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Readdressing the Fertilizer Problem

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The production literature has shown that inputs such as fertilizer can be defined as risk-increasing. However, farmers also consistently overapply nitrogen. A model of optimal input use under uncertainty is used to address this paradox. Using experimental data, a stochastic production relationship between yield and soil nitrate is estimated. Numerical results show that input uncertainty may cause farmers to overapply nitrogen. Survey data suggest that farmers are risk averse, but prefer small chances of high yields compared to small chances of crop failures when expected yields are equivalent. Furthermore, yield risk and yield variability are not equivalent.

Key Words: corn, nitrogen fertilizer, risk-increasing, yield risk

Introduction

Researchers have used several approaches to explore optimal input use under uncertainty, often leading to different conclusions. Pope and Kramer (1979) defined an input to be marginally risk-increasing (decreasing) if the marginal risk premium is positive at the optimum for the risk-averse firm.¹ Under certain conditions, an input is risk-increasing (decreasing) if the risk-averse firm's optimal demand for the input is less (more) than that of the risk-neutral firm. Empirical evidence indicates risk plays a role in farmers' input decisions, including fertilizer use (e.g., Roumasset et al., 1989).

Approaches based on the agronomic theory of a limiting-input technology show input over-application may be viewed as a mode of self-protection, as defined by Ehrlich and Becker (1972). The limiting-input theory was first proposed by Von Liebig (1840), who stated that agricultural production can be defined by a fixed-proportions technology where crop yields are determined by the most limiting input. This production specification can be characterized by a simple linear response and plateau (LRP) production function (Cate and Nelson, 1971). Under the limiting-input view, farmers may overapply nitrogen as self-protection even though the theoretical foundations for this argument imply that yield variability is nondecreasing in the level of inputs used.² As an example, Babcock (1992) notes U.S. farmers tend to over-apply fertilizer based on expectations of ex post realizations of other uncertain factors, despite the fact that yield data imply yield variability is increasing in the level of fertilizer applied.

Empirical evidence reveals that farmers consistently overapply fertilizer (SriRamaratnam et al., 1987; Below and Brandau, 2001; Sawyer et al., 2006). The National Research Council

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¹ The terms marginally risk-increasing (decreasing) and risk-increasing (decreasing) are used interchangeably throughout the paper.

² As the level of use for any single input increases from a binding state to a nonbinding state, the set of possible yield realizations becomes larger, increasing the variability of output by definition.

(1993) estimated that up to 8 billion pounds of excess nitrogen is left in the soil each year. Based on findings reported by Yadav, Peterson, and Easter (1997), farmers in southeastern Minnesota applied nitrogen at rates exceeding both recommended rates and estimated profit-maximizing levels. Below and Brandau (2001) also discussed the overapplication of nitrogen fertilizer, concluding that excessive application was rationalized as a form of cheap insurance against nitrogen being limiting in the event of excellent growing conditions, further justifying the limiting-input and self-protection arguments.

We refer to this apparent paradox—fertilizer being cited as both risk-increasing and over-applied—as the fertilizer problem. This paper attempts to reconcile these two seemingly opposing viewpoints through a number of contributions. First, we present a corn-yield response model where the application rate of fertilizer chosen by the producer may differ from the amount of fertilizer available in the production process due to exogenous sources of uncertainty. Following Rothschild and Stiglitz (1970, 1971), the impact of this uncertainty on input demand is then illustrated for both risk-neutral and risk-averse producers. Drawing from Just and Pope (1979) and Pope and Kramer (1979), we show that a positive relationship between an input and production variability is not sufficient for that input to be defined as risk-increasing when there is uncertainty associated with the level of input use.

We then turn to an empirical application using nitrogen field trial data from Iowa. A flexible functional form is used to model corn yield response to available soil nitrogen after plant emergence in the spring. We adopt the estimation procedure originally outlined by Just and Pope (1979), which allows for flexibility with respect to the effect of input use on output variability, to estimate the production function. Consistent with previous empirical studies, we show that yield variability is indeed increasing in the amount of soil nitrogen available after emergence in the spring, indicating soil nitrogen would be considered risk-increasing if input uncertainty were ignored. Next, available soil nitrogen is specified as a stochastic function of applied nitrogen fertilizer, the choice variable of the farmer. Using numerical techniques, we calculate the optimal nitrogen application rates and compare for producers across a range of risk preferences. The results indicate that while nitrogen fertilizer would be defined as a risk-increasing input in the presence of input uncertainty, fertilizer may also be overapplied relative to a benchmark, profit-maximizing application rate which does not explicitly account for input uncertainty.

Another contribution of this study is that the findings are derived using traditional methods and concepts from the production literature in which the risk-increasing definition was first established while introducing the potential for uncertainty with respect to input availability. The combination of these applied methods captures the endogeneity of risk and does not rely on the limiting-input theory assumption for the production relationship to explain over-application of a risk-increasing input. Rather, our results are based on the curvatures of the marginal product of the estimated production relationship and the producer's marginal utility.³

In practice, it is unclear whether agricultural producers' decisions are driven by the sign of the third derivatives of their production and utility functions. Thus, to provide some additional behavioral motivations for producers' actions regarding fertilizer use, primary data from a short survey of Midwestern corn producers are also presented. The questionnaire results suggest important behavioral characteristics which could be introduced into modeling

³ The authors are indebted to an anonymous reviewer for significant contributions that helped to clarify our analytic results and illustrate the effect of uncertainty on the relationship between input use and the variance of production.

techniques. The most notable of these is that farmers seem to recognize the positive relationship between fertilizer application rates and yield variability, but do not view additional nitrogen applications as risk-increasing, indicating yield variability may be a poor proxy for risk.

