha}$  is sufficiently above one to make  $D$  a negative number. If population growth is absent,  $D < 0$  is in fact needed for  $g_c > 0$  along a BGP. However,  $D < 0$  implies instability. Hence this would be a case of an instable BGP with fully endogenous growth.<sup>38</sup>

As to Result (v), it is noteworthy that the absence of a scale effect on growth holds for *any* value of  $\tilde{\alpha}$ , including  $\tilde{\alpha} = 1$ .<sup>39</sup>

A pertinent question now is: are the above results just an artifact of the one-sector set-up? This leads us to consider two-sector models.

## 5 Models with a separate R&D sector

### 5.1 The standard approach

The conclusions (i), (ii), (iii) and (v) above (and partly also (iv)) differ from most of the new growth literature,<sup>40</sup> including most of the contributions that deal explicitly with non-renewable resources and endogenous growth (Jones and Manuelli 1997, Aghion and Howitt 1998 (Chapter 5), Scholz and Ziemes 1999, Schou 2000, Schou 2002, Grimaud and Rougé 2003). These contributions extend the first-generation two-sector endogenous growth models referred to in Section 2, by including a non-renewable resource as an essential input in the manufacturing sector. The non-renewable resource does not, however, enter the R&D or educational sector in these models (not even indirectly in the sense of physical capital produced in the manufacturing sector being used in the R&D sector). As we shall now see, this is the reason that these models give results quite similar to those from conventional endogenous models without non-renewable resources.

The following two-sector framework is a prototype of the afore-mentioned contribu-

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<sup>38</sup>Thus, if we do not require  $D > 0$  in the first place, (iv) could be reformulated as: existence of a *stable* optimal BGP with  $g_c > 0$  requires  $n > 0$ . This is not to say that reducing  $n$  from positive to zero renders an otherwise stable BGP instable. Stability-instability is governed solely by the sign of  $D$ . Given  $D > 0$ , letting  $n$  decrease from a level above the critical value,  $\gamma\rho/(\tilde{\alpha} + \beta + \gamma - 1)$ , given from (26), to a level below, changes  $g_c$  from positive to negative, i.e., growth comes to an end.

<sup>39</sup>More commonplace observations are that increased impatience leads to faster depletion and lower growth (in the plausible case  $\tilde{a} < 1$ ). Further, in the log-utility case ( $\theta = 1$ ) the depletion rate  $u$  equals the effective rate of impatience,  $\rho - n$ .

<sup>40</sup>Here we have in mind the fully endogenous growth literature. The results are more cognate with the results in semi-endogenous growth models without non-renewable resources, like Jones (1995b).

tions:

$$Y = A^\varepsilon K^\alpha L_Y^\beta R^\gamma, \quad \varepsilon, \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma = 1, \quad (28)$$

$$\dot{K} = Y - cL - \delta K, \quad \delta \geq 0,$$

$$\dot{A} = \bar{\mu}L_A, \quad \bar{\mu} = \mu A, \quad \mu > 0, \quad (29)$$

$$\dot{S} = -R,$$

$$L_Y + L_A = L, \text{ constant.}$$

Unlike in the previous model, additions to society's "stock of knowledge",  $A$ , are now produced in a separate sector, the R&D sector, with a technology different from that in manufacturing. The only input in the R&D sector is labour (thus taking to the extreme the feature that this sector is likely to be relatively intensive in human capital). The individual research lab, which is "small" in relation to the economy as a whole, takes R&D productivity,  $\bar{\mu}$ , as given. At the economy-wide level, however, this productivity depends positively on the stock of technical knowledge in society,  $A$  (this externality is one of several reasons that the existence of endogenous technical change implies market failures). Usually, there is no depreciation of knowledge, i.e.,  $\delta_A = 0$ . Aggregate employment in the R&D sector is  $L_A$ . Total employment,  $L$ , in the economy is the sum of  $L_A$  and employment,  $L_Y$ , in the manufacturing sector. In that sector, the firms take  $A$  as given and the technology they face at the micro level may involve different capital-good varieties and qualities. There are many interesting details and disparities between the models concerning these aspects as well as the specifics of the market structure and the policy questions considered. Yet, whether we think of the "increasing variety" models (or Romer-style models to which Scholz and Ziemes 1999 and Schou 2002 belong) or the "increasing quality models" (or quality ladder models to which Aghion and Howitt 1998 and Grimaud and Rougé 2003 belong), at the aggregate level these models end up with a formal structure basically like that above.<sup>41</sup> The accumulation-based growth models by Jones and Manuelli (1997) and Schou (2000) are in one respect different - we shall return to this.

