Regional crop supply behaviour in the EU

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Abstract— **The primary objective of this paper is to estimate behavioural parameters of the quadratic regional supply models in the modelling system CAPRI, using the time series data in the CAPRI database. A Bayesian highest posterior density estimator is developed to address the primary objective. After discarding regions with insufficient data, parameters for up to 23 crop production activities with related inputs, outputs, prices and behavioural functions are estimated for 165 regions in EU-15. The results are systematically compared to the outcomes of other studies. For crop aggregates (e.g. cereals, oilseeds etc.) at the national level of nations, the estimated own price elasticities of supply are found to be in a plausible range. On a regional level and for individual crops, the picture is much more diverse. Whether the regional results are plausible or not is difficult to judge, since no other study of similar regional and product coverage is known to the authors.**

Keywords— **Bayesian estimation, errors-in-variables, PMP**

I. INTRODUCTION

Large scale optimization models typically contain parameters from a multitude of sources, including statistics, outcomes of estimations, and assumptions. The parameters and data are made consistent with the assumed model by some calibration procedure which operates on a single or a handful of parameters (e.g. [11], [15]). This article demonstrates a *consistent* and *transparent* method for estimating parameters of a large scale agricultural optimization model (the CAPRI model, see [1]) using econometric techniques to time series of observations. By *consistent* we mean that the estimating equations are equivalent to the equations of the economic model (its optimality conditions, see also [7], [8]). The transportability of the parameter from the estimation to the simulation model is ensured, in contrast to many situations where parameters are gathered from literature. By *transparent* we mean that a uniform methodology is applied to the whole data set. In the case at hand, this means that the same algorithm is applied to each of 252 regional models, with results for 23 different agricultural crops. Transparency also means that prior information and plausibility considerations are formally included in the estimator.

Heckelei and Britz [5] estimated supply parameters of the regional supply models in CAPRI using cross section data for all regions in a single year, and introducing prior information via generalized cross entropy. This work improves on their approach in several ways: Firstly, a more general Bayesian estimator is developed, secondly, time series data is used in the estimation, and thirdly, the regional coverage is extended to EU27. As in Heckelei and Britz, some limitations apply: The estimation only considers the arable annual crop producing part of the representative regional farm, keeping other parts (husbandry, permanent grassland and permanent crops) fixed when necessary or leaving them out altogether when possible. We also ignore the fertilization constraints of the full model, working with Leontief fertilizer input coefficients.

The remaining part of this report contains four sections. Section two describes the structure of the template regional representative farm model that is used for all regions. Section three formulates the Bayesian estimation model and discusses the use of prior information. In section four, results are presented for selected regions, and compared to the results of other studies. Section five concludes the paper.

II. A REGIONAL SUPPLY MODEL

The regional representative farm is assumed to act as if solving a linearly constrained quadratic programming problem (1) in every time period *t*. Throughout this paper we generally use lower case bold face letters to represent items that are column vectors for each *t*, upper case bold face letters to represent matrices and italic letters to represent scalars. The dimensions of vectors and matrices are denoted by upper case letters, where a lower case version of the same letter denotes the indices of the elements in that dimension, so that for instance the "J-vector of acreages $\mathbf{x}$" means a vector of length J, with elements x_j, $j = 1 \ldots J$. The *prime* character (′) denotes the ordinary transpose of a vector or a matrix.

All regional models have identical structure, and no cross-regional constraints or relationships are assumed, in order to keep the regional estimation rather flexible and to limit the complexity of the estimation. Thus, indices for regions can be omitted. The producer is assumed to solve the optimization problem in each period independently of other periods, and items that change across periods obtain an index t. For example $\mathbf{x}_t$ denotes the vector $\mathbf{x}$ in period t, but $\mathbf{x}$ (without index) denotes the whole 3-dimensional array with dimensions (J,1,T), where the "1" indicates that that dimension is not used in this case.