Background

There is an extensive literature on the relationship between risk and input use in agriculture. The effect of price and production risk on optimal input use has been studied under a wide range of technological assumptions (Ratti and Ullah, 1976; MacMinn and Holtmann, 1983; Sandmo, 1971; Ishii, 1977; Hartman, 1976). In two widely cited examples, Rothschild and Stiglitz (1970, 1971) show that the effect of risk increases (defined by a mean-preserving spread) on optimal input use levels depends on the curvature of the marginal utility curve with respect to the stochastic shock.⁴

Following these classic theoretical findings, examples of agricultural inputs with both risk-increasing and risk-decreasing characteristics have since been reported. For instance, pesticides have been shown to provide protection against production uncertainty, implying a risk-decreasing effect (Feder, 1979), while more recently, Hurley and Babcock (2003) conclude pesticides should be defined as risk-increasing. Just and Pope (1979) conclude that both corn and oat yield variability are increasing functions of nitrogen fertilizer application rates. SriRamaratnam et al. (1987) use an experimental approach to elicit farmers' subjective beliefs about the responsiveness of sorghum yields to nitrogen fertilizer in Texas, finding that farmers generally overestimate the response of yields to fertilizer, causing them to overapply. Ramaswami (1992) provides more generalized conditions under which inputs could be characterized according to the Pope and Kramer (1979) definition, reporting nitrogen fertilizer to be risk-increasing at low (high) application rates for cotton (corn).⁵

Hurley, Mitchell, and Rice (2004) emphasize the endogeneity of risk caused by input choices. Applying their conceptual model of the adoption of Bt corn hybrid technology to two Midwestern counties, they find that Bt corn, an input used as protection against corn borer infestation, could be defined as risk-increasing or risk-decreasing depending on the price of Bt seed. Specifically, they conclude Bt corn is risk-increasing when the expected benefit of adoption is greater than the cost (i.e., when Bt corn *increases* expected profits).

Other studies approach the issue of fertilizer use from the limiting-input perspective. Lanzer and Paris (1981) estimate linear response and plateau (LRP) production functions for Brazilian wheat and soybean production. By incorporating nutrient carry-over, they demonstrate the inefficiency of fertilizer application rate recommendations based on traditional polynomial fertilizer response functions. Using experimental data on corn response to nitrogen and phosphorus fertilizers, Paris (1992) concludes that the LRP specification provides the best interpretation of the production relationship. Babcock (1992) reports that input uncertainty can lead to the overapplication of nitrogen fertilizer under certain nitrogen response and price conditions when the LRP relationship is assumed to characterize production. Babcock and Blackmer (1992) employ an LRP specification to model corn yield response

⁴ Rothschild and Stiglitz (1971) assume risk aversion for the optimizing agent. For a risk-neutral agent under production uncertainty, their curvature result applies to the marginal product curve.

⁵ Ramaswami (1992) derives conditions under which an input could be characterized as risk-increasing (decreasing) based purely on technological assumptions, noting that Pope and Kramer's (1979) original definition includes assumptions on both preferences and technology. His result yields the weakest condition necessary to define an input as risk-increasing (decreasing).

as a function of available soil nitrogen. They find that side-dressing nitrogen based on the results from post-emergence soil nitrate tests could reduce application rates by up to 38%. In a later study, Babcock and Blackmer (1994) also use an LRP specification with experimental data on Iowa corn production to illustrate a positive relationship between optimal nitrogen fertilizer application rates and growing conditions.

However, the LRP approach to modeling production in agriculture has not been adopted extensively in the literature (Lanzer and Paris, 1981), and support for the specification has been limited in both the economic and agronomic literatures (Hennessy, 2009). Moreover, the notion of a limiting input is based on soil science theory at the plant level, while economists tend to examine problems from a more aggregated viewpoint at a field or whole-farm level. Berck and Helfand (1990) reconcile the opposing views of the LRP and differentiable production function specifications by demonstrating that traditional functional forms can be derived as aggregations of the LRP model across many heterogeneous inputs.

The Model

Consider a risk-neutral farmer who chooses to apply an amount of fertilizer x^a at unit price w to produce stochastic output q in maximizing expected profits $E[\pi]$. Furthermore, assume the amount of fertilizer relevant to production is the amount available in the soil x , which is assumed to be a stochastic function of applied fertilizer, such that:

$$(1) \quad x = x^a + \phi\mu,$$

where $E[\mu] = 0$, and $\phi \geq 0$. The special case where $\phi = 0$ is that of input certainty with respect to applied fertilizer. Increasing the value of ϕ increases input uncertainty by way of a mean-preserving spread, with $\phi > 0$ implying the fertilizer available in the soil is equal to applied fertilizer “plus noise” (Rothschild and Stiglitz, 1970). Available fertilizer may differ from the amount applied for a variety of reasons, such as nutrient losses (or gains) from weather events,⁶ heterogeneity with respect to soil types and previous practices (i.e., crop rotations and tillage), and uncertainties related to application technologies.

