



**Consumer Demand for
Food Commodities in the United States
With Projections for 1980**

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This study analyzes the demand for food commodities in the United States in the postwar period using both time-series and cross-section data. Income-consumption relationships are based on data from the 1955 and 1965 USDA household food consumption surveys.

The analysis of cross section data emphasized: (1) effects of grouping observations, (2) choice between expenditures and quantities as the dependent variable, (3) effects of household size on income-consumption relationships, (4) shifts in the regression coefficients (intercepts and income elasticities) between 1955 and 1965, and (5) regional variations in the income-consumption relationships.

A demand interrelationship matrix was developed for 49 commodities or commodity groups at the retail level. Commodities were classified into 15 separable groups and all direct and cross elasticities for commodities within a group were estimated directly. The cross elasticities corresponding to commodities outside a given group were estimated through assumptions of cardinal separability. The synthesis of demand interrelationships was achieved by the use of restrictions on demand equations for an individual consumer as suggested by Frisch (1959) and quantified by Brandow (1961). Consideration also was given to the measurement of time trends on consumption. Marketing margins were analyzed and demand interrelationships were developed at the farm level.

Projections of 1980 consumption per capita were developed for individual commodities and group aggregates. These projections are based on a specification of constant real prices, exogenous projections of real income per capita, and continuation of past time trends for certain commodities.

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CONSUMER DEMAND FOR FOOD COMMODITIES IN THE UNITED STATES WITH PROJECTIONS FOR 1980¹

INTRODUCTION

A NUMBER OF STUDIES have estimated demand characteristics and explained the factors that influence consumer behavior. Individual farm commodities such as beef, corn, cotton, potatoes, sugar, and wheat have been the subject of investigation. Most of these studies are partial analyses without considering the complete interdependent nature of demand. A notable exception is Brandon's (1961) study of the demand interrelationships among all food commodities. One basic reason for the absence of a large number of studies in this area seems to be the gap between the theory of demand and empirical analysis using statistical procedures of estimation. Kuznets (1963) observes, "... to obtain confrontation between theory and fact, large bodies of micro data are needed which refer directly to decision units whose economic behavior is the object of the study. Such data are costly and we have much to learn about how to collect and analyze this information." Recent advances in economic theory, statistical procedures of estimation, and testing hypotheses have overcome some of these problems. The present study reviews a few approaches to bridge the gap between theory and empirical analysis.

These concepts are used to measure income-consumption relationships, demand interrelationships at the retail level, and the nature of price spreads between retail and farm levels.

A demand relationship obtained as a result of maximizing consumer's satisfaction subject to a budget restraint, is expressed as a function of prices of the commodities and consumer income. To analyze the effects of prices and income on the quantity consumed, it is necessary to isolate the effects of other non-economic elements such as psychological, sociological, cultural, and regional factors that determine the level of consumption of a given commodity.

Generally, prices remain unchanged during a short period of time and, therefore, data obtained from cross-section surveys provide a basis for obtaining the effects of income on consumption free from price effects. However, within a cross section, it is difficult to keep the psycho-socio-cultural factors constant and, therefore, the effects of these factors on the income coefficient have to be determined before deciding on the reliability of the income coefficient obtained from cross-section data. Unfortunately, it is often difficult or even impossible to

¹ Submitted for publication June 30, 1970.

quantify the effects of many noneconomic factors. Household consumption survey data for the United States can be used to obtain the effects of household size and region on the income-consumption relationship. Also, changes in income coefficient over time can be evaluated using cross-section data from two points in time, and the effects of redistribution of income on food consumption can be analyzed.

The effect of a change in the price of one commodity will influence the consumption of all commodities in the consumer's choice relationship. Often policy decisions require estimates of demand interrelationships and estimates of the probable level of consumer demand in the future. Recent contributions in the area of economic theory and statistical methods of estimation and testing hypotheses have made it possible to improve procedures used in estimating demand interrelationships. In particular, the assumptions of want independence and neutral want association can be used to separate the commodities in the utility function to obtain separable groups. The estimates obtained for particular groups can be used to synthesize a demand interrelationship matrix showing all the direct- and cross-price elasticities corresponding to the commodities entering the utility function.

Retail prices are determined as an outcome of the decisions made at different levels in the marketing system. In many cases, only a small percentage of the retail prices go to the producers of raw materials, and the difference between prices realized at the retail level and at the farm level can be treated as charges for the marketing services. Such charges are often known as price spreads or marketing margins. It can be hypothesized that certain relationships exist between price spreads and prices

at the retail or farm levels. It is possible to derive the elasticities at the farm level from the elasticities at the retail level based on the relationship assumed between price spreads and actual prices.

Considering the above factors, the objectives of the present study can be summarized as follows:

1. To estimate the effects of different factors that influence the consumption of food items in the United States through the use of cross-section data. In particular, attempts are made to estimate the effects of income, household size, and region on quantities consumed. Also, the income elasticities obtained from 1955 and 1965 cross-section data are compared, considering the effects of income redistribution during this period.

2. To estimate demand interrelationships among the major items in the family food budget at the retail level. Here, the different commodities are classified into separable groups and the demand coefficients within a group are estimated independently. The restraints provided by demand theory are imposed upon the coefficients to synthesize the demand interrelationship matrix.

3. To specify a relationship between price spreads and retail prices and to derive the demand interrelationship at the farm level from corresponding estimates for the retail level. It was specified that a linear relationship exists between price spreads and retail prices. (This specification incorporates the commonly assumed absolute spread and the percentage spread.)

4. To specify procedures for projecting consumption levels in the future and derive estimates, by commodity, for 1980. The projections are based on the assumption of prices held at the 1962-1966 level.

The overall objective of this study is

to contribute toward the understanding of demand characteristics for food commodities in the United States and to estimate the effect of various factors on the consumption of commodities included in the analysis.

I. ECONOMIC AND STATISTICAL FRAMEWORK

Basic Concepts in Demand Theory

The economic framework for the study of consumer behavior generally is through the theory of utility-maximizing individuals (Stigler, 1965), while some recent studies have attempted to make certain generalizations based on observed consumer behavior (see Houthakker, 1961). The purpose of this section is to emphasize theoretical developments of importance for empirical research in the present context. Therefore, some aspects are omitted here, especially those related to welfare economics. Following the general practice in static consumption theory, the beginning point will be the choice problems of an individual consumer with given tastes, attempting to purchase the most preferred combination of commodities subject to a budget restraint.² We shall trace out the consumer theory from an axiomatic point of view and show the derivation of demand functions and the constraints imposed upon them by the assumptions made in orthodox economic theory. This is followed by a brief description of the revealed preference approach, to show the formal equivalence of the two approaches.

Preference axioms

The static theory of consumer behavior begins with a choice problem facing an individual consumer with given income, prices, tastes, and preferences. The consumer is confronted with a set

of goods from which he is supposed to make a choice of items. His choice will be governed by certain behavioral factors, the most important among them being that he is "assumed to choose among the alternatives available to him in such a manner that the satisfaction derived from consuming commodities (in the broadest sense) is as large as possible" (Henderson and Quandt, 1958, p. 6). The extent of satisfaction derived from a given set of goods is assumed to depend upon the individual's preference relationship. Economists often refer to this preference relationship as a utility indicator. Because utility has been a central point of controversy in the theory of consumer behavior, we shall briefly describe this concept.

According to Chipman (1960, p. 221), "Utility in its most general form is a lexicographic ordering represented by a finite or infinite dimensional vector with real components unique only up to an isotone (order preserving) homogeneous transformation, and these vectors (lexical numbers) are ordered lexicographically like decimal numbers or words in a dictionary." However, the historic significance attached to utility is not exactly the same as the above definition. For example, Bentham, who brought the principle of utility into a prominent position, assumed it to be a cardinal measure of pleasure. (See Stigler, 1965, and Dorfman, 1964). The founders of

² We maintain Arrow's (1958, p. 1) distinction between "choice" and "decision making." "Decision making" is usually applied only when "conscious reflective choice is involved." However, it is possible to have selection of commodity bundles with unconscious or unreflective choices. Therefore, the term, "choices," will be used instead of decision making.

the utility theory (Jevons, Walras, and Menger) accepted the existence of utility as a fact of "common experience" and they spoke of utility as an absolute magnitude. (See Stigler, 1965, p. 85; also Samuelson, 1966, pp. 90-96). The early authors considered that the utility derived from the consumption of a commodity was a function of the quantity consumed alone. Thus, if q_1, q_2, \dots, q_n are n commodities belonging to a commodity bundle, the utility function is explicitly written as

$$U = U_1(q_1) + U_2(q_2) + \dots + U_n(q_n).$$

The cardinal nature of the utility function was relaxed by Fisher (1892) and Pareto (1896). They realized that, if a utility function reaches a maximum with a certain basket of commodities, then any order-preserving transformation of that function also reaches a maximum at that particular basket. In other words, the above maximum involves only ordinal properties. In most of the recent works, this ordinal property of the utility indicator is used to avoid the restrictive assumptions of a cardinal utility approach. Therefore, here it is not required that the consumer assigns units or measurements of utility associated with the possession and use of individual commodities. However, it is required that the preference relationship of the individual satisfy the following axioms (Debreu, 1965; Wold, 1952; and Uzawa, 1959).

Axiom 1. Ability to rank (axiom of comparability, see Newman, 1965, p. 10).

This axiom asserts that the consumer

facing different commodity bundles in a given commodity space³ is able to rank⁴ the bundles according to his order of preferences. Given two bundles q^0 and q^1 , the individual is capable of making one of the following three choices:

- (a) q^0 is preferred to q^1 , or ($q^0 p q^1$),
- (b) q^0 and q^1 are equally satisfactory, or ($q^0 I q^1$), and
- (c) q^0 is not preferred to q^1 , or ($q^0 \bar{p} q^1$).

There is no specification as to the extent to which q^0 is preferred to q^1 or otherwise—only that one is preferred to the other or that both are equally preferred.

Axiom 2. Antisymmetry

This axiom avoids any ambiguity in the preference ordering.

- If $q^0 p q^1$, it is not possible that $q^1 p q^0$ holds simultaneously. (2)

Axiom 3. Consistency of Ranking (Transitivity)

Suppose that there are three bundles (q^0, q^1 , and q^2) and that the following conditions hold:

- (a) $q^0 p q^1$ and
- (b) $q^1 p q^2$.

The axiom of consistency asserts that a set of preference relationships satisfying (3) will satisfy the relationship $q^0 p q^2$. Newman (1965) points out two cases where this axiom may not hold: First, there are certain threshold effects where "the combination of two gaps

³ Following Debreu (1965, p. 32), the term "commodity" is used to denote "a good or a service completely specified physically, temporally and spatially." The existence of only a finite number of commodities is assumed. The quantity of any one of the commodities can be any real number. A subset of elements belonging to the commodity space is referred to as a "commodity bundle."

⁴ A given individual does not need to rank all items, but just those of relevance to his experience.

each of which is below the individual's threshold (of preference perception), might itself be above that threshold." Second, the relevant bundles may contain numerous commodities whose preference relationships are such that comparison among the bundles may not be possible.

The axioms of comparability, anti-symmetry, and transitivity are necessary and sufficient conditions for a complete ordering of the commodities in the commodity space.

Axiom 4. Monotonicity

This axiom assures that the consumer has not achieved satiation. In the preference ordering, if the commodity bundles are ranked in an increasing order of preference, the preference relationship remains monotonically increasing.

Axiom 5. Convexity

The consumption set is convex (see Dorfman, pp. 396-97) if, for any two bundles, q^0 and q^1 belonging to the set, their weighted average $tq^0 + (1-t)q^1$ where $0 < t < 1$, also belongs to the same set. A fundamental property of the (ordinal) utility indicator is that the indifference curves represent convex sets, and the minimal property of all utility indicators is quasi-concavity. The assumption of quasi-concave utility indicators is important. Arrow and Enthoven (1961, p. 792) have shown that if the utility function is quasi-concave and monotonic, the usual first-order conditions are necessary and sufficient for obtaining a solution for a constrained maximization problem.

To sum up, the preference axioms mentioned in this section enable us to define an order-preserving, quasi-concave utility indicator which is monotonic and continuous.

Concept of demand

The concepts of demand, as stated in the middle of the nineteenth century by Cournot and Dupuit, were popularized by Marshall. (See Hicks, 1962). Marshall's theory focusing on the quantity-price relation for a single commodity, holding income and all other prices constant, provided a demand function uncompensated for income effects. The work of Pareto and Walras focused on the more general case in which all prices and income are variable. However, the basic theory was clarified by Hicks (1939), in his famous mathematical appendix, which explicitly links utility theory with demand analysis. His work drew on the article written in 1915 by Slutsky (1952) who distinguished between income and substitution effects due to a price change and between a compensated and uncompensated demand function.

Earlier, it was shown that the axioms on the preference relationship of an individual consumer lead to the existence of a monotonic order-preserving utility indicator which, by assumption, is a function of quantities of commodities belonging to the commodity bundle chosen. (See also Kalman, 1968). The behavioral axioms of the individual consumer are such that he makes his choices of the commodity bundle in order to obtain the maximum satisfaction subject to this budget limitation.

Suppose that a consumer with a given income, y , makes a choice of quantities, q_1, q_2, \dots, q_n , from a commodity space with n elements. Then the utility function can be specified as

$$U = U(q_1, q_2, \dots, q_n). \quad (4)$$

If p_1, p_2, \dots, p_n represent the unit prices of these commodities, $p_1q_1 + p_2q_2 + \dots$

+ $p_n q_n$ will be the total expenditure and this should not exceed the income; or

$$p_1 q_1 + p_2 q_2 + \cdots + p_n q_n \leq y. \quad (5)$$

So the choice problem reduces to finding a maximum of $U(q_1, q_2, \cdots, q_n)$ subject to the restriction (5). When we retain the inequality sign in restriction (5), the usual calculus method of finding constrained maximum becomes complicated and, therefore, we have to resort to programming techniques (see Dantzig, 1963; Hadley, 1964). Quadratic utility indicators were used in a few studies (Wegge, 1968). However, for explaining the theoretical concepts, it is sufficient to retain the equality sign in (5).

The consumer's choice of q_1, q_2, \cdots, q_n will correspond to the quantities consistent with maximization of

$$U(q_1, q_2, \cdots, q_n) + \lambda(y - p_1 q_1 - \cdots - p_n q_n). \quad (6)$$

Differentiating with respect to q_1, q_2, \cdots, q_n and λ , we get the following normal equations:

$$\begin{cases} U_j(q_1, q_2, \cdots, q_n) - \lambda p_j = 0, \\ (j = 1, \cdots, n) \\ y - p_1 q_1 - p_2 q_2 - \cdots - p_n q_n = 0, \end{cases} \quad (7)$$

where

$$U_j = \frac{\partial U}{\partial q_j}.$$

The system (7) provides $(n + 1)$ equations in $(n + 1)$ variables (q_1, q_2, \cdots, q_n and λ) when all the prices and income are given. Therefore, under given prices and income, we can solve for the quantities that provide the individual with the highest possible level on his preference

relationship. The solutions will be of the form

$$q_j = q_j(p_1, p_2, \cdots, p_n, y). \quad (8)$$

$(j = 1, 2, \cdots, n)$

The quantity purchased of each commodity is expressed as a function of its price, price of other commodities, and income; hence, the relationships in (8) represent a set of demand functions.

The demand functions (8) obtained from (7) should satisfy the second-order condition of utility maximization. Here the quasi-concavity assumption of utility indicators plays its role. The conditions expressed in (7) only assures that the consumer is neither on the uphill nor the downhill side of the preference function; rather, it leaves him at a stationary point, which may correspond to either a maximum or a minimum point of satisfaction. The necessary conditions, as expressed in (7) are sufficient if $U(q_1, q_2, \cdots, q_n)$ is a twice differentiable quasi-concave function in the neighborhood of the optimum. This condition can be elaborated to show the restrictions on the marginal rate of substitution. Here we shall use the properties of concave and quasi-concave functions. The Hessian matrix, H of U , can be written as

$$H = \begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ \cdot & \cdot & \cdots & \cdot \\ U_{n1} & \cdot & \cdots & U_{nn} \end{bmatrix} \quad (9)$$

where

$$U_{ij} = \frac{\partial^2 U}{\partial q_i \partial q_j}.$$

For a quasi-concave function, the bordered Hessian matrix (bordered with

the marginal utilities $\partial U/\partial q_i$ will have alternating principal minors starting with negative. Therefore,

$$(-1)^r D_r \geq 0 \quad (r = 1, 2, \dots, n) \quad (10)$$

where

$$D_r = \begin{vmatrix} U_{11} & \dots & U_{1r} U_1 \\ U_{21} & \dots & U_{2r} U_r \\ \dots & \dots & \dots \\ U_1 & \dots & U_r 0 \end{vmatrix}$$

and where

$$U_j = \frac{\partial U}{\partial q_j}, \quad (j = 1, \dots, n)$$

In particular, when $n = 2$,

$$\begin{bmatrix} U_{11} & U_{12} & U_1 \\ U_{21} & U_{22} & U_2 \\ U_1 & U_2 & 0 \end{bmatrix} > 0. \quad (11)$$

If we consider any movement along a given indifference curve, the following must hold:⁵

$$U_1 dq_1 + U_2 dq_2 = 0,$$

and the slope of the line tangent to this curve is defined as:

$$\frac{dq_2}{dq_1} = -\frac{U_1}{U_2} \quad (12)$$

The change in the slope of the indifference curve is given by

$$\begin{aligned} \frac{d\left(-\frac{U_1}{U_2}\right)}{dq_1} &= \frac{\left(U_{11} + U_{12} \frac{dq_2}{dq_1}\right) U_2 - \left(U_{21} + U_{22} \frac{dq_2}{dq_1}\right) U_1}{U_2^2}, \\ &= \frac{-U_{11} U_2^2 + U_{12} U_1 U_2 + U_{21} U_1 U_2 - U_{22} U_1^2}{U_2^2}. \end{aligned} \quad (13)$$

Here the numerator is the same as the ^{of the} matrix in (11). Using the relationship, (11), the sign of (13) depends on the sign of U_2 alone. The relationship, (13), is positive if $U_2 > 0$ and it is negative if $U_2 < 0$. Also, when the marginal utilities are positive, from the relationships (11), (12), and (13), we have the indifference curves downward sloping and convex to the origin. This convexity of indifference curves assumes diminishing marginal rate of substitution. The marginal rate of substitution is diminishing when we have diminishing marginal utility. Therefore, concavity of utility

function (diminishing marginal utility) is sufficient for diminishing marginal rate of substitution. However, this is not necessary, because the sign of the rate of change of the marginal rate of substitution depends only on the sign of U_2 as long as condition (11) holds. It is possible for (11) to be true even if U_{11} or U_{22} or both may be positive. Therefore only diminishing marginal rate of substitution is necessary for assuring a maximum of solution in (8).

To sum up, quasi-concave functions guarantee that if the necessary first-order conditions are satisfied at a certain

⁵ Indifference curve, in cardinal analysis, represents the locus of points in the commodity space which provide the same level of satisfaction. For our purpose, it is enough to assume that two commodity bundles fall on the same indifference curve if the consumer is indifferent towards these two bundles in an ordinal sense, (q^0, q^1). It is not necessary to introduce any cardinal utility assumption in this section.

point, it corresponds to a point of maximum satisfaction (see proofs in Arrow and Enthoven, 1961; Wold and Jureen, 1964). There will be a maximum for a quasi-concave function even if it exhibits increasing marginal utility without violating the property of diminishing marginal rate of substitution. Incidentally, this property helps us to release the preference relationship from a restrictive assumption of diminishing marginal utility.

Restrictions on demand functions

The demand functions satisfy a number of important relationships. Hicks (1962), Wold and Jureen (1964), Frisch (1959), Brandow (1961), and Pearce (1961) have summarized the properties of the demand functions. These conditions follow from differentiation of the first-order conditions.

Homogeneity Condition (row restraint)—The first-order conditions in (7), from which demand functions are derived, imply that if prices and income are changed by the same proportion, the quantity demanded remains the same. In the case of two commodities, the first-order conditions are

$$\begin{cases} U_1 - \lambda p_1 = 0 \\ U_2 - \lambda p_2 = 0 \\ y - p_1 q_1 - p_2 q_2 = 0. \end{cases} \quad (14)$$

This leads to a condition that

$$\frac{U_1}{U_2} = \frac{p_1}{p_2}$$

When income and prices are changed by the same proportion (say k), the first

* Elasticities are defined by $\epsilon_{ij} = (p_i/q_i)(\partial q_i/\partial p_i)$, ($i, j = 1, 2, \dots, n$) and $\epsilon_{iy} = (y/q_i)(\partial q_i/\partial y)$.

order conditions become

$$\begin{cases} U_1 - \lambda k p_1 = 0, \\ U_2 - \lambda k p_2 = 0, \\ k y - k p_1 q_1 - k p_2 q_2 = 0, \text{ or} \\ y - p_1 q_1 - p_2 q_2 = 0. \end{cases} \quad (15)$$

Equations (15) also lead to the conditions

$$\frac{U_1}{U_2} = \frac{p_1}{p_2} \text{ and } y - p_1 q_1 - p_2 q_2 = 0.$$

Since the first order conditions essentially remain the same, the optimum commodity bundle is unaltered. This property is known as the homogeneity condition, implying that demand functions are homogeneous of degree zero in prices and income. Consider the demand function for a single commodity i :

$$q_i = q_i(p_1, p_2, \dots, p_n, y). \quad (16)$$

Using Euler's theorem for homogeneous functions of degree zero, we have

$$\begin{aligned} p_1 \frac{\partial q_i}{\partial p_1} + p_2 \frac{\partial q_i}{\partial p_2} + \dots \\ + p_n \frac{\partial q_i}{\partial p_n} + y \frac{\partial q_i}{\partial Y} = 0 \cdot q_i. \end{aligned} \quad (17)$$

Converting (17) into elasticities by dividing throughout by q_i , we obtain

$$\epsilon_{i1} + \epsilon_{i2} + \dots + \epsilon_{in} + \epsilon_{iy} = 0. \quad (18)$$

This implies that the direct- and cross-price elasticities and income elasticities add to zero.

Engel aggregation.—The budget restraint is denoted by

$$p_1q_1 + p_2q_2 + \dots + p_nq_n = y. \quad (19)$$

$$p_1 \frac{\partial q_1}{\partial y} + p_2 \frac{\partial q_2}{\partial y} + \dots + p_n \frac{\partial q_n}{\partial y} = 1 \quad \text{or} \quad (20)$$

$$\frac{p_1q_1}{y} \cdot \frac{y}{q_1} \frac{\partial q_1}{\partial y} + \dots + \frac{p_nq_n}{y} \cdot \frac{y}{q_n} \frac{\partial q_n}{\partial y} = 1. \quad (21)$$

It will be convenient to express (21) in terms of budget proportions and elasticities as

$$W_{1e_{1y}} + W_{2e_{2y}} + \dots + W_{ne_{ny}} = 1 \quad (22)$$

where

$$W_i = (p_iq_i)/y \text{ represents the share of}$$

The effect of a change in income on consumption can be obtained by differentiating (19) with respect to y to obtain

expenditures (budget proportions) on j^{th} commodity.

Equation (22) implies that the income elasticities weighted by the respective expenditure proportions add to one.

Cournot aggregation.—The effect of a change in the price of j^{th} commodity, with all prices remaining the same, can be obtained by differentiating the budget restraint (19) with respect to p_j , or

$$p_1 \frac{\partial q_1}{\partial p_j} + p_2 \frac{\partial q_2}{\partial p_j} + \dots + q_j + p_j \frac{\partial q_j}{\partial p_j} + \dots + p_n \frac{\partial q_n}{\partial p_j} = 0 \quad \text{or} \quad (23)$$

$$p_1 \frac{\partial q_1}{\partial p_j} + p_2 \frac{\partial q_2}{\partial p_j} + \dots + p_j \frac{\partial q_j}{\partial p_j} + \dots + p_n \frac{\partial q_n}{\partial p_j} = -q_j.$$

Expressing (23), in terms of elasticities and budget proportions, we have

$$\frac{p_1q_1}{p_jy} \frac{p_j}{q_1} \frac{\partial q_1}{\partial p_j} + \dots + \frac{p_nq_n}{p_jy} \frac{p_j}{q_n} \frac{\partial q_n}{\partial p_j} = -\frac{q_j}{y}, \quad (24)$$

$$\frac{p_1q_1}{y} \frac{p_j}{q_1} \frac{\partial q_1}{\partial p_j} + \dots + \frac{p_nq_n}{y} \frac{p_j}{q_n} \frac{\partial q_n}{\partial p_j} = -\frac{p_jq_j}{y} \quad \text{or} \quad (25)$$

$$W_{1e_{1j}} + W_{2e_{2j}} + \dots + W_{ne_{nj}} = -W_j.$$

Thus, the weighted sum of the elasticities in the j^{th} column is equal to the negative of the expenditure proportion on the j^{th} commodity.

Slutsky condition.—The Slutsky (1952) relationship incorporates a fundamental relationship between changes in

quantities and the marginal utility of income. The effects of simultaneous changes in prices and income can be obtained by taking total derivatives of the first-order conditions in (7). As developed in a later section, a change in the consumption of the i^{th} commodity as a

result of a change in j^{th} commodity price can be represented as

$$\frac{\partial q_i}{\partial p_j} = \left(\frac{\partial q_i}{\partial p_j} \right) U_{\text{const}} - q_j \left(\frac{\partial q_i}{\partial y} \right) \quad (26)$$

$$= k_{ij} - q_j \frac{\partial q_i}{\partial y}$$

The first term on the right-hand side (k_{ij}) is the substitution effect and the second term is the income effect.

It is also shown that the compensated cross-price derivatives are symmetric.

$$\frac{\partial q_i}{\partial p_j} + q_j \left(\frac{\partial q_i}{\partial y} \right) = \frac{\partial q_j}{\partial p_i} + q_i \left(\frac{\partial q_j}{\partial y} \right) \quad (27)$$

Converting (27) into elasticities,

$$e_{ij} = \frac{w_j}{w_i} e_{ji} + w_j (e_{iy} - e_{iy}) \quad (28)$$

When the price changes of the same commodity are considered, the substitution effect, $k_{ii} = \partial q_i / \partial p_i + q_i (\partial q_i / \partial y)$, will be negative.

The demand restrictions, expressed in terms of elasticities, are summarized in table 1.

Other aspects of demand theory

A few other aspects of demand theory are summarized briefly to round out the discussion. Such aspects as multiperiod consumption decisions are not used in this study. The concept of separability used in the study is discussed in a later section.

Indirect utility indicator.—Many of the classical properties of the demand functions discussed can be derived from a dual representation of the utility indicator, often known as the indirect utility indicator. The notion of duality has become popular in recent years, especially in the framework of linear program-

TABLE 1
MATRIX OF DEMAND ELASTICITIES

q_i	p_i					
	p_1	p_2	p_3	...	p_n	y
q_1	e_{11}	e_{12}	e_{13}	...	e_{1n}	e_{1y}
q_2	e_{21}	e_{22}	e_{23}	...	e_{2n}	e_{2y}
q_3	e_{31}	e_{32}	e_{33}	...	e_{3n}	e_{3y}
...
q_n	e_{n1}	e_{n2}	e_{n3}	...	e_{nn}	e_{ny}

Row Restraint (Homogeneity Condition)

$$\sum_i e_{ij} + e_{iy} = 0$$

$$\text{(for } i = 1) \quad e_{11} + e_{12} + e_{13} + \dots + e_{1n} + e_{1y} = 0$$

Engel Aggregation

$$\sum_i w_i e_{iy} = 1$$

$$w_1 e_{1y} + w_2 e_{2y} + w_3 e_{3y} + \dots + w_n e_{ny} = 1$$

Cournot Aggregation

$$\sum_i w_i e_{ij} = -w_j$$

$$\text{(for } j = 3) \quad w_1 e_{13} + w_2 e_{23} + w_3 e_{33} + \dots + w_n e_{n3} = -w_3$$

Slutsky

$$e_{ij} = e_{ij}(w_i/w_j) + w_i(e_{iy} - e_{iy})$$

$$\text{(for } j = 3, i = 2) \quad e_{22} = e_{22}(w_2/w_3) + w_2(e_{2y} - e_{2y})$$

Substitution Effect

$$e_{ii} < 0$$

$$e_{ii} + w_i e_{iy} < 0$$

ming. The utility indicator, as defined earlier, is a function of quantities of commodities consumed. Also from (8), the quantity consumed of a given commodity is a function of its price, price of other commodities, and income.

$$U = U(q_1, q_2, \dots, q_n)$$

$$q_i = q_i(p_1, p_2, \dots, p_n, y)$$

Therefore,

$$U = U[q_1(p_1, p_2, \dots, p_n, y),$$

$$q_2(p_1, p_2, \dots, p_n, y), \dots,$$

$$q_n(p_1, p_2, \dots, p_n, y)]$$

$$= \phi(p_1, p_2, \dots, p_n, y)$$

which is known as the indirect utility indicator. Though the existence of an utility indicator dependent upon prices and income was pointed out by Hotelling

(1932), its full implications were explored only later by Court (1941), Houthakker (1952), and Samuelson (1960). The function ϕ is a dual representation of U . The properties of demand functions can be obtained from ϕ by obtaining the minimum level of expenditures necessary to reach a given level of individual satisfaction, say $\bar{\phi}$. That is, obtain

$$\begin{aligned} & \text{Min. } p_1q_1 + p_2q_2 + \dots + p_nq_n + \\ & \lambda[\phi(p_1, p_2, \dots, p_n, y) - \bar{\phi}] \\ & q_i = -\lambda(\partial\phi/\partial p_i) \quad (i = 1, 2, \dots, n) \end{aligned}$$

give the first order conditions.

Considering two commodities q_i and q_j ,

$$q_i = -\lambda \frac{\partial\phi}{\partial p_i} \quad \text{and}$$

$$q_j = -\lambda \frac{\partial\phi}{\partial p_j}$$

Therefore,

$$\frac{q_i}{q_j} = \frac{\partial\phi}{\partial p_i} / \frac{\partial\phi}{\partial p_j}$$

Samuelson (1965) provides a proof for the complete e quality of direct and indirect utility indicators.

The theory of revealed preference.—

An alternative to the utility approach to demand theory is revealed preference, as originally suggested by Samuelson (1966). The consumer with a given income is placed in a market situation and his choices of baskets at different market conditions are observed. These observations reveal the consumer's preferences which serve as the basis for arriving at certain conclusions regarding the choice function of the individual.

Suppose that in a price situation $p^0(p_1^0, p_2^0, \dots, p_n^0)$, the consumer buys

a commodity bundle q^0 . The commodity bundle q^0 is said to be preferred to an alternative bundle q^1 if, $p^0q^1 \geq p^0q^0$. In the price situation p^1 in which q^1 is bought, the q^0 basket must be at least as expensive. Therefore, $p^0q^1 \leq p^0q^0$ implies $p^1q^1 \leq p^1q^0$. This is known as the weak axiom of revealed preference. Samuelson (1938) asserts that the postulates of weak axiom "is logically equivalent to the reformulation (of consumer theory) of Hicks and Allen." Arrow (1959) has demonstrated the complete equivalence of the weak axiom of revealed preference with the existence of an ordering from which the choice function can be derived.

The results of homogeneity of the demand functions of degree zero and the negativity of the substitution term (k_{ij}) can be obtained from the weak axiom. However, to prove the symmetry of the substitution term ($k_{ij} = k_{ji}$), the weak axiom has to be extended to what is known as the strong axiom of revealed preference (Houthakker, 1950). In fact, the weak axiom deals with only pairwise choices, while the strong axiom incorporates the consistency idea that if $q^0p^1q^1$ and $q^1p^2q^2$, then $q^0p^2q^2$. This can be extended to any number of commodity bundles. A consumer whose choices are governed by these axioms will always possess an indifference map. Further, Debreu (1965) has shown that if the preference ordering is reflexive, transitive, and the "no worse than" and "no better than" sets are closed, a continuous utility function exists. In fact, Houthakker has established the formal equivalence of the revealed preference and utility function approaches to the theory of consumer behavior by showing that "a theory based on semi-transitive revealed preference entails the existence of ordinal utility, while the property of semi-transitivity itself was derived from

utility consideration.⁷ Though a few corrections are provided for Houthakker's proof, his result is still valid.

Multiperiod consumption decisions.—

The choice problems discussed in the previous sections were based on decisions made for a single period of time. For the consumption of food items it may be true that the amount demanded adjusts itself within each period to the changes in prices, income, and other variables affecting demand in that period. However, for many durables adjustments may not be complete during the same period because of such factors as inertia, expectations about future prices and income, and the influence of past incomes (see Stone 1954a, p. 272). Also, in the framework of a static analysis, savings are just like any other item of expenditure, while a dynamic framework requires the introduction of borrowings and lendings which alters the individual's distribution of income over the different periods. It is possible to extend the static theory of consumer choices to a dynamic framework by introducing a multi-period utility index and budget restraint (Tintner, 1938). In this case, a multiperiod ordinal utility index can be assumed to be dependent upon the planned consumption of each commodity over the entire planning horizon. With n commodities and t time periods, the utility function can be written as

$$U = U(q_{11} \cdots q_{n1}, q_{12} \cdots q_{n2}, \cdots, q_{1t} \cdots q_{nt}),$$

where

q_{it} represents the consumption of i^{th} commodity in period t .

The consumer maximizes his utility subject to a budget restraint to obtain a demand function (Henderson and Quandt, 1958, pp. 229–234):

$$q_{it} = D_{it}(p_{11} \cdots p_{nt}, i_1 \cdots i_{t-1})$$

where

p_{it} is the price of i^{th} commodity during period t and

i_t is the interest rate during period t .

Choices under uncertainty.—The choice problems discussed so far are based on the assumption that prices and income were known with certainty. However, when purchases of many durables are based on multiperiod decision processes, it may not be possible to know future prices and income with certainty. Consumer choices under stochastic situations also can be analyzed systematically using the Von Neumann and Morgenstern (1947) approach. (Also, see Friedman and Savage (1948), Luce and Raiffa (1957), and Samuelson (1952). Here, we are interested in determining the choice pattern of consumers under uncertain situations. If an individual is consistent in his preference ordering, it is possible to construct an index which describes his preference numerically. Henderson and Quandt (1958, pp. 34–38) provide five axioms that allow construction of an ordinal utility index which can also be used to predict choices in uncertain situations. The utility numbers associated with each possible stochastic component is determined as expected values. However, choices under uncertainty involve some elements of cardinal utility, although strict cardi-

⁷ Uzawa (1959) has shown the equivalence of these two approaches, and also that the strong axiom of revealed preference can be replaced by the weak axiom if the demand functions satisfy a regularity condition. As Houthakker (1961, p. 713) points out, "The economic meaning of this (regularity) condition has not been spelled out."

nality could be avoided by taking two arbitrary starting points and successively confronting the consumer with other choice situations in order to evaluate them on a relative basis. Instead of the strict cardinal measure of utility, this involves only an operational meas-

ure of satisfaction associated with different risky situations. The marginal utility that can be derived from the utility index obtained in this manner is similar to the marginal rate of substitution.

Empirical Models of Consumer Demand

While the theory of consumer demand was being developed through the utility approach, demand characteristics based on observed consumer behavior were analyzed. In certain cases, this empirical analysis was used to test the conclusions based on the utility approach and in others to obtain meaningful theories of consumer behavior. Here are some significant highlights of these studies.

The effects of income and prices on consumption have been analyzed in many studies. One of them, using family budgets to analyze the differences in consumption by poor and rich families, led Engel to his proposition that "The poorer a family, the greater the proportion of its total expenditure that must be devoted to the purchase of food." (Stigler, 1965, p. 203). Long before data on quantity and prices were available on market performance over time, data on quantities harvested and prices prevailing at harvest period were used to arrive at an inverse relationship between these two factors.⁸ However, statistical analysis of demand became popular only in the current century after the pioneering attempts of Moore (1917) and Leffeldt (1914). Stigler (1962, p. 1) points out that "Mathematical analysis became increasingly common after Walras's first edition . . . but statistical economics, the name given by Henry Moore, is the one important modern development. Henry

Moore was its founder . . . Moore's basic contribution was not to invent this field, but he made statistical estimation of economic functions an integral part of modern economics." Working's study of potatoes (1922) prompted a series of studies on individual commodity demand functions. Schultz (1938) combined a review of economic theory with a large number of empirical studies. The objective of most studies have been (1) to develop equations that can be used to forecast prices or quantities and (2) to approximate the demand curves of economic theory. Empirical studies have been facilitated by developments in data availability, improved data-processing facilities and developments in econometric theory.

The developments in the field of statistics and econometrics have made substantial contributions to demand analysis. Correlation and curve-fitting techniques have made it possible to estimate the parameters of a postulated functional relationship among prices, income, and quantities. Statistical testing procedures provide the framework for testing various hypotheses regarding the behavior of these variables. Early studies, based on least-squares estimation procedures using time-series data, were handicapped by problems like autocorrelation, multicollinearity, heteroskedasticity, and simultaneous equation

⁸ This is often recognized as Gregory King's Law that, "... a defect in harvest may raise the price" (of corn).

bias. The dilemma of applying ordinary least-squares analysis to time-series data was expressed by Working (1927) and Haavelmo (1943) but improvements have overcome at least some of these problems. Others remain in spite of advances in econometric methods.

Often studies using macro data to estimate the relationship among demand variables (quantities, price, income, and other attributes) make an implicit assumption that demand relationships, applicable to a "representative consumer" in a given area and time period, are a true representation of the consumption pattern in that area and for that period. Both time-series and cross-section data will be used in this study to analyze demand characteristics.

Cross-section analysis

Cross-section data relate to the consumption behavior of a given sample of the population at a given period of time. Early studies, based on household food consumption data from cross-section surveys, were used for estimating income elasticities. In most cases, published data on consumption patterns give the quantities of food items consumed or expenditures on these items by certain income classes. Based on these grouped data, it is possible to obtain weighted regressions to estimate the aggregate income elasticity.⁹ A systematic application of this procedure is available in Wold and Jureen (1964, p. 216). They had four family-size groups, and each group was divided into four income classes according to annual income per consumer unit or the family unit standardized according to size and age distribution. Assuming constant income elasticity E , a regression equation of the

form, $d = C y^E$ was fitted. (Here, d = quantity consumed and y = annual income.) Using a linear logarithmic regression, the income elasticity for the i^{th} family-size group can be obtained as

$$E^i = \frac{\sum H_v(X_v - M_x)y_v}{\sum H_v(X_v - M_x)^2} \quad (29)$$

where

$$M_x = \frac{\sum U_v X_v}{\sum U_v}$$

All summations run over four income classes,

N_v = total number of households in V^{th} income class,

C_v = consumer units per household,

$U_v = N_v C_v$ = total number of consumer units,

X_v = logarithm of income per consumer unit, and

Y_v = logarithm of food expenditure per consumer unit.

After obtaining elasticity coefficients for the four groups, the aggregate elasticity was found, using the formula

$$E = \frac{\sum K^i E^i}{\sum K^i} \quad (30a)$$

where

$K^i = \sum H_v y_v^i$ and

y_v = food expenditure per consumer unit.

Stone's (1954) analysis of British family budgets was based on the assumption that all income changes were in the same proportion. (See also Prais and Houthakker, 1955). If q_v represents the quantity of the i^{th} commodity consumed by the v^{th} individual and y_v the income

⁹ For a discussion on weighted regressions see Draper and Smith (1966, pp. 77-81) and Johnston (1963, pp. 207-211). Instead of using weighted regression, it is possible to use simple regression on group averages to obtain an unbiased estimate, though the variance will differ (see Malinvaud, 1966, pp. 242-46).

of the r^{th} individual, and if the relationship between the logarithms of q_{ir} and y_r is linear with slope b , market income elasticity is obtained by

$$\frac{b}{q_i} \sum_r q_{ir} \frac{\partial y_r}{\partial y} \frac{y}{y_r} \quad (30b)$$

Most of these studies were based on the assumption that the elasticity was constant over the whole range of observations and that income and quantities were the only relevant variables. To overcome these drawbacks, a number of modifications have been suggested. One of these is to construct a general model incorporating additional variables like family size and composition, urbanizations, regions, and other qualitative variables. Herrmann's (1964) analysis of 1955 U.S. consumption data shows that the most important variables in explaining food expenditure patterns in their order of importance are income, urbanizations, region, life-cycle stage, education of homemaker, and social class.

Another approach to explain variations of the type mentioned above is to obtain regression equations for each subgroup separately. (See Sosnick, 1962, and Foytik, 1951, for use of this procedure in estimating seasonal demand functions). The equality of these regression coefficients can be tested, using standard tests of significance. However, in many cases, when it is required to test the equality of regression coefficients for factors like regions, seasons, and urbanizations, individual regression equations may not always be appropriate because of multiplicity of equations and small number of degrees of freedom associated with each equation. Analysis of covariance techniques are useful to handle such problems.

Time series analysis

Demand theory specifies that the

quantity consumed of a particular commodity is a function of its price, prices of other commodities, and income. This static theory of demand for the individual, with all income spent on commodities including savings, takes prices and income as "given." As will be discussed in the second part of this study, the move from the individual to market demand requires simplified assumptions to maintain essentially the same model for empirical estimation. For example, the assumption of prices as predetermined variables may be appropriate for the individual but not for the market. Stone (1954a) in the United Kingdom utilized a model paralleling the classic model mentioned above, arguing that prices were essentially determined in the world markets. On the other hand, commodity analysts in the United States have argued that, for many agricultural products, quantities available at the end of the harvesting season are essentially exogenous and that prices at farm and retail are functionally related to quantities and income-shift variables. Even for the analysis of a single commodity, it is often necessary to specify multi-equation models to account for export demand, domestic demand, and stocks. The approach adopted by the analyst of time-series data depends on a great variety of questions relating to the scope of the model and problems of estimation. Here are some considerations of direct importance to the present study with its emphasis on estimation of demand at the retail level.

Single equation models.—Consider the estimation of demand function parameters for a simple model in which one commodity can be singled out for analysis. Theoretically, the following model might be specified as a first approximation:

$$q_{it} = f_i(p_{it}, z_{it}, y_t, u_{it}) \quad (31a)$$

where

q_{it} = per capita consumption of the i^{th} commodity,

P_{it} = price of the i^{th} commodity,

z_{it} = other factors affecting demand (assumed exogenous),

y_t = per capita disposable income, and

u_{it} = a random disturbance.

A number of problems arise from the estimation of the desired parameters:

Error specification. Estimation by ordinary least squares requires that the error term is not correlated with prices and income, the absence of autocorrelation, constant variance over time, and, in addition, sufficient observations in relation to the number of parameters to be estimated. When these conditions are not met, more complex formulations are required, as is discussed in texts on econometrics such as Johnston (1963), Goldberger (1964), or Malinvaud (1966).

Multicollinearity. Often prices and income move together over time, resulting in problems of multicollinearity which, in the extreme case, results in a singular matrix. In the case of income and prices, writers have suggested combining time-series data with income slopes or elasticities estimated from cross-section data. With high multicollinearity among prices, problems exist as to the significance of coefficients. Where the goal is to approximate a complete demand matrix, other methods must be introduced.

Relevant variables. In the more general models, quantity consumed is related to all commodity prices and income which is an impossible task, when time-series data are used. The concepts of separable utility functions provide guides for meaningful methods to simplify the estimation procedures.

Mathematical form of the equation. No *a priori* guideline exists for the functional form of the relationship among quantity, prices, and income that is appropriate in all cases. Time-series data provide only samples from a limited range of observations. Some of the commonly used functions belong to one of the following:

Linear:

$$q = a + by + cp + u,$$

Semi-logarithmic:

$$q = a + b \log y + c \log p + u,$$

Double-logarithmic:

$$\log q = a + b \log y + c \log p + u, \text{ or}$$

Inverse-logarithmic:

$$\log q = a + by + cp + u.$$

Static versus dynamic functions. A static model, as specified in equation (31a), may be incorrect in several ways. Shifts in consumers' tastes may affect the slope or position of the demand curve; past levels of consumption may affect consumption patterns; or levels of inventory of consumer goods may be important although this is probably of minor importance for food commodities. The analyst must choose among alternative approaches, realizing the limitations inherent in any given approach.

Simultaneous equation models. In a theory of general equilibrium, the quantities consumed are equated to the quantities produced. Similarly, the quantity consumed of a given commodity is functionally related to all commodity prices and income (or factor prices). Concentration on a demand matrix alone limits a realistic analysis of individual commodity markets. This is unfortunate but necessary within the scope of this study. However, even within the demand matrix, simultaneous relationships that are recognized are difficult to estimate.

Demand theory asserts that consump-

tion of all commodities are interrelated. But, in a single-equation approach, it is not possible to incorporate this simultaneous nature of the demand relationship and, therefore, the coefficients obtained from the single equation method will be subject to possible bias.

Different estimation procedures, like two-stage least squares, full information maximum likelihood, limited information maximum likelihood, and three-stage least squares, are available for handling simultaneous estimation problems. Though a number of problems associated with simultaneous estimation procedure have been solved, there is still some doubt regarding the advantages of simultaneous equations over single equation procedures (see Christ, 1960).

The most important problem of using simultaneous equation methods for estimating demand coefficients is not one of estimation procedures but, rather, the problem of defining an identifiable model (see Hood and Koopmans 1953, Fisher 1966, and Wegge 1965). Suppose that there are n endogenous variables, y_1, y_2, \dots, y_n and m exogenous variables, z_1, z_2, \dots, z_m . A structural equation can be represented as

$$BY + CZ = U.$$

The reduced form is given by

$$Y = -B^{-1}CZ + B^{-1}U.$$

The problem of identification is one of deducing the value of the parameters of the structural relations from a knowledge of the reduced-form parameters. The order condition of identifiability states

that the number of variables excluded from each equation must be at least as great as the number of endogenous variables in the model less one. Therefore, in the present example, the maximum number of variables that can appear in any equation is

$$[n + m - (n - 1)] = m + 1$$

As far as the system of demand equations for all commodities are concerned, it is difficult to meet this identification criteria.

Dynamic models of demand

A number of attempts have been made to modify the static formulation of demand functions.¹⁰ Distributed lag models and recursive systems have been especially useful to incorporate some of the dynamic elements. A distributed lag model may be estimated, using any of the following three methods:

(1) Make no assumption as to the relationship among regression coefficients. Tinbergen (1951) suggested the typical form of the demand equation to be

$$q_t = a + b_0 p_t + b_1 p_{t-1} + \dots + b_i p_0 \quad (31b)$$

$$= a + \sum_0^{\infty} b_i p_{t-i}$$

where

q_t is the quantity demanded in period t and

P_{t-i} is the price in period $(t - i)$.

Here, we do not make any assumption

¹⁰ Bieri (1966) classifies different approaches to incorporate time into demand analysis as: (1) models introducing the time dimension through dating of the variables in the utility function (Henderson and Quandt, 1958; Modigliani and Brumberg, 1954; and Strotz, 1956); (2) models introducing dynamic aspects into demand function (Houthakker and Taylor, 1966; Nerlove, 1958b; and Farrell, 1952; and (3) models introducing dynamic aspects into the utility function (Cramer, 1957; Stone, 1954; Stone, *et al.*, 1964; Basmann, 1956; and Teujimura and Sato, 1964).

regarding the relationship among b_t . No restriction is imposed on the distribution of the lag effect of past prices on the quantity demanded. In practice, lagged prices will be added successively to the least-squares regression of quantity on prices until the coefficients become insignificant or the signs become erratic. (2) Assume a general form for the distribution of lag and estimate the parameters. Fisher (1925) and Koyck (1954) have suggested that the form of the distribution of the lag may be approximately assumed and then the specific characteristics of the distribution may be estimated. If time is treated as a continuous variable, the demand relationship is

$$q_t = a + \int_0^{\infty} b(u)P(t-u)du. \quad (32)$$

Koyck calls the distribution of $b(u)$ the "time shape of an economic reaction" and the cumulative distribution of $b(u)$ "the adjustment path." The distribution of $b(u)$ can be assumed; for example, Fisher assumed a logarithmic normal distribution. (3) Develop an explicit dynamic model which implies the distributed lag only incidentally. This approach is basically similar to Nerlove's (1958b) static expectation model. The current quantity consumed will change in proportion to the long-run equilibrium quantity and current quantity.

$$q_t - q_{t-1} = \gamma(\bar{q}_t - q_{t-1}) \quad (33)$$

where

- q_t is the quantity consumed in period t ,
- \bar{q}_t is the quantity demanded in long-run equilibrium, and
- γ is a constant of proportionality.

Nerlove (1958b, p. 308) compares these three approaches as follows:

"Because of the finite length of, and degree of auto-correlation in most economic time series, the first approach where nothing is assumed is not always feasible. On the other hand, the second approach must necessarily contain a somewhat arbitrary assumption concerning the form of the distribution of lag. The third approach leads to a direct interpretation of the distribution of lag in terms of producer or consumer behavior and, therefore, in terms of the difference between short- and long-run elasticities of supply or demand."

Models combining cross-section and time-series data

Because both time-series and cross-section analysis have certain inherent disadvantages, attempts have been made to supplement one method with the other. The "conditional regression analysis" used by Wold and Jureen (1964) and the "extraneous estimators" used by Stone (1954a) are based on the utilization of income elasticities obtained from budget studies in conjunction with time-series data.¹¹ Gourevix's (1960) analysis of consumption behavior, based on data derived from household surveys and time series of national averages, consisted of three kinds of comparisons: (1) household surveys, consumption of households at a given period; (2) international comparisons, average consumption in different countries at a given period; and (3) time series, change in the average consumption over the last decade. To estimate regional and sectoral demand characteristics, analysis of covariance can be used effectively. Hoch (1962), Mundlak (1961 and 1963), and Paris and Hoch (1966) have used this approach for estimating production-function parameters. Balestra and Nerlove (1966) and Ben-David and Tomek

¹¹ Durbin (1953) explains a procedure for handling extraneous information. Kuh and Meyer (1957) warn against giving undue importance to extraneous estimation procedures. Also see Kuh (1959).

(1965) have applied similar approaches for estimating demand functions.

Mundlak model.—Assume that there are observations on quantity, price, and income from m regions for t periods. If the regression coefficients in all regions are the same and if regional characteristics can be isolated, the demand function can be written as

$$q_i^{(r)} = B_0 + B_{1i}P_{1i}^{(r)} + \dots + B_{ki}P_{ki}^{(r)} + B_y Y^{(r)} + CR^{(r)} + \epsilon r^t \quad (34)$$

where

$q_i^{(r)}$ = quantity demanded in region r during period t ,
 $p_{it}^{(r)}$ = price of commodity, i , in region r during period t ,
 $R^{(r)}$ = regional effect in region r ,
 (commodity) $i = 1, 2, \dots, k$,
 (region) $r = 1, 2, \dots, m$, and
 (period) $t = 1, 2, \dots, t$.

The usual procedure in cross-section data is to fit a regression to data collected for regions at a period of time, ignoring the contribution of regional factors, $CR^{(r)}$. When we are interested in regional variations also, we obtain the estimates by minimizing the sum of squares

$$S = \sum_r \sum_t (q_i^{(r)} - B_0 - B_{1i}P_{1i}^{(r)} - B_{ki}P_{ki}^{(r)} - B_y Y^{(r)} - A^{(r)})^2 \quad (35)$$

where

$$A^{(r)} = CR^{(r)}.$$

If the demand function is complete,

$\hat{C} = 1 - \sum_j \hat{\beta}_j$. Therefore, $\hat{R}_i = A^{(r)}/\hat{C}$, which gives an estimate of regional effect.

Ben-David and Tomek model.—If the regression coefficients take different values for different regions, the following model can be used:

$$q_i^{(r)} = a_0 + \sum_{j=1}^m a_j D_{ji}^{(r)} + b_0 P_i^{(r)} + \sum_{j=1}^m b_j S_{ji}^{(r)} + U_i^{(r)} \quad (36)$$

where

$r = j = 1, 2, \dots, m$,

$t = 1, 2, \dots, t$,

q = endogenous variable,

P = exogenous variable,

D_j = intercept-shifting variable with

$$D_j^{(r)} = \begin{cases} 1, & \text{when } r = j, \\ 0, & \text{when } r \neq j, \end{cases} \text{ and}$$

S_j = slope containing dummy variables, with

$$S_{ji}^{(r)} = D_{ji}^{(r)} P_i^{(r)} = \begin{cases} P_i^{(r)}, & \text{when } r = j \\ 0, & \text{when } r \neq j \end{cases}$$

The above equation can be estimated, using certain assumptions, and the equality of regression coefficients can be tested. Also, these models can be generalized to handle variations in both slopes and intercepts of demand over different regions.

Shifts over time

The time-series analysis assumes that the structure of demand and the values of coefficients remain stable over the period under consideration. It is possible that the structure over time may gradually change. The effect of such

shifts on demand coefficients have been recognized (see Daly, 1956). If the change in structure is clearly identifiable, a shift variable can be introduced in the regression equation. One possible method of identifying shifts may be through a preliminary graphical analysis. Shift variables could be introduced in the demand function through dummy variables whose values are one and zero, depending upon the period of observation. According to Rojko (1961, p. 44), another way to handle changes in structure is to break the period into sub-periods during which no change in structure has occurred. One difficulty with this approach is that the number of observations per subperiod may be small for statistical analysis.

Introducing time as an explicit variable in the demand equation is another feasible approach. The coefficient of time may absorb most of the unexplained variations and an interpretation of the coefficient may become difficult. Sometimes, time-series and cross-section data may be combined to analyze the secular and cyclical changes of demand. (For example, see Douglas, 1967.)

Foytik (1951) suggests two alternative methods that could be used to allow for systematic shifts in the regression coefficients over time. The first approach is essentially the same as fitting separate demand functions for each period and

the second approach considers all weekly observations together and estimates an equation of the form:

$$P_i = a + (b + b'w)Q_i + (C + C'w)D_i \\ + (d + d'w)Q_{i-1} \\ + (e + e'w)W \quad (37)$$

where

$i = w =$ week of season,

$P =$ price,

$Q =$ quantity sold, and

$D =$ consumer income.

To find out whether the demand equation changed systematically over time, the coefficients b' , c' , d' , and e' were tested to see whether they differed significantly from zero.

Though the above approaches can be used to study the shifts over time of a single commodity, they may not be sufficient to analyze stochastic elements in consumer behavior. Barten (1966) has specified a model which takes into account the shifts in utility function over time. Rosenberg (1968) provides different approaches to incorporate stochastic variations among parameters during the period of observation. However, certain conceptual and empirical problems are yet to be solved before such procedures can be applied to a large number of commodities.

The Gap Between Demand Theory and Empirical Analysis

In theoretical development, we specify certain postulates and deduce the behavior of the variables through logic. In contrast, empirical studies deal with quantifiable phenomena. Often theoretical developments and empirical analysis complement each other—empirical analysis can be used to verify the validity of certain theories. Sometimes certain

theories are reached by starting from an empirical analysis. In the field of demand analysis, econometricians have often built empirical models based on the significance of economic variables like prices and quantities, and justified their findings through economic theory. On the other hand, some models in consumption theory are not subject to

empirical verification because of deficiencies in data or in statistical procedures. As a result of this, we are faced with a situation of "... insufficient predictive power, inappropriate basis for empirical analysis, and difficulties in establishing empirical confrontation..." (deJanvry, 1968, p. 4) which is often referred to as "the gap between theory and empirical analysis." This section describes attempts to bridge this gap and, also, points out certain difficulties encountered in the process. The discussion of concepts of separability owes much to the excellent treatment in deJanvry's (1966) award-winning thesis.

The fact that, according to economic theory, consumption of a particular commodity is dependent upon its price, prices of other commodities, and income, suggests that the demand for all commodities are interrelated and that any empirical study should consider the demand for all commodities simultaneously. If there are n commodities, this involves $(n \times n)$ price elasticities and n income elasticities for a total of $n(n + 1)$ parameters to be estimated. For the model to be estimable by known techniques, the number of observations should be at least equal to the number of parameters to be estimated which, in this case, is $n(n + 1)$. When a large number of commodities are considered, this condition cannot be satisfied and we run into the so-called "problem of degrees of freedom." (For further discussion, see deJanvry and Bieri, 1968; Clarkson, 1963; Mishan, 1961; and Boutwell and Simmons, 1968.) If we impose the restrictions on consumer demand, the number of parameters to be estimated directly can be reduced. The symmetry condition provides $n[(n - 1)/2]$ restrictions, the homogeneity condition provides n restrictions, and the Engel aggregation provides one restriction for

a total of $n[(n - 1)/2] + n + 1 = (n^2 + n + 2)/2$ restrictions. Therefore, the number of parameters to be estimated independently is reduced to

$$\begin{aligned} n(n + 1) - \frac{(n^2 + n + 2)}{2} \\ = \frac{1}{2}(n^2 + n - 2) \end{aligned}$$

which still remains to be a big number to permit direct estimation of a system of equations involving large numbers of commodities. Realizing this difficulty, two different approaches have been adopted—a single commodity or a sub-sector analysis, and the "integrationist's approach" (Boutwell, 1965, p. 8).