Two key features emphasised by new growth theory are immediately apparent. First, because technological ideas - sets of instructions - are *non-rival*, what enters both in the production function for  $Y$  and that for  $\dot{A}$  is *total*  $A$ . This is in contrast to the *rival* goods: capital, labour and the resource flow. For example, a given unit of labour can be used no more than one place at a time. Hence, only a fraction of the labour force

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<sup>41</sup>Essentially this structure also characterizes the two-sector models by Robson (1980) and Takayama (1980), although these contributions do not fully comprehend the non-rival character of knowledge, since they have  $L_A/L$  in (29) instead of  $L_A$ .

enters manufacturing, the remaining fraction entering R&D. Second, there is a tendency for *increasing returns to scale* to arise when knowledge is included in the total set of inputs. At least when we ignore externalities, the well-known replication argument gives reason to expect constant returns to scale w.r.t. the *rival* inputs (here  $K$ ,  $L_Y$  and  $R$  in the manufacturing sector and  $L_A$  in R&D). Consequently, as we double these rival inputs and *also* double the amount of knowledge, we should expect more than a doubling of  $Y$  and  $\dot{A}$ . An additional key feature of new growth theory, apparent when the above technology description is combined with assumptions about preferences and market structure, is the emphasis on incentives as driving R&D investment. When the resource becomes more scarce and its price rises, the value of resource-saving knowledge increases and R&D is stimulated.<sup>42</sup>

Using the principle of growth accounting on (28), taking  $n = 0$  into account, we get, along a BGP,<sup>43</sup>

$$(1 - \alpha)g_c = \varepsilon g_A - \gamma u, \quad (30)$$

where

$$g_A = \mu \ell_A L, \quad \ell_A \equiv \frac{L_A}{L}, \text{ constant.}$$

We have  $g_A > 0$  if  $\ell_A > 0$ . The essential non-renewable resource implies a drag on the growth of consumption. Yet, by sufficient conservation of the resource (implying a small  $u \equiv R/S$ ) it is always possible to obtain  $g_c > 0$ . And it is possible to increase  $g_c$  without decreasing  $u$ , simply by increasing  $\ell_A$ . These two last conclusions have a quite different flavour compared to the results (i) and (ii) from the extended D-H-S-S model.

The fraction,  $\ell_A$ , of the labour force in R&D will depend on parameters such as  $\alpha$ ,  $\varepsilon$ ,  $\mu$  and those describing preferences and the allocation device, whether this is the market mechanism in a decentralized economy or the social planner in a centralized economy. To be specific, let us again consider a social planner and the criterion (22). Along a BGP we get once more (25) (from the Ramsey rule and the Hotelling rule). Further, efficient allocation of labour across the two sectors and across time leads to  $\ell_A = 1 - \beta u / (\varepsilon \mu L)$ .

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<sup>42</sup>Using patent data, Popp (2002) finds a strong, positive impact of energy prices on energy-saving innovations.

<sup>43</sup>In this two-sector framework a BGP means a path along which  $Y$ ,  $C$ ,  $K$  and  $N$  grow at constant rates (not necessarily positive). It is understood that the path considered is *efficient* and thus leaves nothing of the resource unutilized forever.

Combining this with (30) and (25) we find, along a BGP,

$$\begin{aligned} \ell_A &= \frac{\varepsilon\mu L(\beta + \theta\gamma) - \beta(1 - \alpha)\rho}{\varepsilon\mu L\theta(1 - \alpha)}, \\ g_c &= \frac{\varepsilon\mu L - (1 - \alpha)\rho}{\theta(1 - \alpha)}, \quad \text{and} \\ u &= \frac{(\theta - 1)\varepsilon\mu L + (1 - \alpha)\rho}{\theta(1 - \alpha)}. \end{aligned}$$

This is an example of fully endogenous growth: given  $(1 - \theta)\varepsilon\mu L < (1 - \alpha)\rho < \varepsilon\mu L$ ,<sup>44</sup> per capita growth is positive along a BGP without support of growth in any exogenous factor. A caveat is that this result relies on the knife-edge assumption that the growth engine (the R&D sector) has exactly constant returns to the producible input(s), here  $A$ . The problematic scale effect on growth ( $\partial g_c / \partial L > 0$ ) crops up again (although often hidden by the labour force being normalized to one). Indeed, this is why these models assume a constant labour force; with  $n > 0$  the growth rate will be forever rising. In any event, contrary to the implication of (26), sustained positive growth is conceivable without population growth and whether  $\rho = 0$  or  $\rho > 0$ .

