Modeling the Cattle Replacement Decisions

by

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Abstract

In this paper we evaluate the performance of a dynamic model of cattle replacement and culling decisions. We derive the price of cattle when it is treated as a unit of capital and evaluate various rates of adjustment of the cattle herd to determine the length of the cattle cycle. Replacement decision is modeled as the solution to a dynamic optimization problem where the breeding herd is viewed as a capital asset that is capable of producing two outputs: calves and culled cows. The own-price, replacement and interest rate elasticities calculated for both the short-run and long-run time-frames suggest fairly rapid adjustment rates. Tests of cycle length revealed a 14-year cattle cycle.

Key Words: cow-calf, dynamic duality, nonstatic expectation, cattle cycles

Introduction

The biological constraints of cattle production ensure that there is a lag between economic decisions and actual production. For example, producers often respond to price changes by adjusting the size of the breeding herd which, in turn, influences the number of calves or cattle put on the market one or more years hence. These lags and the biological nature of the livestock species prevents supply responses from fitting comfortably into a stylized modeling framework. In introducing his model, Buhr (1993) points out that "attempts to model the livestock industry have been a mixture of economic theory and ad hoc techniques." The numerous models of the cattle industry convey this point quite well (see Nerlove (1958), Antonovitz and Green (1990), Rosen (1986), Trapp (1986), Weimer and Stillman (1990), Marsh (1994), and Nerlove and Fornari (1998)).

This paper focuses on the crucial decision to control the size of the breeding herd. Following Nerlove and Fornari (1998), Buhr (1997) and Msafiri and Coyle (2001), we portray cow-calf operators as profit maximizers who manage their cattle assets over time. Thus, we present a dynamic optimization problem for cow-calf operators. The empirical application in this paper differs from Nerlove and Fornari's model by using a continuous time model to portray cow-calf operators. It differs from Msafiri and Coyle's model by assuming cow-calf operators have risk neutral preferences. Our model differs from the typical dynamic cattle model in that we assume cow-calf operators have nonstatic expectations of cattle prices and allow interest rates to vary.

We also portray both the replacement (investment) and culling equations as part of the dynamic decision.

Our goal in this paper is to evaluate the performance of a dynamic model of the replacement and culling decisions. Within this context, we investigate two issues: First, we derive the price of cattle when cattle are treated as a unit of capital. We argue that the level of aggregation of the data determines how the price of capital should be used in the model and experiment with the way producers respond to this capital price. Second, since our model does not assume that producers instantly adjust the size of their breeding herd, we evaluate various rates of adjustment of the cattle herd to determine the length of the cattle cycle. Then, we report both short-run and long-run elasticities for replacement and culling with respect to price.

The Long-run Replacement Decision

The replacement decision, the decision to set aside heifers for breeding purposes, is modeled as the solution to a dynamic optimization problem. In this model, the breeding herd is viewed as a capital asset that is capable of producing two outputs: calves and culled cows. Producers maximize profits over time by using replacement heifers and culled cows to manage the size of their breeding herds. Production in time period "t" is represented by the following transformation function:

$$F_t(X(t), Y_1(t), Y_2(t), L(t), B(t), \overset{\bullet}{B}(t), \tau) = 0 \tag{1}$$

where X represents feed inputs, Y_1 represents calves born, Y_2 represents culled cattle, L is pasture land, B is the capital asset and the dot over the B represents the equation of motion for B. The term τ represents the level of technology. In our model, there are two outputs: calves and culled cows. There are two variable inputs: hay and the breeding herd (capital input), and one quasi-fixed factor, pastureland.

The transformation function is assumed to have the properties of a typical transformation function with adjustment costs: It is continuous, twice differentiable, convex, and a closed set in $Y_1 Y_2, X, B$, and $\dot{B}$. It is strictly increasing in the outputs (Y_1 and Y_2) and strictly decreasing and convex in B. It is increasing (decreasing) in $\dot{B}$ and convex in $\dot{B}$.

The last assumption represents the adjustment costs. An increase in breeding stock diverts resources and can temporarily reduce output. For example, new breeding cows may compete for resources from existing breeding stock. First-calf heifers, that are not yet culled, may on average be less productive. A more explicit adjustment cost is the direct decline in calves sent to feedlots.