Single commodity or sector models

A single equation is formulated for a commodity to estimate the direct price and a few other cross-price elasticities. The effect of all other omitted variables is implied to be zero. The choice of other prices to be included is often based on subjective judgments of researchers. Also, the number of parameters to be included in each equation depends upon the required number of degrees of freedom which can be increased either through increasing the number of observations or through decreasing the number of parameters in the equation. The number of observations can be increased by extending the period of observation or reducing the interval between successive observations in the case of time series data, and enlarging the sample space in the case of cross-section data. In general, the effect of using enlarged time-series and cross-section data is to increase the variability in the data and it may be important to test whether any structural change or heterogeneity has occurred in the process of enlargement. The number of parameters in the model can be reduced by defining composite

commodities. Although the estimation problem can be solved by aggregating commodities, this procedure introduces a number of "aggregation problems." Also, for policy analysis, often information on individual commodities is required and an aggregate derived from heterogeneous items may not reflect the characteristics of individual commodities belonging to the set.

The "integrationist's approach"

The integrationist's approach recognizes the interrelationships among all commodities. To overcome the problems of degrees of freedom and identification, a number of assumptions regarding the interaction of commodities and the nature of utility functions are introduced. Also, attempts are made to incorporate the theoretical restrictions into the statistical model. Strotz' (1957, 1959) utility tree and Houthakker's (1960) additive preferences and various forms of separability ideas belong to this group. Frisch (1959) in a pioneering article, proposed that the demand relationships derived from utility theory could be used in computing all direct price and cross elasticities under an assumption of want independence.

Frisch model.—Frisch considers the implications for estimation of a matrix

$$\sigma_{ij} = \frac{\partial q_i(u_1, u_2, \dots, u_n)}{\partial u_j} \cdot \frac{u_j}{q_i} \quad (\text{want elasticity}) \text{ and}$$

$$\phi = \frac{\partial \lambda}{\partial y} \cdot \frac{y}{\lambda} \quad (\text{money flexibility}).$$

For income elasticities, the conventional row restraint, or Slutsky-Schultz condition, is that the elasticities for prices and income sum to zero, or

$$-e_{iy} = \sum_j e_{ij} \quad (40)$$

($i, j = 1, \dots, n$)

of demand coefficients for the case in which the utility of some or all commodities are independent of the quantity of others. The idea of *want independence* is explained by Frisch (1959, p. 178) by referring to commodities where, for example, "... the marginal utility of using more electricity in the home can safely be regarded as independent of the quantity of Swiss cheese consumed." Similarly, he discusses the case where commodity groups may be want independent, but dependence is assumed among commodities within a group. The major argument is for the case of want-independent commodities and can be compared with the classical case in which the Slutsky relation is given as

$$e_{ij} = e_{ji} \frac{w_j}{w_i} + w_j(e_{jy} - e_{iy}). \quad (38)$$

The Frisch statement of this relation expresses price elasticities (e_{ij}) as a function of want elasticity (σ_{ij}), budget proportions (w_i), income elasticities (e_{iy}), and the flexibility of the marginal utility of income with respect to income (ϕ).

$$e_{ij} = \sigma_{ij} - w_j e_{iy} - \frac{1}{\phi} w_j e_{jy} e_{iy} \quad (39)$$

where

The Frisch statement in terms of want elasticities and the money flexibility coefficient is

$$e_{iy} = \phi \sum_j \sigma_{ij}. \quad (41)$$

Consider the case where a good is want

independent of all other goods, or where $\sigma_{ij} = 0$ for all $i \neq j$. The cross-price and income elasticities may then be expressed as

$$e_{ij} = -w_j e_{iw} - \frac{1}{\phi} w_j e_{jw} e_{iw}, \quad (42)$$

$$= -e_{ij} w_j \left(1 + \frac{e_{jw}}{\phi} \right), \quad (\text{cross-price elasticity}), \text{ and} \quad (43)$$

$$e_{iw} = \phi \sigma_{ii} \quad (\text{income elasticity}).$$

To obtain the direct price elasticity σ_{ii} in equation (43), substitute the term under want independence, we solve for ϕ in equation (42), and obtain

$$e_{ii} = \frac{e_{iw}}{\phi} - w_i e_{iw} - \frac{1}{\phi} w_i e_{iw} e_{iw}, \quad (44)$$

$$= -e_{iw} w_i - \frac{1 - w_i e_{iw}}{\phi} \quad (\text{own price elasticity}).$$

Under want independence, we may solve for ϕ , or

$$\phi = \frac{e_{iw} - w_i e_{iw} e_{iw}}{e_{ii} + w_i e_{iw}} \quad (\text{money flexibility}). \quad (45)$$

If a value of ϕ is known, equation (42) may be used to obtain estimates of cross elasticities under the assumption of want independence ($\sigma_{ij} = 0$). Money flexibility may be estimated from equation (45) for any commodity where the direct-price and income elasticity coefficients are known. Further, estimates for various commodities or commodity groups should provide similar values of ϕ if the assumption of want independence is valid. Thus, if we know all income elasticities, expenditure weights, and direct-price elasticities for a single commodity, all the remaining parameters can be derived.

The assumption of want independence for all commodities implies complete additivity of the direct utility function, or

$$u(q_1, q_2, \dots, q_n) = u_1(q_1) + u_2(q_2) + \dots + u_n(q_n). \quad (46)$$

Houthakker (1960) refers to this case of independent utilities as "direct additivity" and shows that the cross derivatives of demand are proportional to the derivatives with respect to income. Thus, under independent utilities, the commodities are still related through the budget restraint but with demand interrelationships of a much less complex form than with conventional theory when complete dependence is allowed. Barten (1964, 1967) suggests a model incorporating a weaker assumption on the utility function, with complete additivity replaced by conditions of "almost additive preferences." For a detailed comparison of the Hicks, the Frisch, and the Barten models, see Appendix B.

Barten model.—The essential contribution of Barten's model is to develop methods of estimating demand functions with somewhat less restrictive conditions

being imposed on the preference function. Consider the Hessian matrix U in which elements are $U_{ij} = \partial^2 U / (\partial q_i \partial q_j)$. For the Frisch case of want independence for all commodities, the Hessian matrix has zeros except for diagonal elements. The Hicks matrix would allow values in all cells. Barten would limit values to diagonal elements plus certain nondiagonal elements of a nature that the consumer's preferences are "almost additive." The term for a cross-price elasticity for Barten is of the form

$$e_{ij} = \frac{p_j}{q_i} U_{ij}^{-1} - w_j e_{iv} \left(1 + \frac{e_{jv}}{\phi} \right). \quad (47)$$

For Frisch, the first term on the right of the equality is σ_{ij} ; otherwise, the equations are identical. Barten's formulation incorporates a term (U_{ij}^{-1}) that is invariant under transformations whereas the Frisch term (σ_{ij}) is not. For practical considerations, these two derivations can be considered as equivalent as pointed out by Ayanian (1969).

For estimation purposes, Barten specifies a regression equation of the form

$$\begin{aligned} \Delta \log q_i^{(t)} = & \alpha_i + \sum_{j=1}^n \left(\epsilon_{ij} - \frac{1}{\phi} e_{iv} e_{jv} w_j - e_{iv} w_j \right) \Delta \log p_j^{(t)} \\ & + e_{iv} \Delta \log y(t) + u_i(t) \end{aligned} \quad (48)$$

where the constant term represents trend due to changes in tastes or other factors, ϵ_{ij} represents that part of the substitution effect that is directly related to the interaction of commodities in the utility function, the compound term (money flexibility, budget proportion, and income elasticities) represents the remaining substitution effects, e_{iv} the income elasticity, and u_i a random error term. The first difference of logarithmic specification is justified on pragmatic grounds. Details of the estimation procedures are available in Barten (1964).

A different approach to the complete additivity assumption used in Frisch's formulation and to the almost additivity assumption used in Barten's formulation is obtained by introducing different concepts of separability, developed from the "utility tree" concept discussed by Strotz (1957, 1959).

The role of separability in demand analysis

The concept of a utility tree.—The

basic idea is that the elements belonging to the commodity bundle can be partitioned into different groups (similar to the branches of a tree). It is assumed that consumers follow a budget allocation process in such a way that, in the first stage, the total expenditure is divided into different subgroups and then, at the second stage, the amount allotted to each subgroup is allotted to individual commodities belonging to that subgroup. In other words, consumers follow a two-stage budgeting process by which total expenditure is first split into group expenditures at the first stage and then each group expenditure is split into individual commodity expenditures at the second stage. According to this procedure, it is possible to calculate all the parameters involved in the system of demand equations from a knowledge of certain parameters.

To permit the use of the two-stage allocation process, the utility function must satisfy certain properties. Strotz starts with the classical utility function

$U(q_1, q_2, \dots, q_n)$ and assumes that it is separable in the branches 1, 2, \dots , s if it can be written as¹²

$$U = F[U^1(q^1) + U^2(q^2) + \dots + U^s(q^s)] \quad (49)$$

where

$$U^i(q^i) = U^i(q_1^i, q_2^i, \dots, q_{n_i}^i)$$

n_i = number of commodities in the i^{th} group such that
 $n_1 + n_2 + \dots + n_s = n$.

As usual, maximizing this utility function subject to the budget restriction

$$\sum_{i=1}^n p_i q_i = y^i,$$

we can arrive at a demand function of the form,

$$q_j^{(i)} = \alpha_j^{(i)} + \sum_{i \in I} \beta_{ji}^{(i)} P_i + \sum_{k \in I} \beta_{jk}^{(i)} P_k + \gamma_j^{(i)} y \quad (50)$$

where

$q_j^{(i)}$ = j^{th} commodity belonging to the i^{th} commodity group,

$j = 1, 2, \dots, n_i$, and

$i = 1, 2, \dots, s$.

Taking two commodities belonging to the same group, the relationship can be written as

$$\begin{aligned} q_{j_1}^{(i)} &= \alpha_{j_1}^{(i)} + \sum_{i \in I} \beta_{j_1 i}^{(i)} P_i + \sum_{k \in I} \beta_{j_1 k}^{(i)} P_k + \gamma_{j_1}^{(i)} y \\ q_{j_2}^{(i)} &= \alpha_{j_2}^{(i)} + \sum_{i \in I} \beta_{j_2 i}^{(i)} P_i + \sum_{k \in I} \beta_{j_2 k}^{(i)} P_k + \gamma_{j_2}^{(i)} y \end{aligned} \quad (51)$$

for any such two commodities belonging to a group, the coefficients $\beta_{j_1 k}^{(i)}$ and $\beta_{j_2 k}^{(i)}$ will be in fixed proportion for k not in

branch i . Strotz has shown that the ratio of such price slopes is equal to the ratio of income slopes, $\gamma_{j_1}^{(i)} / \gamma_{j_2}^{(i)}$. Therefore,

$$\frac{\beta_{j_1 k}^{(i)}}{\beta_{j_2 k}^{(i)}} = \frac{\beta_{j_1 k_1}^{(i)}}{\beta_{j_2 k_1}^{(i)}} = \frac{\beta_{j_1 k_2}^{(i)}}{\beta_{j_2 k_2}^{(i)}} = \frac{\gamma_{j_1}^{(i)}}{\gamma_{j_2}^{(i)}} \quad (k_1, k_2, \dots, gI). \quad (52)$$

Using the relationship (52), a number of parameters in the system of demand equations can be calculated with the knowledge of income coefficients and at least one other interbranch coefficient.

Concepts of separability.—To separate commodities in the utility function, the ratio of marginal utilities of a pair

of commodities i and j is assumed to be unaffected by the level of consumption of a third commodity k . In other words, it can be assumed that

$$\frac{\partial \frac{U_i}{U_j}}{\partial q_k} = 0 \text{ for } k \neq i, j \quad (53)$$

¹²This is the "strong" definition of a separable utility function. Strotz admitted that, in his original formulation, he had defined a weak form of separable utility function, though his results were derived based on this strong form of separability (see Strotz, 1959). Expressing the utility function in this manner implies additivity among groups.

holds for at least some of the goods. While the concept of complete additivity used by Frisch assumes that the marginal utilities of i and j are unaffected, the concept of separability assumes that the marginal utilities are changed equally because of a change in the consumption of the k^{th} commodity. Depending on the assumptions, four types of separability can be defined.

Weak separability. This concept implies that the utility function can be divided into subgroups such that the marginal rate of substitution between two commodities i and j from the same group (g) is independent of the quantities of commodities not belonging to group g . That is

$$\frac{\partial U_i}{\partial U_j} = 0 \quad (54)$$

for all $i, j \in$ group g and $k \notin$ group g . Goldman and Uzawa (1964) have shown that the utility function $U(q_1 \cdots q_n)$ is weakly separable with respect to a grouping of the n commodities into a class of s mutually exclusive and exhaustive subgroups (with n_1, n_2, \dots, n_s commodities in each of the s groups respectively) if, and only if, the utility function is of the non-additive form

$$U(q_1 \cdots q_n) = F[U^1(q^1), U^2(q^2), \dots, U^s(q^s)] \quad (55)$$

where

$$U^i(q^i) = U^i(q_1^i, q_2^i, \dots, q_{n_i}^i); \quad i = 1, 2, \dots, s; \text{ and} \\ n_1 + n_2 + \dots + n_s = N.$$

Weak homogeneous separability. Green (1964) adds one property to the weak separability concept—that each subgroup is homogeneous of degree one. In other words,

$$U(q_1^i, q_2^i, \dots, q_{n_i}^i)$$

is homogeneous of degree one for all i .

Strong separability implies that the utility function can be partitioned into subgroups such that the marginal rate of substitution between two commodities i and j from two different subsets does not depend upon the quantities of commodities that do not belong to the same groups as i and j .

$$\frac{\partial U_i}{\partial U_j} = 0 \quad (56)$$

for all $k \in$ group g ; i from group m , and j from group s .

Strong separability is applicable only to those utility functions which are additive among groups.

$$U(q_1 \cdots q_n) = F[U^1(q^1) + U^2(q^2) + \dots + U^s(q^s)]. \quad (57)$$

Earlier it was shown that strong separability was required for the establishment of the concept of a utility tree.

Pearce separability. Pearce introduced the concept of neutral association which has been later referred to as "Pearce Separability" by Goldman and Uzawa (1964). Pearce defined the existence of neutral want association between goods i and j and good k if the marginal rate of substitution between two commodities i and j from the same subset is independent of the quantities of all other commodities.

$$\frac{\partial U_i}{\partial q_k} = 0; \quad (58)$$

$i, j \in \text{group } g \text{ and } k \neq i, j.$

$$U(q_1 \cdots q_n) = U\{U^1[f_1^1(q_1^1) + \cdots + f_n^1(q_n^1)] + \cdots + U^s[f_1^s(q_1^s) + \cdots + f_n^s(q_n^s)]\}. \quad (59)$$

From the description of Pearce separability, it is evident that it includes both weak separability and strong separability.

The two-stage maximization process.—Imposition of the assumption of separability reduces the number of parameters to be estimated directly. Suppose that there are n commodities belonging to s separable groups. In the first stage, the total expenditure y has to be allotted among the s commodity groups. Let y_1, y_2, \dots, y_s represent the amounts spent on different groups ($y = y_1 + y_2 + \dots + y_s$). Therefore, the first stage is to determine y_1, y_2, \dots, y_s such that the utility is maximized. That is,

$$U(y_1, y_2, \dots, y_s) - \lambda(\sum_{i=1}^s y_i - y)$$

should be a maximum. As usual, the first order conditions are

$$\frac{\partial U}{\partial y_i} - \lambda = 0 \quad \text{and} \quad (60)$$

$$\sum_{i=1}^s y_i - y = 0 \quad (i = 1, 2, \dots, s).$$

Equations (60) presumably can be solved to obtain the group expenditures y_1, y_2, \dots, y_s . But expenditures for a particular group is a function of the group price indices P_1, P_2, \dots, P_s .

This implies that any two commodities from the same group should be in neutral want association with all other commodities in that group. Under Pearce separability, the utility function $U(q_1 \cdots q_n)$ should be separated into a form

Therefore, the solutions obtained from (60) can be written as a function of group price indices and income

$$y_i = y_i(P_1, P_2, \dots, P_s, y) \quad (61)$$

$(i = 1, 2, \dots, s).$

Thus, at the end of the first stage, the group expenditures are expressed as a function of price indices of all groups.

In the second stage,

$$U^i(q_1^i, q_2^i, \dots, q_n^i)$$

is maximized for all groups i subject to the restriction that group expenditures equal the amount determined at the first stage. That is,

$$U^i(q_1^i, q_2^i, \dots, q_n^i) + \lambda_i(\sum_{j=1}^n P_j^i q_j^i - Y_i)$$

should be maximized. Taking the first order conditions and solving for the quantities,

$$q_j^i = q_j^i(P_1^i, P_2^i, \dots, P_{n_i}^i, y_i(P_1, P_2, \dots, P_s, y)) \quad (62)$$

for all $j = 1, 2, \dots, n_i$
 $i = 1, 2, \dots, s.$

Thus all the demand equations can be specified.

Under certain conditions the two-stage maximizations of a separable utility function provide the same equilibrium solutions as direct maximization. Gorman (1959) and Green (1964, p. 22) have provided the conditions under which two-stage maximization is consistent. Accordingly, two-stage maximization of a utility function will be consistent if the weak separability conditions are satisfied together with any one of the following conditions: (1) only two groups exist, (2) strong separability exists, (3) weak homogeneity conditions are satisfied, (4) all functions U^i except one (say the first) are homogeneous and U can be written as

$$U = U[U^1, \phi(U^2, U^3, \dots, U^r)]; \text{ and}$$

(5) the functions U^i beyond m are homogeneous and U can be written as

$$U = U[U^1 + U^2 + \dots + U^m + H(U^{m+1}, \dots, U^r)].$$

When the two-stage maximization is consistent, the maximum levels of utility reached by direct and two-stage maximization should be the same. It is possible to extend the concept of two-stage maximizations to a number of stages, if necessary.

Identifying separable groups.—So far it was assumed that it will be possible to identify separable groups. In actual practice, it will not be possible to look upon the marginal utilities to determine the nature of separability; deJanvry has demonstrated the use of factor and cluster analysis to identify homogeneous groups. However, the grouping obtained through factor analysis may not be unique because the criteria for obtaining the groupings are, to some extent, based on value judgments.

Practical considerations

Attempts to bridge the gap between demand theory and empirical work have often run into difficulties because of the restrictive assumptions used in the theory and the inadequate nature of the data. More specifically, we run into difficulties regarding the concept of demand, its time dimension and aggregation problems, marketing margins, and the nature of recorded data, all of which are discussed below.

Concept of demand.—As Baumol (1965, p. 210) points out, "Demand functions, as they are defined in economic analysis are queer creatures, somewhat abstract, containing generous elements of the hypothetical and, in general, marked by an aura of unreality." Although the Walras-Pareto-Hicks concept of demand serves as a basis for defining demand in pure economic theory, it is not adapted to the measurement of demand for statistical analysis.

The relationship between price and quantity through demand functions provides answers to questions like, "What would consumers do if price were different from the present level?" In static analysis, a demand relationship is specified for a particular period of time. In practice, each time an observation is made, we get one point on a demand curve and, by the time another observation is made, the curve might have shifted because consumer tastes have changed or for other reasons influencing demand. These shifts may influence the nature of functions obtained from time-series analysis and, at times, it will be difficult to isolate the effects of such shift variables from purely economic variables such as prices and income. Sometimes this difficulty may lead to the conclusion that economic variables may not be significant factors in explain-

ing demand. An example of this type is obtained in Prest's (1949) analysis of consumption patterns in the United Kingdom for 1870-1938 where only 1 per cent of the variance of the consumption of tea and tobacco was explained by income and price. Farrell (1952) explains Prest's results in terms of irreversible demand functions. He assumes, "That a man who has been induced by a rise in income or a fall in the price of tobacco to take up smoking, or to smoke heavily, will form a habit, and will not, when prices or income returns to its former level, cut his consumption to the former level." Often irreversibility will tend to be a matter of differential lags. For example, Goodwin, *et al.* (1968) derive irreversible demand functions for beef with different lagged-response coefficients for the increasing and decreasing phases of a consumption cycle.

Aggregation.—According to Green (1964, p. 1), "Aggregation is a process whereby a part of the information available for the solution of a problem is sacrificed for the purpose of making the problem more easily manageable." Most of the demand relationships specified earlier apply to an individual consumer with a given income facing given market conditions. But, in empirical work, the behavior of a single consumer (in a perfectly competitive market) is not interesting; we study the behavior of the market which is an aggregation of all individual consumers. Therefore, we are faced with the question of deriving theories on aggregate (macro) relationships based on individual (micro) relationships. In particular, it is necessary to specify a consistent procedure for aggregation and then determine the nature of the aggregation bias involved in the procedure adopted. Green (1964, p. 1) defines aggregation to be consistent,

"... when the use of information more detailed than that contained in the aggregates would make no difference to the results of the analysis of the problem at hand." On aggregation bias, Grunfeld and Griliches (1960, p. 1) argue that aggregation may sometimes reduce the specification error and thus bring some gain in accuracy. According to them,

"... *in practice* we do not know enough about micro behavior to be able to specify micro equations perfectly. Hence empirically estimated micro relations, whether those of individual consumers or of individual producers, should not be assumed to be perfectly specified either in an economic sense or in a statistical sense. Aggregation of economic variables can, and in fact, frequently does, reduce these specification errors. Hence, aggregation does not only produce an aggregation error, but may also produce an aggregation gain."

Theil's (1954) work on linear aggregation is one of the early systematic approaches adopted. Allen (1964, Ch. 20) discusses the case of (1) aggregation over individuals, (2) aggregation over commodities, and (3) aggregation over individuals and commodities. Theil (1959, p. 14) points out that, "... broadly speaking, if linear micro relations are aggregated in terms of linear aggregates to a linear macro relation, the resulting macro parameters are weighted sums of all micro parameters." Here, we shall illustrate this problem using some simple examples.

Aggregation over individuals. Earlier it was deduced that an individual's demand for a commodity is a function of the price of the commodity, price of other commodities, and income. Assuming a linear relationship, the demand for a commodity for the j^{th} individual can be written as

$$q_{ij} = a_{ij} + \sum_i b_i p_i + c_j y_j \quad (63)$$

where

q_{ij} = quantity of i^{th} commodity demanded by j^{th} individual,
 p_i = price of i^{th} commodity, and
 y_j = income of j^{th} individual.

In a given market situation, it can be assumed that the price is the same for all individuals and that only income is variable. Under this simplifying assumption, (63) can be written as

$$q_{ij} = a_{ij} + c_j y_j. \quad (64)$$

If we specify an aggregate relation of the form

$$q_i = a_i + c y \quad (65)$$

between aggregate demand q_i and aggregate income y , the problem of aggregation is to find a consistent method of relating (64) and (65).

Summing (64) over all individuals,

$$\sum_{j=1}^n q_{ij} = \sum_j a_{ij} + \sum_j c_j y_j. \quad (66a)$$

To establish a relationship between y and individual incomes, let us define the mean value of micro slopes (marginal propensity to consume) as

$$\bar{c} = \frac{1}{n} \sum c_j. \quad (66b)$$

Now let us define the aggregate income y as the sum of individual marginal propensity to consume multiplied by individual income, divided by the mean value of the micro slopes, or

$$y = \frac{1}{\bar{c}} \sum_j c_j y_j. \quad (67)$$

$$y = \frac{n}{\sum c_j} \sum c_j y_j \quad (\text{from } 66b \text{ and } 67). \quad (68)$$

From (65) and (68), we obtain

$$q_i = a_i + \frac{c}{\bar{c}} \sum_j c_j y_j. \quad (69)$$

Comparing (66) and (69), we get a consistent aggregation if

$$q_i = \sum_{j=1}^n q_{ij} \\ a_i = \sum_j a_{ij}, \text{ and} \quad (70)$$

$$\frac{c}{\bar{c}} = 1, \text{ or } c = \bar{c} = \frac{1}{n} \sum c_j.$$

From (70), it is clear that the aggregate quantity is the sum of individual quantities, the intercept of the aggregate relationship is the sum of individual intercepts, and the aggregate marginal propensity to consume is the average of individual marginal propensities. Aggregate income is defined as a weighted average of individual incomes, the weights being proportional to the individual marginal propensities and, therefore, the aggregate income, as defined here, will differ from commodity to commodity. However, this is only one form of aggregation—other approaches are available in Allen (1964), Stone (1954), and Wold and Jureen (1964).

Aggregation over commodities. Often it will be required to aggregate individual commodities to commodity groups or it may be that a given commodity appears in different forms or varieties and it will be difficult to handle all varieties separately. In such cases, it is necessary to aggregate over commodities. To consider a simple example, suppose that there is only one individual in the market facing k varieties of a given commodity. The demand relationship, ignoring other prices for simplification, can be written as

$$q_i = a_i + b_i p_i \quad (71)$$

where

q_j = quantity of j^{th} variety demanded
and
 p_j = price of j^{th} variety.

$$\frac{\sum w_{j0}}{w_{j0}} q_j^* = a_j \frac{w_{j0}}{\sum w_{j0}} + b_j p_j^* \quad (73)$$

$$= a_j^* + b_j p_j^*$$

where

$$a_j^* = \frac{w_{j0}}{\sum w_{j0}} a_j.$$

If an aggregate relationship

$$q = a + bp \quad (74)$$

is specified between q and p , the results in (70) can be applied to (73) and (74) to arrive at the aggregates as

$$\begin{aligned} a &= \sum_{j=1}^k q_j^* \\ &= \sum_{j=1}^k \frac{w_{j0}}{\sum w_{j0}} \frac{q_j}{q_{j0}} \\ &= \frac{\sum w_{j0} q_j}{\sum w_{j0} q_{j0}}, \text{ and} \\ &= \frac{\sum p_{j0} q_j}{\sum p_{j0} q_{j0}}. \end{aligned} \quad (75)$$

Also, using (68), p can be obtained as

$$\begin{aligned} p &= \frac{k}{\sum b_j} \sum b_j p_j^* \\ &= \frac{k}{\sum b_j} \sum b_j \frac{w_{j0}}{\sum w_{j0}} \frac{p_j}{p_{j0}}, \quad (76) \\ &= \frac{k}{\sum b_j} \frac{\sum b_j q_j p_j}{\sum p_{j0} q_{j0}}. \end{aligned}$$

As before, it is possible to modify these aggregation procedures by using other assumptions.

When different varieties of the same commodity are considered, quantity and prices are in comparable units and they can be aggregated, as in the case of different individuals in the previous example. The only difference is that, instead of summing over individuals, here, summation is over varieties.

When it is necessary to add up a mixed bag of commodities, the homogeneity property is no longer valid and it is not possible to add up the quantities or prices. In this case, a standard approach is to construct index numbers of quantities and prices with respect to some basic quantities and prices (say q_{j0} and p_{j0}). With reference to the base value, (71) can be formulated as

$$\frac{q_j}{q_{j0}} = a_j + b_j \frac{p_j}{p_{j0}}. \quad (72)$$

Using a transformation of variables applicable when using Laspeyres index numbers,

$$q_j^* = \frac{w_{j0}}{\sum w_{j0}} \frac{q_j}{q_{j0}} \text{ and}$$

$$p_j^* = \frac{w_{j0}}{\sum w_{j0}} \frac{p_j}{p_{j0}}$$

where

$$w_{j0} = p_{j0} q_{j0}$$

(72) can be written as

$$\frac{\sum w_{j0}}{w_{j0}} q_j^* = a_j + b_j \frac{\sum w_{j0}}{w_{j0}} q_j^*.$$

Dividing by

Aggregation over individuals and commodities. In this case, the simple treatment presented in the previous two cases is no longer applicable. The presence of cross effects make it difficult to obtain the aggregate relationship as a sum of individual relationships. Allen (1964, pp. 714-15) shows that it is not permissible to write the extended Keynesian consumption function of the form $q = ay + bp + c$ as the result of simple aggregation over individuals and commodities from a similar micro form. He suggests that "the alternatives open are to ignore micro theory, taking the consumption function ($q = ay + bp + c$) as the basic construction or, on the other hand, to stick to micro theory and to avoid macro relations like ($q = ay + bp + c$) except as a rough approximation or as appropriate to particular circumstances in particular time periods."

Marketing margins.—The choice of price and quantities entering demand relationships is an important consideration. Though often consumers make their purchasing decisions based on prices at the retail level, the product has to go through a number of channels before it reaches the consumer. In fact, modern markets are undergoing rapid changes from a period when the consumer used to bake the bread for home consumption to now when a number of intermediaries exist between the producer and the ultimate consumer. Normally, consumer reactions at the retail level will be transmitted to the producers through the intermediaries.¹³ The prices at the farm level and at the retail level will be separated through marketing margins,

the magnitude of which will be dependent upon the nature of the product, the number of intermediaries, and other related factors. As Waugh (1964, p. 20) points out, a complete theory of demand would have to explain the factors that influence retail prices and price spreads between farmers (producers) and consumers. For the sake of exposition, the term "marketing group" is used here to denote all the intermediaries.¹⁴ In this case, it is possible to identify four types of behavioral relationships: (1) the consumer demand, (2) marketing group's supply, (3) marketing group's demand, and (4) producer supply. To provide an economic model of these behavioral relationships, we shall use the following symbols:

q^r = quantity consumed at the retail level,

q^f = quantity produced at the farm level,

p^r = price at retail level,

p^f = price at farm level,

$M = p^r - p^f$, charges realized by the marketing group, and

y = consumer income.

Ignoring all other factors influencing the behavior of different groups involved, the following implicit relations can be specified:

$$F_1(q^r, p^r, y) = 0 \quad (77)$$

(consumer demand),

$$F_2(q^r, p^r, M) = 0 \quad (78)$$

(marketing group's supply),

$$F_3(q^f, p^f, M) = 0 \quad (79)$$

(marketing group's demand),
and

¹³ Producers will use this information to modify their product. This is why modern theories of marketing management give great importance to consumers, in contrast to the old belief that "supply creates its own demand." See Kotler (1967, Ch. 1) or a discussion of this "new concept of marketing."

¹⁴ The number of stages identified explicitly will partly depend upon the purpose of the analysis (see Hildreth and Jarrett, 1955, p. 107).

$$F_4(q', p') = 0 \quad (80)$$

(producer supply—supply at farm level).

Because M is a function of p^r and p^f , it is possible to eliminate M from (78) and (79) and write these relationships as

$$F_2(q^r, p^r, p^f) = 0 \text{ and}$$

$$F_3(q^f, p^r, p^f) = 0.$$

Some of the variables defined above can be adjusted to eliminate certain variables and to arrive at what is known as derived demand relationships. Also, it is possible to make certain assumptions regarding the relationship between farm prices and retail prices and derive the relationships between elasticities at different levels as will be demonstrated later.

Nature of recorded data (demand and supply interactions).—There has been much controversy regarding the sig-

nificance of the prices shown in the records and whether this price level corresponds to the intersection of supply and demand. The experimental markets of Chamberlin (1957, pp. 226–249) and Smith (1962) have shown that the supply and demand do no more than place bounds on quantity and price. According to the Cournot-Walras theory of general equilibrium, there exists a tendency that the total quantity purchased by the consumers (market demand) is equal to the total quantity produced by entrepreneurs (market supply). Often consumption and production are not in equilibrium in the short run. The notions of supply and demand should be kept apart and, for demand analysis, quantities should refer to quantities purchased. When consumption data are not available, adjustments have to be made in production data to incorporate exports, imports, and changes in stock levels.

II. EMPIRICAL ANALYSIS OF DEMAND

Demand Interrelationships at the Retail Level

The model

Here, an attempt is made to obtain a demand interrelationship matrix for 49 major food commodities (or commodity groups) in the United States. Also, an attempt is made to obtain estimates of the linear effects of time on consumption of a number of commodities.

Relation to the Brandow study.—Brandow (1961) took 24 food items and obtained the complete structure of demand relationships in terms of direct and cross-price elasticities and income elasticities in a synthesized model. To construct the retail demand model, Brandow used the direct elasticities obtained from a number of studies and invoked most of the theoretical restric-

tions on demand coefficients. Although the estimates obtained from Brandow's study are not claimed to be precise, he has demonstrated the use of Frisch's (1959) procedure to obtain all the direct and cross-price elasticities. However, the estimates can be modified in a number of respects:

(1) Brandow derived most of his direct elasticities from "statistical estimates obtained from other studies" and, as such, they do not follow a consistent pattern of estimation procedures. Different studies may have used data belonging to different periods and obtained from various sources. Also, the estimation procedures used may vary. Thus, it is difficult to obtain consistency in dif-

ferent studies which is also reflected in the estimates.

(2) With 24 commodities or commodity groups, it is necessary to have a considerable degree of aggregation. Because wide variations exist among commodities of a group and we may be interested, for policy purposes, in a single commodity rather than in a commodity group, it is desirable to obtain a detailed breakdown of commodity groups according to individual commodities belonging to a group. Variations that exist for a given commodity as to grade, size, and other characteristics are of recognized importance, but are not considered in this study.

(3) Demand coefficients of many commodities may have changed between the postwar and the prewar period. Although inclusion of the prewar period may provide more variation in the data compared to the relatively stable consumption pattern during the postwar period, it is doubtful whether the estimates obtained from a long time series, without considering structural changes in the variables, is useful for policy analysis in a future period. At present, we have more than 20 observations of annual data for the postwar period, and thus can obtain estimates from the postwar demand structure alone.

In the present study, the coefficients were obtained using a uniform estimation procedure; the data of the 49 commodities relate to the postwar period; and estimates from other studies, especially Brandow's, were used only when data were not available for estimating the coefficients independently.

Nature of assumptions.—As mentioned, the assumptions of Frisch and Barten imply cardinal utility and those of Strotz and Pearce ordinal utility. As Hallberg (1968, pp. 378-79) states "... if the proper combination of commodi-

ties are involved, either of these propositions (neutral-want association of Pearce and want-independence of Frisch) will probably be acceptable as reasonable approximations to actual consumer behavior." Also, Pearce (1961) points out that, under certain conditions, it is possible to derive the conclusions obtained by Frisch from both neutral-want association and the want-independent assumptions. For an empirical analysis of the type considered here, identification of proper commodity groups and estimation procedures requires more attention than choosing between neutral-want association and want independence. While using the assumption of ordinally separable utility indicators (neutral-want association), it may often become necessary to estimate parameters from a nonlinear system and the methods available for their estimation may become very complicated when commodities are numerous. Therefore, for practical considerations, we had to adopt a procedure which implied elements of want independence and neutral-want association as follows:

(1) Because of their large number, we grouped the commodities into different separable commodity groups.

(2) Following the principle of two-stage maximization, the demand equation for a single commodity within a group was specified as a function of prices of all commodities within the group, prices indices for other groups, and income.

(3) It was assumed that commodities belonging to a particular group were want independent of commodities belonging to another group so we could use the procedure suggested by Frisch to obtain the cross elasticities corresponding to all commodities outside a given group. Considering the cardinality assumptions involved in want independence, we would have preferred to avoid

it so that we could deal with ordinal assumptions alone. However, the estimation problems associated with handling large numbers of commodities was such that we had to impose the assumptions of want independence for all commodities outside a given group.

Choice of commodities.—Our choice of commodities was based on two major considerations:

- (1) The commodity included should account for at least .3 per cent of the food budget, based on U. S. Bureau of Labor Statistics (1963). Since total food expenditures account for 22.79 per cent of all expenditures, included commodities should occupy at least .68 per cent of total expenditures. However, we made exceptions for onions, carrots, and sweet potatoes, which accounted for only .263, .263, and .175 per cent, respectively, because data were available for these items.
- (2) Commodities were included where data were available on prices, quantities consumed, and an income-consumption pattern. In general, time-series data on prices were taken from U. S. Bureau of Labor Statistics (1963) reports, quantities consumed from Hiemstra (1968), and cross-section data on food consumption was from 1955 and 1965 food consumption survey reports (see Appendix A.)

Among the 49 selected commodities we have five commodity groups—"other fresh fruit," "other fresh vegetables," "other canned fruits and vegetables," "bread and other cereal products," and "other beverages." These groups were included because it was difficult to obtain individual commodities within these groups satisfying the above criteria, while their combined effects were not negligible.

¹⁶ These proportions were originally derived by the Bureau of Labor Statistics. The quantity weights used in the derivation of these figures were based on the spending pattern of wage earner and clerical worker families as derived from the 1960-1961 Surveys of Consumer Expenditures.

Determination of expenditure weights.—The expenditure proportions of different items are derived from both time-series and cross-section data. Hiemstra (1968, p. 172) reports the expenditure proportions¹⁶ on major commodity groups in December of 1963 and 1966. Because these proportions included meals eaten away from home also, they were adjusted to remove this effect and then these two periods were averaged to obtain group expenditure proportions. To obtain the budget proportions for individual commodities included in this study, the group proportions were calculated as above and a breakdown, according to the expenditure proportions, was obtained from the 1965 household food-consumption survey. For example, the adjusted time-series estimate of the budget proportions for dairy products accounted for 3.8277 per cent of total expenditures. According to the 1965 food consumption survey, butter, fresh milk, evaporated milk, cheese, and ice cream accounted for 8.451, 58.873, 2.817, 17.465, and 12.394 per cent, respectively, of the total dairy products. Therefore, the budget proportions for butter, fresh milk, evaporated milk, cheese, and ice cream are obtained as .003235 (i.e., $.038277 \times .08451$), .022535 (i.e., $.038277 \times .58873$), .001078, .006685, and .004744, respectively. The expenditure proportions for all the other commodities are calculated in the same manner (table 2). To facilitate comparison with other studies, table 2 also gives the budget proportions used in Brandow's study. (Brandow used weights for 1955-1957, obtained by adjusting 1947-1949 data.)

Choice of regression equations.—The demand for a commodity, expressed in terms of all prices and income, is repre-

TABLE 2
EXPENDITURE PROPORTIONS

Commodity and group*	This study's proportions expressed as percentage of		Brandow proportions expressed as percentage of all expenditures
	Food expenditures	All expenditures	
Meat, poultry, and fish (group 1)			
Beef.....	11.555	2.8337	2.834
Veal.....	.890	.2005	.422
Pork.....	13.223	3.0149	2.194
Lamb.....	.890	.2005	.180
Chicken.....	3.818	.8703	.919
Turkey.....	.789	.1789	.229
Fish.....	2.701	.6166	.583
Total.....	33.861	7.7186	7.831
Eggs (group 2)			
Total.....	3.375	.8933	1.128
Fats and oils (groups 3, 4, and 5)			
Butter.....	1.419	.3235	.390
Lard.....	.299	.0681	.136
Shortening.....	1.375	.3133	.281
Margarine.....	.635	.1224	.188
Salad dressing.....	1.135	.2588	.320
Total.....	4.766	1.0923	1.261
Dairy products (groups 6 and 7)			
Fresh milk.....	9.887	2.2535	2.796
Evaporated milk.....	.473	.1078	.172
Cheese.....	2.923	.6685	.375
Ice cream.....	2.061	.4744	.590
Total (excluding butter).....	15.374	3.5042	3.923
Potatoes (group 8)			
White.....	1.226	.2795
Sweet.....	.175	.0399
Total.....	1.401	.3194	.469
Sweeteners, excluding noncalorics (group 9)			
Sugar.....	2.573	.5861
Corn syrup.....	1.009	.2300
Total.....	3.581	.8162	1.840
Fruit (group 10)			
Fresh.....	4.845	1.1045
Canned.....	.217	.5063
Frozen.....	.745	.1700
Dried.....	.373	.0850
Total.....	6.182	1.8648	1.859
Vegetables (group 11)			
Fresh.....	4.285	.9721
Canned.....	3.072	.6842
Frozen.....	.745	.1700
Dried.....	.373	.0850
Total.....	8.386	1.9113	2.535
Cereal and bakery products (group 12)			
Rice.....	.588	.1333
Wheat flour.....	1.039	.2399
Breakfast cereals.....	2.869	.6516
Corn meal.....	.390	.0889
Bread and other bakery products.....	9.746	2.2212
Total.....	14.672	3.3319	1.818
Beverages and soup (groups 13, 14, and 15)			
Coffee.....	2.445	.5873
Nonalcoholic beverages.....	1.963	.4475
Soups.....	1.556	.3538
Total.....	5.963	1.3886	1.031
TOTAL ALL FOOD.....	100.00	22.79	23.177†
TOTAL NONFOOD.....		77.21	76.823

* See text for discussion of commodity groups. See table 5, last row, for expenditure proportions for certain individual items in the fruit and vegetable categories.

† Includes dry beans, peas, and nuts of .392.

sented by

$$q_i = q_i(p_1, p_2, \dots, p_n, y). \quad (81)$$

Taking total differentials of (81),

$$\begin{aligned} dq_i &= \frac{\partial q_i}{\partial p_1} dp_1 + \frac{\partial q_i}{\partial p_2} dp_2 \\ &+ \dots + \frac{\partial q_i}{\partial p_n} dp_n \\ &+ \frac{\partial q_i}{\partial y} dy. \end{aligned} \quad (82)$$

Dividing (82) by q_i ,

$$\begin{aligned} \frac{1}{q_i} dq_i &= \frac{1}{q_i} \frac{\partial q_i}{\partial p_1} dp_1 + \dots \\ &+ \frac{1}{q_i} \frac{\partial q_i}{\partial p_n} dp_n \\ &+ \frac{1}{q_i} \frac{\partial q_i}{\partial y} dy. \end{aligned} \quad (83)$$

But

$$\frac{d(\log q_i)}{dq_i} = \frac{1}{q_i},$$

therefore,

$$\begin{aligned} \frac{dq_i}{q_i} &= d(\log q_i), \\ \frac{1}{q_i} \frac{\partial q_i}{\partial p_j} dp_j &= \frac{p_j}{q_i} \frac{\partial q_i}{\partial p_j} \frac{dp_j}{p_j} \\ &= e_{ij} d(\log p_j). \end{aligned}$$

Similarly,

$$\frac{1}{q_i} \frac{\partial q_i}{\partial y} dy = e_{iy} d(\log y).$$

Substituting these results in (83), we have

$$\begin{aligned} d(\log q_i) &= e_{i1} d(\log p_1) + \dots \\ &+ e_{in} d(\log p_n) \\ &+ e_{iy} d(\log y). \end{aligned} \quad (84)$$

When we take only finite differences, the differentials in (84) can be replaced by first differences and, therefore, equation (84) can be written as

$$\begin{aligned} \Delta \log q_i &= e_{i1} \Delta \log p_1 + \dots \\ &+ e_{in} \Delta \log p_n \\ &+ e_{iy} \Delta \log y. \end{aligned} \quad (85)$$

While using time-series data,

$$\Delta \log q_i = (\log q_{i,t} - \log q_{i,t-1})$$

whereas, using equation (85) for cross-section data, price changes vanish and, therefore,

$$\Delta \log q_i = e_{iy} \Delta \log y. \quad (86)$$

The present study makes extensive use of equations of type (85) and (86). The use of first differences may reduce the effect of serial correlation if the serial correlation coefficient is approximately one. However, in some cases, a double logarithmic function gave statistically better estimates than the first difference equation and, in such cases, we have selected the estimates with better statistical properties.

Grouping of commodities into separable groups.—To obtain a regression equation similar to (85), it is necessary to restrict the number of commodities to be included in any given equation. Grouping of commodities according to certain characteristics permits us to obtain a consistent estimate through two-

stage maximization. From the previous section, we have a demand equation for the j^{th} commodity belonging to group i , expressed as

$$q_j^i = q_j^i(p_1^i, p_2^i, \dots, p_n^i, y_i(p_1, p_2, \dots, p_n, y)) \quad (87)$$

for

$$j = 1, 2, \dots, n_i; i = 1, 2, \dots, s$$

where

$p_1^i, p_2^i, \dots, p_n^i$ represent the individual commodity prices,

P_1, P_2, \dots, P_s represent group price index, and

y_i is the expenditure on the i^{th} group.

Equation (87) can also be expressed as

$$q_j^i = q_j^i(p_1^i, p_2^i, \dots, p_n^i, P_1, P_2, \dots, P_s, y) \quad (88)$$

To use this two-stage maximization procedure, we had to classify our 49 commodities into a number of separable groups. Instead of depending on arbitrary groupings, we adopted a grouping developed by deJanvry (1966, p. 112). DeJanvry took the price elasticities, income elasticities, and budget shares used in Brandow's study and developed the proportionality factors¹⁶ represented by

$$\theta_{ij} = \frac{y}{e_{iw}} \frac{e_{iw}}{w_j e_{iw}} + 1$$

Using the proportionality factors, separable groups were identified such that

the between group θ_{ik} 's are maximally equal for all $i \in I$ and $k \in K$. Eventually, the following 15 groups were established:

1. Beef, veal, pork, lamb, chicken, turkey, fish
2. Eggs
3. Butter
4. Lard
5. Shortening, margarine, salad dressing and cooking oil
6. Milk, evaporated milk
7. Cheese, ice cream
8. Potatoes, sweet potatoes
9. Sugar, corn syrup
10. Apples, bananas, oranges, other fresh fruits, canned peaches, canned pineapple, frozen fruits
11. Lettuce, tomatoes, beans, onions, carrots, other fresh vegetables, canned peas, canned corn, canned tomatoes, dry vegetables, frozen vegetables, other canned fruits, and vegetables
12. Rice, wheat flour, breakfast cereals, corn meal, bread, and other cereals
13. Coffee
14. Soup
15. Other beverages

The list of all 49 commodities are contained in table 5 on page 46.

Estimation of the demand matrix

Having grouped the 49 commodities into 15 groups, a demand equation similar to (88) was used for each commodity in a given group. The functional form of the demand equations, in many cases, was similar to (85) so that we were using the first difference of variables. In many cases, the coefficients of the group prices turned out to be highly insignificant and, therefore, in actual estimation, we have omitted a number of price indices. Also,

¹⁶ DeJanvry used the proportionality factors to arrive at groupings based on factor analysis. "The criteria for the allocation of variables to composites will be that these reproduce for each variable its intercorrelations with the other variables. This corresponds to the building of cluster of variables that have maximal correlations in terse and maximally proportional correlations with outside variables." (1966, p. 57)

both annual and quarterly data were used, where available, to estimate the demand coefficients. Thus, for many commodities, there were a number of equations using both annual and quarterly data and, also, using logarithmic form and the first difference of logarithms of variables. Demand equations were specified with quantities as dependent variables and prices of all commodities belonging to the same group, together with price indices of other groups and income, as the independent variables. Therefore, it was possible to obtain the direct price elasticity and cross-price elasticities with respect to all the commodities in the same group from a given demand equation.

A number of equations existed for the same commodity, and one coefficient had to be chosen. In most cases, the criteria for selection of coefficients were based on the statistical properties such as the fit of the equation (as determined by the multiple correlation coefficient), the significance of the individual coefficients (determined by the *t*-statistic), and the existence of serial correlation (determined by the Durbin-Watson test statistic). The theoretically anticipated sign of the regression coefficient was also considered when we had estimates satis-

fying more or less the same statistical properties. All the estimates of direct and cross-price elasticities for commodities belonging to the same group are obtained in this manner. However, the cross-elasticity coefficients were adjusted to satisfy the symmetry condition. Quantity and price data on five commodities (turkey, fish, frozen fruits, frozen vegetables, and bread) were not available on a comparable basis. Therefore, we used estimates obtained from other sources for the direct price elasticities of these commodities.¹⁷

Synthesis of demand interrelationships.—We have seen how the direct and cross-price elasticities for all commodities belonging to each group are determined. The section on income and consumption shows how various measures of income elasticities are obtained. Using these figures and the restrictions imposed upon an individual's demand coefficients by the theory of consumer behavior, we can calculate all the other remaining coefficients. All nonfood items are treated in one category. The demand interrelationships are represented in table 3.

We can summarize the restrictions on demand coefficients specified in table 1 as follows:

(a) Row Restraint (Homogeneity Condition)

$$e_{i1} + e_{i2} + \dots + e_{i49} + e_{i50} + e_{iY} = 0 \quad (i = 1, 2, \dots, 50) \quad (89)$$

(b) Symmetry

$$e_{ij} = w_{ij} e_{ji} - w_j(e_{ii} - e_{jY}) \quad (i, j = 1, 2, \dots, 50) \quad (90)$$

(c) Cournot Aggregation

$$w_1 e_{1j} + w_2 e_{2j} + \dots + w_{49} e_{49j} + w_{50} e_{50j} = -w_j \quad (j = 1, 2, \dots, 50) \quad (91)$$

(d) Engel Aggregation

$$w_1 e_{1Y} + w_2 e_{2Y} + \dots + w_{49} e_{49Y} + w_{50} e_{50Y} = 1 \quad (92)$$

¹⁷ Brandow's estimates were used for turkey, fish, and bread. The direct price elasticity estimate for frozen vegetables was based on an unpublished study of Ben C. French, Department of Agricultural Economics at Davis.

TABLE 3
DEMAND INTERRELATIONSHIP MATRIX

	Commodities				All food	Nonfood	Income
	1,	2,	...	49			
1.....	e_{11}	e_{12}	...	$e_{1,49}$	e_{1f}	$e_{1,40}$	$e_{1,y}$
2.....	e_{21}	e_{22}	...	$e_{2,49}$	e_{2f}	$e_{2,40}$	$e_{2,y}$
.....
.....
49.....	$e_{49,1}$	$e_{49,2}$...	$e_{49,49}$	e_{49f}	$e_{49,40}$	$e_{49,y}$
All food.....	$e_{f,1}$	$e_{f,2}$...	$e_{f,49}$	e_{ff}	$e_{f,40}$	$e_{f,y}$
Nonfood.....	$e_{40,1}$	$e_{40,2}$...	$e_{40,49}$	e_{40f}	$e_{40,40}$	$e_{40,y}$
Expenditure weight	w_1	w_2	...	w_{49}	w_f	w_{40}	1

NOTE: Subscripts 1-49 denote the 49 food items, "40" denotes nonfood, "f" denotes all food, and "y" denotes income.

(e) Frisch Equations

$$e_{ij} = -\frac{1}{\phi} e_{iv} e_{jv} w_j - e_{iv} w_j \text{ for } i \neq j, \text{ and} \quad (93)$$

$$e_{ii} = -e_{iv} w_i - \frac{1 - w_i e_{iv}}{\phi} \quad (94)$$

These restrictions were used in the following manner to obtain all the elements in the demand interrelationship matrix.

(a) Our starting point is the estimates of direct-price elasticity (e_{ii}), income elasticity (e_{iv}), and expenditure proportions (w_i) for all commodities. We have already explained the estimation procedure adopted in obtaining all these elements. Also, we have seen that the cross elasticities for commodities belonging to the same food group basically are obtained through direct estimation process. The particular coefficients used in this analysis were chosen from a large number of estimates, using certain desirable properties of the estimates and some element of intuitive judgment.

(b) Income elasticity for all food (e_{fv}) is obtained as a weighted average of the

income elasticities for individual commodities, the weights being the expenditure proportions, or

$$e_{fv} = \frac{w_1 e_{1v} + w_2 e_{2v} + \dots + w_{49} e_{49v}}{w_1 + w_2 + \dots + w_{49}} \quad (95)$$

$$= \frac{w_1 e_{1v} + w_2 e_{2v} + \dots + w_{49} e_{49v}}{w_f}$$

because the expenditure weight of all food is the sum of expenditure weights for individual food items. The elasticity obtained in this manner was .176265. Although this estimate is lower than Brandow's (.25667), it compares favorably with independent estimates of income elasticities for all food in other studies. For example, Hiemstra (1968, p. 29) obtained the following equations by fitting annual data for the 1948-1966

period through the use of the single equation least-square model.

$$\begin{aligned}
 x_1 &= 2.227 - 0.182 x_2 && (.093) \\
 &+ 0.174 x_3 - 0.104 x_4 && (.060) \quad (.125) \\
 &&& (R^2 = .733)
 \end{aligned}
 \tag{96}$$

where

- x_1 = log per capita food consumption index,
 x_2 = log retail food price index,
 x_3 = log per capita disposable personal income index, and
 x_4 = log consumer price index (all items less food). All the indices are with reference to the base period, 1957-1959 = 100.

For the period 1948-1962, Waugh (1964, p. 16) obtained a regression equation

$$x_1 = 2.19 - 0.24 x_2^* + 0.14 x_3^* \tag{97}$$

(.15) (.50)

where the variables are as defined above except that x_2 and x_3 are deflated by the consumer price index to obtain x_2^* and x_3^* , respectively. Thus, the estimate, .176265 (obtained in this study) can be compared with .174 in (96) and .14 in (97). However, income elasticity obtained from 1965 cross-section data using expenditure as the dependent variable was .277111 (see table 13). The estimate of .176265 corresponds to quantities and, therefore, it is consistent with the general notion that income elasticity with expenditure as the dependent variable is higher than the corresponding estimate with quantity as the dependent variable.

(c) Income elasticity of nonfood (e_{soy}). From equation (92), we know that the weighted sum of all the income elasticities is unity. We have already estimated all the expenditure weights and the income elasticities for commodities other than nonfood. Therefore, in equation (92), only e_{soy} is unknown.

$$e_{soy} = \frac{1 - (w_1 e_{1y} + w_2 e_{2y} + \dots + w_n e_{ny})}{w_{so}} \tag{98}$$

Using equation (95), this can be written as

$$e_{soy} = \frac{1 - w_f e_{fy}}{w_{so}} \tag{99}$$

Using (99), the income elasticity of nonfood is estimated as

$$\frac{1 - (.227918)(.176265)}{.772082} = 1.243165.$$

This estimate is fairly consistent with Brandow's estimate of 1.22426.

(d) Direct elasticity for all foods (e_{ff}). Using (94), we can write e_{ff} as

$$e_{ff} = -e_{fy} w_f - \frac{1 - w_f e_{fy}}{\phi} \tag{100}$$

On the right-hand side of (100), all the terms except the value of the money flexibility coefficient (ϕ) are known. Since the value for equation (99) approximates that of Brandow, it seems to be appropriate to use Brandow's estimate of ϕ at $-.86$. Thus, from equation (100), the value of e_{ff} is $-.236845$, which compares favorably with results obtained independently. Using the results obtained by Girshick and Haavelmo (1947), Buchholz, *et al.*, (1962) have calculated the elasticity of food consumption with respect to retail prices to be $-.25$. In (97), Waugh had obtained the value of e_{ff} as $-.24$. Considering the rounding of decimals in Waugh's results, the figure

obtained from our estimation procedure is consistent and we are justified in taking the value of ϕ at $-.86$. Brandow (1961, p. 18), himself, agrees that his estimate of $-.34137$ "appears on the basis of time-series analysis to be slightly too high."

(e) Direct elasticity for nonfood (e_{f50}) is also obtained in a procedure similar to that of direct elasticity of all food.

$$\begin{aligned} e_{f50} &= -e_{50y}w_{50} - \frac{1 - w_{50}e_{50y}}{\phi} \\ &= -1.243165 \times .818796 \\ &= -1.017898. \end{aligned}$$

This estimate is consistent with -1.02556 from Brandow's study.

(f) Cross elasticity of all food with re-

spect to nonfood prices (e_{f50}) is obtained using the homogeneity restraint that

$$e_{f50} + e_{ff} + e_{fv} = 0. \quad (101)$$

From steps (b) and (d), we obtained e_{fv} and e_{ff} as $.176265$ and $-.236845$, respectively. Therefore, from equation (101),

$$e_{f50} - .236845 + .176265 = 0.$$

Thus, we obtain an estimate of e_{f50} as $.060580$, which is considerably lower than Brandow's corresponding figure ($.08470$). However, our estimate is greater than at least one independent estimate with the coefficient, $.052$, reported in Brandow (1961, p. 18).

(g) Cross elasticity of nonfood with respect to food prices (e_{50f}). Using the symmetry relations in equation (90)

$$\begin{aligned} e_{50f} &= \frac{w_f}{w_{50}} e_{f50} - w_f(e_{50y} - e_{fv}) \\ &= \frac{.227918}{.772082} (.060580) - .227918 (1.243165 - .176265) \\ &= -.225266. \end{aligned}$$

(h) Cross elasticities showing the effects of nonfood prices on consumption of individual foods (e_{i50}) for $i = 1, 2, \dots, 49$. From equation (93) we have

$$\begin{aligned} e_{i50} &= -\frac{1}{\phi} e_{iy}e_{jv}w_j - e_{iv}w_j \\ &= -e_{iv}w_j \left(1 + \frac{e_{50y}}{\phi} \right). \end{aligned}$$

Assuming j to be nonfoods, we have

$$e_{i50} = -e_{iv}w_{50} \left(1 + \frac{e_{50y}}{\phi} \right).$$

Therefore,

$$\frac{e_{i50}}{e_{iv}} = -w_{50} \left(1 + \frac{e_{50y}}{\phi} \right). \quad (102)$$

Here, it is implied that each food is want independent of nonfood items. The right-hand side of (102) is independent of i , which implies that the ratio of cross elasticities showing the effect of nonfood prices on consumption of an individual food item and the income elasticity of the same food do not vary from commodity to commodity. In particular, if we consider the total food, using the results from (f) and (b), the ratio e_{f50}/e_{fv} becomes $.060580/.176265 = .343687$. We have assumed that the com-

mon ratio e_{i50}/e_{iy} , for all i to be the same as e_{j50}/e_{jy} . Therefore,

$$e_{i50} = .343687 e_{iy} \quad (103)$$

$$(i = 1, 2, \dots, 49).$$

Thus, the cross elasticities showing the effects of nonfood prices on the consumption of individual foods is 34.3687 per cent of the corresponding income elasticities. Brandow obtained the corresponding percentage as 33.

