Pooling, Separating, and Cream-Skimming In Relative-Performance Contracts

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Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Providence, Rhode Island, July 24-27, 2005

Abstract: Existing research on tournament-style contests suggests that mechanisms to sort contestants by ability level are unnecessary in the case of linear relative-performance contracts. This paper suggests that this result stems from uniform treatment of workers' marginal returns from effort, marginal disutilities of effort, and reservation wages. Here, we investigate relative-performance contracts with a model that allows these three factors to vary by growers' unobservable ability. Given this framework, we find that it is possible for processors to improve expected profits and total expected welfare by replacing a single contract offering meant to pool all growers with an offering of two contracts meant to separate growers by ability. Under some circumstances, a "cream-skimming" contract offering designed to attract only workers above a minimum ability level can also improve expected profits.

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When output is stochastic, conditioned on workers' efforts or abilities, and correlated across workers, tournament-style relative-performance contracts are found to exhibit numerous efficient properties (Lazear and Rosen 1981; Nalebuff and Stiglitz 1983; Green and Stokey 1983; Holmström 1982; Malcomson 1986). While tournament-style contests are incorporated into many labor-market settings, such as those where salespersons compete for bonuses or employees compete for promotions, perhaps the best known and studied setting involves the use of contracts between poultry companies and broiler growers (e.g., Knoeber and Thurman, 1994; Levy and Vukina, 2002 and 2004; Goodhue, 2000; Tsoulouhas and Vukina, 2001). Within a particular settlement group, broiler growers are offered a single, pooled contract containing an incentive clause based on the growers' relative performance within the group. Standard results suggest that when growers' performances are random but conditioned on their efforts, pooling contracts with relative-performance clauses outperform piece-rate contracts if risks common to all growers in the group dominate individual risks. In other words, by basing an incentive payment on an individual grower's relative performance, broiler contracts are designed to provide high-powered individual incentives while minimizing the noisiness of the performance measure.

When heterogeneous and hidden grower abilities are taken into account, however, some of the efficiency properties of broiler contracts are lessened and, in fact, depend on the exact structure of the contract. For example, Lazear and Rosen (1981) show that heterogeneous, mixed-type contests create inefficiencies in rank-order tournaments. For these tournament schemes, Lazear and Rosen (1981, p. 858) suggest that handicapping or non-price rationing can alleviate some of the inefficiencies, but that a "pure price," incentive pay mechanism that induces self-selection cannot be sustained.[1] On the other hand, Knoeber and Thurman (1994) suggest that linear relative-performance evaluation schemes – those where the incentive payment is proportional to the gap between individual and group average performance – do not lose their efficiency with heterogeneous contests because higher-ability growers have "no incentive to rest on their laurels" and lower-ability growers have "no incentive to accept their fate" (Knoeber and Thurman, 1994, p.170). These claims are echoed by Levy and Vukina, who say that relative-performance contracts do not provide principals an incentive to sort mixed-ability agents into homogeneous groups (Levy and Vukina, 2004, p. 354).[2] The general implication of this line of research is that mechanisms to sort different ability contestants are either unnecessary, as in the case of relative-performance contracts, or costly, as in the case of the rank-order tournament contracts.

This paper suggests that the Knoeber and Thurman finding, however, depends on several commonly employed assumptions used in relative-performance contracting models. First, most models of relative-performance contracts specify that agents' marginal rewards for improved performance are identical no matter whether agents are more or less able (e.g., Knoeber and Thurman 1994, p. 161; Levy and Vukina 2004, p.354-355). Second, many models likewise assume that agents' marginal disutility of effort does not vary by agents' ability. And third, to the best of our knowledge, all models investigating relative-performance contracts assume that all agents within a contest have identical outside opportunities (or reservation utilities), even if they are also assumed to have heterogeneous abilities (e.g., Levy and Vukina 2002; Tsoulouhas and Vukina 2001). In the standard principal-agent framework, these assumptions manifest

[1] Bhattacharya and Guasch (1988) offer a solution to this inefficiency by constructing a contest that compares one ability group's performance against another's.
[2] Levy and Vukina (2004) focus on a separate problem associated with mixing agents of heterogeneous abilities: namely, they investigate the risk associated with random composition of agents – i.e., random league composition.

themselves most importantly in the specification of workers' incentive and participation constraints. More to the point, these assumptions help lead to the finding that sorting by ability is unnecessary when relative-performance contracts are used.

In this paper, we expand on the relatively standard model of relative-performance contracts in a number of ways, most importantly by allowing an agent's ability to affect his marginal reward, marginal disutility, and outside opportunities. To the best of our knowledge, these model extensions represent important departures from previous studies of incentives in relative-performance contracts.[3] Despite the extensions, however, our principal-agent model, described in the next section, is largely consistent with the standard features of existing contracting models. The standard model often allows for either moral hazard when agents' efforts are assumed to be unobservable (e.g., Knoeber and Thurman 1994; Levy and Vukina 2002), adverse selection when agents' abilities are likewise assumed to be unobservable (e.g., Goodhue[4] 2000), or both (e.g., Lazear and Rosen 1981; Yun 1997). Our model incorporates both moral hazard and adverse selection. The standard model also often specifies growers' output as having both a deterministic portion based on growers' efforts (and maybe their abilities) and a stochastic portion with both individual and common shocks (e.g., Levy and Vukina 2004). Our model also has these standard components; however, we employ a relatively uncommon output specification to allow for the marginal reward from effort to vary by ability level. Finally, we restrict our model in two important ways: First, we restrict the contract type to linear contracts with a base-pay component and an incentive component that depends on gap between individual

[3] They may have precedent, however, in other lines other research areas. For example, heterogeneous outside opportunities are often employed in the more general context of investigating labor mobility, unemployment, and so-called "efficiency wages" (e.g., Weiss, 1990; Arnott, Hosios, and Stiglitz, 1988; Kahn, 1988; McLaughlin, 1991; Arvin and Arnott, 1992; Ito, 1988; and Bloemen and Stancanelli, 2001).

[4] Goodhue (2000) discusses but does not model the case with both adverse selection and moral hazard.

and group-average output. And second, we assume the number of agents is fixed, and that an offering of separating contracts is feasible only if it is acceptable to all agents.

Using this expanded model, we find that it is generally possible for a principal (e.g., a poultry processor) to improve profitability by replacing its usual single offering of a contract that pools all heterogeneous agents (e.g., poultry growers) with an offering of two contracts that separates growers into two groups of abilities – high and low.[5] As opposed to the single, pooling contract, we find that two separating contracts can reduce expected utility for low-ability growers but increase it for high-ability growers. These results are sensitive, however, to how grower abilities are distributed.

Under some circumstances, we also find that a single "cream-skimming" contract offering that pools only growers above a minimum ability level, rather than pooling all growers, can sometimes improve processor profits over a pooling contract (but not over separating contracts).[6] The cream-skimming contract often increases the average utility of participating growers' over the average utility of growers in the pooling contract. While we find that a cream-skimming contract generally leads to lower processor profits than two separating contracts, extra transaction costs or political costs (not modeled) associated with a second contract offering could justify its use. In fact, we may have little way of knowing that a cream-skimming contract is not already employed by some poultry processors. For example, industry observers witness a single contract offering and generally assume it is a pooling contract; but grower turnover could be

[5] A number of background assumptions are behind this result: First, a sorting equilibrium is by no means guaranteed. In this paper, we focus on the results from specifications where a sorting equilibrium is possible rather than investigating the sufficient conditions for an equilibrium's existence. Second, we also do not examine the case when a second principal offers a competing set of contracts to the same set of agents. And third, we only investigate the case where agents are separated into two groups, where the arbitrary number two is chosen for simplicity.

[6] The term "cream skimming" is often used in settings of asymmetric information where the party with hidden information is able to use that information to his or her advantage. For example, it is often used in the health insurance sector where it may be possible for insurers or health maintenance organizations to target only the healthiest customers. For more examples of how cream skimming may be used, see, for example, Asheim and Nilssen (1996); Barros (2003); Lewis and Sappington (1995); and Marchand, Sato, and Schokkaert (2003).

evidence that cream-skimming is currently occurring. The less than full employment outcome that results from the cream-skimming contract offering is consistent with numerous efficiency wage and labor studies where adverse selection is present (e.g., Weiss 1990; Shapiro and Stiglitz 1984; Carmichael 1985 and 1990; Jovanovich 1979). After the model development and some analytic manipulations, these results are established with the help of a numerical example. We conclude the paper with a discussion of limitations and policy implications.

The Model

For a starting point, we rely on a standard principal-agent model that bases individual incentive compensation on a group benchmark. While the model is general, it is most concretely applied to the broiler industry. We will therefore use the term "growers" to mean agents working on behalf of a single poultry processor, i.e., the principal. In the broiler industry, grower compensation clauses in standard broiler contracts contain a base payment and a bonus or discount payment depending on how well growers perform against the average performance of a specified group of n growers. The calculation of the group's average performance includes all growers whose flocks were harvested at approximately the same time. In addition, for computational purpose, we assume that a fixed comparison group is adopted by the processor.[7] Given this setup, a simplified payment function for grower i under a relative-performance contract can be constructed as:

(1) $$w_i = \alpha + \beta[x_i - \frac{1}{n}\sum_{j=1}^{n} x_j],$$

[7] Levy and Vukina (2004) distinguish broiler contracts with a random comparison league and fixed comparison league. For this study, a fixed comparison league is assumed only for computational purposes. However, it may be plausible to assume that broiler processors usually contract with relatively fixed broiler growers due to regional restrictions. Also note that the group average performance benchmark, described in (1), includes grower i.

where x_i is grower i's live output of broilers, α is the base payment, and β is the incentive bonus. We further assume that the live output produced by each grower is given by $x_i = x(e_i, a_i, z, u_i)$, where e_i is grower i's effort, a_i is grower i's ability realized before the contract is signed, z is a common shock borne by all growers, and u_i is an idiosyncratic shock faced by grower i. We assume that z is an i.i.d. normal random variable with mean zero and variance σ_z^2, u_i is an i.i.d normally distributed random variable across growers with mean zero and variance σ_u^2, and a_i is randomly distributed in the range $[\underline{a}, \overline{a}]$ with $0 < \underline{a} < \overline{a} < \infty$.[8] Recall that we assume that neither growers' abilities nor efforts are directly observable by the processor. However, the distributions of the random shocks and growers' abilities are public information to both the processor and all growers, and the processor can only observe growers' outputs at the time of transaction.

The following output structure is used for this study:

(2) $$x_i = \phi a_i^p e_i^q + z + u_i \quad ,$$

where the coefficients p and q can be thought of as returns-to-scale parameters common to all broiler growers, and the scalar ϕ is an arbitrary constant. Unlike previous research that specifies (2) as additive in ability and effort, the multiplicative form allows for growers' marginal product of effort to vary by ability.[9] Assuming fixed comparison groups over time, the variance of x_i is thus $\text{var}(x_i) = \sigma_z^2 + \sigma_u^2$ and the covariance between any x_i and x_j is $\text{cov}(x_i, x_j) = \sigma_z^2$. While agents' abilities are assumed to be known to growers before the contract is signed, the processor cannot distinguish grower abilities, and each grower cannot distinguish other growers' abilities either before or after the contract is signed.