Output is assumed to be an increasing concave function in the amount of available fertilizer, $q_x \geq 0$, $q_{xx} \leq 0$, $\forall x \geq 0$. All other production inputs, denoted by vector \mathbf{z} with unit price vector \mathbf{r} , are taken as given. Thus, the farmer maximizes expected profits with respect to fertilizer conditional on all inputs (other than fertilizer) being exogenous:

$$(2) \quad \pi = q(x(x^a, \phi), \varepsilon; \mathbf{z}) - wx^a - \mathbf{r}'\mathbf{z}.$$

Output has a stochastic component, ε , which is assumed to be a mean-zero disturbance whose variance may or may not depend on inputs x , x^a , and \mathbf{z} . The stochastic disturbance serves as a proxy for growing conditions throughout the production period, where better growing conditions (higher levels of outputs) derive from larger draws of ε . The risk-neutral farmer’s profit-maximization problem is summarized in equation (3), where all inputs are constrained to be nonnegative:

⁶ Corn producers in the Midwest primarily apply nitrogen fertilizer in the fall after harvest or in the spring prior to planting. In either case, there is a period of time between application and nutrient uptake by the plant when random events, such as rainfall, may affect available nutrient levels in the soil.

$$(3) \quad \max_{x^a} E[\pi | \mathbf{z}] = E\left[q(x(x^a, \phi), \varepsilon; \mathbf{z})\right] - wx^a - \mathbf{r}'\mathbf{z},$$

$$\text{s.t.: } x^a \geq 0, \text{ and equation (1).}$$

Denoting the expected profit-maximizing level of applied fertilizer by $x^{a*}(w, \phi)$, the first-order condition for a maximum is given by equation (4), where derivatives are denoted with a subscript and the first-order condition holds with equality if $x^{a*} > 0$:

$$(4) \quad E[\pi_x] = E\left[q_x(x(x^{a*}, \phi), \varepsilon; \mathbf{z})\right] - w \leq 0, \quad x^{a*} \geq 0.$$

The second-order sufficient condition requires the expected value of the second derivative of output with respect to fertilizer to be negative; i.e., $E[q_{xx}(x(x^{a*}, \phi), \varepsilon; \mathbf{z})] \leq 0$. This condition is satisfied by the curvature assumptions on q . The application rate given in (4) when $\phi = 0$, $x^*(w, 0)$, will be used as the benchmark or recommended application rate in determining whether input uncertainty may lead to overapplication. This benchmark application rate is consistent with agronomic approaches to establishing recommended application rates for fertilizer where yield response functions are estimated by field trial data, and rate recommendations are those which maximize expected profits (Sawyer et al., 2006).

Now consider a risk-averse farmer who chooses fertilizer inputs to maximize the expected utility of profits:

$$(5) \quad \max_{x^a} E[U(\pi | \mathbf{z})] = E\left[U\left(q(x(x^a, \phi), \varepsilon; \mathbf{z}) - wx - \mathbf{r}'\mathbf{z}\right)\right],$$

$$\text{s.t.: } x \geq 0, \text{ and equation (1),}$$

where the utility function U is assumed to be increasing and concave; that is, $U_\pi \geq 0$, $U_{\pi\pi} \leq 0$, $\forall \pi$. Let the expected utility-maximizing level of fertilizer be denoted by $x^{a**}(w, \phi)$. The first- and second-order conditions are now:

$$(6) \quad E[U_x] = E\left[U_\pi(\pi)\left(q_x(x(x^{a**}, \phi), \varepsilon; \mathbf{z}) - w)\right)\right] \leq 0, \quad x^{a**} \geq 0$$

and

$$(7) \quad E[U_{xx}] = E\left[U_{\pi\pi}(\pi)\left(q_x(x(x^{a**}, \phi), \varepsilon; \mathbf{z}) - w)\right)^2 + U_\pi(\pi)q_{xx}(x(x^{a**}, \phi), \varepsilon; \mathbf{z})\right] \leq 0.$$

The second-order condition in (7) is also satisfied given the curvature assumptions for U and q .

Suppose interior solutions exist for both the risk-neutral and risk-averse farmer so that (4) and (6) both hold with equality. Then, using $\text{Cov}(x, y)$ to denote the covariance between x and y , (6) can be rewritten as:

$$(8) \quad E\left[q_x(x(x^{a**}, \phi), \varepsilon; \mathbf{z})\right] - w = -\frac{\text{Cov}\left(U_\pi(\pi), q_x(x(x^{a**}, \phi), \varepsilon; \mathbf{z})\right)}{E[U_\pi(\pi)]},$$

where the term on the right-hand side of (8) is what Pope and Kramer (1979) defined as the marginal risk premium (MRP). The input x is defined to be risk-increasing (decreasing) if the MRP is positive (negative). Alternatively, $x^{a**}(w, \phi) \leq (>)x^{a*}(w, \phi)$ if MRP is positive (negative) when evaluated at the optimum. Given positive marginal utility, the denominator is positive so that the sign of the MRP is the opposite of the sign of the covariance between marginal utility and marginal product.

Taking the total differential of the first-order conditions (4) and (6) yields the following comparative static results:

$$(9) \quad \frac{\partial x^{a^*}(w, \phi)}{\partial \phi} = - \frac{\text{Cov}\left(q_{xx}\left(x\left(x^{a^*}(w, \phi)\right), \varepsilon; \mathbf{z}\right), \mu\right)}{E\left[q_{xx}\left(x\left(x^{a^*}(w, \phi)\right), \varepsilon; \mathbf{z}\right)\right]}$$

and

$$(10) \quad \frac{\partial x^{a^{**}}(w, \phi)}{\partial \phi} = - \frac{\text{Cov}\left(U_{\pi\pi}\left[q_x\left(x\left(x^{a^{**}}(w, \phi)\right), \varepsilon; \mathbf{z}\right) - w\right], \mu\right) + \text{Cov}\left(U_{\pi}\left[q_{xx}\left(x\left(x^{a^{**}}(w, \phi)\right), \varepsilon; \mathbf{z}\right) - w\right], \mu\right)}{E\left[U_{\pi\pi}\left(q_x\left(x\left(x^{a^{**}}(w, \phi)\right), \varepsilon; \mathbf{z}\right) - w\right)^2 + U_{\pi}q_{xx}\left(x\left(x^{a^{**}}(w, \phi)\right), \varepsilon; \mathbf{z}\right)\right]}.$$

