Levels or Differences in Meat Demand Specification

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Abstract

We estimated a wholesale demand system for beef, pork, lamb, chicken, and turkey using quarterly U.S. data and a dynamic, CBS system (Keller and Van Driel). The CBS system is a differential system, which means that it might be more appropriately applied in those situations where the data have unit roots. If there are unit roots, differencing the data can improve the properties of the estimates. If the data do not have unit roots, differencing the data might harm the properties of the estimates.

We tested the specification of the model's error terms using state-space techniques. State-space units allow one to deal with roots on the unit circle without filtering the data (See Durbin and Koopman). The demand system has only four independent error terms. The state-space model we used could have decomposed these four independent error terms into four errors with unit roots and four with 0 roots. Adding state-space features to the model greatly improved its performance as measured by the likelihood ratio statistics. The estimates imply that the raw demand data have two unit roots and three 0 roots. Our mixed approach improves the properties of the estimates.

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Introduction

This paper is a preliminary report on our research into quarterly meat demand in the United States. We estimated a wholesale demand system for beef, pork, lamb, chicken, and turkey using dynamic, CBS system (Keller and Van Driel). Our raw data ran for 88 quarters, 1970 to 2000 inclusive. We had 86 usable observations after dealing with the lags and differences inherent in our specification. We assume that the demand for these five meats is separable from the demands for other goods. We selected the CBS system because it has a number of attractive features. Its greatest advantage from our point of view is that it is linear in its parameters, although making the model dynamic introduces some non-linearities. The equality restrictions of constrained optimization theory can be incorporated using linear restrictions.

The endogenous and exogenous variables in the CBS system are functions of differences in the logarithms of quantities, prices, and expenditures. The CBS and other differential systems might be more appropriately applied in those situations where the underlying price and quantity data have unit roots. Differencing or filtering data will improve the statistical properties of estimates if the data has unit roots. It will degrade the estimates' properties if the raw data does not have unit roots, or if the wrong type of filter is used.

In this paper, we deal with two types of roots on the unit circle, 1 and –1. Differential demand systems will eliminate roots equal to one if the raw data have that unit root. If the "raw" data do not have roots equal to 1, transforming it to the CBS specification might induce roots equal to –1. We deal with the unit root cases by using state-space econometrics to specify the error terms of the demand system. Our application has some special features that allow us to simplify the more general state-space model. The state-space approach allows us to take the two unit-root cases and build a model that nests both roots of 1 and –1.

Econometric Models

We focus on the state-space features of the model and briefly describe the CBS model. More details about CBS can be found in Keller and Van Dreil, or in Barten and Bettendorf. The CBS model relates a function of the changes in quantities demanded to changes in prices and changes in the scale of quantity demanded. The CBS system was designed as a consumer demand system, and can be made consistent with all the theoretical properties of consumer demand system. Our data is wholesale data, and represents a derived demand system. However, this derived demand system is conditional on the overall "scale" of meat output. These types of conditional derived demand systems have the same types of economic restrictions as consumer demand systems. The data is quarterly U.S. disappearance of beef, pork, lamb, chicken, and turkey. The prices for beef and pork are Economic Research Service estimates of wholesale values. The lamb price is based on the carcass lamb price published by the Agricultural Marketing Service (AMS). Chicken and turkey prices are the whole-bird prices also published by AMS.

The "straight" CBS model is linear in its parameters and the equality restrictions of optimization theory can be imposed using linear restrictions. We allowed for dynamic adjustment, using a procedure developed by Anderson and Blundell. Their structure makes it relatively easy to recover the long-run coefficients from a general, dynamic model. Our CBS model is written as:

(1) $ddY(t) = -a*dY(t-1) + (ddX(t-1)*c + dX(t-1)*a)*B + Z(t)*C + w(t).$

The term ddY(t) is the differenced vector of endogenous variables and dY(t-1) is the lagged endogenous variable. We use "dY" instead of "Y" because the CBS endogenous variables are functions of the first differences of quantities. The CBS is one of those demand systems with a singular covariance matrix, and is estimated by deleting an equation. The estimates are invariant to the equation deleted; we deleted the beef equation. Likewise, ddX(t) is the vector of differences in the CBS exogenous variables and dX(t-1) is the lagged exogenous vector. CBS exogenous variables are functions of the differences in prices and scale. The terms "a" and "c" are scalar adjustment coefficients and "B" is a matrix of long-run coefficients. Note that the lagged endogenous and exogenous terms are multiplied by the same scalar, adjustment coefficient (except for sign). This model is a special case of Anderson and Blundell's most general model. In this form, the long-run elasticities of demand are a function of the B vector. The short-run price and scale responses are c*B. If "c" is equal to 1, then (1) is consistent with first-order autoregression. We imposed the equality restrictions implied by constrained optimization on the B matrix estimates. Optimization theory also requires that the compensated price effects be negative semi-definite, the negativity constraint. Keller and Van Driel showed that the negativity constraint held if the price terms of the CBS coefficient matrix are also negative, semi-definite. We imposed these non-linear, inequality restrictions on the B estimates.