(i) Cross elasticities showing the effects of individual food prices on non-food quantity (e_{50i}) for $i = 1, 2, \dots, 49$. Since we have obtained e_{i50} from (h), we can use the symmetry relationship in (10) to obtain e_{50i} as

$$e_{50i} = \frac{w_i}{w_{50}} e_{i50} - w_i(e_{50y} - e_{iy}) \quad (104)$$

for $i = 1, 2, \dots, 49$. From (a), (c), and (h), all the coefficients on the right-hand side of (104) are known and, hence, the values of e_{50i} can be calculated.

(j) Cross elasticities showing the effect of all food prices on the consumption of individual foods (e_{fi}), $i = 1, 2, \dots, 49$.

$$e_{fi} = \frac{w_1 e_{1i} + w_2 e_{2i} + \dots + w_{49} e_{49i}}{w_1 + w_2 + \dots + w_{49}},$$

$$w_1 e_{1i} + w_2 e_{2i} + \dots + w_{49} e_{49i} = (w_1 + w_2 + \dots + w_{49}) e_{fi} \quad (107)$$

$$= w_f e_{fi}.$$

Substituting (107) in (106)

$$e_{fi} w_f + e_{50i} w_{50} = -w_i, \quad (108)$$

$$e_{fi} = -\frac{w_i + w_{50} e_{50i}}{w_f}.$$

From (a) and (i), all the terms on the right-hand side of (108) are known.

From the homogeneity restraint (89), we have $e_{i1} + e_{i2} + \dots + e_{i49} + e_{i50} + e_{iy} = 0$. But the sum, $e_{i1} + e_{i2} + \dots + e_{i49}$ represents e_{if} . Therefore,

$$e_{if} + e_{i50} + e_{iy} = 0 \quad (i = 1, 2, \dots, 49)$$

or

$$e_{if} = -(e_{iy} + e_{i50}). \quad (105)$$

Both terms on the right-hand side are known from (a) and (h). Therefore, the cross elasticities showing the effect of all food prices on the consumption of a given commodity is obtained as the negative of the sum of the income elasticity and the cross elasticity showing the effect of nonfood prices on the consumption of that commodity.

(k) Cross elasticity showing the effect of individual commodity prices on food consumption (e_{fi}), $i = 1, 2, \dots, 49$. From the Cournot aggregation in (91), we have

$$w_1 e_{1i} + w_2 e_{2i} + \dots + w_{49} e_{49i} + w_{50} e_{50i} = -w_i. \quad (106)$$

If we define e_{fi} as

(l) Cross elasticity showing the effect of individual commodity prices on the commodities outside the group (e_{ij}) $i \neq j$, $i \in I$, and $j \notin I$. Suppose that we are considering the i^{th} commodity. We have already estimated the direct elasticity, cross elasticities with respect to commodities in the same group, and income elasticity, using (a). The nonfood cross

elasticity is obtained from (h). Thus, using the homogeneity restraint, we can obtain the sum of the remaining unknown coefficients in the same row. For convenience, if we assume that the first k commodities belong to a group, for any commodity belonging to that group, e_{i1} , e_{i2} , \dots , e_{ik} , e_{i50} , and e_{iy} are known. Since we have

$$(e_{i1} + e_{i2} + \dots + e_{ik}) + e_{ik+1} + \dots + e_{i49} + e_{i50} + e_{iy} = 0,$$

the sum of the cross elasticities corresponding to the commodities outside the group is $-(e_{i1} + \dots + e_{ik} + e_{i50} + e_{iy})$, which is the negative of the sum of all known coefficients. Let us denote this sum by R_i . Having known the value of R_i , our next job is to see how this value is distributed over the individual commodities. In (93), we have seen that the cross elasticities can be obtained as

$$e_{ij} = -e_{iy}w_j \left(1 + \frac{e_{iy}}{\phi} \right).$$

Since all the values on the right-hand side are known, we can calculate the remaining $(49-k)$ cross elasticities showing the effects of their prices on the consumption of the i^{th} commodity. It is possible that the sum of the coefficients obtained in this manner will not add to R_i . So we will have to adjust the individual coefficients such that their sum adds to R_i . Thus, the cross elasticities are taken only as proportional to $-e_{iy}w_j(1 + e_{iy}/\phi)$, without assuming strict equality. By the time we complete this step, all the coefficients in the i^{th} row are known.

In practice, we start with the first row and obtain all the coefficients in that row, then, using these figures and the symmetry relationship, we can calculate the remaining elements in the first

column. Now, we complete the second row and obtain the remaining elements of the second column, using symmetry relationship. Thus, continuing a row and column operation, we can obtain all the unknown coefficients in the demand interrelationship matrix (table 5). The steps taken in obtaining the demand interrelationship matrix are given in table 4.

Linear effects of time on consumption

The specification of demand functions in terms of prices and income excludes any possible continuous systematic variations in the demand as a result of factors other than those specified. Often, in demand analysis, such variations are handled by incorporating time as a variable with linear or nonlinear effects. If the purpose of the analysis is only for making projections, inclusion of time may result in more reliable projections than when time is excluded. However, it may be possible that variables such as income and prices are highly correlated with time so that introduction of time as a variable may result in estimation problems. An estimate of the coefficient associated with time may often be an overestimate because most unexplained variations might be absorbed by this coefficient in addition to the effect of time. Further, it will be hard to provide an interpretation for the coefficient associated with time. Considering these factors, we have avoided the practice of using time as a variable in our demand equations.

Use of the first difference of the variables in the demand equations provides an opportunity to test the presence of any linear effect of time. If we add a constant to the demand equation in (85),

TABLE 4
SYNTHESIS OF DEMAND INTERRELATIONSHIPS
(SUMMARY)

Commodity		Commodity group				All food	Nonfood	Income
Group	Item	I,	II, ...	XV				
I	1	1, 2, ..., n ₁	1, 2, ..., n ₂	1, 2, ..., n ₁₅	(j)	(b)	(a)	
	2	(a)	(c) (c)	(c)	(j)	(b)	(a)	
	.							
	n ₁							
II	1		(a) (c) (c)		(j)	(b)	(a)	
	2	(f)						
	.							
	n ₂							
.		(c)	(c) (a) (b)		(j)	(b)	(a)	
XV	1		(c) (c) (a)		(j)	(b)	(a)	
	2	(c)						
	.							
	n ₁₅							
All food.....		(k)	(k) (k) (k)		(d)	(f)	(b)	
Nonfood.....				(i)	(g)	(e)	(c)	
Expenditure weight.....		(a)	(a) (a) (a)		(a)	(a)	(1)	

NOTE: The letters in the boxes represent the steps mentioned in the text.

specified in terms of the first difference of the variables, it can be written as

$$\Delta \log q_i = a_i + e_{i1} \Delta \log p_1 + e_{i2} \Delta \log p_2 + \dots + e_{in} \Delta \log p_n + e_{iV} \Delta \log y. \quad (109)$$

When the independent variables (prices and income) do not change over the years, (109) reduces to $\Delta \log q_i = a_i$. If the value of a_i is significantly different from zero, it implies that some change in consumption will take place from the preceding year even if there was no change in prices and income. Thus, when first differences of variables are used in

the demand equation, the constant term in the regression equation represents the linear effect of time. We have used first difference equations similar to (109) for 39 commodities and the constant term was significantly different from zero at the 5 per cent level for ten commodities—eggs, butter, lard, evaporated milk, ice cream, sweet potatoes, beans, canned peas, wheat flour, and soup. At the 10 per cent level, the coefficients of six more commodities—cheese, canned peaches, dried fruit, carrots, corn meal, and coffee—became significantly different from zero (see Appendix table A-6).

Foote (1958, p. 43) suggests that the percentage effect of time in each year can be obtained by taking the anti-

TABLE
DEMAND INTERRELATIONSHIPS

Column	Commodities	Beef 1	Veal 2	Pork 3	Lamb & mutton 4	Chicken 5	Turkey 6	Fish 7	Eggs 8
Row									
1	Beef.....	-0.643800	0.027953	0.082609	0.045433	0.067560	0.007828	0.003128	0.001338
2	Veal.....	0.359295	-1.717737	0.197665	0.096017	0.173394	0.013810	0.003210	0.007393
3	Pork.....	0.076284	0.014062	-0.413013	0.080183	0.036251	0.005037	0.004830	0.002556
4	Lamb and mutton.....	0.589451	0.066057	0.891362	-2.825500	0.233588	0.018092	0.002810	0.003085
5	Chicken.....	0.197067	0.048577	0.120766	0.054801	-0.777330	0.063745	0.003722	0.002383
6	Turkey.....	0.097580	0.014702	0.065260	0.017900	0.400000	-1.558329	0.001078	0.002145
7	Fish.....	0.020910	0.002203	0.026578	0.002052	0.007780	0.001690	-0.230000	0.010679
8	Eggs.....	0.010179	0.002748	0.011093	0.001940	0.003235	0.001720	0.007339	-0.318280
9	Butter.....	0.001944	0.001671	0.000304	0.001116	0.000315	0.001100	0.003195	0.007614
10	Lard.....	0.013436	0.003164	0.015369	0.002264	0.004603	0.001972	0.008338	0.018530
11	Shortening.....	0.010978	0.002849	0.012144	0.002019	0.003605	0.001809	0.007729	0.014990
12	Margarine.....	0.011898	0.002966	0.013319	0.002110	0.003935	0.001858	0.008184	0.016220
13	Salad dressing.....	0.003491	0.002014	0.002780	0.001332	0.000948	0.002229	0.004921	0.010354
14	Fresh milk.....	0.006400	0.002138	0.004993	0.001474	0.001604	0.001260	0.005007	0.010714
15	Evaporated milk.....	0.011784	0.002915	0.013001	0.002083	0.003867	0.001843	0.007045	0.016783
16	Cheese.....	0.004107	0.001954	0.003150	0.001232	0.001085	0.001272	0.004295	0.009479
17	Ice cream.....	0.001525	0.001617	-0.000239	0.001073	0.000138	0.001078	0.002995	0.007231
18	Potatoes.....	0.008248	0.002492	0.012071	0.001748	0.002603	0.001198	0.006372	0.013035
19	Sweet potatoes.....	0.008064	0.001364	0.004961	0.001242	0.001793	0.001430	0.000847	0.010358
20	Sugar.....	0.010892	0.002838	0.012026	0.002011	0.003872	0.001782	0.007696	0.015377
21	Corn syrup.....	0.003489	0.002260	0.006218	0.001867	0.001945	0.001448	0.005475	0.011826
22	Apples.....	0.007822	0.002390	0.007607	0.001669	0.002234	0.001529	0.007669	0.012453
23	Bananas.....	0.007528	0.002368	0.007610	0.001673	0.002339	0.001529	0.008997	0.012428
24	Oranges.....	0.003504	0.001804	0.002131	0.001241	0.000816	0.001213	0.003628	0.008277
25	Other fresh fruit.....	0.000622	0.001337	-0.003058	0.000860	-0.000649	0.000917	0.001929	0.005376
26	Canned peaches.....	0.001230	0.001691	-0.000626	0.001087	0.000033	0.000950	0.002350	0.006970
27	Canned pineapples.....	-0.002098	0.001141	-0.000017	0.000710	-0.001198	0.000800	0.001194	0.004104
28	Dried fruits.....	0.001998	0.001693	0.000407	0.001124	0.000233	0.001116	0.003255	0.007676
29	Frozen fruits.....	-0.008793	0.000271	-0.013764	0.000042	-0.003651	0.000903	-0.002146	-0.001689
30	Lettuce.....	0.007286	0.001338	0.007312	0.001650	0.002283	0.001514	0.005675	0.012220
31	Tomatoes.....	0.008585	0.002321	0.008372	0.001579	0.001999	0.001456	0.005526	0.011613
32	Beans.....	0.011887	0.002830	0.013244	0.002114	0.003947	0.001857	0.008176	0.018217
33	Onions.....	0.011777	0.002993	0.013143	0.002066	0.003892	0.001845	0.008178	0.015292
34	Carrots.....	0.001906	0.001630	0.000234	0.001111	0.000268	0.001108	0.003197	0.007688
35	Other fresh vegetables.....	0.007903	0.002357	0.007190	0.001642	0.002221	0.001504	0.005648	0.013174
36	Canned peas.....	0.010902	0.002653	0.012042	0.002013	0.003870	0.001782	0.007709	0.018409
37	Canned corn.....	0.011181	0.002968	0.012358	0.002036	0.003684	0.001800	0.007812	0.015890
38	Canned to- matoes.....	0.006445	0.002258	0.006208	0.001566	0.001946	0.001447	0.005423	0.011444
39	Dry vege- tables.....	0.005114	0.001923	0.004436	0.001432	0.001445	0.001346	0.004809	0.010374
40	Frozen vege- tables.....	-0.007383	0.000390	-0.011909	0.000185	-0.003128	0.000408	-0.001450	-0.000499
41	Other canned fruits and vegetables.....	0.006639	0.002193	0.005143	0.001462	0.001847	0.001387	0.005097	0.010818
42	Rice.....	0.010145	0.002741	0.011069	0.001938	0.003301	0.001727	0.006487	0.014585
43	Wheat flour.....	0.009293	0.002631	0.009942	0.001862	0.002987	0.001662	0.006890	0.013979
44	Breakfast cereals.....	0.010069	0.002723	0.010974	0.001931	0.003280	0.001721	0.007289	0.014372
45	Corn meal.....	0.010663	0.002726	0.007379	0.001934	0.003282	0.001722	0.007279	0.014359
46	Bread and other cereals.....	0.011694	0.002969	0.013339	0.002111	0.006941	0.001867	0.008198	0.016340
47	Coffee.....	0.010417	0.002778	0.011380	0.001964	0.003402	0.001746	0.007456	0.014963
48	Soup.....	0.004429	0.003006	0.003655	0.001372	0.001230	0.001301	0.004494	0.006641
49	Other bever- ages.....	0.004697	0.002030	0.008914	0.001391	0.001200	0.001316	0.004590	0.009992
50	All food.....	-0.042031	-0.006135	-0.025047	-0.006998	-0.009140	-0.006247	-0.001207	-0.003931
51	Nonfood.....	-0.021707	-0.000780	-0.031655	-0.006838	-0.008374	-0.000496	-0.007617	-0.010281
52	Expenditure propor- tions.....	0.026337	0.002005	0.030149	0.002005	0.008703	0.001789	0.006186	0.006833

AT THE RETAIL LEVEL

Butter 9	Lard 10	Shortening 11	Margarine 12	Salad dressing 13	Fresh milk 14	Evapora- ted milk 15	Cheese 16	Ice cream 17	Potatoes 18
0.00083	0.000118	0.000489	0.000198	0.000329	0.002782	0.000189	0.000768	0.000471	0.000301
0.001813	0.000638	0.002891	0.001089	0.001807	0.015300	0.000930	0.004226	0.002298	0.002148
0.000630	0.000222	0.000635	0.000378	0.000828	0.008315	0.000323	0.001469	0.000901	0.000746
0.000982	0.000346	0.001457	0.000590	0.000979	0.008286	0.000604	0.002288	0.001404	0.001163
0.000560	0.000187	0.000831	0.000336	0.000555	0.004723	0.000287	0.001304	0.000800	0.000663
0.000538	0.000189	0.000835	0.000338	0.000556	0.004587	0.000278	0.001252	0.000769	0.000637
0.002708	0.000982	0.004012	0.001625	0.002699	0.023226	0.001387	0.006301	0.003364	0.003206
0.003460	0.001279	0.005394	0.002184	0.003429	0.006088	0.001854	0.008471	0.005198	0.004313
-0.562438	0.000302	0.001271	0.000500	0.000855	0.007233	0.000429	0.001997	0.001235	0.001017
0.002825	-0.400000	0.280000	0.100479	0.000832	0.002813	0.000171	0.000776	0.000476	0.000398
0.002847	0.000333	-1.015830	0.002385	0.170090	0.102849	0.006415	0.028374	0.017878	0.014378
0.424534	0.058758	0.133704	-0.846301	0.115826	0.009049	0.000667	0.002690	0.001884	0.001271
0.009149	-0.000241	0.208188	0.054820	-0.694401	0.002451	0.000156	0.000650	0.000417	0.000346
0.001409	-0.000088	0.013882	0.000243	0.000498	-0.345545	0.010047	0.000673	0.000413	0.000342
0.000347	0.000074	0.018735	0.000645	0.001109	0.214888	-0.318906	0.001236	0.000759	0.000628
0.001190	-0.000124	0.012862	0.000183	0.000355	0.001250	-0.000089	-0.460142	0.005229	0.002327
0.000798	-0.000181	0.010314	-0.000010	0.000107	-0.000615	-0.000135	0.002235	-0.527608	0.001494
0.001829	-0.000009	0.015842	0.000414	0.000758	0.004718	0.000116	0.006473	0.003635	-0.308563
0.002399	0.000079	0.018740	0.000645	0.001107	0.007898	0.000234	0.009273	0.004485	0.420164
0.002226	0.000081	0.017938	0.000510	0.001011	0.009327	0.000235	0.007674	0.004226	0.000896
0.001358	-0.000064	0.014423	0.000308	0.000584	0.003221	0.000036	0.006682	0.003073	0.000138
0.001697	-0.000036	0.015263	0.000368	0.000585	0.004107	0.000084	0.006144	0.003349	0.000246
0.001715	-0.000036	0.015394	0.000370	0.000588	0.004127	0.000085	0.006155	0.003354	0.000249
0.001088	-0.000140	0.012274	0.000130	0.000322	0.000947	-0.000066	0.004431	0.002356	-0.000142
0.000462	-0.000247	0.008813	-0.000146	0.000169	-0.002717	-0.000281	0.002450	0.001234	-0.000662
0.000747	-0.000180	0.010381	-0.000098	0.000080	-0.001162	-0.000199	0.003291	0.001714	-0.000401
0.000226	-0.000285	0.007644	-0.000222	-0.000242	0.005113	-0.000348	0.001777	0.000846	-0.000744
0.000899	-0.000178	0.010908	0.000309	0.000156	-0.000500	-0.000162	0.002654	0.001923	-0.000318
-0.000798	-0.000414	0.002330	-0.000662	-0.000688	-0.006374	-0.000642	-0.001264	-0.000894	-0.001427
0.001878	-0.000042	0.016089	0.000356	0.000604	0.008970	0.000074	0.008091	0.003290	0.000228
0.001859	-0.000061	0.014809	0.000308	0.000566	0.003310	0.000041	0.008711	0.003100	0.000322
0.002385	0.000077	0.018736	0.000680	0.001110	0.007776	0.000381	0.008181	0.004437	0.000628
0.002375	0.000074	0.018628	0.000687	0.001097	0.007643	0.000272	0.008072	0.004448	0.000688
0.000908	-0.000181	0.011382	-0.000335	0.000187	-0.000600	-0.000169	0.003792	0.001981	-0.000319
0.001867	-0.000044	0.015614	0.001819	0.000655	0.003846	0.000070	0.006000	0.003266	0.000218
0.002239	0.000061	0.017937	0.000737	0.001011	0.006838	0.000236	0.007676	0.004227	0.000696
0.002275	0.000068	0.018151	0.000808	0.001037	0.007189	0.000247	0.007793	0.004294	0.000623
0.001845	-0.000064	0.014639	0.000302	0.000584	0.003226	0.000036	0.006671	0.003078	0.000137
0.001347	-0.000098	0.013284	0.000024	0.000456	0.002087	-0.000028	0.006050	0.002726	0.000001
0.000678	-0.000422	0.003448	-0.000672	-0.000732	-0.008393	-0.000686	0.000625	-0.000526	-0.001291
0.001427	-0.000085	0.018773	0.000250	0.000504	0.002533	0.000000	0.006289	0.003689	0.000063
0.002119	0.000032	0.017389	0.000525	0.000941	0.006351	0.000263	0.007348	0.004048	0.000619
0.001989	0.000010	0.016674	0.000491	0.000857	0.005698	0.000183	0.006946	0.003811	0.000490
0.002111	0.000031	0.017306	0.000631	0.000933	0.006267	0.000199	0.007312	0.004017	0.000618
0.002108	0.000030	0.017295	0.000630	0.000934	0.006275	0.000201	0.007329	0.004014	0.000618
0.002889	0.000077	0.018738	0.000645	0.001108	0.007781	0.000280	0.008131	0.004486	0.000699
0.002182	0.000039	0.017664	0.000553	0.000963	0.006541	0.000214	0.007469	0.004102	0.000547
0.001261	0.000114	0.012873	0.000178	0.000396	0.001881	-0.000051	0.004773	0.002664	-0.000004
0.001281	0.000109	0.013028	0.000191	0.000414	0.001742	-0.000043	0.004863	0.002815	-0.000044
-0.000606	0.000139	-0.000999	-0.000281	-0.000403	-0.004682	-0.000190	-0.006324	-0.008831	-0.002990
-0.000834	-0.000923	-0.003763	-0.001524	-0.002182	-0.021383	-0.001340	-0.006006	-0.008686	-0.002098
0.002335	0.000081	0.003183	0.001226	0.000588	0.023635	0.001078	0.000688	0.004744	0.002798

TABLE 5

Column	Commodities	Sweet potatoes	Sugar	Corn syrup	Apples	Bananas	Oranges	Other fresh fruits
		19	20	21	22	23	24	25
<i>Row</i>								
1	Beef.....	0.000066	0.000913	0.000297	0.000325	0.000189	0.000393	0.000299
2	Veal.....	0.000355	0.006021	0.001633	0.001788	0.000404	0.002160	0.001646
3	Pork.....	0.000124	0.001745	0.000627	0.000620	0.000351	0.000780	0.000571
4	Lamb and mutton.....	0.000193	0.002719	0.000884	0.000987	0.000563	0.001170	0.000991
5	Chicken.....	0.000110	0.001550	0.000504	0.000551	0.000321	0.000667	0.000508
6	Turkey.....	0.000106	0.001499	0.000484	0.000530	0.000309	0.000641	0.000468
7	Fish.....	0.000533	0.007481	0.002437	0.002687	0.001551	0.003224	0.002463
8	Eggs.....	0.000717	0.010070	0.003276	0.003585	0.002085	0.004335	0.003298
9	Butter.....	0.000169	0.002374	0.000772	0.000845	0.000491	0.001022	0.000777
10	Lard.....	0.000066	0.000923	0.000300	0.000329	0.000191	0.000397	0.000202
11	Shortening.....	0.000275	0.003576	0.010626	0.011938	0.006975	0.015741	0.011010
12	Margarine.....	0.000210	0.000959	0.000682	0.001055	0.000617	0.001392	0.000973
13	Salad dressing.....	0.000057	0.000909	0.000263	0.000287	0.000168	0.000379	0.000268
14	Fresh milk.....	0.000057	0.000799	0.000260	0.000284	0.000166	0.000375	0.000262
15	Evaporated milk.....	0.000105	0.001487	0.000478	0.000522	0.000305	0.000690	0.000481
16	Cheese.....	0.000396	0.005456	0.001778	0.001941	0.001134	0.002569	0.001790
17	Ice cream.....	0.000246	0.003498	0.001128	0.001233	0.000720	0.001626	0.001137
18	Potatoes.....	0.059934	0.000753	0.000245	0.000268	0.000156	0.000353	0.000247
19	Sweet potatoes.....	-0.130414	0.001401	0.000455	0.000499	0.000291	0.000667	0.000460
20	Sugar.....	0.000082	-0.241856	0.082022	0.001392	0.000813	0.001835	0.001284
21	Corn syrup.....	0.000010	0.131678	-0.442900	0.000425	0.000248	0.000660	0.000392
22	Apples.....	0.000027	0.002773	0.000456	-0.730000	0.124629	0.256814	0.000305
23	Bananas.....	0.000027	0.002780	0.000487	0.213601	-0.614987	0.000460	0.000305
24	Oranges.....	-0.000035	0.001496	0.000140	0.181264	0.000000	-0.663184	0.000305
25	Other fresh fruits.....	-0.000107	0.000020	-0.000260	0.001339	0.000982	-0.600000	-0.000000
26	Canned peaches.....	-0.000076	0.000646	-0.000090	0.003642	0.029481	0.000833	0.000984
27	Canned pineapples.....	-0.000131	-0.000478	-0.000396	0.122799	0.000000	0.000000	0.000519
28	Dried fruits.....	-0.000063	0.000913	-0.000017	0.000000	0.111785	0.000000	0.000701
29	Frozen fruits.....	-0.000241	-0.002747	-0.001008	0.000004	0.001767	0.005284	0.005874
30	Lettuce.....	0.000034	0.002897	0.000465	0.000182	0.000009	0.000030	0.002332
31	Tomatoes.....	0.000011	0.002463	0.000398	0.000119	0.000636	0.004793	0.002232
32	Beans.....	0.000099	0.004262	0.000686	0.000675	0.000942	0.006539	0.003175
33	Onions.....	0.000097	0.004212	0.000873	0.000664	0.000630	0.006493	0.003151
34	Carrots.....	-0.000062	0.000997	-0.000032	-0.000270	0.000241	0.003437	0.001462
35	Other fresh vegetables.....	0.000022	0.002666	0.000457	0.000498	0.000601	0.005001	0.002344
36	Canned peas.....	0.000082	0.003916	0.000796	0.000490	0.000824	0.006299	0.003996
37	Canned corn.....	0.000087	0.004007	0.000815	0.000611	0.000687	0.006296	0.003044
38	Canned tomatoes.....	0.000009	0.002420	0.000630	0.000111	0.000643	0.004763	0.002316
39	Dry vegetables.....	-0.000012	0.001958	0.000266	-0.000004	0.000337	0.004316	0.001572
40	Frozen vegetables.....	-0.000218	0.002366	-0.000880	-0.001121	-0.000465	0.000224	-0.000239
41	Other canned fruits and vegetables.....	-0.000004	0.002137	0.000813	0.000040	0.000488	0.004488	0.002067
42	Rice.....	0.000070	0.003566	0.000728	0.000437	0.000216	0.000929	0.002867
43	Wheat flour.....	0.000088	0.003375	0.000648	0.000363	0.000762	0.005687	0.002715
44	Breakfast cereals.....	0.000069	0.003644	0.000731	0.000421	0.000910	0.005947	0.002856
45	Corn meal.....	0.000069	0.003640	0.000722	0.000420	0.000810	0.005946	0.002856
46	Bread and other cereals.....	0.000069	0.004249	0.000887	0.000675	0.000641	0.006538	0.003175
47	Coffee.....	0.000076	0.003735	0.000751	0.000449	0.000284	0.006084	0.002914
48	Soup.....	-0.000023	0.001753	0.000209	-0.000067	0.000405	0.004117	0.001366
49	Other beverages.....	-0.000019	0.001820	0.000227	-0.000064	0.000420	0.004181	0.001901
50	Alcohol.....	-0.000071	-0.001963	-0.002328	-0.002090	-0.001198	-0.005606	0.002733
51	Nonfood.....	-0.000496	-0.007014	-0.002293	-0.002494	-0.001457	-0.002288	-0.002308
52	Expenditure proportions.....	0.000399	0.006861	0.002201	0.002396	0.001398	0.003794	0.003468

(Continued)

Canned peaches 26	Canned pine- apples 27	Dried fruits 28	Frozen fruits 29	Lettuce 30	Tomatoes 31	Beans 32	Onions 33	Carrots 34	Other fresh vegetables 35
0.000138	0.000098	0.000087	0.000084	0.000187	0.000233	0.000120	0.000097	0.000061	0.000665
0.000763	0.000482	0.000479	0.000349	0.001031	0.001282	0.000712	0.000831	0.000334	0.003326
0.000264	0.000167	0.000186	0.000122	0.000358	0.000446	0.000247	0.000184	0.000116	0.001158
0.000412	0.000261	0.000259	0.000189	0.000858	0.000985	0.000388	0.000287	0.000181	0.001801
0.000236	0.000149	0.000148	0.000108	0.000318	0.000396	0.000220	0.000184	0.000108	0.001027
0.000226	0.000143	0.000142	0.000104	0.000206	0.000380	0.000211	0.000167	0.000099	0.000986
0.001132	0.000722	0.000714	0.000525	0.001824	0.001912	0.001058	0.000798	0.000800	0.004984
0.001622	0.000971	0.000860	0.000706	0.002063	0.002870	0.001473	0.001070	0.000673	0.006673
0.000359	0.000229	0.000226	0.000166	0.000486	0.000606	0.000335	0.000252	0.000159	0.001573
0.000139	0.000089	0.000088	0.000085	0.000189	0.000236	0.000130	0.000098	0.000062	0.000612
0.000888	0.000238	0.000202	0.000239	0.000989	0.000878	0.004766	0.003647	0.002360	0.022262
0.000460	0.000286	0.000283	0.000207	0.000610	0.000758	0.000420	0.000314	0.000208	0.001967
0.000123	0.000078	0.000077	0.000067	0.000166	0.000207	0.000115	0.000086	0.000067	0.000535
0.000121	0.000077	0.000076	0.000066	0.000164	0.000204	0.000113	0.000084	0.000066	0.000530
0.000222	0.000141	0.000140	0.000103	0.000301	0.000375	0.000209	0.000154	0.000103	0.000973
0.000827	0.000526	0.000521	0.000380	0.001121	0.001394	0.000773	0.000677	0.000382	0.003817
0.000825	0.000334	0.000331	0.000241	0.000712	0.000985	0.000461	0.000386	0.000243	0.002297
0.000114	0.000073	0.000072	0.000052	0.000155	0.000182	0.000107	0.000080	0.000063	0.000499
0.000212	0.000138	0.000134	0.000097	0.000289	0.000367	0.000198	0.000149	0.000098	0.000929
0.000593	0.000378	0.000373	0.000273	0.000804	0.001000	0.000554	0.000414	0.000274	0.002594
0.000181	0.000118	0.000114	0.000084	0.000245	0.000305	0.000168	0.000128	0.000084	0.000792
0.002347	0.008416	0.008149	0.008036	0.000118	0.000144	0.000080	0.000069	0.000040	0.000378
0.000446	0.000349	0.000319	0.000306	0.000519	0.000799	0.000427	0.000318	0.000211	0.001996
0.011621	0.000212	0.000047	0.000306	0.001695	0.002108	0.001109	0.000872	0.000678	0.004570
0.002776	0.002222	0.001666	0.000332	0.000601	0.000747	0.000414	0.000308	0.000205	0.001938
-0.769157	0.129909	0.084183	0.018497	0.000999	0.001231	0.000682	0.000509	0.000337	0.003193
0.186849	-0.828286	0.000090	0.001183	0.000086	0.000082	0.000048	0.000034	0.000023	0.000231
0.106957	0.000160	-0.855307	0.007920	0.000329	0.000409	0.000227	0.000189	0.000112	0.001063
0.018910	0.000526	0.008966	-1.000000	0.000389	0.000970	0.001847	0.001229	0.000814	0.007797
0.001277	0.000394	0.000343	0.000760	-0.181400	0.048788	0.031481	0.000000	0.008715	0.008107
0.001211	0.000636	0.000317	0.008645	0.002627	-0.384600	0.000000	0.000172	0.007044	0.006831
0.001691	0.000572	0.000509	0.004829	0.005388	0.000306	-0.255000	0.000008	0.002088	0.000709
0.001879	0.000585	0.000504	0.004609	0.000199	0.000813	0.000000	-0.250000	0.039463	0.004228
0.000827	0.000188	0.000540	0.002892	0.000100	0.000865	0.000377	0.001411	-0.487100	0.008767
0.001268	0.000390	0.000473	0.003762	0.002498	0.002828	0.000005	0.000472	0.001261	-0.320000
0.001601	0.000533	0.000085	0.004442	-0.123520	0.000248	0.000000	0.000362	0.000172	0.000895
0.001625	0.000543	0.000482	0.004462	0.000172	0.000264	0.004203	0.000000	-0.049995	0.000827
0.001203	0.000362	0.000314	0.000830	-0.218945	0.000000	0.004486	0.004042	0.008820	0.000476
0.001060	0.000309	0.000285	0.003378	0.028191	0.000000	0.000000	-0.000127	0.000081	0.001115
-0.000044	-0.000005	-0.000016	0.001071	0.012495	0.016082	0.007142	0.000364	0.003584	0.004880
0.001127	0.000330	0.000195	0.000473	0.000402	0.000440	-0.000136	-0.000011	0.000292	0.000230
0.001535	0.000504	0.000447	0.004307	0.000401	0.000395	0.000060	0.000023	0.000243	0.000539
0.001487	0.000471	0.000416	0.004148	0.000363	0.000339	0.000025	0.000005	0.000223	0.000700
0.001599	0.000502	0.000444	0.004295	0.000398	0.000391	0.000048	0.000021	0.000241	0.000829
0.001523	0.000601	0.000444	0.004294	0.000397	0.000390	0.000048	0.000021	0.000241	0.000830
0.001692	0.000671	0.000552	0.004828	0.000497	0.000508	0.000101	0.000060	0.000282	0.001119
0.001559	0.000614	0.000456	0.004386	0.000414	0.000412	0.000058	0.000028	0.000248	0.000882
0.001023	0.000288	0.000242	0.003284	0.000088	0.000028	-0.000115	-0.000097	0.000115	-0.000072
0.001043	0.000294	0.000249	0.003300	0.000097	0.000041	-0.000109	-0.000093	0.000120	-0.000040
-0.002349	-0.000296	-0.000280	-0.005806	-0.001010	-0.001814	-0.000142	-0.000122	-0.001043	-0.001246
-0.001170	-0.000079	-0.000078	-0.000488	-0.001486	-0.001792	-0.000083	-0.000070	-0.000466	-0.000468
0.001416	0.001133	0.000850	0.001700	0.001398	0.001797	0.000799	0.000599	0.000599	0.004829

TABLE 5

Column	Commodities	Canned peas 36	Canned corn 37	Canned tomatoes 38	Dry vegetables 39	Frozen vegetables 40	Other canned fruits and vegetables 41	Rice 42
Row								
1	Beef.....	0.000221	0.000178	0.000109	0.000103	0.000078	0.000739	0.000201
2	Veal.....	0.001215	0.000978	0.000608	0.000564	0.000429	0.004063	0.001108
3	Pork.....	0.000422	0.000340	0.000206	0.000196	0.000149	0.001411	0.000365
4	Lamb and mutton.....	0.000658	0.000530	0.000326	0.000306	0.000233	0.002200	0.000800
5	Chicken.....	0.000375	0.000302	0.000186	0.000174	0.000133	0.001254	0.000342
6	Turkey.....	0.000380	0.000290	0.000178	0.000167	0.000127	0.001203	0.000329
7	Fish.....	0.001813	0.001490	0.000894	0.000845	0.000640	0.006063	0.001549
8	Eggs.....	0.002438	0.001968	0.001202	0.001138	0.000660	0.008151	0.002217
9	Butter.....	0.000573	0.000463	0.000283	0.000258	0.000203	0.001922	0.000523
10	Lard.....	0.000223	0.000180	0.000110	0.000104	0.000079	0.000747	0.000203
11	Shortening.....	0.008111	0.006558	0.004040	0.003785	0.002863	0.027178	0.007421
12	Margarine.....	0.000717	0.000580	0.000357	0.000335	0.000254	0.002402	0.000656
13	Salad dressing.....	0.000195	0.000158	0.000097	0.000092	0.000069	0.000654	0.000179
14	Fresh milk.....	0.000193	0.000156	0.000096	0.000090	0.000068	0.000647	0.000177
15	Evaporated milk.....	0.000354	0.000287	0.000176	0.000165	0.000125	0.001188	0.000325
16	Cheese.....	0.001319	0.001066	0.000657	0.000615	0.000466	0.004417	0.001207
17	Ice cream.....	0.000838	0.000677	0.000417	0.000391	0.000295	0.002805	0.000757
18	Potatoes.....	0.000182	0.000147	0.000090	0.000085	0.000064	0.000609	0.000166
19	Sweet potatoes.....	0.000339	0.000274	0.000158	0.000158	0.000119	0.001133	0.000309
20	Sugar.....	0.000945	0.000765	0.000471	0.000441	0.000335	0.003168	0.000885
21	Corn syrup.....	0.000289	0.000232	0.000144	0.000135	0.000102	0.000667	0.000264
22	Apples.....	0.000137	0.000110	0.000069	0.000064	0.000043	0.000453	0.000123
23	Bananas.....	0.000727	0.000588	0.000382	0.000339	0.000257	0.002437	0.000648
24	Oranges.....	0.001894	0.001512	0.000983	0.000930	0.000705	0.008879	0.001824
25	Other fresh fruits.....	0.000705	0.000571	0.000352	0.000329	0.000250	0.002356	0.000546
26	Canned peaches.....	0.001184	0.000941	0.000580	0.000543	0.000412	0.003900	0.001068
27	Canned pineapples.....	0.000078	0.000063	0.000039	0.000036	0.000028	0.000261	0.000071
28	Dried fruits.....	0.000387	0.000313	0.000193	0.000181	0.000137	0.001288	0.000354
29	Frozen fruits.....	0.002809	0.002371	0.001400	0.001311	0.000994	0.009411	0.002570
30	Lettuce.....	-0.125083	0.000000	-0.133999	0.017200	0.015992	0.002027	0.000250
31	Tomatoes.....	0.000000	0.000000	0.000003	0.000040	0.015853	0.001833	0.000140
32	Beans.....	0.000045	0.000087	0.000078	0.000184	0.016243	0.000177	0.000158
33	Onions.....	0.049332	0.000021	0.005979	0.000000	0.018234	0.001057	0.000119
34	Carrots.....	0.000000	-0.094900	0.006870	0.000000	0.015699	0.000192	0.000188
35	Other fresh vegetables.....	0.000113	0.000094	0.000106	0.000062	0.015997	0.000000	0.000121
36	Canned peas.....	-0.186000	0.047200	0.033962	0.014854	0.016187	0.003801	0.000220
37	Canned corn.....	0.069001	-0.265000	0.000127	0.069132	0.016202	0.003807	0.000214
38	Canned tomatoes.....	0.068376	0.000000	-0.176000	0.000037	0.015398	0.001904	0.000099
39	Dry vegetables.....	0.024200	0.079500	0.000000	-0.480000	0.018573	0.003116	0.000089
40	Frozen vegetables.....	0.012656	0.010127	0.007597	0.007597	-1.084400	0.003182	0.001702
41	Other canned fruits and vegetables.....	0.000614	0.000430	0.000249	0.000459	0.015902	-0.400000	0.001080
42	Rice.....	0.000201	0.000148	0.000144	0.000194	0.003134	0.005681	-0.320000
43	Wheat flour.....	0.000183	0.000108	0.000119	0.000169	0.006002	0.003832	0.049226
44	Breakfast cereals.....	0.000195	0.000143	0.000143	0.000162	0.008114	0.005656	0.005371
45	Corn meal.....	0.000195	0.000142	0.000142	0.000192	0.008112	0.005652	0.000000
46	Bread and other cereals.....	0.000295	0.000221	0.000193	0.000285	0.008358	0.003345	0.000258
47	Coffee.....	0.000214	0.000187	0.000162	0.000202	0.003150	0.000781	0.000388
48	Soup.....	-0.000109	-0.000100	-0.000019	0.000028	0.002838	0.003822	0.000025
49	Other beverages.....	-0.000097	-0.000092	-0.000013	0.000033	0.002856	0.003498	0.000078
50	All food.....	-0.000471	-0.000320	-0.000762	-0.000914	-0.008437	-0.006875	-0.000508
51	Nonfood.....	-0.001895	-0.001370	-0.000876	-0.000631	-0.000600	-0.000677	-0.001550
52	Expenditure proportions.....	0.001416	0.001133	0.000860	0.000850	0.001700	0.008980	0.001333

(Continued)

Wheat flour 43	Breakfast cereals 44	Corn meal 45	Bread and other cereals 46	Coffee 47	Soup 48	Other beverages 49	All food 50	Nonfood 51	Income 52
0.000346	0.000983	0.000134	0.003562	0.000682	0.000415	0.000530	-0.289553	0.099839	0.289914
0.001906	0.005407	0.000739	0.019758	0.004690	0.002283	0.002915	-0.794299	0.263185	0.991134
0.000662	0.001878	0.000257	0.006864	0.001624	0.000793	0.001013	-0.179327	0.045868	0.123499
0.001032	0.002928	0.000401	0.010702	0.002539	0.001236	0.001879	-0.767470	0.196303	0.571167
0.000658	0.001859	0.000238	0.006102	0.001447	0.000705	0.000900	-0.239835	0.061345	0.178490
0.000658	0.001503	0.000219	0.006880	0.001391	0.000677	0.000886	-0.920213	0.167324	0.768389
0.002839	0.003025	0.001099	0.029472	0.006900	0.003405	0.004348	-0.005411	0.001384	0.094027
0.003815	0.010842	0.001478	0.039620	0.006898	0.004577	0.005846	-0.073743	0.016882	0.054851
0.000900	0.002556	0.000348	0.006339	0.002215	0.001079	0.001378	-0.427444	0.106331	0.313113
0.000350	0.000994	0.000135	0.003622	0.000861	0.000420	0.000539	0.067184	-0.017184	-0.060600
0.012738	0.036177	0.004983	0.132187	0.031344	0.018271	0.019608	-0.039093	0.167324	0.629094
0.001124	0.003199	0.000436	0.011687	0.002771	0.001360	0.001726	0.000000	0.000000	0.000000
0.000307	0.000870	0.000119	0.003198	0.000738	0.000398	0.000470	-0.382785	0.097908	0.284877
0.003003	0.000861	0.000118	0.003146	0.000748	0.000384	0.000484	-0.273638	0.068991	0.202847
0.000566	0.001581	0.000217	0.005778	0.001370	0.000668	0.000852	0.000000	0.000000	0.000000
0.002070	0.005881	0.000802	0.214548	0.008095	0.002482	0.003171	-0.334389	0.085630	0.248889
0.001315	0.003735	0.000309	0.013645	0.003236	0.001576	0.002014	-0.448186	0.113682	0.331296
0.000285	0.000811	0.000111	0.002964	0.000702	0.000342	0.000437	-0.156716	0.040085	0.116831
0.000830	0.001500	0.000207	0.005812	0.001308	0.000636	0.000813	0.000000	0.000000	0.000000
0.001485	0.004218	0.000575	0.015412	0.003655	0.001871	0.002275	-0.043167	0.011041	0.032128
0.000453	0.001287	0.000176	0.004703	0.001115	0.000543	0.000694	-0.232411	0.039702	0.173709
0.000214	0.000909	0.000083	0.002225	0.000828	0.000257	0.000329	-0.188015	0.048009	0.139925
0.001142	0.003244	0.000443	0.011883	0.002810	0.001369	0.001749	-0.187133	0.047605	0.139268
0.001311	0.003893	0.001213	0.032443	0.007795	0.003784	0.004795	-0.349973	0.089512	0.260457
0.001109	0.003151	0.000430	0.011812	0.002730	0.001330	0.001699	-0.537476	0.137475	0.400000
0.001823	0.005192	0.000798	0.018971	0.004499	0.002192	0.002900	-0.399212	0.088462	0.340670
0.000122	0.000347	0.000047	0.001268	0.000301	0.000147	0.000187	-0.524135	0.076881	0.447274
0.000408	0.001728	0.000236	0.006313	0.001497	0.000729	0.000939	-0.398423	0.054173	0.315250
0.004411	0.012330	0.001709	0.043783	0.010856	0.005289	0.006757	-0.588801	0.227734	0.861427
0.000448	0.001265	0.000173	0.004624	0.001096	0.000534	0.000682	-0.172288	0.028286	0.147023
0.000240	0.000682	0.000083	0.002495	0.000492	0.000288	0.000388	-0.228234	0.068831	0.170988
0.000271	0.000771	0.000105	0.002818	0.000668	0.000326	0.000416	-0.000000	0.000000	0.000000
0.000204	0.000578	0.000079	0.002114	0.000501	0.000244	0.000312	-0.006319	0.001616	0.004703
0.000323	0.000816	0.000125	0.003349	0.000794	0.000387	0.000491	-0.420128	0.169782	0.319366
0.000208	0.000591	0.000081	0.002158	0.000512	0.000249	0.000318	-0.201583	0.051583	0.160000
0.000377	0.001071	0.000146	0.003145	0.000928	0.000452	0.000678	-0.043072	0.011017	0.032056
0.000367	0.001043	0.000142	0.003100	0.000903	0.000440	0.000582	-0.001771	0.000126	0.028445
0.000119	0.000339	0.000046	0.001237	0.000293	0.000143	0.000183	-0.203221	0.028801	0.173420
0.000163	0.000434	0.000089	0.001587	0.000378	0.000184	0.000234	-0.264191	0.037275	0.216916
0.000291	0.000897	0.000131	0.003035	0.001189	0.000502	0.000474	-0.827866	0.211748	0.618108
0.001884	0.005267	0.000718	0.010244	0.004683	0.002223	0.002840	-0.268737	0.068737	0.200000
0.088792	0.028367	0.000092	0.003053	0.001462	0.000707	0.000904	-0.074553	0.019089	0.056484
-0.300000	0.300000	0.004488	0.010000	0.003358	0.001636	0.002089	-0.111703	0.028571	0.083132
0.000060	-0.220000	0.000000	0.006280	0.002977	0.001450	0.001853	-0.077484	0.019821	0.056783
0.011838	0.000000	-0.220000	0.004648	0.002202	0.001073	0.001371	-0.077672	0.019867	0.057805
0.001263	0.002218	0.000237	-0.150000	0.001384	0.000822	0.001179	0.000000	0.000000	0.000000
0.001612	0.003549	0.000381	0.006500	-0.282181	-0.21403	0.027341	-0.063469	0.016234	0.047235
0.000732	0.001506	0.000111	0.000538	0.002600	-0.450000	0.026323	-0.317611	0.081238	0.236878
0.000758	0.001574	0.000119	0.000743	0.003031	0.002034	-0.438899	-0.309048	0.079043	0.230000
-0.001383	-0.001414	-0.000408	-0.003918	-0.002270	-0.004790	-0.005837	-0.203646	0.060690	0.178265
-0.002680	-0.007356	-0.001031	-0.027613	-0.006548	-0.003189	-0.004076	-0.226266	-1.017898	1.243165
0.002369	0.006516	0.000889	0.022212	0.008573	0.003538	0.004675	0.227018	0.720882	1.000000

TABLE 6
CLASSIFICATION OF COMMODITIES ACCORDING TO LINEAR TREND

Annual decrease and increase of consumption in per cent						
Over -5	-5 to -3	-3 to -1	-1 to 1	1 to 3	3 to 5	Over 5
Eggs Lard	Veal Butter	Milk Ice cream	Pork Lamb	Beef Sugar	Margarine Salad and cooking oil Cheese	Beans
Sweet potatoes Dried fruits	Evaporated milk Oranges Canned peas	Corn syrup Apples Canned peaches Tomatoes Carrots Canned corn Dried vegetables Wheat flour Corn meal Coffee	Chicken Shortening Potatoes Canned pineapple Onions Breakfast cereals	Bananas Lettuce Rice Soup		
TOTAL COMMODITIES: 4	6	12	8	6	3	1

logarithm of the constant term plus 2, if all variables in the first difference analysis are converted to logarithms and if the constant term differs significantly from zero. For example, in the case of cheese, a_1 is equal to .017222 and we took the antilogarithm of 2.017222 as 104.20. So we could say that time trend alone would increase consumption of cheese by 4.20 per cent annually. For eggs, we have a_1 equal to -0.025019 and, therefore, we found the antilogarithm of 1.974981 as 94.40, which means that time trend alone would decrease the consumption of eggs by 5.60 per cent annually. Though some of the commodities appearing in Appendix table A-6 showed coefficients not significantly different from zero, we took these coefficients also to obtain the percentage effects of time in each year on the consumption of these commodities. Summary table 6 is based on the percentage changes given in Appendix table A-6.

Seasonal effects on time trend

When quarterly rather than annual data are used, considerable seasonal vari-

ation is possible. Therefore, it may not be appropriate to assume that the effect of season is the same for all the seasons. To test the seasonal effects on time trend, we specified two different types of regression equations:

(a) The first type is the same as in (109), except that the first differences correspond to differences in observations from season to season. The constant term in this equation can be interpreted as an indication of the linear effect of seasons, assuming that the trend during different seasons does not vary.

(b) The second type assumed that the time trend varied from season to season. Instead of putting a single constant, we specified four constants corresponding to four seasons (using dummy variables) and all the other variables were retained as in (109). In this case, the coefficients corresponding to each dummy variable can be interpreted as the time trend associated with that particular season. Having estimated both types of equations, we tested the significance of individual coefficients, using t -values, and the difference in the two specifications,

using F -values. For this study, we have confined this analysis to five commodities—butter, margarine, shortening, salad and cooking oil, and lard. In table 7, column a_i corresponds to the constants obtained when no dummy variable was introduced and $s_1, s_2, s_3,$ and s_4 correspond to the coefficients of the four dummy variables, denoting four quarters of the year. While the linear effect

of time was not significantly different from zero in the case of all commodities when season was not considered, these coefficients became significantly different from zero in eight cases when season was introduced. The F -test showed that three out of five commodities showed variations in the constants according to season.

Analysis of the Farm-retail Price Spread

The retail market is only one link in a chain of institutions or agencies affecting the marketing process and the prices at these levels are linked by marketing charges (defined to include costs such as transportation, processing, packaging, and profits). Often public policy decisions may be influenced by the behavior of marketing margins and an economic analysis of factors influencing prices will not be complete without proper consideration of this aspect. In many situations demand theory has overlooked the spread between prices at the retail level and at the farm level, and only a limited number of studies have treated margins explicitly. The lack of understanding of the nature of margins and their behavior on the market seriously limits our understanding of demand theory. This section analyzes the nature of price spreads between the retail and farm levels, and explores the possibilities of obtaining demand elasticities at one level of the marketing system from a knowledge of these measures at another level.

Marketing margins and the elasticity of demand

The Concept of the Farm-retail Price Spread — A farm-retail price spread (marketing margin or marketing charge)

“is the difference between the retail price of a product and its farm value—the payment (adjusted for by-product values) to farmers for an equivalent quantity of farm products” (U. S. Department of Agriculture, 1957, p. 1). Generally the spread includes the costs incurred and profits enjoyed by all agencies involved in the transfer of products from farmers to consumers. Often, these charges include the payments for the services such as assembly of raw materials from the farm, processing, storage, transportation, wholesaling, and retailing.

This study divides consumer expenditures on food items into two parts: payments to the farmers as returns for production of raw food products (farmer's share) and payments to the agencies that assemble and process the raw materials and distribute the final product to consumers (farm-retail spread). Both of these together equal the retail price. For measurement purposes, if any two of these factors are known, the third can be deduced. The practice of the U. S. Department of Agriculture is to obtain retail price and farmer's share independently and to deduce the farm-retail spread. Three measurements used in obtaining estimates of farmer's share are discussed by Ogren (1956). These measures, together with the retail prices

TABLE 7
SEASONAL EFFECTS ON TIME TREND

Commodity	Equation	Seasonal coefficient						Sum of squares of deviations	Degrees of freedom	F-statistic
		S ₁	S ₂	S ₃	S ₄	S ₅	S ₆			
Butter.....	General	-.004941 (-.5888)						.082143	38	
	Dummy variable		-.077341 (3.1843)	-.024824 (-3.8288)	.033078 (4.3172)	-.037730 (-2.8970)		.048188	35	9.8411*
Margarine.....	General	.006021 (.5082)						.002574	38	
	Dummy variable		-.040530 (-4.9088)	.006864 (0.6047)	.039221 (4.3090)	.010886 (.1929)		.028228	35	14.18800*
Shortening.....	General	-.008041 (-.4430)						.061866	38	
	Dummy variable		-.013115 (-1.0046)	.008273 (.4672)	.010942 (.8706)	-.018086 (-1.4200)		.047470	35	1.0313†
Salted and cooking oil	General	-.008700 (-.7866)						.132140	38	
	Dummy variable		-.029223 (-1.4420)	-.006613 (-.3360)	-.020437 (-1.8721)	.027054 (1.2032)		.116228	35	1.7980†
Lard.....	General	.000538 (.0477)						.112303	38	
	Dummy variable		-.019052 (-.9480)	-.004794 (-.3109)	.031201 (3.3433)	-.069910 (-3.4882)		.070909	35	8.7941*

* Significant.
† Non-significant.

published by the Bureau of Labor Statistics, provide us with data on farm-retail spread.

The importance of marketing services varies for different commodities. In general, the farmer's share decreases as the number of intermediate operations increases. Fox (1951, p. 68) points out that marketing margins for food crops show great variation. Grain products undergo much processing between farms and consumers. Meat products move through the marketing system in a short time. Fruits and vegetables, during the producing seasons, may go directly from the processor to the consumer without much processing. In other crops, transportation costs may account for a substantial portion of the marketing spread. In the present study, we could obtain data on marketing margins for 32 commodities; among them, the farmer's share varied widely. Table 8 shows that the highest percentage of farmer's share is for butter (about 75 per cent) and the lowest for breakfast cereals (about 10 per cent).

Pricing methods.—Because the basis of calculating farm-retail spread is the retail prices paid by the consumers and the payments received by farmers for equivalent quantities of products, an understanding of pricing practices is necessary (see discussions in Oxenfeldt, 1966, Chs. 9 and 10 and Kotler, 1967, Ch. 15). No single method of setting price is applicable to all the products. Oxenfeldt (1966, pp. 289–294) discusses two types of pricing methods—complete and partial methods.

Complete pricing methods. The prices at each level may be set, using any one of the following methods:

- (a) Cost-plus pricing and average-cost pricing, calling for the addition of some base cost as a margin to cover profit.
- (b) Flexible mark-up method, calling for

TABLE 8
FARMER'S SHARE OF RETAIL PRICE

Commodity	Farm price/retail price	
	Quarterly data*	Annual data†
per cent		
<i>Meat</i>		
Beef.....	59.44	
Pork.....	52.27	
Lamb.....	53.75	
Chicken.....	52.46	
<i>Eggs</i>	65.16	
<i>Fats and oils</i>		
Butter.....	71.00	75.95
Shortening.....	32.52	37.05
Margarine.....	28.33	
Salad dressing.....	20.70	
<i>Milk products</i>		
Milk.....	44.04	
Evaporated milk.....	42.59	46.12
Cheese.....	43.62	44.60
Ice cream.....	37.13	
<i>Potatoes</i>		
White.....	31.66	
Sweet.....		37.52
<i>Sugar</i>	35.99	37.90
<i>Fruit</i>		
Apples.....	34.22	32.58
Oranges.....	32.10	
Canned peaches.....	17.40	
Dried fruit.....		35.05
<i>Vegetables</i>		
Lettuce.....	33.94	
Fresh tomatoes.....	24.80	
Beans.....	43.41	
Onions.....	22.59	
Carrots.....	26.83	
Canned peas.....	14.17	14.17
Canned corn.....	12.59	13.79
Canned tomatoes.....	18.22	16.91
Dried vegetables.....		44.93
<i>Other</i>		
Wheat flour.....	35.35	
Breakfast cereal.....	9.97	10.82
Corn meal.....	27.21	
Bread.....		18.14

* Data for 1955 to 1967 obtained from USDA (1965b, various issues).

† Data for 1947 to 1964 taken from USDA (1965) and data for 1965 and 1966 obtained from USDA (1965b, various issues).

markup to be varied on the basis of several possible considerations, including demand conditions.

(c) Trial-and-error or experimental pricing method, calling for trying one or more prices that seem to be appropriate and choosing the one which gives best results.

(d) Research method of pricing which is based on actual market trials through experimental markets.

(e) Intuitive method of pricing which is based on the price setter's intuitive knowledge. It may not involve any systematic techniques.

Partial pricing methods. The prices at each level may be fixed through price maintenance or price followership. Price maintenance implies that a constant price that has been proven effective is retained for a long period. Price followership implies that prices charged by the "followers" will, in some way, be related to the price charged by a "leader."

In addition to complete and partial pricing methods, two other forms of pricing are known as price-line pricing and multi-stage pricing. In price-line pricing, the price charged for a commodity is retained at the same level for a long time but adjustments are made in quality to offset increased cost. Multi-stage pricing is done in different stages such as selection of market target, choice of brand image, choice of marketing mix, selection of pricing policy, selection of pricing strategy, and choice of specific price.

Hoos (1954, pp. 123-141) classifies the farm-retail spread as systematic and nonsystematic. Systematic methods of setting margins include: (1) fixed absolute margin, (2) fixed percentage margin, and (3) costs-per-pound margin which varies with the store's purchase price. Nonsystematic methods include: (1) following closely the price of near or strong competitors by "shading" under or "padding" over the price set by a

"leader" or a "competitor" and (2) short-run profit maximization.