Overall, we have a more optimistic perspective than in the extended D-H-S-S model. Indeed, the conclusions are quite different from the results (i), (ii) and (v) above (and partly also different from (iv)). The conclusions are, however, pretty much in conformity with those of the fully endogenous growth models without non-renewable resources. With the exception of the scale effect on growth we get similar results in the model by Jones and Manuelli (1997). They consider an economy with a sector producing consumption goods with labour, capital and the non-renewable resource and a sector producing capital goods with only capital (not even labour). The model by Schou (2000) is a Lucas-style human-capital-based model extended with a non-renewable resource entering only the manufacturing sector (with the addition of pollution from this resource). Since in both models it is the accumulation of a *rival* good that drives growth, the scale effect on growth does not appear, but this is the only difference in relation to the questions considered here.

The explanation of the optimistic results in all these models is that the growth-generating sector is presumed not to depend on the non-renewable resource (neither directly nor indirectly). In reality, however, most sectors, including educational institutions and research laboratories, use fossil fuels for heating and transportation purposes,

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<sup>44</sup>The first inequality ensures  $u > 0$  (equivalent with the necessary transversality condition in the optimal control problem being satisfied), the second ensures  $g_c > 0$ .

or at least they use indirectly minerals and oil products via the machinery, computers etc. they employ. The extended D-H-S-S model in the previous section *did* take this dependency of the growth engine (in that model the manufacturing sector itself) on the natural resource into account and therefore gave substantially different results. In the next section we shall see that a two-sector model with the resource entering (also) the R&D sector leads to results similar to those of the extended D-H-S-S model from Section 4, but quite different from those of the above two-sector model.

## 5.2 Growth-essential non-renewable resources

When a natural resource is an essential input (directly or indirectly) in the growth-engine, we shall call the resource *growth-essential*.

### 5.2.1 The resource as input in both sectors

Extending the above two-sector framework as in Groth (2005), we consider the setup:

$$Y = A^\varepsilon K^\alpha L_Y^\beta R_Y^\gamma, \quad \varepsilon, \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma = 1, \quad (31)$$

$$\dot{K} = Y - cL - \delta K, \quad \delta \geq 0, \quad (32)$$

$$\dot{A} = \bar{\mu} L_A^\eta R_A^{1-\eta}, \quad \bar{\mu} = \mu A^\varphi, \quad \mu > 0, \quad 0 < \eta < 1, \quad (33)$$

$$\dot{S} = -R, \quad (34)$$

$$L_Y + L_A = L = L(0)e^{nt}, \quad n \geq 0, \quad (35)$$

$$R_Y + R_A = R. \quad (36)$$

There are three new features. First, only a fraction of the resource flow  $R$  is used in manufacturing, the remainder being used as an essential input in R&D activity. Second, the knowledge elasticity,  $\varphi$ , of research productivity is allowed to differ from one; as argued in the section on the Jones critique, even  $\varphi < 0$  should not be excluded *a priori*. Third, population growth is not excluded.

Along a BGP, using the principle of growth accounting on (31) yields

$$(1 - \alpha)g_c = \varepsilon g_A - \gamma(n + u). \quad (37)$$

Applying the same principle on the R&D equation (33) (after dividing by  $A$  and presupposing the R&D sector is active) and assuming balanced growth we get, after substituting

into (37),

$$(1 - \alpha)g_c = \left( \frac{\varepsilon\eta}{1 - \varphi} - \gamma \right) n - \left( \frac{\varepsilon(1 - \eta)}{1 - \varphi} + \gamma \right) u. \quad (38)$$

Since  $u > 0$ , from this<sup>45</sup> follows that a BGP with  $g_c > 0$  is technologically feasible only if

$$\varphi < 1 + \frac{\varepsilon(1 - \eta)}{\gamma} \text{ and either } (n > 0 \text{ and } \varepsilon\eta > (1 - \varphi)\gamma) \text{ or } \varphi > 1.$$