One unique feature of our cattle model, vis-à-vis other dynamic models of cow-calf operators, is that we allow cow-calf operators to anticipate price changes into the future when making their long-run decisions. That is, we portray cow-calf operators as having non-static price expectations. To implement this assumption, we use the method of Luh and Stefanou (1996) for incorporating non-static price expectations into a dynamic model. Thus, producers, who

optimize over time, plan for prices to change into the future. While this seems to be a reasonable assumption, a majority of the dynamic cattle models assume static price expectations.

Under the assumptions listed above, Luh and Stefanou show that equations of motions got prices must be included as constraints into the profit maximization problem. In this case, the profit maximization problem for cattle producers can be written as:

$$J(P_1, P_2, P_3, B, w, t, L)$$

$$= \underset{Y_1 \ Y_2 X, I}{Max} \int_t^\infty e^{-rt}[P_1(t)Y_1(t) + P_2(t)Y_2(t) - W_1(t)X_1(t) - P_3(t)B(t)]dt$$

$$Subject\ to:$$

$$F(Y_1(t)\ Y_2(t), X(t), L(t), B(t), \overset{\square}{B}(t), \tau) = 0$$

$$\overset{\bullet}{B}(t) = I(t) - Y_2 - \alpha B(t) \tag{2}$$

$$\overset{\bullet}{P}_i(t) = \vartheta(P_i(t), t), \quad for\ i = 1, 2, 3$$

where P_1 is the price of feeder calves, P_2 is the price of culled cattle, P_3 is the opportunity cost of breeder cows, W is the price of hay, t is time, L represents the quasi-fixed factor land and 'r'' represents the rate of interest. The dot above an equation represents an equation of motion describing the evolution of a variable. The term α represents the depreciation rate of the breeding herd. There is one equation of motion for the cattle herd (the state variable) as in the typical dynamic optimization problem. What stands out about the above problem, is that it includes an equation of motions for future prices. This uses the suggestion of Luh and Stefanou regarding incorporating producer price expectations into a dynamic model. Note that each equation of motion is a function of the level of the variable and time.

The above problem represents a standard dynamic optimization problem for producers that maximize profits when investment in capital assets creates adjustment costs. As in any dynamic profit maximization problem, it is assumed that production plans are continuously revised as new prices and information are observed. The control variables are Y_1, Y_2, X, and I and the state variable, B, is the cattle herd. The value function $J(.)$ represents the solution to the dynamic profit maximization problem and is a function of output prices, input price(s), the price of the breeding asset, and the level of technology, t, as well as the quasi-fixed factors such as land and management labor. The first constraint confines cow-calf operators to the technology as represented by the transformation function.

One distinct aspect of the above problem (relative to the majority of cow-calf models) is that producer expectations regarding output prices and the price of breeding stock follow a dynamic process over time. That is, equations describing the evolution of price for both outputs and the price of capital are included as constraints in the optimization problem. This represents Luh and Stefanou's method for modeling nonstatic price expectations in a dynamic optimization model. The equations of motion for price ($\overset{\square}{P}_i$) account for future price expectations of livestock producers. Hay prices, in contrast, are modeled with static expectations since hay prices often are a function of unpredictable weather variables.

The dynamic choice model can be converted into a static equivalent called the Hamiltonian-Jacobi equation (Kamien and Schwartz (1991)). The Hamiltonian equivalent of Equation 2 is:

$$rJ(P_1, P_2, P_3, B, w, t) = \underset{Y_1, Y_2, X, I}{Max} \{P_1(t)Y_1(t) + P_2(t)Y_2(t) - W_1(t)X_1(t) - P_3(t)B(t)\}\}$$

$$+J_{p1}\vartheta(p_1, t) + J_{p2}\vartheta(p_2, t) + J_{p3}\vartheta(p_3, t)$$

$$\tag{3}$$

$$+J_b(I - Y_2 - \alpha B) + J_t$$

where the derivatives of the value function *J(.)* with respect to P_i, (i=1,2,3), and *B* are represented as J_{pi}, and J_b, respectively. In the above problem, the term r represents the rate of interest and *rJ(.)* equals the value of profits in one period.[1]