The model can then be written for each period as

Maximise w.r.t. $\mathbf{x}$

$$\mathbf{x}_t'\left[\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t\right] - \mathbf{x}_t'\left[q_t\mathbf{c} - \tfrac{1}{2}l_t\left[\mathbf{D} + \mathbf{GBG}'\right]\mathbf{x}_t\right] \tag{1}$$

subject to

$$\mathbf{R}_t\mathbf{x}_t = \mathbf{v}_t$$

where for each t,

$\mathbf{x}_t$ vector of acreages for each of J land uses
$\mathbf{Y}_t$ $J \times J$ diagonal matrix of yields
$\mathbf{p}_t$ J vector of prices
$\mathbf{s}_t$ J vector of direct subsidies
$\mathbf{A}_t$ $J \times I$ matrix of input coefficients for I inputs
$\mathbf{w}_t$ I vector of input prices
q_t price index
$\mathbf{c}$ J vector of parameters
l_t land availability index (described below)
$\mathbf{D}$ $J \times J$ diagonal matrix of parameters
$\mathbf{G}$ $J \times M$ matrix that sums up land use by each of M = 6 crop groups, i.e. with $g_{jm} = 1$ if crop j belongs to group m, else $g_{jm} = 0$
$\mathbf{B}$ 6×6 matrix of parameters
$\mathbf{R}_t$ $2 \times J$ matrix of constraint coefficients, where $r_{1j} = 1$ for $j = 1\ldots J$ and r_{2j} is the net set-aside contribution of crop j
$\mathbf{v}_t$ 2 vector with v_1 total land available, $v_2 = 0$.

The model implies that the producer maximises the sum of gross margins (the first term) minus a quadratic function (the second term), subject to a land constraint and set-aside requirement. The quadratic function in the objective function is a *behavioural function* (and $\mathbf{c}$, $\mathbf{D}$ and $\mathbf{B}$ behavioural parameters) in the tradition of *positive mathematical programming* (PMP, see e.g. [10] or [11]) that is intended to capture the aggregated influence of economic factors that are not explicitly included in the model [6]. It is the objective of this work to estimate the behavioural parameters.

In order to reduce the number of behavioural parameters to estimate, we assume that cross-crop effects are only permitted between *groups of crops*. That is achieved using a vector $\mathbf{c}$ of linear effects, a diagonal matrix $\mathbf{D}$ of quadratic own-crop effects, and a matrix $\mathbf{B}$ of cross-group effects. The $J \times M$ matrix $\mathbf{G}$ is used to sum the acreages within each group, substantially reducing the number of parameters compared to estimation of a full $J \times J$ matrix. The appendix lists groups, crops and inputs.

The prices $\mathbf{p}$ and $\mathbf{w}$ in the model are nominal, and since the quadratic function is assumed to capture, among other things, the opportunity cost of resources not explicitly modelled, it should be inflated. This is obtained by multiplication of $\mathbf{c}$ by the general price index q_t.

The total amount of land fluctuates slightly between years, in general with a downward trend due to migration of land into other sectors (fallow land is modelled explicitly as a land use activity). We do not know if it is productive or unproductive land that migrates, so to avoid that land migration strongly influences land rent (the dual value of the first constraint), we use *land shares* in the quadratic term. This is equivalent to scaling the matrix [$\mathbf{D}$ + $\mathbf{GBG}'$] by the square inverse of total land available in each period. For scaling purposes, it is also multiplied by ½ times square of total land available in year 2000, or $(v_1)_{2000}$. Thus $l_t = ((v_1)_{2000}/(v_1)_t)^2$.

The optimization model (1) can be equivalently described by the following first- and second order conditions for optimal $\mathbf{x}$

$$\begin{aligned}&\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c}\\ &- l_t\left[\mathbf{D} + \mathbf{GBG}'\right]\mathbf{x}_t - \mathbf{R}_t'\boldsymbol{\lambda}_t = \mathbf{0}\end{aligned} \tag{2}$$

$$\mathbf{R}_t\mathbf{x}_t = \mathbf{v}_t \tag{3}$$

$$\mathbf{B} = \mathbf{U}'\mathbf{U} \tag{4}$$

$$d_{jj} \geq 0 \text{ for } j = 1\ldots J \text{ (and } d_{ij} = 0 \text{ for } i \neq j) \tag{5}$$

$\boldsymbol{\lambda}_t$ is the 2×1 vector of dual values for the constraints. Note that for positive semi-definiteness of the Hessian matrix, it is sufficient that $\mathbf{B}$ is positive semi-definite, which is satisfied by the Cholesky factorisation with the upper triangular matrix $\mathbf{U}$, and that all elements of $\mathbf{D}$ are non-negative[1].

