MODELLING THE DYNAMICS OF WEED MANAGEMENT TECHNOLOGIES

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Abstract

An appropriate economic framework for valuing the benefits of weed management technologies is to treat weeds as a renewable resource stock problem. Consequently, the weed seed bank is defined as a renewable resource that changes through time due to management and seasonal conditions. The goal of decision-makers is to manage this (negative) resource so as to maximise returns over some pre-specified period of time. A modelling framework is presented for evaluating the biological and economic effects of weed management. The framework includes population dynamics, water balance, crop growth, pasture growth and crop/pasture rotation models for measuring the physical interactions between weeds and the environment. These models link in with numerical optimal control, dynamic programming and stochastic dynamic programming models for determination of optimal decision rules and measuring economic impact over time of policy scenarios.

Key words: weeds, modelling, dynamic analysis.

1. Introduction

Weeds are an important issue to Australian agricultural production systems. Weeds impose costs through reductions in yield and quality of production, increases in the input requirements for control and, in extreme weed-affected situations, the cost of adjustment to new production systems. One of the major reasons that weeds impose economic costs is because they compete for light, nutrients and water in agricultural systems. The effect of this competition is the consumption of these resources that would otherwise be available for crop growth, thus resulting in a reduction in crop yield. To alleviate the weed competition problem, farmers use a range of options for weed control.

The concept of integrated weed management (IWM) has been developed in response to the problems of herbicide resistance and for longer-term weed management and represents a management system that combines a range of appropriate weed control options. There has only been limited adoption of IWM by farmers and some sectors of the weed science community. Although the greater costs of IWM are readily apparent, the benefits have not been as easily demonstrated. Whether this is because of poor selection of weed control options that make up an IWM strategy or because of the use of an inappropriate framework for measuring the economic benefits is unclear.

The annual loss in economic surplus due to the presence of weeds in Australian winter cropping systems is estimated to be $1,277 million (Jones, et al. 2000). This cost is a significant value as it represents 18 percent of the value of total Australian grain and oilseed

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production for the 1998-99 season. Consequently, substantial economic gains can be achieved through new weed control technologies or management strategies that either reduce the extent of weed losses or result in more efficient control outcomes from a given level of weed expenditure. Annual ryegrass, wild oats and wild radish were determined to be the most important weeds in terms of their economic impact. Other species were of regional and zonal importance but are largely insignificant at the national level.

The aim of the paper is to present a framework to model the economic benefits of weed management technologies. Such a framework can then be used to determine the features of an optimal weed management strategy for Australian cropping systems. To address this aim, the study will specifically focus upon the issues of the potential economic benefits of IWM and whether economic frameworks for assessing the benefits of weed control technologies (including IWM) should be static (ie. single year) or dynamic (ie. multiple years).

2. Economic Frameworks

Kennedy (1987) presented the case for using a multi-period optimisation framework for evaluating weed problems. Dynamic optimisation models have been applied to a number of weed management problems, including Pandey and Medd (1990, 1991), Pandey, Lindner and Medd (1993), Fisher and Lee (1981), Taylor and Burt (1984), Sells (1995), Gorddard, Pannell and Hertzler (1995, 1996), Wu (2001) and Jones and Medd (1997, 2000).

The theoretical justification of a dynamic model is that including the inter-temporal effects of weed control into a decision-making framework will result in a different level of optimum weed control and economic benefit than would result from a static model. The main feature of dynamic models is that they explicitly account for any carryover effects from weed management. Any weed that escapes treatment in one period has the potential to reproduce and deposit seed to the weed seed bank, leading to potential future yield losses or increased costs of weed control.

When using a dynamic framework, it is useful to view the weed problem as one of optimally managing a resource stock through time (McInerney 1976, Conrad and Clark 1987, Clark 1990). Viewing weeds as a resource stock involves a modification to the economic framework for valuing the benefits from weed control. At issue is how much of the weed stock to consume or deplete in the current period and how much should be left for the future. This involves a change in the assumption of profit maximisation from weed control for a single season or year to an assumption of maximising returns over a longer time period so as to capture the carryover effects.

The renewable resource framework that represents the weed management problem is illustrated in Figure 1. The horizontal axis represents the level of weed control in period t_0 and the curves *MB* and *MC* represent the marginal benefits and marginal costs of control in period t_0 given a specific initial seed-bank stock. The single period optimum level of control is represented by u_0^*. Weed control in period t_0 will reduce the initial seed bank stock in period t_1 from what it otherwise would have been in the absence of control and, therefore, will have a positive effect upon total revenue in period t_1. The curve *MFB* represents the marginal future benefit from control in period t_0 due to the reduction in the seed bank from what it otherwise would have been. Combining *MB* and *MFB* determines the total marginal benefit (*TMB*) from control in period t_0, and the optimal inter-temporal level of control in t_0

(u_1^*) is determined by the intersection of *TMB* and *MC*. This results in a larger level of control being optimal than if only the current period benefits were considered.