The advantage of the static version of the model, (the Hamiltonian) is that the principles of duality can be applied to derive the properties of the value function (see Epstein, 1981). However, the key advantage of the static version to the dynamic problem is that the envelope theorem can be used to derive the output supply, input demand, and investment equations. Luh and Stefanou show that when future prices are expected to change, the two outputs can be derived from the following derivatives of the value function:

$$y_1 = rJ_{p1} - [J_{p1,p1}\vartheta(p_1, t) + J_{p1,p2}\vartheta(p_2, t) + J\vartheta(p_3, t)$$
$$+J_{p1}(\delta\vartheta(p_1, t)/\delta p_1) + J_{p1,b}(I - \alpha B - Y_2) + J_{p1,t}]$$

$$\tag{4}$$

$$y_2 = rJ_{p2} - [J_{p2p2}\vartheta(p_2, t) + J_{p2p1}\vartheta(p_1, t) + J_{p2,p3}\vartheta(p_3, t)$$
$$+J_{p2}(\delta\vartheta(p, t)/\delta p_2) + J_{p2,b}(I - \alpha B) + J_{p2,t}]/(1 - J_{p2,b})$$

$$\tag{5}$$

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[1] The above problem is a standard constrained optimization problem. However, following the convention established by Epstein (1981) we represent the Lagrange multipliers by their economic equivalent. For example, J_b, represents the shadow price of the breeding herd and is equivalent to the Lagrange multiplier.

$$\dot{B} = J_{b,p3}^{-1}[rJ_{p3} + B - J_{p1,p3}\vartheta(p_1,t) - J_{p2,p3}\vartheta(p_2,t)$$

$$-J_{p3,p3}\vartheta(p_3,t) - J_{p3}(\delta\vartheta(p_3,t)/\delta p_3)] \tag{6}$$

In the above model, equation 4 is a calf equation, equation 5 is a cull cow equation, and equation 6 is an equation of motion for the cattle herd. Note that culled cattle (y_2) are represented as an output: since culls are supplied to the market and slaughtered for meat. However, culled cattle also influence the size of the breeding herd. Because of the twofold role played by potentially culled cows, there is an additional term in the cull equation. Therefore, the culling equation is slightly different from the standard supply function represented in dynamic models. This explains the difference in the functional form between the first and second supply equations. Finally, the herd adjustment equation ($\dot{B}$) should be viewed no differently from a standard capital difference equation in a dynamic optimization model.

Given a particular specification for the value function for each of the above three equations, ($y_1, y_2, \dot{B}$) can be derived by applying the above envelope theorem to the value function. The 3 equations represent a system, which can be jointly estimated. Once estimated, the response of replacements (investment) to a price change can be obtained by solving the equation of motion for replacement. That is:

$$B(t) = I(t) - C(t) - \alpha B(t) \tag{7}$$

So that replacement (investment) can be written as:

$$Rpl = I(t) = \overset{\bullet}{B}(t) + Y_2 + \alpha B(t) \tag{8}$$

And the effect of prices on replacement is:

$$\partial Rpl / \partial p_i = \partial I / \partial p_i = \partial(\overset{\bullet}{B}) / \partial p_i + \partial(Y_2) / \partial p_i \tag{9}$$

A Diversion on Prices and Aggregation

The model presented in this paper includes three separate cattle prices: the price of calves, the price of culled cows, and the replacement price (the opportunity cost of setting aside breeding cows). The price of feeder calves and the price of cows (culls) were readily available from USDA. The replacement price is another matter. There are two possible ways to represent the opportunity cost of setting aside a heifer to be used as a breeding cow. The correct way to represent this opportunity cost depends on (1) the level of development of the breeder cattle market and (2) the level of aggregation in a cow-calf model.

If breeding cows are sold back and forth among cow-calf operators, the cost of setting aside heifers to be used for breeding is equivalent to the cost of purchasing one's own asset (see appendix). This represents the opportunity cost of not selling the cow to another cow-calf operator who would view the breeding animal as a capital asset and be willing to pay capital asset prices for the animal (see Mathews and Short, 2001). If no such market exists for

exchanging breeding cows among cow-calf operators then the off-ranch value for a heifer, the

feeder calf price, is the best representation of the cost of investment.

However, the level of model aggregation also determines how to represent the price of a

breeding cow in a cattle replacement model. For example, even if an internal market exists for

breeding cows, if the entire cow-calf sector is modeled as an aggregate, the cost of setting aside

heifers for breeding is the opportunity cost to the entire cow-calf sector. This would be equal to

the feeder calf price.