The Z(t) variables include quarterly dummies and intercepts. The intercept and full set of quarterly dummies are linearly related. To eliminate colinearity, we required the quarterly dummies to sum to 0 across quarters for each of the species' meat. The singularity of the CBS system makes the intercepts and quarterly dummies sum to 0 across meats. In these differential models, the intercept is often called “a taste-shifter.” A non-zero intercept implies that the demand for a product will change even if prices and expenditures do not change.

An intercept gives a constant drift in tastes or technology over time. An intercept implies that the underlying shift is nothing more than a linear function of time. We wanted to allow for a more flexible kind of shift. We replaced the linear function of time with a quadratic function of time. We also divided the sample into four 22-quarter periods, and allowed for different intercept, trend, and trend-squared effects in each of the sub-periods. We required that the adjoining functions imply the same values at the end periods. The first function and the second function have the same value in quarters 22 and 23; the second and the third are the same in quarters 44 and 45, while the third and fourth match in quarters 66 and 67. Our taste-shift is a discrete, quadratic spline function. We required the spline terms to start at 1 in quarter 1 and end at 88 in quarter 88, just as a trend term would. We used all these restrictions to reduce the 12 spline terms to 5 restricted terms. The taste-shift variable effect can be written as:

(2) $D*[dS(t)*F]$

In equation (2) D is a vector of intercept-like parameters, dS(t) is a vector of changes in the spline terms for quarter “t”, while F is another vector that determines the pattern of the taste shift.

The last term in equation (1) is “w(t),” the random error term of the model. We considered two extreme cases for the time-series properties of w(t). If w(t) is independently and identically distributed (iid) over time, we called it e(t). Because the CBS model is based on differenced data, iid error terms in the CBS model imply that the stochastic parts of the raw data have unit roots. The second case is where the stochastic parts of the raw data are iid. The differencing underlying the CBS model will then induce moving-average autocorrelation into equation (1). We will write this error term as:

(3) w(t) = u(t) – u(t-1).

For the purposes of comparing the two extreme cases, it would be helpful to have an intermediate case that nests both. This is our state-space model, where the error term can now be written as:

(4) w(t) = e(t) + u(t) – u(t-1).

In specifying equation (4) we assume that the e(t) and the u(t) are both iid over time and the e(t), and u(t) are independent of each other. The structure of equation (4) implies that w(t) is going to exhibit some autocorrelation, which implies that knowing w(t) will help one predict what w(t+1) is going to be. We derived functions that give us the optimal predictor of w(t+1), given the information available in time "t." Durbin and Koopman derived formulas for very general state-space models. We used their solutions, and thus refer the reader to their book for the derivations.

The general procedure in state-space modeling is to use the information available in time period t-1 to make the best, conditional forecast in time period t. We began with the assumption that we know the covariance matrix for the u(t) and e(t): σ_u and $\sigma_{u,}$ respectively. The variance matrix for the prediction w(t) was called σ_W(t); in general, this matrix changes over time. Along the way, we derived estimates of u(t) at time "t," u(t|t). The estimate of u(t) is likely to differ from its true value, and we can calculate the variance of the difference between u(t) and u(t|t). The variance of the difference between our estimate of u(t) and its true value is σ_z(t). Again, this matrix can vary over time.

In the first time period, call it "1," we had no information to help us predict what w(1) would be. Our prediction of w(1) was 0. The variance of our prediction of w(1) is:

(5) $\sigma_w(1) = \sigma_e + 2*\sigma_u$

The variance in equation (5) is the variance of the e(1) error term and twice the variance of u(). It is twice the variance of the u as w(1) has both u(0) and u(1) in it. Once we actually saw w(1), we then used this information to get an estimate of u(1). Using Durbin and Koopman's general rules, we then estimate:

(6) $u(1|1) = \sigma_u *[\sigma_w(1)]^{-1} w(1)$,

The variance matrix for the difference between the true u(1) and its estimate is:

(7) $\sigma_z(1) = \sigma_u - \sigma_u *[\sigma_w(1)]^{-1} \sigma_u$.