[1] In fact, we will use a stronger restriction of $d_{jj} \geq \delta_{jj} > 0$ in estimations to avoid numerical problems when estimating elasticities.

A. Outline of a Bayesian estimator

The Bayesian estimator outlined below is based on a measurement error model (see e.g. [2]), where in general, no parameter values are known with certainty. We do not include errors in optimisation, however, i.e. we assume that the agent modelled has perfect information about the true parameters, and is able to determine the optimal production decision exactly. Thus no such errors enter the model equations, thereby influencing production. A more general error model, as discussed by in [13] and [16] would also take into account the possibility that the producer may not correctly appreciate the true parameters and/or is not able to determine exactly the optimal supply decision. Since the general error model requires an increased amount of prior information and is anyway difficult to distinguish from the measurement error model in many cases, we choose to only consider measurement errors.

The basic assumption underlying the data sampling model is that there exists a set of true parameters $\boldsymbol{\psi} = (\mathbf{p},\mathbf{Y},\mathbf{s},\mathbf{A},\mathbf{w},\mathbf{q},\mathbf{l},\mathbf{c},\mathbf{D},\mathbf{B},\mathbf{R},\mathbf{v})$ of the model, satisfying the second order conditions (4-5), a vector of true planned acreages $\mathbf{x}^*$ and a vector of dual values $\boldsymbol{\lambda}^*$ such that $(\mathbf{x}^*,\boldsymbol{\lambda}^*)$ is the unique optimal solution to the model parametrized by $\boldsymbol{\Psi}$. We may thus implicitly write $\mathbf{x}^* = \mathbf{x}^*(\boldsymbol{\psi})$ and $\boldsymbol{\lambda}^* = \boldsymbol{\lambda}^*(\boldsymbol{\psi})$. Furthermore, the values $\mathbf{z} = (\mathbf{x}^{obs},\mathbf{p}^{obs},\mathbf{Y}^{obs},\mathbf{s}^{obs},\mathbf{A}^{obs},\mathbf{w}^{obs},\mathbf{q}^{obs},\mathbf{l}^{obs},\mathbf{R}^{obs},\mathbf{v}^{obs})$ in the CAPRI database are considered the outcome of a random variable vector $\mathbf{Z}$ that is conditional on $\boldsymbol{\Psi}$, i.e. there exists a probability density function $f(\mathbf{z}|\boldsymbol{\Psi})$.

If we express our prior information and "beliefs" about the parameter vector $\boldsymbol{\Psi}$ as a *prior density function* $\xi(\boldsymbol{\Psi})$, we may use Bayes's rule to derive the posterior density function of $\boldsymbol{\Psi}$ conditional on the outcome $\mathbf{z}$:

$$\xi(\boldsymbol{\Psi}|\mathbf{z}) \propto f(\mathbf{z}|\boldsymbol{\Psi})\xi(\boldsymbol{\Psi})$$

We desire an estimation method that chooses as an estimate the parameter vector $\boldsymbol{\Psi}$ that maximises the conditional density $\xi(\boldsymbol{\Psi}|\mathbf{z})$. DeGroot [3] calls this estimator the generalised maximum likelihood estimator[2] and it extensively discussed in [8]. In what follows, we derive the function f from an error model relating $\mathbf{z}$ to $\boldsymbol{\Psi}$, and derive the unconditional (prior) density function ξ from prior beliefs regarding elasticites and dual values of the (implied) model.

[2] Other authors have called it the posterior mode estimator, the maximum a-posteriori estimator or the highest posterior density estimator. We consider this estimator superior to Maximum or Cross Entropy formulations in our context for computational and transparency reasons.