Anecdotal evidence concerning the cow-calf sector suggests either opportunity cost could be

viable for a model using non-aggregate data. However since we model the entire cow-calf sector

in the aggregate, this suggests that the feeder calf price may represent the best proxy for the

opportunity cost of setting aside a heifer to be used as a breed cow. In any case, we choose to

estimate two models, one where the opportunity cost of replacement is represented by the asset

price of cattle and one where it is represented by the feeder calf prices.

Empirical Model

Our first step was to specify a value function. This is specified as:

$$
\begin{aligned}
J(p_1, p_2, p_3, L, B, tr) &= \sum_{i=1}^{3} \beta_i \, p_i + 0.5 \sum_{1}^{3} \sum_{1}^{3} \beta_{ij} \, p_i \, p_j + \sum_{i=1}^{3} \beta_{ib} \, p_i \, B \\
&+ \sum_{i=1}^{3} \gamma_{it} p_i tr + \sum_{i=1}^{3} \gamma_{itl} p_i tr * ld + \gamma_{tl} \, tr \, ld + \gamma_{tt} \, tr^2 + \gamma_{bb} \, B^2
\end{aligned}
\tag{10}
$$

where p_1 is the expected price of the calf output, p_2 is the expected price of cull cows, and p_3 is the investment price or opportunity cost of breeder cows. All prices were normalized by the hay price. The stock of breeder cattle is represented by B, technology by tr, and land by ld.[2]

Given this specification, the calf equation can be written as:

$$y_1 = (r-v_1)\beta_1 + \beta_{11}(r-v_1)\,p_1 + \beta_{12}(r-v_1)\,p_2$$

$$+ \beta_{13}(r-v_1)\,p_3 + \beta_{bl}(r-v_1)\,B + \gamma_{it}(r-v_1-1)*tr - \beta_{ib}\dot{B} \qquad (11)$$

$$- \beta_{11}*\mathcal{G}(p_1,t) - \beta_{12}*\mathcal{G}(p_2,t) - \beta_{13}*\mathcal{G}(p_3,t) - \gamma_{il}*ld$$

Where the term v_i is the derivative of the equation of motion for p_i with respect to own price :

$$\delta\mathcal{G}(p_i,t)/\delta p_i = v_i$$

$$for \quad i = 1,2,3.$$

The culling equation can be written as:

$$y_2 = \{\beta_2(r-v_2) + \beta_{21}(r-v_2)\,p_1 + \beta_{22}(r-v_2)\,p_2$$

$$+ \beta_{23}(r-v_2)p_3 + \beta_{2b}(r-v_2)\,B + \gamma_{2t}(r-v_2-1)tr \qquad (12)$$

$$- \beta_{2b}*\dot{B} - \beta_{21}*\mathcal{G}(p_1,t) - \beta_{22}*\mathcal{G}(p_2,t) - \beta_{23}*\mathcal{G}(p_3,t) - \gamma_{2tl}Ld\}$$

and the asset difference equation becomes:

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[2] As noted earlier there are various possible ways to represent p_3, the opportunity cost of setting aside a heifer to be used a breeding cow.

$$\dot{B} = \{\beta_{b3}\}^{-1}[\beta_3(r-v_3) + \beta_{31}(r-v_3)\,p_1 + \beta_{32}(r-v_3)\,p_2$$
$$+ \beta_{33}(r-v_3)\,p_3 + \beta_{b3}(r_1-v_3)\,B + \gamma_{3t}(r-v_3-1)tr + B$$
$$- \beta_{31}\mathcal{G}(p_1,t) - \beta_{32}\mathcal{G}(p_2,t) - \beta_{3b}\mathcal{G}(p_3,t) - \gamma_{3tl}Ld]$$

(13)

Luh and Stephanou represent the discrete version of a differential equation in prices with a first-order differential equation in prices. The prices evolve as:

$$\dot{P} = v_i + \tilde{v}_i\, p$$

so that:

(14)

$$\delta\mathcal{G}(p_i,t)/\delta t) = v_i = \tilde{v}_i$$

Substituting I-Y_2 in for $\overset{\square}{B}{}^3$ and solving for y_2 produces:

$$y_2 = \{\beta_2(r-\tilde{v}_2) + \beta_{21}(r-\tilde{v}_2)\,p_1 + \beta_{22}(r-\tilde{v}_2)\,p_2$$
$$+ \beta_{23}(r-\tilde{v}_2)p_3 + \beta_{2b}(r-\tilde{v}_2)\,B + \gamma_{2t}(r-\tilde{v}_2)*tr$$

