Bias and Efficiency of Uniform Bid Design in Contingent Valuation

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Abstract: While contingent valuation (CV) methods have experienced growing popularity for estimating the willingness to pay for nonmarket goods and services, optimal bid designs for CV that provide guidance in bid point placement often render themselves impractical by relying on pretest or prior information about the true distribution for willingness to pay. We investigate the use of a practical alternative to existing optimal or robust bid designs called the uniform design. Uniform design randomly draws bid points from a predetermined uniform distribution. Analytics and simulations show that the uniform design has higher low-bound of relative efficiency at 84 percent of D-optimum than a robust design. Simulations also demonstrate that uniform design outperforms other optimal designs when initial information about true parameters is poor and even outperforms robust designs when the true values of parameters are known.

A Uniform Experimental Design for Binary Contingent Valuation Response Models: A Comparison with D-optimal and Robust Designs

1. Introduction

Binary response experiments have been widely used in fields as different as biology and economics. For example, in biological assay studies, clinical trial participants receive a randomly assigned 'dose,' and are then observed at some point in the future for their 'response'. In many cases, the response variable takes the form of a binary indicator: alive or not, cancer-free or not. The varying dose information combined with the binary response variable forms the necessary information to estimate the dose-response function. In economics, the contingent valuation method (CV) closely mimics the biological assay framework. CV measures consumer willingness to pay (WTP) for goods or services for which traditional markets do not exist. Hypothetical markets, in which survey participants must decide whether to purchase a good or service (binary response) at a randomly offered bid (dose), act as a proxy for market based decisions. The dose-response function estimated from the survey responses gives a measure of WTP (or demand) for the good or service.

Such examples describe the unique statistical problem of binary response experimental design. In previous literature, ad hoc designs or optimal design rules based on prior knowledge of the true response function have been used in choosing experimental design points. However, the bias of parameter estimates is analytically a function of experimental points and unknown true parameters (Copas 1988), and the choice of experimental points results in dramatically different point estimate (Cameron and Huppert 1991, Cooper and Loomis, 1992, Kanninen 1995). While parameter estimates converge asymptotically to the true parameter, the standard error of parameter estimates still depends on both experimental design points and unknown true parameters (e.g. Abdelbasit and Plackett 1983, Sitter 1992). A pressing question in such binary response experiment becomes, what is the optimal set of dose from which the experimental point should be drawn and offered to subjects to get the most information about the population response function (Abdelbasit and Plackett 1983, Cameron and Huppert, 1991, Sitter 1992, Nyquist

1992, Sitter and Wu 1993a,b, Cooper 1993, Alberini 1995)? Furthermore, which set of dose provides the estimation result less sensitive to prior knowledge of true response function?

In this paper, we propose a practical and viable alternative to existing experimental designs to solve the efficiency loss problem and to provide a practical design. The proposed experimental design focuses on the problem of designing the optimal bid set in dichotomous choice contingent valuation among applications to many fields. The new experimental design, named uniform design, draws experimental points from a pre-specified continuous uniform distribution[1]. Researchers can implement an experiment by simply deciding the range of uniform distribution as $\mu_0 \pm 2.72\sigma_0$ where μ_0 is the initial information of the population mean of WTP and σ_0 is the initial information of the standard deviation of WTP. Boyle et al. (1988) suggest a similar continuous bid design named as the "method of complementary random numbers," that constructs an empirical cumulative distribution function from prior information on the distribution of WTP. The difference between the uniform design and the method of complementary random numbers is that the uniform design selects random bid points from a predetermined uniform distribution instead of the empirical distribution of the method of complementary random numbers.

We compare the efficacy of the uniform design with those of D-optimal design and Sitter's robust design. D-optimal design is widely used as a benchmark bid design but it relies on the quality of information about the true parameter estimates to get optimal efficiency. Sitter's (1992) robust design aims to reduce the dependence on true parameters. Section 2 briefly reviews them and other existing experimental designs. Section 3 provides an analytical context for D-optimal, robust and uniform designs and section 4 compares them in terms of efficiency and relative efficiency. Section 5 shows the simulation result comparing experimental designs in contingent valuation study. Section 6 summarizes the analysis and provides further discussion.

2. Overview

[1] The uniform design draws upon the work of Lewbel et al. (2003) which assumes a continuous bid distribution to solve an identification problem in nonparametric estimation of willingness to pay in contingent valuation.

Suppose that we estimate willingness to pay (WTP) for a good (G) from individual responses to the stylized contingent valuation question: Would you be willing to pay $\$b_i$ for G? Individual responses are of the binary form: $y_i = 1$ if $WTP_i > b_i$ (a 'yes' response) and $y_i = 0$ otherwise (a 'no' response). In this dichotomous choice contingent valuation setting, the bid design problem is simply how to determine the set of b_i's to get the most efficient estimates of the parameters of the willingness to pay function.

To obtain maximum efficiency, numerous optimality criteria have been discussed in the statistical and experimental literature: e.g. A-, C- and D-optimality, Fiducial interval, and Mean Squared Error (MSE). All optimal criteria aim to minimize or maximize a variance-related criterion function of the relevant parameter estimates. For instance, A-optimal design minimizes the trace of the inverse of the information matrix, i.e., trace of variance-covariance matrix (Sitter and Wu 1993a, Mathew and Sinha 2001). C-optimal design minimizes the variance or the asymptotic variance of the summary statistics of interest, such as mean or median of willingness to pay (Wu 1988, Ford et al. 1992). Instead of the asymptotic confidence interval, the Fiducial design minimizes the length of the fiducial interval proposed by Finney (1971) using Fieller's theorem (Abdelbasit and Plackett 1983, Sitter and Wu 1993b, Alberini 1995). D-optimal design minimizes the volume of the confidence ellipsoid of parameter estimates by maximizing the determinant of the information matrix (Abdelbasit and Plackett 1983, Minkin 1987, Ford et al. 1992, Nyquist 1992, Mathew and Sinha 2001).

Optimal designs, except the MSE-based design, typically consist of one, two or three bid values that depend on the correct model specification, true parameters of the underlying response function and the number of observations. The fundamental paradox of the optimal bid design literature is that the information required for achieving the optimum is exactly the information to be estimated. If such information is available, estimation is unnecessary (Haab and McConnell 2002). In practice, this fact implies that the efficacy of each design hinges on the quality of that prior information. Poor initial information of the true parameter results in loss of efficiency[2].