Types of price spread.—These pricing practices provide guidelines to determine the relationship between price spreads and prices at different levels of the marketing system. In an analysis of this nature, it is possible to incorporate only "systematic" margin policies. In many studies on price spreads (see Dalrymple, 1961, for a review), it is assumed that price spreads are determined in one of the following ways:

Constant percentage spread assumes that the margin is a percentage of prices at the farm level or at the retail level. Although it is not necessary to assume that the percentage remains the same for all levels of volume, in many cases it is assumed to be constant. Let $p^{(r)}$ denote the retail price, $p^{(f)}$ the farm level price, and M the margins. Under constant percentage margins (say, k percentage of retail prices),

$$M = kp^{(r)} \quad (110)$$

Therefore,

$$p^{(r)} = p^{(f)} + kp^{(r)} \quad (111)$$

or

$$p^{(f)} = (1 - k)p^{(r)}$$

Absolute spreads add a specified amount to the farm-level price to obtain the retail price. In some cases, the amount to be added may be a function of variables like price and quantity. However, the simplest case is to assume a fixed quantity as the margin (M^0).

$$p^{(r)} = p^{(f)} + M^0 \quad (112)$$

Price spread and quantity handled may have certain relationship. In this case, it is often assumed that the price spread

is a linear function of the quantity handled (q).

$$M = a + bq. \quad (113)$$

Therefore, the relationship between farm price and retail price can be written as

$$p^{(r)} = a + bq + p^{(f)}. \quad (114)$$

Though these three assumptions regarding the behavior of marketing margins may be applicable to certain situations, it seems appropriate to assume that the price spreads are determined as a combination of percentage and absolute margins. Dalrymple (1961, pp. 5-6) points out that wholesalers appear to use a constant percentage markup and retailers appear to make use of an absolute margin so that when we consider the market as a combination of wholesalers and retailers, it may be appropriate to consider margins as a combination of these two approaches. Also, studies by Thomsen (1951, pp. 221-223), Shepherd (1955, pp. 253-254), and Rojko (1957, p. 157) reveal that margin is composed of elements of which about 50 per cent are absolute and 50 per cent are percentages. Waugh (1964, p. 20) reports that, ". . . many studies of this matter (percentage and absolute spread) in the [U.S.] Department of Agriculture suggest that the price spreads are neither constant percentages nor constant absolute amounts, but somewhere in between the two." Here, the margins are specified as a linear function of retail prices.

$$M_j = \alpha_j + \beta_j p_j^{(r)} \quad (115)$$

where

j stands for the j^{th} commodity.

If $p_j^{(r)}$ and $p_j^{(f)}$ denote the retail- and farm-level prices,

$$p_j^{(r)} = p_j^{(f)} + M_j. \quad (116)$$

From (115) and (116)

$$p_j^{(r)} = p_j^{(f)} + \alpha_j + \beta_j p_j^{(r)}.$$

Therefore,

$$p_j^{(f)} = -\alpha_j + (1 - \beta_j)p_j^{(r)} \quad (117)$$

$$= \alpha_j + b_j p_j^{(r)}$$

where

$$a_j = -\alpha_j \text{ and}$$

$$(1 - \beta_j) = b_j.$$

Using the data reported by U. S. Department of Agriculture, we have fitted an equation of type (117) for 32 commodities and the results are presented in table 9.

The table shows that 19 commodities had both slope and intercept significantly different from zero: beef, pork, lamb, chicken, eggs, butter, shortening, milk, cheese, ice cream, potatoes, sugar, oranges, canned peaches, lettuce, beans, onions, carrots, and breakfast cereals. In 11 commodities the intercepts were not significant but the slopes were: margarine, salad dressing, sweet potatoes, apples, dried fruit, tomatoes, canned peas, canned tomatoes, dry vegetables, wheat flour, and corn meal. The non-significance of a_j implies that α_j also is nonsignificant. Therefore, for these commodities, the hypothesis that margin is a linear function of retail prices is not different from a hypothesis that margin is a fixed proportion of retail prices. Finally, in two commodities—evaporated milk and canned corn—the inter-

TABLE 9
RELATIONSHIP BETWEEN FARM PRICE AS A FUNCTION OF RETAIL PRICE

Commodities grouped as to significance* of:	Data†	Intercept ($-a_i$)	Slope (b_i)	R ²
<i>Slope and intercept</i>				
Beef.....	Q	-24.52	.9188	.75
Pork.....	Q	-21.58	.9014	.90
Lamb.....	Q	-21.50	.8447	.78
Chicken.....	Q	- 6.12	.6769	.95
Eggs.....	Q	-14.08	.9214	.93
Butter.....	Q	18.59	.4841	.44
	A	-23.55	1.0611	.77
Shortening.....	Q	7.79	.2330	.37
	A	-30.74	.9410	.78
Fresh milk.....	Q	- .89	.4897	.99
Cheese.....	Q	18.46	.1891	.21
	A	4.96	.3005	.58
Ice cream.....	Q	- 3.06	.3179	.93
Potatoes.....	Q	-22.13	.6630	.86
Sugar.....	Q	9.03	.1965	.41
	A	8.90	.2145	.70
Oranges.....	Q	-10.48	.4674	.49
Canned peaches.....	Q	- 3.35	.2752	.60
Lettuce.....	A	- 3.17	.5018	.79
Beans.....	A	- 0.73	.4724	.98
Onions.....	A	- 4.00	.7070	.80
Carrots.....	A	- 6.31	.7080	.39
Cereals.....	Q	7.57	- .1973	.86
<i>Slope but not intercept</i>				
Margarine.....	Q	- 1.68	.3457	.38
Salad dressing.....	Q	- 0.37	.2173	.24
Sweet potatoes.....	Q	- 0.19	.3934	.95
Apples.....	Q	1.81	.2298	.23
	A	- 0.33	.3479	.65
Dried fruit.....	A	- 0.95	.3922	.90
Fresh tomatoes.....	A	- 0.87	.3769	.72
Canned peas.....	Q	0.59	.1091	.24
	A	- 0.06	.1447	.52
Canned tomatoes.....	Q	1.49	.0659	.04
	A	.30	.1495	.44
Dried vegetables.....	A	- 0.37	.4759	.90
Wheat flour.....	Q	- 4.32	.4337	.17
Corn meal.....	Q	0.04	.2881	.77
<i>Intercept but not slope</i>				
Evaporated milk.....	Q	3.80	.1863	.06
	A	2.65	.2929	.18
Canned corn.....	Q	- 2.84	- .0110	.01
	A	2.13	.0224	.03

* Significant at the 5 per cent level.

† Quarterly data (Q) relate to 1955-1967 period, and annual data (A) to 1947-1966 period.

cepts were significantly different from zero but not the slopes. Nonsignificance of the slope ($b_i = 1 - \beta_i$) implies that β_i is not significantly different from one, implying that the farm level prices may not change with a change in retail price.

The negative slope for cereals, given in table 9, illustrates one problem in the above analysis. The negative slope may

be caused by (a) widening margins and higher retail prices over time due to more elaborate packaging, changes in the products themselves, the exercise of market power, or similar factors; and (b) a general downward trend in the farm prices of cereal grains that may be associated with markets other than domestic use in cereals. If grain prices

were to rise, holding prices of all other marketing costs constant, cereal prices would tend to rise in time, given the expected positive relation between farm and retail price. A limitation of the above analysis is that a detailed study of marketing margins and costs was not undertaken that would allow analysis of the relation of farm and retail prices, holding marketing costs constant.

Effect of margins on derived demand.—In many empirical studies it may be necessary to derive demand parameters at one level of the marketing system from a knowledge of corresponding parameters at another level. When we consider the intermediaries such as processors, wholesalers, and retailers, it is possible to derive demand functions for all these levels. The quantity demanded by processors, quantity consumed, retail price, and farm-level price can be determined simultaneously using a simplified model consisting of the following elements.

(a) Consumer demand.

$$f_1(q_c, p^{(r)}, y) = 0 \quad (118)$$

where

q_c = quantity consumed,
 $p^{(r)}$ = retail price, and
 y = consumer income.

(b) Marketing group behavior. This consists of the supply and demand for the marketing group and can be reduced to a single equation of the form

$$f_2(q_p, p^{(r)}, p^{(f)}, V_2) = 0 \quad (119)$$

where

q_p , $p^{(r)}$, and y are defined as in (118) and V_2 represents all other variables

influencing marketing group behavior. (c) Producer supply. Let q_p be the quantity supplied. The producer-supply relationship can be expressed as

$$f_3(q_p, p^{(f)}, V_3) = 0 \quad (120)$$

where

V_3 represents all other exogenous variables influencing supply.

If equilibrium conditions are assumed, $q_p = q_c = q$. Using (118), (119), and (120), it is possible to eliminate $p^{(r)}$ and a derived demand equation¹³ can be obtained of the form

$$f_4(q, p^{(f)}, y) = 0. \quad (121)$$

Thus, using the behavioral equations of various marketing groups, we can obtain derived demand equations for one level from the corresponding demand equation for another level in the marketing system.

Bearing this in mind, let us analyze the implications of the behavioral assumption of margins specified in (115) on elasticities at the farm level and elasticities at the retail level. Here, it is assumed that there are n commodities q_1, q_2, \dots, q_n with retail price $p_1^{(r)}, p_2^{(r)}, \dots, p_n^{(r)}$. The elasticity at the retail level is defined by

$$e_{ij} = \frac{\partial \sigma_i}{\partial p_j^{(r)}} \cdot \frac{p_j^{(r)}}{q_i} \quad (122)$$

($i, j = 1, 2, \dots, n$).

Let the corresponding farm-level prices be $p_1^{(f)}, p_2^{(f)}, \dots, p_n^{(f)}$, and the corresponding margins be M_1, M_2, \dots, M_n .

¹³ This is similar to the derived demand relation specified in Hildreth and Jarrett (1955, p. 103). Also see Foote (1968, pp. 100-102). This formulation assumes that farm price and processing price are determined in the same time period, which is not necessarily the case. A more realistic formulation might deal with expected, rather than equilibrium, prices.

If it is assumed that the margin is a linear function of retail prices, using (117), it is possible to specify a relationship between farm-level prices and retail prices as

$$(1 - \beta_j)p_j^{(r)} = \alpha_j + p_j^{(f)}. \quad (123)$$

Therefore,

$$p_j^{(r)} = \frac{1}{(1 - \beta_j)} (\alpha_j + p_j^{(f)}). \quad (124)$$

The elasticities obtained from a derived demand equation can be represented as

$$E_{ij} = \frac{\partial q_i}{\partial p_j^{(f)}} \cdot \frac{p_j^{(f)}}{q_i} \quad (125)$$

which, in this case, can be treated as the farm-level elasticity. Now $\partial q_i / \partial p_j^{(f)}$ can be expressed as

$$\frac{\partial q_i}{\partial p_j^{(f)}} = \frac{\partial q_i}{\partial p_j^{(r)}} \cdot \frac{\partial p_j^{(r)}}{\partial p_j^{(f)}} \quad (126)$$

From (124), the value $\partial p_j^{(r)}$ can be obtained as

$$\frac{\partial p_j^{(r)}}{\partial p_j^{(f)}} = \frac{1}{(1 - \beta_j)}. \quad (127)$$

Substituting (126) and (127) in (125),

$$\begin{aligned} E_{ij} &= \frac{1}{(1 - \beta_j)} \frac{\partial q_i}{\partial p_j^{(r)}} \cdot \frac{p_j^{(f)}}{q_i}, \\ &= \frac{1}{(1 - \beta_j)} \cdot \frac{\partial q_i}{\partial p_j^{(r)}} \cdot \frac{p_j^{(r)}}{q_i} \cdot \frac{p_j^{(f)}}{p_j^{(r)}}, \\ &= \frac{1}{(1 - \beta_j)} \cdot e_{ij} \cdot \frac{p_j^{(f)}}{p_j^{(r)}} \text{ and} \\ &= e_{ij} \cdot \frac{p_j^{(f)}}{(1 - \beta_j)p_j^{(r)}}. \end{aligned} \quad (128)$$

Again, we can express (128) in terms of α_j by substituting the value of $(1 - \beta_j)p_j^{(r)}$ from (123) as

$$E_{ij} = e_{ij} \cdot \frac{p_j^{(f)}}{\alpha_j + p_j^{(f)}}. \quad (129)$$

The relationship in (129) can be used to derive conditions under which the elasticity at retail is higher than the elasticity at the farm level. Special cases of this result are shown in Hildreth and Jarrett (1955, p. 111) and Foote (1958, p. 105). From (129), we have

$$E_{ij} \leq e_{ij} \text{ if } \alpha_j \geq 0. \quad (130)$$

Special cases.—*Constant percentage spread.* When the price is a constant percentage of retail price, we can write

$$p_j^{(f)} = k_j p_j^{(r)}. \quad (131)$$

Comparing (131) with (123),

$$\alpha_j = 0 \text{ and } (1 - \beta_j) = k_j. \quad (132)$$

From (129) and (132),

$$E_{ij} = e_{ij}. \quad (133)$$

Therefore, the elasticities at the farm level and at the retail level are the same.

Constant absolute spread. Here, we have

$$M_j = \alpha_j \text{ and } \beta_j = 0. \quad (134)$$

From (128) and (134) we can write

$$E_{ij} = e_{ij} \cdot \frac{p_j^{(f)}}{p_j^{(r)}}. \quad (135)$$

The elasticity at the farm level is obtained as a product of elasticity at the retail level and the proportion of farm price to retail price. Because in most

cases retail prices are higher than the farm price, the elasticity at the farm level is lower than the elasticity at the retail level as discussed, for example, by Waugh (1964, p. 78).

Elasticity of price transmission.—Elasticity of price transmission is the ratio of relative change in retail price to the relative change in the farm-level price. If we denote this elasticity for the j^{th} commodity as η_j ,

$$\eta_j = \frac{\partial p_j^{(r)}}{\partial p_j^{(f)}} \cdot \frac{p_j^{(f)}}{p_j^{(r)}} \quad (136)$$

When we assume that the marketing margin is a linear function of retail prices, from (127) we have

$$\frac{\partial p_j^{(r)}}{\partial p_j^{(f)}} = \frac{1}{(1 - \beta_j)}$$

Substituting this in (136), η_j is obtained as

$$\eta_j = \frac{1}{(1 - \beta_j)} \cdot \frac{p_j^{(f)}}{p_j^{(r)}} \quad (137)$$

Using the average prices at the farm and retail levels, we have calculated the elasticity of price transmission for 32 commodities included in the present analysis. In the majority of the cases, the elasticity of price transmission was less than one. Hildreth and Jarrett (1955, p. 111) explain the implications of an elasticity of price transmission of less than one as follows: “. . . if producers' price rises while quantity processed and such other factors as prices of inputs used by processors remain fixed, the relative change in consumer price will not exceed the relative change in producers' price. This would certainly be true if effective competition

existed in processing, and might be expected to be typical of other instances as well.”

Derived farm level elasticities

As stated before, elasticities at one level of the marketing system can be derived from a knowledge of the elasticities at another level; the direct and cross elasticities for many commodities can also be obtained at the retail level. This section shows how elasticity at the farm level can be obtained from elasticity at the retail level and the elasticity of price transmission. In (128), it was established that the relationship between retail-level elasticities and farm-level elasticities follows the pattern,

$$E_{ij} = e_{ij} \frac{p_j^{(f)}}{(1 - \beta_j)p_j^{(r)}}$$

Also, from (137),

$$\eta_j = \frac{1}{(1 - \beta_j)} \frac{p_j^{(f)}}{p_j^{(r)}}$$

A combination of these two results

$$E_{ij} = e_{ij} \cdot \eta_j \quad (138)$$

shows that the elasticity at the farm level is the product of elasticity at the retail level and the elasticity of price transmission. Since, for most commodities, elasticity at the farm level is lower than the elasticity at the retail level, the elasticity of price transmission should be less than one. Table 10 shows that the elasticity of price transmission was less than one for 24 commodities. In two commodities (corn meal and canned tomatoes) the margin behavior is such that one of the special cases (percentage margin) is applicable and when we have constant percentage spread, the elasticity of price transmission is one. Table 11 presents the elasticities at the farm

TABLE 10
ELASTICITY OF PRICE
TRANSMISSION, EXPRESSING THE
PERCENTAGE CHANGE IN RETAIL
PRICE TO A 1 PER CENT CHANGE
IN FARM PRICE

Commodity	Elasticity
<i>Meats</i>	
Beef.....	.648917
Pork.....	.583224
Lamb.....	.636246
Chicken.....	.774894
<i>Eggs</i>	
	.707173
<i>Fats and oils</i>	
Butter.....	.706389
Shortening.....	1.400023
Margarine.....	.819340
Saled dressing.....	.953760
<i>Milk products</i>	
Milk.....	.937509
Evaporated milk.....	2.561220
Cheese.....	2.740635
Ice cream.....	.855069
<i>Potatoes</i>	
White.....	.484788
Sweet.....	.953880
<i>Sugar</i>	
	1.804844
<i>Fruit</i>	
Fresh apples.....	.939406
Fresh oranges.....	.686718
Canned peaches.....	.523480
Dried fruits.....	.893730
<i>Vegetables</i>	
Lettuce.....	.676210
Fresh tomatoes.....	.923259
Beans.....	.918910
Onions.....	.460910
Carrots.....	.378923
Canned peas.....	.979548
Canned corn.....	6.185602
Canned tomatoes.....	1.130774
Dried vegetables.....	.944150
<i>Other</i>	
Wheat flour.....	.814939
Breakfast cereals.....	— .606390
Corn meal.....	1.014740

level obtained as the product of retail-level elasticities in table 5 and the elasticities of price transmission corresponding to 26 commodities. The commodities are retained as at the retail level and no attempt is made to convert them to the equivalent commodities at the farm level.

Seasonal variation in margin behavior

Seasonal variation of prices of many agricultural products differ widely among commodities and commodity groups. For example, fresh fruits and vegetables grown locally during the summer and fall may move directly from farmers to consumers while in winter transportation costs on fresh crops absorb a substantial portion of the consumer's payments. Gale (1961, p. 5) observes, "... seasonal changes (in prices and price spreads) for most fruits and vegetables were relatively large, but those for dairy products were small. Eggs, frying chicken, and meat products tended to fall into an intermediate group in magnitude of seasonal variation. Tomatoes showed the widest seasonal fluctuations in prices and spreads of those products reported on, and fluid milk showed the smallest fluctuations." This study uses a covariance analysis to test the significance of the intercept and slope of the relationship expressing price spreads as a linear function of retail prices in different seasons.

Covariance analysis.—Using the quarterly data for the period 1955–1967, we have specified the following three relationships:

First, all the four seasons are considered together to obtain a single regression equation without incorporating any seasonal factors, or

$$(a) \quad p^{(t)} = a + bp^{(r)}$$

Second, all the four seasons are considered together and it is assumed that the intercept is different in the four seasons but the slope remains the same, or

$$(b) \quad p^{(t)} = d_1D_1 + d_2D_2 + d_3D_3$$

$$+ d_4 D_4 + c p^{(r)}$$

where

$D_1, D_2, D_3,$ and D_4 correspond to four dummy variables such that

$$D_j = \begin{cases} 1 & \text{in the } j^{\text{th}} \text{ quarter} \\ 0 & \text{elsewhere.} \end{cases}$$

Third, it is assumed that both slope and intercept are different during the four seasons, or

$$p_1^{(f)} = a_1 + b_1 p_1^{(r)},$$

$$p_2^{(f)} = a_2 + b_2 p_2^{(r)},$$

(c)

$$p_3^{(f)} = a_3 + b_3 p_3^{(r)}, \text{ and}$$

$$p_4^{(f)} = a_4 + b_4 p_4^{(r)}$$

where

$p_j^{(f)}$ and $p_j^{(r)}$ represent the farm-level and retail-level prices, respectively, in the j^{th} season ($j = 1, 2, 3, 4$).

If $s_1, s_2,$ and s_3 correspond to the sum of squares of deviations corresponding to these three different specifications, we can calculate F -values and test the significance of the differences in these specifications. Only four commodities (chicken, potatoes, apples, and carrots) showed significant variations in margin behavior in both slope and intercept (table 12). In three commodities (pork, shortening, and salad dressing), the intercept did not vary from season to season but the slope did. Two commodities (milk and tomatoes), showed seasonal variation in intercept without any variation in slopes. The remaining 18 commodities included in the covariance analysis showed no change in either slope or intercept over the seasons.

Alternative specifications of margin behavior

In the previous analysis, specification of the marketing margins as a linear function of retail prices was based on the hypothesis that the pricing practices followed by the marketing group was in accordance with this specification. Although, in many cases, this hypothesis gave a reasonable explanation of margin behavior, there is no reason why all commodities should follow the same pattern. Another relationship may give a better result. The approach used in the previous sections to obtain the nature of changes at different levels in the market is still valid under alternate specifications of margin relationship. For example, if we have reason to believe that the price spread is determined as a combination of an absolute spread and a percentage of farm-level price, we can write the margin relationship as

$$m_j = \alpha_j^* + \beta_j^* p_j^{(f)}. \quad (139)$$

As before,

$$p_j^{(f)} + m_j = p_j^{(r)}. \quad (140)$$

That is,

$$p_j^{(f)} + \alpha_j^* + \beta_j^* p_j^{(f)} = p_j^{(r)},$$

$$(1 + \beta_j) p_j^{(f)} = -\alpha_j^* + p_j^{(r)}, \text{ or}$$

$$p_j^{(f)} = -\frac{\alpha_j^*}{(1 + \beta_j)} + \frac{1}{(1 + \beta_j)} p_j^{(r)}, \quad (141)$$

$$= a_j^* + b_j^* p_j^{(r)}.$$

Equation (141) is similar to equation (117). All the remaining relationships can be developed similarly. Again, the relationship assumed here is for aggre-

TABLE
 DEMAND INTERRELATIONSHIPS

Column	Commodities	Beef 1	Pork 2	Lamb & mutton 3	Chicken 4	Eggs 5	Butter 6	Margarine 7	Salad dressing 8
Row									
1	Beef	0.416485	0.048180	0.028207	0.052356	0.000946	0.000233	0.000162	0.000313
2	Veal	0.232434	0.115283	0.042003	0.134494	0.005205	0.001281	0.000892	0.001722
3	Pork	0.049349	-0.240879	0.038272	0.027318	0.001808	0.000445	0.000310	0.000596
4	Lamb and mutton	0.381226	0.519364	-1.670464	0.181006	0.002818	0.000691	0.000483	0.000033
5	Chicken	0.127486	0.070429	0.034397	-0.002348	0.001014	0.000396	0.000275	0.000532
6	Turkey	0.063126	0.038073	0.011325	0.309958	0.001518	0.000380	0.000265	0.000511
7	Fish	0.013527	0.015501	0.001306	0.005254	0.007764	0.001913	0.001331	0.002571
8	Eggs	0.006585	0.005470	0.001234	0.002577	-0.225079	0.002571	0.001789	0.003458
9	Butter	0.001258	0.000177	0.000710	0.000244	0.005384	-0.450860	0.131504	0.000915
10	Lard	0.008593	0.008958	0.001410	0.003489	0.012900	0.001854	0.082326	0.000316
11	Shortening	0.007102	0.007083	0.001285	0.002793	0.016593	0.001587	0.042869	0.120205
12	Margarine	0.007691	0.007708	0.001342	0.003049	0.011470	0.299878	-0.693572	0.110354
13	Salad dressing	0.002252	0.001901	0.000847	0.000735	0.007322	0.000463	0.014670	-0.561597
14	Fresh milk	0.004140	0.002912	0.000938	0.001243	0.007577	0.000995	0.000199	0.000470
15	Evaporated milk	0.007610	0.007582	0.001325	0.002997	0.011144	0.001858	0.000528	0.001057
16	Cheese	0.002057	0.001837	0.000847	0.000841	0.006703	0.000841	0.000125	0.000330
17	Ice cream	0.000939	-0.000139	0.000893	0.000107	0.005114	0.000560	-0.000008	0.000102
18	Potatoes	0.005336	0.007040	0.001111	0.002017	0.009253	0.001292	0.000339	0.000720
19	Sweet potatoes	0.005217	0.002893	0.000790	0.001389	0.011568	0.001895	0.000528	0.001055
20	Sugar	0.007094	0.007014	0.001279	0.002768	0.010874	0.001579	0.000425	0.000263
21	Corn syrup	0.002257	0.003626	0.000997	0.001507	0.008151	0.001997	0.000247	0.000556
22	Apples	0.004866	0.004137	0.001062	0.001899	0.008913	0.001199	0.000302	0.000653
23	Bananas	0.004870	0.004438	0.001064	0.001812	0.008789	0.001211	0.000303	0.000655
24	Oranges	0.002267	0.001243	0.000790	0.000632	0.005853	0.000747	0.000107	0.000307
25	Other fresh fruit	0.000402	-0.001783	0.000547	-0.000503	0.003802	0.000328	-0.000119	0.000181
26	Canned peaches	0.000796	-0.000305	0.000692	0.000026	0.001929	0.000528	-0.000023	0.000076
27	Canned pineapples	-0.001357	-0.002926	0.000462	-0.000927	0.002902	0.000107	-0.000180	-0.000231
28	Dried fruits	0.001293	0.000237	0.000715	0.000250	0.005428	0.000614	0.000018	0.000149
29	Frozen fruits	0.005688	-0.000822	0.000027	-0.002829	-0.001191	-0.000562	-0.000542	-0.000848
30	Lettuce	0.004713	0.004265	0.001050	0.001746	0.003642	0.001185	0.000291	0.000633
31	Tomatoes	0.004247	0.003714	0.001005	0.004511	0.008212	0.001108	0.000252	0.000587
32	Beans	0.007690	0.007732	0.001345	0.003059	0.011468	0.001685	0.000705	0.001058
33	Onions	0.007610	0.007665	0.001334	0.003010	0.011472	0.001678	0.000522	0.001045
34	Carrots	0.001233	0.000138	0.000707	0.000208	0.005366	0.000684	-0.000274	0.000150
35	Other fresh vegetables	0.001660	0.001192	0.001045	0.001721	0.008609	0.001178	0.000145	0.000624
36	Canned peas	0.007053	0.007023	0.001281	0.002773	0.010897	0.001582	0.000604	0.000963
37	Canned corn matoes	0.007214	0.007207	0.001295	0.002839	0.011018	0.001607	0.000552	0.000968
38	Canned to- matoes	0.004169	0.003021	0.000996	0.001508	0.008093	0.001091	0.000247	0.000558
39	Dry vegeta- bles	0.003308	0.002587	0.000911	0.001120	0.007336	0.000651	0.000020	0.000434
40	Frozen vegeta- bles	-0.001776	-0.006946	0.000118	-0.002424	-0.000346	0.000408	-0.000469	-0.000716
41	Other canned fruits and vegetables	0.003648	0.003000	0.000949	0.001276	0.007650	0.001008	0.000205	0.000480
42	Rice	0.006563	0.006450	0.001232	0.002558	0.010335	0.001497	0.000438	0.000897
43	Wheat flour Breakfast cereals	0.006012	0.005798	0.001178	0.002315	0.008886	0.001405	0.000394	0.000817
44	Corn meal Bread and other cereals	0.006527	0.006100	0.001229	0.002542	0.010376	0.001491	0.000435	0.000889
45	Corn meal Bread and other cereals	0.006523	0.004304	0.001230	0.002543	0.010366	0.001489	0.000434	0.000890
46	Coffee	0.007094	0.007780	0.001343	0.003054	0.011484	0.001688	0.000528	0.001056
47	Soup	0.006739	0.006637	0.001250	0.002836	0.010581	0.001627	0.000452	0.000913
48	Other beverage ages	0.003010	0.002132	0.000873	0.000953	0.006818	0.000884	0.000146	0.000376
49	All food	-0.027184	-0.014608	-0.003791	-0.007053	-0.002780	-0.000862	-0.000206	-0.000873
50	Nonfood	-0.014043	-0.016462	-0.000533	-0.000641	-0.002720	-0.001790	-0.001249	-0.002050

AT THE FARM LEVEL.

Fresh milk 9	Ice cream 10	Potatoes 11	Sweet potatoes 12	Apples 13	Oranges 14	Canned peaches 15	Dried fruits 16	Lettuce 17	Tomatoes 18
0.002608	0.000403	0.000190	0.000062	0.000305	0.000270	0.000087	0.000078	0.000126	0.000215
0.014314	0.002217	0.001041	0.000339	0.001678	0.001483	0.000482	0.000428	0.000697	0.001183
0.004983	0.000770	0.000362	0.000118	0.000582	0.000515	0.000157	0.000148	0.000242	0.000412
0.007768	0.001201	0.000564	0.000184	0.000908	0.000803	0.000261	0.000231	0.000477	0.000642
0.004427	0.000681	0.000321	0.000105	0.000518	0.000458	0.000149	0.000132	0.000215	0.000366
0.001253	0.000658	0.000309	0.000101	0.000498	0.000440	0.000142	0.000127	0.000207	0.000351
0.021400	0.003304	0.001555	0.000508	0.002305	0.002214	0.000716	0.000628	0.001037	0.001765
0.028768	0.004142	0.002091	0.000684	0.003368	0.002977	0.000963	0.000858	0.001395	0.002173
0.006781	0.001017	0.000493	0.000161	0.000794	0.000702	0.000227	0.000202	0.000329	0.000559
0.002837	0.000407	0.000191	0.000063	0.000309	0.000273	0.000089	0.000079	0.000127	0.000218
0.005353	0.014839	0.006970	0.002265	0.011215	0.010810	0.003218	0.002862	0.004564	0.007918
0.008184	0.001312	0.000616	0.000200	0.000931	0.000956	0.000285	0.000253	0.000412	0.000700
0.002307	0.000357	0.000168	0.000054	0.000270	0.000260	0.000078	0.000069	0.000112	0.000191
-0.022952	0.000353	0.000166	0.000054	0.000270	0.000258	0.000077	0.000068	0.000111	0.000189
0.201272	0.000049	0.000394	0.000100	0.000490	0.000473	0.000140	0.000125	0.000204	0.000346
0.001172	0.001725	0.001123	0.000368	0.001823	0.001757	0.000523	0.000466	0.000758	0.001287
-0.000858	-0.451141	0.000719	0.000234	0.001158	0.001116	0.000332	0.000296	0.000481	0.000817
0.004423	0.003023	-0.149882	0.057158	0.000252	0.000242	0.000072	0.000064	0.000105	0.000177
0.003730	0.003835	0.203090	-0.496308	0.000469	0.000451	0.000134	0.000120	0.000195	0.000330
0.005503	0.003613	0.000288	0.000078	0.001308	0.001260	0.000375	0.000333	0.000544	0.000923
0.003020	0.002828	0.000067	0.000010	0.000309	0.000385	0.000114	0.000102	0.000166	0.000282
0.003850	0.002864	0.000120	0.000026	-0.678372	0.175674	0.001541	0.000133	0.000078	0.000133
0.002899	0.002868	0.000121	0.000028	0.200558	0.000316	0.019000	0.000878	0.000419	0.000710
0.000898	0.002025	-0.000069	-0.000033	0.151492	-0.455418	0.007360	0.000042	0.001146	0.001946
-0.002547	0.001055	-0.000287	-0.000102	0.001258	0.001823	0.001756	0.001485	0.000406	0.000690
-0.001089	0.001466	-0.000194	-0.000072	0.000421	0.021173	-0.480152	0.057362	0.000689	0.001137
0.004793	0.000725	-0.000391	-0.000125	0.115358	0.000000	0.008571	0.000000	0.000045	0.000076
-0.000409	0.001644	-0.000151	-0.000060	0.000000	0.000000	0.007648	-0.585668	0.000222	0.000378
-0.008976	-0.000784	-0.000097	-0.000230	0.002522	0.002603	0.005798	0.003276	0.001615	0.002742
0.003675	0.002813	0.000199	0.000023	0.000171	0.003454	0.000308	0.000307	-0.095810	0.043187
0.003103	0.002651	0.000253	0.000010	0.000412	0.003291	0.000766	0.000283	0.024592	-0.355085
0.007290	0.003837	0.000400	0.000061	0.000540	0.004490	0.001070	0.000455	0.007386	0.000283
0.007165	0.003803	0.000333	0.000093	0.000530	0.004459	0.001062	0.000450	0.000135	0.000751
-0.000469	0.001894	-0.000158	-0.000059	-0.000254	0.002360	0.000523	0.000304	0.013592	0.019264
0.003606	0.002793	0.000104	0.000021	0.000168	0.003434	0.000802	0.000423	0.001689	0.002426
0.006504	0.003614	0.000289	0.000078	0.000460	0.004284	0.001013	0.000590	-0.083321	0.000226
0.008712	0.003672	0.000302	0.000083	0.000480	0.004324	0.001028	0.000431	0.000116	0.000244
0.003024	0.002630	0.000066	0.000009	0.000104	0.003271	0.000761	0.000251	-0.145053	0.000000
0.001967	0.002330	0.000000	-0.000011	-0.000004	0.002964	0.000683	0.000237	0.018063	0.000000
-0.007860	-0.000449	-0.000625	-0.000208	-0.001053	0.000154	-0.000028	-0.000107	0.008449	0.014829
0.002375	0.004445	0.000026	-0.000004	0.000038	0.003082	0.000713	0.000174	0.000272	0.000406
0.005935	0.003453	0.000252	0.000067	0.000401	0.004099	0.000971	0.000399	0.000271	0.000265
0.005248	0.003259	0.000209	0.000053	0.000332	0.003905	0.000922	0.000371	0.000239	0.000313
0.005875	0.003435	0.000249	0.000066	0.000285	0.004084	0.000967	0.000297	0.000268	0.000361
0.005586	0.003432	0.000249	0.000066	0.000285	0.004083	0.000966	0.000297	0.000268	0.000360
0.007295	0.003836	0.000339	0.000094	0.000540	0.004490	0.001070	0.000493	0.000336	0.000469
0.006132	0.003507	0.000265	0.000072	0.000422	0.004157	0.000986	0.000408	0.000250	0.000380
0.004482	0.002192	-0.000021	-0.000022	-0.000054	0.002827	0.000645	0.000216	0.000058	0.000020
0.001633	0.002237	-0.000021	-0.000018	-0.000038	0.002371	0.000660	0.000223	0.000066	0.000038
-0.023140	-0.007295	-0.001013	-0.000088	-0.001929	-0.003782	-0.001422	-0.001126	-0.000683	-0.001675
-0.020947	-0.003160	-0.001456	-0.000173	-0.002343	-0.002258	-0.000740	-0.000632	-0.001005	-0.001654

TABLE 11 (Continued)

Column	Commodities	Beans 19	Onions 20	Carrots 21	Canned peas 22	Canned tomatoes 23	Dry vegetables 24	Wheat flour 25	Corn meal 26
Row									
1	Beef	0.000110	0.000045	0.000023	0.000216	0.000100	0.000097	0.000262	0.000131
2	Veal	0.000854	0.000245	0.000127	0.001190	0.000602	0.000533	0.001552	0.000739
3	Pork	0.000237	0.000085	0.000041	0.000413	0.000209	0.000135	0.000539	0.000257
4	Lamb and mutton	0.000355	0.000122	0.000069	0.000645	0.000326	0.000289	0.000811	0.000401
5	Chicken	0.000203	0.000076	0.000039	0.000367	0.000186	0.000164	0.000479	0.000228
6	Turkey	0.000194	0.000072	0.000038	0.000353	0.000178	0.000156	0.000460	0.000219
7	Fish	0.000972	0.000367	0.000189	0.001776	0.000894	0.000798	0.002314	0.001099
8	Eggs	0.001308	0.000403	0.000255	0.002398	0.001302	0.001073	0.003191	0.001478
9	Butter	0.000308	0.000116	0.000060	0.000563	0.000263	0.000263	0.000733	0.000348
10	Lard	0.000119	0.000045	0.000023	0.000218	0.000110	0.000098	0.000285	0.000135
11	Shortening	0.004360	0.001635	0.000590	0.007345	0.004010	0.003574	0.010379	0.004933
12	Margarine	0.000386	0.000145	0.000076	0.000702	0.000357	0.000316	0.000918	0.000436
13	Salad dressing	0.000196	0.000040	0.000022	0.000194	0.000097	0.000087	0.000250	0.000119
14	Fresh milk	0.000104	0.000039	0.000021	0.000189	0.000096	0.000085	0.000217	0.000118
15	Evaporated milk	0.000184	0.000071	0.000039	0.000347	0.000176	0.000158	0.000453	0.000217
16	Cheese	0.000710	0.000266	0.000145	0.001292	0.000657	0.000581	0.001687	0.000802
17	Ice cream	0.000451	0.000169	0.000092	0.000821	0.000417	0.000369	0.001072	0.000509
18	Potatoes	0.000098	0.000037	0.000020	0.000178	0.000090	0.000080	0.000232	0.000111
19	Sweet potatoes	0.000183	0.000069	0.000037	0.000332	0.000168	0.000149	0.000432	0.000207
20	Sugar	0.000509	0.000191	0.000104	0.000927	0.000471	0.000416	0.001210	0.000575
21	Corn syrup	0.000155	0.000059	0.000032	0.000283	0.000144	0.000127	0.000369	0.000178
22	Apples	0.000074	0.000028	0.000015	0.000134	0.000068	0.000060	0.000174	0.000083
23	Bananas	0.000392	0.000147	0.000080	0.000712	0.000362	0.000320	0.000931	0.000443
24	Oranges	0.001074	0.000402	0.000219	0.001253	0.000993	0.000878	0.002552	0.001213
25	Other fresh fruit	0.000380	0.000142	0.000078	0.000692	0.000352	0.000311	0.000904	0.000430
26	Canned peaches	0.000827	0.000235	0.000128	0.001140	0.000580	0.000513	0.001490	0.000708
27	Canned pineapples	0.000042	0.000016	0.000009	0.000076	0.000039	0.000034	0.000099	0.000017
28	Dried fruits	0.000209	0.000078	0.000042	0.000379	0.000193	0.000171	0.000496	0.000236
29	Frozen fruits	0.001513	0.000566	0.000308	0.002752	0.001400	0.001238	0.003595	0.001709
30	Lettuce	0.028928	0.000090	0.003305	-0.122524	-0.133009	0.016239	0.000363	0.000173
31	Tomatoes	0.000090	0.000079	0.002569	0.000000	0.000003	0.000035	0.000196	0.000092
32	Beans	-0.234322	0.000001	0.001151	0.000044	0.038778	0.000174	0.000221	0.000105
33	Onions	0.000000	-0.115228	0.012808	0.018323	0.008979	0.000000	0.000166	0.000079
34	Carrots	0.003489	0.015736	-0.168363	0.000000	0.000670	0.000000	0.000263	0.000125
35	Other fresh vegetables	0.000005	0.000218	0.000478	0.000111	0.000109	0.000006	0.000170	0.000081
36	Canned peas	0.000000	0.000611	0.000065	-0.181218	0.033982	0.015854	0.000307	0.000146
37	Canned corn	0.003662	0.000000	-0.018944	0.057794	0.000127	0.058829	0.000299	0.000142
38	Canned to- matoes	0.022485	-0.001863	0.001565	0.055223	-0.176000	0.000035	0.000097	0.000040
39	Dry vegeta- bles	0.000000	0.000059	0.000023	0.023705	0.000000	-0.453192	0.000125	0.000051
40	Frozen vegeta- bles	-0.006563	-0.002168	0.002029	0.012397	0.007597	0.007178	0.002390	0.001134
41	Other canned fruits and vegetables	0.000125	0.000005	0.000111	0.000601	0.000249	0.000433	0.001511	0.000718
42	Rice	0.000046	0.000011	0.000092	0.000197	0.000144	0.000183	0.072360	0.000002
43	Wheat flour	0.000023	0.000002	0.000054	0.000150	0.000119	0.000160	-0.244382	0.000466
44	Breakfast cereals	0.000044	0.000010	0.000091	0.000192	0.000143	0.000181	0.000049	0.000090
45	Corn meal	0.000044	0.000010	0.000091	0.000192	0.000142	0.000181	0.000647	-0.220090
46	Bread and other cereals	0.000093	0.000028	0.000107	0.000281	0.000195	0.000241	0.001029	0.000237
47	Coffee	0.000053	0.000013	0.000094	0.000210	0.000152	0.000191	0.001232	0.000361
48	Soup	-0.000106	-0.000045	0.000044	-0.000106	-0.000019	0.000026	0.000597	0.000111
49	Other bever- ages	-0.000100	-0.000043	0.000045	-0.000095	-0.000013	0.000031	0.000615	0.000119
50	All food	-0.000130	-0.000056	-0.000035	-0.000461	-0.000762	-0.000863	-0.001127	-0.000408
51	Nonfood	-0.000912	-0.000341	-0.000177	-0.001660	-0.000876	-0.000785	-0.002168	-0.001031

gate spreads. If we are interested to study the breakdown of aggregate spreads to different elements in the marketing system, we can specify appro-

appropriate behavioral assumptions for price spreads at each of these stages and carry out an analysis similar to that for the aggregate spread.

Income-Consumption Relationships

This section illustrates different approaches to income-consumption relationships, using the 1955 and 1965 household food-consumption data obtained from the USDA (1956, 1968). In particular, it analyzes the effects of income, household size, and region in determining the consumption pattern, and explains the changes in demand coefficients as a result of using quantity consumed or expenditures on different items as the dependent variable.

Estimation of income elasticities

Choice of data.—Often the effect of income changes on consumption is measured by the elasticity of demand with respect to income (income elasticity) defined as the ratio of relative change in the quantity consumed to the relative change in income, holding other factors influencing demand at a constant level. Thus, the income elasticity for the i^{th} commodity is given by

$$e_{iy} = \frac{\partial q_i}{\partial y} \cdot \frac{y}{q_i}$$

where

- $q_i = f(p_1, p_2, \dots, p_n, y)$
 q_i = consumption of i^{th} commodity,
 p_i = price of i^{th} commodity, and
 y = income.

Both time-series data from market statistics and cross-section data from

¹⁹ Strictly speaking, in the cross-section data, apart from income, we can expect influences of variables like household size, education, social status, and similar other variables. Often classification of data according to all these factors may be difficult. (See Clark *et al.* (1954) and Rockwell (1959).

household food consumption surveys are used to obtain income elasticities. For time-series data, we specify a form of the demand function in terms of prices and income and obtain the coefficients through a regression equation. From this demand function, the income elasticity can be obtained as a partial derivative w.r.t. income when a double logarithmic form is used. For cross-section data, prices remain approximately the same for all consumers and, therefore, we can specify the demand relationship omitting prices.¹⁹ However, it is possible to

TABLE 12
SEASONAL VARIATION IN THE
RELATION BETWEEN FARM-LEVEL
PRICE AND RETAIL-LEVEL PRICE

Commodities with variation in slope and/or intercept	Commodities without variation in slope or intercept
<i>Intercept and slope</i>	Beef
Chicken‡	Lamb
Potatoes*	Eggs
Apples*	Butter
Carrots‡	Margarine
<i>Intercept only</i>	Evaporated milk
Milk†	Cheese
Tomatoes*	Ice cream
<i>Slope only</i>	Sugar
Pork†	Oranges
Shortening*	Canned peaches
Salad dressing*	Lettuce
	Onions
	Canned peas
	Canned corn
	Canned tomatoes
	Wheat flour
	Breakfast cereal

* Significant at 5 per cent level.

† Significant 10 per cent level.

‡ Significant at 5 per cent for intercept, 10 per cent for slope.

give two different interpretations for the demand elasticities obtained from these two methods. In a static analysis, we assume that the consumer makes a change in consumption, if there is any, as soon as there is a change in income. A lag may exist in consumption adjustments as a result of income changes. Generally, the items included in family budgets have different durability and the purchases may represent a long-term consumption pattern. On the other hand, a short-term expansion in income may lead the consumer to acquire goods for immediate consumption. Wold and Jureen (1964, pp. 227-228) point out that, for the majority of consumers, the income level is fairly stable. For a group of families included in the cross-section data, the changes in income over time are, on the whole, small and infrequent compared with existing income differences between the families in the group. Therefore, "the families have usually adapted themselves to the income level at which they have been recorded, so that the budget data primarily reflect the demand pattern in the sense of long run income changes. In other words, the income elasticities derived from family budget data can most immediately be interpreted as long run elasticities." From the point of view of practical applications of demand analysis, these long-term elasticities are more relevant for many policy decisions than the short-term elasticities obtained from time-series data.²⁰ This is one reason why we chose income elasticities obtained from budget studies over those from time-series data. Other reasons were:

(1) Time-series data on prices and income are highly correlated which makes it difficult to obtain structural income elasticities from time-series data.

(2) Estimates of income elasticities may differ, depending on whether we use quantity or expenditure as the measure of demand. Comparing the estimates obtained from these two measures of demand, we can draw certain conclusions regarding the quality consciousness of consumers. It is often difficult to obtain consistent time-series data on these two measures of demand because of limitations in collecting and reporting data while, in a budget study, it is easy to incorporate these two measures.

(3) Demand elasticity may shift over time. When using time-series analysis, often we assume fixed coefficients if we do not include some shift variables. It is difficult to estimate the coefficients, using an assumption of varying parameters. If we have data for different cross sections, it may be possible to estimate one set of coefficients for each cross section and, then, taking these coefficients, we can estimate the trends in the coefficients if they follow a systematic patterns.

(4) To compare the elasticity measures for different commodities, the data must be comparable in other respects. If we are using time-series data to obtain income elasticities for different commodities, it is difficult to assure consistency in a number of other factors. Therefore, to obtain meaningful comparisons of income elasticities that can be used to assess the changes of different commodities due to an income change, it is desirable to keep a number of other variables constant. This is effectively handled in cross-section analysis because it provides a measure of the reaction of consumers' demand to changes in income without complications of changes in distribution of income, family size, and other social, economic, and demo-

²⁰ Wold and Jureen (1964, pp. 228-230) point out that irreversibility of demand functions and continued introduction of new products tend to make the income elasticities of family budget data smaller than the income elasticities obtained from time-series data.

graphic factors which are present in the time-series data.

We also chose to use cross-section data because detailed data on consumption and income were available from at least two nationwide surveys (in 1955 and 1965) on food consumption. The 1965 survey had a broad objective and, at the time of this analysis, only data relating to the spring quarter were available. Therefore, the present analysis is restricted to these data.

Choice of function.—To calculate the income elasticities, the first job is to specify a functional relationship of consumption and income. Various functional forms may be appropriate as for time-series analysis. Assuming that the effect of all other variables are negligible, a linear relationship between demand and income can be specified as

$$q = a + by. \quad (142)$$

In particular, when the variables q and y are in logarithms, the coefficient b is the income elasticity. If the data show the presence of a unit serial correlation coefficient, first differences of the variables will get rid of this serial correlation. In this case, it is convenient to use a regression equation of the form

$$\Delta q = a + b \Delta y \quad (143)$$

where

Δq and Δy represent the first differences of the variables.²¹

Again, the income elasticities are obtained directly if the variables are specified in logarithms as

$$\Delta (\log q) = a + b \Delta (\log y). \quad (144)$$

²¹ In time-series data, we define $\Delta q_t = (q_t - q_{t-1})$ while, using cross-section data, we have to take the differences between successive observations (samples) during the same period. If the autocorrelation coefficient is other than unity, other methods of estimation are more appropriate.

²² The per-capita income is obtained by dividing the family income by the number of persons in the family. The family income represents the family's 1964 money income after deduction of state and federal income taxes. See Appendix A for a description of the data and their sources.

Ordinary versus weighted regression.—Published data on food consumption from the cross-section surveys are reported by income groups. The effects of induced heteroskedasticity can be eliminated by using a weighted regression. In the present study, we have calculated the income elasticity for different commodities, using both ordinary and weighted regression procedures. We have used the per-capita quantity consumed and per-capita income²² for obtaining a regression equation of the form

$$\log q = a + b \log y. \quad (145)$$

On the whole, the estimates of income elasticity obtained from both ordinary and weighted regressions were not very different, but in many cases where the elasticities were positive, coefficients from weighted regressions were slightly higher than those from ordinary regressions, a result in conformity with that of Iyengar (1964). Table 13 gives the estimates obtained from these two methods. Further analysis of these data, with household size an added variable, indicates somewhat different estimates, as is discussed later.

Consumption differentials.—To show the relationship between the consumption differences in the upper and lower-income groups and the income elasticity, a bivariate classification of commodities was constructed as table 14. The income elasticities used are the same as in column 2 of table 13. The quantity index is obtained by expressing the average per-capita consumption as a percentage of the per-capita consumption of persons in the lowest income-group (0-1,000

TABLE 13
COMPARISON OF INCOME ELASTICITIES OBTAINED FROM ORDINARY AND
WEIGHTED REGRESSIONS BASED ON HOUSEHOLD FOOD CONSUMPTION
SURVEY DATA, SPRING, 1965

Commodity	Income elasticity		
	Ordinary regression	Weighted regression	Deviations*
Beef	.270	.290	.020
Veal	.551	.591	.040
Pork	.008	.008	.000
Lamb and mutton	.591	.571	-.020
Chicken	-.034	-.037	-.003
Fish	-.060	-.036	-.024
Turkey	.788	.789	.001
Eggs	-.072	-.078	-.006
Butter	.269	.318	.049
Lard	-1.297	-1.437	-.140
Shortening	.029	-.008	-.037
Margarine	-.006	-.022	-.016
Salad dressing	.284	.285	.001
Fresh milk	.367	.377	.010
Evap. milk	-.610	-.674	-.064
Cheese	.227	.249	.022
Ice cream	.323	.331	.008
Potatoes	.016	.008	-.008
Sweet potatoes	-.504	-.587	-.083
Sugar	-.169	-.190	-.021
Corn syrup	-.708	-.756	-.050
Apples	.142	.140	-.002
Oranges	.227	.260	.033
Bananas	.133	.139	.004
Canned peaches	.012	-.002	-.014
Canned pineapples	.408	.447	.039
Dry fruit	-.031	-.043	-.012
Frozen fruit	.424	.061	.037
Fresh tomatoes	.161	.170	.009
Fresh beans	-.481	-.495	-.014
Onions	.005	-.003	-.008
Carrots	.313	.319	.006
Lettuce	.424	.446	.022
Canned peas	.043	.032	-.011
Canned corn	.054	.029	-.025
Canned tomatoes	.165	.173	.008
Dry vegetables	-.818	-.914	-.096
Frozen vegetables	.577	.616	.039
Wheat flour	-.631	-.685	-.054
Rice	-.605	-.651	-.046
Breakfast cereals	.056	.058	.002
Corn meal	-1.059	-1.143	-.084
Coffee	-.047	.047	.094
Soup	.216	.236	.020
All food	.277	.304	.027

* Deviations are calculated by taking the difference between weighted regressions and ordinary regressions.

dollars). Thus, a quantity index of less than 100 means that the per-capita consumption in the lowest-income group is higher than average consumption levels. The results in table 14 are useful to persons engaged in marketing farm com-

modities because it provides a framework to determine the nature of emphasis to be placed on different income groups while designing a program for promoting the sale of a particular commodity.

TABLE 14
BIVARIATE CLASSIFICATION OF COMMODITIES ACCORDING TO INCOME
ELASTICITY AND CONSUMPTION INDEX*

Consumption Index	Income Elasticity						Total
	Negative	0-.1	.1-.2	.2-.3	.3-.4	>.4	
0-50	Lard, evaporated milk, corn syrup, fresh beans, dry vegetables, wheat flour, rice, corn meal (8)						(8)
50-100	Chicken, fish, eggs, sweet potatoes, sugar, margarine, dry fruits (7)						(7)
100-150		Pork, shortening, onions, canned peas, canned corn, breakfast cereals, coffee, potatoes, canned peaches (9)	Apples, bananas, canned tomatoes (3)	Butter, cheese, oranges (3)			(15)
150-200			Fresh tomatoes (1)	Beef, salad dressing, soup (3)	Ice cream, carrots (2)	Canned pineapple (1)	(7)
200-250					Fresh milk (1)		(1)
250-300						Lettuce, veal, frozen vegetables (3)	(3)
>300						Lamb, turkey, frozen fruits (3)	(3)
Total	(15)	(9)	(4)	(6)	(3)	(7)	(44)

* The consumption index is derived by expressing the average quantity consumed as a percentage of the quantity consumed by the lowest income group.

Quantity versus quality elasticity.— While discussing the income elasticity, we have deliberately defined income elasticity as the ratio of relative change in the quantity consumed and the rela-

tive change in income. It is possible to define demand in terms of expenditures on a particular commodity or in terms of quantities of the commodity consumed. When we use these different variables as

the dependent variable in the demand equation, we get two different estimates of income elasticities, namely, "the elasticity of expenditures with respect to income" and the "elasticity of quantities consumed with respect to income." The difference between these two types of elasticities can be interpreted as a measure of the quality consciousness of the consumers, because the quality of a product and its price can be assumed to be directly correlated. Quality here merely assumes that the consumer has at least a subjective reason to rank different varieties as superior and inferior.

Let x denote the expenditure on a commodity which is the product of the quantity consumed (q) and its price (p).

$$x = pq. \quad (146)$$

The elasticity of expenditures with respect to income (hereafter referred to as expenditure elasticity) is defined as

$$e_{xy} = \frac{\partial x}{\partial y} \cdot \frac{y}{x}. \quad (147)$$

From (146) and (147),

$$e_{xy} = \frac{\partial(pq)}{\partial y} \frac{y}{pq} \quad (148)$$

But

$$\frac{\partial(pq)}{\partial y} = p \frac{\partial q}{\partial y} + q \frac{\partial p}{\partial y}.$$

Substituting in (148)

$$\begin{aligned} e_{xy} &= \left(p \frac{\partial q}{\partial y} + q \frac{\partial p}{\partial y} \right) \frac{y}{pq} \\ &= \frac{\partial q}{\partial y} \frac{y}{q} + \frac{\partial p}{\partial y} \frac{y}{p}. \end{aligned} \quad (149)$$

The first term on the right-hand side of (149), by definition, is the elasticity of quantity consumed with respect to income (quantity elasticity). The second term gives the relative change of price with respect to the relative change in income. As we have assumed that the quality of a commodity and its prices are directly correlated, the higher the price, the higher the quality and therefore, the relative changes in prices can be interpreted as relative changes in qualities.²³ Thus, the second term on the right-hand side of (149) can be taken as the ratio between the relative change in quality and relative change in income, which can be referred to as the quality elasticity. So we have split the expenditure elasticity as the sum of two terms—one representing quantity elasticity and the other representing quality elasticity (See Gerra 1959, p. 149). The quality elasticity can be taken as a measure of consumers' desire for improved quality, given the present average or standard quality. In general, it is expected that the quality elasticity is positive, because higher-income groups tend to consume more expensive or fancy grades and varieties. Also, upgrading of diets, with increases in income, is reflected in the fact that changes in quantities consumed

²³ We have excluded the possibility that consumption of food commodities is influenced by snob appeal or scarcity. Although we recognize that, in reality, higher prices need not necessarily reflect higher quality, we assume that, for the market as a whole, higher average prices imply higher average quality. Scitovsky (1945, p. 100) supports this view as, "Economists are wont to minimize the importance of this factor (price) fearing the havoc it may wreak with the whole theory of choice. But 'mass observation' of one's friends and [their] wives shows that more often than not people judge quality by price. The word 'cheap' usually means inferior quality nowadays; and in the United States 'expensive' is in the process of losing its original meaning and becoming a synonym for superior quality."

may not be so large as changes in expenditures on food items. In the present analysis, the majority of the commodities met this expectation, though some of them did not (see table 15).

TABLE 15
COMPARISON OF QUANTITY AND QUALITY ELASTICITIES BASED ON
HOUSEHOLD FOOD CONSUMPTION SURVEY DATA, SPRING, 1965*

Commodity	Expenditure Elasticity	Quantity Elasticity	Quantity Elasticity
Beef	.380	.270	.110
Veal	.698	.551	.147
Pork	.130	.608	.122
Lamb and mutton	.876	.591	-.015
Chicken	.056	-.034	.090
Fish	.140	-.080	.200
Turkey	.869	.768	.101
Eggs	-.016	-.072	.066
Butter	.274	.268	.005
Lard	-1.238	-1.297	.081
Shortening	-.040	.026	-.060
Margarine	.051	-.006	.057
Salad dressing	.401	.294	.117
Fresh milk	.349	.367	-.018
Evap. milk	-.640	-.610	-.030
Cheese	.241	.227	.014
Ice cream	.335	.323	.012
Potatoes	.030	.016	.020
Sweet potatoes	-.293	-.504	.311
Sugar	-.178	-.169	-.009
Corn syrup	-.569	-.706	.137
Apples	.207	.142	.065
Oranges	.192	.227	-.035
Bananas	.140	.125	.005
Canned peaches	.009	.012	-.003
Canned pineapples	.475	.408	.067
Dry fruits	-.037	-.021	-.006
Fresh tomatoes	.278	.161	.117
Fresh beans	-.421	-.491	.050
Onions	.023	.005	.017
Carrots	.268	.313	-.045
Lettuce	.348	.424	-.076
Canned peas	.110	.043	.067
Canned corn	.065	.064	.011
Canned tomatoes	.151	.185	-.014
Dry vegetables	-.641	-.818	.177
Frozen vegetables	.619	.577	.042
Wheat flour	-.611	-.631	.020
Rice	-.319	-.605	.286
Breakfast cereals	.195	.056	.139
Corn meal	-1.012	-1.058	.046
Coffee	.011	.047	-.036
Soup	.228	.216	.012

*Based on ordinary regression.