(15)

$$- \beta_{2b}*I - \beta_{21}\tilde{\mathcal{G}}_1 - \beta_{22}\tilde{\mathcal{G}}_2 - \beta_{23}\tilde{\mathcal{G}}_3\}/(1+\beta_{2b})$$

where ~ over the v and equation of motion terms represent the estimated values of these terms.
Similary, substituting expectations into the herd difference equation and rearranging obtains:

$$\dot{B} + B(\tilde{v}_3 - r) = \{\beta_{b3}\}^{-1}[\beta_3(r-\tilde{v}_3) + \beta_{31}(r-\tilde{v}_3)\,p_1 + \beta_{32}(r-\tilde{v}_3)\,p_2$$
$$\beta_{33}(r-\tilde{v}_3)\,p_3 + \gamma_{3t}(r-\tilde{v}_3)*tr - \gamma_{3tl}*Ld - \beta_{31}\tilde{\mathcal{G}}_1 - \beta_{32}\tilde{\mathcal{G}}_2 - \beta_{33}\tilde{\mathcal{G}}_3] + \{\beta_{b3}\}^{-1}B$$

(16)

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[3] At this point, we choose to ignore death loss, an assumption, which probably has little effect on our model.

Estimation

Following Luh and Stefanou, the price difference equations were estimated *a priori* and used to obtain expected prices as well as the v_i parameters. The three-equation system can be estimated through iterative seemingly unrelated regressions (SUR). We estimated two specifications: one in which we used the asset price of cattle to represent the replacement price and one with the feeder calf price as a replacement price.

Each three-equation model was nonlinear in the parameters and the parameters failed to converge when estimating the model.[i] This left two options. One option was to estimate a reduced form model, that would be linear in the parameters, but that would prevent us from imposing symmetry. The second option was to impose a few parameters of the structural form model. We chose the latter. That is, we set the β_{2b} parameter close to its final value in the non-convergent model and parametrically varied the β_{v1} parameter.[ii] Both models reached the highest likelihood function at the same value of β_{v1} (discussed below).

Having set the value of the β_{v1} parameter, next we proceeded to test the cattle asset price model against the opportunity cost model. To do this, we applied the Davidson and Mackinnon systems test for non-nested models. Our objective was to determine the correct specification: a model that used an asset price in the replacement equation or a model that used the opportunity cost (feeder price) in the replacement equation. Similar to most non-nested tests, the Davidson and Mackinnon test is equivalent to setting up a compound model, which is a weighted average of two models, and testing the weighting parameters. In the Davidson and Mackinnon systems test,

a T-test is applied to a transformed variable, which is equivalent to applying a test on the parameter that weighs both models.

The Davidson and Mackinnon test is performed twice using each model as the null. Either model can be rejected in favor of the other, or both models can be rejected. Using the capital price model as the null, we obtain a test statistic of 21.63, which rejects the capital price model in favor of the opportunity cost (feeder price) model. However, when reversing the test and using the opportunity cost (feeder price) model as the null, we obtained a test statistic of 21.03. This led to a rejection of the feeder price model in favor of the capital price model. In light of this ambiguous result, we used informal criteria to select models. The feeder price model was chosen since in the asset price model both supply curves were downward sloping while in the feeder price model supply curves were upward sloping. While a dynamic model does not preclude a downward sloping supply curve (as do static models), we choose to report the opportunity cost (feeder price) model.

Adjustment rate.

The β_v parameter is critical to determining the rate of herd size adjustment. When the adjustment rate of the breeding herd was left unrestricted, either model became so nonlinear (in the parameters) that they failed to converge. This inability to nest the various adjustment rates within a more general model without introducing convergence problems precludes reporting a formal test on the rates of adjustment. In light of this, we ran various restricted models several times over, imposing various adjustment rates. This exercise can be viewed as parametrically

varying the adjustment rate to determine which adjustment rate produced a model with the highest likelihood function.

We first imposed an adjustment rate equal consistent with an 8-year cycle, a 10-year cycle, a 12-year cycle, and then a 14-year cycle. Then we allowed the cycle to change after 1987, the mid-period of the database and a period during which some livestock analysts believe the traditional cattle cycle may have become longer. Then we imposed a base cycle of 8 years (and then 10, 12, and 14 years) and allowed this base cycle to change to each of the other 3 cycles after 1987.