[2] Efficiency is defined as the ratio of the determinant of the information matrix of a design to the optimal determinant. The definition is discussed in detail in section 4. Abdelbasit and Plackett (1983) show the efficiency loss in optimal designs with poor information.

An obvious solution for efficiency loss due to poor initial information is a sequential design (Abdelbasit and Plackett 1983, Minkin 1987, Nyquist 1992). The sequential design divides the experiment into a series of sub-experiments. The bid design is updated after each iteration based on estimates of parameters garnered from the previous stage. Consequently, sequential designs have more design points than optimal designs. Successive updates improve the efficiency of the design for poor initial estimates (Abdelbasit and Plackett 1983) and the procedure can be designed more efficiently by considering how good the initial estimate turns out to be once the previous estimation is conducted (Minkin 1987). In spite of the intuitive appeal, however, the practicality of a sequential method in contingent valuation applications is still in question.

Alternatively, Sitter (1992) introduces a minimax procedure to obtain designs robust to the uncertainty of the initial information. D-optimal design is a special case of the robust design when the experimenter is confident with his data. In general, the robust design, however, has more design points over wider range than optimal designs. Sitter argues that "the less knowledge of the parameter values one has prior to the experiment, the more spread out the design should be and the more design points should be used." Although Sitter's design is robust to poor initial parameter estimates and the implementation for a specific application is straightforward, the robust design relies critically on the experimenter's confidence about the quality of the information.

3. The Value of Initial Information in Experimental Designs

To demonstrate the effect of poor initial information on experimental designs, we first derive the general expression for the determinant of the information matrix for a standard CV problem. Suppose that WTP_i for G has a constant mean (μ) and an additive *i.i.d.* error component (ε_i) following a logistic distribution with zero mean and constant variance (σ^2): $WTP_i = \mu + \varepsilon_i$. The probability of a "yes" response is

(1) $$\Pr_i(yes) = \Pr_i[\mu + \varepsilon_i > b_i] = F(\beta(\mu - b_i))$$

where $F(t_i) = \exp(t_i)[1+\exp(t_i)]^{-1}$, $t_i = \beta(\mu - b_i)$ and β is the inverse of the standard deviation. Then, the log likelihood function of binary response becomes

$$\log L = \sum_i \left\{ \left(1 - y_i\right) \ln\left[1 - F\left(t_i\right)\right] + y_i \ln F\left(t_i\right) \right\}$$

where y_i is the binary response vector. The analytics of the design problem are simplified for the case of the logistic distribution by taking advantage of the logistic relation: $f(t) = F(t)\left(1 - F(t)\right)$. Define a weight w_i as

(2) $$w_i \equiv \frac{\exp\left(t_i\right)}{\left\{1 + \exp\left(t_i\right)\right\}^2},$$

then the information matrix of the log-likelihood function becomes

(3) $$I\left(\mu, \beta\right) = \begin{bmatrix} \sum_i w_i \beta^2 & \sum_i w_i \beta\left(\mu - b_i\right) \\ \sum_i w_i \beta\left(\mu - b_i\right) & \sum_i w_i \left(\mu - b_i\right)^2 \end{bmatrix}.$$

Most of optimal designs choose the bid-vector to optimize criteria functions derived from the information matrix. D-optimal design maximizes the determinant of the information matrix since the determinant represents the volume of the inverse the k-dimensional variance-covariance. When the number of bid points is *J* and observations (*N*) are distributed evenly across the *J* bid points, the determinant of the information matrix becomes

(4) $$\det\left[I\left(\mu, \beta\right)\right]_J = \frac{\left(n\beta\right)^2}{2} \sum_{i=1}^{J} \sum_{j=1}^{J} w_i w_j \left(b_i - b_j\right)^2 .$$

where $n = N/J$. The determinant in equation (4) depends on the squared distance between each pair of bid points, the weight evaluated at each point, and the true parameter β. The weight is also a function of true parameters $\left\{\mu, \beta\right\}$.

In application of bid designs to CV, the researcher chooses actual bid points by selecting normalized design points d_i from a pre-specified bid distribution and then calculating the actual bid points based on d_i and the prior information μ_0 and β_0 about the true parameter values[3]:

(5) $$b_i = \mu_0 + d_i / \beta_0 .$$

[3] Most practitioners directly choose b_i when implementing a CVM survey. For generality in design, the optimal design literature focuses on choosing the normalized bid points, d_i. Conditional on the prior information, there is a one-to-one mapping between normalized bid points and actual bid points.

By substituting (5) into (4) and using the definition of the weight in the equation (2), the determinant of a *kk*-design with *J* points becomes[4]

$$\text{(6)} \qquad \det_J = \frac{1}{2}\left[n\left(\frac{\beta}{\beta_0}\right)\exp\left\{\beta\left(\mu-\mu_0\right)\right\}\right]^2 \sum_{i=1}^{J}\sum_{j=1}^{J}\left\{\frac{\left(d_i-d_j\right)}{A_i A_j}\right\}^2 \exp\left\{\frac{\beta}{\beta_0}\left(d_i+d_j\right)\right\}$$

where $A_k = \exp\left\{\frac{\beta}{\beta_0}d_k\right\}+\exp\left\{\beta\left(\mu-\mu_0\right)\right\}$. The determinant of any symmetric design is expressed as a function not only of the choice of design points (d_i), but also of the quality of the initial information through $\beta\left(\mu-\mu_0\right)$ and β/β_0. From equation (6), the determinant of the symmetric two-point design is expressed by[5]

$$\text{(7)} \qquad \det_D = \left[\frac{Nd_0}{AB}\left(\frac{\beta}{\beta_0}\right)\exp\left\{\beta\left(\mu-\mu_0\right)\right\}\right]^2$$

where $A = \exp\left(\frac{\beta}{\beta_0}d_0\right)+\exp\left\{\beta\left(\mu-\mu_0\right)\right\}$ and $B = \exp\left(-\frac{\beta}{\beta_0}d_0\right)+\exp\left\{\beta\left(\mu-\mu_0\right)\right\}$ and $\pm d_0$ are two symmetric design points. When the researcher has the correct prior information about μ and β, the two-point design has its maximum determinant of $5.01\cdot10^{-2}N^2$ at $d_0 = 1.54$ for the logistic distribution, which is called D-optimum[6].