The signs of (9) and (10) are equal to the signs of the numerators given the second-order conditions of the risk-neutral and risk-averse farmers' optimization problems. More specifically, the sign of equation (9) depends on the sign of q_{xxx} , with $\partial x^{a^*}(w, \phi)/\partial \phi > (<) 0$ as $q_{xxx} > (<) 0$. Similarly, a sufficient condition for $\partial x^{a^{**}}(w, \phi)/\partial \phi > (<) 0$ is that q_{xxx} and $U_{xxx} > (<) 0$. These are analogous to the results reported by Rothschild and Stiglitz (1971). Assuming convex (concave) marginal utility and convex (concave) marginal product implies the optimal level of input use when $\phi > 0$ is greater (less) than when $\phi = 0$. Note that the mean-preserving spread is introduced with respect to available nitrogen—not production. Thus, given the concavity assumption for the production function, from Jensen's inequality a mean effect is also introduced by available nitrogen uncertainty:

$$E\left[q\left(\phi > 0 \mid x^a\right)\right] \leq E\left[q\left(\phi = 0 \mid x^a\right)\right].$$

In the empirical application, we implement the production relationship suggested by Just and Pope (1979), where the production function is the sum of nonnegative mean and variance components, which are both deterministic functions of soil nitrogen, x . The error term enters multiplicatively with the variance component and is assumed to be distributed according to the standard normal distribution; i.e.:

$$(11) \quad q = f(x) + h(x)\varepsilon, \quad \varepsilon \sim N(0, 1).$$

The error term in the production function is assumed to capture the effects of exogenous forces, such as weather, while the scale of this variability is determined endogenously by input levels through the variance function component. Furthermore, the marginal productivity of an input, captured by f_x , does not impose any a priori restrictions on that input's effect on yield variability.

As shown by Pope and Kramer (1979), if input x is known with certainty, the variance of output is equal to $h^2(x)$, and the effect of input use on output variability is given by $\partial \text{Var}(q)/\partial x = 2h_x(x)h(x)$. Note that if there is no input uncertainty (i.e., $\phi = 0$), $h_x > 0$ is a sufficient condition for $\text{MRP} \geq 0$ and the input to be defined as risk-increasing (Pope and Kramer, 1979; Ramaswami, 1992). However, this condition is not sufficient if there is input uncertainty. The sign of the MRP is the opposite of the sign of its numerator, given the assumption of positive marginal utility, which can be rewritten as:

$$(12) \quad \text{Cov}\left(U_\pi(\pi), q_x\left(x\left(x^{a^{**}}(w, \phi)\right), \varepsilon\right)\right) = \\ E\left[U_\pi(\pi)\left(f_x\left(x\left(x^{a^{**}}(w, \phi)\right)\right) + h_x\left(x\left(x^{a^{**}}(w, \phi)\right)\right)\varepsilon\right)\right] \\ - E[U_\pi(\pi)]E\left[f_x\left(x\left(x^{a^{**}}(w, \phi)\right)\right) + h_x\left(x\left(x^{a^{**}}(w, \phi)\right)\right)\varepsilon\right].$$

Using the definition of variance, the relationship between output variance and applied fertilizer is given by:

$$(13) \quad \frac{\partial \text{Var}\left(q\left(x\left(x^a\right), \varepsilon\right)\right)}{\partial x^a} = \\ 2E\left[\left(f\left(x\left(x^a\right)\right) + h\left(x\left(x^a\right)\right)\varepsilon\right)\left(f_x\left(x\left(x^a\right)\right) + h_x\left(x\left(x^a\right)\right)\varepsilon\right)\right] \\ - 2E\left[f\left(x\left(x^a\right)\right) + h\left(x\left(x^a\right)\right)\varepsilon\right]E\left[f_x\left(x\left(x^a\right)\right) + h_x\left(x\left(x^a\right)\right)\varepsilon\right].$$

By rearranging (13) and substituting in (12), we can show that

$$(14) \quad \text{Cov}\left(U_\pi(\pi), q_x\left(x\left(x^{a^{**}}(w, \phi)\right), \varepsilon\right)\right) = \\ E\left[U_\pi(\pi)\left(f_x\left(x\left(x^{a^{**}}(w, \phi)\right)\right) + h_x\left(x\left(x^{a^{**}}(w, \phi)\right)\right)\varepsilon\right)\right] \\ - \frac{E[U_\pi(\pi)]E\left[\begin{array}{l} \left(f\left(x\left(x^{a^{**}}(w, \phi)\right)\right) + h\left(x\left(x^{a^{**}}(w, \phi)\right)\right)\varepsilon\right) \\ \left(f_x\left(x\left(x^{a^{**}}(w, \phi)\right)\right) + h_x\left(x\left(x^{a^{**}}(w, \phi)\right)\right)\varepsilon\right) \end{array}\right]}{E\left[f\left(x\left(x^{a^{**}}(w, \phi)\right)\right) + h\left(x\left(x^{a^{**}}(w, \phi)\right)\right)\varepsilon\right]} \\ + \frac{\frac{\partial \text{Var}\left(q\left(x\left(x^{a^{**}}(w, \phi)\right), \varepsilon\right)\right)}{\partial x^a} E[U_\pi(\pi)]}{2E\left[f\left(x\left(x^{a^{**}}(w, \phi)\right)\right) + h\left(x\left(x^{a^{**}}(w, \phi)\right)\right)\varepsilon\right]}.$$

Both the first and second terms in (14) are positive by assumption, while the sign of the third term is determined by the relationship between input use and the variability of output. Therefore,

$$\frac{\partial \text{Var}\left(q\left(x\left(x^a\right), \varepsilon\right)\right)}{\partial x^a} > 0$$

is not a sufficient condition for the MRP to be positive and fertilizer to be risk-increasing when there is input uncertainty. This is because input uncertainty removes the deterministic characteristic of the mean component from the Just-Pope production function.