Table 1 reports the likelihood function of these various restricted models. The diagonal of table 1 represents models that do not change. For example, the upper left hand corner represents a model with an 8-year cycle throughout, the next diagonal element represents a model with a 10-year cycle throughout, until we reach the right hand corner which reports the likelihood function for a model with a 14-year cycle throughout. The best performing model is the one with a 14-year cycle through the whole period, which is longer than the cycle length found in previous studies by Rosen (1986) and Trapp (1986) who found 10-12-year cycles in studies from an earlier time period.

Finally, cattle models often assume that the calving rate is a proportion of herd size. Our model, like that of Msafiri and Coyle and others, portrays calves as an output subject to economic factors. We tested whether economic variables (prices and interest rates) influence the calf equation by setting the coefficients on price and interest rate terms equal to zero. A system likelihood test was applied and produced a χ^2 coefficient of 108, which was significant at the .01

percent confidence level. Imposing the restriction that economic factors did not influence the calving rate significantly reduced the fit of the model. This indicates that these economic factors can influence the calving rate.

Elasticities

The short-run supply elasticities were obtained by taking the derivative of the supply equations with respect to price and evaluating the elasticity at the means of the data. The short-run replacement elasticities were obtained by taking the derivative in Equation 8 and evaluating the data at the means. This derivative involved parameters from both $\dot{B}$ and cull equations. The long run represents the steady state or when the herd size does not change. The long-run elasticities are obtained by the formula:

$$\varepsilon_{ij}^{L} = \{(\partial Y_i / \partial p_j)_{B=\overline{B}} + (\partial Y_i / \partial \overline{B})(\partial \overline{B} / \partial p_j)\}(p_j / Y_i) \tag{17}$$

where $\overline{B}$ represents cattle herd in the steady state. It is derived by setting $\dot{B}$ to zero and solving for B.

Table 2 reports various short-and long-run elasticities. The price elasticities in the first output supply equation represent the feeder calf elasticities and are quite low as would be expected. The long-run elasticities are only slightly higher. A rise in the cull-cow price increases the number of culls while a rise in replacement cattle price decreases the number of culls. This should be

expected. However, unexpectedly, an increase in feeder calf prices also decreases the number of culls. As with the calf equation, the long-run elasticities are slightly higher.

The replacement elasticities were calculated using Equation (8), which required that we combine the derivatives of the $\overset{\square}{B}$ and Y_2 (cull) equations. This explains why some of the symmetric elasticities do not appear to be of the same sign. The short-run price elasticities in the replacement equation are fairly high relative to those of the other two equations. Yet the own-price and cross-price elasticities with respect to culls have the expected signs. The price elasticities of the calf output were negative, which was an unexpected result. However, the coefficients used to calculate this elasticiticity were not signficant.

Interestingly, the long-run and short-run elasticity of replacement with respect to a change in cull price works out to be the same. The long-run replacement own-price elasticity is slightly higher in the short run, providing some indication that cow-calf operators may overshoot, in the short run, in their response to a change in the opportunity cost of replacement.

Finally, since our model allows interest rates to vary, we are able to report interest rate elasticities. Viewing cattle as an asset, with the foregone value being the interest on money assets, one would expect the interest rate elasticity to be negative in the replacement equation. Our interest rate elasticity was negative and quite elastic. However, our cull elasticity, while

smaller, was also negative, which is counterintuitive.[4] If the rate of return on money assets rose, one would expect the number of culls to increase.

Conclusion

This paper evaluates how a dynamic model of the beef cow replacement decision performs. The replacement decision is modeled as the solution to a dynamic optimization problem where the breeding herd is viewed as a capital asset that is capable of producing two outputs: calves and culled cows. We represent producers as maximizing profits over time by using replacement heifers and culled cows to manage the size of their breeding herds. The empirical model was evaluated as a value function where the output prices of feeder calves and culled cows were represented by expected prices, and the replacement calf price was treated as the asset price in one model and an opportunity cost (feeder calf price) in another model.