For equally spaced *kk*-designs, the determinant in equation (6) can be expressed using the order of bids and the distance since the distance between adjacent points is fixed. Let h_J be the distance between adjacent points of *J* bid points and suppose that design points are rearranged in the order from the lowest. Then, using $d_i - d_j = \left(i-j\right)h_J$, $d_i + d_j = \left(i+j-J-1\right)h_J$, and $d_i = \left[i-\left(J+1\right)/2\right]h_J$, the determinant of Sitter's robust design becomes

$$\text{(8)} \qquad \det_R = \frac{1}{2}\left[nh_J\left(\frac{\beta}{\beta_0}\right)\exp\left\{\beta\left(\mu-\mu_0\right)\right\}\right]^2 \sum_{i=1}^{J}\sum_{j=1}^{J}\left\{\frac{\left(i-j\right)}{\tilde{A}_i\tilde{A}_j}\right\}^2 \exp\left\{\frac{\beta}{\beta_0}\left(i+j-J-1\right)h_J\right\}$$

where $\tilde{A}_k = \exp\left\{\frac{\beta}{\beta_0}\left[k-\left(J+1\right)/2\right]h_J\right\}+\exp\left\{\beta\left(\mu-\mu_0\right)\right\}$.

[4] A *kk*-design has *k* design points symmetric around μ and equal number of observations at each point.
[5] The formula can be easily derived from equations (4) and (5). See Abdelbasit and Plackett (1983)
[6] Kalish and Rosenberg (1978) showed, in their unpublished technical report, that two-point designs symmetric with respect to μ are optimal for several design criteria including D-optimality. Also, see Ford et al. (1992) for the optimal probability mass point of various distributions.

When bid points are randomly selected from a continuous distribution, the determinant of information matrix in equation (3) is expressed by an asymptotic determinant similarly to the case of constant information design in Abdelbasit and Plackett (1983). Let the asymptotic density of *b* be $h(b)$ and take the limit of the information matrix as $J \to \infty$ so that the summation is replaced by the integral and n_j by $h(b)db$ in the information matrix:

$$I(\mu,\beta)=\begin{bmatrix} \beta^2 \int w(\beta(\mu-b))h(b)db & \int \beta(\mu-b)w(\beta(\mu-b))h(b)db \\ \int \beta(\mu-b)w(\beta(\mu-b))h(b)db & \int (\mu-b)^2 w(\beta(\mu-b))h(b)db \end{bmatrix}.$$

Let also *t* be $\beta(\mu-b)$ and substitute the asymptotic density function with a continuous uniform density function, then the asymptotic determinant becomes

(9) $$\det I(\mu,\beta)=\frac{1}{\beta^2}\left[\left\{\int w(t)dt\right\}\left\{\int w(t)t^2dt\right\}-\left\{\int w(t)tdt\right\}^2\right]$$

since $dt=-\beta db$. Note that $w(t)$ and $w(t)t^2$ are symmetric around zero. While the constant information design has a determinant independent of true parameters when the design measure is uniform (Abdelbasit and Plackett, 1983), the determinant in equation (9) is inversely dependent on the true variance. Equation (9) implies that the experimental design with a continuous uniform distribution for its bid selection provides more information as the true WTP is widely distributed.

Since the new experimental design utilizes a continuous uniform design, we name equation (9) by the asymptotic determinant of uniform design. In uniform design, poor information in the choice of bid range by researcher distorts the asymptotic determinant of the information matrix. Let the normalized endpoints of uniform distribution be $\pm\alpha$. Then, the nominal bid range becomes $\underline{b}=\mu_0-\alpha/\beta_0$ and $\overline{b}=\mu_0+\alpha/\beta_0$. Plugging two endpoints into equation (9) provides the asymptotic determinant of uniform design conditional on initial information. Researcher' problem is how to choose α to maximize the optimal criterion function, which is explained in the next section.

4. Efficiency and Relative Efficiency of Uniform Design

Following Abdelbasit and Plackett (1983), the loss of information of uniform design due to poor initial information is measured in terms of efficiency. The efficiency of experimental design is defined by the ratio of the determinant of a design evaluated at μ_0 and β_0 to D-optimum. For the comparison purpose, the determinant of D-optimal design and its D-optimum are also expressed in the asymptotic form. Note that the determinant of D-optimal design in the equation (7) is the square of rectangular area with the height of $d_0(\beta/\beta_0)\exp\{\beta(\mu-\mu_0)\}/AB$ and the width of *N* and that D-optimum is $5.01\cdot 10^{-2}N^2$. Suppose that true parameters are available for initial information. Then, by taking the same sample size as uniform design, the asymptotic D-optimum becomes $(0.1\bar{b})^2$.[7] Now, the asymptotic efficiency of uniform design is defined as the ratio of the asymptotic determinant of uniform design to the asymptotic D-optimum:

$$(12) \qquad Eff_U = \left(\frac{1}{0.1\bar{b}\beta}\right)^2 \left[\left\{\int_{\underline{b}}^{\bar{b}} w(t)\,dt\right\}\left\{\int_{\underline{b}}^{\bar{b}} w(t)t^2 dt\right\} - \left\{\int_{\underline{b}}^{\bar{b}} w(t)\,t dt\right\}^2\right].$$

The integration in the asymptotic efficiency is computationally calculated for a given range of design.

The optimal bid range of uniform design is defined by the range maximizing the asymptotic efficiency. Figure 1 and Table 1 show computationally and through simulation that given the true parameter, the maximum efficiency of uniform design is 84 percent of D-optimum when the range of uniform distribution is approximately $[-2.72,\ 2.72]$. The optimal range of uniform design corresponds to 6.2th and 93.8th percentiles in the logistic distribution. In considering that D-optimal design has design points at 17.6th and 82.4th percentiles of the logistic distribution, uniform design suggests wider range for the bid distribution[8]. Narrowing the range of the uniform distribution reduces the efficiency more rapidly than does broadening the range.

[Figure 1 located here]

[Table 1 located here]

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[7] As the determinant of D-optimum depends only on the sample size *N*, the asymptotic determinant of D-optimum depends only on the range.

[8] The result that uniform design has wide range of bid is consistent with previous studies suggesting wider range for the robust estimate (Kanninen 1995 and Alberini 1995). However, uniform design provides much wider than others provide.