Naturally, the least upper bound for  $\varphi$ 's that allow non-explosive growth is here higher than when the resource is not a necessary input in the R&D sector. We also see that for the technology to allow steady positive per capita growth, *either*  $\varphi$  must be above one *or* there must be population growth (to exploit increasing returns to scale) *and* an elasticity of  $Y$  w.r.t. knowledge large enough to overcome the drag on growth caused by the inevitable decline in resource use. Not surprisingly, in the absence of population growth, sustained per capita growth requires a higher elasticity of research productivity with respect to knowledge than when the growth engine does not need the resource as an input. The “standard” two-sector model of the previous section relied on the aggregate invention production function having exactly constant returns (at least asymptotically) to produced inputs, that is,  $\varphi = 1$ . Slightly increasing returns w.r.t.  $A$  would in that model lead to explosive growth, whereas slightly decreasing returns lead to growth petering out. Interestingly, when the resource is growth-essential, the case  $\varphi = 1$  loses much of its distinctiveness. Yet, the “bad news” for fully endogenous growth is again that  $\varphi > 1$  seems to be a too optimistic and strong assumption. The reason is similar to that given in Section 4.1 for doubting that  $\tilde{\alpha} > 1$ , namely that whenever a given technology has  $\varphi > 1$ , it can sustain *any* per capita growth rate no matter how high - a rather suspect implication. Thus, once more we are left with semi-endogenous growth ( $\varphi \leq 1$ ) as the only appealing form of endogenous growth (as long as  $n$  is exogenous).

In parallel to Result (ii) above, (38) shows that when  $\varphi < 1$ , only policies that decrease the depletion rate  $u$  along a BGP, can increase the per capita growth rate  $g_c$ . For example, embedding the just described technology in a Romer (1990)-style market structure, Groth (2006) shows that a research subsidy, an interest income tax and an investment subsidy do not affect long-run growth whereas taxes that impinge on resource extraction do. The point is that whatever market forms might embed the described technology and whatever policy instruments are considered, the growth-accounting relation (38) *must* hold (given the assumed Cobb-Douglas technologies).

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<sup>45</sup>For ease of interpretation we have written (38) on a form analogue to (37). In case  $\varphi = 1$ , (38) should be interpreted as  $(1 - \varphi)(1 - \alpha)g_c = [\varepsilon\eta - (1 - \varphi)\gamma]n - [\varepsilon(1 - \eta) + (1 - \varphi)\gamma]u$ .

Let us again consider a social planner and the criterion (22). Then, along a BGP we have once more (25) (from the Ramsey rule and the Hotelling rule). Combining this with (38) we find, along a BGP,

$$g_c = \frac{\varepsilon n - [\varepsilon(1 - \eta) + (1 - \varphi)\gamma]\rho}{\tilde{D}}, \quad \text{and}$$

$$u = \frac{[(\theta - 1)\varepsilon - \tilde{D}]n + (1 - \varphi)(1 - \alpha)\rho}{\tilde{D}},$$

where  $\tilde{D} \equiv (1 - \varphi)(\beta + \theta\gamma) + (\theta - 1)\varepsilon(1 - \eta)$  is assumed positive (this seems to be the empirically relevant case and it is in any event necessary, though not sufficient, for stability).<sup>46</sup> We see that in the plausible case  $\varphi < 1 + \varepsilon(1 - \eta)\gamma$  the analogy of the results (iii), (iv) and (v) from the extended D-H-S-S model of Section 4 go through.<sup>47</sup>

The conclusion is that when a non-renewable resource is an essential input in the R&D sector, quite different and more pessimistic conclusions arise compared to those of the previous section. Sustained growth without increasing effort (i.e., without  $n > 0$ ) now requires  $\varphi > 1$  in contrast to  $\varphi = 1$  in the previous section. Now policies aimed at stimulating long-run growth generally have to go via resource conservation.

### 5.2.2 Capital in the R&D sector

The results are essentially the same in the case where the resource is a direct input only in manufacturing, but the R&D sector uses capital goods (apparatus and instruments) produced in the manufacturing sector. Thus, indirectly the resource is an input also in the R&D sector, hence still growth-essential. The model is:

$$Y = A^\varepsilon K_Y^\alpha L_Y^\beta R^\gamma, \quad \varepsilon, \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma = 1, \quad (39)$$

$$\dot{K} = Y - cL - \delta K, \quad \delta \geq 0,$$

$$\dot{A} = \bar{\mu} K_A^{1-\eta} L_A^\eta, \quad \bar{\mu} = \mu A^\varphi, \quad \mu > 0, \quad 0 < \eta < 1, \quad (40)$$

$$\dot{S} = -R,$$

$$K_Y + K_A = K, \quad (41)$$

$$L_Y + L_A = L = L(0)e^{nt}, \quad n \geq 0.$$

<sup>46</sup>A possible reason for the popularity of the model of the previous section is that it has transitional dynamics that are less complicated than those of the present model (four-dimensional dynamics versus five-dimensional).