Data and Estimation

The data used for this study come from the earlier works of Binford, Blackmer, and Cerrato (1992) and Blackmer et al. (1989). The data set contains information on corn yields, nitrogen fertilizer application rates, and results from a late spring soil nitrogen test collected from 15 experiment stations across the state of Iowa between 1985 and 1990. A significant amount of weather variability is included in the data, with years of excellent growing conditions and high yield levels (1987, 1990) and years of extremely poor growing conditions and low yield levels (1988 drought). All input levels, other than applied nitrogen fertilizer, were held constant at nonlimiting levels across years and sites to isolate the effects of nitrogen fertilizer on corn yields. Additionally, both continuous corn (corn-corn) and corn following soybeans (corn-soybean) rotations were examined.

The corn-soybean rotation data consisted of a total of 750 observations, while the continuous corn data included 1,248 observations. Data on continuous corn covered all six years and all 15 experiment station locations in the full data set. Data for the corn-soybean rotation were only available for eight experiment station sites over a four-year period (1987–1990). Nitrogen fertilizer rates ranged from 0 to 300 pounds per acre in 25–50 pound increments, with three repetitions of each application rate performed annually at each experiment station site. A late spring soil nitrate test was also performed to determine the level of nitrogen in parts per million (ppm) available in the soil for plant growth. The available soil nitrate levels were highly correlated with fertilizer application rates (correlation of 0.70). Soil nitrate levels ranged from 3.8 to 134.6 ppm, with an average of 27.5 (30.7) ppm and standard deviation of 18.1 (16.3) ppm in the corn-corn (corn-soybean) data. Yields ranged from 4 to 218 bushels per acre, with an average of 118.4 (143.2) bushels per acre and a standard deviation of 45.9 (39.7) bushels per acre in the corn-corn (corn-soybean) data. Overall, average soil nitrate and yield levels were higher and less variable for the corn-soybean rotation data, which is consistent with previous findings regarding the benefits of crop rotation.

Using the three-stage approach outlined by Just and Pope (1979), a variety of functional forms were estimated for the mean and variance components in the production function. The translog form was compared to alternative specifications, including Cobb-Douglas, linear, and quadratic functions. A likelihood-ratio test rejected the Cobb-Douglas form, while the linear and quadratic specifications were found to provide fits inferior to the translog form when plotted against the data. Although basic soil characteristics may have differed between experiment sites, Blackmer et al. (1989) and Binford, Blackmer, and Cerrato (1992) note that care was taken to make each of the observations as comparable as possible. Tillage, planting, and harvest practices were coordinated to be nearly identical across the experiment sites, with regard to both methods and timing. Of course, heterogeneity may exist across experiment sites due to factors such as weather variability and differences in soil types.

Dummy variables (d_s) were included to capture site-specific effects.⁷ The first stage of the process provides consistent estimates of the mean yield component parameters by estimating the following equation using a nonlinear least squares (NLS) estimator:⁸

⁷ Dummies could also be included to capture year effects. However, this would control for some of the exogenous risk farmers face when they choose input levels (i.e., weather events). Site dummies are included, assuming site effects represent farm-specific measures which would be known by the producer.

⁸ The nonlinear least squares estimation was carried out in MatLab using code written by the authors. The model was estimated using the linearized approach and convergence criterion discussed in Greene (2003).

$$(15) \quad q_{i,s} = f(x_{i,s}) + \varepsilon_{i,s}^* = \alpha_0 x_{i,s}^{\alpha_x} \exp\left(\frac{1}{2} \alpha_{xx} \log(x_{i,s})^2\right) + \alpha'_s d_s + \varepsilon_{i,s}^*,$$

where

$$\varepsilon_{i,s}^* = h^{1/2}(x_{i,s}) \varepsilon_{i,s} \quad \text{and} \quad E[\varepsilon_{i,s}] = E[\varepsilon_{i,s} | x] = 0.$$

Given the assumptions on ε , the composed error term ε^* is normally distributed with a zero mean, but is heteroskedastic, implying inefficiency of the first-stage estimates (Greene, 2003). Noting that

$$E[(\varepsilon_{i,s}^*)^2] = E[h(x_{i,s}) \varepsilon_{i,s}^2] = h(x_{i,s}) \quad \text{and} \quad (\varepsilon_{i,s}^*)^2 = E[(\varepsilon_{i,s}^*)^2] \xi_i,$$

where ξ_i is such that $E[\xi_i] = 1$, consistent parameter estimates for the mean component from the first stage can be used to obtain consistent estimates of the first-stage residuals, $\hat{\varepsilon}_{i,s}^*$. The squared residuals can then be regressed, in a nonlinear framework, on nitrogen levels in a translog functional form to obtain consistent estimates of the parameters for the variance component of the production function.

Finally, the third stage of estimation is conducted within a generalized NLS estimation procedure, where the covariance matrix is estimated using the second-stage parameter estimates for the variance component. The third stage of the procedure reestimates the parameters of the mean yield component, providing unbiased, consistent, and efficient parameter estimates. The use of dummy variables in the model implies a simple heteroskedastic covariance structure where the nondiagonal elements of the estimated covariance matrix are equal to zero. Production functions for the continuous corn and corn-soybean rotations were estimated to separate the rotational effects.

Estimation Results

Tables 1 and 2 present the parameter estimates (t -statistics) for the corn-soybean rotation data and the continuous corn data, respectively. The first and third columns of each table report the first-stage and third-stage parameter estimates, respectively, for the mean component of the production function. The second columns in the tables report the coefficient estimates for the variance component of the production function.