[8] A number of explicit distributions for a_i will be specified later for numerical simulation.
[9] Goodhue (2000) represents an exception to the additive specification: she assumes that $p = 1$, $q < 1$, and adds a variable that reflects purchased inputs. Later in the paper, we examine a special case of (2) where $p = 0$, $q = 1$, and $\phi = 1$, which corresponds to the standard additive specification.

The processor is risk neutral and has a profit function, $\pi(x,w)=\sum_{i=1}^{n}(x_i - w_i)$, where w_i is specified in (1), and the processor's output price is normalized to one. Each grower with ability a_i has a time-separable utility function $U_i(w_i,e_i,a_i)=u(w_i)-C(e_i,a_i)$, where the utility function is strictly concave and the disutility function takes the form $C(e_i,a_i)=\frac{1}{2a_i}e_i^2$, a specification that allows for heterogeneous marginal disutilities of effort. Further, we adopt the commonly used assumption that growers' utility functions have the property of constant absolute risk aversion, $u(w_i)=1-\exp(-rw_i)$, where r is the Arrow-Pratt coefficient of absolute risk aversion. Thus expected utility $E[U_i(\cdot)]$ is tantamount to $E[U_i(\cdot)]\propto Ew_i-\frac{1}{2}r\,\mathrm{var}(w_i)-\frac{1}{2a_i}e_i^2$.

Here and throughout the essay, we use E to represent the mathematical expectation operator conditional on information available at the beginning of the period.

A Single Offering of a Pooling Contract

In this case, a processor offers a single contract $C_P=\{\alpha_P,\beta_P\}$ intended to be acceptable to all n heterogeneous growers even though it is the growers' individual choice to accept or reject the contract. As usual, the offered contract specifies a payment schedule depending on $\{\alpha_P, \beta_P, x\}$. Given the pooling contract and the assumptions described above, the processor maximizes its expected profits, Π_P, subjected to incentive-compatibility constraints and growers' participation constraints.

Thus, the processor solves the problem:

(3) $$\Pi_P=\max_{\alpha,\beta}E_a\{\sum_{i=1}^{n}(Ex_i-Ew_i)\}$$

subject to

(4) $$EU_i = Ew_i - \frac{1}{2} r \operatorname{var}(w_i) - \frac{1}{2a_i} e_i^2 \geq u_0(a_i) \qquad \forall i, \quad \text{and}$$

(5) $$e_i \in \arg\max \{Ew_i - \frac{1}{2} r \operatorname{var}(w_i) - \frac{1}{2a_i} e_i^2\}, \qquad \forall i.$$

The participation constraint (4) states that each grower of heterogeneous ability obtains at least his reservation utility $u_0(a_i)$ under the pooling contract offered by the processor, while the incentive-compatibility constraint (5) requires that each grower optimally chooses his effort by maximizing his expected utility. An important departure from most models is that each heterogeneous grower is associated with a different reservation utility. Here we assume that $u_0(a_i)$ increases with a_i because higher-ability growers are likely to face better outside options.[10]

The solution to this problem, which relies on the specifications of w_i in (1) and x_i in (2), is fairly standard. One assumes that one of the participation constraints is binding because, otherwise, the processor can always reduce the payment to the growers until it reaches their reservation utility level.[11] Without loss of generality, it is assumed here that the processor offers a contract such that only the grower with the lowest ability, $\underline{a}$, obtains his reservation utility, and all other grower types capture strictly greater utility relative to their reservation utility. Given this setup, we offer the standard solution next.

From (1), $\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} (\alpha + \beta(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j)) \equiv n\alpha$. Thus, the processor's total expected profit can be written as $\Pi_P = E_a\{\sum_{i=1}^{n} Ex_i - n\alpha\}$. In addition, the total expected welfare, W_P, of all growers and the processor is

[10] Heterogeneous outside opportunities are more common in efficiency wage literature (e.g., Weiss). When we later specify the exact shape of $u_0(a_i)$, we will assume that outside opportunities vary by ability in the same general way that a contract grower's expected utility does. To be specific, $u_0(a_i)$ will have the same order in a_i (i.e., a_i will be raised to the same power) as the contract grower's expected utility. The order of a_i will depend on p and q.

[11] Good references on this topic include Mas-Collel, Whinston, and Green (1995) and Salanié (1997).

$$W_P = \Pi_P + \sum_i EU_i = E_a\{\sum_{i=1}^n (Ex_i - Ew_i + Ew_i - \frac{1}{2} r \operatorname{var}(w_i) - \frac{1}{2a_i} e_i^2)\}$$

$$= E_a\{\sum_{i=1}^n (Ex_i - \frac{1}{2} r \operatorname{var}(w_i) - \frac{1}{2a_i} e_i^2)\}.$$

With the optimal pooling contract denoted as $C_P = \{\alpha_P, \beta_P\}$, the payment to each grower becomes $w_i = \alpha_P + \beta_P[x_i - \frac{1}{n}\sum_{j=1}^n x_j] = \alpha_P + \beta_P[x_i - \bar{x}]$, $\forall i$. Hence, we could compute the expected payment and the variance for each grower type:

$$Ew_i = \alpha_P + \beta_P[\phi a_i^p e_i^q - E_a(\frac{1}{n}\sum_{j=1}^n (\phi a_j^p e_j^q))],\text{ and}$$

$$\operatorname{var}(w_i) = \beta_P^2 \operatorname{var}(x_i - \frac{1}{n}\sum_{j=1}^n x_j])$$

$$= \beta_P^2[(\frac{(n-1)^2}{n^2} + \frac{n-1}{n^2})(\sigma_z^2 + \sigma_u^2) - 2\frac{(n-1)^2}{n^2}\sigma_z^2 + \frac{(n-1)(n-2)}{n^2}\sigma_z^2] = \beta_P^2 \frac{n-1}{n}\sigma_u^2.$$

Note that the variance of each grower's payment depends only on the idiosyncratic shock without being affected by the common shock.

Substituting Ew_i and var(w_i) into the processor's problem (3) - (5) yields

$$\Pi_P = \max_{\alpha_P, \beta_P} E_a\{\sum_{i=1}^n Ex_i - n\alpha_P)\}$$

subject to

$$EU_i = \alpha_P + \beta_P[\frac{n-1}{n}(\phi a_i^p e_i^q) - E_a(\frac{1}{n}\sum_{j=1, j\neq i}^n (\phi a_j^p e_j^q))] - \frac{1}{2} r\beta_P^2 \frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_i^2 \geq u_0(a_i), \ \forall i$$

$$e_i \in \arg\max\{\alpha_P + \beta_P[\frac{n-1}{n}(\phi a_i^p e_i^q) - E_a(\frac{1}{n}\sum_{j=1, j\neq i}^n (\phi a_j^p e_j^q))] - \frac{1}{2} r\beta_P^2 \frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_i^2\}, \ \forall i.$$

From the incentive-compatibility constraint, each grower chooses the optimal effort such that

$$e_i = [\phi \frac{n-1}{n} a_i^{p+1} \beta_P q]^{\frac{1}{2-q}} \qquad (q \neq 2).$$

Hence, from (2), the expected output for each grower type a_i is

$$Ex_i = \phi a_i^p e_i^q = \phi a_i^p [\phi \frac{n-1}{n} a_i^{p+1} \beta_P q]^{\frac{q}{2-q}} = \phi[\phi \frac{n-1}{n} \beta_P q]^{\frac{q}{2-q}} a_i^{\frac{2p+q}{2-q}} \qquad (q \neq 2),$$

and the cost function of each grower type can be written as

$$C(e_i, a_i) = \frac{1}{2a_i} e_i^2 = \frac{1}{2a_i} [\phi \frac{n-1}{n} a_i^{p+1} \beta_P q]^{\frac{2}{2-q}} = \frac{1}{2} [\phi \frac{n-1}{n} \beta_P q]^{\frac{2}{2-q}} a_i^{\frac{2p+q}{2-q}} .$$

Therefore, the expected utility of each grower type a_i has the order of $O(a_i^{\frac{2p+q}{2-q}})$ in its own type. Based on this observation, we assume that each grower type's reservation utility has the same order in a_i as its expected utility function.[12] More specifically, each type's reservation utility takes the following form: $u_0(a_i) = k a_i^{\frac{2p+q}{2-q}}$, where k is a positive constant.

When the grower with the lowest ability, $\underline{a}$, obtains his reservation utility, one participation constraint becomes binding:

$$E[U_i \mid a_i = \underline{a}] = \alpha_P + \beta_P[\frac{n-1}{n}(\phi a_i^p e_i^q) - E_a(\frac{1}{n}\sum_{j=1, j\neq i}^{n}(\phi a_j^p e_j^q))] - \frac{1}{2} r \beta_P^2 \frac{n-1}{n} \sigma_u^2 - \frac{1}{2a_i} e_i^2 = u_0(\underline{a}).$$

If we denote the optimal effort exerted by ability type $\underline{a}$ as $\underline{e}$, we find that

$$\alpha_P = u_0(\underline{a}) - \beta_P[\frac{n-1}{n}(\phi \underline{a}^p \underline{e}^q) - E_a(\frac{1}{n}\sum_{j=1, a_j \neq \underline{a}}^{n}(\phi a_j^p e_j^q))] + \frac{1}{2} r \beta_P^2 \frac{n-1}{n} \sigma_u^2 + \frac{1}{2\underline{a}} \underline{e}^2 \quad .$$

The processor's total profit under this contract becomes

[12] If each grower's expected utility and reservation utility have different orders in ability, then the participation constraints could easily be violated for some range of grower types. An economic rationale for this mathematical necessity is that ability may be expected to have similar effects on utility, both inside and outside the contract poultry market.

$$\Pi_P = \max_{\alpha_P, \beta_P} E_a\{\sum_{i=1}^{n} Ex_i - n\alpha_P\}$$

$$= E_a\{\sum_{i=1}^{n}(\phi[\phi\frac{n-1}{n}\beta_P q]^{\frac{q}{2-q}} a_i^{\frac{2p+q}{2-q}}) - nu_0(\underline{a}) + n\beta_P[\frac{n-1}{n}(\phi\underline{a}^p \underline{e}^q) - E_a(\frac{1}{n}\sum_{j=1, j\neq i}^{n}(\phi a_j^p e_j^q))]$$

$$-\frac{1}{2}r\beta_P^2(n-1)\sigma_u^2 - n\frac{1}{2}[\phi\frac{n-1}{n}\beta_P q]^{\frac{2}{2-q}} \underline{a}^{\frac{2p+q}{2-q}}\}$$