B. Data sampling process

The distribution of $\mathbf{Z}$ is based on the following assumptions:

1. All elements in $\mathbf{Z}$ are independent
2. Subsidies, price index, set-aside rate and total land constraint are known with certainty. Thus, outcomes of those items in the random vector $\mathbf{Z}$ will be the corresponding items of $\boldsymbol{\Psi}$ itself, and from now on removed from $\mathbf{Z}$. An outcome of $\mathbf{Z}$ is thus written $\mathbf{z} = (\mathbf{x}^{obs},\mathbf{p}^{obs},\mathbf{w}^{obs},\mathbf{Y}^{obs},\mathbf{A}^{obs})$.
3. Errors are additive. We write an outcome of $\mathbf{Z}$ as the sum of its conditional expectation $\boldsymbol{\mu}(\boldsymbol{\psi}) = (\boldsymbol{\mu}_x,\boldsymbol{\mu}_p,\boldsymbol{\mu}_w,\boldsymbol{\mu}_Y,\boldsymbol{\mu}_A)$, (with appropriate dimensions), and the random error vector $\boldsymbol{\varepsilon}$, so that $\mathbf{Z} = \boldsymbol{\mu}(\boldsymbol{\psi}) + \boldsymbol{\varepsilon}$.
4. Producers have naïve price expectations. The (statistical) expectation of the price measurement in period t-1 equals the producer price in that period, or conversely,

   $$\mathbf{p}_t = (\boldsymbol{\mu}_p)_{t-1}$$
   $$\mathbf{w}_t = (\boldsymbol{\mu}_w)_{t-1}$$

   where the expression on the right hand side denotes the expected value of the output and input prices for all crops in period t-1.
5. Expected yields and input requirements follow linear trends. We thus have

   $$\mathbf{Y}_t = (\boldsymbol{\mu}_y)_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 T_t \qquad (6)$$

   $$\mathbf{A}_t = (\boldsymbol{\mu}_A)_t = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 T_t$$

   with T being a linear trend and $\boldsymbol{\beta} = (\boldsymbol{\beta}_0,\boldsymbol{\beta}_1)$ and $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_0,\boldsymbol{\alpha}_1)$ new parameters to estimate. Prior means or input allocation come from estimates $\mathbf{A}^{obs}$ available in the CAPRI data base.

C. Augmented parameter vector and its prior distribution

When observations have been made, the outcome $\mathbf{e}$ of the error vector $\boldsymbol{\varepsilon}$ has also been determined, but the outcome is unknown—we don't know what part of an observation is error and what part is parameter. We can thus choose to consider $\mathbf{e}$ yet another unknown parameter, the distribution of which may be subject to prior information. If the density function f for the random vector $\mathbf{Z}$ is conditional also on $\mathbf{e}$ and the yield and input parameters $\boldsymbol{\beta}$, and $\boldsymbol{\alpha}$ defined above, then there are no random components left, and f becomes the degenerate density function,

$$f(\mathbf{z}\,|\,\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e}) = \begin{cases} 1: & \begin{bmatrix} \mathbf{z} = \boldsymbol{\mu}(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha}) + \mathbf{e} \\ g(\boldsymbol{\psi},\mathbf{x}^*,\lambda^*) = 0 \end{bmatrix} \\ 0: & \text{else} \end{cases}.$$

A large number of different parameter vectors ($\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e}$) give the density value "1" for almost any outcome $\mathbf{z}$ of $\mathbf{Z}$. Without further information, there is no way of discriminating between any two such vectors by saying that one is any more likely than the other to be the true parameter vector. Thus, we require a prior distribution $\xi(\boldsymbol{\Psi},\mathbf{e},\boldsymbol{\alpha},\boldsymbol{\beta})$ that assigns a probability to each parameter vector, i.e. allows for unique identification of the parameters, and that we define based on the following assumptions:

1. $\xi(\boldsymbol{\Psi},\mathbf{e},\boldsymbol{\alpha},\boldsymbol{\beta}) = \xi(\mathbf{e})\xi(\boldsymbol{\lambda}^*(\boldsymbol{\Psi},\boldsymbol{\alpha},\boldsymbol{\beta}))\xi(\boldsymbol{\eta}(\boldsymbol{\Psi},\boldsymbol{\alpha},\boldsymbol{\beta}))$, with $\boldsymbol{\eta}(\boldsymbol{\Psi},\boldsymbol{\alpha},\boldsymbol{\beta})$ denoting the vector of implied own price supply elasticities. That is, we assign prior distributions to error terms, dual values and implied point price elasticities of supply, and assume that those are functionally independent.
2. The errors $\mathbf{e}$ are independent and normally distributed with standard deviations equal to a fix share of the observed value of the respective parameter. Specifically we assume $\mathbf{e} \sim N(0,\boldsymbol{\Sigma}_e)$ with $\boldsymbol{\Sigma}_e$ a diagonal matrix with $\sigma_{ei}^2 = (0.20/3z_i)^2$ on the i^{th} position. This means that we assume that errors are independent normally distributed with mean zero covariance matrix such that three standard deviations cover 20% of the observed value (or prior mode) of the related parameter.
3. The dual values $\boldsymbol{\lambda}$ are independent, with means proportional to average observed gross margins over all crops in each region each year, and standard deviations proportional to a fix share of that. Specifically, the prior mode is assumed to be 25% of the average observed gross margin $\overline{m}_t$ in the respective year taken over all crops except sugar beet[3], and that three standard deviations cover 20% of the prior mode. (compare empirics)
4. We asssume that the parameter vector is such that the implied point price elasticity of supply matrix $\boldsymbol{\eta}(\boldsymbol{\Psi},\boldsymbol{\alpha},\boldsymbol{\beta})$ is normally distributed with mean depending on the crop mix (rotational shares) and standard deviation independent for each item. Means and standard deviations are derived below.

Most studies (see comparison to other studies below) find supply elasticities in the range of, say, 0.1 to 5, and that the elasticity typically around unity for major crops, but higher for crops that occupy a small share of the total area. The main argument for such a relation is that expanding a major crop by a given percentage under land and rotational constraints is difficult compared to expanding a crop with small current acreage by the same percentage. Letting r_j denote the share of land allocated to crop j, we assume that the own price supply elasticities have means $0.5r_j^{-1/3}$. That assumption further discussed in the results section. Standard deviations are such that three standard deviations cover 1000% of the mean. The standard deviation relative to mean is thus fifty times that of the acreages, prices or yields.

The explicit expression for supply elasticities in our constrained model can be obtained by solving the first order conditions for $\mathbf{x}_t$ (repeated here for convenience),

$$\mathbf{x}_t^*(\mathbf{p}_t,\boldsymbol{\lambda}_t) = l_t^{-1}\left[\mathbf{D}+\mathbf{GBG}'\right]^{-1}\left[\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c} - \mathbf{R}_t'\boldsymbol{\lambda}_t\right]. \quad (7)$$

Let $\mathbf{E}_t = l_t\left[\mathbf{D}+\mathbf{GBG}'\right]$ and insert that expression into the constraints to obtain a solution for $\boldsymbol{\lambda}$,

$$\boldsymbol{\lambda}_t^*(\mathbf{p}_t) = \left[\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{R}_t'\right]^{-1}\left[\mathbf{R}_t\mathbf{E}_t^{-1}\left(\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c}\right) - \mathbf{v}_t\right]. \quad (8)$$

Computing $\mathbf{x}^*_t(\mathbf{p}_t,\boldsymbol{\lambda}^*_t(\mathbf{p}_t))$ by inserting (8) into (7), taking derivatives and multiplying the result by yield gives us the expression for marginal production[4], which finally can be used in the general definition of a price elasticity to obtain:

$$\boldsymbol{\eta}_t = \mathbf{X}_t^{-1}\left(\mathbf{E}_t^{-1}\mathbf{Y}_t - \mathbf{E}_t^{-1}\mathbf{R}_t'\left[\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{R}_t'\right]^{-1}\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{Y}_t\right)\mathbf{P}_t \quad (9)$$

[3] Because sugar quota rents are missing in the model

[4] In this case, the marginal production could be solved for directly. In the general case with continuous derivatives, the implicit function theorem may be used instead.

where upper case $\mathbf{X}_t$ means the square diagonal matrix with $\mathbf{x}_t$ on the diagonal, and similar for upper case $\mathbf{P}_t$ and $\mathbf{Y}_t$.