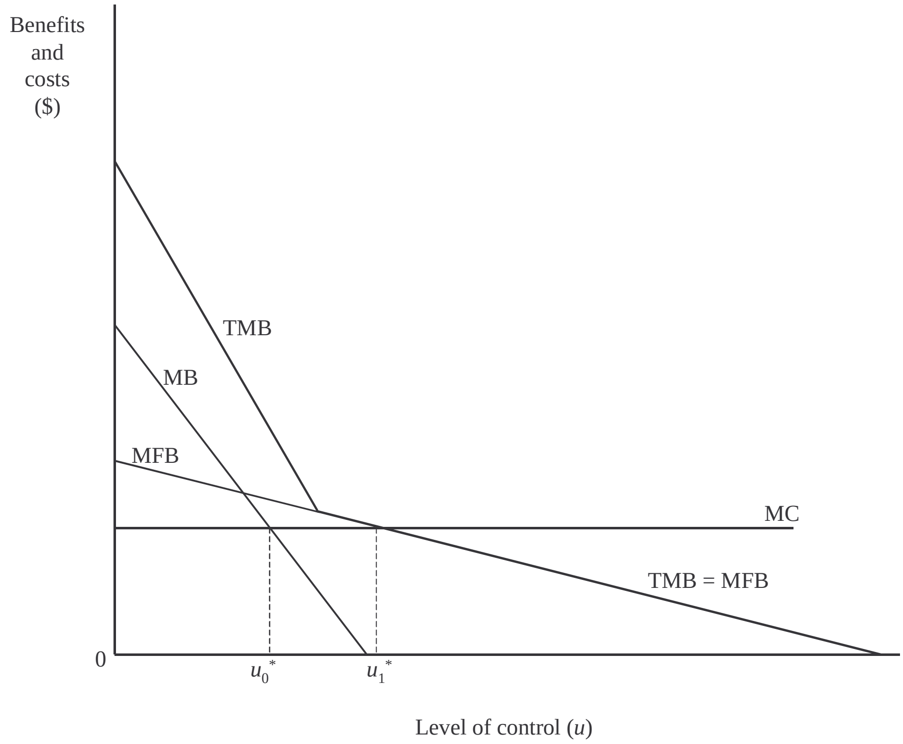

Figure 1 Derivation of optimal inter-temporal level of weed control when future.

Although the two-period diagram provides the basic principles of the inter-temporal resource allocation problem, in most cases decision-makers are concerned with the optimal allocation of resources over a much longer time horizon. The principles of optimal resource allocation can be used develop the framework further, which is essentially a dynamic optimisation one (Kamien and Schwartz 1981; Conrad and Clark 1987; Clark 1990; Chiang 1992).

In a dynamic setting the objective of the farmer is to determine the level of depletion of the stock of the seed resource (*x*) from weed control (*u*) in each season or year that maximises profit over a period of *T* years. Therefore, the typical dynamic allocation problem is as follows.

$$\max_{u_t} \sum_{t=0}^{T} \pi_t(x_t, u_t)\beta^t + F(x_T) \tag{1}$$

subject to

$$x_{t+1} - x_t = g(x_t, u_t) \tag{2}$$
$$x_0 = a$$

Where $\pi_t = Y(x_t,u_t)\, p_y - c_y - u_t\, P_u$ is the annual profit function for the crop yield Y_t, crop price p_y, and variable cost c_y and weed-control cost P_u; $\beta=1/(1+r)$ is the discount factor for the discount rate *r*; *F* is the terminal value at the end of the planning period and is a function of the remaining stocks of the seed resource. The function $g(x_t,u_t)$ is a difference equation called the equation of motion and defines the change in the state variable from period t to period t+1, and a is the initial value of x in period t_0. The problem is one of determining the optimal values for u_t and x_t, which are obtained from the equation of motion.

The problem can be represented by the application of optimal control theory. This involves the solution of a Hamiltonian function (H).

$$H_t = \pi(x_t, u_t) + \beta\lambda_{t+1} g(x_t, u_t) \tag{3}$$

The Hamiltonian function is the net profit obtained from an existing level of the state and control variables plus the discounted value of any change in the stock of the state variable valued at the costate variable, λ_{t+1}. For the weed management problem the costate variable represents the shadow price of a unit of the stock of the seed bank and is also referred to as the user cost (benefit) from a stock increase (depletion). In the last term on the right hand side of equation (3), the $g(x_t,u_t)$ function indicates the rate of change of the seed bank corresponding to the control variable. When the function is multiplied by the costate variable, λ_{t+1}, it is converted to a monetary value and represents the rate of change in the economic value of the seed bank corresponding to the control variable. In effect this term can be viewed as the future profit effect of weed population changes. The dynamic maximisation problem presented in equation (3) thus differs from the standard static maximisation problem as the future income effects from current period decisions are explicitly included in the return function.

The first order conditions for this problem, as developed by Pontryagin *et al*. (1962), are as follows.

$$\frac{\partial H}{\partial u_t} = P_y \frac{\partial Y}{\partial u_t} - P_u + \beta\lambda_{t+1} \frac{\partial g}{\partial u_t} = 0 \tag{4}$$

$$\beta\lambda_{t+1} - \lambda_t = -\frac{\partial H}{\partial x_t} = -P_y \frac{\partial Y}{\partial x_t} - \beta\lambda_{t+1} \frac{\partial g}{\partial x_t} \tag{5}$$

$$\frac{\partial H}{\partial \lambda_t} = x_{t+1} - x_t = g(x_t, u_t) \tag{6}$$

Equation (4) is the maximum principle, the standard condition for maximisation with respect to u_t. Equation (5) is the adjoint equation and denotes the rate of change of the shadow price over time. Equation (6) is a re-statement of the equation of motion. This set of equations allows the solution of the three unknown optimal trajectories x_t^*, u_t^* and λ_t^*. These trajectories depend critically on the initial state of the system and, although x_0 is generally given, λ_0 is unknown and an additional condition called the transversality condition is required to obtain a unique solution. In this particular problem where terminal time (T) is given and the terminal state (x_T) is free the transversality condition is $\lambda_{T+1} = F^{'}(x_{T+1})$.