<sup>47</sup>Although a scale effect on growth is absent, a positive scale effect on levels remains, as shown in Groth (2005). This is due to the non-rival character of technical knowledge.



Possibly,  $1 - \eta < \alpha$  (since the R&D sector is likely to be relatively intensive in human capital), but for our purposes here this is not crucial.

Using the growth accounting principle on (39) again gives (37) along a BGP. Applying the same principle on the R&D equation (40) (presupposing the R&D sector is active) and assuming balanced growth, we find

$$(1 - \varphi)g_A = (1 - \eta)g_K + \eta n = (1 - \eta)g_c + n, \quad (42)$$

in view of  $g_K = g_C = g_c + n$ . This shows that existence of a BGP with positive growth requires  $\varphi < 1$ .<sup>48</sup> Both  $K$  and  $A$  are essential producible inputs in the two sectors; hence, the two sectors together make up the growth engine.

Substituting (42) into (37) yields

$$[(1 - \varphi)(1 - \alpha) - \varepsilon(1 - \eta)]g_c = [\varepsilon - (1 - \varphi)\gamma]n - (1 - \varphi)\gamma u. \quad (43)$$

Since  $u > 0$ , we see that a BGP with  $g_c > 0$  is technologically feasible only if, in addition to the requirement  $\varphi < 1$ ,

$$\varepsilon > (1 - \varphi)\gamma, \text{ which, if } n = 0, \text{ can be strengthened to } \varepsilon > \frac{(1 - \varphi)(1 - \alpha)}{1 - \eta}.$$

That is, given  $\varphi < 1$ , the knowledge elasticity of manufacturing output should be high enough. These observations generalize Result (i) from the extended D-H-S-S model and, when  $\varepsilon > (1 - \varphi)(1 - \alpha)/(1 - \eta)$ , also Result (ii). The combined accumulation of  $K$  and  $A$  drives growth, possibly with the help of population growth.

Again, let us consider a social planner and the criterion (22). Along a BGP we get once more (25) (from the Ramsey rule and the Hotelling rule). Combining this with (43) yields, along a BGP,

$$\begin{aligned} g_c &= \frac{\varepsilon n - (1 - \varphi)\gamma\rho}{D^*}, \quad \text{and} \\ u &= \frac{[(\theta - 1)\varepsilon - D^*]n + [(1 - \varphi)(1 - \alpha) - \varepsilon(1 - \eta)]\rho}{D^*}, \end{aligned}$$

where  $D^* \equiv (1 - \varphi)(\beta + \theta\gamma) - \varepsilon(1 - \eta)$  is assumed positive. The results (iii), (iv) and (v) from the extended D-H-S-S model immediately go through.

Thus, also when the non-renewable resource is only indirectly growth-essential, do we get conclusions in conformity with those in the previous subsection, but quite different

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<sup>48</sup>As soon as  $\varphi \geq 1$ , growth becomes explosive.

from those of standard endogenous growth models with non-renewable resources entering only the manufacturing sector. This is somewhat at variance with the section on growth and non-renewable resources in Aghion and Howitt (1998). They compare their two-sector Schumpeterian approach (which in this context is equivalent to what was above called “the standard approach”) with a one-sector AK model extended with an essential non-renewable resource and no population growth (which is equivalent to the extended D-H-S-S model with  $\alpha = 1$  and  $n = 0$ ). Having established that sustained growth is possible in the first approach, but not in the second, they ascribe this difference to “the ability of the Schumpeterian approach to take into account that the accumulation of intellectual capital is ‘greener’ (in this case, less resource intensive) than the accumulation of tangible capital” (p. 162). However, as the above example shows, even allowing the R&D sector to be “greener” than the manufacturing sector, we may easily end up with AK-style results. The crucial distinction is between models where the non-renewable resource is growth-essential - directly or indirectly - and models where it is not. To put it differently: by not letting the resource enter the growth engine (not even indirectly), Aghion and Howitt’s “Schumpeterian approach” seems biased toward sustainability.