Given initial estimates of the mean and variance of WTP, the optimal choice of uniform bid range may be $\underline{b}_0 = \mu_0 - 2.72/\beta_0$ and $\overline{b}_0 = \mu_0 + 2.72/\beta_0$. Plugging two endpoints into equation (12) provides the asymptotic efficiency of the optimal uniform design conditional on initial information. By definition, the asymptotic efficiency shows the relative increase of confidence volume of parameter estimates due to poor information. Poor information deteriorates the asymptotic efficiency of uniform design through $\beta(\mu - \mu_0)$ and β/β_0. Figure 2 exhibits the asymptotic efficiency with poor initial information. The maximum efficiency of uniform design is 84 percent at correct initial information where D-optimal design has the maximum determinant, and poor information of μ_0 reduces symmetrically the asymptotic efficiency given β/β_0.

[Figure 2 located here]

To compare the efficacy of uniform design with other designs under poor initial information, efficiencies of D-optimal and robust designs are derived below. From equation (7) and D-optimum, the efficiency of D-optimal design is expressed as

(10) $$Eff_D = \left[\frac{C}{A \cdot B}\left(\frac{\beta}{\beta_0}\right)\exp\left\{\beta\left(\mu - \mu_0\right)\right\}\right]^2 = \frac{\det_D}{5.01 \cdot 10^{-2} N^2}.$$

where *A* and *B* are defined in equation (7), and $C = \left\{\left[1 + \exp\left(d_0\right)\right] \cdot \left[1 + \exp\left(-d_0\right)\right]\right\}$ and $d_0 = 1.54$. Note that the efficiency of D-optimal design does not depend on the sample size[9]. As shown by Abdelbasit and Plackett (1983), the effect of poor initial estimates of μ is symmetric and overestimating β results in a greater loss of efficiency than underestimating. As the size of the true β is larger, i.e., as the true variance becomes smaller, the effect of poor information is more serious.

The second design scheme for the comparison with uniform design is Sitter's robust design with D-optimal criterion, which is one of equally spaced *kk*-design. From equation (8) and D-optimum, the efficiency of Sitter's robust design is expressed as

[9] The efficiency (standard error) of parameter estimate increases as the number of sample size increases. However, since the efficiency of a design, by definition, represents the relative size of information matrix, the number of different bid points instead of the sample size affects the efficiency through the denominator and summation.

$$Eff_R = \frac{1}{2}\left[\frac{C}{J}\frac{h_J}{d_0}\left(\frac{\beta}{\beta_0}\right)\exp\{\beta(\mu-\mu_0)\}\right]^2 \sum_{i=1}^{J}\sum_{j=1}^{J}\left\{\frac{(i-j)}{\tilde{A}_i\tilde{A}_j}\right\}^2 \exp\left\{\frac{\beta}{\beta_0}(i+j-J-1)h_J\right\} \tag{11}$$
$$= \frac{\det_R}{5.01\cdot 10^{-2} N^2}.$$

Like the efficiency of D-optimal design, the efficiency of Sitter's design, in fact the efficiency of equally spaced *kk*-design, does not depend on the sample size but on total number of different bids, distance between adjacent bid points and poor initial information. Table 2 reports the efficiency of Sitter's robust design in equation (11) when the researcher has correct information ($\mu_0 = \mu$ and $\beta_0 = \beta$)[10]. Table 2, therefore, shows the result of the best case contrary to the Sitter's table 1 that reports the worst case. By construction, Sitter's robust design with correct information has the highest efficient when experimental points are evenly distributed in two D-optimal design points and loses the efficiency as it has more bid points.

[Table 2 located here]

Table 3 exhibits the efficiency contour of Sitter's design when the prior information deviates from the true value. By construction, the top-left figure of Table 3 is the efficiency of D-optimal design. As can be seen in Table 3, the highest efficiency of robust design does not correspond to the situation when the researcher has the true information. For example, the robust design with $J = 4$ and $h = 2.23$ at $\beta_U / \beta_L = 1.5$ and $\mu_\Delta = 2.0$, has 65.4 percent efficiency of D-optimum at correct initial information, but the efficiency increases as β / β_0 is smaller than one and μ_0 is close to μ. Given β / β_0, the poor information of μ_0 has the symmetric effect on the efficiency in all cases.

[Table 3 located here]

Figure 2 and Table 3 show that the asymptotic efficiency of uniform design is relatively flat compared to that of D-optimal design, which implies that problem from poor initial information is not as serious in the uniform design as in the D-optimal design. Compared with robust design, uniform design has lower efficiency when initial information is correct or close to correct, but efficiency of robust design deteriorates more rapidly than

[10] The number of observation in point is normalized to be one when total experimental points are 13, thus *n* is the relative size of observation in each design.

uniform design in many cases. More seriously, however, the efficacy of robust design relies heavily on the confidence of researcher.

For more clear comparison, we define the relative efficiency of a design by the ratio of the efficiency of a design to the efficiency of D-optimal design, i.e. the determinant of a design with μ_0 and β_0 to the determinant of D-optimal design evaluated at the same information rather than D-optimum:

$$Rff_J = Eff_J \,/\, Eff_D = \det_J / \det_D \,.$$

The relative efficiency of robust designs, therefore, is the ratio of equation (8) to equation (7) with $d_0 = 1.54$ or the ratio of (11) to (10). The asymptotic relative efficiency of uniform design is defined as the ratio of the asymptotic efficiency of the uniform design to the asymptotic efficiency of D-optimal design. By definition, the relative efficiency implies how slowly a design loses the efficiency compared with D-optimal design as initial information becomes worse.

Figure 3 and Table 4 show the relative efficiency of uniform design and robust design, respectively. The relative efficiency of D-optimal design is always one by definition. Figure 3 implies that uniform design outperforms D-optimal design especially when the initial information about μ is poor. The minimum of the asymptotic relative efficiency of uniform design is 84 percent at the point of the maximum efficiency, i.e. at $\mu_0 = \mu$ and $\beta_0 = \beta$. The robust design has relatively higher advantage than D-optimal design in most cases, as the initial information is getting poorer. However, uniform design guarantees the lower bound of relative efficiency at 84 percent even though uniform design has lower relative efficiency than robust design when initial information seriously deviates from the true value.