Denoting $E_a[a_i^{\frac{2p+q}{2-q}}] = a_m^{\frac{2p+q}{2-q}}$, then differentiating Π_P with respect to β_P yields

$$\frac{\partial \Pi_P}{\partial \beta_P} = n\phi[\phi\frac{n-1}{n}q]^{\frac{q}{2-q}}\frac{q}{2-q}a_m^{\frac{2p+q}{2-q}}\beta_P^{\frac{2q-2}{2-q}} + n\phi\frac{2}{2-q}\beta_P^{\frac{q}{2-q}}\frac{n-1}{n}[\phi\frac{n-1}{n}q]^{\frac{q}{2-q}}(\underline{a}^{\frac{2p+q}{2-q}} - a_m^{\frac{2p+q}{2-q}})$$

$$-r\beta_P(n-1)\sigma_u^2 - n\frac{1}{2-q}[\phi\frac{n-1}{n}q]^{\frac{2}{2-q}}\beta_P^{\frac{q}{2-q}}\underline{a}^{\frac{2p+q}{2-q}} = 0$$

Hence, the following equation solves the optimal bonus under a single tournament contract.

$$\beta_P \in \arg\{n\phi[\phi\frac{n-1}{n}q]^{\frac{q}{2-q}}\frac{q}{2-q}a_m^{\frac{2p+q}{2-q}}\beta_P^{\frac{2q-2}{2-q}} + n\phi\frac{2}{2-q}\beta_P^{\frac{q}{2-q}}\frac{n-1}{n}[\phi\frac{n-1}{n}q]^{\frac{q}{2-q}}(\underline{a}^{\frac{2p+q}{2-q}} - a_m^{\frac{2p+q}{2-q}})$$

$$-r\beta_P(n-1)\sigma_u^2 - n\frac{1}{2-q}[\phi\frac{n-1}{n}q]^{\frac{2}{2-q}}\beta_P^{\frac{q}{2-q}}\underline{a}^{\frac{2p+q}{2-q}} = 0\}$$

In particular, when $p = q = 1$, the optimal bonus payment becomes

$$\beta_P = \frac{\phi^2 a_m^3}{\phi^2\frac{n-1}{n}(2a_m^3 - \underline{a}^3) + r\sigma_u^2}.$$

The most prominent feature of this bonus payment is that it is independent of the common shock. In addition, it is negatively related to the risk aversion coefficient and the variance of idiosyncratic shocks. An increase in mean ability has an ambiguous effect on the bonus payment, all else equal.

Further, using e_i and substituting β_P into α_P solves the optimal base payment,

$$\alpha_P = u_0(\underline{a}) - \beta_P[\frac{n-1}{n}(\phi \underline{a}^p \underline{e}^q) - E_a(\frac{1}{n}\sum_{j=1,j\neq i}^n(\phi a_j^p e_j^q))] + \frac{1}{2}r\beta_P^2\frac{n-1}{n}\sigma_u^2 + \frac{1}{2\underline{a}}e_i^2$$

$$= u_0(\underline{a}) - \beta_P^{\frac{2}{2-q}}(\phi\frac{n-1}{n})^{\frac{2}{2-q}} q^{\frac{q}{2-q}}(\underline{a}^{\frac{2p+q}{2-q}} - a_m^{\frac{2p+q}{2-q}}) + \frac{1}{2}r\beta_P^2\frac{n-1}{n}\sigma_u^2 + \frac{1}{2}[\phi\frac{n-1}{n}q]^{\frac{2}{2-q}}\beta_P^{\frac{2}{2-q}}\underline{a}^{\frac{2p+q}{2-q}}$$

Hence, we can compute the processor's total profit, Π_P, under the optimal single-tournament relative-performance contract:

$$\Pi_P = E_a[\sum_{i=1}^n(\phi[\phi\frac{n-1}{n}\beta_P q]^{\frac{q}{2-q}} a_i^{\frac{2p+q}{2-q}}) - n\alpha_P] = n[\phi(\phi\frac{n-1}{n}\beta_P q)^{\frac{q}{2-q}} a_m^{\frac{2p+q}{2-q}} - \alpha_P],$$

and the total expected welfare, W_P, of both the processor and all growers,

$$W_P = E_a\{\sum_{i=1}^n(Ex_i - \frac{1}{2}r\,\mathrm{var}(w_i) - \frac{1}{2a_i}e_i^2)\}$$

$$= n\phi(\phi\frac{n-1}{n}\beta_P q)^{\frac{q}{2-q}} a_m^{\frac{2p+q}{2-q}} - \frac{1}{2}r\beta_P^2(n-1)\sigma_u^2 - \frac{1}{2}n[\phi\frac{n-1}{n}q]^{\frac{2}{2-q}}\beta_P^{\frac{2}{2-q}} a_m^{\frac{2p+q}{2-q}}.$$

An Offering of Two Separating Contracts

In this section, we assume that the processor offers all growers two contracts intended to separate them by ability levels. The processor thereby constructs two tournaments, with each tournament customized to a different group of growers based on their abilities. Recall that growers' ability is randomly distributed in the range $[\underline{a}, \bar{a}]$. We suppose the processor offers a menu of two contracts, $C_S = \{C_G, C_B\} = \{\{\alpha_G, \beta_G\}, \{\alpha_B, \beta_B\}\}$, where the contract C_G attracts more able grower types with $a_i \in [\hat{a}, \bar{a}]$, the contract C_B attracts less able growers type with $a_i \in [\underline{a}, \hat{a})$, and $\hat{a}$ is the ability level that separates higher-ability growers from lower-ability growers. Aided by the revelation principle, we examine the case where C_G is accepted "truthfully" by growers of $\hat{a}$ ability and above and C_B is accepted by growers lower ability than $\hat{a}$.

Given the contracts, the processor rewards each grower accepting C_G with w_i^G, where

$w_i^G = \alpha_G + \beta_G[x_i^G - \bar{x}^G]$. Hence, the expected payment for each grower of type $a_i^G > \hat{a}$ is:

$Ew_i^G = E\{\alpha_G + \beta_G[x_i^G - \bar{x}^G]\} = \alpha_G + \beta_G[Ex_i^G - E\bar{x}^G] = \alpha_G + \beta_G[\phi(a_i^G)^p(e_i^G)^q - E\bar{x}^G]$. Denote the number of growers in the high-ability group as *G*. Since each grower cannot foresee the exact number of growers in the group he participates, the expected relative standard that each grower *i* competes against takes the following form:

$$\begin{aligned} E\bar{x}^G &= E_G\{E_a[\bar{x}^G \mid G]\} = E_G\{E_a[\frac{1}{G}\sum_G(\phi(a_j^G)^p(e_j^G)^q) \mid G]\} \\ &\cong \{\frac{1}{E_G G}\phi(a_i^G)^p(e_i^G)^q) + \frac{1}{E_G G}\sum_{E_G G-1} E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q) \mid G]\} \\ &= \frac{1}{n^G}(\phi(a_i^G)^p(e_i^G)^q) + \frac{(n^G-1)}{n^G}E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q), \end{aligned}$$

where $n^G = n\ prob(a_i \in [\hat{a}, \bar{a}]) = np^G$ is the expected number of growers in the more able league. Note that the last two equalities are only applicable asymptotically, which requires a sufficiently large number of participants in the contracts. Hence,

$$Ew_i^G = E\{\alpha_G + \beta_G[x_i^G - \bar{x}^G]\} = \alpha_G + \beta_G[\frac{n^G-1}{n^G}(\phi(a_i^G)^p(e_i^G)^q) - \frac{(n^G-1)}{n^G}E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q)]$$

and $\text{var}(w_i^G) = \beta_G^2 \text{var}(x_i^G - \bar{x}^G) = \beta_G^2 \frac{n^G-1}{n^G}\sigma_u^2$.

Similarly, the payment to each grower in the low-ability group accepting C_B is given by $w_i^B = \alpha_B + \beta_B[x_i^B - \bar{x}^B]$. Hence,

$Ew_i^B = E\{\alpha_B + \beta_B[x_i^B - \bar{x}^B]\} = \alpha_B + \beta_B[\frac{n^B-1}{n^B}(\phi(a_i^B)^p(e_i^B)^q) - \frac{n^B-1}{n^B}E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q)]$, and

$\text{var}(w_i^B) = \beta_B^2 \text{var}(x_i^B - \bar{x}^B) = \beta_B^2 \frac{n^B-1}{n^B}\sigma_u^2$, where $n^B = n\ prob(a_i \in [\underline{a}, \hat{a})) = np^B$ is the expected number of growers in the less able league.

Given the offering of $C_S = \{C_G, C_B\}$, the processor obtains total expected profit, Π_S, where

$$\begin{aligned}\Pi_S &= \max_{\alpha_B,\beta_B,\alpha_G,\beta_G,\hat{a}} \{E_a[\sum_G (Ex_i^G - Ew_i^G) \mid a_i \in [\hat{a},\bar{a}]] + E_a[\sum_B (Ex_i^B - Ew_i^B) \mid a_i \in [\underline{a},\hat{a})]\} \\ &= E_a\{\sum_G (\phi(a_i^G)^p (e_i^G)^q) - n^G\alpha_G \mid a_i \in [\hat{a},\bar{a}]]\} + E_a\{\sum_B (\phi(a_i^B)^p (e_i^B)^q) - n^B\alpha_B \mid a_i \in [\underline{a},\hat{a}]]\}\end{aligned} \tag{6}$$

The optimal contracts must satisfy the following set of constraints. First, the participation constraint for each grower type must be satisfied: $EU_i^G \mid a_i \in [\hat{a},\bar{a}] \geq u_0(a_i^G)$, or

$$E_a[\alpha_G + \beta_G[\frac{n^G-1}{n^G}\phi(a_i^G)^p(e_i^G)^q - \frac{(n^G-1)}{n^G}E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q)] - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2}r\beta_G^2\frac{n^G-1}{n^G}\sigma_u^2 \mid a_i \in [\hat{a},\bar{a}]] \geq u_0(a_i^G), \tag{7}$$

and $EU_i^B \mid a_i \in [\underline{a},\hat{a}] \geq u_0(a_i^B)$, or

$$\alpha_B + \beta_B[\frac{n^B-1}{n^B}\phi(a_i^B)^p(e_i^B)^q - \frac{n^B-1}{n^B}E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q)] - \frac{1}{2a_i^B}(e_i^B)^2 - \frac{1}{2}r\beta_B^2\frac{n^B-1}{n^B}\sigma_u^2 \mid a_i \in [\underline{a},\hat{a}]] \geq u_0(a_i^B)\cdot \tag{8}$$

As with the single contract offering, we assume that each grower type $a_i \in [\underline{a},\bar{a}]$ has a reservation utility $u_0(a_i) = ka_i^{\frac{2p+q}{2-q}}$. Because it applies to both the single pooling contract and two separating contracts, this assumption will guarantee that each type-specific grower type faces the same outside opportunities no matter which contract is actually offered. It also guarantees that the growers' expected utility and reservation utility will have the same order in a_i, a condition that aids in finding a solution. Intuitively, this assumption implies that growers' utility will take similar forms, both inside and outside the contract sector.