The two assumptions, that prices reflect quality, and that income and quality of the goods consumed are positively correlated, lead to the conclusion that persons in the higher-income group pay more than those in lower-income groups for the same quantity. To see whether

this is true, we took the 1965 consumption data in terms of quantities and expenditures and assumed that their ratio represents the price. Taking this estimate of price, we obtained indices of prices paid by different groups with the lowest group price as the base. We found

TABLE 16

CLASSIFICATION OF COMMODITIES ACCORDING TO QUALITY ELASTICITY AND DIFFERENCE IN PRICES PAID BY UPPER AND LOWER INCOME GROUPS

Differential Price Index*	Quality Elasticity				Total
	Negative	0-.05	0.051-.10	>.10	
Negative	Lamb, shortening, fresh milk, evaporated milk, sugar, oranges, dry fruits, carrots, lettuce, canned tomatoes, coffee (11)	Ice cream, corn meal (2)			(13)
0-15	Canned peaches (1)	Butter, cheese, potatoes, bananas, fresh beans, canned corn, wheat flour, soup (8)	Eggs, margarine (2)	Turkey, onions, corn syrup (3)	(14)
15-30		Frozen vegetables (1)	Apples, canned peas (2)	Pork (1)	(4)
30-50			Chicken, canned pineapple (2)	Beef, veal, salad dressing, fresh tomatoes, breakfast cereals (5)	(7)
>50			Lard (1)	Fish, sweet potatoes, dry vegetables, rice (4)	(5)
Total	(12)	(11)	(7)	(13)	(43)†

* See the text for the definition of differential price index.
† Frozen fruits excluded.

large variations among these indices for different commodities. Table 16 compares the quality elasticities obtained from table 15 and the calculated price indices.

The price index in table 16 represents the difference between the prices paid by the upper income group and the lower income group expressed as a percentage of the prices paid by the lower income group. Thus, a negative price differential implies that the price paid by the higher income group is less than the price paid by the lower income group. Incidentally, most of the commodities with negative quality elasticity are those

with a negative price differential. Although there is an explanation for the negative quality elasticity from this behavior, it may not be proper to conclude that high income groups pay lower unit prices for these commodities because of lower quality. It is possible that quality may be only one factor influencing prices—the prices paid by consumers may be a function of many other factors like quantity purchased, nature of demand for the commodity, the availability of substitute products, and family economies of scale. For example, if distributors offer quantity discounts, persons in the higher income group can buy

in-bulk quantities to obtain a lower unit cost compared to persons in the lower income group who can afford to buy only small quantities. In other words, it is not possible to ascertain whether the price variation from one income group to the next is due to such influences as differences in quality of the products purchased, extensive buying habits, or heavy purchases from farms to avoid regular marketing channels. Since the effects of these different factors are not the same for all commodities, it would be useful to analyze each case separately to explain the factors influencing the negative price differential. However, because such detailed analysis would require data collected specifically for this purpose, we have excluded it from the present study.

Effect of household size

So far, the analysis is based on the assumption that income is the only variable influencing consumption in a cross-section analysis. Although a number of other factors may also influence the level of consumption, the data available for this analysis contained only information on quantity consumed, expenditures, income, and household size. In studies using earlier cross-section data, it was found that there were economies of scale in food use in large households. Analyzing 1955 consumption survey data for individual households, Rockwell (1959, p. v) observes, "total household consumption of food increases with increases in the number of persons in the household, but consumption per person declines as household

size increases." Therefore, we included the household size variable to test whether the 1965 survey data show a similar result. We used both expenditure and quantity as dependent variables and obtained a regression equation of the type

$$\begin{aligned} \log q &= a + b \log y + c \log s, \\ &\text{and} \\ \log x &= a' + b' \log y + c' \log s \end{aligned} \quad (150)$$

where

- q = per capita quantity consumed,
- x = per capita expenditure,
- y = per capita income, and
- s = household size. (measured in terms of adult equivalent units, with one unit equal to 21 meals eaten at home.)

For comparison, the results obtained from the two different forms of regression equations are tabulated for quantity and expenditure data in Appendix tables A-1 and A-2.

Significance of the coefficients.—

Table 17 presents a summary of the results from Appendix tables A-1 and A-2. A negative coefficient indicates that here exist economies of scale in food use associated with household size while a positive sign indicates the opposite tendency.²⁴ As expected, total food consumption per capita has a negative coefficient associated with household size although the coefficient itself is not significantly different from zero. When using quantity as the dependent variable, 24 coefficients were positive and 20

²⁴ Such economies can be attributed to a number of factors: (a) holding household income constant, income per person decreases as family size increases with a higher number of children in such family units with different consumption patterns. For example, in table 17, milk has a positive coefficient associated with family size whereas coffee has a negative coefficient; this pattern probably is associated with family composition; (b) there may be savings in expenditures per person associated with buying large quantities at lower unit prices; and (c) there may be a reduction in leftover food on a per-capita basis for large family units.

TABLE 17
EFFECT OF HOUSEHOLD SIZE ON CONSUMER PURCHASES, QUANTITY,
AND EXPENDITURES

Commodity group	Sign of coefficient associated with household size with dependent variable as quantity or expenditure			
	Both positive	Quantity (+) Expenditure (-)	Quantity (-) Expenditure (+)	Both negative
Meats	Beef Veal	Turkey	Pork	Lamb and mutton Chicken Fish
Eggs.....		Eggs		
Fats and oils.....	Lard Shortening Salad dressing	Margarine		Butter
Milk	Fresh milk Evaporated milk Ice cream			Cheese
Potatoes.....	White potatoes Sweet potatoes			
Sugar and syrup.....	Sugar			Corn syrup
Fruit	Frozen fruit (quantity only)	Oranges Canned peaches		Apples Bananas Canned pineapple Dried fruit
Vegetables.....	Carrots Canned peas Canned corn	Canned tomatoes Dried vegetables		Fresh tomatoes Fresh beans Onions Lettuce Fresh vegetables
Other.....	Wheat flour Soup			Rice Breakfast cereals Corn meal Coffee
All food.....				All food (expenditure only)

coefficients were negative. Only five coefficients were significant at the 5 per cent level and another six coefficients were significant at the 10 per cent level (see table 18).

Existence of economies in consumption was more revealing in the case of consumption expenditures of different food items—26 coefficients were negative with 6 of them being significant. Only three positive coefficients were significant.

Effects on quality elasticity. A comparison of tables 16, A-1, and A-2 shows that, as a result of the introduction of household size in the regression equation, the sign of quality elasticity has changed from negative to positive for seven commodities (lamb, fresh milk, sugar, corn syrup, oranges, canned peaches, and canned tomatoes.) However, six commodities (shortening, evaporated milk, dried fruit, carrots, lettuce, and coffee) did not change the negative

TABLE 18
 COMMODITIES WITH SIGNIFICANT COEFFICIENTS ASSOCIATED WITH
 HOUSEHOLD SIZE

Sign of coefficients	Quantity as dependent variable		Expenditure as dependent variable			
	Level of significance		Total	Level of significance		Total
	5%	10%		5%	10%	
Positive....	Salad dressing Frozen fruit Canned corn	Veal Shortening Fresh milk Potatoes	7	Salad dressing	Shortening Canned corn	3
Negative.....	Dried fruit Coffee	Lamb Breakfast cereal	4	Lamb Canned pineapple Dried fruit Coffee	Chicken Apples	6
TOTAL.....	5	6	11	5	4	9

quality elasticity reported in table 16, and corn meal, which had a small positive quality, has changed its sign to negative. Thus, when household size also is included as a variable in the regression equation, the number of commodities with negative quality elasticity is reduced from 13 to 7. Incidentally, this suggests that the presence of negative quality elasticity, in the case of these commodities, can be explained in terms of some other variables excluded from the demand equations. Isolation of such additional variables influencing the demand for these commodities may require an exhaustive analysis of individual commodity demand.

Comparison of 1955 and 1965 consumption

When data from different cross sections are compared, it is necessary to take into account any differences in the population characteristics, sampling procedures, and other structural changes that have occurred during the interval between the cross sections. In the United States, nationwide surveys on food consumption of families were conducted in

1936 (U. S. Bureau of Labor Statistics, 1940, and USDA, 1941a and 1941b), 1942 (USDA, 1944), 1948 (Clark, *et al.*, 1954), 1955 (USDA, 1956), and 1965 (USDA, 1968). There are a number of conceptual and methodological problems, discussed in detail by Clark, *et al.*, 1954, and Bush, 1961, such as differences in the universe covered, sampling design, and the type of information gathered associated with comparing the data from these cross sections. In particular, it is difficult to adjust the data from 1936, 1942, and 1948 to a comparable form with those of 1955 and 1965. There are a number of factors like the season, universe, and size of sample that are in common with the 1955 and 1965 spring cross-section surveys. Therefore, we have used only these two cross-section data for comparison purposes.

Changes in the level of consumption.—Using the per-capita consumption of different commodities during this period (Appendix table A-3), table 19 shows the direction of change in consumption of different commodities for the aggregate of all households and by low, medium, and high income groups.

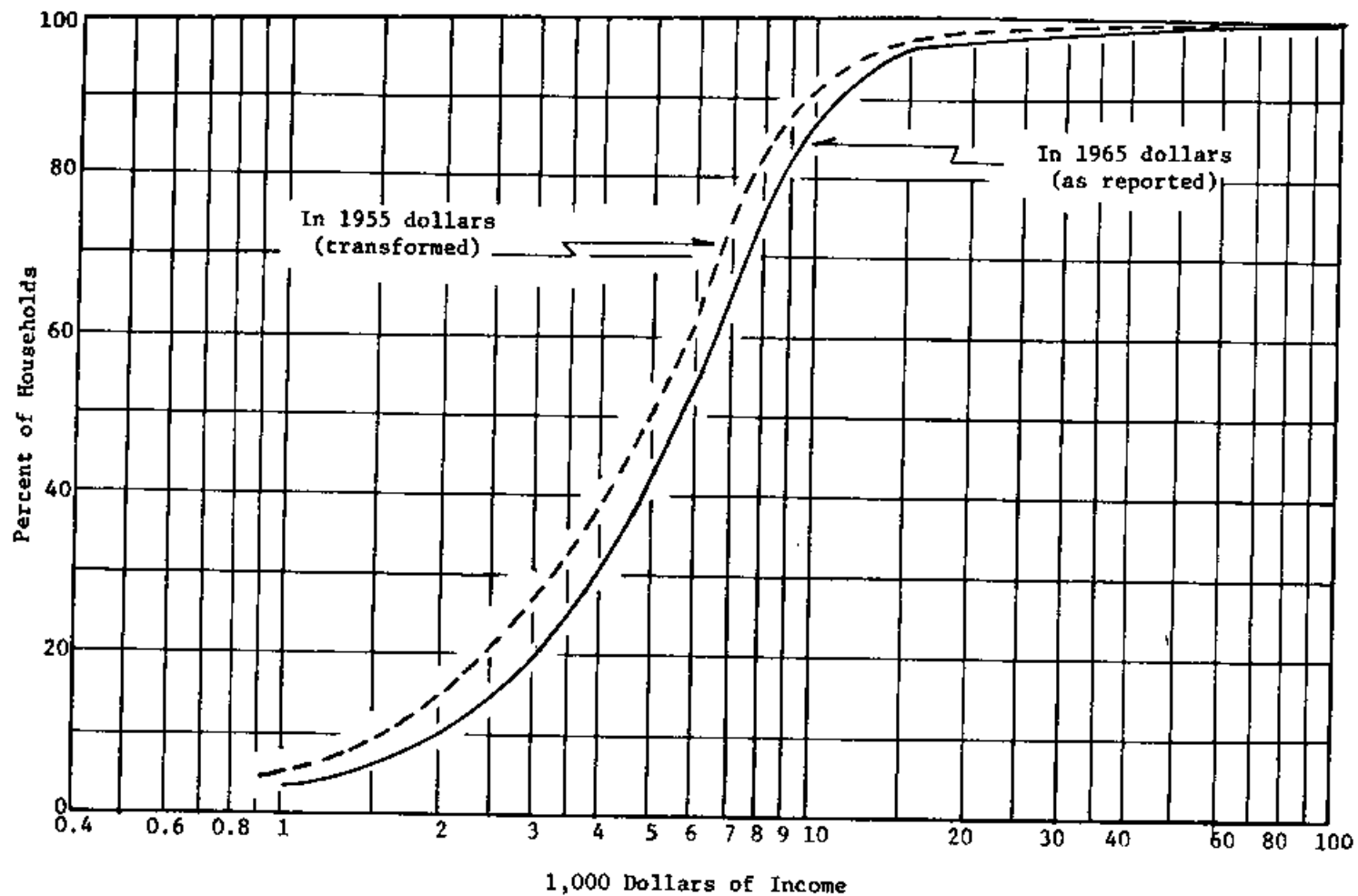


Fig. 1. Classification of shifts in regression coefficients.

TABLE 19
CHANGE IN PER CAPITA
CONSUMPTION, 1955 TO 1965

Commodities with consumption over all households*	
Increasing	Decreasing
<i>All income groups: (11)</i> Beef, chicken, fish Margarine Apples Canned corn, canned tomatoes, frozen vegetables Rice, cereals, soup	<i>All income groups: (16)</i> Veal, lamb, turkey, eggs Butter, shortening Milk, evaporated milk, potatoes Oranges, canned pineapple, frozen fruit Tomatoes, onions, carrots Wheat flour
<i>Two income groups only: (3)</i> Canned peaches (L, M) Cheese (L, H) Ice cream (L, H)	<i>Two income groups only: (6)</i> Lard (L, H) Sugar (L, H), coffee (M, H) Beans (L, H), canned peas (L, M) Sweet potatoes (M, H)
<i>One income group only: (3)</i> Bananas (L) Salad dressing (H) Lettuce (H)	<i>One income group only: (5)</i> Pork (L) Dried vegetables (H), dried fruit (H) Corn meal (L), corn syrup (H)
Total commodities (17)	Total commodities (27)

* Income groups are designated low (L), medium (M), and high (H), and all households.

Of the 44 commodities, consumption increased in all income groups for 11 and decreased for 16.²⁶ For the remaining 17 commodities, increases or decreases were shown for at least one income group. Summarizing by income group, the low income group had 20 increases and 24 decreases, the medium income group had 20 increases and 24 decreases, the high income group had 18 increases and 26 decreases, and all households had 17 increases and 27 decreases in per-capita consumption in 1965 as compared with 1955.

We can further analyze these changes by considering the relative changes given in table A-3, summarized in table 20. The shifts in consumption behavior were

not uniform; wide fluctuations existed in the extent of variations in the consumption of different commodities. All income groups reduced the per-capita consumption of three commodities—veal, butter, and frozen fruit—by more than 40 per cent. If we consider that an arbitrary 5 per cent change may be due to reporting errors in the data, 19 commodities showed a reduction in consumption by more than 5 per cent and 17 commodities showed an increase by more than 5 per cent in the low income group. In the medium and high income groups, 21 and 24 commodities, respectively, showed a reduction of more than 5 per cent while 15 and 13 commodities, respectively, showed an increase of more than 5 per cent.

Shifts in slope and constant coefficients.—To compare the nature of shifts in regression coefficients, we obtained both ordinary and weighted regression equations of the type

$$\log q = a + b \log y$$

for the two periods. Looking at the size of the coefficients, we classified the commodities into four different groups shown graphically in figure 1.

Group I (increase in the intercept and decrease in the slope). The diagram shows that the gap between the regression lines corresponding to 1955 and 1965 decreases as income increases up to y^* . Whether these two lines will meet at a finite horizon will depend upon the relative size of changes. Lower income groups experienced a relatively higher percentage increase in consumption than higher income groups. In fact, beyond the point of intersection, people in the higher income group reduced their consumption of the commodity under consideration.

²⁶ This is consistent with the time-series data on consumption reported in Hiemstra (1968) except for the commodities, apples, shortening, and onions.

TABLE 20
COMPARISON OF CHANGES IN PER CAPITA CONSUMPTION OF DIFFERENT
COMMODITIES DURING SPRING 1955 AND 1956

Income Group	Percentage Change in 1955 over 1956						
	<-40	-40 to -20	-20 to -5	-5 to 5	5 to 20	20 to 40	>40
dollars	per cent						
<3000	Veal, butter, frozen fruits (3)	Lamb, turkey, lard, shortening, fresh milk, oranges, canned pineapple, wheat flour, corn meal (9)	Salad dressing, potatoes, sugar, tomatoes, beans, carrots, lettuce (7)	Eggs, pork, evap. milk, sweet potatoes, apples, onions, canned peas, canned tomatoes (8)	Fish, cheese, ice cream, corn syrup, bananas, canned corn, dry vegetables, coffee (8)	Beef, chicken, margarine, dry fruits, breakfast cereals (5)	Canned peaches, frozen vegetables, rice, soup (4)
3000 to 4999	Veal, lamb, butter, oranges, canned pineapples, frozen fruits (6)	Milk, carrots (2)	Turkey, shortening, evap. milk, ice cream, potatoes, sweet potatoes, bananas, tomatoes, onions, lettuce, canned peas, wheat flour, coffee (13)	Pork, eggs, salad dressing, cheese, sugar, apples, beans, frozen vegetables (8)	Beef, fish, lard, corn syrup, canned tomatoes, soup (6)	Margarine, canned peaches, canned corn, dry vegetables, breakfast cereals (5)	Chicken, dry fruits, rice, corn meal (4)
>5000	Lamb, veal, turkey, butter, sweet potatoes, oranges, frozen fruits, dry vegetables (8)	Lard, shortening, evap. milk, canned pineapples, carrots (5)	Eggs, milk, potatoes, sugar, corn syrup, dry fruits, tomatoes, beans, onions, wheat flour, coffee (11)	Pork, cheese, ice cream, bananas, canned peaches, canned peas, frozen vegetables (7)	Salad dressing, apples, lettuce, canned tomatoes, soup (5)	Beef, chicken, fish, margarine, canned corn (5)	Rice, breakfast cereals, corn meal (3)
All Households	Lard, dry fruits, frozen fruits, dry vegetables (4)	Veal, lamb, butter, shortening, evap. milk, beans, sweet potatoes, oranges, wheat flour, corn meal, canned pineapples (11)	Turkey, eggs, milk, potatoes, sugar, corn syrup, onions, carrots, canned peas (9)	Pork, bananas, tomatoes, coffee (4)	Fish, salad dressing, cheese, ice cream, apples, canned peaches, lettuce, canned corn, canned tomatoes, breakfast cereals (10)	Beef, chicken, margarine, frozen vegetables, soup (5)	Rice (1)

Group II (decrease in the intercept and increase in the slope). Here, the change is exactly opposite to group I. The diagram shows a similar pattern except that

the lines corresponding to the two years are interchanged. Here, the lower income group experienced a reduction in the quantities consumed while the upper

income groups might have experienced an increase in consumption. Again, the boundary between increase and decrease in consumption (the point of intersection) depends upon the slopes and intercepts of different commodities in these two periods.

Group III (increase in both intercept and slope). In this case, both upper and lower income groups increased consump-

tion and the percentage of increase may be higher for the upper income group compared to the lower income group.

Group IV (decrease in both intercept and slope). This group's behavior is opposite to that of group III. Here, both upper and lower income groups have reduced consumption. The 1965 regression lines lie everywhere below the 1955 regression line.

TABLE 21
CLASSIFICATION OF COMMODITIES ACCORDING TO CHANGES IN
REGRESSION COEFFICIENTS FROM 1955 TO 1965

Group I: Change in intercept positive change in slope negative		Group II: Change in intercept neg- ative change in slope positive		Group III: Change in intercept posi- tive change in slope positive	
Regression		Regression		Regression	
Weighted	Nonweighted	Weighted	Nonweighted	Weighted	Nonweighted
Beef	Beef	Veal	Veal	Cheese	Fish
Chicken	Chicken	Pork	Pork	Onions	Cheese
Eggs	Eggs	Fish	Lamb	Breakfast cereals	Onions
Lard	Lard	Turkey	Turkey	Corn meal	Breakfast cereals
Margarine	Margarine	Butter	Butter	(4)	(4)
Evap. milk	Evap. milk	Salad dressing	Shortening	Group IV: Change in intercept neg- ative change in slope negative	
Sweet potatoes	Sweet potatoes	Fresh milk	Salad dressing	Regression	
Sugar	Sugar	Ice cream	Fresh milk	Weighted	Nonweighted
Corn syrup	Corn syrup	Potatoes	Ice cream	Lamb	Oranges
Oranges	Bananas	Apples	Potatoes	Shortening	Carrots
Bananas	Canned peaches	Canned pineapples	Apples	Frozen fruits	Wheat flour
Canned peaches	Dry fruits	Fresh tomatoes	Canned pineapples	Fresh tomatoes	(3)
Dry fruits	Fresh beans	Fresh lettuce	Frozen fruits	Lettuce	
Fresh beans	Canned peas	Canned peas	Fresh tomatoes	Canned tomatoes	
Canned corn	Dry vegetables	Canned tomatoes	Lettuce	Corn meal	
Dry vegetables	Frozen vegetables	(15)	Canned tomatoes	(17)	
Frozen vegetables	Rice		Corn meal	(4)	
Wheat flour	Coffee				
Rice	Soup				
Coffee	(19)				
Soup					
(21)					

Table 21 gives the classification of commodities into these four groups.

Testing the equality of coefficients.—Table 21 gives only a qualitative estimate of the nature of the changes in regression coefficients. To make a statistically sound statement on the variability of regression coefficients obtained from these two cross sections, it is necessary to apply certain testing procedures. Here are two such procedures: (1) To test the equality

of coefficients obtained from two cross sections, we can pool the data from them and obtain a covariance analysis to test the significance of the difference in income elasticities during these two periods.

(2) The second approach is similar to that of Middelhook (1968) to test the stability of input-output coefficients over two different periods. If \bar{a}_1 and \bar{a}_2 are two estimates, with standard deviations σ_a ,

and $\hat{\sigma}_a$, obtained from two cross sections of size n_1 and n_2 , the difference between these estimates is significant if $|\hat{a}_1 - \hat{a}_2| < t_1\hat{\sigma}_{a_1} + t_2\hat{\sigma}_{a_2}$, where t_1 and t_2 correspond to the t -values α_1 and α_2 , respectively, at the desired level of tolerance.

We used the second approach to test the equality of income elasticities obtained from 1955 and 1965 survey data. At the 5 per cent level, only four commodities (beef, eggs, fresh milk, and canned peaches) showed significant deviations in the income elasticities. At the

10 per cent level, three more commodities, (sweet potatoes, lettuce, and soup) are added. Beef, eggs, sweet potatoes, and soup decreased in income elasticity, the others showed an increase.

The income elasticity of all foods remained relatively stable during this period. There is close similarity between the coefficients of regression equations, using expenditures from 1955 and 1965 cross-section surveys when the dependent variable is the per-capita expenditure on total food. The regression equations obtained for these two periods were

$$1955: \quad \log x = .132 + .266 \log y; \quad R^2 = .92 \text{ and} \\ (1.49) \quad (9.09)$$

$$1965: \quad \log x = .132 + .277 \log y; \quad R^2 = .88, \\ (3.66) \quad (3.43)$$

where

x = per-capita consumption expenditure on all food items,

y = per-capita income (the figures in the parentheses are t -values).

A comparison with some of the previous cross-section estimates shows that the income elasticity has been falling over the last three decades and has become relatively stable. For example, using earlier cross-section data, Burk²⁶ (1951) estimated the income elasticities in 1935-1936, 1941 (two), 1944, and 1947 as .49, .49, .58, .33, and .31, respec-

tively. From these five estimates and the two estimates obtained in this study for 1955 and 1965, it appears that the income elasticity for all food expenditures has stabilized in between .25 and .30.

Effect of a change in income distribution.—The comparison of income elasticities obtained from 1955 and 1965 cross-section data is proper only if the structure of the populations was comparable in these two periods. A closer look at the samples used in both surveys reveals that there was an improvement in the distribution of families in different income groups. In the 1955 sample, only 32 per cent belonged to the group with

²⁶ Burk (1951) reports the following regression equations:

1935-1936	$\log x_1 = .88 + .49 \log y_1; R^2 = .99$
1941	$\log x_1 = .93 + .49 \log y_1; R^2 = .99$
1941	$\log x_2 = .64 + .58 \log y_2; R^2 = .99$
1944	$\log x_2 = 1.47 + .33 \log y_2; R^2 = .95$
1947	$\log x_2 = 1.61 + .31 \log y_2; R^2 = .96$

where

x_1 = average expenditure per capita for food and alcoholic beverages, total population;

y_1 = average per capita disposable income, current dollars, total population;

x_2 = food expenditure per capita, urban families; and

y_2 = disposable income per capita, urban families, current dollars.

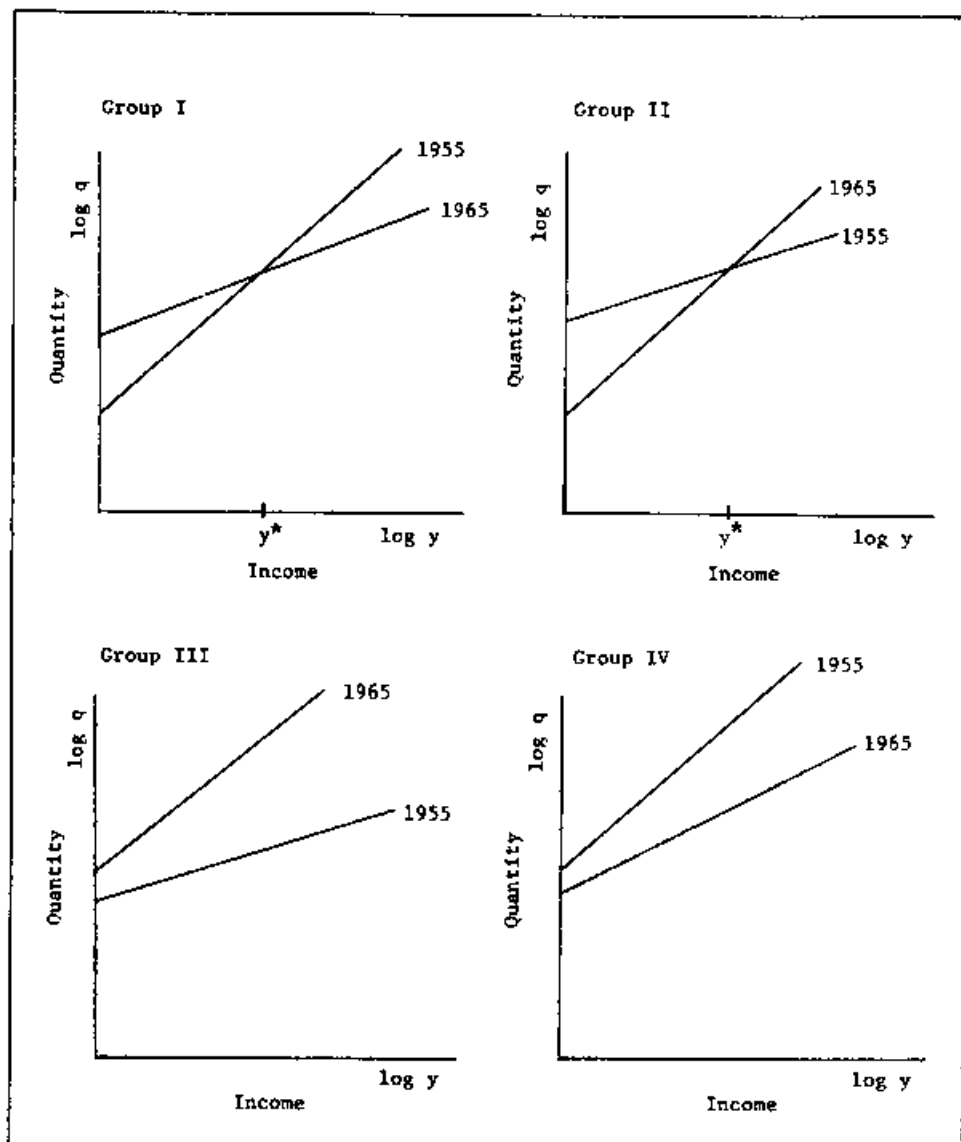


Fig. 2. Cumulative frequency distribution of families by income level in 1965 and transformation to 1955 dollars.

household income greater than \$5,000, in the 1965 sample, 58 per cent. If the samples during these two periods were representative, this increase might have been the result of two factors—a general increase in the price level and a general improvement in the income status of families. To make meaningful comparisons of data from these two samples, we isolated the price effect from the increases in general income. In other words, we adjusted the 1965 income-size distribution to a distribution among income classes, in terms of 1955 dollars, without a change in relative distribution of income. To achieve this, we used an approach suggested by Burk (1961, p. 56). The procedure is illustrated in figure 2 and involves the following steps:

(1) Take the distribution of samples in 1965 and plot the percentage cumulative frequencies on the ordinary vertical axis of a semilogarithmic paper against the upper class limit of each income class taken along the logarithmic horizontal axis.

(2) To adjust the distribution from the 1965 price level, move the curve obtained above to the left by the ratio of consumer price index in 1955 to that in 1965 (86.586 per cent).

(3) Read off the cumulative frequencies for the adjusted curve at the class limits and calculate the frequencies for each class by subtraction. The adjusted frequencies are shown as in table 22. Comparing the distribution of samples in 1955 and in 1965 adjusted at 1955 price levels, it appears that the movement was mainly from the middle income group to the high income group with only minor changes in the low income group.

Implications of the redistribution of sample size on aggregation.—Here, we shall demonstrate the implications of the redistribution of sample proportions on income elasticities obtained as an aggre-

TABLE 22
THE TRANSFORMATION OF 1965
SAMPLES TO 1955 PRICE LEVELS

Income group	1955 Survey (actual)	1965 Survey (actual)	1965 Survey (adjusted to 1955 price levels)
	dollars	per cent	
< 3,000	29.3	22.2	27
3,000-4,999	37.8	19.8	25
> 5,000	32.3	58.0	48

TABLE 23
TOTAL FOOD EXPENDITURES IN
1955 AND 1965

Income group	Weekly per-capita total food expenditures		
	1955 (actual)	1965 (actual)	1965 (at 1955 price level)
	dollars		
< 3,000	6.31	7.50 (18.85)	6.49 (2.35)
3,000-4,999	8.20	8.90 (8.53)	7.70 (-6.10)
> 5,000	10.95	11.90 (8.67)	10.30 (-5.94)

NOTE: The figures in the parentheses represent percentage changes over the expenditure levels in 1955.

gate of subgroups within a cross section with reference to total food expenditures. Comparing the total food expenditures in 1955 and 1965, the low income group has registered the highest percentage increase in food consumption expenditures—about 18 per cent—compared to about 8 per cent in other groups (see table 23).

To estimate the aggregate elasticity for all groups with known income elasticities (E_i), the aggregate elasticity E can be written as

$$E = \frac{\sum k_i E_i}{\sum k_i} \quad (151)$$

where

$$k_i = n_i e_i,$$

n_i = number of observations in group i , and

e_i = per-capita food expenditure in group i (in the present case, $i = 1, 2, 3$).

Since k_i for a given group is defined as the product of the number of observations and the per-capita food expenditure, the effect of a unilateral reduction in the sample size in a group is to reduce k_i while the effect of a unilateral increase in per-capita expenditure is to increase k_i . Therefore, the changes in the value

TABLE 24
WEIGHTS ASSOCIATED WITH INCOME GROUP
(VALUES OF k_i FOR 1955 AND 1965)

Income group	1955*	1955 (actual)†	1965 (distribution and expenses at 1955 level)‡	1965 (distribution at 1965 levels and expenses at 1965 price)§
<i>dollars</i>				
< 3,000	198.04 (22.08)	166.50 (16.12)	175.23 (20.32)	202.60 (20.32)
3,000-4,999	309.96 (36.39)	176.22 (17.00)	192.50 (22.33)	222.50 (22.33)
> 5,000	353.68 (41.53)	690.20 (66.82)	493.30 (57.35)	571.20 (57.35)
TOTAL..	861.68	1,032.92	862.13	896.20
(Σ <i>k_i</i>).....	(100.00)	(100.00)	(100.00)	(100.00)

* Column 2 in table 22 multiplied by column 2 in table 23.

† Column 3 in table 22 multiplied by column 3 in table 23.

‡ Column 4 in table 22 multiplied by column 4 in table 23.

§ Column 4 in table 22 multiplied by column 3 in table 23.

NOTE: The figures in the parentheses represent percentages of total weight.

of k_i for each year is determined as an outcome of these two opposite effects. The values of k_i for each group in 1955 and 1965 are presented in table 24.

The combined effects of the redistribution of the samples and the changes in per capita expenditures during the period, 1955-1965, is to reduce the value of k_i for low and medium groups and to increase it for the high income group. In the case where the lower income group has higher income elasticities, the changes in k_i , described above reduce the weight given to higher income elasticities (and increase the weight given to lower income elasticities), resulting in an aggregate income elasticity which is lower in the latter period than in the original period so long as the income

elasticities within different groups remain unchanged during the period. Therefore, when comparing the income elasticities obtained as the aggregates of subgroups in two cross-section surveys, the effects of the following factors are important:

- (1) Distribution of samples into subgroups in both cross sections after removing the effects of price changes.
- (2) Changes in per-capita consumption among subgroups belonging to the two cross sections.
- (3) Changes in income elasticities in each subgroup during the interval between the survey.

In comparing any of the above factors the effect of changes in other factors must be removed. Further, it is possible

TABLE 25
REGIONAL CONSUMPTION INDEX (U. S. AVERAGE = 100)

Quantity Index Region	0-50	50-75	75-95	95-105	105-125	125-150	>150
North-east (R ₁)	Corn syrup, dry vegetables, wheat flour, corn meal (4)	Lard, shortening, beans (3)	Pork, eggs, margarine, salad dressing, evap. milk, sweet potatoes, sugar, canned peaches, canned pineapples, dry fruits, frozen fruits, tomatoes, canned corn (13)	Beef, chicken, fish, milk, cheese, ice cream, potatoes, apples, bananas, carrots, lettuce, canned peas, breakfast cereals, coffee (14)	Oranges, onions, rice, soup (4)	Butter, canned tomatoes, frozen vegetables (3)	Veal, lamb, turkey (3)
West (R ₂)	Lard, beans, corn meal (3)	Veal, sweet potatoes, corn syrup (3)	Pork, fish, butter, shortening, ice cream, potatoes, sugar, canned peas, dry vegetables, wheat flour, rice (11)	Chicken, eggs, milk, evap. milk, bananas, tomatoes, onions, coffee (8)	Beef, turkey, margarine, salad dressing, cheese, apples, oranges, canned peaches, frozen fruits, canned corn, canned tomatoes, breakfast cereals, soup (13)	Canned pineapples, carrots, lettuce, frozen vegetables (4)	Lamb, dry fruits, (2)
North Central (R ₃)	Lamb, lard, evap. milk, rice, corn meal (5)	Veal, sweet potatoes, corn syrup, dry vegetables (3)	Chicken, turkey, fish, shortening, salad dressing, milk, tomatoes, beans, onions, canned tomatoes, frozen vegetables, wheat flour (12)	Eggs, margarine, sugar, dry fruits, canned peas (5)	Beef, pork, cheese, ice cream, potatoes, apples, oranges, bananas, canned peaches, canned pineapple, frozen fruits, carrots, lettuce, canned corn, breakfast cereals, coffee, soup (17)		Butter (1)
South (R ₄)	Lamb (1)	Veal, turkey, butter, cheese, oranges, carrots, soup (7)	Beef, milk, ice cream, potatoes, apples, canned peaches, canned pineapples, dry fruits, frozen fruits, lettuce, canned corn, canned tomatoes, breakfast cereals, coffee, frozen vegetables (18)	Bananas, onions (2)	Pork, chicken, eggs, margarine, salad dressing, sugar, fresh tomatoes, canned peas (5)	Fish (1)	Lard, shortening, evap. milk, sweet potatoes, corn syrup, beans, dry vegetables, wheat flour, rice, corn meal (10)

to obtain the same income elasticity from two sets of cross-section data as a result of compensating adjustments in the above factors.

Regional variations

When we discuss the U. S. food consumption pattern, it would be useful to

see the extent of variation among regions. Unfortunately, detailed data on consumption levels by regions are not available for extended periods. The 1965 consumption survey data is tabulated for four regions—Northeast, West, North Central, and South. From the average consumption and size of the household

TABLE 26
VARIABILITY IN REGIONAL CONSUMPTION INDICES

Commodity	Outside the interval 50-150	Outside the interval 75-125	Outside the interval 95-105	Range
Beef			✓	26.22
Veal	✓	✓	✓	148.15
Pork			✓	20.25
Lamb	✓	✓	✓	161.61
Chicken			✓	17.35
Turkey	✓	✓	✓	118.21
Fish		✓	✓	49.60
Eggs			✓	21.60
Butter	✓	✓	✓	204.06
Lard	✓	✓	✓	159.98
Shortening	✓	✓	✓	103.29
Margarine			✓	27.60
Salad dressing			✓	26.62
Fresh milk			✓	17.47
Evaporated milk	✓	✓	✓	100.97
Cheese		✓	✓	80.64
Ice cream			✓	18.64
Potatoes			✓	31.10
Sweet potatoes	✓	✓	✓	88.32
Sugar			✓	39.24
Corn syrup	✓	✓	✓	160.98
Apples			✓	28.57
Oranges		✓	✓	46.87
Bananas			✓	9.23
Canned peaches			✓	35.01
Canned pineapples		✓	✓	52.40
Dry fruits	✓	✓	✓	77.89
Frozen fruits			✓	37.08
Tomatoes			✓	35.87
Beans	✓	✓	✓	146.56
Onions			✓	26.64
Carrots		✓	✓	72.08
Lettuce		✓	✓	53.86
Canned peas			✓	20.47
Canned corn			✓	30.48
Canned tomatoes		✓	✓	61.71
Dry vegetables	✓	✓	✓	124.73
Frozen vegetables		✓	✓	44.59
Wheat flour	✓	✓	✓	114.54
Rice	✓	✓	✓	115.18
Breakfast cereals			✓	31.80
Corn meal	✓	✓	✓	240.08
Coffee			✓	16.98
Soup		✓	✓	49.60
Total	15	24	44	

NOTE: A ✓ mark indicates that consumption index in at least one region fell outside the specified interval.

given in the regional data, the average per-capita consumption of the commodities were derived for each region. Using the per-capita consumption obtained in this manner and the per-capita consumption for the United States as a whole, we derived a consumption index for each region by expressing the per-capita consumption in regions as a percentage of

the per-capita consumption for the United States (Appendix table A-4, summarized in table 25).

Table 25 shows that the importance given to different food items in the four regions differs considerably. The consumption pattern in the Northeast is closest, and that of the southern regions farthest from the U. S. average (see table

TABLE 27
REGIONAL INDICES OF PRICES PAID (U. S. AVERAGE = 100)

Price Index Regions	0-50	50-75	75-95	95-105	105-125	125-150	> 150
Northeast (R ₁)	Turkey (1)		Canned pineapples, dry fruits, canned corn, frozen vegetables, corn meat (5)	Lamb, butter, salad dressing, evap. milk, potatoes, sugar, corn syrup, apples, oranges, beans, tomatoes, onions, carrots, lettuce, canned peas, soup (16)	Beef, veal, pork, chicken, fish, eggs, lard, shortening, milk, margarine, cheese, ice cream, sweet potatoes, bananas, canned peaches, canned tomatoes, breakfast cereals, dry vegetables, wheat flour, rice, coffee (21)	Frozen vegetables (1)	
West (R ₂)			Veal, lamb, carrots, lettuce, canned tomatoes, frozen vegetables (6)	Beef, pork, chicken, eggs, shortening, margarine, milk, cheese, ice cream, potatoes, sugar, corn syrup, canned pineapples, dry fruits, tomatoes, frozen fruits, beans, onions, canned corn, wheat flour, breakfast cereals, coffee, soup (23)	Turkey, fish, lard, salad dressing, evap. milk, apples, oranges, bananas, canned peaches, canned peas (10)	Butter, dry tables, rice (3)	Sweet potatoes, corn meal (2)
North Central (R ₃)	Soup (1)	Beef, veal, turkey, eggs, lard, cheese, apples, bananas, canned peaches, canned pineapples, canned peas, canned corn, dry vegetables, coffee (14)	Pork, chicken, fish, butter, shortening, margarine, evap. milk, ice cream, potatoes, sugar, oranges, dry fruits, tomatoes, wheat flour, breakfast cereals (15)	Lamb, salad dressing, fresh milk, corn syrup, beans, onions, carrots, lettuce, canned tomatoes, rice, corn meal (11)	Sweet potatoes, frozen fruits (2)	Frozen vegetables (1)	
South (R ₄)	Beans, dry vegetables (2)	Beef, veal, pork, chicken, fish, ice cream, salad dressing, corn syrup, apples, oranges, bananas, dry fruit, tomatoes, canned peas, rice (15)	Lamb, turkey, eggs, butter, lard, shortening, margarine, sweet potatoes, sugar, canned peaches, canned pineapples, onions, breakfast cereals, corn meal, coffee, soup (16)	Fresh milk, evap. milk, cheese, potatoes, canned corn, frozen vegetables, wheat flour (7)	Carrots, lettuce, canned tomatoes (3)	Frozen fruits (1)	

26). Of the 44 commodities considered in the analysis, 15 had a consumption index falling outside the range (50-150) in at least one region, 24 fell outside the range (75-125), and all the 44 commodities had at least one index falling outside the range (95-105). Thus, even after we make allowances for a 5 per cent deviation from the U. S. average consumption

pattern due to variations in reporting data, all the commodities showed some regional variations in their consumption pattern. The range of consumption index (difference between the highest and lowest regional index) varied from 8.23 for bananas to 240.08 for corn meal. Thirteen commodities (veal, lamb, turkey, butter, lard, shortening, evaporated

milk, corn syrup, beans, dry vegetables, wheat flour, rice, and corn meal) had a range of more than 100.

The presence of such wide variations in the quantity consumed in different regions could be explained in terms of (1) differences in prices, (2) differences in income-consumption relationship, and (3) other regional factors. We attempted to analyze the influence of prices and income in regional consumption behavior.

Differences in prices paid.—The expenditure and quantity data published in the survey reports were used to find the prices paid by consumers in different regions, and these were expressed as a percentage of average U. S. figures to obtain the index of prices paid. Using these indices (Appendix table A-5) we have prepared table 27 corresponding to table 25 for quantity indices. The spread or price indices in different regions is not so wide as the spread of quantity indices.

Although the cross-section data do not provide information to study the effects of price changes on quantity consumed, it denotes one point on the demand curve for each commodity. For each commodity, the quantity index in Appendix table A-4 and the price index in Appendix table A-5 provide four observations—one for each region—on price-quantity relationships. These points can be located on a two-dimensional space and, if the demand curve is assumed to be downward sloping, there will be an inverse relationship between the price index and the quantity index. Since it was assumed that the average U. S. quantities and prices form a basis of comparison, this point will be represented by (100, 100). We can locate the points corresponding to the price-consumption relationship in the four regions with reference to the basis, and if the demand relationship follows the conventional

shape, a positive deviation of price index from 100 will be associated with a negative deviation of quantity index and vice versa. Therefore, if the deviations of price index and quantity index from 100 are opposite in sign, it can be concluded that a high or low consumption index in a particular region is effected as a result of low or high price in that region. Also, if the deviations of price and quantity indices follow the same sign, it could mean either that the demand curve is not downward sloping or that the regional difference in consumption is not influenced by some factors other than prices. In table 28, deviations of price indices and quantity indices in different regions are expressed in terms of their signs—a positive sign denoting an index greater than 100 and a negative sign denoting an index less than 100.

Assuming that the demand curves for all these commodities are downward sloping, a positive sign can be expected to be associated with a negative sign if the consumption difference is influenced by price differences alone. However, this was not true for 16 commodities in the Northeast, 14 commodities in the West, 17 commodities in the North Central, and 13 commodities in the South.

Table 29 shows that the regional difference in consumption can be explained, at least partly, in terms of prices for seven commodities—pork, shortening, margarine, sweet potatoes, corn syrup, beans, and frozen vegetables. Consumption in all other commodities was influenced by factors other than prices in at least one region. Therefore, it is not possible to reject a hypothesis that certain regional factors influence consumption of various commodities in different regions.

To analyze the second aspect (variations in income-consumption relation-

TABLE 28
DEVIATIONS OF PRICE AND QUANTITY INDICES FROM BASE IN DIFFERENT
REGIONS

Commodity	Northeast		West		North Central		South	
	P	Q	P	Q	P	Q	P	Q
Beef.....	+	-	-	+	-	+	-	-
Veal.....	+	+	-	-	-	-	-	-
Pork.....	+	-	+	-	-	+	-	+
Lamb.....	+	+	-	+	+	-	+	-
Chicken.....	+	+	+	-	-	-	-	+
Turkey.....	-	+	+	+	-	-	+	-
Fish.....	+	+	+	-	-	-	-	+
Eggs.....	+	-	-	+	-	-	-	+
Butter.....	+	+	+	-	-	+	+	-
Lard.....	+	-	+	-	-	-	+	+
Shortening.....	+	-	+	-	+	-	-	+
Margarine.....	+	-	-	+	+	-	-	+
Salad dressing.....	+	-	+	+	+	-	-	+
Fresh milk.....	+	+	+	-	+	-	+	-
Evaporated milk.....	+	-	+	+	+	-	+	+
Cheese.....	+	+	-	+	-	+	+	-
Ice cream.....	+	+	-	-	-	+	-	-
Potatoes.....	-	+	-	-	-	+	+	-
Sweet potatoes.....	+	-	+	-	+	-	-	+
Sugar.....	+	-	-	-	-	-	-	+
Corn syrup.....	+	-	+	-	+	-	-	+
Apples.....	-	+	+	+	+	+	-	-
Oranges.....	+	+	+	+	+	+	-	-
Bananas.....	+	-	+	-	-	+	-	-
Canned peaches.....	+	-	+	+	-	+	-	-
Canned pineapples.....	-	-	+	+	-	+	-	-
Dry fruits.....	+	-	-	+	+	-	-	-
Frozen fruits.....	+	-	+	+	+	+	+	-
Tomatoes.....	+	-	+	-	-	-	-	+
Beans.....	+	-	+	-	+	-	-	+
Onions.....	-	+	-	-	+	-	-	+
Carrots.....	-	+	-	+	+	+	+	-
Lettuce.....	+	+	-	+	+	+	+	-
Canned peas.....	-	+	+	-	-	-	-	+
Canned corn.....	-	-	-	+	-	+	+	-
Canned tomatoe.....	+	+	-	+	+	-	+	-
Dry vegetables.....	+	-	+	-	-	-	-	+
Frozen vegetables.....	-	+	-	+	+	-	+	-
Wheat flour.....	+	-	+	-	-	-	+	+
Rice.....	+	+	+	-	+	-	-	+
Breakfast cereals.....	+	-	+	+	-	+	-	-
Corn meal.....	-	-	+	-	+	-	-	+
Coffee.....	+	+	-	-	-	+	+	-
Soup.....	-	+	-	+	+	+	+	-

NOTE: P stands for prices and Q for quantities. A + indicates that the index is greater than 100 (so that the deviation is positive) and a - indicates that the index is less than 100.

ship over the regions), we have used a covariance analysis.

Covariance analysis for testing regional variation.—Using the consumption data from four regions, we specified three types of regression equations as follows:

Model 1

$$\log q_{ij} = a_i + b_i \log y_j \quad (152)$$

where

q_{ij} = consumption of i^{th} commodity
in the j^{th} region and
 y_j = income in the j^{th} region.

Here, for a given commodity, the data from the four regions are pooled to obtain a single regression equation. With

11 income groups, we have 44 observations to fit the regression equation in (152).

Model 2

$$\log q_{ij} = b_i \log y_j + c_j \log d_j \quad (153)$$

where

the variables q_{ij} and y_j are as in the previous model and d_j is a dummy variable associated with the j^{th} region such that $\log d_j = 1$ in region j
0 elsewhere.

An alternative way to express equation (153) is to specify three dummy variables associated with three regions and to retain the constant in the regression equation. The difference between model 1 and model 2 is that the latter incorporates different intercepts for different regions while the former assumes the same intercepts for all the regions.

Model 3

$$\log q_{ij} = a_{ij} + b_{ij} \log y_j \quad (154)$$

where

the variables are defined as above.

While models 1 and 2 used all the 44 observations of a commodity to obtain a single regression equation, model 3 provides four regression equations corresponding to four regions. Therefore model 3 incorporates variations in both the intercepts and slopes of regression equations for a commodity in different regions.

Thus, model 1 assumed that the intercept and slope of the demand equation remained the same over the four regions; model 2 assumed that the intercept was different in different regions, but the slopes remained the same; and model 3

TABLE 29
COMMODITIES WITH REGIONAL
PRICE-CONSUMPTION
RELATIONSHIP NOT CONSISTENT
WITH EXPECTATION

Northeast	West	North Central	South
Veal	Veal	Veal	Veal
Lamb			Beef
Chicken		Chicken	
	Turkey	Turkey	
Fish		Fish	
		Eggs	
Butter		Lard	Lard
	Salad dressing		
Fresh milk			
	Evaporated milk		Evaporated milk
Cheese			
Ice Cream	Ice Cream		Ice Cream
	Potatoes		
	Sugar	Sugar	
Oranges	Apples	Apples	Apples
	Oranges	Oranges	Oranges
			Bananas
	Canned peaches		Canned peaches
Canned pineapples	Canned pineapples		Canned pineapples
Dry fruits			Dry fruits
		Frozen fruits	
		Tomatoes	
	Onions		
Lettuce		Carrots	
		Lettuce	
Canned corn		Canned peas	
		Dry vegetables	
		Wheat flour	Wheat flour
Rice			
	Breakfast cereals		Breakfast cereals
Corn meal			
Coffee	Coffee		
		Soup	
16	14	17	13

assumed that both intercept and slope varied over the regions. Let s_1 , s_2 , and s_3 be the sum of squares of residuals associated with models 1, 2, and 3, respectively, and n_1 , n_2 , n_3 , respectively, be the degrees of freedom. Because the number of restrictions are least in the case of the third model, s_3 will be the smallest. Similarly, it can be shown that $s_1 > s_2 > s_3$. For testing the hypothesis that variations exist in intercepts alone,

we use the sum of squares from the first and second models and define an F -ratio given by²⁷

$$F_{(n_1-n_2), n_2} = \frac{(S_1 - S_2)/(n_1 - n_2)}{S_2/n_2} \quad (155)$$

$$= \frac{(S_1 - S_2)n_2}{(n_1 - n_2)S_2}$$

If the calculated F is higher than the table value of F at the desired level of significance, we reject the hypothesis of zero variability in the intercepts over the different regions and conclude that there exists significant variation at least in two of the intercepts in different regions.

Similarly, for testing the significance of variation in slopes alone, we can use the second and third models and define an F -test given by

$$F_{(n_2-n_3), n_3} = \frac{(S_2 - S_3)/(n_2 - n_3)}{S_3/n_3} \quad (156)$$

As before, if the F -values obtained in (156) are higher than the tabled value of F , we reject the hypothesis of non-differences in the slopes (in this case, income elasticities) of the regression equations in the four regions. In other words, a rejection of the hypothesis that income elasticities are not different in the four regions makes it necessary to use different income elasticity measures for different regions.

Combining the two tests, if a particular commodity shows significance in both cases, variations exist in both intercept and slope. Also, it is possible to repeat the whole analysis, using expenditures on commodities in different regions instead of quantities consumed. In the present analysis, we have used both

quantities and expenditures as dependent variables to test the regional variations. In most cases, the intercepts showed significant regional differences. Only three commodities (bananas, canned peas, and breakfast cereals) provided nonsignificant differences in intercepts when quantity was the dependent variable, while the intercepts were nonsignificant for five commodities (bananas, canned peas, sweet potatoes, canned peaches, and frozen fruits) when expenditure was the dependent variable. When using quantity as the dependent variable, the income elasticities were significantly different for 31 commodities (25 at the 5 per cent level of significance and 6 at the 10 per cent level) and nonsignificant for 13 commodities. However, the number of commodities with nonsignificant regional differences in income elasticities rose to 20 when expenditure was used as the dependent variable. It may be possible to attribute this change to the influence of prices, that is, a significant regional difference in income elasticities, with quantity as the dependent variable, might have been offset by price differences within the regions to show a nonsignificant regional difference in income elasticities, with expenditure as the dependent variable. These factors are revealed in table 30 where we have adopted a cross classification according to the significance of the slopes and intercepts of income-consumption relationships in different regions.

When quantities were the dependent variables, 28 commodities showed significant regional variations in both intercept and slope, 13 commodities showed significant variation in intercept but nonsignificant variation in slope, and the remaining 3 commodities showed

²⁷ When we are interested in testing the significance of a few dummy variables, a t -test on individual coefficients will suffice. However, here an F -test is used to test the significance of the coefficients simultaneously.

TABLE 30
CLASSIFICATION OF COMMODITIES ACCORDING TO REGIONAL VARIATIONS
IN INTERCEPTS AND SLOPES OF INCOME CONSUMPTION

Intercept and slope significant*		Intercept significant slope non-significant		Intercept non-significant slope significant		Intercept and slope non-significant	
Quantity dependent	Expenditure dependent	Quantity dependent	Expenditure dependent	Quantity dependent	Expenditure dependent	Quantity dependent	Expenditure dependent
Beef	Beef		Chicken		Bananas		
Veal	Veal		Turkey		Bananas		
Pork	Pork				Canned peas		
Lamb	Lamb				Canned peas		
Chicken		Fish	Fish				
Turkey		Butter	Butter				
Eggs	Eggs						
Shortening	Shortening	Lard	Lard				
Margarine	Margarine		Salad dressing				
Salad dressing		Milk	Milk				
Cheese	Cheese	Evaporated milk	Evaporated milk				
Ice cream	Ice cream	Sweet potatoes					Sweet potatoes
Potatoes	Potatoes		Corn syrup				
Sugar	Sugar	Oranges	Oranges				
Corn syrup		Canned peaches					Canned peaches
Apples	Apples		Canned pineapples				
Canned pineapples			Dry fruits				
Dry fruits		Frozen fruits					Frozen fruits
Fresh tomatoes	tomatoes						
Beans	Beans	Onions	Onions				
Carrots	Carrots		Frozen vegetables				
Lettuce	Lettuce	Wheat flour	Wheat flour				
Canned corn	Canned corn	Rice	Rice				
Canned tomatoes	Canned tomatoes	Corn meal	Corn meal				
Dry vegetables	Dry vegetables						
Frozen vegetables	Breakfast cereals				Breakfast cereals		
Coffee	Coffee						
Soup	Soup						
(28)	(22)	(13)	(17)	(3)	(2)	(0)	(3)

* Significant category includes significance at 5% and 10% levels.

nonsignificant variation in intercept but significant variation in slope. There was no commodity with both intercept and slope showing nonsignificant regional

variation. However, there were some modifications when expenditure was the dependent variable. Here, 22 commodities had significant regional variation in

both slope and intercept, 17 had significant variations in intercept but non-significant variation in slope, 2 commodities had nonsignificant variation in intercept but significant variation in slope, and the remaining 3 commodities had nonsignificant variation in both intercept and slope.

Seasonal variations

Seasonality in food consumption originates from factors like crop production, climatic factors, and similar other reasons. Though modern processing, transportation, and refrigeration facilities have reduced the incidence of seasonality

in food consumption, many food items, especially fruits and vegetables, still show seasonal consumption patterns. Since the 1965 consumption survey was spread over the four seasons, it was originally planned to analyze the income-consumption pattern during the four seasons, using an approach similar to the one used for analyzing regional variations. Later on, it was revealed that these data will be available only after some time and, therefore, it was not possible to incorporate this aspect in the present analysis. For the same reason, it became necessary to exclude a discussion on the interaction of regions and seasons.

Demand Projections for 1980

This section reviews alternative approaches to demand projections and also projects consumption by commodity to 1980 on the assumption of constant real prices, time trends, and a particular increase in real income. To facilitate exposition, the commodities are discussed under ten groups—meat, eggs, fats and oils, dairy products, potatoes, sugar, fruits, vegetables, cereal and bakery products, and other commodities.

Alternative approaches to demand projections

Presented are possible uses of the demand interrelationship matrix for projecting future levels of demand.

Long-run changes in per-capita consumption of different food commodities occur as a result of individual and joint effects of many factors such as changes in relative prices and income, changes in tastes and preferences, introduction of new products, changes in occupation and urbanization, and changes in the age composition of the population. In the short run, it is convenient to assume that socioeconomic factors other than prices

and income remain constant. Therefore short-run changes in per-capita consumption will be influenced by price and income changes. Price changes result mainly from supply conditions such as production cycles, seasonal variations, weather conditions, and technological changes. It may be possible to obtain forecasts of these variables for short-run periods. These forecasts of supply conditions can be used to project the price levels in the immediate future and, thus, changes in consumption levels for the immediate future can be projected. However, projections for longer periods of time cannot be handled in the same manner, because we may not assume that all the other factors remain the same. The following discussion indicates several alternative possibilities as to the use of the demand interrelationships for projecting consumption levels to 1980.

Projections with constant demand matrix plus time trend.—The demand interrelationship matrix was developed under conditions of static equilibrium. The only dynamic element introduced

into the analysis is the measurement of the time trend of consumption for certain commodities obtained from the constant coefficient of the first difference of logarithmic estimating equation. One approach to projecting consumption in a future period is to use the demand matrix plus the time trend.

If the demand matrix is denoted by M , we have

$$Q = M \cdot P \quad (157)$$

$(n \times 1)$ $(n \times n + 1)$ $(n + 1 \times 1)$

where

Q = vector of quantities, expressed in logs;

M = matrix of price and income elasticities; and

P = vector of prices and income, expressed in logs.