### 5.2.3 The case of limited substitutability in the R&D sector

One might argue that, at least in the R&D sector, the elasticity of substitution between labour (research) and other inputs must be low. Hence, let us consider the limiting case of zero substitutability in the models of the two previous subsections. First, we replace (33) in the model of Section 5.2.1 by

$$\dot{A} = \mu A^\varphi \min(L_A, A^\psi R_A), \quad \psi > 0.$$

Then, along any efficient path with  $g_A > 0$  we have  $L_A = A^\psi R_A$  so that  $g_A = \mu A^{\varphi-1} L_A = \mu A^{\varphi+\psi-1} R_A$ . Log-differentiating this w.r.t.  $t$  and setting  $\dot{g}_A = 0$  gives, along a BGP,  $(\varphi - 1)g_A + n = 0 = (\varphi + \psi - 1)g_A - u$ . Since  $n \geq 0$  and  $u > 0$ ,  $1 - \psi < \varphi \leq 1$  is required (if  $\varphi > 1$ , growth becomes explosive). In the generic case  $\varphi < 1$ ,  $g_A = n/(1 - \varphi)$  so that  $g_A > 0$  requires  $n > 0$ ; we end up with

$$\begin{aligned} g_c &= \frac{\varepsilon - \gamma\psi}{(1 - \alpha)(1 - \varphi)} n, \\ u &= \frac{\varphi + \psi - 1}{1 - \varphi} n. \end{aligned}$$

Thus, both the per capita consumption growth rate and the depletion rate  $u$  along a BGP are in this case technologically determined. As an implication, preferences and economic

policy can have only level effects, not long-run growth effects. If  $n = 0$ , no BGP with  $g_c > 0$  exists in this case.

The singular case  $\varphi = 1$  is different. This is the only case where there is scope for preferences and policy to affect long-run growth. Indeed, in this case, where  $n = 0$  is needed to avoid a forever increasing growth rate, along a BGP we get  $g_c = (\varepsilon - \gamma\psi)\mu L_A$  and  $u = \psi\mu L_A$ .

We get similar results if in the model of Section 5.2.2 we replace (40) by

$$\dot{A} = \mu A^\varphi \min(K_A, A^\psi L_A), \quad \psi > 0.$$

Along any efficient path with  $g_A > 0$ , now  $K_A = A^\psi L_A$  so that  $g_A = \mu A^{\varphi-1} K_A = \mu A^{\varphi+\psi-1} L_A$ . Log-differentiating this w.r.t.  $t$  and setting  $\dot{g}_A = 0$  gives, along a BGP,  $(\varphi - 1)g_A + g_K = 0 = (\varphi + \psi - 1)g_A + n$ . Since  $n \geq 0$ ,  $\varphi \leq 1 - \psi$  is required (if  $\varphi > 1 - \psi$ , growth becomes explosive). In the generic case  $\varphi < 1 - \psi$ , both the depletion rate  $u$  and the per capita consumption growth rate become technologically determined:

$$\begin{aligned} g_c &= \frac{\psi}{1 - \varphi - \psi} n, \\ u &= \frac{\varepsilon - \beta\psi - \gamma(1 - \varphi)}{(1 - \varphi - \psi)\gamma} n, \end{aligned}$$

where the inequalities  $n > 0$  and  $\varepsilon > \beta\psi + \gamma(1 - \varphi)$  are presupposed. If  $n = 0$ , no BGP with  $g_A > 0$  exists in this case.

Only in the singular case  $\varphi = 1 - \psi$  can preferences and policy affect long-run growth. Indeed, in this case, where  $n = 0$  is needed to avoid a forever increasing growth rate, along a BGP we find  $g_c = \psi\mu L_A$  and  $u = (\varepsilon - (1 - \alpha)\psi)\mu L_A$ , where  $\varepsilon > (1 - \alpha)\psi$  is presupposed.

To conclude, with zero substitution between the production factors in the R&D sector, one “degree of freedom” is lost. As an implication, in the generic case there is no scope for preferences and policy affecting growth. Only in a knife-edge case can preferences and policy affect growth. Thus, the robust case is in this regard in conformity with semi-endogenous growth models without non-renewable resources à la Jones (1995b), and the non-robust case is in conformity with fully endogenous growth models without non-renewable resources à la Romer (1990).

## 6 Discussion

New growth theory suggests that costly innovation is the key factor in overcoming the inevitable decline in use of non-renewable resources. Yet what innovations, together with accumulation of capital, can achieve, depends on the returns to producible inputs, including technical knowledge. We have argued for the neoclassical view that diminishing returns are the most likely case. Then the growing technical knowledge that is needed for continued economic growth requires sustained growth in research effort to countervail the diminishing returns. With a rising population there is scope for a rising number of researchers and the growth prospects seem relatively fine. However, the general conception is that economic and cultural conditions are likely to put an end to population growth within 40-80 years and as early as 20-25 years in the now more developed regions (United Nations, 2005). Thus, according to the theory above we should expect a slowdown of long-run per capita growth.