For each rotation, an unrestricted model was estimated including a full set of site dummies. A restricted model was then estimated, eliminating the site dummies that were not statistically significant in the unrestricted model. Comparisons of yield plots across years and sites were consistent with the statistical significance of the dummy estimates. Site 9 was arbitrarily chosen as the baseline site. Negative effects for site 13 were shown for corn following soybeans, while negative effects at sites 4 and 13 were significant for the continuous corn data.

Average yield levels were found to be increasing and concave functions of available soil nitrate for both rotations. Yield variability was also found to be an increasing function of available soil nitrate for both rotations. The yield functions for soil nitrate tend to flatten out as the level of soil nitrate increases beyond 60 ppm. For the continuous corn data, yield variability increases almost linearly with soil nitrate, while yield variability is estimated to be increasing but concave in soil nitrate. The estimation results confirm that yield variability does indeed increase with the level of available soil nitrate (i.e., $h_x > 0$) for both crop rotations. Therefore,

Table 1. Production Function Parameter Estimates, Corn-Soybean Rotation

Variable	First Stage		Second Stage		Third Stage	
	Unrestricted	Restricted	Unrestricted	Restricted	Unrestricted	Restricted
α_0	56.89 (0.30)	62.16 (0.26)	164.46 (0.002)	71.19 (0.003)	63.01 (0.28)	67.92 (0.24)
α_x	0.522*** (23.71)	0.482*** (19.90)	0.779 (0.578)	1.465 (0.902)	0.458*** (21.43)	0.426*** (17.94)
α_{xx}	-0.145*** (-69.23)	-0.130*** (-56.35)	-0.115 (-1.003)	-0.338** (-2.412)	-0.126*** (-61.91)	-0.112*** (-49.79)
Site Dummies:						
d_3	9.93 (0.445)		—	—	12.40 (0.546)	
d_{10}	31.07 (1.282)		—	—	30.07 (1.247)	
d_{11}	-7.23 (-0.391)		—	—	-8.02 (-0.436)	
d_{12}	-7.27 (-0.393)		—	—	-6.50 (-0.349)	
d_{13}	-31.80* (-1.689)	-38.01** (-2.275)	—	—	-31.51* (-1.669)	-38.22** (-2.288)
d_{16}	43.55 (0.777)		—	—	43.74 (0.779)	
d_{17}	-3.94 (-0.081)		—	—	-1.83 (-0.037)	

Note: Single, double, and triple asterisks (*, **, ***) denote statistical significance at the 10%, 5%, and 1% levels, respectively. Values in parentheses are *t*-statistics.

one might conclude that available nitrogen is a risk-increasing input to corn production if input uncertainty is ignored. We now introduce the relationship between available nitrogen and applied nitrogen to investigate the risk implications of input uncertainty.

Optimal Application Rates

Following Babcock and Blackmer (1992), maximum-likelihood methods were used to fit three-parameter gamma distributions to soil nitrate availability, conditional on the amount of fertilizer applied, assuming the parameters were linear functions of the application rate x^a . Alternative specifications were also examined, but likelihood-ratio tests indicated the linear specifications best fit the experimental data. Parameter restrictions were imposed to ensure a lower bound of zero ppm for available soil nitrate when the application rate was zero. Both the mean and variance of soil nitrate levels are increasing in the level of fertilizer applied for both rotations. The skewness exhibited in the soil nitrate distributions also increases with the amount of nitrogen fertilizer applied.

Using the gamma distribution parameter estimates, 1,000 soil nitrate draws were generated for each nitrogen application rate over a grid ranging from 0 to 300 pounds per acre. For each soil nitrate draw, expected profit and utility were computed assuming a constant absolute risk aversion (CARA) utility function for the risk-averse farmer, using a random draw of 1,000

Table 2. Production Function Parameter Estimates, Corn-Corn Rotation

Variable	First Stage		Second Stage		Third Stage	
	Unrestricted	Restricted	Unrestricted	Restricted	Unrestricted	Restricted
α_0	26.07 (1.09)	33.39 (0.95)	584.85 (0.003)	474.30 (0.004)	27.29 (1.07)	35.71 (0.91)
α_x	0.776*** (61.85)	0.651*** (53.28)	0.175 (0.747)	0.372* (1.668)	0.750*** (61.16)	0.606*** (50.66)
α_{xx}	-0.178*** (-150.64)	-0.142*** (-123.07)	0.050** (2.41)	-0.011 (-0.554)	-0.172*** (-148.25)	-0.128*** (-113.10)
Site Dummies:						
d_1	-13.31 (-0.60)		—	—	-23.66 (-0.57)	
d_3	11.82 (0.55)		—	—	15.25 (0.70)	
d_4	-96.51 (-2.12)	-100.71** (-2.051)	—	—	-94.18** (-2.06)	-98.68** (-2.01)
d_5	22.22 (0.77)		—	—	21.34 (0.74)	
d_6	11.45 (0.44)		—	—	8.52 (0.33)	
d_8	2.41 (0.10)		—	—	1.39 (0.06)	
d_{10}	-1.90 (-0.09)		—	—	-1.33 (-0.07)	
d_{11}	10.92 (0.48)		—	—	8.80 (0.40)	
d_{12}	-7.10 (-0.37)		—	—	-4.81 (-0.24)	
d_{13}	-50.36*** (-2.79)	-54.63*** (-3.255)	—	—	-47.74*** (-2.60)	-52.11*** (-3.10)
d_{14}	-8.06 (-0.14)		—	—	-4.89 (-0.09)	
d_{15}	-5.76 (-0.10)		—	—	-5.60 (-0.10)	
d_{16}	50.79 (0.79)		—	—	52.03 (0.80)	
d_{17}	-0.809 (-0.02)		—	—	0.051 (0.001)	

Note: Single, double, and triple asterisks (*, **, ***) denote statistical significance at the 10%, 5%, and 1% levels, respectively. Values in parentheses are *t*-statistics.

standard normal deviates for ε with the estimated production function.^{9,10} This defined profit and utility distributions for each nitrogen application rate in the grid. The nitrogen application rates that maximized expected profits and expected utility were evaluated for both crop

⁹ Following Babcock, Choi, and Feinerman (1993), the coefficient of absolute risk aversion was calibrated to 0.017 to yield a risk premium ratio of 25% for both the corn-soybean and continuous corn rotations. Risk-aversion levels above and below this calibrated level were also examined.