When estimates of P are available for a future period, an estimate of Q can be

obtained from (157) under the assumption of constant demand elasticities. Let this estimate for the i^{th} commodity be \hat{q}_i . Also, let the annual time trend for the i^{th} commodity be S_i , expressed as a percentage change per year. If the projection is for K periods ahead of the present period, the time trend alone will introduce a multiplier of $(1 + S_i)^K$. This multiplier, together with the projected static level (\hat{q}_i), provides the following equation for consumption for K periods as:

$$\hat{q}_{iK} = \hat{q}_i(1 + S_i)^K. \quad (158)$$

Projections with changing demand matrix.—Another method of incorporating dynamic elements into the projection framework is by assuming a varying demand interrelationship matrix. Here, it is possible to assume that the elements in the matrix M vary over time in a certain manner. For example, consider the demand equations

$$\Delta \log q_i = \sum_{j=1}^n e_{ij} \Delta \log p_j + e_{iy} \Delta \log \bar{y} \quad (i, j = 1, 2, \dots, n) \quad (159)$$

where

q_i = quantity of the i^{th} commodity demanded,

p_j = price of j^{th} commodity, and

\bar{y} = a measure of real income.

prices. Then the expenditure proportion on the i^{th} commodity is given by

$$W_i = \frac{p_i q_i}{y} \quad (160)$$

Let y denote the income at current

Multiplying (159) by (160) we obtain:

$$\begin{aligned} W_i \Delta \log q_i &= \sum_{j=1}^n W_i e_{ij} \Delta \log p_j + W_i e_{iy} \Delta \log \bar{y} \\ &= \sum_{j=1}^n \pi_{ij} \Delta \log p_j + \pi_{iy} \Delta \log \bar{y} \end{aligned} \quad (161)$$

where

$$\pi_{ij} = W_i e_{ij} \text{ and}$$

$$\pi_{iy} = W_i e_{iy}.$$

If a regression equation of the type (161)

is fitted, using time-series data on quantities, prices, income, and expenditure proportions, it is implied that π_{ij} and π_{iy} remain the same over the years whereas, in (159), it was assumed that

TABLE 31
ELASTICITIES IN DIFFERENT YEARS

Commodity	Value of η_{ij}	Direct Elasticities (e_{ij}) in			
		1955	1960	1963	1970*
Beef.....	-4.09595	-0.68268	-0.60235	-0.63015	-0.63999
Veal.....	-2.23395	-3.72325	-3.19136	-2.79244	-2.48217
Pork.....	-3.18303	-0.44844	-0.40820	-0.43026	-0.42452
Lamb and mutton	-1.07680	-1.79467	-1.79467	-2.15360	-2.15360
Chicken.....	-1.37835	-0.41788	-0.55134	-0.57431	-0.59928
Lard.....	-0.07850	-0.19628	-0.26667	-0.26667	-0.26667
Ice cream.....	-0.68074	-0.60067	-0.66074	-0.66074	-0.66074
Potatoes.....	-0.34812	-0.28779	-0.21758	-0.24566	-0.28779
Oranges.....	-1.22042	-0.87173	-0.71789	-0.64233	-0.61021
Coffee.....	-0.93738	-0.22882	-0.33477	-0.36052	-0.37494

* Hypothetical.

e_{ij} and e_{iv} remain the same over the years. But η_{ij} and η_{iv} are obtained from e_{ij} and e_{iv} by multiplying with W_i . Since the budget proportion W_i varies over time, as a result of changes in the prices and consumption pattern, e_{ij} and e_{iv} also vary over time. Thus, if expenditure proportions are known for different years, it is possible to obtain an estimate of the demand matrix M for the different periods. For projection of future consumption levels, two different approaches could be used.

Estimation of changing demand coefficients. Using the values corresponding to each element of M , obtain a time trend for the coefficient and use this trend to project the value of the coefficient for a future period. That is, if M_{ij}^t corresponds to the $(ij)^{\text{th}}$ element of the interrelationship matrix for the period t ($i, j = 1, 2, \dots, n; t = 1, 2, \dots, s$), the s values of M_{ij} could be used to establish a trend equation for M_{ij} and this could be used to obtain M_{ij}^{t+K} where K is the desired number of years for which projection is required. Having obtained the value of M_{ij}^{t+K} for all i and j , equation (157) can be used to project the value of Q if the price and income levels are known. For practical reasons, it is impossible to isolate trend values

from price effects from every element of M_{ij} .

Estimation of expenditure proportions. Instead of obtaining a projection for M_{ij}^{t+K} directly, it is possible to estimate the expenditure proportions and to derive the value of M_{ij}^{t+K} . From the estimation procedure used, η_{ij} is assumed to be the same for all time periods or

$$\eta_{ij} = W_i^{(t)} M_{ij}^{(t)}.$$

Therefore,

$$M_{ij}^{(t)} = \frac{\eta_{ij}}{W_i^{(t)}}. \quad (162)$$

To illustrate this procedure, η_{ii} was calculated for a few commodities using a regression equation similar to (161).

Table 31 shows the value of η_{ii} and the direct price elasticities (e_{ii}) for 1955, 1960, and 1963. These are obtained by dividing η_{ii} with the expenditure proportions corresponding to these commodities during 1955, 1960, and 1963, respectively. The last column (5) gives a projected value of the direct elasticities, assuming hypothetical values of the expenditure proportions in 1970. The same procedure could be used to obtain projected values of M_{ij}^{t+K} for all i and

j and, once these values are known, as before, equation (158) can be used to project the consumption levels for K periods ahead.

To use (157) for projection of prices, it can be rewritten as

$$P = M^{-1}Q. \quad (163)$$

When the objective of the study is to predict prices, it is more appropriate to fit demand equations with prices as dependent variables and to obtain a price flexibility matrix corresponding to M^{-1} . Most of the restrictions in deriving the demand interrelationships in terms of elasticities can be derived in terms of price flexibilities (see, for example, Houck, 1966). However, the inverse of the elasticity matrix, M , can be taken as a rough approximation to the price flexibility matrix and can be used for projecting future prices if the quantities consumed and income are known. For agricultural commodities, however, price prediction equations generally require specification of alternative outlets (fresh, processed), stock changes, and exports.

The above discussion indicates that projections of per-capita consumption levels in 1980 could be obtained either by using a projected demand inter-relationship matrix for 1980 or by using projected expenditure proportions for 1980. Both these methods require that prices and income levels are available for 1980. In the absence of projections of future price levels, we have obtained projections of demand assuming constant retail prices, increasing income levels, and a time trend factor.

Demand projections by commodity group

These demand projections are based on (a) a particular assumption as to increased real income by 1980, (b) the income

elasticities used in the demand matrix, and (c) the time-trend coefficients obtained from time-series analyses of individual commodities. In particular, per-capita consumption levels for individual commodities are projected for 1980, assuming constant real prices (1962-1966 average); real income per capita will increase from \$2,298 (the 1962-1966 average) to \$3,261 in 1980 as reported by Daly and Egbert (1966); and the estimated time trend, where statistically significant, will continue to 1980. The average annual per-capita consumption during 1962-1966 was taken as the base period. The cumulative effect of the time trend on this base consumption level was obtained as a product of the base consumption level and the multiplier $(1 + S_t)^K$ where S_t is the annual percentage change in consumption due to trend and K is the number of years to 1980. For many commodities, the time trend was not statistically significant and projections are based only on the income elasticity. Results are presented by commodity group, given estimates of consumption and expenditures per capita for 1962-1966 and 1980.

Meat, poultry, and fish.—These commodities accounted for about one-third of the 1962-1966 average food expenditures. Detailed analyses of time-series data were made for only five items—beef, veal, pork, lamb and mutton, and chicken. Other items, such as turkey, fish, and edible meat offals are included for completeness but time-trend coefficients are not available for these commodities. Data on per-capita consumption are shown in table 32 for the base period (1962-1966) and for 1980, and are in terms of retail weight. For purposes of comparison, estimates by Daly and Egbert (1966) were converted to retail weight and also are given in the table.

TABLE 32
MEAT, POULTRY, AND FISH: CONSUMPTION, PRICES, AND EXPENDITURES,
1962-1966 AVERAGE AND PROJECTED 1980

Commodity	Consumption per capita			Retail price, 1962-1966	Expenditures per capita (1962-1966 dollars)	
	1962-1966	Estimated 1980 based on:			1962-1966	1980
		Income elasticity	Daly-Egbert			
	pounds (retail weight)			cents	dollars	
Meat						
Beef	72.08	80.84	88.6*	74.10	53.41	59.90
Veal	4.20	3.04		87.65	3.68	2.66
Pork	57.76	60.82	53.9	75.50	43.67	45.98
Lamb and mutton	3.90	4.37	3.1	60.00	2.34	2.62
Edible offals	10.24†	11.16		61.90	6.34	6.91
Total meat	148.18	160.23			109.44	118.07
Poultry						
Chicken	32.18	33.22		40.90	13.18	16.04
Turkey	7.28	9.62		54.10	3.94	5.20
Total poultry	39.46	48.84	15.5		17.10	21.24
Fish						
Fresh and frozen	5.88			70.20	4.13	
Canned	4.28			75.25	3.22	
Cured60			73.40	.37	
Total fish	10.66	10.65			7.72	7.72
TOTAL	198.30	219.75			134.26	147.03

* Includes veal.

† Increase estimated at 9 per cent corresponding to projected increase in beef plus pork consumption.

The income elasticities for beef, veal, lamb and mutton, and turkeys are weighted regression coefficients based on the 1965 survey data. Those for pork, chicken, and fish are based on the 1955 survey data, with estimates from the 1965 data negative in sign and not statistically significant for chicken and fish. Income elasticity coefficients and annual time-trend factors are summarized in table 33. Because none of the time-trend variables were statistically significant, projections are based on the income elasticity coefficient.

Data on expenditures per capita, given in table 32, are in terms of constant (1962-1966) dollars. Expenditures are projected to increase by 9.5 per cent in real terms. The level of beef consumption in recent years would suggest that

the projected 1980 level may be too low. (See Appendix A figures A-1 to A-14 for postwar trends in actual prices and consumption per capita.)

Eggs account for about 3 per cent of consumer food expenditures. The income elasticity is close to zero. The estimate, based on the 1965 survey data (using weighted regression), was $-.076$ but the log difference equation result of $.055$ was used in this study (see table 33). There was a statistically significant negative time trend of more than 5 per cent per year but consumption appears to have leveled off at about 40 pounds per capita. Thus, the estimates of consumption and expenditures for 1980 are based on the income effect only. The estimate is considered as an upper limit and is somewhat higher than that reported by Daly

TABLE 33
RETAIL INCOME ELASTICITIES AND TIME TRENDS FOR MAJOR FOOD
COMMODITIES†

Item	Income elasticity	Annual time trend‡	Item	Income elasticity	Annual time trend‡
		per cent			per cent
<i>Meat, poultry, and fish</i>			<i>Canned</i>		
Beef290	2.90	Peaches341	-2.55**
Veal591	-4.82	Pineapple447	-0.84
Pork133	-0.02	Other200	
Lamb and mutton571	-0.33	Dried prunes315	-8.60**
Chicken178	0.90	Frozen681	
Turkey768				
Fish004		<i>Vegetables</i>		
<i>Eggs</i>	.055	-5.60*	<i>Fresh</i>		
<i>Fats and oils</i>			Lettuce147	1.10
Butter318	-3.22**	Tomatoes170	-1.56
Lard	-.050	-5.09**	Beans (snap and lima)	0.0	5.71*
Shortening029	-0.16	Onions006	0.0
Margarine	0.0	3.70	Carrots319	-2.42**
Mayonnaise and salad dressing285	3.20	Other160	
<i>Dairy products</i>			<i>Canned</i>		
Milk204	-1.42	Peas032	-4.90
Evaporated milk	0.0	-2.69*	Corn023	-1.12*
Cheese249	4.20**	Tomatoes and products173	
Ice cream331	-2.74*	Other200	
<i>Potatoes</i>			Dry edible beans217	-1.30
Potatoes (fresh)117	-0.71	Frozen vegetables610	
Sweet potatoes (fresh)	0.0	-6.46*			
<i>Sugar and sweeteners</i>			<i>Cereals and bakery products</i>		
Sugar032	1.80	Rice (milled)056	1.30
Corn syrup174	-2.12	Wheat flour (white and whole wheat)083	-1.26
<i>Fruit</i>			Breakfast cereals068	-0.11
<i>Fresh</i>			Corn meal and hominy058	-1.86**
Apples140	-1.02	Bread and other products	0.0	
Bananas140	2.00			
Oranges260	-3.02	<i>Beverages and soup</i>		
Other400		Coffee047	-2.14**
			Soup236	2.70*
			Other beverages220	

† Income elasticities are based on cross-section data and time trends on time-series analysis (see Appendix table A-1).

‡ Figures without subscript indicate coefficient not statistically significant.

* Significant at the 5 per cent level.

** Significant at the 10 per cent level.

and Egbert (1966) as shown in table 34.

Fats and oils account for about 4 per cent of consumer food expenditures. Income elasticities, derived mainly from the 1965 survey data, are relatively low except for butter (.318) and mayonnaise and salad dressing (.285). The trend coefficients were negative and statistically significant for butter and lard (see table 33). Projections for 1980 for butter and lard are based on income and trend effects which result in consumption levels below those for 1962-1966.

The consumption of margarine is pro-

jected at current levels if 1962-1966 average price relationships hold. This estimate may be low, however, if the current trend in prices and consumer preferences continues. The consumption estimate for all fats and oils of 47 pounds per capita is below that of Daly and Egbert (1966) of 49.5 pounds, as shown in table 35.

Dairy products (excluding butter) account for about 15 per cent of consumer food expenditures. In this study, only major dairy products were analyzed, including fluid whole milk, American

TABLE 34
EGGS: CONSUMPTION, PRICE, AND EXPENDITURES 1962-1966 AVERAGE
AND 1980

Commodity	Consumption per capita			Retail price 1962-1966	Expenditures per capita (1962- 1966 dollars)	
	1962-1966	1980 estimate based on:			1962-1966	1980
		Income elasticity	Daly- Egbert			
	pounds (retail weight)			cents	dollars	
Eggs	40.34	41.27	38.95*	35.0	14.12	14.44

* Source: Daly-Egbert (1968) estimate of consumption is 80 per cent of the 1957-1959 level.

cheese, evaporated whole milk, and ice cream.

The income elasticity for milk obtained from 1965 survey data is .20 which compares favorably with that reported by Rojko (1957). Further, Rojko reports the percentage effect of time per year as -1.32 as compared with -1.42 in this study. Although our coefficient is not statistically significant, a negative time trend appears consistent with observed trends and with Rojko's study. Thus, the combined income and trend effects are used in projecting consumption to 1980 (as indicated in table 36).

For American cheese, Rojko reports a positive trend of 4.27 per cent which

compares with 4.20 in this study. However, our income elasticity from cross-section data of .249 differs markedly from Rojko's time-series estimate of -.99, an estimate that was not statistically significant. For projections to 1980, the estimated level, based on the income elasticity and significant time-trend effect, was 12.04 pounds and only 6.78 pounds based on income effect alone. These estimates were averaged to obtain the 1980 consumption level given in table 36.

Estimated consumption of evaporated milk is based on a statistically significant negative time trend obtained from time-series analysis. The income elasticity

TABLE 35
FATS AND OILS: CONSUMPTION, PRICE, AND EXPENDITURES, 1962-1966
AVERAGE AND 1980

Commodity	Consumption per capita			Retail price 1962-1966	Expenditures per capita (1962-1966 dollars)	
	1962-1966	1980 estimate based on:			1962-1966	1980
		Income elasticity	Income and trend			
	pounds (retail weight)			cents	dollars	
Butter	6.62	4.92	77.65	5.14	3.83
Lard	6.35	3.04	20.15	1.28	.61
Shortening	14.10	14.27	29.40	4.15	4.20
Margarine	9.80	9.80	26.65	2.83	2.83
Other	13.40*	15.00	38.35	5.14	5.75
TOTAL	59.28	47.04	49.5		18.54	17.22

* Includes all other fats and oils of which mayonnaise and salad dressing equal 3.9 pounds.

TABLE 36
DAIRY PRODUCTS EXCLUDING BUTTER: CONSUMPTION, PRICES, AND
EXPENDITURES, 1962-1966 AND 1980

Commodity	Consumption per capita			Retail price 1962-1966	Expenditures per capita (1962-1966 dollars)	
	1962-1966	1980 estimate based on:			1962-1966	1980
		Income and trend	Daly- Egbert			
	pounds (retail weight)			cents	dollars	
Fluid milk and cream						
Fluid whole milk (A)	271.40	242.18	..	11.9	(32.30)	
Other	40.96	36.55*		
TOTAL fluid milk equivalent	(303.6)			11.9	36.13	32.24
Cheese						
American (B)	6.14	9.41†	..	84.10	5.16	7.91
Other	7.88	12.08‡	..	36.15¶	2.87	4.40
Evaporated and condensed						
Evaporated whole milk (C)	8.90	5.06	..	17.5	1.56	.89
Other	7.12	4.06§	..	22.4	1.59	.92
Ice cream (D)	18.14	17.55**	..	35.0	6.51	6.30
Dry milk products						
Nonfat dry milk	5.86		..			
Other	1.06		..			
	6.92	7.00††		49.01‡‡	3.39	3.43
Total						
Product weight	367.46				57.21	56.09
Sum of items A, B, C, D	304.58	274.20				
Milk equivalent: fat content basis	025		570			

* Percentage increase assumed equal to that for milk.

† Average of estimates based on income (8.73) and income and trend (12.04).

‡ Percentage increase assumed equal to that for American cheese.

§ Estimate for cottage cheese.

¶ Percentage increase assumed equal to that for evaporated milk.

|| Estimate for condensed milk.

** Average of estimates based on income (20.66) and income and trend (14.44).

†† Estimated at about 1962-1966 level.

‡‡ Estimate for nonfat dry milk.

(shown in table 36) was set at zero although the cross-section data for 1965 provided a negative income elasticity of $-.674$. It is expected that consumption will continue to decline in the future.

Estimated consumption of ice cream, based on an income coefficient, was 20.66 pounds and that for income and a statistically significant time trend was 14.44 pounds. These estimates were averaged in making projections for 1980.

Potatoes.—Per capita consumption of potatoes and sweet potatoes showed a

declining trend during the postwar period (see Appendix figure A-5). The decline in fresh potatoes were somewhat offset by increases in the consumption of processed potato products, especially chips (included with fresh) and frozen french fries. Retail prices for fresh potatoes and sweet potatoes increased over the period, contributing to the downward trend in consumption.

The income elasticity coefficients are from 1955 survey data. The elasticity coefficients, based on the 1965 survey

TABLE 37
 POTATOES: CONSUMPTION, PRICES, AND EXPENDITURES, 1962-1966 AND 1980

Commodity	Consumption per capita			Retail price 1962-1966	Expenditures per capita (1962-1966 dollars)	
	1962-1966	1980 estimate based on:			1962-1966	1980
		Income elasticity	Income and trend			
	pounds (retail weight)			cents	dollars	
Potatoes						
Fresh.....	92.40	96.92	87.59	7.80	7.21	6.83
Canned.....	42					
Frozen.....	5.38		9.00*	30.05	1.61	2.70
TOTAL, product weight.....	98.18		96.59		8.82	9.53
fresh equivalent.	106.20		106			
Sweet potatoes						
Fresh.....	4.40	4.40	1.54	15.30	.67	.24
Canned.....	1.16		1.16	22.00	.26	.26
TOTAL, product weight.....	5.56		2.70		.93	.50
fresh equivalent.....	5.72					
TOTAL, product weight	103.74		99.29		9.75	10.03

* Rough estimate based on trends in consumption.

data, were .008 for potatoes and $-.587$ for sweet potatoes. For projections to 1980, the combined effect of income and time trend is used for fresh potatoes and sweet potatoes (table 37). The estimates for potato products are rough estimates

based on recent trends in consumption
Sweeteners (excluding noncalorics).—
 Sugar is the major item in this group that accounts for more than 3 per cent of consumer food expenditures. Consumption per capita has remained rela-

TABLE 38
 SUGAR AND SWEETENERS: CONSUMPTION, PRICES, AND EXPENDITURES,
 1962-1966 AND 1980

Commodity	Consumption per capita			Retail price 1962-1966	Expenditures per capita (1962- 1966 dollars)	
	1962-1966	1980 estimate based on:			1962-1966	1980
		Income elasticity	Daly-Egbert			
	pounds (retail weight)			cents	dollars	
Sugar, refined	96.32*	98.12	97.2†	12.80	12.10	12.27
Corn syrup	13.00	13.95		21.25	2.75	3.90
Other sweeten- ers.....	6.64	6.64‡		27.00§	1.80	1.80
TOTAL.....	116.46				16.55	17.03

* Includes 5.3 pounds in processed fruits and vegetables.

† Converted to refined from raw as reported by Daly-Egbert (1966), using factor 793.5.

‡ Assumed equal to 1962-1966 average.

§ Price of other sweeteners (1957-1959) adjusted to 1962-1966 level (Hiamstra 1968).

TABLE 39
FRUITS: CONSUMPTION, PRICE, AND EXPENDITURES, 1962-1966 AND 1980

Item	Consumption per capita			Retail price 1962-1966	Expenditures per capita (1962-1966 dollars)	
	1962-1966	1980 estimate based on:			1962-1966	1980
		Income elasticity	Income and trend			
	pounds (retail weight)			cents	dollars	
Fresh						
Apples	17.20	18.21		17.35	2.98	3.16
Bananas	16.96	17.95		15.90	2.70	2.85
Oranges	14.46	16.04		18.10	2.62	2.90
Other	29.16	29.16*		22.05	6.43	6.43
TOTAL	77.78	81.36			14.73	15.34
Canned fruit						
Peaches	4.42		5.28	18.55	1.19	.98
Pineapples	3.98	3.66		27.20	.94	1.00
Other	13.46	13.45†		20.00	2.69	2.69
TOTAL	22.86	22.57			4.72	4.67
Other items						
Canned juice	11.66	11.66*		18.40‡	2.15	2.15
Chilled fruit and juice	1.99	1.99*		26.90‡	.53	.53
Chilled citrus segments37	.37*		22.10‡	.08	.08
Frozen fruit	3.70	4.72		66.20	2.08	2.65
Frozen citrus juice	4.82	5.90		69.40	3.21	4.09
TOTAL	22.54	24.64			8.05	9.50
Dried fruits						
Prunes60		.23	42.43	.26	.10
Other	2.36	2.02†		39.50‡	.93	.80
TOTAL	2.96	2.25			1.19	.90
TOTAL, Retail					28.69	30.41
Product weight	126.04	130.64				
Fresh equivalent	171.64		206			

* Assumed equal to 1962-1966 level.

† Based on income elasticity of $-.002$ from 1965 survey data weighted regression analysis.

‡ Weighted average price for 1967-1969 (Hiesentra, 1968, p. 51).

§ Based on income elasticity of $-.043$ from 1965 survey data weighted regression analysis.

tively constant at about 97 pounds per capita. The income elasticity for sugar (and also for corn syrup) is based on 1965 survey data with the coefficient obtained by a log difference equation. The annual time trend obtained from the time-series data analysis was not statistically significant. Projections, therefore, are based on the income elasticity coefficient and the projected in-

crease in per capita disposable income (table 38). The same approach was used for corn syrup. The exclusion of non-caloric sweeteners, the demand for which has been increasing, is an important limitation on the possible accuracy of these projections.

Fruits include a large number of commodities, but statistical analyses were made on only a few of the important

TABLE 40
VEGETABLES: CONSUMPTION, PRICE, AND EXPENDITURES, 1962-1966 AND
1980

Commodity	Consumption per capita			Retail price 1962-1966	Expenditures per capita (1962-1966 dollars)	
	1962-1966	1980 estimate based on:			1962-1966	1980
		Income elasticity	Income and trend			
	pounds (retail weight)			cents	dollars	
Vegetables						
Fresh						
Lettuce.....	19.04	20.21		16.40	3.12	3.31
Tomatoes.....	10.44	11.19		32.90	3.43	3.68
Beans, snap and lima.....	2.30	1.67		24.40	.58	.41
Onions.....	10.92	10.94		12.00	1.31	1.31
Carrots.....	6.64		7.31	15.25	1.01	1.15
Other.....	42.44	45.10		18.00	7.64	8.12
TOTAL.....	91.78	96.42			17.07	17.98
Canned						
Pears.....	4.12		2.00	18.70	.77	.39
Corn.....	5.48	5.53		19.50	1.07	1.08
Tomatoes, whole.....	4.50	4.97		13.90	.63	.69
Tomato products.....	14.10	15.10*		26.90	3.79	4.06
Other.....	17.88	19.24		26.95	4.81	5.21
TOTAL.....	46.06	46.94			11.07	11.42
Frozen.....	8.18	10.27		36.50	2.99	3.75
TOTAL, Product Weight,						
Retail.....	140.02	153.63			31.13	33.15
Fresh Equivalent, Retail	194.06	..	208†			
Dry beans.....	7.16	2.30		24.4	.17	.13
TOTAL.....			31.40	33.28

* Estimated at .25 as compared to .17 for canned whole tomatoes.

† Converted from farm to retail using factor of 98.2.

fruits. About 8 per cent of consumer food expenditures are for fruits in fresh, canned, dried, and frozen form. The existence of many commodities and multi-outlets prohibited a detailed analysis of each item with the adopted approach to demand analysis. The fresh fruits studied include apples, bananas, oranges, and all others. The demand for prunes was selected as an important dried fruit and frozen fruit was taken as an aggregate.

All items showed positive income coefficients based on weighted regressions

of the 1965 survey data except for canned peaches (-.002) and dried fruit (-.043) for which alternative estimates were used.

Projected consumption levels for included items are based on income elasticity estimates except where trend coefficients also were statistically significant (canned peaches and prunes). Other items were assumed equal to 1962-1966 per capita consumption levels (table 39).

Vegetables account for more than 8 per cent of consumer food expenditures.

TABLE 41
CEREAL AND BAKERY PRODUCTS: CONSUMPTION AND EXPENDITURES,
1962-1966 AND 1980

Commodity	Consumption per capita			Expenditures per capita	
	1962-1966	1980 estimate based on:		Average 1963 and 1966*	1980†
		Income elasticity	Income and trend		
pounds			dollars		
Rice, milled	7.18	7.35		2.34	2.40
Wheat flour, white and whole wheat	108.20		93.35	4.16	3.59
Breakfast cereals	7.88		7.94	11.45	11.54
Corn meal and hominy	0.76		7.62	1.56	1.22
Bread and other products				39.02	39.02
TOTAL				58.53	57.77

* Based on the relative importance of expenditures on cereals and bakery products in the BLS consumer surveys of 1963 and 1966 (Heimstra, 1968, p. 172) of 2.48 per cent of total expenditures. This percentage was applied to the average of disposable income in 1963 and 1966 of \$2,350. The distribution of expenditures by item is based on data from the 1965 survey.

† With constant prices, the change in expenditures is proportional to the change in per-capita consumption.

As with fruit, vegetables are sold in many varieties and forms but only a limited number of commodities were analyzed separately. *Fresh vegetables* include lettuce, tomatoes, snap and lima beans, onions, carrots, and "all other fresh vegetables." *Canned vegetables* include peas, corn, canned whole tomatoes and tomato products (product weight), and "all other canned vegetables." *Dry*

edible beans and *frozen vegetables* are considered also.

Income elasticities show considerable variation, with frozen vegetables estimated at .610 as compared with negligible income effect, for example, for canned peas. Several vegetable items have statistically significant negative time trends—such as canned peas, fresh beans, and carrots (table 40).

TABLE 42
BEVERAGES AND SOUP: CONSUMPTION AND EXPENDITURES, 1962-1966
AND 1980

Commodity	Consumption per capita			Expenditures per capita	
	1962-1966	1980 estimate based on:		Average 1963 and 1966	1980*
		Income elasticity	Daly- Egbert		
pounds			dollars		
Coffee, green bean equivalent	15.24	15.54	16.00	9.82	10.01
Soup	16.16	17.76		7.89	8.67
Other beverages				6.24	6.00
TOTAL				23.95	26.68†

* With constant prices, the change in expenditures is proportional to the change in per-capita consumption.

† Based on relative importance of expenditures on nonalcoholic beverages in the BLS consumer surveys of 1963 and 1966 (Heimstra, 1968, p. 172) of 1.915 per cent of total expenditures. This percentage was applied to the average of disposable income per capita in 1963 and 1966 of \$2,350. The distribution of expenditures by item is based on data from the 1965 survey.

TABLE 43
PER CAPITA FOOD EXPENDITURES, 1962-1964 AND PROJECTED 1980,
ASSUMING COMMODITY PRICES REMAIN AT 1962-1964 LEVELS

Commodity group	Food expenditures per capita			
	1962-1966	1980		
		Estimated	Changes from 1962-1966	Percentage of all food expenditures
	dollars		per cent	
Meat, poultry, fish.....	134.26	147.03	+ 9.5	33.0
Eggs.....	14.12	14.44	+ 2.2	3.2
Fats and oils (including butter).....	18.54	17.22	- 7.1	3.9
Dairy products (excluding butter).....	57.21	56.09	- 2.0	12.6
Potatoes.....	9.75	10.03	+ 2.9	2.2
Sweeteners (excluding non-caloric).....	16.55	17.03	+ 2.9	3.8
Fruits.....	28.69	30.41	+ 6.0	5.8
Vegetables.....	31.40	33.28	+ 6.6	7.5
Cereal and bakery products.....	53.53	57.77	+ 1.3	13.0
Beverages and soup.....	23.93	26.63	+11.4	6.0
TOTAL included items.....	392.73	409.85	+ 4.4	92.0
Excluded food expenditures.....	34.27*	35.75†	+ 4.4	8.0
Expenditures				
Food.....	427	446	+ 4.4	100
Nonfood.....	1,656	2,572	+52.6	..
TOTAL.....	2,113	3,018 ‡	+42.8	..
Disposable personal income.....	2,298	3,281 ¶	41.01	..

* Includes underestimation of expenditures for meals away from home since consumption is evaluated at retail prices. Also, the following expenditures (1957-1959) are not included: molasses (\$2.07), baby food (\$1.55), peanuts (\$2.24), tree nuts and coconuts (\$2.30), and frozen desserts (\$2.46).

† Estimated at 104.4 per cent of 1962-1964 level, equal to the increase for included items.

‡ Estimated at 92 per cent of disposable personal income, equal to that for 1962-1964.

¶ Expressed in 1964 dollars, as given by Daly-Figbert (1966).

Consumption levels in 1980, assuming constant prices at 1962-1966 levels, are projected to increase for most vegetable products—especially for frozen vegetable items. Decreases are expected in consumption per capita of dried, edible beans and items such as canned peas, where frozen products will replace market demand for the product. Expenditures in 1980 are projected to increase by 6.48 per cent.

Cereals and bakery products account for about 15 per cent of consumer food expenditures. This group posed difficulties for analysis because certain items such as wheat flour are purchased as

such by the consumer but also is used in processed products such as bread and bakery products. We analyzed four items—milled rice, white and whole wheat flour, breakfast cereals (taking the price of corn flakes as representative of all items), and corn meal and hominy (using corn meal prices). Bread and other bakery products were included by using the direct elasticity of $-.150$ for cereals and bakery products used in Brandow (1961).

The income elasticity related to cross-section data for wheat flour purchased as such. The time-trend coefficients, obtained from time-series analyses of each

commodity using log first difference equations, are negative except for milled rice. Although not statistically significant, these time-trend coefficients plus income elasticities are used to adjust base period expenditure levels to 1980 expenditures. This appeared reasonable because of consumption trends in recent years. These estimates are given in table 41.

Beverages and soup.—Only two commodities, coffee and soup, were analyzed based on survey and time-series data. Another item, "other nonalcoholic beverages," was included since expenditures are important as based on the U. S. Bureau of Labor Statistics consumer expenditure surveys of 1963 and 1966 (see table 42). In projecting expenditures to 1980, the income elasticity coefficient

alone was used for coffee and soup, although the time-trend coefficients were statistically significant. This judgment factor was based on inspection of recent trends for these commodities (see Appendix figure A-15).

The summary of per-capita expenditures for 1980 is given in table 43. The distribution of expenditures changes slightly with the major dollar increase in expenditures (in 1962-1966 dollars) for meat. In 1962-1966, food expenditures accounted for 18.6 per cent of disposable personal income. This proportion is expected to decline to 13.5 per cent in 1980, assuming real income per capita increases 42.8 per cent and real food expenditures per capita increase by only 4.4 per cent.

Conclusions and Implications

Conclusions

The major conclusions, based on the results of this study, are listed below in the order in which they are discussed in the text.

(1) It is possible to obtain a demand interrelationship matrix incorporating the ideas of neutral want association and want independence.

(2) For most commodities included in this study, the behavior of marketing margins appeared to be a linear function of the retail prices. One advantage of this specification is that it includes both the constant absolute spread and the constant percentage spread as special cases.

(3) Both the intercept and slope in the relationship expressing price spread as a linear function of retail price did not show significant variations over seasons for most commodities. Only four commodities showed seasonal variations in both intercepts and slopes.

(4) In general, higher income groups tended to consume high-quality products as far as quality can be measured in terms of prices. In the cases where quality elasticity was negative, the prices paid by the upper income group was less than the prices paid by the lower income group. This may indicate that quality is only one of the factors that influence average prices paid; other factors are ability to obtain quantity discounts, facilities to shop from low priced areas, and direct bulk purchases from farms.

(5) With per-capita quantity consumed as the dependent variable, economies of food use were revealed for four commodities and diseconomies of scale were revealed for seven commodities. However, when per-capita expenditure was the dependent variable, the number of commodities with economies and diseconomies was six and three, respectively. Economies in food use were more

predominant in per-capita expenditures than in per-capita quantities.

(6) For most commodities, income elasticities did not change significantly between 1955 and 1965. Though there were some readjustments in the quantities consumed by persons in different income groups over this period, a redistribution of population in the income groups offset some of this effect and the net result was that only seven commodities showed significantly different income elasticities for these two years.

(7) For most of the commodities, a relationship, linear in logarithms, between consumption and income showed significant regional variation in the intercept over the four regions. When quantities were the dependent variable, 31 commodities showed significant regional variations in income elasticities, while the corresponding number was reduced to 24 when expenditure was the dependent variable. The existence of such wide variations in the coefficients of income-consumption relationship over the different regions makes it necessary to place emphasis on regional estimates rather than an approximation to national estimates when a study is intended to analyze regional demand characteristics.

(8) Projections of per-capita consumption levels and expenditures are developed for all major food items. The assumption of constant prices at the 1962-1966 level allowed estimation of consumption and expenditures in 1980. If further research on supply response at farm, processing, and retail were available, the constant price assumption could be relaxed to allow more realistic price consumption and expenditure estimates.

Implications of the Results

Implications on methodology.—The implications on the methodology of de-

mand analysis presented here are not based on theoretical considerations alone, rather, they provide empirical support to some of the existing theoretical analysis.

(1) It was concluded that grouping of households according to income groups does not constitute serious loss of precision on the estimates. As Malinvaud (1966, pp. 242-46) points out, the loss of precision will be small if the exogenous variables are homogeneous within a group and if there are at least ten groups. The results indicate that the classification of households, according to family income, into twelve income groups in the 1965 household food consumption survey is appropriate. For such cross-section studies, if the objective is only to obtain the income-consumption relationship, it is enough to retain data according to income groups.

(2) Although it can be argued that income elasticities obtained from time-series data may be more appropriate for projection purposes for most of the commodities included in this analysis, income and prices were highly correlated and, therefore, estimates of income elasticities obtained from the time-series data seemed to be inappropriate. Also, the income elasticities obtained from time-series data reflect short-run elasticities. Cross-section data provide estimates of income elasticities that are generally considered more accurate than those obtained from time-series data. This indicates that it may be appropriate to use a combination of cross-section and time-series data for estimating demand interrelationships. In other words, estimates of income elasticities can be obtained from cross-section data and it can be used as an extraneous estimate for time-series analysis.

(3) Comparison of income elasticities obtained from different cross-section sur-

veys requires that the effects of changes in general price levels and the effects of a redistribution of income over the cross sections should be removed before shifts in income elasticities can be analyzed. The estimated income elasticities will be influenced by the level of prices and relative proportion of individuals in different income classes. In this analysis, a graphic approach was used to convert the 1965 distribution of samples into the 1955 price levels and an illustration was provided to show the implications of redistribution of income on aggregate income elasticity.

(4) Analysis of covariance was used to test the variations in income elasticities over the regions and to test the seasonal variations in margin relationships. Aggregate income elasticities can be obtained as a weighted sum of income elasticities for different regions. However, given an aggregate income elasticity, if there is reason to believe that regional variations exist in income elasticities, there is no method to disaggregate for regional elasticities. In such circumstances, for regional demand analysis, it may be necessary to obtain regional estimates instead of approximating it from the aggregate estimate. In this case, disaggregation of an aggregate income elasticity into regional elasticities is a more important and serious problem than the generally recognized aggregation problems.

(5) Use of the first differences of the variables sometimes may improve the reliability of the coefficients appearing in the demand functions. In the present study, the demand functions were specified as linear in logarithms of the variables and linear in the first differences of the logarithms of the variables. In a number of cases, the first difference specification gave demand equations satisfying *a priori* expectations of signs

and significant coefficients. In most cases, a higher R^2 was obtained when the variables were in logarithms rather than in log differences. However, the higher R^2 , in the case of some equations with logarithms of original variables, could be partially explained by the high intercorrelation among some variables appearing in the equations. Also, in some cases, when the logarithms of the original variables were used, the Durbin-Watson statistic showed either the presence of serial correlation or it fell in an inconclusive range. When the first difference of the logarithms of the original variables were used in the regression equations, both the intercorrelations and serial correlations were reduced. Thus, specification of the regression equations with the first differences of the logarithms of the original variables improved the statistical properties of the estimates. This result is in conformity with Parks' (1968) empirical comparison of alternative functional forms of demand where he concluded that the specification with variables in the first differences of logarithms (referred to as the Rotterdam model of Theil and Barten) gave the best statistical properties among the models compared.

(6) Frisch's procedure for calculating all the direct and cross elasticities implies that the utility function is additive in a cardinal sense. Although this assumption may hold for commodity groups, it may not be true for all the individual items of food. Therefore, when the goal is to obtain a demand interrelationship matrix of the type discussed in this study, Frisch's approach alone is not satisfactory. It was demonstrated here that concepts of ordinal separability can be utilized to identify separable groups and, among the separable groups, an assumption of strong separability permits the use of Frisch's

approach. Still, the approach used here does not handle the simultaneous nature of demand relationships since the procedure used in the present study is based on a single equation approach. The solution of demand equations incorporating the simultaneous nature of demand characteristics and the theoretical restrictions on demand parameters may often lead to the solution of a nonlinear system of equations. When the number of commodities included in the study is large, as in the present case, the iterative procedures used for the solution of the nonlinear system will often become cumbersome though not impossible. In such cases, often it may become necessary to determine a trade-off between including more commodity details and estimating demand parameters through simultaneous relationships.

Implications on demand parameters.—

In addition to the factors pointed out in the text, here are some aspects of the results which can be generalized:

(1) Although the income elasticity for total food is small, wide variations exist among different food items. For commodities such as lard, margarine, evaporated milk, sweet potatoes, beans, and bread, the income elasticities were very close to zero, while frozen fruits and frozen vegetables had income elasticities of more than 0.6. The classification of income elasticities, according to their magnitude, presented in table 14 may be useful for marketing managers in terms of planning their marketing strategy although more detailed analyses may be needed. Similarly, the regional characteristics of income consumption relationship will help marketing managers to determine the nature of emphasis to be placed upon different regions in the overall marketing program. Also, for those commodities showing economies or diseconomies in consumption pattern,

changes in family structure also will be an important consideration.

(2) An estimate of demand interrelationships is required for many policy decisions, especially in the areas of supply control and demand adjustments. Our demand interrelationship matrix is for the United States as a whole and, as such, it is applicable only as an aggregate relationship. Variations among regions and problems of disaggregation make it difficult to derive similar demand interrelationships for different regions directly from the estimates for the United States as a whole. Therefore, when policy decisions are restricted to a state or to a region, it may be necessary to obtain separate regional estimates of demand interrelationships. Another aspect to be kept in mind is that the process of synthesis adopted in obtaining the individual coefficients appearing in the interrelationship matrix is such that the estimates cannot be considered as precise. Also, it is not possible to obtain a measure of accuracy associated with many coefficients. However, these limitations may not reduce the usefulness of the results as they are intended to be only approximate relationships. The assumptions used in obtaining a static demand interrelationship as in table 4 could be modified to incorporate dynamic elements and these modified estimates could be used for projecting the demand interrelationships in a future period.

(3) The nonrejection of the hypothesis that farm retail spreads can be expressed as a linear function of retail prices opens a number of interesting aspects. Demand theory has often emphasized only retail markets and, therefore, most theories are based on retail prices and quantities consumed by the final consumers. The retail market is only one link in the pipeline joining producers and con-

sumers, and a theory of markets will be complete only if all the different links are considered. In particular, it would be convenient if the demand characteristics at one level of the market could be estimated from a knowledge of corresponding characteristics at another level of the market. It was in this connection that the farm level elasticities were derived from retail level elasticities using the elasticity of price transmission. Since margin data were not available for all the commodities included in the present study, farm level elasticities were derived for only a few commodities. However, with margin data for all commodities, it is possible to obtain a complete demand interrelationship matrix at the farm level similar to table 4.

Suggestions for further research

This study has drawn heavily from other demand studies and it is hoped to serve as a basis for further studies. Considering the large number of commodities included in the present analysis, it was possible only to analyze some broad characteristics of demand which are applicable to all commodities. However, individual commodities may often possess certain special characteristics of demand which cannot be brought into the framework of a general study of this nature. Limitations of data and other facilities restricted the scope of this study to certain aspects of demand analysis. Here are some of the factors omitted from the present study which offer great potential for further research.

- (1) While defining the quality elasticity, it was assumed that it represented the difference between expenditure elasticity and quantity elasticity. The implications of the assumption that quality factors account for the difference in these two kinds of elasticities are discussed in the text. Since the estimates of quality

elasticity are derived estimates, the reliability of these estimates will depend upon the reliability of expenditure elasticity and quantity elasticity. Existence of omitted variables will affect both expenditure and quantity elasticity. For example, inclusion of household size in the regression equations has explained some of the negative quality elasticity. If data were available, it would be possible to identify other significant variables. Thus, it may be useful to study the effect of other variables influencing demand relationships on quality elasticity.

Apart from the problem of refining the estimates of quality elasticity from cross-section data, it may be important to study consumer behavior in terms of reactions to quality changes for individual products. Food processing firms and marketing organizations often face the problem of determining the characteristics of their final products. To make effective decisions on the ultimate quality of the product offered in the market, it is important to observe how consumers in different income and social classes react to products with different qualities. It is, therefore, necessary to handle each product separately. For a given commodity, different varieties (qualities) can be identified and specific studies can be undertaken to isolate the salient features influencing buyer decisions. Most existing time-series data are not suited for this type of detailed analysis and, therefore, it is necessary to devise special surveys of buying habits and consumer reactions towards quality. Also, it is important to obtain proper measures of quality which can be quantified.

- (2) The effects of income, household size, and region on quantities consumed were analyzed in detail. It was also pointed out that some other variables such as

season, age distribution, and education also influence consumption. Although it may be difficult to obtain data on all these factors, some of them can be obtained. For example, the 1965 consumption survey obtained data for the four seasons and the complete set of data may be available in the next few years. These data will provide an excellent source of materials for analyzing the seasonal variations in demand and the interactions of seasons and regions on consumption behavior. Isolation of similar other factors will contribute towards better understanding of consumption behavior.

(3) Another area of theoretical and empirical relevance is the application of separability concepts in obtaining demand interrelationships. Our analysis combined cardinal separability and ordinal separability. The major problem in using ordinal separability assumption alone in estimating a simultaneous system of demand equations is that when the number of commodities is large, the resulting nonlinear system is difficult to solve. Also, it is important to obtain

suitable criteria to group the commodities into separable groups. Thus, identification of proper separable groups and estimation criteria are two promising areas of applied statistical research. Comparison of results obtained from such different methods to determine their relative accuracy may be a related topic.

(4) Ample opportunities exist for research in the area of price spreads. Our analysis was based on one particular behavioral relationship between farm level prices and retail prices. It is possible to extend this analysis to include a number of other intermediaries specifying their behavioral characteristics. Also, it may be possible to try other forms of behavioral relationships and to compare these forms with the results obtained from this study. Possible generalizations from these behavioral relationships would be a valuable addition to the present understanding of behavior of different marketing groups and the mechanism of setting prices in the market.

APPENDIX A

Source of Data

Cross-section data

In the present study, we have made extensive use of data from the household consumption surveys in 1955 and 1965. These two surveys were conducted in a systematic manner and they have many aspects in common.

The 1955 survey. (USDA, 1956) provides data on food consumption by all households during the spring of 1955. The objective of the survey was listed as "to obtain current information on patterns of food consumption, expenditures, dietary levels, and household food practices. The households were grouped

(1) by region—Northeast, North Central, South, and West, . . . ; (2) by urbanization—rural farm, rural non-farm, and urban within each region; and (3) by several family income classes within region-urbanization categories." A description of the survey, procedures for working with the data, and examples of use of the data in economic analysis are available in Burk and Lanahan (1958).

The sample included 6,060 households, selected from a housekeeping population of about 153 million civilians. (Out of a total of 162 million, 9 mil-

lion people were excluded because they lived in a household not having at least one person who ate ten or more meals from the household during the survey week or because they lived in rooming houses, hotels, or institutions.) The 6,060 sample households were classified into two groups: the first, containing 4,605 households, was selected on a percentage-probability basis and served as the basic survey group; the second group of 1,455 farm households were taken as a supplementary sample to assure reliable data on farm-consumption patterns.

Data collection.—The survey was conducted by a private marketing research firm under the direction of statisticians and economists from the U. S. Department of Agriculture. Actual data collection was done in personal interviews by trained interviewers.

Period of observation.—Earlier studies had indicated that the spring season was the most representative period for the consumption of many food items. Therefore, the interviews were conducted in the April-June period and the data collected related to food consumption in the week preceding the interview.

Types of information.—The questionnaires used in the survey were designed to cover information about family membership and household composition, money income in 1954, use of individual food items at home in the seven days preceding the interview, and expenditures for meals and snacks away from home by members of the family. A number of reports have been published, using the basic data collected. Reports 1 to 5 contain information on income, household size, expenditures on food items, quantity consumed, and percentage of households consuming the particular item. These five reports correspond to five areas—the entire United States, and the Northeast, North Central, South,

and West; for each region, details are available according to urbanization and income classes. Reports 6 to 10 contain information on (a) less detailed tables on the quantities of foods used than contained in reports 1 to 5, (b) on nutritive value of foods used, and (c) distribution of persons into age and sex groupings. Report 11 provides data on home canning and freezing, Report 12 provides data on home production in 1954, and Report 13 is on home-baking practices. In the present study, we were mainly interested in the structure of the consumption of food commodities and, therefore, we have used the quantities of individual foods used from all sources. Since the present study has emphasized demand interrelationships and shifts in consumption patterns, we considered per-capita quantities of foods to be more appropriate to calculate income elasticities and other measures than expenditure data. However, we have used expenditure data when we were considering the quality aspects.

The 1965 survey was similar to that in 1955. As the report (USDA, 1968, p. 3) points out, "The chief difference between the 1965-66 nationwide survey and the earlier surveys is that the 1965-66 survey is the only one which covered all the four seasons of the year."

The sample for the 1965 survey consisted of 15,101 households of one or more members. As before, it was selected from all households excluding (a) about 5 per cent of the population who were not housekeeping; (b) about 1.5 per cent who were living in group quarters such as rooming houses, hospitals, and prisons; and (c) about 3 to 4 per cent of the population who lived in households in which no member ate at least ten meals from the home supplies. Half the sample (7,532) was collected in the spring of 1965 and the other half was

distributed equally among the other seasons (summer and fall, 1965 and winter, 1966). "The sample design provided for a national self-weighting basic sample plus a supplementary farm sample which overweights the number of farm households in the approximate proportion of 5 : 1." (USDA, 1968, p. 204)

The households to be interviewed were selected in accordance with a multistage area sample design with added control by season. For the basic sample, 144 first-stage units of expected size of 10,000 households were selected at random. Within each first-stage unit, second-stage units of 30 expected housing units were selected at random. Each of the second-stage units was visited and a list of housing units was prepared. By systematic selection, housing units were chosen for interview in the spring in sufficient numbers to yield an average of three households per second-stage unit after allowing for vacancies and other omissions. The lists were updated in summer, fall, and winter, and a sufficient number of households were chosen to yield an average of one schedule per second-stage unit in each of these seasons. Selection of housing units from the second-stage units was independent for each season and no substitutes were provided for households unable or unwilling to participate in the survey.

The data collection was by means of personal interview with members of the household conducted by experienced interviewers who were specially trained for this survey. A detailed list of food items was used to help the respondents to recall the items consumed during the seven days preceding the interview.

Period of observation.—Interviews for collecting data were conducted in all regions during the period, April 3, 1965 to April 2, 1966. Since no substitutions were made in the samples, interviewers

were instructed to call as many as three times if necessary to make the original contact in rural areas, four times in the urban areas, and six times in 281 second-stage sample units in 15 large cities where collection difficulties were anticipated.

Types of information.—As before, the data contained the kinds, quantities, and costs of food items used at home during the seven days preceding the interview. Expenditures for meals and snacks away from home paid by the family members were also outlined. In addition to family income, data were collected on age, education, and employment of the homemaker. Food consumption was measured at the level at which the foods came into the kitchen and, therefore, the data correspond to economic rather than physical consumption. Although the survey covered all the four seasons, at the time of the present study, only five reports relating to spring, 1965 were released. These five reports gave data on household size and consumption of different food items in terms of quantities and values for the United States and the four regions—Northeast, North Central, South, and West.

Comparison with the 1955 survey.—One of the objectives of the 1965 survey was to obtain comparable data with those obtained in the 1955 survey. Therefore, the survey methods had a number of similarities. Here, we shall point out some differences that might affect comparability (this is taken from Report 1 [USDA, 1968, pp. 202-04]).

Modification of schedule. (a) To facilitate machine computation, the design of the 1965 schedule was different from that of 1955.

(b) In 1955, a figure for income was derived from a set of questions asked by the interviewer. For the 1965 survey, a "global" figure for income was obtained

by asking the respondent to estimate 1964 money income after first asking about specific sources of income.

(c) Separate information on donated food issued to low-income families was not obtained in 1955. In 1965, this was obtained separately.

(d) The 1955 questionnaire contained a section on home baking. This was not included in the 1965 questionnaire. On the other hand, the 1965 survey obtained data on the food intake of individuals—a section which was not included in the 1955 survey.

Treatment of households of single individuals. The 1955 data by income were for households of two or more persons. In addition, data on one-person households were shown separately on each table. In 1965, the income classification included all households regardless of size.

Exclusion of money value of food used by boarders and help. In 1955, the money value of food used at home was adjusted to exclude the value of food used by boarders and farm help. In 1965, this adjustment was not made because the effect had been found to be slight.

Difference in the handling of homemade mixtures. Homemade mixtures on hand at the beginning of the seven-day period and used during the survey week in 1965 are included in prepared form whereas, in 1955, such mixtures were included as individual ingredients.

Change in grouping of food items. "Half and half" and "baby cereals" were treated differently in the two cross sections.

Time-series data

Length of time interval.—Published data on consumption and prices are generally on an annual basis. If demand conditions are fairly homogeneous during the year, it is enough to use annual

data to estimate the demand equations. If demand conditions vary widely within a year, it is necessary to use data with shorter intervals. Quarterly data on consumption and prices are available for a few commodities. The choice between annual and quarterly data depends upon the homogeneity in the demand relationship and the purpose for which the model is built. Hiemstra (1967, p. 9) points out that "use of annual data appears most useful for the immediate future, that is, for the next two or three years. Short-term outlook up to and including one year ahead, probably should be based on quarterly data to account for near term variations such as the effects of stock changes." In the present study, we have used both quarterly and annual data to estimate demand equations and a choice of coefficients was made based on the properties of the estimates.

Years considered.—It is generally believed that, for most commodities, the consumption pattern before and after World War II has changed. If we have to handle data from different structures in one regression equation, special devices such as the inclusion of dummy variables have to be adopted. During the immediate period following the war years, enough time-series data were not available to use only the postwar years. Now enough data have been accumulated since the War to permit analysis of the postwar period alone and, therefore, we have used data starting from 1946.

Consumption data.—Often consumption of food items is expressed in terms of three different measures—(1) weight of food items consumed, (2) expenditure on different food items, and (3) nutritive value of food items expressed in terms of calories, proteins, fats, and other vitamins and minerals. When we are interested in the demand for an individual

commodity, the most appropriate measure of demand would be the quantity of the commodity being used. However, when we deal with aggregates of individual commodities, it is difficult to aggregate different commodities if they are expressed in terms of physical units and, therefore, we have to convert them into comparable units. In this situation, it becomes convenient to measure demand in terms of expenditure or nutritive values. Also, when consumption is measured in terms of quantities, aggregation can be made using index measures of consumption of individual foods. While comparing the trend in the consumption of commodities, it is possible to obtain different directions of trend from measures of consumption in terms of quantities and expenditures on account of the influence of price movements. So long as the demand curve is downward sloping, a decrease in the price will cause an increase in quantity consumed, resulting in its upward trend. Whether or not the trend in expenditure also shows the same direction depends upon the elasticity of the commodity. Using the elasticity theorems (Baumol, 1965, p. 179), "if a demand curve has elasticity less than unity (it is inelastic), a rise in price will increase consumer expenditure and vice versa." Also, "if the curve has an elasticity greater than unity (it is elastic), a fall in price will increase consumer expenditure and vice versa." In the present analysis, our choice of the quantities measured in terms of physical units as the measure of demand was based on the following considerations: (1) in most cases, we were interested in the demand for a single commodity and, as such, aggregation problems were not important; (2) for many policy decisions regarding supply adjustments in agriculture, we are interested in the nature of changes

in quantities demanded rather than re-allocation of family food budget according to the changes in prices of different commodities; (3) demand theory specifies quantity consumed as a function of prices and other variables; and (4) since the expenditure on a commodity is the product of its price and quantity consumed, inclusion of both expenditure and prices in the same equation may create some statistical problems of estimation.

Quantity data is available at different levels of the marketing system. Quantity produced and quantity consumed domestically are available in separate tabulations. In our study, we were only interested in domestic consumption and, therefore, we have used data corresponding to domestic food consumption alone. Most of the per-capita consumption figures used in this study are taken from Hiemstra (1968).

Trends in commodity consumption.—

Trends in per-capita food consumption is available in Hiemstra (1968, pp. 7-15) and only a brief discussion is given here. Consumption of beef and poultry have increased considerably during the post-war period while pork and veal consumption have declined. Lamb consumption has remained fairly stable. Per-capita consumption of eggs and total dairy products have declined; those of processed fruits and vegetables have increased at the expense of fresh fruits and vegetables. Consumption of total fats and oils have remained fairly stable and the consumption of total cereal products has dropped. Increases in consumption of instant coffee has offset the decline in the consumption of regular coffee. These trends are shown in figures A-1 to A-14. Figure A-15 gives trends in actual per-capita food expenditures (undeflated) and expenditures expressed as a percentage of personal disposable income.

Appendix figures A-1 to A-14 illustrate 1946–1968 prices for selected commodities, and their per-capita consumption with projection to 1980.