The Davidson and Mackinnon test was used determine whether the capital price model or the opportunity cost model was the best representation of the transformation model. The test was inconclusive. Thus, the feeder price model was chosen since it had an upward sloping supply curve while the asset price model had a downward sloping supply curve. This was considered an informal method of model selection since a dynamic model does not preclude a downward sloping supply curve.

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[4] Numerous parameters were used to calculate the interest rate elasticity and it was not possible to establish a significance level.

The length of the cattle cycle was also determined. By parametrically varying the adjustment rate over an 8 to 14 year time-period, we observed that the 14-year cycle performed best by producing a model with the highest likelihood function. The 14-year cycle is consistent with the views of many analysts. The own-price, replacement and interest rate elasticities were calculated for both the short-run and long-run time-frames.

The feeder calves price elasticities are quite low as would be expected, and long-run elasticities are only slightly higher, suggesting fairly rapid adjustment. As expected, the increase in the cull-cow price increases the number of culls, while an increase in the replacement cattle price decrease the number of culls. This latter result was counterintuitive.

The short-run price elasticities in the replacement equation were fairly high relative to that of the other two equations with expected signs for own price and cross price elasticities with respect to culls. The price elasticities of the calf output was negative which was an unexpected result.

The interest rates elasticities were also calculated. Viewing cattle as an asset with the forgone value being the interest on money assets, one would expect the interest rate elasticity to be negative in the replacement equation. Our interest rate elasticity was negative and quite elastic. However, our cull elasticity, while smaller, was also negative, which is also counterintuitive.

Several items remain to be explored. First, alternative functional forms should be tried to deal with convergence problems. Second, alternative forms of more sophisticated or perhaps, even more workable price expectations should be explored. Finally, there may be some gains to

modeling the decisions of cow-calf operators with downstream operators such as feedlot

operators.

Table 1 Likelihood function at Different Adjustment rates:

	Years to Adjust			
	Eight	Ten	Twelve	Fourteen
Eight	-1962.64	-1958.94	-1968.94	-1957.60
Ten	-1963.30	-1895.82	-1935.96	-1902.51
Twelve	-1933.42	-1890.90	-1888.50	-1893.65
Fourteen	-1931.60	-1893.32	-1883.34	-1882.89

1/ The diagonals represent the likelihood function when there is
no change. The off diagonals represent the likelihood function when there is
a change after 1987. For example the number –1962.64 is the likelihood function when
an eight year cycle was imposed. The number –1958.94 is the likelihood function when a eight
cycle was imposed until 1987 and then a ten year cycle is imposed afterwards.

Table 2: Elasticities

	Y1-calves	Y2-cull	Y3-replacement
P1-(calves) Short run	.391	-.183	-.88
P1-calves Long Run	.417	-.22	-1.06
P2-cull Short run	-0.014	.43	2.15
P2-cull Long run	-.03	.46	2.15
P3-Rplacement	.063	-0.54	-2.68
P3-Rplacement	.107	-.62	-2.30
Interest rates	.25	-.524	-2.77

1/ The replacement elasticities are derived from equations 2 and 3 Hence
cross price elasticities need not be same or opposite sign.

Table 3 --The estimated equations

Calves Born

		Parameter	T-stat
B1	R	-5144	-0.19
B11	P_1(r-v)	-1435.8	-1.28
B12	P_2(r-v-1)	842.54	0.99
B13	P_3(r-v-1	-202.19	-0.99
B1b	R*B-dB	2.47	1.45
.	Past	.0035	0.035
γ_{it}	T*(r-v-1)	-4.94	-0.20
Bt11	Dm	-7210.1	-8.10
Bt12	D2	-4770.2	-4.60
Bt13	D3	3896.5	7.71

Cull Cows

		Parameter	T-stat
B2	R	-66031	-3.52
B21	P_1(r-v-1)	842.54	0.99
B22	P_2(r-v-1)	-1435.1	-2.20
B23	P_3(r-v-1)	2166.8	4.29
B2b	R*B-dB	-4.1	Na
-B2b	Past	-.036	-2.81
γ_{it}	T*(r-v-1)	71.54	9.53
Bt21	Dm	472.62	5.22
Bt22	D2	584.22	6.37
Bt23	D3	66.436	0.76