Our problem has two advantages over Durbin and Koopman's more general problems. First, we have fewer problems initializing our estimates. Our initial estimate of u(0) is a vector of zeros, and we know that the variance of this estimate is σ_u. Durbin and Koopman spend a great deal of space in their book on initializing the state variable and its variance. The second advantage we have is that we need to keep track of fewer variance matrices. One of the things that wedo not show is the new, improved, estimate of u(0), or u(0|1). In more general state-space models, we would need it and its variance to help forecast next period's value. In our case, u(0) is never seen again, so we did not calculate its updated value or variance. In theory, updating u(0) is not a problem. In practice, it is another set of calculations for our estimation routine that we can eliminate without affecting the end result.

Our forecast of w(2) given our information in time period 1 was –u(1|1). This produced 3 sources of error in our forecast of w(2). Both e(2) and u(2) are unpredictable, and our estimate of u(1) is (possibly) inaccurate. Our forecast variance was the variance of w(2)+u(1|1), which is equal to:

(8) $\sigma_w(2) = \sigma_e + \sigma_u + \sigma_z(1)$.

Our estimate of u(2) and its variance are:

(9) $u(2|2) = \sigma_u *[\sigma_w(1)]^{-1} (w(2) + u(1|1))$

(10) $\sigma_z(2) = \sigma_u - \sigma_u *[\sigma_w(2)]^{-1} \sigma_u$.

We continued to loop through the time periods in this manner, getting new predictions for u(t|t) and the time-varying variance matrices. As Durbin and Koopman noted, in problems of this type, the matrices $\sigma_w(t)$ and $\sigma_z(t)$ will approach some steady-state values. To further reduce the size of the estimation program, we used the steady-state values of σ_w and σ_z, rather than the time-varying values. In our application, these two variance matrices converge to 10 decimal places in 3 periods, so the use of steady-state rather than time-varying variance matrices was likely to have small effects on the outcome.

Estimation: Procedures and Results

Implementation of the state-space model requires replacing the "real" parameters with estimates of the parameters. We were interested in comparing the more general state-space specification with the two alternatives. The state-space specification can be made into the alternatives by setting either σ_u or σ_e to a zero matrix. We ended up estimating 15 alternative models with different restrictions on the σ_u and σ_e matrices. By definition, covariance matrices have to be positive, semi-definite. We imposed this restriction on the estimated matrices using Cholesky decompositions. Making either σ_u or σ_e a zero matrix makes their rank equal to 0. The endogenous variables' cross-product matrix has a rank of four. Using the Cholesky decomposition allowed us to restrict the rank of any of the matrices to less than four. The σ_w matrix will not have its full rank unless the sum of the ranks of σ_u and σ_e is four or greater. We ran all models with all combination of the ranks of σ_u and σ_e that exceeded four.

We estimated the model using maximum likelihood estimation. We compared the various models using their likelihoods. These likelihoods are reported in Table 1.

Many of the likelihoods in Table 1 are the same. Imposing equality restrictions on models invariably lowers the likelihood. Reducing the rank of either the σ_u or σ_e matrices in some way restricts the model, except that forcing the σ_u and σ_e matrices to be positive, semi-definite is imposing an inequality restriction. In these cases, there is always a chance that the estimates will go to the boundary of the inequality restriction, which will then become binding. It is common in state-space applications for the estimated covariance matrices to fail to have full rank. When we ran the least-restricted model, the e()'s matrix had a rank of 2, and u()'s a rank of 3. All the likelihoods where the rank of σ_u is three or greater and the σ_e two or greater are the same. All the likelihoods where the rank of σ_u was two are the same, as are all those where the rank of σ_u was one.

When testing equality restrictions, the differences in twice the likelihoods has an asymptotic chi-square distribution. Going from the two most restrictive alternatives to the general state-space model adds 10 independent covariance terms. The difference between the likelihood of the general and most restrictive models is statistically significant. One problem with using the likelihood ratio test in this case is that we are in part, testing inequality restrictions, and there is a finite probability that the difference in the likelihoods is zero. We actually had several tests that worked out to 0 in this case. It is likely that the true distributions of the difference in likelihoods has more of its probability at lower values than the chi square distribution. The chi square's probabilities probably underestimate the odds of rejecting the null hypothesis when it is true.