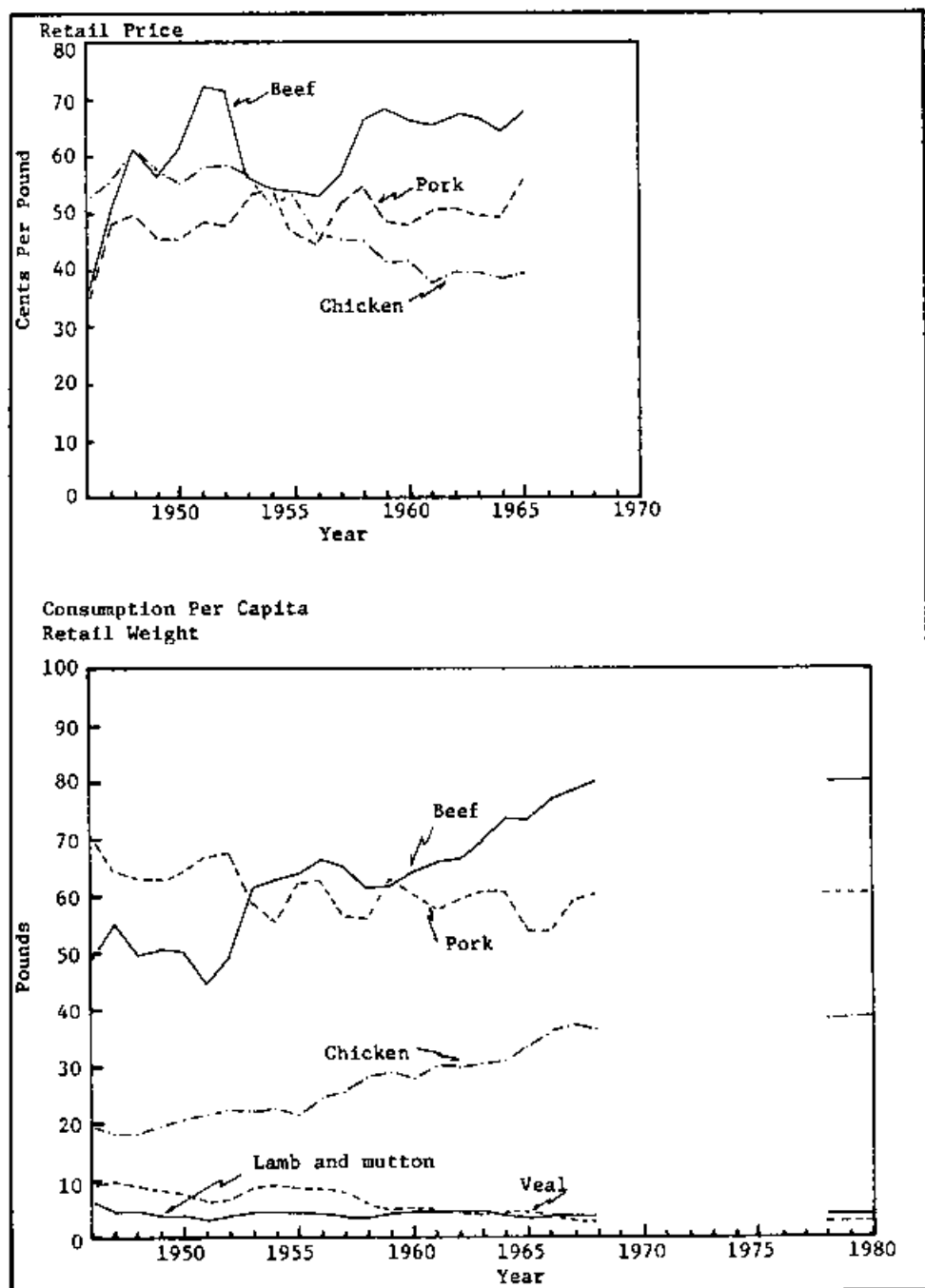


Fig. A-1. Meat and chicken.

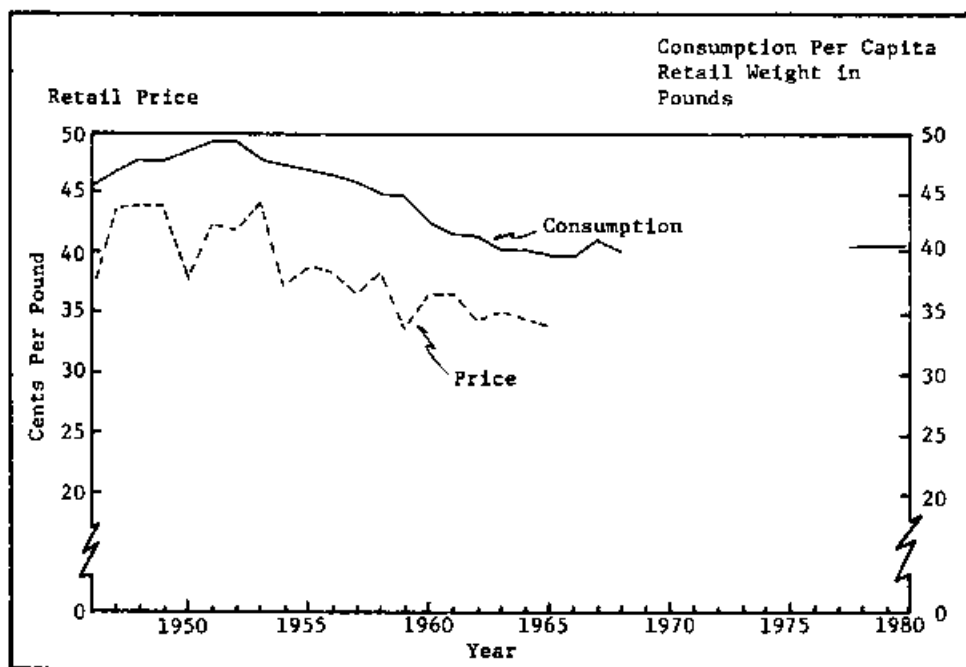


Fig. A-2. Eggs.

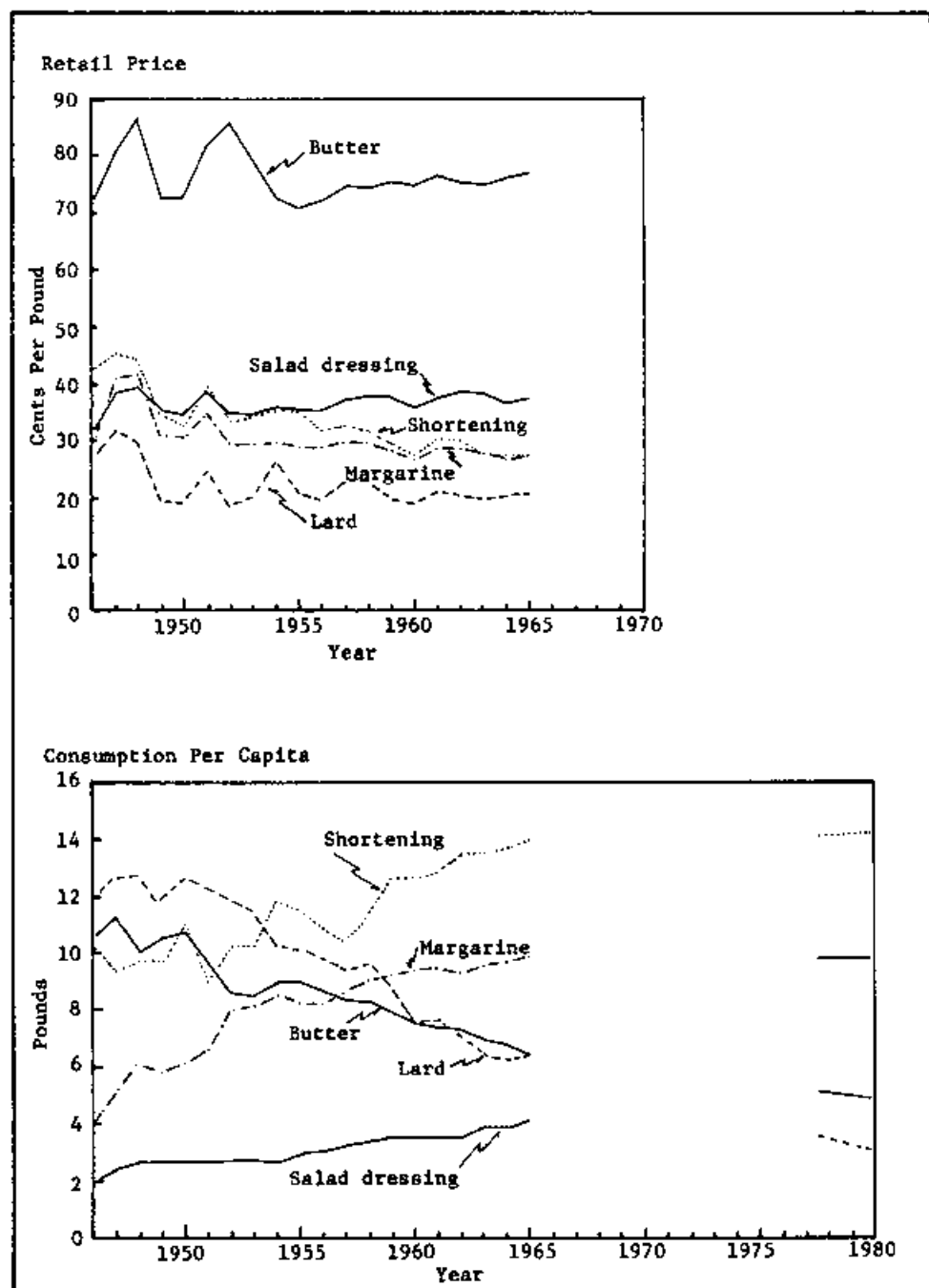


Fig. A-3. Fats and oils.

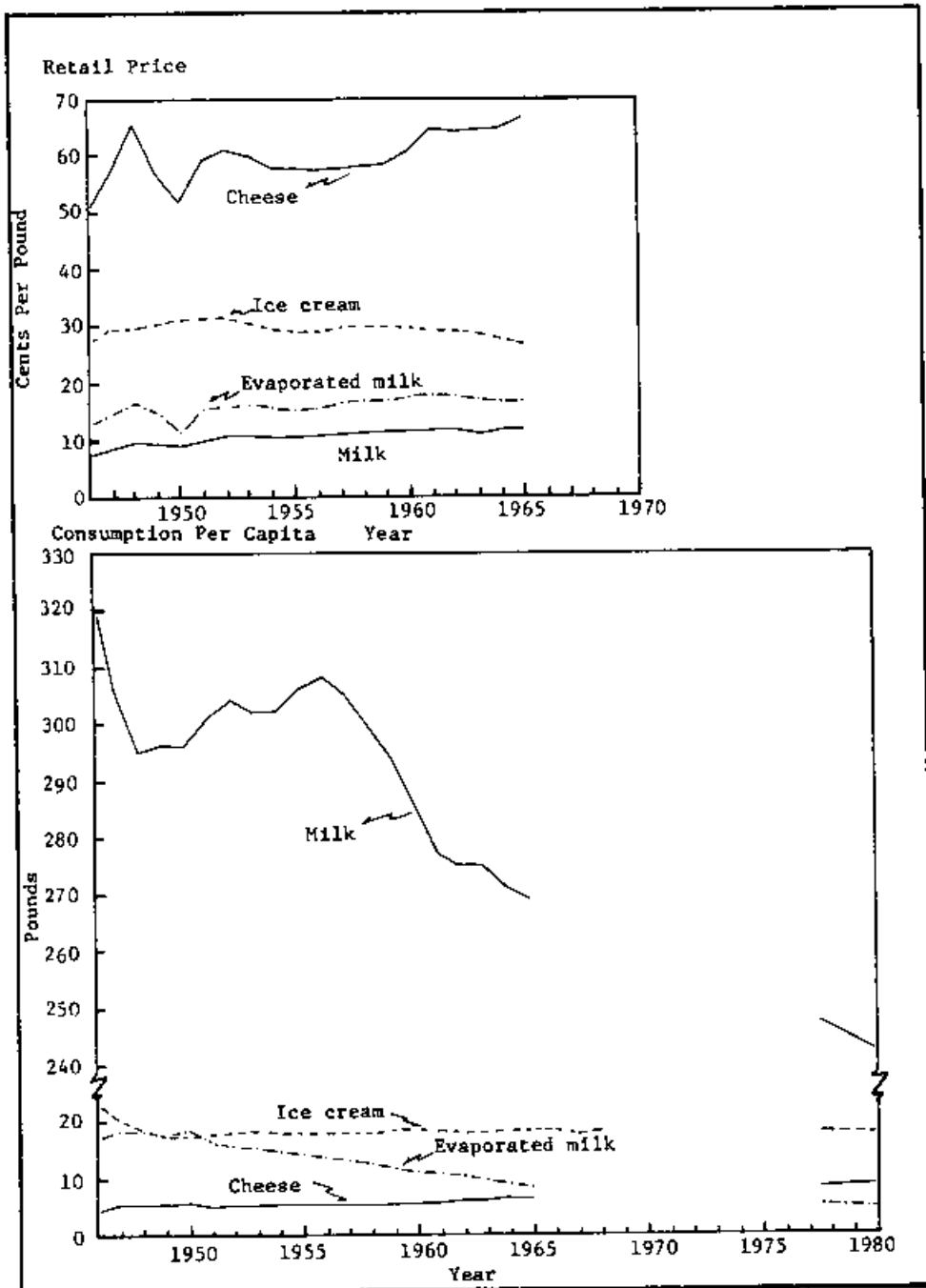


Fig. A-4. Dairy products, excluding butter.

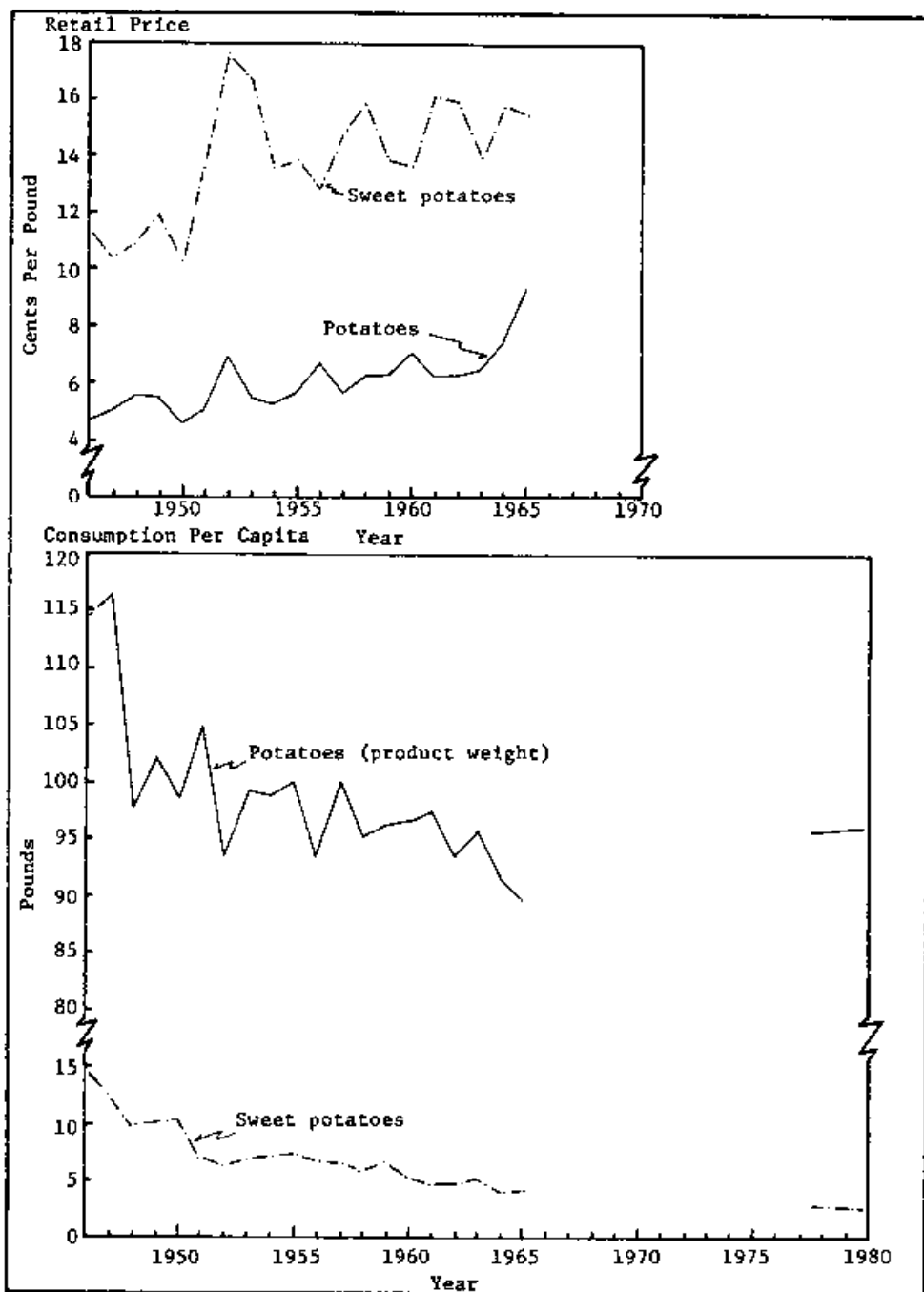


Fig. A-5. Potatoes.

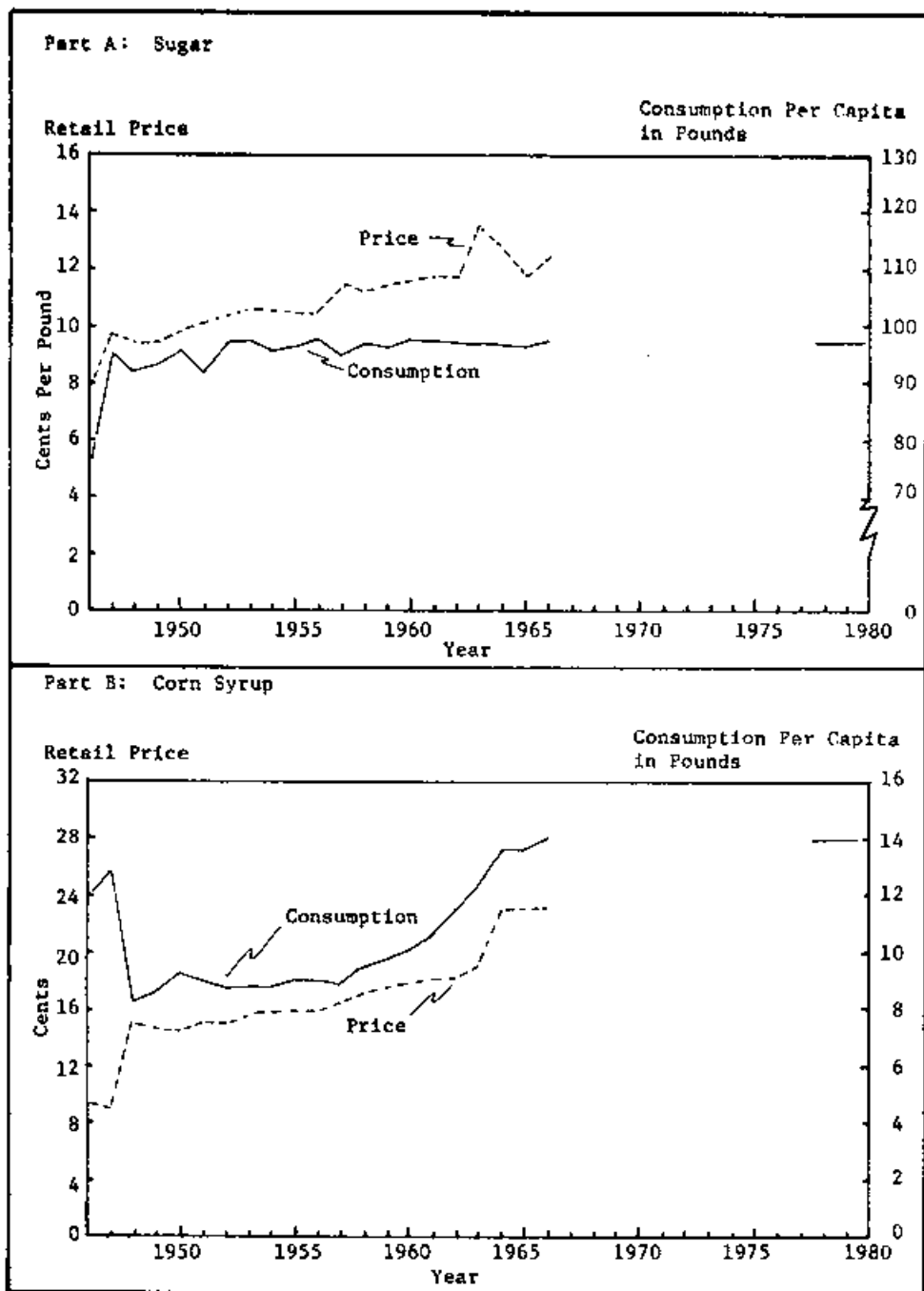


Fig. A-6. Sugar and corn syrup.

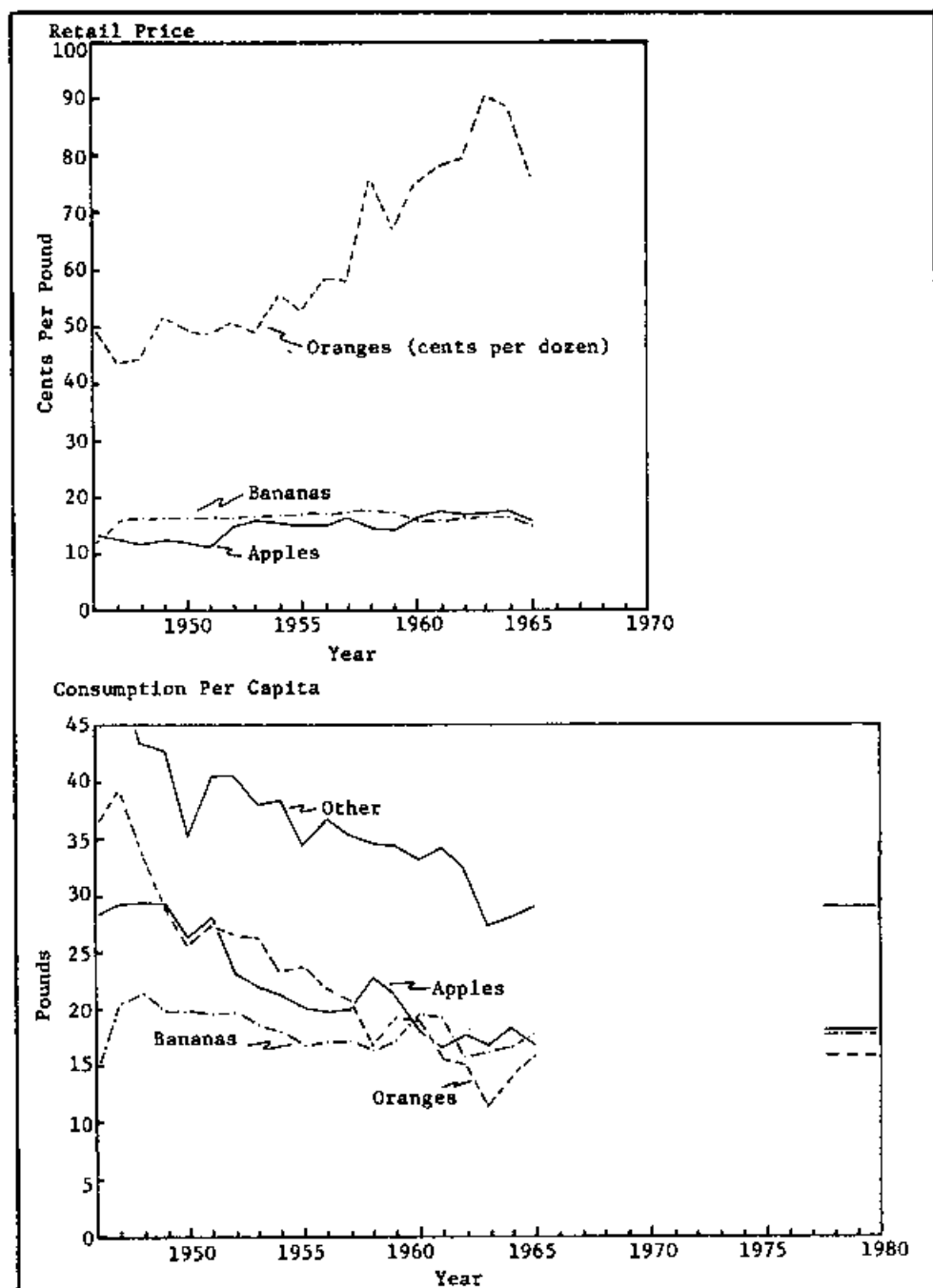


Fig. A-7. Fresh fruit.

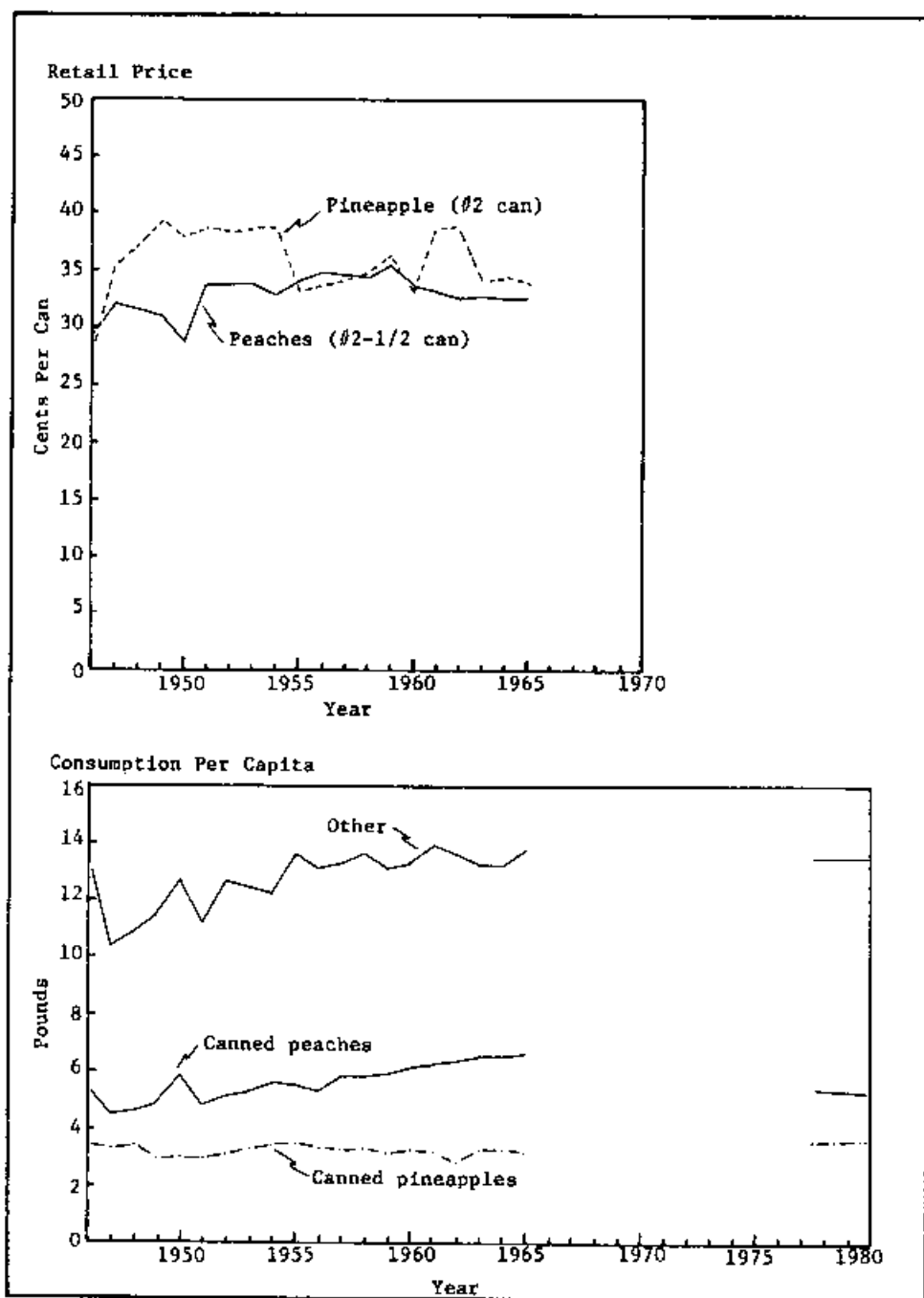


Fig. A-8. Canned fruit.

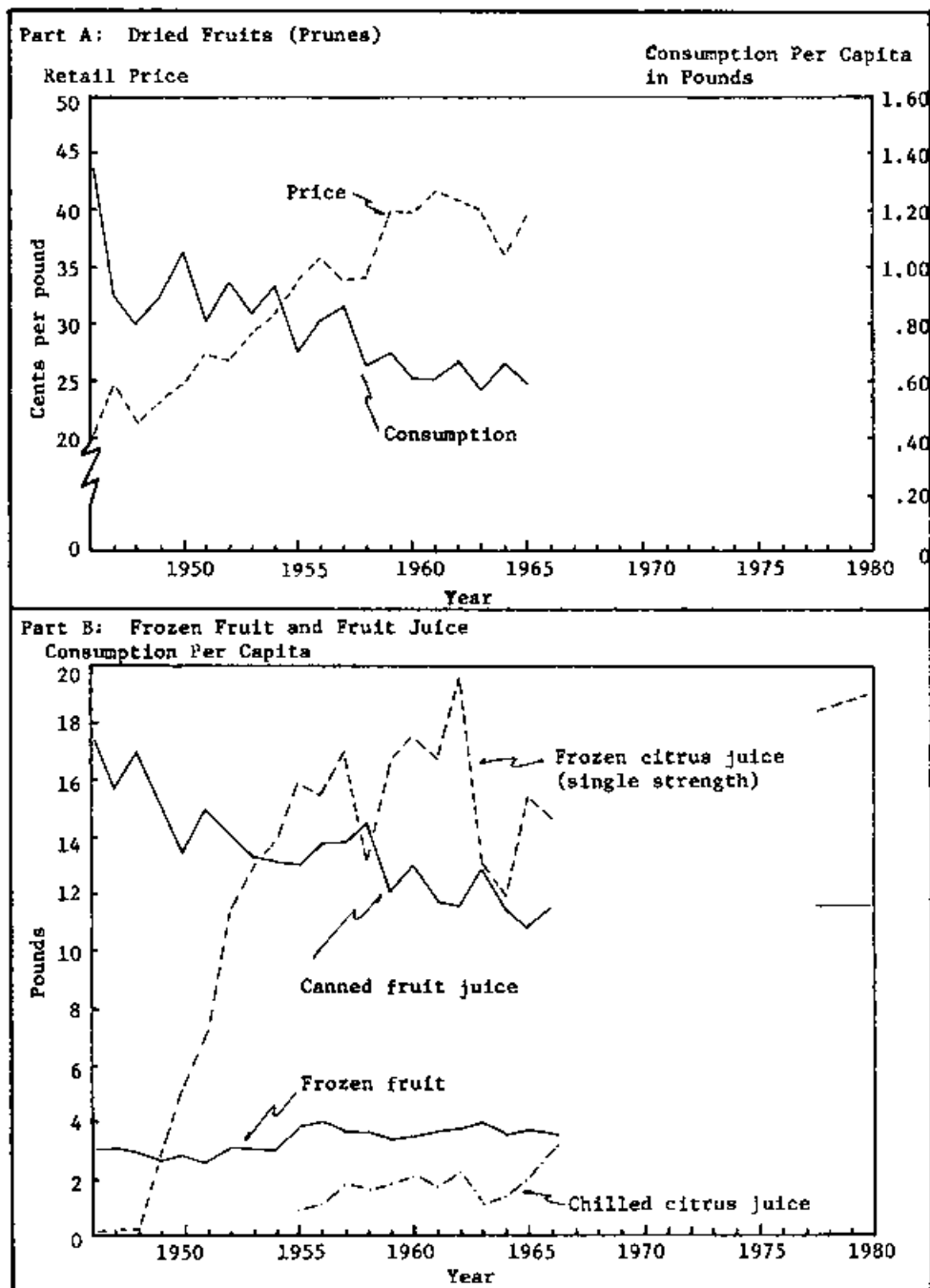


Fig. A-9. Dried fruit (prunes), frozen fruit, and fruit juices.

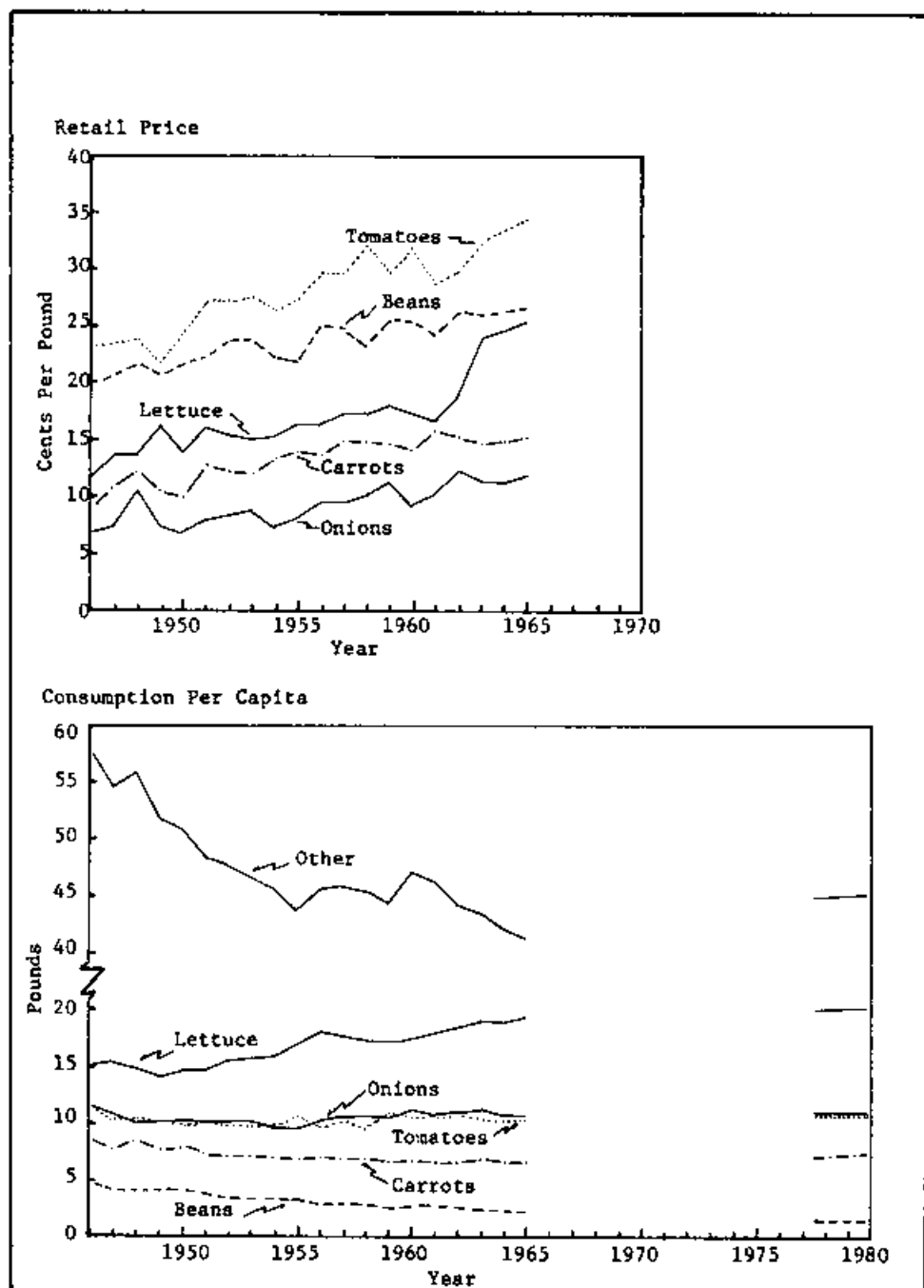


Fig. A-10. Fresh vegetables.

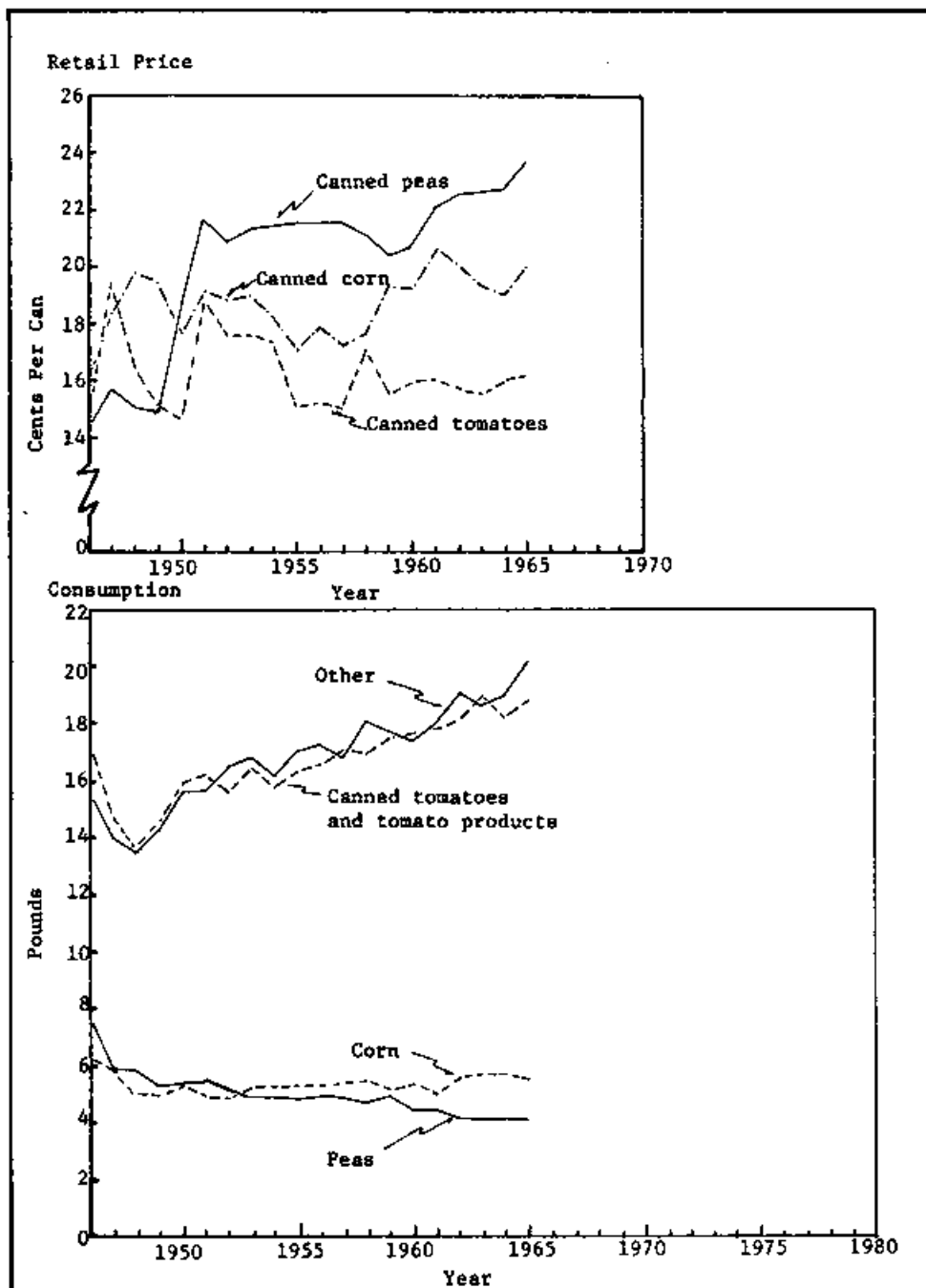


Fig. A-11. Canned vegetables.

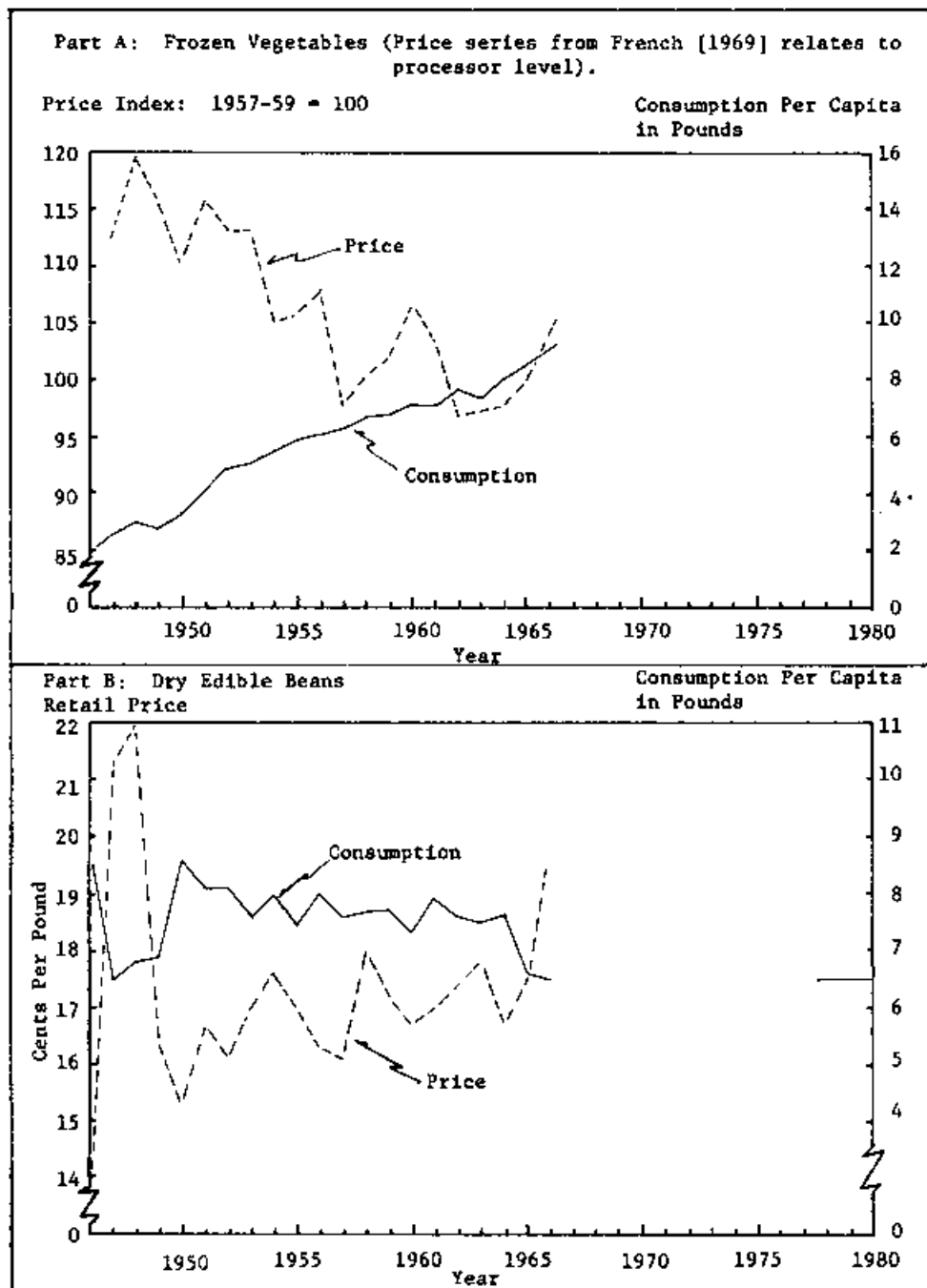


Fig. A-12. Frozen vegetables and dry edible beans.

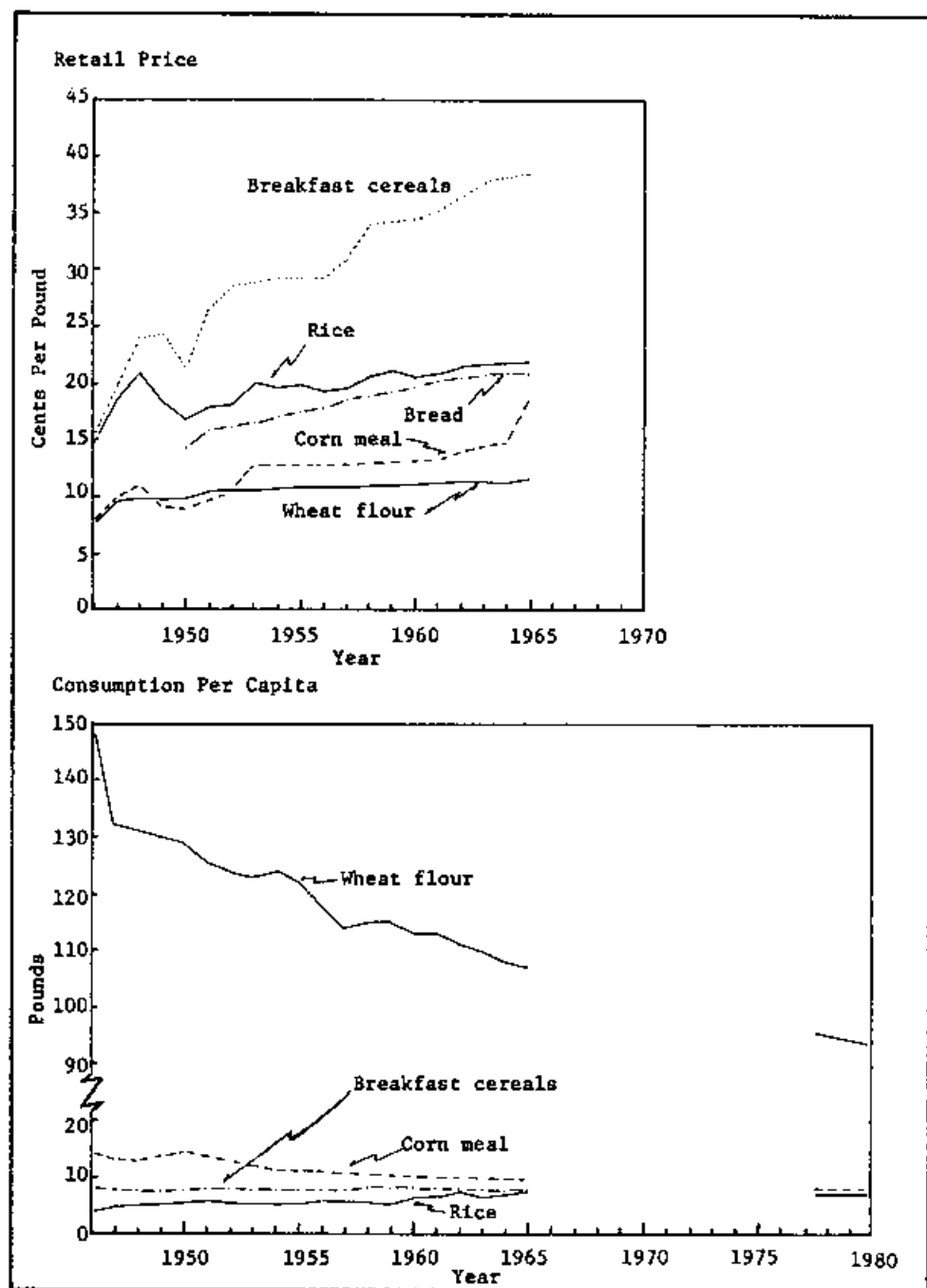


Fig. A-13. Selected cereal and bakery products.

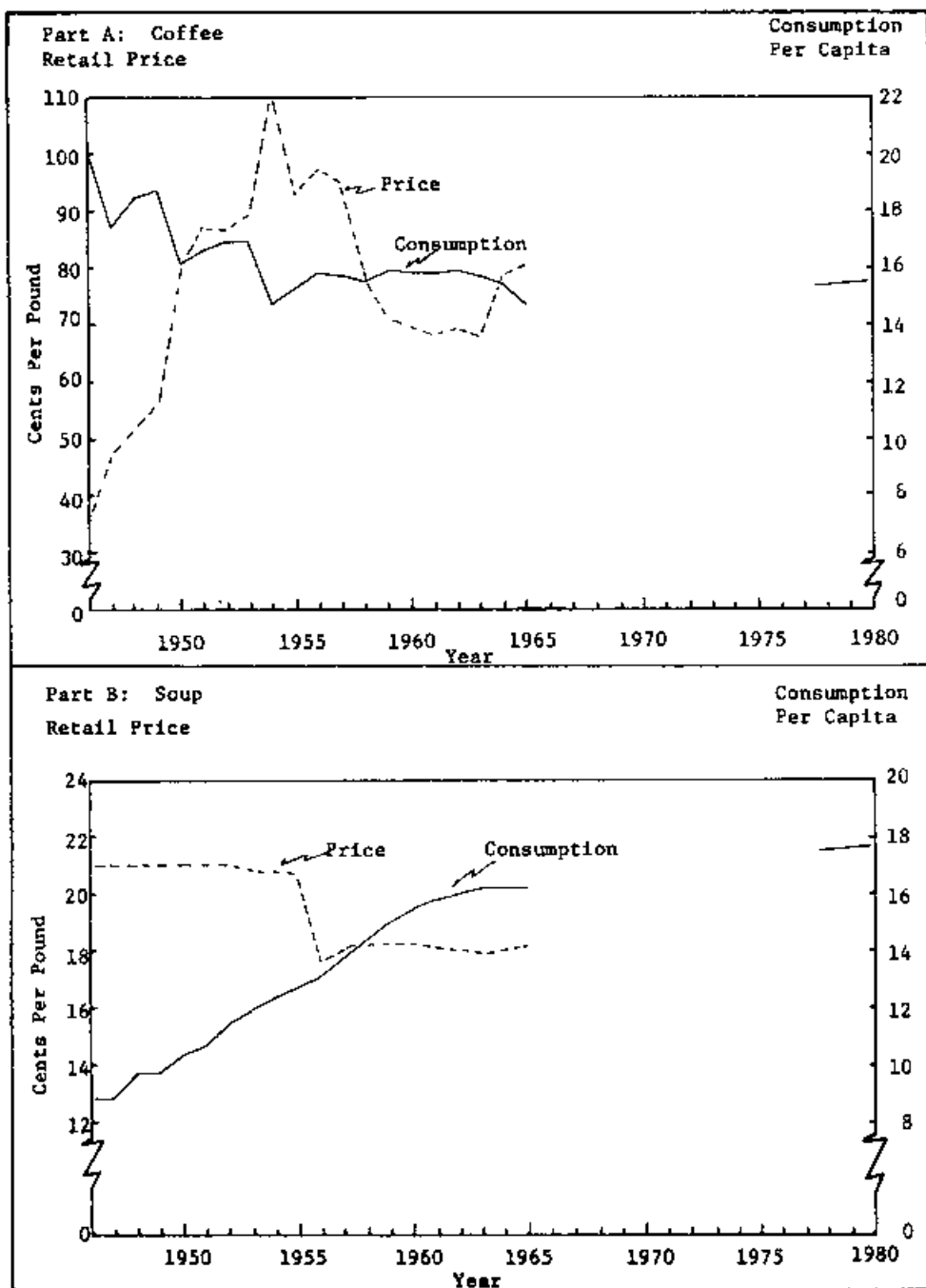


Fig. A-14. Coffee and soup.

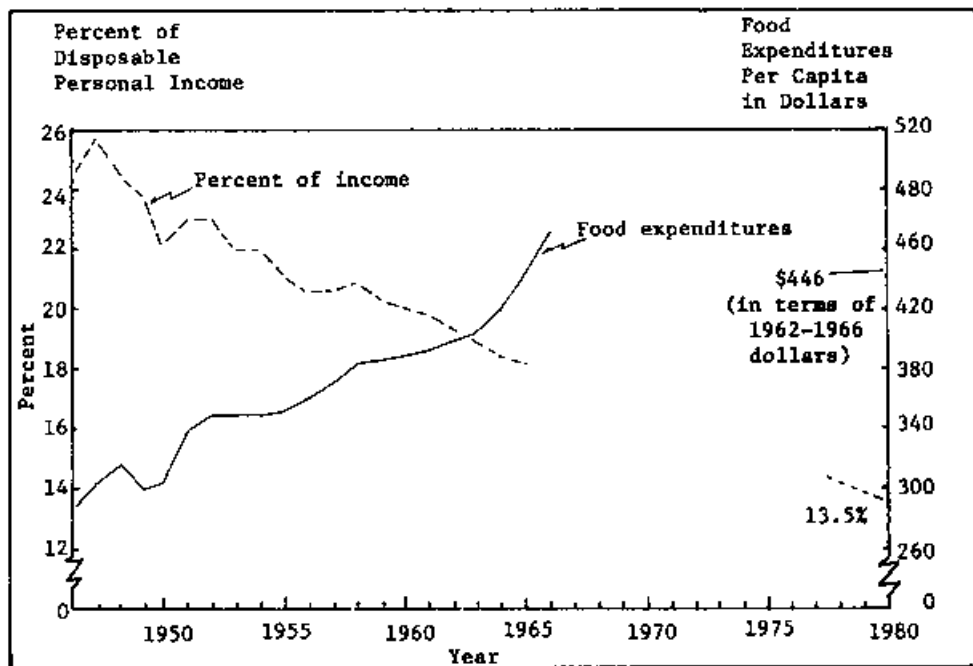


Fig. A-15. Food expenditures: Actual per-capita expenditures 1946-1968 and expenditures as percentage of personal disposable income. (Source: U. S. Department of Commerce as reported by Hiemstra, 1968, p. 181.)

Prices.—We have used retail prices (annual and quarterly) for obtaining the demand relationships, and prices at the farm level (annual and quarterly) for obtaining the relationship between retail prices and farm level prices. Annual data on retail prices were derived from various reports of the U. S. Bureau of Labor Statistics (1963), and annual data on farm level prices from the USDA report on farm-retail spreads (USDA, 1965). Quarterly data on both retail prices and farm-level prices were derived from various issues of marketing and transportation situation (USDA, 1955).

Trends in retail prices.—In general, prices of most commodities have increased. However, there was substantial reduction in the price of poultry. Also, prices of ice cream and bananas have declined. Prices of coffee, margarine, and eggs remained fairly stable. The highest increase of retail prices was experienced by fresh fruits and vegetables. The trends in prices for major commodities are shown in figures A-1 to A-14. These prices are in actual rather than deflated values.

Deflation of prices.—Time-series data on prices represent the actual prices in each year. The price level in a given year is influenced by supply and demand factors along with changes in general price levels. Different approaches have been suggested to remove the effects of inflation and deflation in the economy from the prices reported:

(1) A general approach to remove the effect of changes in price level is to deflate the observed data using the consumer price index. (For problems associated with deflation, see Foote (1958) and Shepherd (1963, pp. 121-31).

(2) A second approach is to include the general price level as a separate variable in the regression equation. Kuh and Meyer (1955) suggest different criteria

to determine whether the data should be deflated or the deflator should be included as a separate variable in the analysis.

(3) Waugh (1964, p. 11) suggests that when the analysis concerns only two variables (the quantity consumed and some kind of deflated prices), "it may often be convenient to deflate by dividing prices by consumer income."

As Shepherd (1963) points out, no standard technique of deflation is applicable to all problems. Considering the merits of all the above three approaches, while using time-series data, we have followed the practice of deflating prices and income by the consumer price index.

Price-quantity relationships are static in nature since we have assumed that the various coefficients remain the same over the different years. Stochastic elements in the coefficients and other dynamic factors can be incorporated only with a number of estimation problems and the large number of commodities included in our analysis prevented us from introducing any dynamic considerations. We could only isolate the shown effects of linear trends.

Marketing margins

For the purpose of this study, we have defined marketing margin as the difference between the prices paid by consumers and the prices received by the producers. Thus, the data correspond to aggregate price spreads in the sense producers. Thus, the data correspond to aggregate price spreads in the sense that they include processing charges, handling charges, and profits earned by different marketing agencies.

A statistical note on grouped-data regressions

Consider a general linear model in which the dependent variable y is a func-

tion of k predetermined variables X_i and an error term u . There are n observations. In matrix notation, the model is expressed as:

$$Y = X\beta + u. \quad (164)$$

The assumptions required for estimation are the well-known conditions, Ω , that:

$$\begin{aligned} E(u) &= 0 \\ E(uu') &= \sigma^2 I_n \\ X &\text{ is a set of fixed numbers} \\ X &\text{ has rank } k < n. \end{aligned} \quad (165)$$

The estimate of β is given by $\hat{\beta} = (X'X)^{-1}X'y$.

In the case of two variables, the regression equation can be written as

$$y_i = \alpha + \beta X_i + U_i \quad (166)$$

and the estimate of β is given by

$$\hat{\beta} = \frac{\sum_{i=1}^n (X_i - \bar{X})(y_i - \bar{y})}{\sum_{i=1}^n (X_i - \bar{X})^2} \quad (167)$$

Often, in cross-section analysis, the size of the sample is large, and it is difficult to calculate the sum of squares and sum of cross products of the deviations in (167) for all observations. Also, most of the published data give the averages of a number of observations after grouping the observations according to certain characteristics. Therefore, it is important to examine whether the grouping of the observations introduces any bias in the estimates.

Grouped data.²⁸—Now assume that the n observations are grouped into k

groups with n_1, n_2, \dots, n_k observations in each group ($n_1 + n_2 + \dots + n_k = n$). Also, let X_{ij} and y_{ij} be defined as

$$\begin{aligned} X_{ij} &= i^{\text{th}} \text{ observation of } X \text{ belonging} \\ &\text{to } j^{\text{th}} \text{ group and} \\ y_{ij} &= i^{\text{th}} \text{ observation of } y \text{ belonging} \\ &\text{to } j^{\text{th}} \text{ group.} \end{aligned}$$

The group averages of X and y are given by

$$\bar{X}_j = \frac{\sum_{i=1}^{n_j} X_{ij}}{n_j} \quad (j = 1, 2, \dots, k). \quad (168)$$

$$\bar{y}_j = \frac{\sum_{i=1}^{n_j} y_{ij}}{n_j}$$

When a regression equation of the type (166) is specified for the group means as variables, we have

$$\bar{y}_j = \alpha + \beta \bar{X}_j + V_j. \quad (169)$$

If (166) satisfies the Ω assumptions given in (165), V_j will equal

$$E(V_j) = 0 \text{ and} \quad (170)$$

$$V(V_j) = \frac{\sigma^2}{n_j}$$

Using the least-square estimation procedure, from (169) an estimate of β can be obtained as

$$b = \frac{\sum_{j=1}^k (\bar{X}_j - \bar{\bar{X}})(\bar{y}_j - \bar{\bar{y}})}{\sum_{j=1}^k (\bar{X}_j - \bar{\bar{X}})^2} \quad (171)$$

²⁸ This section is partly based on Malinvaud (1966, pp. 242-46).

where

$$\bar{X}_j = \frac{1}{k} \sum_j \hat{X}_j \quad \text{and}$$

$$\bar{y}_j = \frac{1}{k} \sum \hat{y}_j.$$

The expected value of b will be the same as β and, hence, b is an unbiased linear estimator of β . However, the variance of the estimates will be different in both cases (i.e., using original data and grouped data). The extent that grouping causes loss of precision will depend upon the variability within groups. It is possible to group the observations in such a manner that the loss in precision is brought to a minimum. For example, the loss in precision is zero when all the y values within a given group are the same. However, this may be difficult to achieve in practice and, therefore, the next best thing is to obtain groups such that values of y within a group are close together. If the sample is designed to have the same number of observations in each group, the observation of y can be arranged in an ascending or descending order and then make the first (n/k) observations to fall in the first group, the next (n/k) observations to fall in the second group, ..., and the last (n/k) observations to fall in the last (k^{th}) group. If it is not necessary to have an equal number of observations in each group, and if the total number of desired groups is given, it is possible to form class intervals of y and to obtain a frequency distribution of observations belonging to each group to derive at group averages.