This expression is strongly non-linear in **D** and **B** (via **E**) and thus difficult to include as constraint in the estimation. In some models, the expression has been simplified by neglecting the second term in the bracket and only computing diagonal elements in **E**. That simplification was previously used in different model to compute only diagonal elements of **E**, e.g. in the CAPRI model (not published), and by Helming (2005) in the DRAM model.

Nevertheless, with appropriate initialisation of the solution algorithm (CONOPT for GAMS) together with reasonable bounds for the variables, equation 9 turns out to be possible to solve simultaneously in the estimation (with inversion and Cholesky factorization of the Hessian), thus enabling us to include our priors regarding elasticities of supply in a transparent way[5].

D. Definition of the estimator

Putting all the pieces together, we can now formulate the estimation problem using Bayes's theorem as above and write

$$\hat{\boldsymbol{\Psi}} = \arg\max \xi(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e}|\mathbf{z}) \propto f(\mathbf{z}|\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e})\xi(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e})$$

With the degenerate density function, this is equivalent to solving

$$\begin{aligned} &\max && \xi(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e}) \\ &\text{subject to} && \mathbf{z} = \boldsymbol{\mu}(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha}) + \mathbf{e} \\ & && \mathbf{g}(\boldsymbol{\psi},\mathbf{x}^*,\boldsymbol{\lambda}^*) = \mathbf{0} \end{aligned}$$

Taking the logarithm of the objective function and replacing the constraints with the equations derived above, we arrive at the following extremum estimation problem:

Minimise

$$\text{vec}\left(\mathbf{e}_x,\mathbf{e}_Y,\mathbf{e}_p,\mathbf{e}_w,\mathbf{e}_A,\left(\boldsymbol{\lambda}-\boldsymbol{\lambda}^{prior}\right),\left(\text{diag}(\boldsymbol{\nu})-\hat{\mathbf{v}}\right)\right)' \times \boldsymbol{\Sigma}_{total}^{-1}\text{vec}\left(\mathbf{e}_x,\mathbf{e}_Y,\mathbf{e}_p,\mathbf{e}_w,\mathbf{e}_A,\left(\boldsymbol{\lambda}-\boldsymbol{\lambda}^{prior}\right),\left(\text{diag}(\boldsymbol{\nu})-\hat{\mathbf{v}}\right)\right)$$

subject to

$$\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c} - l_t\left[\mathbf{D}+\mathbf{GBG}'\right]\mathbf{x}_t - \mathbf{R}_t'\boldsymbol{\lambda}_t - MAC_t\delta = \mathbf{0}$$

$$\mathbf{R}_t\mathbf{x}_t = \mathbf{v}_t$$

$$\mathbf{B} = \mathbf{U}'\mathbf{U}$$

$d_{jj} \geq 0$ for $j = 1\ldots J$ (and $d_{ij} = 0$ for $i \neq j$)

$$\mathbf{x}^{obs} = \mathbf{x} + \mathbf{e}_x$$

$$\mathbf{Y}^{obs} = \mathbf{Y} + \mathbf{e}_Y$$

$$\mathbf{Y}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 T_t$$

$$\mathbf{p}_t = \mathbf{p}_{t-1}^{obs} + (\mathbf{p}_t^{adm} - \mathbf{p}_{t-1}^{adm}) + \mathbf{e}_{pt}$$

$$\mathbf{A}^{exp} = \mathbf{A} + \mathbf{e}_A$$

$$\mathbf{A} = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 T_t$$

$$\nu_t = \mathbf{X}_t^{-1}\left(\mathbf{E}_t^{-1}\mathbf{Y}_t - \mathbf{E}_t^{-1}\mathbf{R}_t'\left[\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{R}_t'\right]^{-1}\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{Y}_t\right)\mathbf{P}_t$$

The dummy variable MAC_t with associated parameter δ in the first order condition was added to control for additional effects of the MacSharry reform. It is equal to 1 for year 1992 and earlier for regions that were member of the EU then, and zero from 1993 and on[6].