To gain further insight into the difference between static and dynamic solutions, equation (4) is rearranged to obtain the following equation.

$$P_y\left(\frac{\partial Y}{\partial u_t}\right)+\beta\lambda_{t+1}\frac{\partial g}{\partial u_t}=P_u \tag{7}$$

or

$$\frac{\partial Y}{\partial u_t}=\frac{P_u-\beta\lambda_{t+1}\dfrac{\partial g}{\partial u_t}}{P_y} \tag{8}$$

Equation (7) represents the derivation the level of weed control where the marginal benefit of control equals the marginal cost. In the dynamic case the marginal benefit is adjusted by including the term $\beta\lambda_{t+1}(\partial g/\partial u_t)$ which is a measure of the future benefit from weed control in the current period. Consequently, it is analogous to the concept of *MFB* in Figure 1.

Equation (8) states that optimal weed management occurs when the marginal product of weed control equals the ratio of input price to output price. But now input price is decreased by the value of its beneficial future effect (the second term in the numerator). This will result in a higher level of control than in the static case. To understand why, note that in equation (8) $\lambda_{t+1} \leq 0$ (ie. weeds have a negative effect on profit) and $\partial g/\partial u \leq 0$ (ie. weed control reduces population growth and reproduction). Since $\beta > 0$ the second term in the numerator must be either negative or zero. Given that $\partial Y/\partial u > 0$ and $\partial^2 Y/\partial u^2 < 0$, the value of u would have to be increased beyond what would occur for a static problem (as $\beta\lambda_{t+1}\partial g/\partial u_t$ is ignored in the static case) to obtain a lower marginal cost and thus satisfy condition (8).

3. Models

There are a number of alternative numerical techniques available for solving the dynamic and stochastic weed management problem. Those that were considered suitable to this problem were numerical optimal control (NOC), dynamic programming (DP), non-linear programming (NLP) and genetic algorithms (GA). An important extension to DP is stochastic dynamic programming (SDP) for dealing with problems that consider uncertainty. For the numerical solution requirements of this study, NLP and GA were rejected in favour of NOC, DP and SDP.

An important component of the study is the measurement of the biological state of the weed system, as identified by the equation of motion (equation 2). An illustration of the dynamic economic modelling system, and the interactions of the NOC, DP and SDP models with the biological system is illustrated in Figure 2.

A biophysical modelling system was developed to deal with the complex biological interactions that influence the equation of motion, yield and the profit function. The biophysical modelling system used in this study is illustrated in Figure 3. The study involves the interaction of water balance, weed population dynamics, crop yield, crop rotation and pasture growth models.

The impact of a variable climate upon crop growth and weed population dynamics is derived by calculating environmental indexes for soil moisture, temperature and light. These indexes are then combined to determine a multi-factor growth index. The water balance model is necessary to calculate soil moisture levels and consequently the moisture and growth indexes.

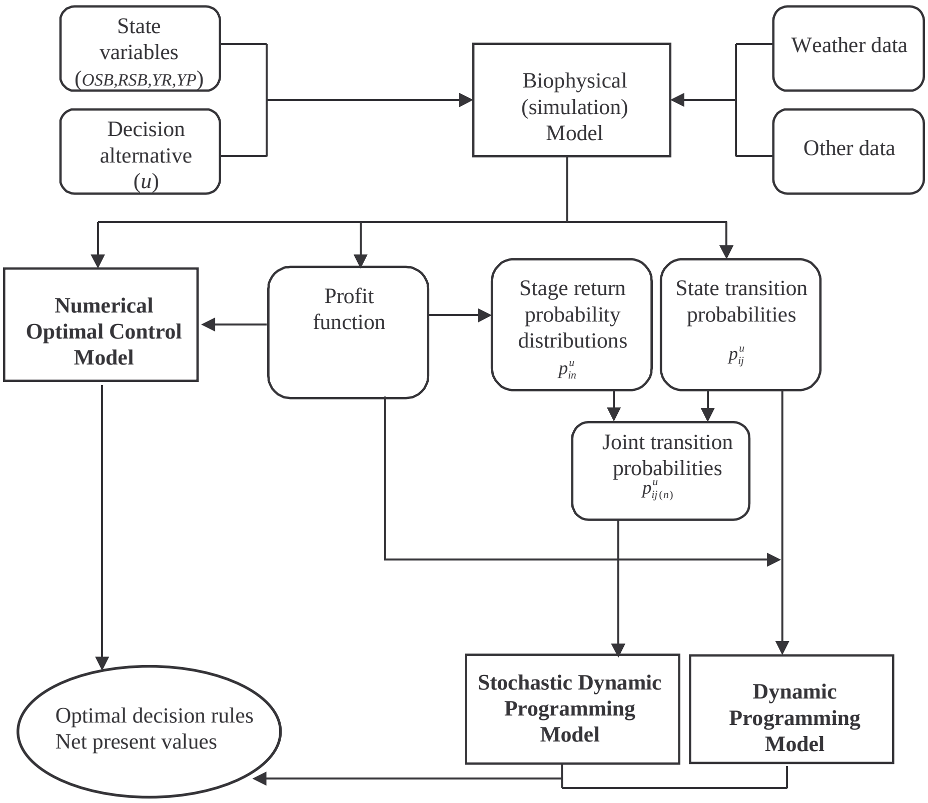

Figure 2 Interactions between the biophysical and economic models.