¹⁰ For each draw of soil nitrate, the yield distribution was determined by the variance component of the yield function and the draw for the exogenous risk component, ε .

Table 3. Optimal Nitrogen Application Rates (lbs./acre)

Coefficient of Absolute Risk Aversion	Corn-Soybean Rotation		Continuous Corn Rotation	
	$\phi = 0$	$\phi > 0$	$\phi = 0$	$\phi > 0$
0	105	113	169	172
0.0001	105	113	169	172
0.001	104	112	167	170
0.01	88	98	153	159
0.02	70	82	138	146

rotations and over a range of risk-aversion levels. The relative price of nitrogen fertilizer was set to calibrate optimal application rates close to current rate recommendations in the Midwest. The results were then compared to the expected profit- and utility-maximizing application rates under certainty with respect to soil nitrate availability.¹¹

Table 3 reports the optimal nitrogen application rates over varying risk preferences for both the continuous corn and corn-soybean crop rotations. The benchmark application rates, $x^{a*}(w, 0)$, were determined to be 105 and 169 pounds per acre for the corn-soybean and continuous corn rotations, respectively. For all levels of risk aversion, the optimal application rate for the risk-averse farmer is less than that for the risk-neutral farmer. These results apply in the cases of both input certainty and uncertainty, and illustrate the fact that nitrogen fertilizer is risk-increasing.

However, comparing the optimal application rates under input certainty and input uncertainty shows that farmers with all risk preferences apply more fertilizer when soil nitrate is stochastic. For the corn-soybean rotation, optimal application rates under input uncertainty are 8% to 17% greater than when soil nitrate is a deterministic function of applied fertilizer. Note also that for farmers with risk-aversion coefficients less than 0.001, the optimal application rates under input uncertainty (112–113 pounds per acre) are greater than the economically optimal rate (105 pounds per acre). Given uncertainty with respect to soil nitrate availability, even moderately risk-averse farmers may optimally overapply nitrogen despite it being risk-increasing. This result is driven by the curvature of the yield response and utility functions.

The results are similar for the continuous corn rotation. As risk aversion increases, optimal application rates decline because fertilizer is a risk-increasing input. However, application rates for farmers with all types of risk preferences increase by 2% to 6% when input uncertainty is introduced. As with the corn-soybean rotation results, farmers with risk-aversion coefficients less than 0.001 have optimal application rates greater than risk-neutral farmers under input certainty (170–172 vs. 169 pounds per acre).

Farmer Survey Results

The results thus far have been based on assumed preference relationships and an estimated production function. In an attempt to gain additional insight, a survey was distributed to Midwestern corn farmers to analyze their risk preferences and subjective opinions on the

¹¹ For the case of input certainty, the level of soil nitrate was set equal to the mean of the gamma distribution implied by the fertilizer application rate.

Table 4. Farmer Survey Results

Question	Option A	Option B
1	50% chance of winning \$1,000 50% chance of winning \$0 (20%)	\$450 with certainty (80% ^{**})
2	4% chance of winning \$12,000 96% chance of losing \$500 (40%)	95% chance of winning \$500 5% chance of losing \$9,500 (60% ^{**})
3	95% chance of 185 bu./acre 5% chance of 85 bu./acre (41%)	4% chance of 300 bu./acre 96% chance of 175 bu./acre (59%)
4	95% chance of 185 bu./acre 5% chance of 85 bu./acre (18%)	180 bu./acre (82% ^{**})
5	4% chance of 300 bu./acre 96% chance of 175 bu./acre (48%)	180 bu./acre (52%)
6	Do you think applying more nitrogen fertilizer increases your yield risk? Yes (28%)	No (72% ^{**})
7	Do you think applying more nitrogen fertilizer increases your yield variability? Yes (56%)	No (44%)

Note: Double asterisks (**) denote statistical significance at the 5% level.

relationship between fertilizer use and risk. The survey was distributed to farmers in Illinois, Iowa, Missouri, and North Dakota in the fall of 2006. The majority of the surveys were completed by corn producers who attended informational meetings held by Decision Commodities.¹² Additionally, a small portion of the surveys were personally administered to farmers delivering grain to local elevators in southeastern Minnesota. A total of 130 responses were obtained, with all respondents filling out the entire survey. Survey results were also compiled at the state level, but did not differ from those of the entire sample at a 5% significance level. (A copy of the survey and a summary of results at the state level can be obtained from the authors upon request.) The survey questions loosely follow those of Kahneman and Tversky's (1979) famous critique of expected utility theory. The survey results are summarized in table 4, where the proportion of surveyed farmers choosing each option is provided in parentheses following each choice.