Second, the optimal effort of each grower the higher-ability group must satisfy an incentive-compatibility constraint,

$$e_i^G \in \arg\max\{\alpha_G + \beta_G[\frac{n^G-1}{n^G}(\phi(a_i^G)^p(e_i^G)^q) - \frac{(n^G-1)}{n^G}E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q)] - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2}r\beta_G^2\frac{n^G-1}{n^G}\sigma_u^2\}, \tag{9}$$

$$\forall a_i^G \in [\hat{a},\bar{a}],$$

from which

$$e_i^G = [\phi\frac{n^G-1}{n^G}(a_i^G)^{p+1}\beta_G q]^{\frac{1}{2-q}}, \forall a_i^G \in [\hat{a},\bar{a}]. \tag{10}$$

Hence, the expected output from each higher-ability grower type is

$$Ex_i^G = \phi(a_i^G)^p (e_i^G)^q = \phi(a_i^G)^p [\phi \frac{n^G - 1}{n^G} (a_i^G)^{p+1} \beta_G q]^{\frac{q}{2-q}} = \phi[\phi \frac{n^G - 1}{n^G} q]^{\frac{q}{2-q}} (\beta_G)^{\frac{q}{2-q}} (a_i^G)^{\frac{2p+q}{2-q}} \qquad (q \neq 2),$$

and its associated cost function is

$$C(e_i^G, a_i^G) = \frac{1}{2a_i^G}(e_i^G)^2 = \frac{1}{2a_i^G}[\phi \frac{n^G - 1}{n^G} (a_i^G)^{p+1} \beta_G q]^{\frac{2}{2-q}} = \frac{1}{2}[\phi \frac{n^G - 1}{n^G} q]^{\frac{2}{2-q}} (\beta_G)^{\frac{2}{2-q}} (a_i^G)^{\frac{2p+q}{2-q}}.$$

Similarly, growers in the lower-ability group face an incentive-compatibility constraint,

(11) $$e_i^B \in \arg\max\{\alpha_B + \beta_B[\frac{n^B - 1}{n^B}(\phi(a_i^B)^p (e_i^B)^q) - \frac{n^B - 1}{n^B} E_{a,j \neq i}(\phi(a_j^B)^p (e_j^B)^q)] - \frac{1}{2a_i^B}(e_i^B)^2 - \frac{1}{2} r \beta_B^2 \frac{n^B - 1}{n^B} \sigma_u^2\},$$

$$\forall a_i^B \in [\underline{a}, \hat{a}),$$

from which

(12) $$e_i^B = [\phi \frac{n^B - 1}{n^B} (a_i^B)^{p+1} \beta_B q]^{\frac{1}{2-q}}, \ \forall a_i^B \in [\underline{a}, \hat{a}).$$

Hence, the expected output from each lower-ability grower type is

$$Ex_i^B = \phi(a_i^B)^p (e_i^B)^q = \phi(a_i^B)^p [\phi \frac{n^B - 1}{n^B} (a_i^B)^{p+1} \beta_B q]^{\frac{q}{2-q}} = \phi[\phi \frac{n^B - 1}{n^B} q]^{\frac{q}{2-q}} (\beta_B)^{\frac{q}{2-q}} (a_i^B)^{\frac{2p+q}{2-q}} \qquad (q \neq 2),$$

with the cost function written as

$$C(e_i^B, a_i^B) = \frac{1}{2a_i^B}(e_i^B)^2 = \frac{1}{2a_i^B}[\phi \frac{n^B - 1}{n^B} (a_i^B)^{p+1} \beta_B q]^{\frac{2}{2-q}} = \frac{1}{2}[\phi \frac{n^B - 1}{n^B} q]^{\frac{2}{2-q}} (\beta_B)^{\frac{2}{2-q}} (a_i^B)^{\frac{2p+q}{2-q}}.$$

In addition to the above sets of constraints, the optimal contracts must satisfy another pair of incentive-compatibility constraints described as truth-telling constraints. More specifically, faced with two separating contracts, it must be optimal for each grower type to choose his own league rather than the other league. Before formulating these constraints, additional notation must be defined.

Since each grower's reward is associated with the relative difference between his performance and the group average, one grower's deviation from choosing his own league would also affect the average performance of the group in which the grower actually participates. Thus, if one high-ability grower *i* deviates and joins the less able league, given all other growers choosing their own league, the average performance of the lower-ability group becomes

$\bar{x}^{BG} = \frac{1}{n^B+1}(x_i^G + \sum_B x_j^B)$. Thus, the deviating grower receives reward,

$w_i^{GD} = \alpha_B + \beta_B[x_i^G - \bar{x}^{BG}]$. Consequently, his expected payment and the variance of the payment are

$$Ew_i^{GD} = \alpha_B + \beta_B[\phi(a_i^G)^p(e_i^{GD})^q - \frac{1}{n^B+1}(\phi(a_i^G)^p(e_i^{GD})^q + \sum_B E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q))] \text{, and}$$

$$\mathrm{var}(w_i^{GD}) = \beta_B^2 \mathrm{var}(x_i^G - \bar{x}^{BG}) = \beta_B^2 \mathrm{var}(\frac{n^B}{n^B+1}x_i^G - \frac{1}{n^B+1}\sum_B x_j^B)$$

$$= \beta_B^2[((\frac{n^B}{n^B+1})^2 + \frac{n^B}{(n^B+1)^2})(\sigma_z^2+\sigma_u^2) - 2(\frac{n^B}{n^B+1})^2\sigma_z^2 + \frac{n^B(n^B-1)}{(n^B+1)^2}\sigma_z^2] = \beta_B^2\frac{n^B}{n^B+1}\sigma_u^2.$$

Hence, a deviating high-ability grower can obtain expected utility

$$EU_i^{GD} = Ew_i^{GD} - \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2}r\,\mathrm{var}(w_i^{GD})$$
$$= \alpha_B + \beta_B[\phi(a_i^G)^p(e_i^{GD})^q - \frac{1}{n^B+1}(\phi(a_i^G)^p(e_i^{GD})^q + \sum_B E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q))] - \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2.$$

Therefore, the deviating high-ability grower optimally chooses the optimal effort by maximizing this expression. Specifically,

$$e_i^{GD} \in \arg\max\{\alpha_B + \beta_B[\phi(a_i^G)^p(e_i^{GD})^q - \frac{1}{n^B+1}(\phi(a_i^G)^p(e_i^{GD})^q + \sum_B E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q))]$$
$$- \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2\},$$

from which $e_i^{GD} = [\phi\frac{n^B}{n^B+1}(a_i^G)^{p+1}\beta_B q]^{\frac{1}{2-q}}$.

On the other hand, if one lower-ability grower i deviates and joins the more able league, the average performance of the more able group is $\bar{x}^{GB} = \frac{1}{n^G+1}(x_i^B + \sum_G x_j^G)$. Thus, the deviating low-ability grower receives reward $w_i^{BD} = \alpha_G + \beta_G[x_i^B - \bar{x}^{GB}]$. Hence, its expected payment and the variance of its payment are,

$$Ew_i^{BD} = \alpha_G + \beta_G[\phi(a_i^B)^p(e_i^B)^q - \frac{1}{n^G+1}(\phi(a_i^B)^p(e_i^B)^q + \sum_G E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q))] \text{, and}$$

$$\text{var}(w_i^{BD}) = \beta_G^2 \text{ var}(x_i^B - \bar{x}^{GB}) = \beta_G^2 \text{ var}(\frac{n^G}{n^G+1}x_i^B - \frac{1}{n^G+1}\sum_G x_j^G)$$

$$= \beta_G^2[((\frac{n^G}{n^G+1})^2 + \frac{n^G}{(n^G+1)^2})(\sigma_z^2+\sigma_u^2) - 2(\frac{n^G}{n^G+1})^2\sigma_z^2 + \frac{n^G(n^G-1)}{(n^G+1)^2}\sigma_z^2] = \beta_G^2\frac{n^G}{n^G+1}\sigma_u^2.$$

Further, the deviating low-ability grower must optimally choose optimal effort,

$$e_i^{BD} \in \arg\max\{\alpha_G + \beta_G[\phi(a_i^B)^p(e_i^{BD})^q - \frac{1}{n^G+1}(\phi(a_i^B)^p(e_i^{BD})^q + \sum_G E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q))] - \frac{1}{2a_i^B}(e_i^{BD})^2 - \frac{1}{2}r\beta_G^2\frac{n^G}{n^G+1}\sigma_u^2\},$$

from which $e_i^{BD} = [\phi\frac{n^G}{n^G+1}(a_i^B)^{p+1}\beta_G q]^{\frac{1}{2-q}}$.

Finally, the additional truth-telling constraints can be formulated. Because the processor offers two separating contracts based on ability types, each grower must prefer his own contract over that designed for the other group. More precisely, for each grower in the higher-ability league, $a_i^G \in [\hat{a}, \bar{a}]$,

$$\begin{aligned}(13)\quad & \alpha_G + \beta_G[\frac{n^G-1}{n^G}(\phi(a_i^G)^p(e_i^G)^q) - \frac{n^G-1}{n^G}E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q)] - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2}r\beta_G^2\frac{n^G-1}{n^G}\sigma_u^2 \geq \\ & \alpha_B + \beta_B[\phi(a_i^G)^p(e_i^{GD})^q - \frac{1}{n^B+1}(\phi(a_i^G)^p(e_i^{GD})^q + \sum_B E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q))] - \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2.\end{aligned}$$

Similarly, for each grower in the lower-ability league, $a_i^B \in [\underline{a}, \hat{a})$,

$$(14) \quad \alpha_B + \beta_B[\frac{n^B - 1}{n^B}(\phi(a_i^B)^p(e_i^B)^q) - \frac{n^B-1}{n^B}E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q)] - \frac{1}{2a_i^B}(e_i^B)^2 - \frac{1}{2}r\beta_B^2\frac{n^B-1}{n^B}\sigma_u^2 \geq$$

$$\alpha_G + \beta_G[\phi\phi(a_i^B)^p(e_i^{BD})^q - \frac{1}{n^G+1}(\phi(a_i^B)^p(e_i^{BD})^q + \sum_G E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q))] - \frac{1}{2a_i^B}(e_i^{BD})^2 - \frac{1}{2}r\beta_G^2\frac{n^G}{n^G+1}\sigma_u^2.$$

Thus, the processor solves the problem in (6) subject to the constraints (7), (8), (9), (11), (13), and (14). However, to find the optimal set of separating contracts, several assumptions are necessary.[13] More specifically,

Assumption 1: Given (13) and (8), condition (7) can be dropped without affecting the optimal contracts. Thus, condition (7) is non-binding for any high-ability type. This assumption requires that a high-ability grower obtains greater utility when deviating than his reservation utility.

Assumption 2: One and only one of the constraints in condition (8) can be binding. Because the processor targets all growers in the low-ability group with contract C_B, only a single base payment α_B can be specified in C_B. Moreover, the processor can always reduce the base payment until one of the constraints in (8) binds. More specifically, we assume that for the lower-ability group, only the participation constraint for grower type $\underline{a}$ is binding. This assumption also implies that condition (14) must be non-binding for lower-ability growers because, otherwise, a different α_B could be found.