[Table 4 located here]

[Figure 3 located here]

5. Simulation using Albemarle and Pamlico Sounds Data

In this section, by simulating of true willingness to pay from actual survey data, we compare D-optimal, robust, uniform designs as well as an ad hoc design using actual data in Huang, Haab and Whitehead (1997)'s contingent valuation study. The purpose of the study was to estimate willingness to pay for a water quality improvement in the Albemarle and Pamlico Sounds in eastern North Carolina. In the study, the random-digit-dial telephone survey asked subjects whether they would be willing to vote for a project to restore Albemarle and/or Pamlico Sounds water quality to 1980 levels at a predetermined price (bid). An ad hoc experimental design in the original study randomly selected bid values from the vector {\$100, 200, 300, 400}. The estimated result from actual data was

$$\ln(WTP) = 3.8623 + 0.1034 \cdot INC - 0.3580 \cdot D + \varepsilon \text{ and } \varepsilon \sim N\left(0, 0.3047^{-2}\right)$$

where *INC* is income level and *D* is a dummy variable for Pamlico sound only.

With assumption that the log willingness to pay in the above is the true willingness to pay, the process of simulation is as follows. Each bid design selected bid points based on population mean and variance of log willingness to pay: $E\left[\ln(WTP)\right] = \mu = 3.99$ and $\sigma = 0.3047^{-1}$.[11] For Sitter's robust design, the simulation chose the design scheme with $\beta_U / \beta_L = 1.5$ and $\mu_\Delta = 2.0$. Consequently, D-optimal bid vector was {\$1.29, \$2288.12} and optimal uniform distribution had a range of [\$0.40, \$7448.07]. Robust design bid vector was {\$0.13, \$7.22, \$408.14, \$23077.07}. For each bid design including the original ad-hoc design, hypothetical binary responses were generated by comparing the simulated true willingness to pay with randomly assigned bid points. The simulation was conducted with 100 iterations.

[Table 5 located here]

Table 5 shows the estimation results for each of the four simulated models. The table also reports determinants and relative efficiency. In this simulation, D-optimal design was expected to provide the most efficient parameter estimates, since the log willingness to pay is a linear model, error term is symmetric in terms of log value, and initial information coincides with the true information. Relative efficiencies, however, show that uniform design yields the highest efficiency followed by D-optimal and the original ad-hoc design. The log linear specification of the true model may explain the reason of higher efficiency of uniform design over D-optimal design. Robust design has the lowest relative efficiency, which may be due to too generous confidence of researcher about the quality of initial

[11] To adjust the analytical solution of the logit model for the normal distribution, bid points of robust and uniform designs were multiplied by $\sqrt{3}/\pi$. D-optimal bid points are $\exp(\mu \pm 1.14\sigma)$ following previous studies.

information. The original ad-hoc design has higher efficiency than the robust design even though the original design is a one-sided design (i.e., all bids are greater than the mean of expected log willingness to pay). Uniform design also outperforms other bid designs in terms of variance of parameter estimates (A-optimality) and the point estimate of the median willingness to pay.

6. Discussion and Conclusions

Both analytically and through Monte Carlo simulations, we compared the performance of uniform bid design with D-optimal design and Sitter's robust designs. D-optimality was chosen for the optimal criterion because of its popularity and usefulness. By construction, optimal bid designs provide optimal efficiency when the underlying true distribution and parameters are known. Unknown true parameter values and uncontrollable response rates to surveys bring difficulty in applying optimal designs to actual survey. Sitter's robust design or ad hoc designs employed in the actual studies reduce the risk from reliance on initial information by dispersing design points over wider ranges than D-optimal design. While Sitter's design is robust to poor information, the design point varies depending on researcher's belief about the quality of information, which generates quite different design efficiency. Response rate is another problem of robust design as well as to other optimal designs.

Analytics and simulations show that uniform design provides higher minimum efficiency than robust designs and outperforms the optimal design with poor initial information[12]. Uniform design scatters bid points by randomly drawing design points from a predetermined uniform distribution, so that each respondent receives a different bid point. Perhaps, this design scheme could be pointed out as the biggest drawback of uniform design since uniform design requires a different copy of the survey for each respondent, which increases the survey cost. However, additional burden of uniform design is outweighed by the potential statistical cost or extensive pretest information of ad hoc design and optimal or robust designs. Consequently, uniform design reduces the dependence of estimation result on design structure and poor information, guarantees a

[12] Simulation results also show that uniform design does not lose efficiency more than other designs under the A- and other optimality criteria.

higher minimum efficiency in any situation, and relieves the difficulty of researcher in choosing arbitrary bid values. Since a design independent of the poor initial information is unavailable, uniform bid design offers a practical and robust alternative to existing bid designs for researchers facing strict budget constraints, or performing a pre-survey to gather better information for the next stage. Furthermore, uniform design provides binary data continuous with respect to bid, enabling the researcher to apply more flexible non- and semi-parametric estimation techniques.

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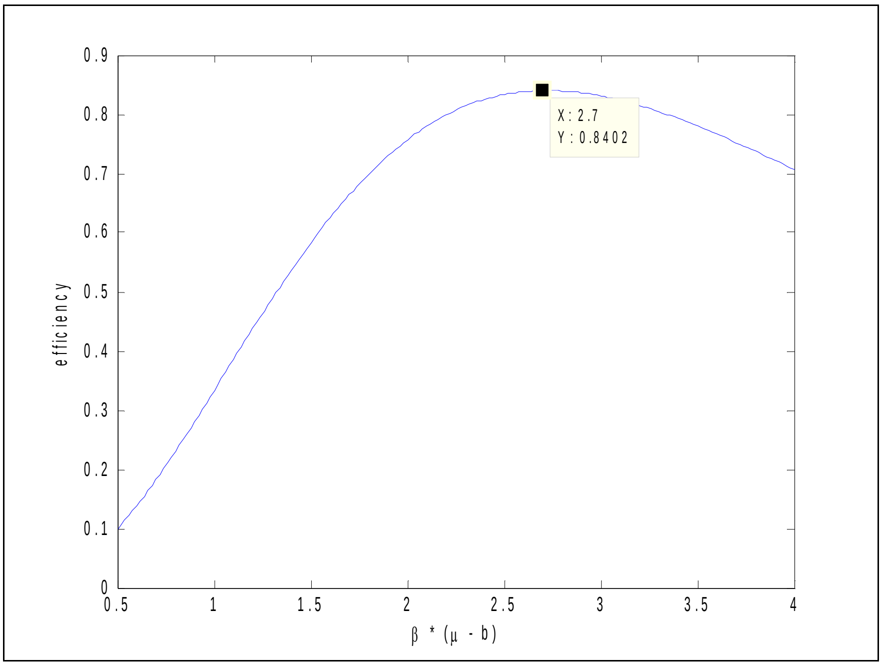