The objective of the weed population dynamics model is to calculate the change in the weed seed bank from one year to the next due to a weed management decision. A number of the population dynamics parameters are dependent upon environmental factors such as soil moisture and temperature. Consequently, the outputs of the water balance model are an important input to the population dynamics model.

The crop yield model is comprised of two components; a crop growth sub-model and a yield-loss sub-model. The growth indexes derived from the water balance model are used in the calculation of crop growth and, therefore, weed-free yield. The yield-loss sub-model is based upon the density-based yield loss equation (Cousens 1985) and is linked to the population dynamics model for the determination of weed density.

Because one of the rotational options available to farmers is to implement a pasture phase, a pasture growth model is included. The purpose of this model is to determine seasonal pasture biomass and potential livestock carrying capacity, and thus economic returns from grazing.

The sequencing of the crop and pasture rotation is kept track of by a simple crop rotation model. This model imposes various crop yield penalties depending on the length of the cropping phase, and imposes a minimum period in the pasture phase when this part of the rotation is selected.

A NOC model was developed to determine optimal weed management decision rules and their effect upon the resource stock (ie. the weed seed bank). Two weed control options were evaluated; post-emergence herbicide and selective spray-topping herbicide.

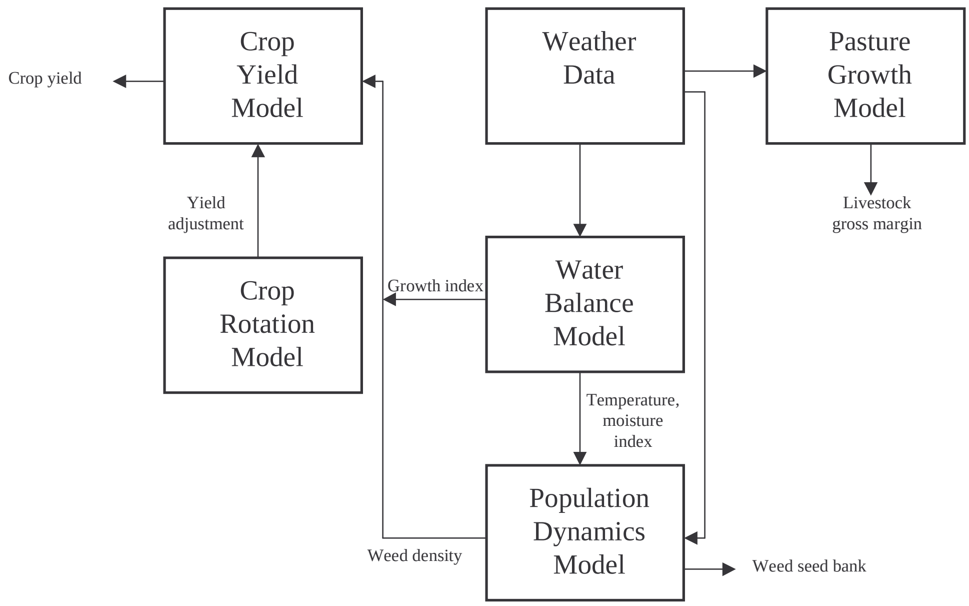

Figure 3 The biophysical model system.

3.1 The numerical optimal control model

Traditional herbicide treatments at the post-emergence stage of the life cycle represent a plant kill approach to weed management (ie. the focus is to maintain and conserve crop yield). A selective spray-topping herbicide treatment is regarded as a seed kill approach to weed management, with the emphasis being to minimise seed input so as to achieve a reduction in the weed seed bank over time. Two scenarios were evaluated involving combinations of the herbicide management options; post-emergence herbicide (PE), and post-emergence herbicide plus a selective spray-topping herbicide (SST). In both cases the control decision (*u*) was the optimal rate of the post-emergence herbicide. By combining plant kill and seed kill approaches, the second scenario provides a measure of the potential economic benefits of IWM. A third scenario (STATIC) was developed which involved a post-emergence herbicide application from a single year simulation of the NOC model.

For the NOC model the dynamic weed problem is stated as follows.

$$\max PV = \max_{u_t}\left[\sum_{t=0}^{T}\beta^t \pi(SB_t, u_t)\right] \tag{9}$$

subject to

$$SB_{t+1} - SB_t = g(SB_t, u_t) \tag{10}$$

$$SB_0 = SB(0) \tag{11}$$

Where *PV* is the present value of net profit, π is the net annual profit and *SB* is the weed seed bank state variable. *SB*(0) is the initial weed seed bank of either wild radish (*RSB*) or wild oats (*OSB*) in period t_0. The first step in solving this problem is to define the current-value Hamiltonian.

$$H_t = \pi(SB_t, u_t) + \beta\lambda_{t+1} g(SB_t, u_t) \tag{12}$$

The Hamiltonian function is the net profit obtained from an existing level of the state and control variables plus the value of any change in the stock of the state variable valued at the costate variable, λ_{t+1}. The solution of the NOC model involves the simultaneous solution of the maximum principle, the adjoint equation and the equation of motion.