Assumption 3: One and only one of the constraints in condition (13) can be binding. Again, because the processor targets all growers in the high-ability group with contract C_G, only a single base payment α_G can be specified in C_G. Therefore, only one of the constraints in condition (13) can be binding to solve α_G. Without loss of generality, we assume that, under the optimal

[13] These assumptions are in general consistent with standard results from contract theory, therefore, no further proof is provided in the text. However, when solutions are later computed, all these assumptions will be verified.

contract offering, only the incentive constraint in (13) for the grower with separating ability $\hat{a}$ will be binding.

Readers should note that, given the assumptions above, only one particular set of optimal contracts can be established. Any other choice of the binding constraints among (7), (8), (9), (11), (13), and (14) would result in a different set of optimal contracts and different optimal separating ability $\hat{a}$. However, in some cases, it may be impossible to find optimal separating contracts because the participation constraints and the incentive compatibility constraints could be violated[14]

Under these assumptions, the processor's problem becomes:

(15)
$$\begin{aligned}\Pi_S &= \max_{\alpha_B,\beta_B,\alpha_G,\beta_G,\hat{a}} \{E_a[\textstyle\sum_G (Ex_i^G - Ew_i^G) \mid a_i \in [\hat{a},\overline{a}]] + E_a[\sum_B (Ex_i^B - Ew_i^B) \mid a_i \in [\underline{a},\hat{a}]]\} \\ &= E_a\{\textstyle\sum_G (\phi(a_i^G)^p (e_i^G)^q) - n^G \alpha_G \mid a_i \in [\hat{a},\overline{a}]\} + E_a\{\sum_B (\phi(a_i^B)^p (e_i^B)^q) - n^B \alpha_B \mid a_i \in [\underline{a},\hat{a}]\},\end{aligned}$$

subject to

(16)
$$[EU_i^B \mid a_i = \underline{a}] = \alpha_B + \beta_B[\frac{n^B-1}{n^B}\phi(\underline{a}^B)^p(\underline{e}^B)^q - \frac{n^B-1}{n^B}E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q)] - \frac{1}{2\underline{a}^B}(\underline{e}^B)^2 - \frac{1}{2}r\beta_B^2\frac{n^B-1}{n^B}\sigma_u^2 = u_0(\underline{a}^B),$$

(17)
$$\begin{aligned}&\alpha_G + \beta_G[\frac{n^G-1}{n^G}(\phi(\hat{a}^G)^p(\hat{e}^G)^q) - \frac{n^G-1}{n^G}E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q)] - \frac{1}{2\hat{a}^G}(\hat{e}^G)^2 - \frac{1}{2}r\beta_G^2\frac{n^G-1}{n^G}\sigma_u^2 = \\ &\alpha_B + \beta_B[\phi(\hat{a}^G)^p(\hat{e}^{GD})^q - \frac{1}{n^B+1}(\phi(a_i^G)^p(e_i^{GD})^q + \textstyle\sum_B E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q))] - \frac{1}{2\hat{a}_i^G}(\hat{e}_i^{GD})^2 - \frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2.\end{aligned}$$

Denote $E_a[(a_i^G)^{\frac{2p+q}{2-q}}] = a_{mG}^{\frac{2p+q}{2-q}}$ and $E_a[(a_i^B)^{\frac{2p+q}{2-q}}] = a_{mB}^{\frac{2p+q}{2-q}}$. Then, from (16), the base payment for the less able group B can be computed:

(18)
$$\begin{aligned}\alpha_B &= u_0(\underline{a}^B) - \beta_B[\frac{n^B-1}{n^B}\phi(\underline{a}^B)^p(\underline{e}^B)^q - \frac{n^B-1}{n^B}E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q)] + \frac{1}{2\underline{a}^B}(\underline{e}^B)^2 + \frac{1}{2}r\beta_B^2\frac{n^B-1}{n^B}\sigma_u^2 \\ &= u_0(\underline{a}^B) - \frac{n^B-1}{n^B}\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}(\beta_B)^{\frac{2}{2-q}}((\underline{a}^B)^{\frac{2p+q}{2-q}} - a_{mB}^{\frac{2p+q}{2-q}}) + \frac{1}{2}[\phi\frac{n^B-1}{n^B}q]^{\frac{2}{2-q}}(\beta_B)^{\frac{2}{2-q}}(\underline{a}^B)^{\frac{2p+q}{2-q}} + \frac{1}{2}r\beta_B^2\frac{n^B-1}{n^B}\sigma_u^2\end{aligned}$$

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[14] Interested readers can investigate the global optimal values of $\hat{a}$ for all other possibilities. Salanié (1997, pp. 26-32) provides a thorough discussion of local versus global incentive constraints, and their relationship to the Spence-Mirrlees condition.

Similarly, from (17),

$$\alpha_G = -\beta_G[\frac{n^G-1}{n^G}(\phi(\hat{a}^G)^p(\hat{e}^G)^q) - \frac{n^G-1}{n^G}E_{a,j\neq i}(\phi(a_j^G)^p(e_j^G)^q)] + \frac{1}{2\hat{a}^G}(\hat{e}^G)^2 + \frac{1}{2}r\beta_G^2\frac{n^G-1}{n^G}\sigma_u^2$$

(19)
$$+(\alpha_B+\beta_B[\phi(\hat{a}^G)^p(\hat{e}^{GD})^q - \frac{1}{n^B+1}(\phi(a_i^G)^p(e_i^{GD})^q + \sum_B E_{a,j\neq i}(\phi(a_j^B)^p(e_j^B)^q))] - \frac{1}{2\hat{a}_i^G}(\hat{e}_i^{GD})^2 - \frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2)$$

$$= -(\phi\frac{n^G-1}{n^G})^{\frac{2}{2-q}}q^{\frac{q}{2-q}}(\beta_G)^{\frac{2}{2-q}}[(\hat{a}^G)^{\frac{2p+q}{2-q}} - a_{mG}^{\frac{2p+q}{2-q}}] + \frac{1}{2}[\phi\frac{n^G-1}{n^G}q]^{\frac{2}{2-q}}(\beta_G)^{\frac{2}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} + \frac{1}{2}r\beta_G^2\frac{n^G-1}{n^G}\sigma_u^2$$

$$+(\alpha_B+(\beta_B)^{\frac{2}{2-q}}\frac{n^B}{n^B+1}[\phi[\phi\frac{n^B}{n^B+1}q]^{\frac{q}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} - \phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}a_{mB}^{\frac{2p+q}{2-q}}] - \frac{1}{2}[\phi\frac{n^B}{n^B+1}q]^{\frac{2}{2-q}}(\beta_B)^{\frac{2}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} - \frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2)$$

Then, substituting the base payments (18) and (19) and efforts into the processor's total expected profit yields,

$$\Pi_S = \max_{\alpha_B,\beta_B,\alpha_G,\beta_G,\hat{a}}\{E_a[\sum_G(Ex_i^G - Ew_i^G)\,|\,a_i\in[\hat{a},\bar{a}]] + E_a[\sum_B(Ex_i^B - Ew_i^B)\,|\,a_i\in[\underline{a},\hat{a})]\}$$

$$= n^G\{\phi[\phi\frac{n^G-1}{n^G}q]^{\frac{q}{2-q}}(\beta_G)^{\frac{q}{2-q}}a_{mG}^{\frac{2p+q}{2-q}}) + (\phi\frac{n^G-1}{n^G})^{\frac{2}{2-q}}q^{\frac{q}{2-q}}(\beta_G)^{\frac{2}{2-q}}[(\hat{a}^G)^{\frac{2p+q}{2-q}} - a_{mG}^{\frac{2p+q}{2-q}}]$$

$$-\frac{1}{2}[\phi\frac{n^G-1}{n^G}q]^{\frac{2}{2-q}}(\beta_G)^{\frac{2}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} - \frac{1}{2}r\beta_G^2\frac{n^G-1}{n^G}\sigma_u^2$$

$$-(\beta_B)^{\frac{2}{2-q}}\frac{n^B}{n^B+1}[\phi[\phi\frac{n^B}{n^B+1}q]^{\frac{q}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} - \phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}a_{mB}^{\frac{2p+q}{2-q}}]$$

$$+\frac{1}{2}[\phi\frac{n^B}{n^B+1}q]^{\frac{2}{2-q}}(\beta_B)^{\frac{2}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} + \frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2\}$$

$$+n^B\{\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}(\beta_B)^{\frac{q}{2-q}}a_{mB}^{\frac{2p+q}{2-q}}\} - n\{u_0(\underline{a}^B) - \frac{n^B-1}{n^B}\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}(\beta_B)^{\frac{2}{2-q}}((\underline{a}^B)^{\frac{2p+q}{2-q}} - a_{mB}^{\frac{2p+q}{2-q}})$$

$$+\frac{1}{2}[\phi\frac{n^B-1}{n^B}q]^{\frac{2}{2-q}}(\beta_B)^{\frac{2}{2-q}}(\underline{a}^B)^{\frac{2p+q}{2-q}} + \frac{1}{2}r\beta_B^2\frac{n^B-1}{n^B}\sigma_u^2\}$$

By treating $\hat{a}$ as a parameter for the moment, the first-order conditions to the above problem are

$$\frac{\partial\Pi_2}{\partial\beta_G} = n^G\{\phi[\phi\frac{n^G-1}{n^G}q]^{\frac{q}{2-q}}\frac{q}{2-q}(\beta_G)^{\frac{2q-2}{2-q}}a_{mG}^{\frac{2p+q}{2-q}}) + (\phi\frac{n^G-1}{n^G})^{\frac{2}{2-q}}q^{\frac{q}{2-q}}\frac{2}{2-q}(\beta_G)^{\frac{q}{2-q}}[(\hat{a}^G)^{\frac{2p+q}{2-q}} - a_{mG}^{\frac{2p+q}{2-q}}]$$

$$-\frac{1}{2}[\phi\frac{n^G-1}{n^G}q]^{\frac{2}{2-q}}\frac{2}{2-q}(\beta_G)^{\frac{q}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} - r\beta_G\frac{n^G-1}{n^G}\sigma_u^2\}$$

$$= n^G(\beta_G)^{\frac{2q-2}{2-q}}\{\phi[\phi\frac{n^G-1}{n^G}q]^{\frac{q}{2-q}}\frac{q}{2-q}a_{mG}^{\frac{2p+q}{2-q}}) + \beta_G[(\phi\frac{n^G-1}{n^G})^{\frac{2}{2-q}}q^{\frac{q}{2-q}}\frac{2}{2-q}[(\hat{a}^G)^{\frac{2p+q}{2-q}} - a_{mG}^{\frac{2p+q}{2-q}}]$$

$$-\frac{1}{2}[\phi\frac{n^G-1}{n^G}q]^{\frac{2}{2-q}}\frac{2}{2-q}(\hat{a}^G)^{\frac{2p+q}{2-q}}] - r\frac{n^G-1}{n^G}\sigma_u^2\beta_G^{\frac{4-3q}{2-q}}\} = 0$$

which solves the bonus payment for the higher-ability group,

(20)
$$\beta_G \in \arg\{\phi[\phi\frac{n^G-1}{n^G}q]^{\frac{q}{2-q}}\frac{q}{2-q}a_{mG}^{\frac{2p+q}{2-q}}) + \beta_G[(\phi\frac{n^G-1}{n^G})^{\frac{2}{2-q}}q^{\frac{q}{2-q}}\frac{2}{2-q}[(\hat{a}^G)^{\frac{2p+q}{2-q}} - a_{mG}^{\frac{2p+q}{2-q}}]$$
$$-\frac{1}{2}[\phi\frac{n^G-1}{n^G}q]^{\frac{2}{2-q}}\frac{2}{2-q}(\hat{a}^G)^{\frac{2p+q}{2-q}}] - r\frac{n^G-1}{n^G}\sigma_u^2\beta_G^{\frac{4-3q}{2-q}} = 0\}$$