Figure 1: Efficiency of Uniform Design with Different Bid Range

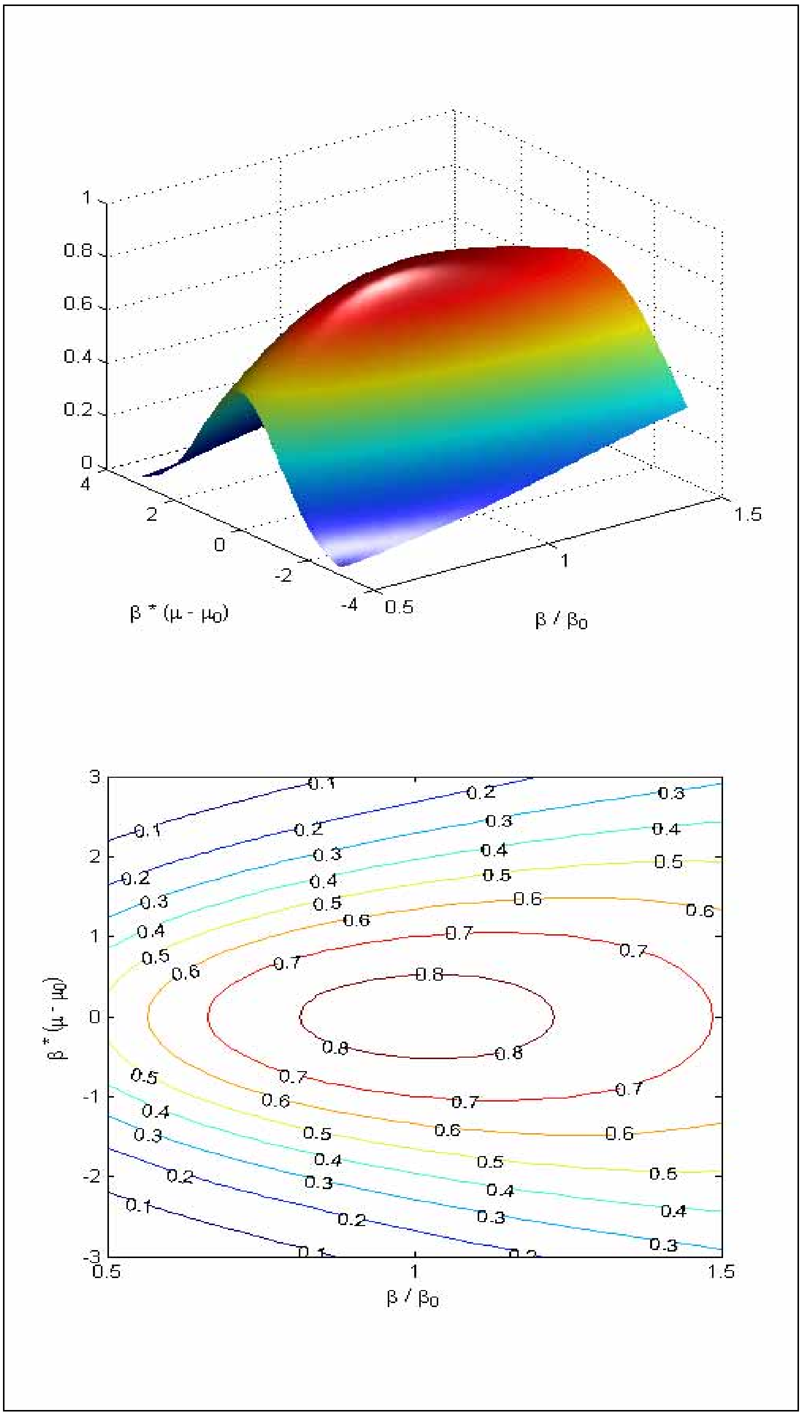


Figure 2: The Asymptotic Efficiency of Uniform Design

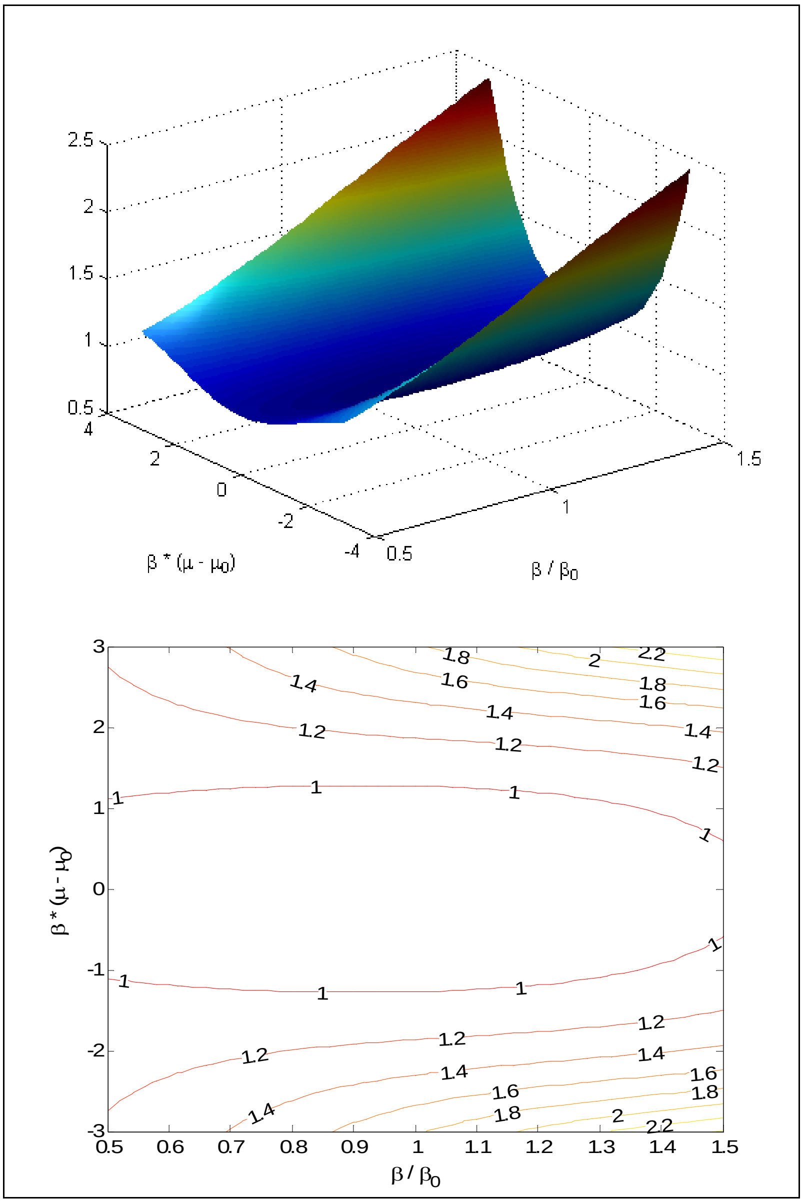


Figure 3: The Relative Efficiency of Uniform Design

Table 1: Different Range of Uniform Distribution with 1,000 Iterations, N = 320

d	1.72	2.22	2.72	3.22	3.72
μ	100.23 (3.80)	100.01 (3.96)	99.89 (4.35)	100.23 (4.42)	99.99 (4.60)
σ	30.27 (4.37)	29.95 (3.72)	29.98 (3.35)	29.80 (3.28)	29.91 (3.11)
Eff	67.57	80.52	84.03	81.10	75.00

The true parameters are $\mu = 100$ and $\sigma = 30$.
The parenthesis reports the standard error in 1,000 iterations
Eff=Efficiency relative to D-Optimum.

Table 2: Efficiency of Sitter's Robust Design using D-optimality Criterion

		μ_Δ									
β_U / β_L		0	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1.0	J	2	2	2	3	3	3	4	4	4	4
	h	3.09	3.15	3.35	2.53	2.99	3.46	2.80	3.17	3.52	3.86
	Eff	1	0.9993	0.9883	0.7500	0.6165	0.4847	0.4907	0.4075	0.3435	0.2917
1.25	J	2	2	3	3	3	4	4	4	5	5
	h	2.75	2.67	1.86	2.30	2.77	2.31	2.86	3.03	2.57	2.84
	Eff	0.9788	0.9673	0.8638	0.8074	0.6815	0.6291	0.5216	0.4369	0.3702	0.3055
1.5	J	2	2	3	3	4	4	4	5	5	6
	h	2.50	2.41	1.69	2.15	1.86	2.23	2.57	2.23	2.46	2.22
	Eff	0.9342	0.9119	0.8544	0.8368	0.7702	0.6543	0.5518	0.4724	0.4006	0.3562
2.0	J	2	2	3	4	4	5	5	6	6	7
	h	2.12	2.02	1.50	1.40	1.76	1.61	1.85	1.72	1.87	1.76
	Eff	0.8188	0.7795	0.8158	0.8547	0.7979	0.7134	0.6149	0.5321	0.4709	0.4034