The control variable for the NOC model was the dose of a post-emergence herbicide. The herbicides were assumed to be Broadside® for wild radish and Hoegrass® for wild oats. The NOC model requires a continuous and completely differentiable function for the control variable. This was achieved by using dose response functions for the two herbicides. The equation of motion of the NOC model was calculated by a version of the population dynamics model.

The net profit (π) component of the Hamiltonian function is calculated as follows.

$$\pi = P_y\left(Y_{WF}\,Y_L\right) - VC_C - P_u u - MCost \qquad (13)$$

or

$$\pi = P_y\left(Y_{WF}\,Y_L\right) - VC_C - P_u u - MCost - CC_{ST} \qquad (14)$$

Where *MCost* is the application cost of a herbicide, VC_C is the variable cost associated with a crop, and CC_{ST} is the control cost associated with selective spray-topping. The profit equation (13) is used for scenarios PE and STATIC, while equation (14) is used for scenario SST. The parameter values used in the three economic models are given in Table 1.

3.2 The deterministic dynamic programming model

The objective of the DP model is to determine the economic benefits of IWM for managing weed populations over the longer-term. The model differs to the NOC model in that it considers a range of discrete weed management options. The decision alternatives in the DP model involve crops (wheat), pasture, and various IWM combinations in the cropping phase such as tillage (ie. autumn tickle), post-emergence herbicide, increased competition (ie. increased sowing density) and a selective spray-topping herbicide. A 20-year planning horizon is assumed with a decision required annually on the crop/pasture/IWM choice.

There are four state variables in the model; wild radish seed bank (*RSB*), wild oats seed bank (*OSB*), the number of years remaining in the crop rotation (*YR*), and previous land use (*YP*). The first two state variables track the number of weed seeds in the soil. The state variables *YR* and *YP* are included when the crop-pasture rotation scenario is included

The state variable *YP* is imposed so as to limit the length of the cropping phase and impose yield penalties as the length of the cropping phase is extended. Yield penalties will occur due to the depletion of soil fertility and the increase in soil-borne diseases as the cropping phase is extended. The purpose of the state variable *YR* is to impose a minimum time period for the pasture phase and to ensure proper sequencing of each pasture year. Once a farmer makes the decision to switch to pasture, this phase of the rotation is unlikely to be limited to one or two years. This is because of the pasture establishment costs, the lower returns in the initial year and the increasing benefits to later crops from the build up in soil nitrogen with each year of pasture. Without this state variable the DP model may result in an unrealistic optimal rotation solution involving a cycling of one or two years crop and a year of pasture.

Table 1 Description and values of parameters in economic models

Parameter	Unit	Description	Value
P_u	$/L	Herbicide cost - Hoegrass	18.75
P_u	$/L	Herbicide cost - Broadside	14.80
$MCost$	$/ha	Herbicide application cost	2.50
r	%	Discount rate	5
P_y	$/tonne	Crop price (ie. wheat)	180
P_s	$/tonne	Price of silage/hay	120
GM_{L}	$/head	Livestock gross margin	27.3
VC_{C}	$/ha	Variable cost - crop	303.8
VC_{S}	$/ha	Variable cost – silage/hay	423.3
VC_{G}	$/ha	Variable cost – green manure	180.0
VC_{F}	$/ha	Variable cost - fallow	15.0
VC_{P}	$/ha	Variable cost - pasture	37.4
CC_{PEO}	$/ha	Control cost – post-emergence herbicide wild oats	30.63
CC_{PER}	$/ha	Control cost – post-emergence herbicide wild radish	17.30
CC_{STO}	$/ha	Control cost – spray-topping herbicide wild oats	25.50
CC_{STR}	$/ha	Control cost – spray-topping herbicide wild radish	17.75
CC_{AT}	$/ha	Control cost – autumn tickle	6.40
CC_{IC}	$/ha	Control cost – increased competition	12.00
CC_{WC}	$/ha	Control cost – winter cleaning in pastures	7.50
Y_s	t/ha	Yield of silage/hay	6.0
SR	hd/ha	Livestock stocking rate	5
SR	hd/ha	Livestock stocking rate with winter cleaning	4

Source: NSW Agriculture (2001), Mullen *et al*. (2002)

The objective function of the DP model is the maximisation of the present value of net returns from the sequence of crops, pasture and weed control over a 20-year time horizon subject to the constraints imposed by the state variables. The recursive equation of the DP model is

$$V_t(OSB_t, RSB_t, YR_t, YP_t) = \max_u [\pi_t(OSB_t, RSB_t, YR_t, YP_t, u_t) + \beta V_{t+1}(OSB_t, RSB_t, YR_t, YP_t)] \quad (15)$$

subject to

$$OSB_{t+1} = OSB_t + g_1(OSB_t, u_t) \quad (16)$$

$$RSB_{t+1} = RSB_t + g_2(RSB_t, u_t) \quad (17)$$

$$YR_{t+1} = YR_t + g_3(YR_t, u_t) \quad (18)$$

$$YP_{t+1} = u_t \quad (19)$$

Where the variable u_t is the IWM decision alternative in year t.