We conclude, therefore, that the state-space model is a statistically significant improvement over the two extreme alternatives. Neither a pure level nor a pure difference model adequately explains the stochastic processes driving wholesale-level meat demand.

The difference in twice the likelihood of the rank-3 σ_u, rank-2 σ_e and the model where the rank of σ_u is dropped to two is only 4.30. Dropping the rank of σ_u from 3 to 2 requires the elimination of two free terms. The value of 4.30 is not significant for a two-degree-of-freedom chi-square. However, there is that finite chance that the chi-square will be 0 if the rank actually is 2, which makes it impossible to say anything about the true significance of this change.

Table 2 shows the short-run and long-run elasticities of demand. The long-run demand is generally less elastic than short-run demand. Thus, the estimated multiplier for the current CBS exogenous variables, the "c" parameter in equation (1), is 1.484, while the lagged multiplier is 0.9875. Current price and scale changes have larger short-run effects than their long-run effect. All five meats in the system have inelastic demands; the demand for lamb is almost perfectly inelastic.

Figure 1 shows how the taste variable evolves over time. Table 3 shows how demands for the meats changes in response to the taste shifter and seasonally. The taste shift decreases demand for the red meats, particularly beef, while increasing poultry demand. The taste-shift term peaks in 1995, then declines, implying an increasing demand for red meat starting in the mid-1990's. The increase in red meat demand since 1995 does not erase the losses from 1979 to 1995. Turkey, as one might expect, shows the strongest seasonal demand pattern.

Conclusions and Future Research

We are confident in concluding that our state-space approach improves the performance of our model. The underlying data seems to have a mix of unit and non-unit roots that cannot be corrected with simple difference filters. The most obvious, unanswered question is about the test of dropping the rank of the u(t) covariance matrix from three to two. Future research will evaluate this test using some type of empirical technique, for instance, Monte Carlo simulations.

Another area for future research is to relate the errors to the structure of the model estimates. This involves imposing restrictions on the covariance matrix for the u(t). For example, the quadratic spline terms are meant to capture the change in tastes over time. The spline terms may only approximate the "true" taste change. The approximation error in the taste change may be one of the sources of the u(t) error terms. Another potential source of u(t) involves the scale/expenditure terms. One of the features of the CBS model is that it is consistent with non-linear aggregation. There is some "representative" level of scale or expenditure that is consistent with the market average share. The representative scale will not generally be the average scale.

The empirical model uses the change in the average scale. The difference between the average and representative scale could be another source of u(t) in the model.

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Table 1—Twice the likelihood under different assumptions about the rank of the covariance matrices

Rank of covariance matrix of u(t)	Rank of covariance matrix of e(t)				
	0	1	2	3	4
0					476.02
1				536.70	536.70
2			574.96	574.96	574.96
3		-794.14	579.26	579.26	579.26
4	505.23	549.23	579.26	579.26	579.26

Table 2—Demand elasticities

shortrun wholesale demand elasticities at mean shares						
	price terms					expenditure
	beef	pork	lamb	chicken	turkey	or scale
beef	-0.595	-0.306	-0.002	-0.105	-0.017	1.026
pork	-0.503	-0.340	-0.015	-0.130	-0.042	1.030
lamb	0.490	-0.209	-0.123	0.257	-0.202	-0.213
chicken	-0.333	-0.256	0.005	-0.301	-0.050	0.935
turkey	-0.150	-0.274	-0.046	-0.164	-0.277	0.911
longrun wholesale demand elasticities at mean shares						
	price terms					expenditure
	beef	pork	lamb	chicken	turkey	or scale
beef	-0.565	-0.307	-0.004	-0.117	-0.025	1.017
pork	-0.503	-0.329	-0.013	-0.133	-0.042	1.020
lamb	0.166	-0.241	-0.086	0.128	-0.150	0.182
chicken	-0.389	-0.273	0.001	-0.248	-0.047	0.956
turkey	-0.265	-0.285	-0.034	-0.156	-0.200	0.940

Table 3—Taste and seasonal effects on wholesale meat demand. Percent change in demand per quarter.

	taste	Q1	Q2	Q3	Q4
beef	-0.14%	4.86%	0.99%	0.08%	-6.01%
pork	-0.04%	-1.57%	-3.66%	-1.36%	6.72%
lamb	-0.02%	-3.48%	-0.89%	3.90%	0.59%
chicken	0.30%	6.48%	2.64%	-1.44%	-7.36%
turkey	1.06%	-64.61%	8.93%	15.23%	38.95%

Figure 1—Taste shift variable compared to trend