From (170), it can be seen that we have introduced heteroskedasticity as a consequence of grouping the data. Though the original data were assumed to have homoskedasticity, grouping has induced heteroskedasticity. To estimate the parameters under such induced hete-

roskedasticity, it is possible to use Aitken's generalized least-squares method. Since the variances are changed by a multiple of the number of observations in each group, this procedure reduces to a weighted regression (Draper and Smith, 1966, pp. 77-81).

Weighted regression.—In the use of Aitken's generalized least squares, the estimates of $\hat{\beta}$ from a model $Y = X\beta + U$ where $E(U) = 0$ and $E(UU') = \sigma^2\Sigma$ is given by

$$\hat{\beta} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Y.$$

In the case of grouped data, using (170)

$$\Sigma = \begin{bmatrix} \frac{1}{n_1} & & & & \\ & \frac{1}{n_2} & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \frac{1}{n_k} \end{bmatrix} \quad (172)$$

Therefore,

$$\Sigma^{-1} = \begin{bmatrix} n_1 & & & & \\ & n_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & n_k \end{bmatrix}$$

Since Σ^{-1} is known, the estimates can be obtained as

$$\hat{\beta} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Y.$$

In particular, if there are only two variables having a relationship of the type (166), the estimate of β is given by

$$\hat{\beta} = \left[\frac{\sum_j n_j (\bar{X}_j - \bar{X})(\bar{y}_j - \bar{y})}{\sum_j n_j (\bar{X}_j - \bar{X})^2} \right] \quad (173)$$

which is of the same form as (171) except

that each term inside the sum is weighted according to the number of observations from which the group average was formed.

The tables A-1 to A-6 on the pages following present certain statistical results of interest.

Statistical Tables

TABLE A-1

COMPARISON OF REGRESSION COEFFICIENTS OBTAINED FROM TWO SPECIFICATIONS WHEN QUANTITY IS THE DEPENDENT VARIABLE

Commodity	log q = a + b log y			log q = a' + b' log y + c' log s			
	a	b	R ²	a'	b'	c'	R ²
Beef.....	-.656 (-7.04)	.270 (9.10)	.90	-.626 (-6.09)	.225 (3.45)	.219 (.78)	.91
Veal...	-2.103 (-14.29)	.651 (7.96)	.88	-2.958 (-14.47)	.326 (2.52)	1.088 (1.90)	.92
Pork.....	-.273 (-4.46)	.008 (.41)	.02	-.311 (-3.18)	.065 (1.71)	-.277 (-1.70)	.28
Lamb and mutton	-3.210 (-11.94)	.591 (8.89)	.84	-.894 (14.17)	.894 (5.85)	-1.468 (-2.24)	.90
Chicken.....	.000 (.18)	-.034 (-2.19)	.38	-.009 (-.17)	-.007 (-.21)	-.132 (-.90)	.41
Fish.....	-.247 (-1.44)	-.060 (-1.24)	.15	-.342 (-2.32)	.082 (.88)	-.687 (-1.73)	.39
Turkey.....	-3.813 (-7.93)	.788 (5.01)	.74	-2.805 (-6.92)	.757 (2.17)	.006 (.04)	.74
Eggs.....	-.027 (-.48)	-.072 (-4.01)	.04	-.028 (-.41)	-.073 (-1.77)	.003 (.02)	.04
Butter.....	-1.776 (-3.33)	.269 (3.96)	.64	-1.779 (-7.29)	.273 (1.76)	-.018 (-.03)	.64
Lard.....	2.824 (4.65)	-1.297 (-6.70)	.83	2.849 (4.31)	-1.488 (-3.42)	.912 (.49)	.84
Shortening.....	-1.088 (-3.71)	.029 (.31)	.01	-.871 (-3.29)	-.298 (-1.77)	1.581 (2.20)	.38
Margarine.....	-.598 (-6.67)	-.006 (-.21)	.01	-.573 (-5.78)	-.044 (-.70)	.186 (.69)	.06
Salad dressing...	-1.709 (-18.27)	.284 (8.47)	.89	-1.601 (-25.05)	.122 (3.01)	.781 (4.50)	.97
Fresh milk.....	-.780 (-9.55)	.367 (13.98)	.95	-.723 (-9.35)	.281 (5.74)	.418 (1.99)	.97
Evaporated milk..	1.166 (3.16)	-.810 (-5.77)	.79	1.207 (3.21)	-.673 (-2.82)	.208 (.30)	.79
Cheese.....	-1.183 (-9.04)	.227 (5.44)	.76	-1.227 (-8.57)	.284 (3.23)	-.326 (-.83)	.79
Ice cream.....	-1.428 (-23.15)	.323 (15.65)	.96	-1.498 (-20.23)	.320 (6.62)	.012 (.06)	.96
Potatoes.....	.104 (1.19)	-.016 (.69)	.04	.103 (1.98)	-.072 (-1.36)	.427 (1.89)	.34
Sweet potatoes....	.074 (.18)	-.504 (-3.82)	.82	.207 (.45)	-.704 (-2.43)	.964 (.78)	.65
Sugar.....	-.285 (3.17)	-.189 (-4.37)	.68	.456 (3.78)	-.278 (-3.59)	.518 (1.67)	.76
Corn syrup.....	.834 (2.16)	-.706 (-5.71)	.78	.734 (1.70)	-.556 (-2.02)	-.728 (-.62)	.79
Apples.....	-.827 (-8.78)	.142 (4.73)	.71	-.845 (-7.95)	.169 (2.51)	-.131 (-.48)	.72
Oranges.....	-1.092 (-6.71)	.227 (4.34)	.68	-1.022 (-5.84)	.122 (1.09)	.508 (1.07)	.72
Bananas.....	-.793 (-9.04)	.135 (4.86)	.72	-.798 (-7.97)	.144 (2.26)	-.089 (-.14)	.72
Canned peaches...	-.911 (-8.59)	.012 (.36)	.01	-.893 (-7.44)	-.015 (-.19)	.127 (.39)	.03

TABLE A-1—Continued

Canned pineapples	-2.724 (-9.05)	.408 (4.24)	.67	-2.876 (-9.15)	.636 (3.19)	-1.103 (-1.20)	.72
Dry fruits.....	-1.424 (-4.69)	-.031 (-.32)	.01	-1.689 (-7.01)	.366 (2.30)	-1.923 (-2.94)	.62
Frozen fruits.....	-3.883 (-12.50)	.624 (6.31)	.62	-3.597 (-15.63)	.185 (1.32)	2.079 (3.32)	.92
Fresh tomatoes.....	-1.004 (-3.49)	.161 (4.27)	.67	-1.078 (-9.36)	-.273 (3.74)	-.543 (-1.74)	.76
Fresh beans.....	.652 (4.19)	-.481 (-9.70)	.91	.589 (3.51)	-.386 (-3.63)	-.464 (-1.02)	.92
Onions.....	-.734 (-7.53)	.005 (.15)	.003	-.750 (-6.79)	.028 (.41)	-.115 (-.38)	.02
Carrots.....	-1.826 (-13.89)	.313 (7.49)	.86	-1.809 (-12.11)	.288 (3.03)	.124 (.31)	.86
Lettuce.....	-1.772 (-13.06)	.424 (9.80)	.91	-1.798 (-11.75)	.464 (4.78)	-.195 (-.47)	.92
Canned peas.....	-1.947 (7.30)	.043 (.94)	.09	-1.005 (-6.32)	-.020 (-.20)	.306 (.71)	.14
Canned corn.....	-.997 (-4.73)	.054 (.80)	.07	-.824 (-4.65)	-.205 (-1.63)	1.256 (2.61)	.50
Canned tomatoes.....	-1.396 (-17.84)	.165 (6.80)	.83	-1.369 (-15.97)	.124 (2.28)	.197 (.84)	.84
Dry vegetables.....	1.660 (3.35)	-.818 (-6.51)	.77	1.690 (2.99)	-.864 (-2.56)	.224 (.15)	.77
Frozen vegetables.....	-2.601 (-13.35)	.577 (9.29)	.91	-2.639 (-12.01)	.634 (4.54)	-.276 (-.46)	.91
Wheat flour.....	1.641 (6.44)	-.631 (-7.76)	.87	1.641 (5.63)	-.631 (-3.41)	.000 (.00)	.87
Rice.....	.999 (3.14)	-.605 (-5.96)	.80	1.028 (2.83)	-.648 (-2.81)	-.209 (-.21)	.80
Breakfast cereals.....	-.661 (-10.88)	.056 (2.87)	.48	-.704 (-12.82)	.119 (3.32)	-.309 (-2.01)	.65
Corn meal.....	2.521 (5.82)	-1.059 (-7.81)	.87	2.453 (5.08)	-.656 (-3.13)	-.488 (-.37)	.87
Coffee.....	-.798 (-11.76)	.047 (2.19)	.35	-.862 (-17.80)	.144 (4.86)	-.466 (-3.54)	.75
Soup.....	-1.287 (-9.36)	.216 (4.93)	.73	-1.208 (-8.72)	.098 (1.11)	.572 (1.52)	.79

TABLE A-2

COMPARISON OF REGRESSION COEFFICIENTS OBTAINED FROM TWO SPECIFICATIONS WHEN EXPENDITURE IS THE DEPENDENT VARIABLE

Commodity	$\log x = a + \beta \log y$			$\log x = a' + \beta \log y + \gamma' \log s$			
	a	β	R^2	a'	β'	γ'	R^2
Beef	-1.153 (-9.07)	.380 (9.39)	.91	-1.152 (-7.93)	.379 (3.11)	.009 (.02)	.91
Veal	-3.615 (-14.76)	.698 (3.94)	.90	-3.502 (-13.52)	.529 (3.22)	.816 (1.18)	.91
Pork	-.568 (-10.66)	.130 (7.63)	.93	-.559 (-9.27)	.116 (3.03)	.066 (.40)	.87
Lamb and mutton	-3.256 (-10.51)	.576 (5.83)	.79	-3.505 (-13.27)	.980 (5.67)	-1.813 (-2.53)	.88
Chicken	-.710 (-11.32)	.086 (2.82)	.47	-.754 (-12.96)	.123 (3.33)	-.322 (-2.04)	.65
Fish	-1.108 (-6.96)	.140 (2.73)	.46	-1.188 (-7.20)	.261 (2.49)	-.587 (-1.31)	.65
Turkey	-4.424 (-11.62)	.669 (7.16)	.85	-4.470 (-10.35)	.947 (3.45)	-.379 (-.32)	.85
Eggs	-.543 (-12.11)	-.016 (-1.11)	.12	-.553 (-10.93)	-.002 (-.06)	-.067 (-.49)	.15
Butter	-1.934 (-8.81)	.274 (3.91)	.63	-1.943 (-7.77)	.294 (1.85)	-.099 (-.15)	.63
Lard	1.958 (2.85)	-1.236 (-5.65)	.78	2.007 (2.57)	-1.317 (-2.65)	.390 (.18)	.78
Shortening	-1.443 (-5.46)	-.040 (-.47)	.02	-1.245 (-5.27)	-.338 (-2.25)	1.444 (2.25)	.40
Margarine	-1.325 (-20.58)	.051 (2.48)	.41	-1.326 (-17.98)	.052 (1.10)	-.093 (-.91)	.41
Salad dressing	-2.529 (-27.62)	.401 (13.73)	.95	-2.451 (-32.77)	2.85 (6.00)	.562 (2.77)	.98
Fresh milk	-1.351 (-17.78)	.349 (1.42)	.96	-1.313 (-16.58)	.293 (5.83)	.271 (1.26)	.97
Evaporated milk486 (1.50)	-.640 (-6.20)	.81	.511 (1.88)	-.677 (-2.89)	.181 (.19)	.81
Cheese	-1.498 (-11.41)	.241 (5.75)	.79	-1.547 (-10.85)	.315 (3.48)	-.359 (-.93)	.81
Ice cream	-1.956 (-17.30)	.335 (9.56)	.91	-1.947 (-15.53)	.321 (4.04)	.066 (.19)	.91
Potatoes	-.941 (-11.86)	.036 (1.42)	.18	-.907 (-10.69)	-.015 (-.28)	.246 (1.07)	.28
Sweet potatoes	-1.325 (-2.36)	-.293 (-1.65)	.23	-1.288 (-2.02)	-.350 (-.87)	.273 (.16)	.24
Sugar	-.512 (-4.42)	-.178 (-4.82)	.72	-.448 (3.80)	-.274 (-3.68)	.473 (1.45)	.78
Corn syrup	-.281 (.90)	-.569 (-5.74)	.79	-.330 (-1.11)	-.420 (-1.93)	-.722 (-.78)	.80
Apples	-1.841 (-20.03)	.207 (7.07)	.85	-1.908 (-22.37)	.308 (5.83)	-.490 (-2.16)	.90
Oranges	-1.883 (9.63)	.192 (3.03)	.51	-1.884 (-8.43)	.194 (1.37)	-.009 (-.02)	.51
Bananas	-1.676 (-17.40)	.140 (4.56)	.70	-1.703 (-15.78)	.177 (2.58)	-.175 (-.60)	.71
Canned peaches	-1.607 (-9.26)	.009 (.16)	.003	-1.680 (-9.03)	.119 (1.01)	-.531 (-1.05)	.12
Canned pineapples	-3.482 (-13.22)	.475 (5.86)	.78	-3.686 (-15.90)	.762 (5.34)	-1.485 (-2.37)	.87
Dry fruits	-1.809 (-0.16)	-.037 (-.40)	.02	-2.059 (-8.67)	.339 (2.25)	-1.821 (-2.82)	.51
Fresh tomatoes	-1.947 (-14.55)	.278 (6.52)	.83	-1.984 (-13.32)	.333 (3.58)	-.278 (-.66)	.83
Fresh beans	-.098 (-.60)	-.431 (-8.13)	.88	-.140 (-.75)	-.308 (-3.18)	-.306 (-.61)	.89
Onions	-1.681 (-15.50)	.022 (.64)	.04	-1.688 (-13.62)	.032 (.40)	-.047 (-.14)	.05
Carrots	-2.519 (-23.98)	.268 (8.01)	.88	-2.515 (-20.96)	.261 (3.43)	.634 (.10)	.88
Lettuce	-2.207 (-14.46)	.348 (7.15)	.85	-2.274 (-14.02)	.450 (4.38)	-.494 (-1.12)	.87
Canned peas	-1.642 (-11.72)	.110 (2.08)	.32	-1.910 (-10.21)	.062 (.52)	.230 (.45)	.34
Canned corn	-1.742 (-8.62)	.065 (1.01)	.10	-1.584 (-8.69)	-.157 (-1.35)	1.077 (2.16)	.43

TABLE A-2—Continued

Canned tomatoes	-.2,015 (-21.93)	.151 (3.17)	.75	-2.035 (-19.69)	.181 (2.76)	-.144 (- .51)	.76
Dry vegetables	.322 (.87)	-.641 (-5.41)	.77	.258 (.61)	-.544 (-2.04)	-.468 (- .41)	.77
Frozen vegetables	-3.191 (-15.06)	.619 (9.17)	.90	-3.288 (-14.89)	.706 (5.40)	-.710 (-1.17)	.92
Wheat flour	.801 (2.53)	-.811 (-8.07)	.88	.804 (2.22)	-.615 (-3.57)	.021 (.03)	.88
Rice	-.538 (-3.30)	-.319 (-6.13)	.81	-.585 (-3.24)	-.247 (-2.15)	-.348 (- .71)	.82
Breakfast cereals	-1.489 (-18.70)	.195 (7.68)	.87	-1.523 (-17.92)	.245 (4.57)	-.250 (-1.08)	.88
Corn meal	1.399 (9.43)	-1.013 (-7.80)	.87	1.398 (9.50)	-1.012 (-3.43)	-.007 (- .01)	.87
Coffee	-.701 (-11.64)	.011 (.59)	.04	-.765 (-22.42)	.106 (4.92)	-.460 (-4.96)	.76
Soup	-1.933 (-15.04)	.228 (8.57)	.77	-1.870 (-13.96)	.132 (1.56)	.464 (1.23)	.81
All food	.132 (3.66)	.277 (3.43)	.88	.111 (.91)	.309 (4.01)	-.163 (- .46)	.88

TABLE A-3
COMPARISON OF WEEKLY PER-CAPITA CONSUMPTION IN 1955 AND 1965
(QUANTITY)

Commodity	Income Group								
	<3000			3000 - 4999			>5000		
	1955	1965	Change	1955	1965	Change	1955	1965	Change
	pounds		per cent	pounds		per cent	pounds		per cent
Beef...	.8493	1.1601	35.42	1.2115	1.4379	18.69	1.4469	1.8301	26.46
Veal.....	.0383	.0220	-42.56	.0753	.0413	-45.51	.0995	.0576	-42.11
Pork.....	1.1061	1.0925	-1.23	1.0818	1.1157	2.28	1.0728	1.0775	0.44
Lamb and mutton.....	.0380	.0297	-21.84	.0678	.0355	-47.04	.1313	.0605	-53.92
Chicken.....	.6011	.8232	35.95	.5878	.8256	45.40	.6471	.7754	21.71
Fish.....	.3584	.4053	13.09	.3125	.3309	7.81	.3029	.3659	20.80
Turkey.....	.0225	.0175	-22.22	.0519	.0483	-5.97	.1059	.0589	-44.38
Eggs.....	.6026	.8023	- 05	.6635	.5905	- 57	.6971	.5348	-10.43
Butter.....	.1819	.0895	-50.80	.1662	.0901	-45.79	.2462	.1445	-41.31
Lard.....	.3252	.2200	-32.35	.1024	.1198	16.99	.0425	.0284	-33.18
Shortening.....	.1244	.0952	-23.47	.1490	.1294	-13.83	.1275	.0970	-23.22
Margarine.....	.1762	.2136	38.25	.2075	.2570	23.86	.1813	.2364	30.39
Salad dressing.....	.1205	.1103	- 8.48	.1590	.1521	- 4.70	.1848	.1758	- 6.55
Fresh milk.....	2.5164	1.5963	-36.56	3.0426	2.2042	-27.56	3.0995	2.8832	- 8.88
Evap. milk.....	.5834	.3375	- 4.50	.3112	.2721	-12.58	.1688	.1209	-28.29
Cheese.....	.2278	.2720	19.40	.2011	.2776	- 4.64	.3372	.3967	2.43
Ice cream.....	.2067	.2403	16.26	.3244	.2953	- 8.97	.3786	.3904	3.66
Potatoes.....	1.5885	1.3647	-19.18	1.7766	1.5651	-11.90	1.5476	1.4225	- 8.08
Sweet potatoes.....	.0509	.0529	3.93	.0479	.0444	- 7.31	.0399	.0232	-41.85
Sugar.....	.9509	.8294	-12.78	.8032	.8863	4.12	.6870	.6425	- 8.43
Corn syrup.....	.0873	.0930	6.53	.0532	.0592	11.28	.0289	.0270	- 6.57
Apples.....	.3595	.3685	2.50	.2697	.3738	1.11	.4311	.4541	5.34
Oranges.....	.5085	.3277	-35.56	.6968	.3734	-46.41	.8394	.4827	-42.49
Bananas.....	.3142	.3749	19.28	.4495	.4193	- 6.72	.4638	.4600	- 0.82
Canned peaches.....	.0725	.1290	77.93	.1117	.1369	21.66	.1400	.1357	- 3.07
Canned pineapples.....	.0392	.0261	-33.42	.0586	.0251	-57.17	.0765	.0479	-37.39
Dry fruit.....	.0265	.0349	31.70	.0160	.0266	60.25	.0325	.0303	- 6.77
Frozen fruit.....	.0107	.0053	-41.12	.0293	.0118	-59.73	.0446	.0167	-62.47
Fresh tomatoes.....	.3026	.2795	- 7.00	.2298	.2987	-18.53	.3332	.3547	- 7.57
Fresh beans.....	.2682	.2372	-10.81	.1543	.1567	1.56	.1204	.1071	-11.05
Onions.....	.1932	.1910	- 1.14	.2107	.2009	- 7.29	.2147	.1895	-12.20
Carrots.....	.1195	.1067	-11.55	.1955	.1417	-27.52	.2249	.1679	-25.34
Lettuce.....	.2189	.2311	- 7.16	.3338	.3085	- 7.58	.4350	.4673	7.28
Canned peas.....	.1156	.1139	- 1.64	.1367	.1270	- 6.42	.1256	.1259	.24
Canned corn.....	.1111	.1308	17.73	.1410	.1832	29.92	.1219	.1466	21.90
Canned tomatoes.....	.1054	.1103	4.65	.1184	.1314	10.98	.1255	.1435	14.34
Dry vegetables.....	.2005	.2383	18.85	.1237	.1866	36.30	.1091	.0581	-46.76
Frozen vegetables.....	.0497	.0919	84.91	.1277	.1328	3.99	.2248	.2282	1.51
Wheat flour.....	1.5064	.9163	-39.17	.6463	.5986	- 7.38	.3925	.3186	-18.83
Rice.....	.1568	.2413	53.89	.0705	.1731	146.63	.0530	.0900	69.81
Breakfast cereals.....	.2287	.3090	35.11	.2474	.2118	-25.94	.2308	.2391	47.24
Corn meal.....	.6669	.5171	-22.46	.1609	.2574	59.98	.9632	.0840	57.89
Coffee.....	.2047	.2160	6.50	.2261	.2127	- 5.93	.2499	.2314	- 7.40
Soup.....	.1191	.1930	62.05	.2247	.2479	10.32	.2510	.2753	9.68
Food expenditures	6.3128	7.5013	18.81	8.1940	8.9042	8.67	10.9527	11.8965	8.62

TABLE A-4
REGIONAL CONSUMPTION INDEX (U. S. AVERAGE = 100)

Commodity	Per-capita consumption				U. S.	Consumption index			
	R ₁	R ₂	R ₃	R ₄		R ₁	R ₂	R ₃	R ₄
Beef.....	1.6310	1.8626	1.8106	1.4298	1.6504	98.52	112.85	109.70	86.63
Veal.....	.0975	.0255	.0364	.0304	.0486	200.61	52.46	79.01	62.35
Pork.....	.9847	.9552	1.1745	1.1768	1.0842	89.99	87.29	107.33	107.51
Lamb and mutton.....	1.067	.0900	.0236	.0213	.0547	200.54	180.98	43.14	38.93
Chicken.....	.8048	.7603	.7307	.8089	.7963	101.06	95.47	91.76	109.11
Turkey.....	.0884	.0607	.0443	.0274	.0516	171.31	117.63	85.85	53.10
Fish.....	.3810	.2162	.2810	.4634	.3677	103.61	85.99	78.42	126.02
Eggs.....	.4878	.5814	.6502	.6067	.6592	87.23	103.96	98.39	109.03
Butter.....	.1737	.1118	.1027	.0640	.1276	136.12	87.61	254.21	50.15
Lard.....	.0396	.0287	.0502	.1524	.0790	50.12	36.32	32.93	192.91
Shortening.....	.0518	.0830	.0976	.1585	.1033	60.14	80.34	94.48	153.43
Margarine.....	.2012	.2683	.2386	.2652	.2431	82.76	110.36	97.32	106.09
Salad dressing.....	.1402	.1757	.1179	.1829	.1610	87.08	109.13	91.86	113.60
Fresh milk.....	2.7347	2.6421	2.4911	2.2621	2.7051	101.09	97.97	92.08	83.62
Evap. milk.....	.1646	.1948	.0887	.2937	.1884	87.36	103.39	47.08	150.95
Cheese.....	.2828	.4376	.4053	.2591	.3525	102.92	124.14	114.97	73.50
Ice cream.....	.3628	.3258	.3780	.3140	.3405	104.70	94.02	109.26	90.62
Potatoes.....	1.4755	1.2044	1.8508	1.3048	1.4316	103.07	84.12	115.81	91.14
Sweet potatoes.....	.0274	.0223	.0236	.0518	.0334	82.03	66.76	70.65	155.08
Sugar.....	.6036	.5910	.6804	.8639	.7082	86.23	83.45	96.07	122.69
Corn syrup.....	.0152	.0319	.0266	.0884	.0455	33.40	70.10	58.46	191.28
Fresh apples.....	.4329	.4568	.4704	.3506	.4194	103.21	108.91	112.16	83.50
Oranges.....	.5000	.5207	.4644	.3170	.4340	115.04	119.81	106.35	72.94
Bananas.....	.4359	.4281	.4644	.4298	.4407	98.91	97.14	105.37	97.52
Canned peaches.....	.1097	.1565	.1538	.1250	.1337	82.04	117.05	115.03	93.49
Canned pineapples.....	.0335	.0511	.0443	.0304	.0395	84.81	129.36	112.15	76.96
Dry fruit.....	.0243	.0479	.0295	.0243	.0303	80.19	158.08	97.35	80.19
Frozen fruit.....	.0121	.0159	.0177	.0121	.0151	80.13	105.29	117.21	80.13
Fresh tomatoes.....	.3109	.3226	.2781	.3963	.3313	93.84	97.37	83.94	119.61
Fresh beans.....	.0701	.0511	.1094	.2580	.1398	50.14	26.55	78.25	183.11
Onions.....	.2225	.1852	.1715	.1920	.1914	116.24	96.81	89.80	100.31
Carrots.....	.1615	.2236	.1604	.1067	.1580	102.21	141.51	114.17	89.43
Lettuce.....	.4065	.5207	.4171	.3079	.3951	103.39	131.78	105.56	77.92
Canned peas.....	.1260	.1080	.1213	.1341	.1246	100.32	87.15	97.35	107.62
Canned corn.....	.1341	.1597	.1801	.1341	.1519	85.28	105.13	118.76	88.28
Canned tomatoes.....	.1890	.1533	.1065	.1067	.1337	141.36	114.65	79.65	79.80
Dry vegetables.....	.0518	.0926	.0828	.1920	.1124	46.08	82.38	73.66	170.81
Frozen vegetables.....	.2378	.2364	.1538	.1585	.1884	126.22	125.47	81.63	84.12
Wheat flour.....	.2164	.3865	.4142	.7560	.4711	46.93	82.04	87.92	80.47
Rice.....	.1524	.1159	.0502	.2042	.1837	113.98	86.01	37.54	152.72
Breakfast cereal.....	.3201	.3769	.3550	.2743	.3221	99.37	117.01	110.21	85.15
Corn meal.....	.0335	.0607	.0532	.4786	.1854	18.06	32.74	28.69	258.14
Coffee.....	.2286	.2236	.2455	.2073	.2249	101.64	99.42	109.15	92.17
Soup.....	.3079	.3071	.2969	.1708	.2533	119.20	115.02	111.07	89.60
Total food exp.....	11.9678	11.4345	10.3668	9.5618	10.6413	112.65	107.45	97.42	89.76

TABLE A-5
REGIONAL INDICES OF PRICES PAID (U. S. AVERAGE = 100)

Commodity	Average prices paid					Index of prices paid			
	R ₁	R ₂	R ₃	R ₄	U. S.	R ₁	R ₂	R ₃	R ₄
Beef.....	8430	7109	6012	6844	7202	115.60	98.31	94.78	93.85
Veal.....	1 0000	7500	8462	8000	8375	106.06	93.75	90.26	85.33
Pork.....	7276	6855	6448	5933	6472	112.42	102.82	99.02	91.87
Lamb and mutton.....	8611	7919	8750	8571	8333	103.33	99.03	105.00	102.85
Chicken.....	4129	3866	3684	3404	3740	121.29	103.36	98.50	91.01
Turkey.....	2021	5760	4606	5556	5294	39.11	109.18	88.13	104.94
Fish.....	6650	7070	6000	5132	6003	115.38	117.18	99.45	85.06
Eggs.....	5375	4396	4247	4550	4619	116.30	95.17	91.84	98.50
Butter.....	7368	1 0000	6909	7143	7142	103.18	140.01	96.73	100.01
Lard.....	2308	2222	1765	2000	1923	126.02	115.84	91.78	104.00
Shortening.....	2941	2682	2727	2822	2847	111.10	101.70	103.02	101.70
Margarine.....	3630	2857	2875	2756	2675	105.38	95.37	100.00	95.96
Salad dressing.....	3696	3818	3800	3333	3564	103.12	106.92	106.02	92.99
Fresh milk.....	2697	2232	2518	2520	2348	114.86	101.44	107.24	107.32
Evap. milk.....	1687	1803	1867	1753	1612	103.41	111.84	103.41	108.74
Cheese.....	4060	5109	4891	5647	5344	113.21	95.60	91.62	105.66
Ice cream.....	4454	3725	3672	3465	3859	115.41	96.52	95.15	90.56
Potatoes.....	0902	1034	1022	1098	1040	95.38	99.42	98.20	105.57
Sweet potatoes.....	2222	2857	2500	1765	1818	122.22	157.15	137.51	97.08
Sugar.....	1312	1189	1174	1158	1201	100.91	99.00	97.75	96.41
Corn syrup.....	2000	2000	2222	1724	2000	100.00	100.00	110.10	86.20
Fresh apples.....	1549	1748	1509	1478	1594	97.17	109.56	94.66	92.72
Oranges.....	1280	1411	1274	1058	1258	101.74	112.16	101.27	84.10
Bananas.....	1469	1493	1274	1277	1379	106.62	108.26	92.38	92.80
Canned peaches.....	2222	2245	1923	1951	2045	106.55	109.77	94.03	95.40
Canned pineapples.....	2727	3125	2667	3000	3076	88.05	101.59	86.70	97.52
Dry fruits.....	3750	4000	4000	3750	4000	98.75	100.00	100.00	93.75
Frozen fruits.....	5000	4000	5000	5000	4000	125.00	100.00	125.00	125.00
Fresh tomatoes.....	3039	2970	2765	2807	2830	107.38	104.94	97.70	81.51
Fresh beans.....	2609	2500	2703	1309	2391	109.11	104.55	113.04	54.74
Onions.....	1238	1207	1379	1209	1260	97.16	95.11	108.66	100.00
Carrots.....	1509	1429	1639	2232	1538	98.11	92.11	106.58	144.47
Lettuce.....	2164	1963	2169	2673	2076	104.23	94.55	105.92	128.75
Canned peas.....	2195	2353	1951	2045	2195	100.00	107.19	88.88	93.18
Canned corn.....	2045	2290	1803	2500	2200	92.93	100.00	81.95	113.63
Canned tomatoes.....	2253	2063	2222	2571	2045	110.41	81.01	108.65	125.72
Dry vegetables.....	2353	2414	1796	1111	1891	124.43	127.85	94.44	68.75
Frozen vegetables.....	0256	0270	0576	0384	0322	79.50	83.85	178.88	119.25
Wheat flour.....	1127	1074	1000	1088	1032	09.20	104.06	96.89	105.42
Rice.....	2200	3058	2353	1791	2045	107.57	149.43	115.06	87.57
Breakfast cereal.....	4476	4322	4083	4000	4150	107.85	104.14	96.28	96.38
Corn meal.....	0909	1579	1111	0955	0983	92.47	180.63	113.02	97.15
Coffee.....	1 0267	9143	9038	9705	9594	107.01	95.29	94.19	101.15
Soup.....	2475	2473	1546	2542	2470	100.20	100.12	92.59	102.91

TABLE A-6
LINEAR EFFECT OF TIME ON CONSUMPTION

Commodity	Constant*	df	Signifi- cance†	Constant + 2	Antilog.	Change per cent
Beef.....	.012741 (1.5709)	12	NS	2.012741	102.90	+2.90
Veal.....	-.021436 (-1.4333)	12	NS	1.978564	95.18	-4.82
Pork.....	-.000755 (-.0747)	12	NS	1.990245	99.82	-0.02
Lamb and mutton.....	-.003587 (-.2062)	12	NS	1.996412	99.17	-0.83
Chicken.....	.004108 (.7213)	12	NS	2.004108	100.90	0.90
Eggs.....	-.025019 (-2.2354)	11	S*	1.974981	94.40	-5.60
Butter (Model 1).....	-.013227 (-1.5525)	14	NS	1.986773	96.78	-3.22
Butter (Model 2) (with oils).....	-.021589 (-2.6228)	12	S*	1.978411	95.15	-4.85
Lard (Model 1).....	-.022715 (-2.7515)	15	S*	1.977285	94.91	-5.09
Shortening.....	-.000650 (-.0381)	12	NS	1.999341	99.86	-0.15
Margarine.....	.015690 (1.2690)	12	NS	2.015690	103.70	3.70
Salad dressing.....	.013932 (1.7760)	12	NS	2.013932	103.20	3.20
Milk.....	-.006189 (-.4809)	14	NS	1.993891	98.58	-1.42
Evaporated milk.....	-.018330 (-3.3554)	14	S*	1.983861	98.31	-3.69
Cheese.....	.017222 (1.8207)	12	S	2.017222	104.20	4.20
Ice cream.....	-.012455 (-2.5840)	12	S*	1.987535	97.26	-2.74
Potatoes.....	-.003074 (-.3983)	14	NS	1.996926	99.29	-0.71
Sweet potatoes.....	-.029886 (-2.6892)	14	S*	1.971014	93.54	-6.46
Sugar.....	.007765 (.9795)	12	NS	2.007765	101.80	1.80
Corn syrup.....	-.009281 (-.6459)	12	NS	1.990739	97.88	-2.12
Apples.....	-.004456 (-.6005)	12	NS	1.995545	98.98	-1.02
Bananas.....	.008464 (.953)	12	NS	2.008464	102.00	2.00
Oranges.....	-.013339 (-1.0410)	12	NS	1.986681	96.98	-3.02
Canned peaches.....	-.011277 (-1.8170)	12	S	1.988773	97.45	-2.55
Canned pineapples.....	-.003676 (-.4870)	12	NS	1.996324	99.18	0.84
Dry fruits.....	-.039020 (-1.0752)	12	S	1.960980	91.40	-8.60
Lettuce.....	.004760 (1.1108)	9	NS	2.004760	101.10	1.10
Tomatoes.....	-.006921 (-1.3422)	9	NS	1.993079	98.41	-1.59
Beans.....	-.025594 (-2.6969)	9	S*	1.974436	94.29	5.71
Onions.....	.000165 (.0310)	9	NS	2.000165	100.00	0
Carrots.....	-.011666 (-1.9235)	9	S	1.988334	97.58	-2.42
Canned peas.....	-.021774 (-3.6389)	9	S*	1.978226	95.10	-4.90
Canned corn.....	-.004901 (-.8200)	9	NS	1.995099	98.88	-1.12
Dry vegetables.....	-.006079 (-.5698)	9	NS	1.994301	98.70	-1.30
Wheat flour.....	-.006452 (-2.8669)	13	S*	1.991548	98.76	-1.25

TABLE A6—Continued

Rice.....	.005452 (.3866)	13	NS	2.006452	101.30	1.30
Breakfast cereals.....	-.000466 (-.2121)	13	NS	1.999534	99.89	-0.11
Corn meal.....	-.008138 (-2.0977)	13	S	1.991862	98.14	-1.86
Coffee.....	-.009409 (-1.8676)	12	S	1.990591	97.88	-2.14
Soup.....	.011282 (2.8288)	12	S*	2.011282	102.70	2.70

* Figures in the parentheses correspond to 't' values.

† NS = non-significant, S = significant at 10 per cent level, S* = significant at 5 per cent level.

APPENDIX B

Mathematical Proofs

Here, we shall introduce a proof for some of the results presented in this monograph. We shall first derive the conditions on demand functions in the classical theory, followed by the modifications introduced by Frisch and by Barten.

Classical model

Conditions for a maximum.—The first order conditions for a maximum for the n commodity case are given by

$$\begin{aligned} U_j - \lambda p_j &= 0 \\ y - q_j p_j &= 0 \end{aligned} \quad (j = 1, \dots, n) \quad (174)$$

where

U_j is the marginal utility of commodity j , p and q are vectors of commodity prices and quantities, y is consumer income, and λ is the marginal utility of income.

The second order conditions for a constrained maximum are that the Hessian matrix bordered with prices has principal minors alternating in sign starting with negative (that is, the bordered matrix is negative definite). The Hessian matrix is designated as A with typical elements $U_{ij} = \partial^2 U / \partial q_i \partial q_j$. This is a symmetric matrix.

Effect of changes in prices and income.—The goal of demand analysis is to determine the effect on quantities consumed of changes in prices and income. Small departures from equilibrium may be evaluated by differentiating the first order conditions given in (174). The total differential is given in matrix form as follows:

$$\begin{bmatrix} A & -p \\ -p' & 0 \end{bmatrix} \begin{bmatrix} Q_p & q_v \\ \lambda_p & \lambda_y \end{bmatrix} = \begin{bmatrix} \lambda I & 0 \\ q' & -1 \end{bmatrix} \quad (175)$$

where

Q_p is a matrix of price slopes where a typical element is $\partial q_i / \partial p_j$, q_v is a vector

of income slopes such as $\partial q/\partial y$, and I is an identity matrix. A typical element in the A matrix is $U_{ij} = \partial^2 U/\partial q_i \partial q_j$.

This system is solved for price and income slopes by obtaining the inverse of the bordered A matrix or

$$\begin{bmatrix} Q_p & | & q_v \\ \hline \lambda_p & | & \lambda_v \end{bmatrix} = \begin{bmatrix} A & | & -p \\ \hline -p' & | & 0 \end{bmatrix}^{-1} \begin{bmatrix} \lambda I & | & 0 \\ \hline q' & | & -1 \end{bmatrix} \quad (176)$$

$$\begin{bmatrix} Q_p & | & q_v \\ \hline \lambda_p & | & \lambda_v \end{bmatrix} = \begin{bmatrix} B & | & b \\ \hline b' & | & b^0 \end{bmatrix} \begin{bmatrix} \lambda I & | & 0 \\ \hline q' & | & -1 \end{bmatrix} \quad (177)$$

where the inverted bordered A matrix is designated as a bordered B matrix.

We may evaluate an income change from (176) for a typical element as

$$\frac{\partial q_i}{\partial y} = -b_i. \quad (178)$$

Similarly, for a price change, we have

$$\frac{\partial q_i}{\partial p_j} = B_{ij}\lambda + b_i q_j. \quad (179)$$

Substituting (178) in (179), we have the Slutsky equation in which the first term is the substitution effect and the second is the income effect or

$$\frac{\partial q_i}{\partial p_j} = B_{ij}\lambda - q_j \frac{\partial q_i}{\partial y}. \quad (180)$$

The *symmetry of the substitution effect* may be proven as follows. The bordered A matrix is symmetric or

$$A_{ij} = A_{ji}. \quad (181)$$

Any element in the bordered B matrix, obtained by inverting the bordered A matrix, also is symmetric or

$$\lambda B_{ij} = \lambda B_{ji} \text{ or} \quad (182)$$

$$K_{ij} = K_{ji} \quad (183)$$

where

$$K_{ij} = \frac{\partial q_i}{\partial p_e} + q_j \frac{\partial q_i}{\partial y} \quad \text{or} \quad (184)$$

$$\frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial y} = \frac{\partial q_i}{\partial p_i} + q_i \frac{\partial q_i}{\partial y}. \quad (185)$$

Converting (185) into elasticities, the relation between any two cross-price elasticities is given by

$$e_{ij} = e_{ii} \frac{W_j}{W_i} + w_j(e_{iy} - e_{iy}). \quad (186)$$

The *own substitution effect* always is negative as can be seen directly. The marginal utility of income, λ_i is always positive. Consider the term, $K_{ii} = B_{ii}\lambda$. The value of B_{ii} is obtained by dividing the cofactor of the bordered Hessian matrix (A_{rr}) by the determinant D . The sign of cofactor A_{rr} must be opposite to that of D since D is of order $n + 1$ and the cofactor is of order n .

The *Engel aggregation* may be seen from the condition that the inverse of a matrix multiplied by a matrix equals an identity matrix or $A^{-1}A = I$ or

$$\left[\begin{array}{c|c} B & b \\ \hline b' & 0 \end{array} \right] \left[\begin{array}{c|c} A & -p \\ \hline -p' & 0 \end{array} \right] = \left[\begin{array}{c|c} I & 0 \\ \hline 0 & I \end{array} \right] \quad (187)$$

Engel aggregation states the price-weighted sum of income slopes equals one or

$$b'(-p) = 1 \quad \text{or}$$

$$\sum p_i \frac{\partial q_i}{\partial y} = 1$$

where

b' is the vector of income slopes.

The *Cournot aggregation* states that the price-weighted sum of a given column of price slopes equals the negative of the budget proportion or

$$B(-p) = 0 \quad \text{or} \quad (188)$$

$$\sum p_i K_{ij} = 0$$

where

the elements of the B matrix are compensated price slopes.

Using the Slutsky condition (184)

$$K_{ij} = \frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial y}, \quad (189)$$

we may express equation (188) as

$$\sum p_i \frac{\partial q_i}{\partial p_i} + \sum p_i \frac{\partial q_i}{\partial y} \cdot q_i = 0. \quad (190)$$

The second term is the Engel aggregation or

$$\sum p_i \frac{\partial q_i}{\partial y} = 1.$$

so we rewrite (190) as

$$\sum p_i \frac{\partial q_i}{\partial p_i} = -q_i. \quad (191)$$

Frisch model

Frisch starts with the first order conditions of utility maximization specified in equations (174).

$$U_j(q_1, q_2, \dots, q_n) - \lambda p_j = 0 \quad (j = 1, 2, \dots, n) \quad (192)$$

where

$$U_j = \frac{\partial U}{\partial q_j} \quad \text{and}$$

$$y - p_1 q_1 - \dots - p_n q_n = 0. \quad (193)$$

The demand functions can be represented as

$$q_i = q_i(p_1, p_2, \dots, p_n, y). \quad (194)$$

As before, the demand elasticities w.r.t. price (e_{ij}) and income elasticities (e_{iy}) are defined as

$$e_{ij} = \frac{\partial q_i}{\partial p_j} \cdot \frac{p_j}{q_i} \quad \text{and} \quad (195)$$

$$e_{iy} = \frac{\partial q_i}{\partial y} \cdot \frac{y}{q_i} \quad (i, j = 1, 2, \dots, n).$$

The proportion of expenditure on the i^{th} commodity is denoted by

$$w_i = \frac{p_i q_i}{y}. \quad (196)$$

The marginal utility of money, λ , is defined as the common ratio

$$\lambda = \frac{U_1}{P_1} = \frac{U_2}{P_2} = \dots = \frac{U_n}{p_n}. \quad (197)$$

Since utility, by definition, is a function of quantities consumed, we have

$$U_i = U_i(q_1, q_2, \dots, q_n), \quad (198)$$

and the inverse function can be written as

$$q_i = q_i(U_1, U_2, \dots, U_n) \quad (i = 1, 2, \dots, n). \quad (199)$$

Now Frisch defines utility accelerators, want elasticities, and money flexibility as

$$F_{ij} = \frac{\partial U_i(q_1, \dots, q_n)}{\partial q_j} \cdot \frac{q_j}{U_i} \quad (\text{utility accelerator}), \quad (200)$$

$$\sigma_{ij} = \frac{\partial q_i(U_1, \dots, U_n)}{\partial U_j} \cdot \frac{U_j}{q_i} \quad (\text{want elasticity}), \quad \text{and} \quad (201)$$

$$\phi = \frac{\partial \lambda}{\partial y} \cdot \frac{y}{\lambda} \quad (\text{money flexibility}). \quad (202)$$

From (175), we have

$$[A \quad -p] \begin{bmatrix} Q_p & y \\ \lambda_p & \lambda_y \end{bmatrix} = [\lambda M \quad 0], \quad (203)$$

or

$$\begin{bmatrix} U_{11} & \dots & U_{1n} - p_1 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ U_{1n} & \dots & U_{nn} - p_n \end{bmatrix} \begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \dots & \frac{\partial q_1}{\partial p_n} \frac{\partial q_1}{\partial y} \\ \frac{\partial q_n}{\partial p_1} & \dots & \frac{\partial q_n}{\partial p_n} \frac{\partial q_n}{\partial y} \\ \frac{\partial \lambda}{\partial p_1} & \dots & \frac{\partial \lambda}{\partial p_n} \frac{\partial \lambda}{\partial y} \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \lambda & 0 \end{bmatrix}. \quad (204)$$

Writing the first equation in full,

$$U_{11} \frac{\partial q_1}{\partial p_1} + \dots + U_{1n} \frac{\partial q_n}{\partial p_1} - p_1 \frac{\partial y}{\partial p_1} = \lambda$$

or

$$\frac{\partial U_1}{\partial q_1} \cdot \frac{\partial q_1}{\partial p_1} + \dots + \frac{\partial U_1}{\partial q_n} \cdot \frac{\partial q_n}{\partial p_1} = p_1 \frac{\partial \lambda}{\partial p_1} + \lambda, \quad \text{or} \quad (205)$$

$$\frac{\partial U_1}{\partial q_1} \cdot \frac{q_1}{U_1} \cdot \frac{U_1}{q_1} \cdot \frac{\partial q_1}{\partial p_1} + \dots + \frac{\partial U_1}{\partial q_n} \cdot \frac{q_n}{U_1} \cdot \frac{U_1}{q_n} \cdot \frac{\partial q_n}{\partial p_1} = p_1 \frac{\partial \lambda}{\partial p_1} + \lambda$$

Using (195), (197), and (200), (205) can be expressed in terms of price elasticities and utility accelerators as

$$F_{11}e_{11} + \dots + F_{in}e_{in} = 1 + \lambda. \quad (206)$$

Similarly, expressing all the other equations in (204) in terms of price elasticities, income elasticities, and utility accelerators, (204) can be rewritten as

$$\begin{bmatrix} F_{11} & \dots & F_{in} \\ \vdots & & \\ F_{n1} & \dots & F_{nn} \end{bmatrix} \begin{bmatrix} e_{11} & \dots & e_{1n}e_{1y} \\ \vdots & & \\ e_{n1} & \dots & e_{nn}e_{ny} \end{bmatrix} = \begin{bmatrix} 1 + \lambda_1 & \lambda_2 & \dots & \lambda_n & \phi \\ \lambda_1 & 1 + \lambda_2 & \dots & \lambda_n & \phi \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_1 & \dots & \dots & 1 + \lambda_n & \phi \end{bmatrix} \quad (207)$$

where

$$\lambda_i = \frac{\partial \lambda}{\partial p_i} \frac{p_i}{\lambda}$$

From (207)

$$\begin{bmatrix} e_{11} & \dots & e_{1n}e_{1y} \\ \vdots & & \\ \vdots & & \\ e_{n1} & \dots & e_{nn}e_{ny} \end{bmatrix} = \begin{bmatrix} F_{11} & \dots & F_{1n} \\ \vdots & & \\ \vdots & & \\ F_{n1} & \dots & F_{nn} \end{bmatrix}^{-1} \begin{bmatrix} 1 + \lambda_1 & \lambda_2 & \dots & \lambda_n & \phi \\ \lambda_1 & 1 + \lambda_2 & \dots & \lambda_n & \phi \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_1 & \dots & \dots & 1 + \lambda_n & \phi \end{bmatrix} \quad (208)$$

Now Frisch shows that

$$[F_{ij}]_{n \times n} \times [\sigma_{ij}]_{n \times n} = I$$

therefore

$$[\sigma_{ij}]_{n \times n} = [F_{ij}]_{n \times n}^{-1} \quad (209)$$

From (208) and (209)

$$\begin{bmatrix} e_{11} & \dots & e_{1n}e_{1y} \\ \vdots & & \\ \vdots & & \\ e_{n1} & \dots & e_{nn}e_{ny} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \dots & \sigma_{1n} \\ \vdots & & \\ \vdots & & \\ \sigma_{n1} & \dots & \sigma_{nn} \end{bmatrix} \begin{bmatrix} 1 + \lambda_1 & \lambda_2 & \dots & \lambda_n & \phi \\ \lambda_1 & 1 + \lambda_2 & \dots & \lambda_n & \phi \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_1 & \lambda_2 & \dots & 1 + \lambda_n & \phi \end{bmatrix}$$

Therefore,

$$e_{iy} = \phi \sum_j \sigma_{ij} \quad \text{and} \quad (210)$$

$$e_{ij} = \lambda_j \sum_j \sigma_{ij} + \sigma_{ij}. \quad (211)$$

Further, Frisch shows that

$$w_i \sigma_{ij} = w_j \sigma_{ji}$$

Summing over j

$$w_i \sum_j \sigma_{ij} = \sum_j w_j \sigma_{ji} \quad \text{or} \quad (212)$$

$$\sum_j \sigma_{ij} = \frac{1}{w_i} \sum_j w_j \sigma_{ji}.$$

From (210) and (212),

$$e_{iw} = \frac{\phi}{w_i} \sum_j w_j \sigma_{ji} \quad \text{and} \quad (213)$$

$$w_i e_{iw} = \phi \sum_j w_j \sigma_{ji}.$$

Summing (213) over i ,

$$\sum_i w_i e_{iw} = \phi \sum_i \sum_j w_j \sigma_{ji}. \quad (214)$$

Using the Engel aggregation,

$$\sum_i w_i e_{iw} = 1.$$

therefore, (214) can be written as

$$\phi = \frac{1}{\sum_i \sum_j w_j \sigma_{ji}} = \frac{1}{\sum_i (w_i \sum_j \sigma_{ij})}. \quad (215)$$

From (211) and (212), we have

$$e_{ij} = \frac{\lambda_j}{w_i} \sum_j w_j \sigma_{ji} + \frac{w_j}{w_i} \sigma_{ji}.$$

From the Cournot aggregation, we have

$$\sum_i w_i e_{ij} = -w_j. \quad (216)$$

From (211) and (216),

$$\sum_i w_i (\lambda_j \sum_j \sigma_{ij} + \sigma_{ij}) = -w_j \quad \text{and} \quad (217)$$

$$\lambda_j \sum_i (w_i \sum_j \sigma_{ij}) + \sum_i \sigma_{ij} w_i = -w_j.$$

From (215) and (217),

$$\begin{aligned}\lambda_j \frac{1}{\phi} + \sum_i w_i \sigma_{ij} &= -w_j \quad \text{and} \\ \lambda_j &= -(w_j + \sum_i w_i \sigma_{ij})\phi.\end{aligned}\tag{218}$$

Frisch also shows that

$$\begin{aligned}e_{iv} &= \phi \sum_j \sigma_{ij} \\ &= \phi \frac{1}{w_i} \sum_j w_j \sigma_{ji} \quad [\text{using (212)}] \\ w_i e_{iv} &= \phi \sum_j w_j \sigma_{ji}.\end{aligned}\tag{219}$$

Interchanging i and j ,

$$w_i e_{jv} = \phi \sum_j w_i \sigma_{ij}.\tag{220}$$

From (218) and (220),

$$\begin{aligned}\lambda_j &= -w_j \phi - w_j e_{jv} \quad \text{and} \\ &= -w_j (\phi + e_{jv}).\end{aligned}\tag{221}$$

Using (211) and (218)

$$\begin{aligned}e_{ij} &= -(w_j + \sum_i w_i \sigma_{ij}) \phi \sum_j \sigma_{ij} + \sigma_{ij} \\ &= \sigma_{ij} - w_j \phi \sum_j \sigma_{ij} - \phi \sum_i w_i \sigma_{ij} \sum_j \sigma_{ij} \quad \text{and} \\ &= \sigma_{ij} - w_j e_{iv} - w_j e_{jv} \frac{e_{iv}}{\phi} \quad [\text{from (219) and (220)}].\end{aligned}$$

Therefore,

$$e_{ij} = \sigma_{ij} - w_j e_{iv} \left(1 + \frac{e_{jv}}{\phi}\right).\tag{222}$$

In particular, when $i = j$,

$$e_{ii} = \sigma_{ii} - w_i e_{iv} \left(1 + \frac{e_{iv}}{\phi}\right).\tag{223}$$

A good i is defined as want independent of good j if $U_{ij} = 0$. Since σ_{ij} is the (ij) th element of the inverse of matrix (U_{ij}) , it follows that $\sigma_{ij} = 0$ for want independent commodities. Therefore, (222) can be written as

$$e_{ij} = -w_j e_{iw} \left(1 + \frac{e_{iw}}{\phi} \right). \quad (224)$$

Also, if a good i is want independent of all other goods,

$$\sigma_{ii} = \frac{e_{iw}}{\phi}$$

and, therefore, from (223),

$$\begin{aligned} e_{ii} &= \frac{e_{iw}}{\phi} - w_i e_{iw} - w_i e_{iw} \frac{e_{iw}}{\phi}, \\ e_{ii} &= -e_{iw} \left(w_i - \frac{1 - w_i e_{iw}}{\phi} \right). \end{aligned} \quad (225)$$

Solving for ϕ , from (225),

$$\phi = \frac{e_{iw}(1 - w_i e_{iw})}{e_{ii} + w_i e_{iw}}. \quad (226)$$

Having obtained a value of ϕ from (226), (224) can be used to obtain any cross elasticity e_{ij} without making any assumption beyond $\sigma_{ij} = 0$ for that particular (ij) combination. To obtain a reliable estimate of ϕ , we need only the direct price elasticity of a single good and income elasticities of all goods. Thus, knowing all income elasticities, expenditure weights and the direct price elasticity of a single commodity, all the remaining parameters can be obtained.

The assumption of want independence imposes complete additivity of the utility function as noted by Barten (1964, 1967). Barten also shows the σ_{ij} 's are not invariant for a transformation of the utility function. Because of these properties, the procedure for computing the complete set of elasticities, using this approach, restricts us to a certain class of utility indicators. Barten suggests an approach which incorporates a weaker assumption on the utility function—the assumption of complete additivity is replaced by an assumption of an almost additive utility indicator.

The Barten approach

Barten also begins his analysis with the classical model where a given consumer maximizes his utility subject to a budget restraint. He expresses (175) as

$$\begin{bmatrix} (U_{ij}) & p' \\ p & 0 \end{bmatrix} \begin{bmatrix} q_v & Q_p \\ -\lambda_v & -\lambda_p \end{bmatrix} = \begin{bmatrix} 0' & \lambda I \\ 1 & -q \end{bmatrix}. \quad (227)$$

From (227),

$$\begin{aligned} \begin{bmatrix} q_v & Q_p \\ -\lambda_v & -\lambda_p \end{bmatrix} &= \begin{bmatrix} (U_{ij}) & p^1 \end{bmatrix}^{-1} \begin{bmatrix} 0^1 & \lambda I \\ 1 & -q \end{bmatrix} \\ &= \frac{1}{p(U_{ij})^{-1}p^1} \begin{bmatrix} p(U_{ij})^{-1}p^1(U_{ij})^{-1} - (U_{ij})^{-1}p^1p(U_{ij})^{-1} & (U_{ij})^{-1}p^1 \\ p(U_{ij})^{-1} & -1 \end{bmatrix} \begin{bmatrix} 0^1 & \lambda I \\ 1 & -q \end{bmatrix} \\ &= \lambda_v \begin{bmatrix} (U_{ij})^{-1}p^1 & \frac{\lambda}{\lambda_v}(U_{ij})^{-1} - \frac{\lambda}{\lambda_v}(U_{ij})^{-1}p^1p(U_{ij})^{-1} - (U_{ij})^{-1}p^1p \\ -1 & p\lambda(U_{ij})^{-1} + q \end{bmatrix} \end{aligned}$$

Therefore,

$$q_v = \lambda_v(U_{ij})^{-1}p^1 \quad \text{and} \quad (228)$$

$$Q_p = \lambda(U_{ij})^{-1} - \left(\frac{\lambda}{\lambda_v}\right)q_vq_v^1 - q_vq. \quad (229)$$

Converting (229) into elasticities, a typical element can be written in the form,

$$e_{ij} = \frac{p_j}{q_i} U_{ij}^{-1} - \frac{1}{\phi} e_{iv}e_{iv}w_j - e_{iv}w_j$$

where

$$U_{ij}^{-1} \text{ is the } (ij)^{\text{th}} \text{ element of } (U_{ij})^{-1}.$$

This can also be written as

$$e_{ij} = \frac{p_j}{q_i} U_{ij}^{-1} - w_j e_{iv} \left(1 + \frac{e_{iv}}{\phi}\right). \quad (230)$$

Equation (230) is identical with (222) except for the first term on the right-hand side. Earlier, it was pointed out that σ_{ij} in (222) was not invariant under transformations but that $(U_{ij})^{-1}$ is invariant under transformations. Thus, from a theoretical point of view, Barten's formulation incorporates less restrictive assumptions than Frisch's formulation.

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