The enterprise choices used in the DP model involve crops, silage/hay, green manure and pastures. Although a range of alternative crops are used in rotations by landholders, such as wheat, barley, oats, canola and pulses, in this study a generic crop based upon wheat is used. This approach has been adopted so as to simplify the analysis and to focus attention upon the weed control options that are available within the cropping phase.

The decision alternatives are a set of IWM options that involve growing crops or pasture with or without various combinations of weed control. The specific weed control options in the cropping phase include post-emergence herbicide and selective spray-topping herbicide for either (or both) wild radish and wild oats, autumn tickle and increased competition.

The options chosen represent weed control at various phases of the weed life-cycle; prior to sowing (autumn tickle), after crop sowing (post-emergence herbicide), in-crop (increased competition) and late in the growing season (selective spray-topping). The options represent a range of non-chemical control options (tillage, competition), traditional chemical control (post-emergence herbicide) and non-traditional chemical control (selective spray-topping).

In the cropping phase the crop could alternatively be cut for hay or silage, or green manured. These operations represent an effective form of weed control in terms of eliminating seed input to the seed bank, particularly when weed infestations are high. In the pasture phase weed control (apart from consumption by livestock) includes a spray-topping herbicide, known as winter cleaning when applied to pasture. Finally, the land may be left fallow for a season.

A total of 76 decision alternatives were derived involving the various combinations of crop, pasture, weed control option, silage/hay, green manure and fallow. The objective of the DP model is to determine the economic benefits of IWM, and to determine if the optimal decision rules and economic benefits differ under conditions of multiple weed populations and alternative cropping systems. The following IWM scenarios were constructed.

PE: Post-emergence herbicide only available
AT: Post-emergence herbicide and autumn tickle available
IC: Post-emergence herbicide and increased competition available
SST: Post-emergence herbicide and selective spray-topping herbicide available
ALL: All control options available

The choice of these scenarios allows for the measurement of the economic impact of any restrictions to IWM. The scenario PE restricted in crop weed control to post-emergence herbicides at the registered rates. This scenario represents the base from which the economic benefits of additional weed control technologies can be valued. Scenario AT involves the restriction of weed control to a post-emergence herbicide and an autumn tickle cultivation. Likewise, the scenarios IC and SST involve weed control being limited to a post-emergence herbicide and either an increased competition technology or a selective spray-topping herbicide respectively. The scenario ALL allowed access to all the decision alternatives and provides the determination of the optimal IWM strategies for any given seed bank.

The scenarios can be grouped according to whether the management focus is upon plant kill or seed kill technologies. The scenarios PE, AT and IC involve plant kill technologies, while SST and ALL include the seed kill technology of selective spray-topping.

The net profit associated with each decision alternative is derived from the following equation.

$$\pi_u = \begin{cases} P_y\left(Y_{WF} \cdot YAF \cdot Y_L\right) - VC_u - CC_u, & \text{for } 1 \le u \le 64 \\ P_s Y_s - VC_u, & \text{for } u = 65 \\ -VC_u, & \text{for } 66 \le u \le 67 \\ GM_L SR_u - VC_u - CC_u, & \text{for } 68 \le u \le 76 \end{cases} \tag{20}$$

Where *YAF* is a yield adjustment factor associated with the *YP* state variable, *VC* is the variable cost, *CC* is the weed control cost, P_S is the price of silage, Y_S is the yield of silage, GM_L is the livestock gross margin and *SR* is the livestock stocking rate. The parameter values for the economic variables in the DP model are given in Table 1.

3.3 The stochastic dynamic programming model

The economic benefits of IWM are expected to be influenced by both the dynamic aspects of the weed problem and by uncertainty in weed population dynamics, weed control efficacy and crop yield. The SDP model was developed so as to measure the effects of this uncertainty on the weed management problem.

In many decision problems the equation of motion depends not only on the state of the system and the decision taken, but also on unpredictable events outside the control of the decision-maker. The return function may likewise depend upon unpredictable events. If random events affecting the net return function and equation of motion at stage *t* are those occurring at stage *t* and not any earlier, the problem may be formulated as a SDP problem without additional state variables. Stochastic processes in SDP models are generally described as a Markov decision process, or Markov chain.

A major advantage of SDP over NOC is that stochastic effects can be readily incorporated into the framework. The stochastic problem is to maximise the expected present value (EPV) of profit from a *T*-period decision process.

$$\max EPV = \max_{u_t}\left[\sum_{t=0}^{T} \beta^t \pi\left(x_t, u_t, e_t\right)\right] \tag{21}$$

Where *e* is an error term and helps to derive the probability distribution for π and EPV. Maximisation of this equation is subject to a set of first-order difference equations for the state variables.

$$x_{t+1} = x_t + g\left(x_t, u_t, \varepsilon_t\right) \tag{22}$$

Where ε is a random variable (or set of random variables) and defines the probability distribution for the state variable. The recursive equation for the weed management SDP problem can be written as follows.

$$V_t\left(SB_t\right) = \max_{u_t}\left\{E\left[\pi\left(SB_t, u_t, e_t\right)\right] + \beta E\left[V_{t+1}\left(SB_{t+1}\right)\right]\right\} \tag{23}$$

Dynamic programming is used as a numerical solution procedure applied to the maximisation of the recursive equation (23) for discretised state and decision variables. The numerical search procedure involves searching over all values of u_t for given SB_t. The equation (23) is often restated as a Markovian dynamic programming recursive equation.

$$V_t(i) = \max_k \left[\pi(i,k) + \beta \sum_j p_{ij}^k V_{t+1}(j) \right] \quad (24)$$

Where i is a 'from' index for states and j is a 'to' index for states, k is an index for the decision alternatives and p_{ij}^k is the (transition) probability of going from state i at stage t to state j at stage (t+1) for the k-th decision alternative. The state transition probabilities are derived from equation (22) and the density function for ε_t. In this study this was achieved by running historical simulations of the biophysical modelling system, where daily interaction between the water balance and population dynamics models determined probabilities for changes in the weed seed bank for each decision.