In particular, when $p = q = 1$,

(21)
$$\beta_G = \frac{\phi^2 a_{mG}^3}{\phi^2(\frac{n^G-1}{n^G})(2a_{mG}^3 - \hat{a}^3) + r\sigma_u^2}.$$

Similarly, differentiating the processor's expect profit with respect to β_B yields

$$\frac{\partial\Pi_2}{\partial\beta_B} = n^G\{-\frac{2}{2-q}(\beta_B)^{\frac{q}{2-q}}\frac{n^B}{n^B+1}[\phi[\phi\frac{n^B}{n^B+1}q]^{\frac{q}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} - \phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}a_{mB}^{\frac{2p+q}{2-q}}]$$
$$+\frac{1}{2}[\phi\frac{n^B}{n^B+1}q]^{\frac{2}{2-q}}\frac{2}{2-q}(\beta_B)^{\frac{q}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} + r\beta_B\frac{n^B}{n^B+1}\sigma_u^2\} + n^B\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}\frac{q}{2-q}(\beta_B)^{\frac{2q-2}{2-q}}a_{mB}^{\frac{2p+q}{2-q}}$$
$$+n\{\frac{n^B-1}{n^B}\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}\frac{2}{2-q}(\beta_B)^{\frac{q}{2-q}}((\underline{a}^B)^{\frac{2p+q}{2-q}} - a_{mB}^{\frac{2p+q}{2-q}})$$
$$-\frac{1}{2}[\phi\frac{n^B-1}{n^B}q]^{\frac{2}{2-q}}\frac{2}{2-q}(\beta_B)^{\frac{q}{2-q}}(\underline{a}^B)^{\frac{2p+q}{2-q}} - r\beta_B\frac{n^B-1}{n^B}\sigma_u^2\} = 0.$$

which solves the bonus payment for the lower-ability group,

(22)
$$\beta_B \in \arg\{n^G\{-\frac{2}{2-q}\beta_B\frac{n^B}{n^B+1}[\phi[\phi\frac{n^B}{n^B+1}q]^{\frac{q}{2-q}}(\hat{a}^G)^{\frac{2p+q}{2-q}} - \phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}a_{mB}^{\frac{2p+q}{2-q}}]$$
$$+\frac{1}{2}[\phi\frac{n^B}{n^B+1}q]^{\frac{2}{2-q}}\frac{2}{2-q}\beta_B(\hat{a}^G)^{\frac{2p+q}{2-q}} + r\beta_B^{\frac{4-3q}{2-q}}\frac{n^B}{n^B+1}\sigma_u^2\} + n^B\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}\frac{q}{2-q}a_{mB}^{\frac{2p+q}{2-q}}$$
$$+n\{\frac{n^B-1}{n^B}\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}\frac{2}{2-q}\beta_B((\underline{a}^B)^{\frac{2p+q}{2-q}} - a_{mB}^{\frac{2p+q}{2-q}})$$
$$-\frac{1}{2}[\phi\frac{n^B-1}{n^B}q]^{\frac{2}{2-q}}\frac{2}{2-q}\beta_B(\underline{a}^B)^{\frac{2p+q}{2-q}} - r\beta_B^{\frac{4-3q}{2-q}}\frac{n^B-1}{n^B}\sigma_u^2\} = 0\}.$$

When $p = q = 1$, the above expression can be simplified as

$$(23)\ \ \beta_B = \frac{(n^B-1)\phi^2 a_{mB}^3}{n^G\phi^2[(\frac{n^B}{n^B+1})^2\hat{a}^3 - 2\frac{n^B-1}{n^B+1}a_{mB}^3] + n\phi^2(\frac{n^B-1}{n^B})^2(2a_{mB}^3-\underline{a}^3) + r\sigma_u^2\frac{(n^B)^3-n}{n^B(n^B+1)}}$$

In this case, it is straightforward to verify that when n^B is large,

$$\beta_B \xrightarrow{n^B\,\text{large}} \frac{(n^B-1)\phi^2 a_{mB}^3}{n^G\phi^2[\hat{a}^3-\underline{a}^3] + n^B\phi^2(2a_{mB}^3-\underline{a}^3) + r\sigma_u^2\frac{(n^B)^3-n}{n^B(n^B+1)}} > 0$$

From (18) and (19), we can obtain the base payments for both groups.

Finally, the processor's total expected profit, Π_S, under the optimal separating contracts can be computed by the following,

$$(24)\quad \begin{aligned}\Pi_S &= E_a[\textstyle\sum_G Ex_i^G - n^G\alpha_G \mid a_i \in [\hat{a},\bar{a}]] + E_a[\textstyle\sum_B Ex_i^B - n^B\alpha_B \mid a_i \in [\underline{a},\hat{a})] \\ &= n^G\{\phi[\phi\frac{n^G-1}{n^G}q]^{\frac{q}{2-q}}(\beta_G)^{\frac{q}{2-q}}a_{mG}^{\frac{2p+q}{2-q}} - \alpha_G\} + n^B\{\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}(\beta_B)^{\frac{q}{2-q}}a_{mB}^{\frac{2p+q}{2-q}} - \alpha_B\}\end{aligned},$$

as can the total expected welfare, W_S, of both the processor and all growers,

$$(25)\quad \begin{aligned}W_S &= E_a\{\textstyle\sum_G(Ex_i^G - \frac{1}{2}r\,\mathrm{var}(w_i^G) - \frac{1}{2a_i^G}(e_i^G)^2)\} + E_a\{\textstyle\sum_B(Ex_i^B - \frac{1}{2}r\,\mathrm{var}(w_i^B) - \frac{1}{2a_i^B}(e_i^B)^2)\} \\ &= n^G\phi[\phi\frac{n^G-1}{n^G}q]^{\frac{q}{2-q}}(\beta_G)^{\frac{q}{2-q}}a_{mG}^{\frac{2p+q}{2-q}} - \frac{1}{2}r\beta_G^2(n^G-1)\sigma_u^2 - \frac{1}{2}n^G[\phi\frac{n^G-1}{n^G}q]^{\frac{2}{2-q}}\beta_G^{\frac{2}{2-q}}a_{mG}^{\frac{2p+q}{2-q}} \\ &\quad + n^B\phi[\phi\frac{n^B-1}{n^B}q]^{\frac{q}{2-q}}(\beta_B)^{\frac{q}{2-q}}a_{mB}^{\frac{2p+q}{2-q}} - \frac{1}{2}r\beta_B^2(n^B-1)\sigma_u^2 - \frac{1}{2}n^B[\phi\frac{n^B-1}{n^B}q]^{\frac{2}{2-q}}\beta_B^{\frac{2}{2-q}}a_{mB}^{\frac{2p+q}{2-q}}\end{aligned}$$

A Single Offering of a Cream-Skimming Contract

The results from the previous section can be used to consider a third case: the offering of a single contract that targets growers above a particular ability level. Conditional on a potentially new choice for $\hat{a}$, the processor officers only one contract that is similar to C_G, which we will now re-label $C_{CS} = \{\alpha_{CS}, \beta_{CS}\}$. This contract is designed to be accepted by growers with ability greater than or equal to $\hat{a}$, and rejected by growers with ability less than $\hat{a}$. As with the separating contracts of the previous section, all the participation and incentive constraints associated with the higher-ability group must still hold, conditional on the potentially new choice of $\hat{a}$. Therefore, the derivation of the optimal contract is omitted. By offering the cream-skimming contract, the processor obtains expected profit, Π_{CS}, where[15]

$$\Pi_{CS} = E_a[\sum_G Ex_i^G - n^G \alpha_G \mid a_i \in [\hat{a}, \overline{a}] = n^G \{\phi[\phi \frac{n^G - 1}{n^G} q]^{\frac{q}{2-q}} (\beta_{CS})^{\frac{q}{2-q}} a_{mG}^{\frac{2p+q}{2-q}} - \alpha_{CS}\},$$

and the total expected welfare of the processor and all high-ability growers is

$$\begin{aligned} W_{CS} &= E_a\{\sum_G (Ex_i^G - \frac{1}{2} r \operatorname{var}(w_i^G) - \frac{1}{2a_i^G}(e_i^G)^2)\} \\ &= n^G \phi[\phi \frac{n^G - 1}{n^G} q]^{\frac{q}{2-q}} (\beta_{CS})^{\frac{q}{2-q}} a_{mG}^{\frac{2p+q}{2-q}} - \frac{1}{2} r \beta_{CS}^2 (n^G - 1)\sigma_u^2 - \frac{1}{2} n^G [\phi \frac{n^G - 1}{n^G} q]^{\frac{2}{2-q}} \beta_{CS}^{\frac{2}{2-q}} a_{mG}^{\frac{2p+q}{2-q}}. \end{aligned}$$

Numerical Solution for Optimal Contracts and Optimal $\hat{a}$

The previous sections derive analytic solutions to the processor's optimization problem conditional on the choice of $\hat{a}$. We are unable, however, to find analytically the optimal $\hat{a}$ and instead turn to numerical solutions. Given the above optimal contract terms along with hypothetical choices for parameters, we numerically establish the optimal value of $\hat{a}$ for each of the three cases – pooling, separating, and cream-skimming contract offerings. Various sensitivity analyses are also conducted by varying model parameters. The general algorithm of the numerical solution consists of the following four steps: (i) For any possible separating ability level, $\hat{a} \in (\underline{a}, \overline{a})$, compute the probability, the mean ability, and the expected number of growers in each group; (ii) For each possible $\hat{a}$, compute the contract terms and then the processor's expected total profit, growers' expected utility, and total expected welfare; (iii) For each $\hat{a}$, check the *ex ante* efficiency conditions (i.e., the incentive-compatibility conditions and

[15] While we conserve notation by using the superscript and subscript G, readers should note that the number of growers in the higher-ability group for the cream-skimming contract, n^G, is likely to be different than the number of growers in higher-ability group for the separating contracts. The same cautionary note also holds for w_G, a_{mG}, and e_G.

participation constraints) for all grower types; and (iv) Compare the processor's expected total profit and total welfare calculations to find the optimal $\hat{a}$ for the separating and cream-skimming contract offerings. Because we assume that each grower has the same utility function and faces the same outside opportunities no matter the contract offering, we are able to compare the pooling, separating, and cream-skimming results are directly comparable without further calibration.