There *are* counteracting forces though. The UN prediction that growth in world population will come to a halt does not necessarily mean that the  $n$  relevant for the technological frontier will be approaching zero equally soon. Even a stationary population does not preclude rising research intensity and educational attainment for a quite long time (Jones 2002a). Longevity is apt to help and so are improved institutional structures. Further, as Solow (1994) remarked “there is probably an irreducibly exogenous element in the research and development process, at least exogenous to the economy. [...] the ‘production’ of new technology may not be a simple matter of inputs and outputs” in the way our models have assumed.

Overall, the abstract character and the insufficient empirical underpinnings of the models call for caution with regard to the big question of limits to growth. But at least it seems safe to infer that endogenizing technical change substantiates the old view that if non-renewable resources are essential, they will ultimately cause a drag on growth. That is, growth ends up smaller than otherwise. In this context one should remember that even if exponential growth ceases, this need not imply absence of growth altogether. Leaving the confines of balanced growth opens up for considering a whole range of less-than-exponential, yet regular, growth paths (with complete stagnation as the limiting case).<sup>49</sup>

There are several complicating factors the above analysis has left aside; and many

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<sup>49</sup>For an exploration of this range, see Groth et al. (2006).

issues at the interface of resource economics and new growth theory have not been considered. Here we list some of these.

1. *Extraction costs and an enriched Hotelling time pattern of energy prices.* Our analysis of endogenous technical change with non-renewable resources share two empirically questionable features with the D-H-S-S model and the original Hotelling (1931) principle. These are the predictions that real resource prices should have a positive trend and resource consumption should have a negative trend. The empirical evidence stretching over more than a century does not confirm this (Nordhaus 1992, Smil 1994, Krautkraemer 1998 and Jones 2002b). Tahvonen and Salo (2001) therefore propose a different approach where there is a gradual transition from (non-essential) non-renewable energy forms to renewable energy forms (hydropower, wind-energy, solar energy, biomass and geothermal energy). There are extraction costs associated with non-renewable energy sources and these costs are decreasing in remaining reserves and extraction knowledge (a by-product of cumulative extraction experience). Knowledge relevant to renewable energy sources is formed as a by-product of physical capital investment. This makes renewable energy forms more and more cost-efficient and an asymptotic AK structure in line with Rebelo (1991) arises, thus making sustained growth feasible. A possible endogenous outcome of all this is a long period of declining resource prices and rising use of non-renewables followed by a shorter period with Hotelling-style trends before finally the renewable resources completely take over.

2. *CES technology with  $\sigma < 1$ . Induced bias.* We have concentrated on one- and two-sector models with Cobb-Douglas technology. In this setting the elasticity of factor substitution,  $\sigma$ , is 1 and technological progress is automatically resource-augmenting. Perhaps this may not be as serious a restriction as one might think at first. Jones (2005) provides microfoundations for the production function being Cobb-Douglas in the long run, though the short-term elasticity of substitution is likely to be less than one. Yet, it is worth considering the possibility that  $\sigma < 1$  also in the long term. In that case technical progress must in the long run be resource-augmenting and labour-augmenting, but not capital-augmenting, to allow for a BGP at least roughly consistent with the empirical evidence. Building on Acemoglu (2003), Di Maria and Valente (2006) show how such bias in technical progress may come about endogenously in a model where both the rate and the direction of technical change are governed by profit incentives. In a similar vein, André and Smulders (2004), extending Smulders and Nooij (2003), demonstrate how induced bias may lead to an U-shaped time pattern for energy prices relative to wages

and an inverted U-shaped pattern for energy use.

Bretschger and Smulders (2006) consider an R&D-based growth model with *two* manufacturing sectors, the “traditional” sector and a “high-tech” sector, both with CES production functions where the elasticity of substitution between intermediate (non-durable) goods and the non-renewable resource is less than one. Provided the elasticity of substitution in the high-tech sector is the highest (and some further conditions), relative price changes shifts consumption demand gradually towards the high-tech sector, and this helps overcoming the decline in the resource input. Yet, what makes sustained *growth* possible is the presumed unitary elasticity of substitution between a man-made input, in this case *knowledge*, and the resource. Thus, the general principle from Section 3 survives.