Note that in SDP problems the stage return can be random as well as the state transitions, thus the recursive equation can be written as follows.

$$V_t(i) = \max_k \left[p_{in}^k \pi(i,k) + \beta \sum_j p_{ij}^k V_{t+1}(j) \right] \quad (25)$$

Where p_{in}^k is the probability for a stage return n corresponding to state i and the k-th decision alternative. The two sets of probabilities can be combined into a set of joint transition probabilities ($p_{ij(n)}^k$). The recursive equation then becomes

$$V_t(i) = \max_k \left\{ p_{ij(n)}^k \left[\pi(i,k) + \beta V_{t+1}(j) \right] \right\} \quad (26)$$

This is the numerical approach adopted in this study for solving the SDP weed management problem.

4. Results

4.1 The numerical optimal control model

The net present values (NPV) for each scenario and a range of initial seed banks were calculated from a 20-year simulation of the NOC model (Table 2). The results were compared for three initial seed banks (100, 1,000 and 5, 000 seeds/m^2) to represent low, medium and high weed infestations. The greatest NPV was associated with the SST scenario for most initial seed banks considered. The PE scenario was economically superior to STATIC in all cases.

There was a significant difference in the economic performance of SST in comparison to PE for wild radish. For example, the NPV of SST for wild radish was 15 percent greater than PE at 1,000 seeds/m^2 and 11 percent greater at 5,000 seeds/m^2. For wild oats, there was little difference in the NPV results between PE and SST.

Table 2 Net present values from the NOC model ($/ha)

Seed bank (seeds/m^2)	STATIC	PE	SST
Wild radish:			
100	4,346	4,495	5,000
1,000	4,210	4,210	4,832
5,000	4,027	4,143	4,588
Wild oats:			
100	4,634	5,276	5,005
1,000	4,316	4,782	4,797
5,000	4,037	4,361	4,508

4.2 The deterministic dynamic programming model

Continuous cropping system

The NPV for a 20-year simulation was calculated for the continuous cropping system and the individual weed states (Table 3). In the case of wild radish, for the low initial weed infestation there was no significant difference in the NPV between the scenarios. For the medium and high initial infestations the greatest NPV was associated with ALL, which was closely followed by either SST or IC. There was no significant difference in the NPV results between PE and AT.

For wild oats, the largest NPV resulted from ALL for each initial seed bank considered. There was no difference in the NPV results between SST and IC, which were superior to PE and AT.

Crop and pasture rotation system

The NPV results for the crop and pasture rotation system are presented in Table 4. For both wild radish and wild oats the NPV results are significantly lower than the corresponding results for the continuous cropping system. This reflects the lower overall returns from including a pasture phase in the rotation.

The benefits of the IWM options of autumn tickle, increased competition and selective spray-topping were dramatically reduced for this system. Although the NPV from ALL was greater than the PE scenario for each initial seed bank, the difference in the NPV was significantly less than for the continuous cropping system.

Table 3 Net present values for continuous cropping system from the DP model ($/ha)

Seed bank (seeds/m^2)	PE	AT	IC	SST	ALL
Wild radish:					
100	5,103	5,103	5,109	5,123	5,134
1,000	4,866	4,866	4,950	4,987	4,991
5,000	4,452	4,500	4,672	4,670	4,754
Wild oats:					
100	5,100	5,231	5,255	5,249	5,255
1,000	4,792	4,872	5,043	5,017	5,086
5,000	4,294	4,346	4,710	4,706	4,828

Table 4 Net present values for crop and pasture system from the DP model ($/ha)

Seed bank (seeds/m^2)	PE	AT	IC	SST	ALL
Wild radish:					
100	3,689	3,689	3,695	3,724	3,732
1,000	3,516	3,519	3,554	3,565	3,570
5,000	3,196	3,214	3,353	3,280	3,375
Wild oats:					
100	3,811	3,811	3,813	3,811	3,813
1,000	3,609	3,609	3,670	3,645	3,680
5,000	3,301	3,301	3,436	3,334	3,443

4.3 The stochastic dynamic programming model

The expected net present value (ENPV) for each weed and scenario combination was derived from the SDP model. A range of summary statistics were calculated for the ENPV, however, for the sake of brevity the mean ENPV for each initial seed bank (Table 5) and the cumulative density functions (CDF) for an initial seed bank of 1,000 seeds/m^2 (Figure 4) are presented.

In the case of wild radish the largest mean ENPV resulted from ALL, closely followed by SST, for all initial seed banks considered. There were no significant differences between AT and PE, while the mean ENPV for IC was significantly greater than PE for initial seed banks of 1,000 and 5,000 seeds/m^2. The difference in the mean ENPV between ALL and PE was significant, being 10 percent greater at 100 seeds/m^2, 11 percent at 1,000 seeds/m^2 and 49 percent at 5,000 seeds/m^2.