The estimator resembles the Bayesian analysis of the measurement error model in [18], but is more complex since it instead of the linear model in [18] (eq. 5.31) has a system of equations representing the optimality conditions of CAPRI, includes nonlinear curvature constraints, and instead of the additive measurement error model for the "exogenous" ([18] eq. 5.30) it relates some model parameters to observations through a simple expectation model

III. DATA

Data for the estimation is provided by the CAPRI database. The part of the dataset relevant for this research has been compiled from the *Economic Accounts for Agriculture* and *New Cronos Regio*, both from Eurostat, completed with policy information from regulations and expert data where necessary. The dataset has been processed by econometric/heuristic software of the CAPRI system to be made complete (no holes in time series) and consistent (with respect to physical and economical interrelations) on NUTS2 level [1]. Despite those efforts, the CAPRI database is still an unbalanced panel.

The panel being unbalanced poses no problem to the estimator, but the estimations require data to be processed prior to estimation by omitting crops,

[5] See [7] for an illustrative example.

[6] A special case of this estimator—if the economic model were a linear equation, the vector e normally distributed for only one of the variables and for the other either uninformative or degenerate zero—is equivalent to an OLS.

regions and years for which the data cube does not satisfy minimum standards. Furthermore, a simple routine for replacing outliers with series mean was applied prior to estimation.

IV. RESULTS

The results are evaluated according to the resulting model behaves in simulation, by comparing the estimated supply elasicities to estimates from literature. Although there are several studies that present elasticities on national level, no other study that the author is aware of publishes elasticities for individual crops on regional level with this crop coverage. Below we compare our point elasticity estimates (for 2002) as well as our priors two studies for France, one for the Netherlands and one for Denmark.

For France, table (1) compares our results to those in [5] (HB00) and [4] (GBC96). GBC96 estimates a model with seven outputs and three inputs based on a restricted profit function, using annual data for France. HB00 estimate a similar model as ours, but they use a cross-section data set of French regions for the year 1994 instead of time series for individual regions as we do.

Table 1. Comparison with other studies of own price supply elasticities in France

Crop	Land share[b]	Prior[c]	Estimate	GBC96[d]	HB00e
Coarse grains[a]	0.034	1.547	2.531	0.758	-.---
Soft wheat	0.273	0.771	1.009	0.715	1.322
Maize	0.102	1.070	1.680	1.630	0.653
Barley	0.092	1.109	2.243	0.351	2.647
Rapeseed	0.045	1.405	1.284	0.418	1.457
Sunflower	0.027	1.664	2.959	0.223	1.126
Soya	0.004	3.276	2.020	3.701	1.861

a: Aggregated from rye, oats and other cereals.
b: Computed from the data in CAPRI for 2002
c: Using the formula for priors reported above
d: Guyomard et al. (1996)
e: Heckelei and Britz (2000)

We see that GBC96 finds smaller elasticities for barley (0.35) and other coarse grains (0.76) than this study (2.24 and 2.53), HB00 (2.65 for barley) or the priors (1.11 and 1.55). For soft wheat the results are much more in line, with the priors (0.77) quite close to GBC96 (0.72) and the estimates (1.01) in between GBC96 and HB00 (1.32). For maize the estimates (1.68) are close to GBC96 (1.63) but much higher than HB00 (0.65), whereas the priors lie in between (1.07). Rapeseed and sunflower occupy small rotational shares, less than 5%, and as a consequence the priors are higher, about 1.5. The elasticity estimates for those crops are also much higher, 1.28 and 2.96, than GBC96, which finds values of 0.42 and 0.22, and more in line with HB00, which finds elasticities greater than unity. All of the three studies find high elasticities for soya, for which the rotational share is less than 0.5%.