3. *Amenity value.* In addition to being valued as inputs in production, natural resources may be assets of value in their own right (amenity value, an argument in the utility function). Although this concern seems more prevailing in relation to environmental goods of a *renewable* resource character, Krautkraemer (1985) and Heal (1998) also study its implications in the context of non-renewable resources and its relation to sustainable development.

4. *Polluting non-renewable resources.* There may be negative externalities associated with the use of non-renewable resources, global warming being a glaring example. In the Suzuki (1976) paper there is a companion model to the one considered in Section 4.1. That companion model links the greenhouse problem to the non-renewable nature of fossil fuels. This is further developed in Sinclair (1994) and Groth and Schou (2006). An analysis closer to the global carbon cycle models of the climatologists is contained in Farzin and Tahvonen (1996). Schou (2000) and Schou (2002) study other aspects of (flow) pollution from use of non-renewable resources.

5. *Other issues.* We have completely passed over the role of *uncertainty* as to size of reserves, outcome of R&D activity, future technology, prices and interest rates. The reader is referred to, e.g., Chichilnisky et al. (1998), Weitzman (1998b, 2001) and Just et al. (2005). The problem of the *non-existence of a complete set of forward markets* (and therefore markets for contingent sales) and the associated stability problems were already intensively discussed in Dasgupta and Heal (1979). The *empirics* of resource scarcity are surveyed in Krautkraemer (1998).

## 7 Summary and conclusion

To the extent that non-renewable resources are necessary inputs in production, sustained growth requires the presence of resource-augmenting technical progress. New growth theory has deepened our understanding of mechanisms that influence the amount and direction of technical change. Applying new growth theory to the field of resource economics and the problems of sustainability yields many insights. The findings emphasized in this article are the following. (1) As expected, in view of the inevitable decline in resource input, whether technical change is exogenous or endogenous, essential non-renewable resources ultimately imply a drag on growth. (2) By calling attention to the non-rivalrousness of technical knowledge, new growth theory has circumvented the relationship between population growth and economic growth; contrary to the teaching implied by both the limits-to-growth exponents and the resource economics of the 1970s, population growth tends to be good for sustainability and economic growth; a possible counteracting factor, outside the framework considered here, might be that increased population density can generate congestion and aggravate environmental problems. (3) Whether or not there is population growth, endogenous technical change *may* bring about the technological basis for a rising per capita consumption in the long run or at least non-decreasing per capita consumption, but we can not be sure. (4) With diminishing returns to producible inputs, including knowledge, the long-run per capita growth rate is pinned down by growth in research effort. (5) Even when sustained growth is technologically feasible, if the rate of impatience is high enough, a utilitarian social planner's solution entails ultimately declining per capita consumption. (6) The standard approach to modelling endogenous technical change in a non-renewable resource set-up ignores that also R&D may need the resource (directly or indirectly). This biases the conclusions in an optimistic direction. Indeed, sustained per capita growth requires stronger parameter restrictions when the resource is "growth essential", than when it is not. (7) When the resource is "growth essential", then a policy aiming at stimulating long-run growth generally has to reduce the long-run depletion rate. In this sense promoting long-run growth and "supporting the environment" go hand in hand. This observation is of particular interest in view of the fact that changing the perspective from exogenous to endogenous technical progress means bringing a source of numerous market failures to light.

New growth theory has usually, as a simplifying device, considered population growth as exogenous. Given this premise, a key distinction - sometimes even controversy -

arises between what is called fully endogenous growth and what is called semi-endogenous growth. In mainstream new growth theory, where non-renewable resources are completely left out of the analysis, this distinction tends to coincide with three other distinctions: (a) that between models that suffer from non-robustness due to a problematic knife-edge condition and models that do not; (b) that between models that imply a scale effect on growth and models that do not; and (c) models that imply policy-dependent long-run growth and models that do not. When non-renewable resources are taken into account and enter the growth engine (directly or indirectly), these dissimilarities are modified: (i) the non-robustness problem vanishes because of the disappearance of the critical knife-edge condition; yet, fully endogenous growth does not become more plausible than before, rather the contrary; (ii) the problem of a scale-effect on growth disappears; (iii) due to the presence of two very different assets, producible capital and non-producible resource deposits, even in the semi-endogenous growth case there is generally scope for policy having long-run growth effects.

The results listed here are, of course, subject to modification to the extent that non-renewable resources may not be essential in the long run. Similarly, a thorough integration of environmental aspects in the analysis deserves much more attention than this review has allowed.

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