We assume that grower ability, a_i, is given by a Beta distribution. From standard distribution theory (Casella and Berger, 1990), a general Beta distribution takes the form:

$$f(x \mid z, w) = \frac{1}{B(z,w)(\overline{a} - \underline{a})^{z+w-1}} (x - \underline{a})^{z-1} (\overline{a} - x)^{w-1}, \quad x \in [\underline{a}, \overline{a}], z > 0,\ w > 0\ ,$$

where $B(z,w) = \int_0^1 x^{z-1}(1-x)^{w-1}dx$. To examine how sensitive the results are to the distribution of grower ability, we consider three different Beta distributions: (i) a symmetric Beta distribution with $z = 2$ and $w = 2$, (ii) a right-skewed Beta distribution with $z = 2$ and $w = 4$, and (iii) a left-skewed Beta distribution with $z = 4$ and $w = 2$.[16] In addition, for each of these three distributions, different returns-to-scale parameters p and q will be considered *ceteris paribus* to investigate their effects on the model results.

For any given $\hat{a}$, the following can be computed numerically for each given distribution:

$$\begin{aligned}
p^B &= prob(a_i \in (\underline{a}, \hat{a})) = \int_{\underline{a}}^{\hat{a}} f(x \mid z, w)dx\,, \\
p^G &= prob(a_i \in [\hat{a}, \overline{a})) = \int_{\hat{a}}^{\overline{a}} f(x \mid z, w)dx\,, \\
n^G &= nprob(a_i \in [\hat{a}, \overline{a}]) = np^G\,, \\
n^B &= nprob(a_i \in [\underline{a}, \hat{a}]) = np^B\,, \\
a_{mG}^s &= E(a_i^s \mid a_i \in [\hat{a}, \overline{a}]) = \int_{\hat{a}}^{\overline{a}} a_i^s \frac{f(x \mid z, w)}{prob[a_i \in [\hat{a}, \overline{a}]]}dx\,, \text{ and}
\end{aligned}$$

[16] A fourth case of Beta distribution, one where $z = w = 1$ resulting in a uniform distribution, leads results that are similar to the symmetric Beta distribution where $z = w = 2$.

$$a_{mB}^{s} = E(a_i^s \mid a_i \in [\underline{a}, \hat{a})) = \int_{\underline{a}}^{\hat{a}} a_i^s \frac{f(x \mid z, w)}{prob[a_i \in [\underline{a}, \hat{a})]} dx,$$

where a_{mG}^{s} and a_{mB}^{s} compute the s^{th} uncentered conditional moments of grower ability types.

To further specify the distribution and other model features, we use parameter values in Table 1. These parameters are not chosen to calibrate to some real-world market; instead, they are chosen in an *ad hoc* fashion to help ensure the feasibility of equilibrium outcomes. While the inability to calibrate the model is an admitted shortcoming, our results focus on whether benefits are possible from separating or cream-skimming, rather than what the level of benefits may actually be. Based on the parameters in Table 1, Tables 2, 3, and 4 display the numerical results for optimal contract terms and associated results given the three distributions of grower ability and a number of different parameter values. For the separating and cream-skimming cases, only the optimal $\hat{a}$'s that maximize total welfare under are shown. In some cases, the optimal $\hat{a}$'s coincide.

Results

Intuitively, our results suggest that increased grower heterogeneity creates opportunities for sorting growers by ability even when a relative-performance contract is used. Four specific results from Tables 2, 3, and 4 are summarized below.

Result 1: When a relatively large proportion of high-ability growers exist and the technology exhibits high returns to scale, an offering of two separating contracts can often improve the processor's expected profit and total welfare of both processor and growers relative to the single pooling contract. However, only growers in the upper portion of the ability distribution are better off under a two-tournament separating contract offering, while all other grower including all lower-ability growers and the rest of the high-ability growers are worse off.

By optimally offering two separating contracts, the processor provides sufficient incentives to have growers with heterogeneous abilities self-select into their own leagues. Given the optimal contracts, high-ability growers reveal their ability types by selecting the contract C_G. Consequently, the relatively high bonus payment specified in this contract induces high-ability growers to exert greater efforts and, hence, the processor obtains more output and extracts more surpluses. Most of the results depicted in Tables 2, 3, and 4 show that separating contracts can raise the processor's expected profit (i.e., *Profit(S)* > *Profit(P)*) and the total welfare of growers and the processor (i.e., *Welf(S)* > *Welf(P)*). The exception to this result occurs when both ability is skewed to the right and returns to scale are low (i.e., p and $q < 1$). The bottom half of Table 3 depicts the exception. In this one exception, a single pooling contract leads to slightly higher total expected welfare but lower expected processor profits.

Compared with the pooling contract, the separating contracts offer greater rewards to growers with relatively higher abilities while offering smaller rewards to relatively less able growers. In other words, the processor extracts more surplus with the separating contracts by assigning stronger penalties to less able growers and stronger rewards to high-ability growers. Using results in the top portion of Table 4 as an example, Figure 1 shows that the optimal separating ability is 2.1. Therefore, growers with ability within the range [1, 2.1) would choose contract C_B, while growers with ability within [2.1, 3] would choose contract C_G. In this case, separating contracts improve the processor's expected profit and total welfare over a pooling contract. However, Figure 1, which is based on the same scenario, also shows that the separating contracts make growers with ability below 2.6 worse off and those with ability above 2.6 better off than with a pooling contract. Thus, by separating growers into different ability groups, the processor provides stronger incentives to have high-ability growers exert more effort and extracts more surplus. In return, more able growers receive extra compensation for revealing their ability.

*Result 2***:** A cream-skimming contract can sometimes improve the processor's expected profit and total welfare over a pooling contract.

Tables 2, 3 and 4 show that, under certain conditions, a cream-skimming contract improves the processor's profit and total welfare over a pooling contract. However, the optimality of a cream-skimming contract depends on the distribution of grower ability and on the returns-to-scale parameters p and q. The cream-skimming contract improves the processor's expected profits in cases where the ability distribution is skewed left (e.g., from Table 4, *Profit(CS) > Profit(P)*); however, when the ability distribution is symmetric or skewed right, cream-skimming improves expected profits only when p and $q = 1$ and not when p and $q < 1$. Cream-skimming improves expected total welfare in even fewer cases, i.e., when p and $q = 1$ and when the ability distribution is either symmetric or left skewed. Intuitively, if the returns-to-scale factors p and q and the proportion of high-ability growers are both small, then the processor's cost of having high-ability growers' types revealed would exceed the benefit of separating more able growers from less able growers. In that case, only a pooling contract would be optimal.

We also find that the cream-skimming contract never improves on the separating contracts. However, if there are positive costs (e.g., political, legal, or financial) of preparing and offering more than one contract, then offering a cream-skimming contract could become justifiable. While this outcome would result is less than full employment for the fixed number of growers, the average utility of each grower remaining under contract is often higher under the cream-skimming contract than under the pooling contract.

*Result 3***:** The optimal bonus and base payment in contract C_G exceed those in contract C_B. Moreover, the contract terms in contract C_G exceed those in the pooling contract C_P, while the contract terms in contract C_B are smaller than those in contract C_P.

The results in Tables 2, 3, and 4 show that the processor offers greater bonus and base payment to the more able group *G* than those to the less able group *B*. Consequently, growers belonging to the low-ability group would prefer their own contract C_B to the contract C_G because of the greater expected penalty associated with joining the high-ability league. On the other hand, given the relatively high bonus and base payment, more able growers would prefer the contract C_G to the contract C_B because they would receive a smaller base payment by joining the low-ability league. The base payment α_B must be smaller than α_G because, otherwise, a more able grower would be always better off by deviating given that β_B is smaller than β_G . Thus, the optimal contract for the high-ability group offers a positive information rent through the base payment α_G. Therefore, the grower with ability $\hat{a}$ is just indifferent between the contract C_G and C_B, and all other grower types in the group *B* strictly prefer the contract C_B to the contract C_G. However, the optimal base payment to the high-ability group must be sufficiently small such that a less able grower in group *B* would not deviate and choose the contract C_G.

Compared to C_P, the contract terms in C_G and C_B also reflect the different incentives that the processor must provide to more able and less able growers. Consistent with standard contract theory, the processor offers a suboptimal contract to less able growers to reduce the information rents rewarded to more able growers.

Result 4: Keeping all other parameters fixed, increases in mean grower ability or returns-to-scale factors (i.e., p and q) raise the optimal processor's profit, growers' total utility, and total welfare.

Tables 2, 3, and 4 show the effects of increasing mean grower ability in various scenarios. In these scenarios, we are careful to change only the mean ability without affecting the variance. Equivalently, this change could be envisioned as a parallel shift of the distribution, or more intuitively it can be thought of as a systematic improvement of grower ability – possibly through exogenous technical change or learning. Hence, growers' output increases, and growers' production costs decrease as well. Consequently, the processor could capture more gains and offer greater rewards to all growers.

The effects of scale factors p and q on growers' and the processor's welfare are similar to changes in growers' mean ability. As these factors increase, growers become more productive and, hence, the processor could capture more surpluses. We note, however, that the effects of p and q depend on numerical values of $\underline{a}$ and $\overline{a}$. Since derivations of optimal contract terms involve moments of growers abilities, the effects of p and q are unambiguous only when $0 \leq \underline{a} < \overline{a} \leq 1$ or $1 \leq \underline{a} < \overline{a} < \infty$. If $0 \leq \underline{a} < 1 < \overline{a} < \infty$, the effects of p and q would be different. For instance, when $a < 1$, a^3 decreases, but $a^{1/3}$ increases with marginal increases in a. When $a > 1$, a^3 increases, but $a^{1/3}$ decreases. Therefore, in this paper, we assume $1 \leq \underline{a} < \overline{a} < \infty$ throughout the numerical simulations.

In addition, Tables 2, 3, and 4 show that existence of optimal separating contracts or cream-skimming contracts depends on the relative distance between the lower and upper limit of grower ability given any distribution function of grower ability. As the mean grower ability increases without changing the variance, the relative distance $(\overline{a} - \underline{a})/\overline{a}$ is also reduced. This change affects the cost of separating more able growers from less able growers. For instance, the cost of separating growers from a distribution with range [10, 11] would exceed that from a distribution with range [0, 1]. As the relative distance $(\bar{a} - \underline{a}) / \bar{a}$ becomes sufficiently small, an optimal separating contract might not exist and only a pooled single-tournament could be optimal.

Special cases: Sensitivity analyses concerning parameter values reveal two special cases where features of the model presented above degenerate to features used in existing research. One special case occurs occur when $k = 0$ so that reservation utilities are uniformly zero for all n growers regardless of ability level; the other special case occurs when $p = 0$, $q = 1$, and $\phi = 1$ so that the output function is additive in effort and independent of ability.