Herd Differences

		Parameter	T-stat
B_1	RRC	-55224	-1.79
B_{31}	P_1(r-v-1)	-202.19	-0.99
B_{32}	P_1(r-v-1)	2166.8	4.29
B_{33}	P_1(r-v-1)	-298.51	-2.86
B_{b3}^{-1}	Hds	1.19	NA
	Past	0.0235	1.62
γ_{it}	RTRB	-12.96	-7.25
Bt31	Dm	-450.11	-4.81
Bt32	D2	-552.50	-5.79
Bt33	D3	-41.12	-0.45

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Appendix

Pricing Cattle as an Asset

Since a classic paper written by Jarvis (1982) economists often have viewed cattle as a capital asset. Yet cattle models do not always use asset-pricing formulas to derive the representative price of cattle. Even if asset prices are available it is not immediately clear how these prices should be incorporated into a dynamic herd management model. This will be discussed below.

First, to derive the cattle asset price the standard capital asset pricing formula can be used (Jorgenson, Mathews and Short (2001)). This is:

$$Pk(t) = \sum_{t=1}^{N} (P_t^e - w_t^e \gamma_1) Q^e /(1+r)^t + P_s^e \qquad (1a)$$

Where P^e is the expected price of a feeder calf price in year "t", w^e the expected hay price in year t, and γ is the amount hay consumer per calf, and r equals the rate of interest.

In order to calculate asset prices we needed to derive market expectations of future calf prices. This would be distinct from the expectations of a representative cow-calf operator who may not use, for example, information relevant to feedlot operators. Expectations of the t+i step ahead feeder calf price were derived by a setting stockyard demand equation for feeder calves equal to a supply equation for feeder calves and then, solving for the feeder calf price, p. [5] Thus to present future expected feeder prices we modeled the following equation:

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$$P_t^e = \pi * P_{t-1} + \beta_1 * pcrn_{t-l} + \beta_2 * psy_{t-l} + \beta_3 * phay_{t-l} +$$
$$\beta_4 * plst_{t-l} + \beta_5 \dot{B}_{t-1} + \alpha_1 dpcrn_{t-l} + \alpha_2 * dpsy_{t-l} + \alpha_3 * dphay_{t-l} + \qquad (2a)$$
$$\alpha_4 * dplst_{t-l} + \alpha_5 d\dot{B}_{t-1} + \gamma_1 * tr^{1/2} + \gamma_2 * tr + \gamma_3 * tr^2 + \gamma_4 * tr^3$$

The letter d in front of a variable represents first differences. The t-l represents a one period lag. For example $pcrn_{t-l}$ represents a corn prices lagged one period. The term $dpcrn_{t-l}$ represents the difference between the corn price in year t and t-l.

In the above equation each explanatory variable is represented in both levels and in differences and a lag value of the endogenous variable is included. This specification is consistent with an error correction model. Thus, we allow for the possibility that markets correct for past forecasting errors which assumes some form of rationality on part of markets[6] Since prediction occurs over time, a polynomial in trend representing the influence of technology on prices was included in the specification.

Once expectations of future feeder prices were derived, the series P_k (the price of capital) was calculated. T hen the P_k series was treated as any other price. That is the capital price variable could be transformed, forecasted "x" periods ahead, or lagged "x" periods.

Output prices

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[6] . The price of corn (prcn), soybeans (psy), and slaughter prices (plst) come from the feedlot demand equation. The hay price and B dot equation come out of the producer supply term.

In our model the price of the output, a calf, is represented by a weighted probability of the value

of its uses and is:

$$P_1 = \eta_{u1} * P_{fl} + \eta_{u2} * P_{fl} + \eta_{u3} * P_{k+} \qquad (3a)$$

where P_{fl} is the expected feeder calf prices 6 quarters ahead, P_k is the price of cattle when viewed

as a capital asset. The term η_{u1} represents the expected probability that the calf will be sent to a

feedlot, η_{u2} represents the expected probability that the calf will be sold to another rancher who

will background it and then sell it to a feedlot, and η_{u3} represents the probability the calf will be

set aside for breeding purposes. Combining terms the price becomes:

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$$P_1 = (1 - \eta_{u3}*) P_{fl} + \eta_{u3} * P_{k+} \qquad (4a)$$

Since approximately half of the calves born are heifers, *a priori* we know that η_{u3} is at least

smaller than 0.5.

Finally, naïve expectations were used to predict hay prices, which often is a proxy for pasture

conditions. Hay prices would be difficult to predict with a model, since weather plays a major

role in determining pasture conditions.[i]