2.5	J	2	3	3	4	5	5	6	7	7	8
	h	1.86	1.18	1.38	1.34	1.29	1.52	1.44	1.38	1.52	1.46
	Eff	0.7103	0.6767	0.7746	0.8535	0.8240	0.7490	0.6627	0.5796	0.5077	0.4400
3.0	J	2	3	4	4	5	6	7	8	8	9
	h	1.66	1.07	.93	1.28	1.23	1.20	1.17	1.16	1.27	1.25
	Eff	0.6145	0.6085	0.7133	0.8481	0.8368	0.7749	0.6971	0.6082	0.5412	0.4661
3.5	J	2	3	4	5	6	7	8	9	10	11
	h	1.51	.98	.90	.96	.98	.99	.99	1.00	.98	1.00
	Eff	0.5381	0.5466	0.6927	0.8304	0.8416	0.7908	0.7173	0.6297	0.5669	0.4817
4.0	J	2	3	4	5	6	7	8	10	11	13
	h	1.38	.90	.88	.92	.94	.95	.96	.87	.89	.81
	Eff	0.4701	0.4879	0.6783	0.8175	0.8456	0.8077	0.7959	0.6532	0.5673	0.5131

μ_Δ is defined as the confidence range of the mean of WTP such that $\beta_L |\mu - \mu_0| \le \mu_\Delta$, where β_L is the lower bound of β that the researcher believe.

Table 3: Efficiency of Sitter's Design when initial information is poor

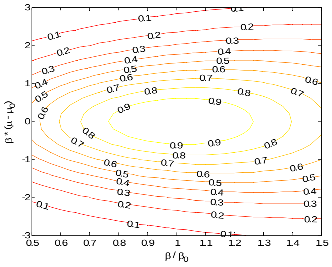
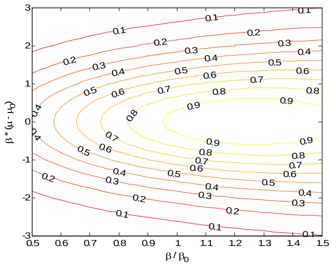
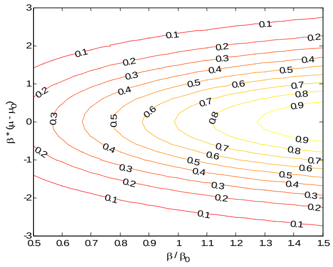
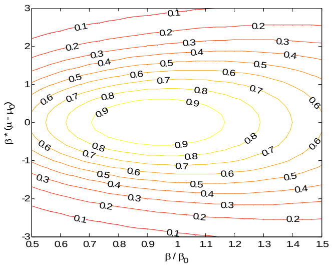
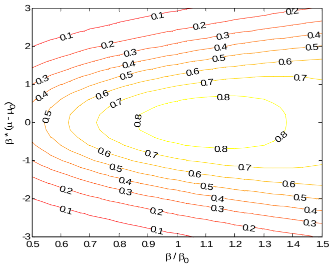
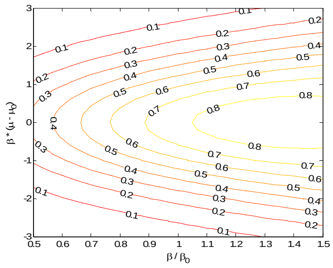
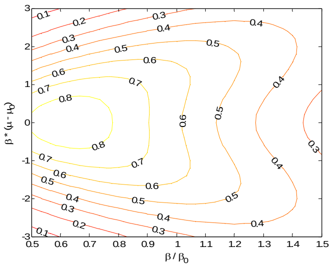
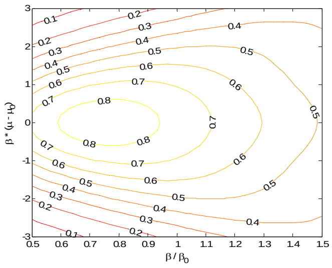
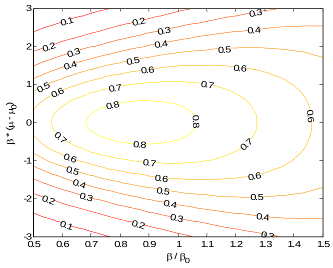
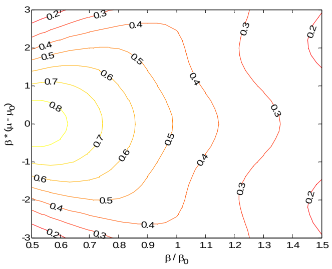
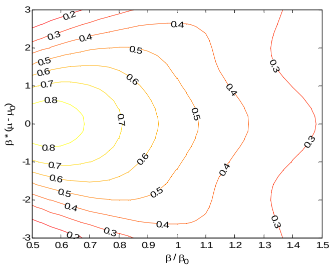
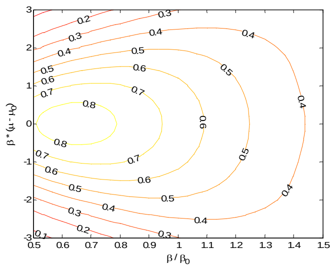
	β_U / β_L		
μ_Δ	1	1.5	2.5
0			
1.0			
2.0			
3.0			