In the case of $k = 0$, model results (not presented) are largely consistent with those presented in Tables 2, 3, and 4. As with the more general cases on the tables, compared to the standard pooling contract offering, higher expected profits for the processor are still possible under an offering of separating contracts regardless of how ability is distributed or the value of the scaling parameters. Higher total welfare for both the processor and growers is also still possible with separating contracts, except in the case when ability is right-skewed and the scaling parameters, p and q are less than 1. The only major difference concerns the feasibility of optimal separating contracts. When $k > 0$, the set of feasible solutions is restricted. An economic interpretation of this result is that k can be thought as reflecting the level of exogenously determined economic activity outside the contract sector. As k increases, the value of outside opportunities increase – particularly for high ability workers, and it becomes increasingly difficult to for a principal to construct a contract targeted to high-ability growers that provides greater expected utility than the non-random reservation utility.

In the case of $p = 0$, $q = 1$, and $\phi=1$, when the output function is additive, model results are again consistent with those in Tables 2, 3 and 4. A pair of separating contracts can be found that improves profitability over the single pooling contract. The same results hold for improving total expected welfare, except when the ability distribution is skewed to the right.

When examined collectively, the results in Tables 2, 3, and 4 tell a fairly consistent story. Most importantly, the results show that, compared to a single contract offering that pools all growers, an offering of separating contracts is able to improve the processor's expected profits in every case examined; such an offering can also improve the processor's and growers' joint welfare in nearly every case. A similar comparison between the single pooling contract and a single cream-skimming contract is not as consistent: A cream-skimming offering can improve processor profitability and joint welfare, but there are numerous cases where the pooling contract may be preferred. Two conditions – sufficiently large returns-to-scale factors and a large proportion of high-ability growers (i.e., a symmetric distribution or a left-skewed distribution of grower ability) – favor both the offering of two separating contracts and a single cream-skimming contract relative to the single pooling contract offering. Another consistent finding in Tables 2, 3, and 4 concerns how a separating contract offering redistributes welfare. In all scenarios, the separating contract offers greater rewards only to the upper portion of the high-ability league and penalizes all other growers relative to the single pooling contract offering. However, when the proportion of high-ability growers is small and the technology exhibits decreasing returns, a separating contract or a cream-skimming contract may be less preferred than the pooling contract offering.

Conclusion and Implications

Conventional wisdom regards linear relative-performance contracts as rendering unnecessary any efforts to sort agents by ability. To the contrary, we find that a uniform treatment of workers' marginal benefits from effort and uniform outside opportunities are responsible for this standard result. When differences in agents' outside opportunities and marginal rewards from effort are instead accounted for, we show that sorting agents into ability groups can help counter this failure and improve processor profits and joint welfare. We show that linear relative-performance contracts can discourage the highest ability growers from putting forth their strongest efforts. In our extended framework, we find that separating contracts and sometimes even cream-skimming contracts can improve profitability and welfare.

These results, which stem from our model development, come with several shortcomings or limitations. Our inability to solve analytically for the optimal $\hat{a}$ leads directly to two major limitations: First, we are unable to solve for the sufficient conditions that guarantee existence of an equilibrium or that guarantee improved profitability. And second, we resort to finding $\hat{a}$ through the use of a numerical example that is not calibrated to match, for example, the broiler industry. Balanced against these limitations is the practicality of our methods, which a broiler processor or some other principal could easily follow to find $\hat{a}$ given proper calibration from proprietary data.

We note above that the improvements from separating contracts come at the expense of less-able workers, who are as a rule worse off. Our results, therefore, begin to encroach on a potentially larger policy issue: namely, the political feasibility and social desirability of tournament-style contracts. In the poultry sector, for example, industry observers often connect tournament-style contracts to grower dissatisfaction. While there may be multiple reasons for grower-voiced complaints about contracts, it is clear that the design of tournament contracts leads to the identification and penalization of under-performers, who very well may be a prime source of complaints. Placed in this setting, an offering of multiple contracts designed to separate growers further may lead to even more complaints. As we note above, lower-ability growers are clearly worse off under a separating contract offering. From a social standpoint, therefore, improvements that accrue to processors and the highest-ability growers may fail to offset the political costs associated with separating grower by ability. In other words, political realities may favor a pooling outcome. A similar argument can be made if multiple contract offerings result in higher legal or financial costs.

The potential political preferences for pooling may inadvertently open the door for cream skimming, however. Like the standard contract, the cream-skimming contract discussed above pools growers; but it causes the lowest-ability growers to reject the contract voluntarily. Politically, a cream-skimming contract may seem fairer than a separating contract. Because we show above that a cream-skimming contract benefits the processor under a number of possible scenarios, processors with political sensitivities may have reason to favor cream-skimming contracts to separating contracts.

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Table 1: Parameters used in the numerical example

Parameters	Values or range
n	50
r	0.5
σ_u^2	0.5
k	0.01
ϕ	1
p, q	1, 1/3
$\underline{a}$ (or al)	See following tables
$\overline{a}$ (or ah)	See following tables

Table 2: Optimal Contracts Under a Symmetric Beta Distribution (z = 2, w = 2)

Symmetric Beta Distribution (*z* = 2, *w* = 2) and *p* = *q* = 1												
Parameters: $\underline{a}=1$ $\overline{a}=3$ $E(a)$=2 var(a)=0.2	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
					0.5317	2.407			56.17	119.4	175.5	1.123
	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
		1.6	39	11	0.622	3.38	0.191	0.0825	51.9	131.8	183.7	1.038
	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		1.5	42		0.602	3.11			52.84	127.6	180.5	1.258
Parameters: $\underline{a}=1.5$ $\overline{a}=3.5$ $E(a)$=2.5 var(a)=0.2	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
					0.5613	4.744			105.7	233.8	339.5	2.114
	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
		2.1	39	11	0.663	6.62	0.238	0.282	94.96	257.7	352.7	1.899
	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		2	42		0.642	6.11			96.68	247.4	344.1	2.302
Symmetric Beta Distribution (*z* = 2, *w* = 2) and *p* = *q* = 1/3												
Parameters: $\underline{a}=1$ $\overline{a}=3$ $E(a)$=2 var(a)=0.2	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
					0.3285	0.1618			4.846	40.1	44.95	0.09691
	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
		1.5	42	8	0.422	0.186	0.14	0.0346	4.019	41.12	45.14	0.0804
	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		1.3	46		0.385	0.173			4.216	38.56	42.77	0.09165
Parameters: $\underline{a}=1.5$ $\overline{a}=3.5$ $E(a)$=2.5 var(a)=0.2	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
					0.3753	0.1903			5.176	47.2	52.38	0.1035
	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
		2	42	8	0.462	0.216	0.183	0.0495	4.342	48.08	52.43	0.0868
	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		1.7	48		0.411	0.197			4.729	46.33	51.06	0.09853

Table 3: Optimal Contracts Under an Asymmetric Beta Distribution (z = 2, w = 4)

Asymmetric Beta Distribution (*z* = 2, *w* = 4) and *p* = *q* = 1												
Parameters: $\underline{a}=1$ $\overline{a}=3$ $E(a)$=1.667 var(a)=0.127	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
					0.5489	1.432			31.5	70.58	102.1	0.63
	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
		1.6	26	24	0.693	2.74	0.347	0.259	27.78	76.44	104.2	0.556
	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		1.2	45		0.588	1.64			29.92	71.93	101.8	0.6649
Parameters: $\underline{a}=1.5$ $\overline{a}=3.5$ $E(a)$=2.167 var(a)=0.127	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
					0.5944	3.243			66.51	158.8	225.3	1.33
	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
		1.8	41	9	0.664	4.09	0.305	0.278	60.95	166.6	227.6	1.219
	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		1.6	48		0.616	3.42			63.99	160	224	1.333
Asymmetric Beta Distribution (*z* = 2, *w* = 4) and *p* = *q* = 1/3												
Parameters: $\underline{a}=1$ $\overline{a}=3$ $E(a)$=1.667 var(a)=0.127	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
					0.3626	0.1463			3.896	36.79	40.68	0.07791
	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
		1.1	48	2	0.387	0.152	0.398	0.0425	3.729	36.94	40.67	0.0746
	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		1.1	48		0.387	0.152			3.702	35.81	39.51	0.07712
Parameters: $\underline{a}=1.5$ $\overline{a}=3.5$ $E(a)$=2.167 var(a)=0.127	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
					0.4141	0.1758			4.149	44.33	48.48	0.08297
	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
		1.6	48	2	0.436	0.182	0.369	0.0402	3.954	44.49	48.44	0.0791
	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		1.6	48		0.436	0.182			3.923	43.01	46.93	0.08173

Table 4: Optimal Contracts Under an Asymmetric Beta Distribution (z = 4, w = 2)

Asymmetric Beta Distribution (*z* = 4, *w* = 2) and *p* = *q* = 1												
Parameters:	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
$\underline{a}=1$					0.5246	3.499			83.57	173.9	257.5	1.671
$\bar{a}=3$	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
$E(a)$=2.333		2.1	37	13	0.715	5.71	0.197	0.215	67.08	213.1	280.2	1.342
var(a)=0.127	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		2	40		0.685	5.22			69.34	202	271.3	1.733
Parameters:	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
$\underline{a}=1.5$					0.546	6.402			147.9	316.7	464.6	2.958
$\bar{a}=3.5$	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
$E(a)$=2.833		2.5	40	10	0.722	9.55	0.19	0.34	118.2	382	500.2	2.364
var(a)=0.127	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		2.5	40		0.722	9.55			116.6	364.9	481.4	2.914
Asymmetric Beta Distribution (*z* = 4, *w* = 2) and *p* = *q* = 1/3												
Parameters:	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
$\underline{a}=1$					0.3048	0.1766			5.659	43.42	49.08	0.1132
$\bar{a}=3$	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
$E(a)$=2.333		1.7	47	3	0.43	0.189	0.059	0.018	4.134	45.94	50.08	0.0827
var(a)=0.127	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		1.7	47		0.43	0.189			4.091	44.36	48.45	0.08705
Parameters:	Pooling				β_P	α_P			*TotUtil(P)*	*Profit(P)*	*Welf(P)*	*AvgUtil(P)*
$\underline{a}=1.5$					0.3472	0.204			6.039	50.12	56.16	0.1208
$\bar{a}=3.5$	Separating	$\hat{a}$	n^G	n^B	β_G	α_G	β_B	α_B	*TotUtil(S)*	*Prof(S)*	*Welf(S)*	*AvgUtil(S)*
$E(a)$=2.833		2.2	47	3	0.467	0.217	0.078	0.0243	4.374	52.52	56.89	0.0875
var(a)=0.127	Cream-skimming	$\hat{a}$	n^G		β_{CS}	α_{CS}			*TotUtil(CS)*	*Prof(CS)*	*Welf(CS)*	*AvgUtil(CS)*
		2.1	48		0.448	0.213			4.575	51.06	55.64	0.0953

Figure 1: Growers' Expected Utility and Reservation Utility Under a Two-tournament Contract and a Single-tournament Contract ($z = 4$, $w = 2$, $p = q = 1$, $\underline{a} = 1$, $\overline{a} = 3$)