The first two questions of the survey were concerned with the respondent's preferences over monetary gambles and were included as "warm-up" questions. The results from question 1 implied a strong preference for the certain amount versus a 50/50 gamble with a greater expected value (\$500 vs. \$450), implying risk aversion among the surveyed corn farmers. In question 2, there was a statistically significant preference for option B. Both gambles A and B in question 2 have expected values of zero. Gamble A provided a small chance of a large gain

¹² Decision Commodities is a company based in Ames, IA, which offers market-based index contracts as risk management and marketing tools for farmers throughout Iowa, Illinois, Missouri, and North Dakota.

and a large chance of a relatively small loss, whereas gamble B had a large chance of a small gain and a small chance of a large loss. Noting that the variance of option A is greater than the variance of option B, farmer preferences within the sample were consistent with expected utility theory, at least within a mean-variance framework.

Questions 3–5 asked the farmer to compare uncertain yield outcomes rather than monetary gambles. The survey asked respondents to answer the questions assuming they did not have access to any type of government support programs, such as federal crop insurance or commodity programs, in an attempt to control for low yield scenarios not being valued at the full level of the potential loss. Question 3 gave the farmer the option of a yield scenario with a large chance of a yield slightly above the expected yield (180 bushels per acre) and a small chance of a very low yield (i.e., a crop failure), or a scenario with a small chance of a very large yield realization and a large chance of a yield realization slightly below the mean. Farmers had a statistically significant preference for the “small chance of a very large yield” scenario (option B) over the “small chance of a very low yield” scenario (option A), even though option B has a higher yield variance.

Questions 4 and 5 asked the farmers to compare the yield gambles from question 3 to certain yields at the mean level of 180 bushels per acre. In question 3, there was a significant preference for the certain expected yield over the “small chance of a very low yield” scenario. However, the results for question 5 showed no strong significant preference for the certain mean yield over the “small chance of a very large yield” gamble. Expected utility maximizers would strictly prefer the certain expected yield over either risky option. While the farmers had a very strong preference for the certain mean yield over the “small chance of a very low yield” scenario, they were relatively indifferent between the certain mean yield and a “small chance of a very high yield” scenario, providing further evidence that farmers view yield “risk” above the mean differently than yield risk below the mean.

Finally, the survey asked two questions regarding the farmers’ subjective beliefs about the effect of fertilizer application on both yield risk and yield variability. While a small majority of the surveyed farmers responded that they believed fertilizer increased yield variability, the significant majority of producers believed fertilizer does not increase yield risk—implying yield risk and variability are not equivalent concepts.

Overall, the survey results provided mixed support for common modeling approaches. Farmers did exhibit some level of risk aversion in that they preferred certain outcomes. Additionally, a large proportion of the surveyed producers believed additional nitrogen increases the variance of yields, which is consistent with nitrogen often being cited as a risk-increasing input. However, two interesting behavioral observations can also be made that differ from standard expected utility theory. First, producers did not necessarily equate variance with risk, as the majority of respondents did not agree that additional nitrogen increased yield risk. Second, there was a strong preference for uncertain outcomes associated with low probabilities of high returns relative to low probabilities of low returns—even when expected returns for the latter case were slightly lower. These results indicate the likelihood of behavioral factors which are impacting fertilizer use choices in addition to those driving the results of this study.

Conclusions

The production literature is rich with studies examining the relationship between production uncertainty and optimal input use. Many authors have concluded that some agricultural inputs,

such as fertilizer, are risk-increasing inputs. These findings apply to empirical work using experimental yield response data (Ramaswami, 1992; Just and Pope, 1979) as well as theoretical analyses (Hurley et al., 2004; Hurley and Babcock, 2003). Empirical evidence tends to support these claims in that yield variability is generally found to be increasing in the amount of fertilizer applied (Roumasset et al., 1989).

However, a significant number of studies also exist which illustrate that farmers consistently overapply fertilizer (Yadav, Peterson, and Easter, 1997; National Research Council, 1993). This is often motivated as an act of self-protection (Ehrlich and Becker, 1972) in response to uncertainty with respect to growing conditions (Below and Brandau, 2001; Babcock, 1992) and/or input availability (Babcock and Blackmer, 1992, 1994), or as the result of biased subjective beliefs regarding yield response (SriRamaratnam et al., 1987).

The results of this study have shown that an input can be simultaneously defined as risk-increasing and overapplied by both risk-neutral and risk-averse producers, where the benchmark application rate is that which maximizes expected profits when input uncertainty is not explicitly accounted for. While the risk-averse farmer will choose application rates below those of the risk-neutral producer, farmers with both types of risk preferences will apply more fertilizer when nitrogen availability is uncertain. These findings contribute to the production literature by reconciling the seemingly confusing notion of an input being both risk-increasing and (rationally) overapplied by producers. Moreover, there exist direct implications for researchers working in the areas of environmental economics and precision agriculture. For example, continued development of precision technologies, such as variable rate applications based on soil sampling, could lead to the more efficient use of fertilizer by crop producers. The development of corn hybrids that use available nitrogen more efficiently could also potentially reduce the magnitude of overapplication and the resulting negative environmental consequences.

In addition to the empirical analysis, the results from a farmer survey were presented. The survey was designed to elicit information on farmer preferences over yield outcomes as well as their subjective beliefs about the relationship between risk and input use. The survey results imply that while farmers do prefer certain outcomes to risky scenarios with equivalent expected values, they discount risk differently when it is primarily above the mean. More specifically, producers tended to prefer scenarios with low probabilities of high payoffs even when expected returns were slightly lower than alternatives. Based on our findings, a number of producers recognize that applying additional nitrogen fertilizer increases yield risk, which is consistent with the risk-increasing nature of fertilizer often cited in the literature. However, farmers do not equate yield variability, or variance, to yield risk, which is an assumption explicitly made in the common application of mean-variance analysis. This indicates that the variance of production, profit, or utility may be a poor proxy for use as a risk metric. At the very least, the results of this study should encourage researchers to continue to develop alternative theoretical frameworks and apply them to optimizing behavior under risk and uncertainty.

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