Table 4: Relative Efficiency of Sitter's Design

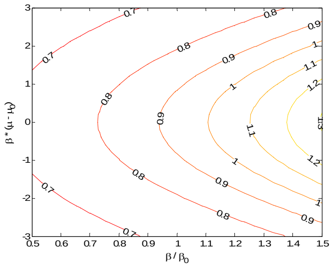
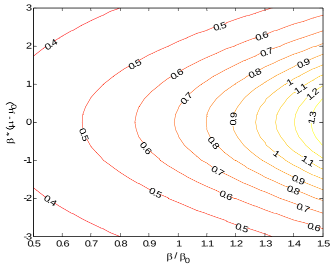
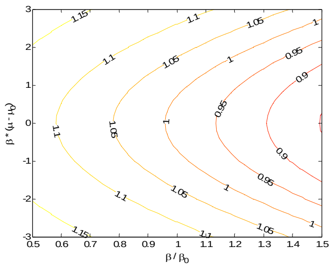
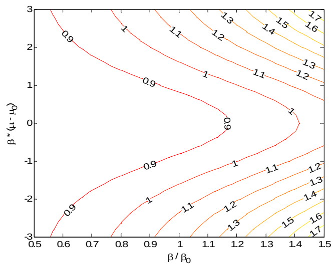
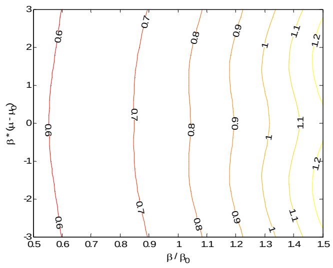
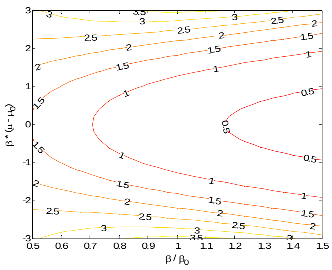
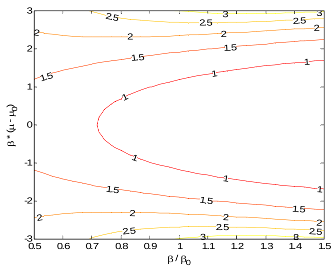
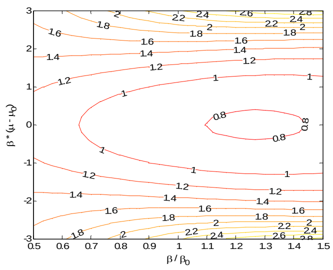
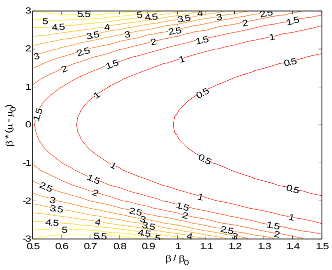
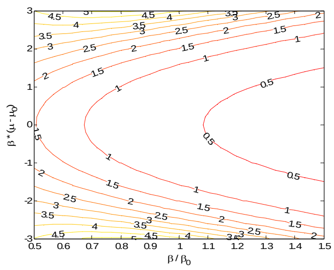
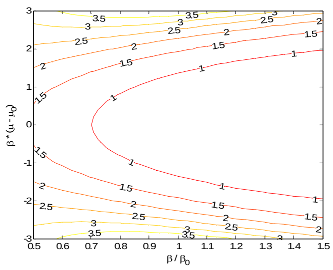
μ_Δ	β_U / β_L		
	1	1.5	2.5
0			
1.0			
2.0			
3.0			

Table 5: Estimation Result with Albemarle and Pamlico Sounds Data

	True	D-optimal	Robust	Original	Uniform
Constant	3.86	4.11 (.40)*	3.45 (.47)*	4.05 (.37)*	4.20 (.35)*
INC	0.10	-0.05 (.09)	0.05 (.10)	0.09 (.06)	-0.03 (.08)
D	-0.36	-0.29 (.37)	-0.16 (.41)	-0.17 (.24)	-0.46 (.33)
ln(Bid)	0.30	0.34 (.02)*	0.30 (.02)*	0.42 (.09)*	0.32 (.02)*
Rff		100	45.81	59.27	122.66
Mean	12340.51 (4682.27)**	3256.79	9943.71	1235.81	5576.14
Median	56.60	45.80 (31.56 64.61)	34.03 (22.49 50.52)	70.59 (41.72 112.59)	48.94 (34.36 66.30)

* Estimates are statistically significant with 95% confidence level.
** The sample average of *WTP*