For the Netherlands, [14] (OLP96) estimate twelve farm type models producing three outputs (CO = Cereals and oilseeds, Rootcrops = Potatoes and sugar beet, and Other = all other crops). They estimate the model using panel data on individual farms, and also have a land constraint and a fixed area of rootcrops. In their table A3 they present supply elasticites, of which the own price effects are compared to our estimates for the Netherlands for similar product aggregates in table (2). To make the comparison, our individual crop elasticities have been aggregated with estimated planned rotational shares for 2002. The “other crops” aggregate in OLP96 could not be formed, since we have three crops, (voluntary and compulsory set-aside and fallow land) for which there is no output price.

Table 2. Comparison with other own price supply elasticity estimates for the Netherlands

Crop group	Land share	Prior	Estimate	OLP96[a]
CO	0.266	0.778	0.937	0.90
Root crops	0.342	0.715	0.909	0.24

a:Oude Lansink and Peerlings (1996)

Our estimates for CO (0.94) are quite close to OLP96 (0.90), but considerably higher for root crops (OLP96 find 0.34, our estimate 0.91). We must then keep in mind that in OLP96, the area used in root crops was fixed, so that the price elasticity can come only from a change in intensity. It then seems reasonable that their estimates for that aggregate turn out lower.

Table 3. Comparison with other own price supply elasticity estimates for Denmark

Crop	Land share	Prior	Estimate	Jensen (1996)
Cereals	0.575	0.601	1.073	0.60
Pulses + rapeseed	0.037	1.498	1.999	0.66
Root crops	0.035	1.522	3.772	3.80

Jensen [12] estimates an econometric model of Danish agriculture, and also presents aggregated supply elasticities for three selected crop groups. In table (3) we have reprinted those elasticities and also our implied estimates for the corresponding aggregates. We see that for the first two groups, our elasticities are higher than those of Jensen., though our prior for cereals is similar to Jensen's estimates. For the last group, root crops, the elasticities are very similar and more than twice as high as our prior.

V. CONCLUSIVE REMARKS

We conclude that the estimated elasticities compare well with estimates from literature in the four cases studied. Nevertheless, only a handful of elasticities from three member states could be compared. The vast amount of estimates are for individual crops in NUTS2 regions, and for them, we have nothing to compare to. Many parameters for small crops at regional level (not published here) are rather high. Such parameter settings will result in a model that reacts strongly on shocks in simulation compared to the current CAPRI model that in the past had inelastic supply.

No confidence regions for the estimates are established. Exact analytical confidence regions are very difficult to deduce. Approximations would in theory be possible. [17] compute approximate probability contours of the posterior in a non-linear errors-in-variables model by iterated linearisations. In our case, analytical deduction of approximate confidence regions is more difficult than in ibid. due to the curvature constraints. Numerical computation by Monte Carlo simulations is not feasible because of the amount of computation time required with the present setup (several hours for a single simulation of all regions).

The estimation produced a large number of results: 1917 elements of the key parameters **c** and **D** respectively, and 5457 elements of the cross group effects matrix **B**. Furthermore, 329 092 price elasticities were computed, including the cross price elasticities. To this comes a very large number of fitted values and all other parameters. The estimation program, data set and full results are shared by the author upon request (electronic). With future application of CAPRI with the new parameters, experiences will be gained regarding the performance of the estimates.

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APPENDIX 1. ACTIVITIES IN ESTIMATION

Table 1. Crop groups and activities modelled

Group	Crops
Cereals:	Soft wheat, durum wheat, Rye, Barley, Oats
Cereals2:	Maize, Other cereals
Oil seeds:	Rapeseed, Sunflower, Soya, Other oilseeds, Non-food rapeseed
Other arable crops:	Potatoes, Pulses, Sugar beet, Fibre crops
Fodder on arable land:	Fodder maize, Silage grass, Fodder root crops
Non-yield crops:	Obligatory set-aside, Voluntary set-aside, Fallow land

List of inputs in the estimation:

Seeds, fuel, pesticides, electricity, lubricants, fertilizers, gas for drying, repairs machinery, repairs buildings, other inputs.