For wild oats, all the IWM scenarios had mean ENPV greater than that derived for the PE scenario. The AT scenario was only marginally greater than PE, however, there were significant differences between IC, SST, ALL and PE. As with the wild radish results, the largest mean ENPV was associated with ALL for each initial seed bank. For 100 seeds/m^2 the mean ENPV of ALL was 16 percent greater than PE, at 1,000 seeds/m^2 it was 63 percent greater and at 5,000 seeds/m^2 ALL was 126 percent greater than PE. These results indicate that there are substantial economic benefits from IWM tactics such as increased competition and selective spray-topping for managing wild oats.

Table 5 Mean expected net present values from the SDP model ($/ha)

Seed bank (seeds/m^2)	PE	AT	IC	SST	ALL
Wild radish:					
100	4,812	4,837	5,005	5,206	5,315
1,000	4,616	4,647	4,828	4,982	5,105
5,000	3,206	3,206	3,735	4,566	4,779
Wild oats:					
100	4,613	4,837	5,302	5,281	5,321
1,000	3,128	3,174	4,315	4,963	5,091
5,000	2,068	2,153	3,719	4,216	4,675

The CDFs of ENPV from a simulation of an initial seed bank of 1,000 seeds/m^2 are illustrated in Figure 11.7. For simplicity only two CDF curves are plotted, a plant kill scenario represented by PE and a seed kill scenario represented by SST. The CDFs show significant differences in the distributions between the scenarios with and without the seed kill technology. For both weeds, the SST distributions exhibit first-degree stochastic dominance over PE. The difference in the CDFs, and the benefits from the seed kill technology, are significantly greater for wild oats than for wild radish.

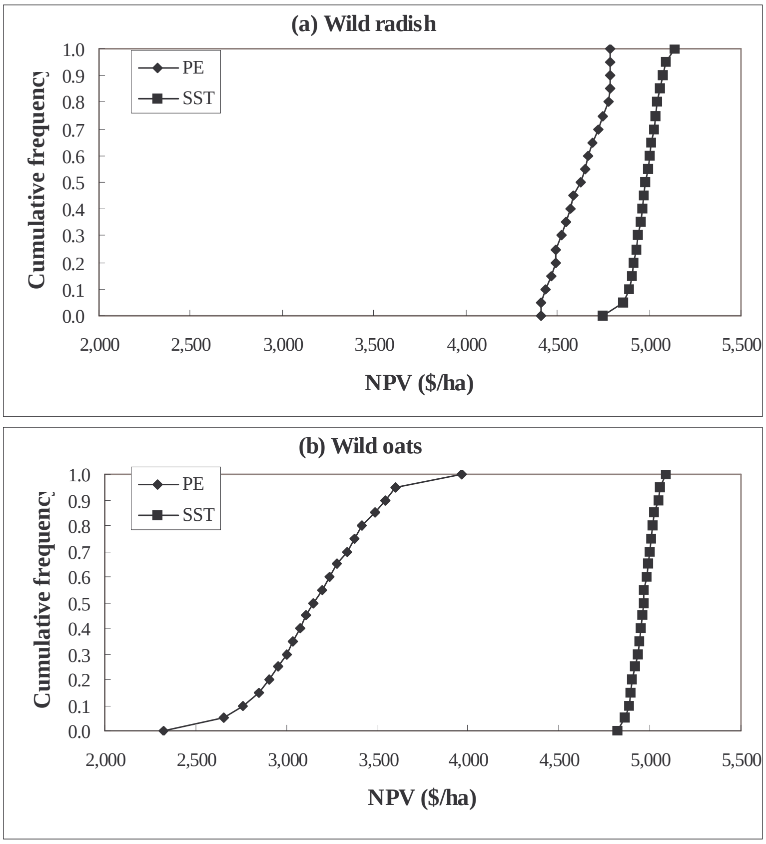

Figure 4 Cumulative density functions of ENPV

5. Conclusions

The results of the analysis from the NOC model showed that there are significant economic differences between static and dynamic decision-making frameworks for the management of wild oats and wild radish. The inclusion of the future economic benefits from weed control leads to a higher optimal level of control than if only the current period benefits is considered. The NOC model also determined the optimal dose rates (data not shown) to be significantly higher than is registered for the post-emergence herbicides. This result suggests that the optimal level of weed control for a given seed bank is considerably higher than that which can be obtained from just the use of these herbicides at the registered rate. This potentially leads to a justification for an IWM approach to weed management, which allows for the aggregation of complementary weed control technologies.

The optimal levels of weed control are unlikely to be achieved through the application of traditional post-emergence herbicides applied at the registered rates. This implies an IWM approach is necessary to achieve long-term benefits from weed management. The inclusion of a selective spray-topping technology combined with a post-emergence herbicide (SST) in the NOC model lead to greater economic benefits than if just the post-emergence herbicide was used (PE). However, an IWM strategy is not limited to these two technologies as other options may include seed capture, increasing competition between the crop and the weed, and tillage to stimulate seed germination. An optimal IWM strategy for a given weed and seed bank may have various combinations of these technologies. A NOC model is unsuitable to address the issue of the optimal IWM combinations given the discrete nature of the problem. The alternative dynamic optimisation techniques of DP and SDP are more suitable for this task.

The major finding from the DP model was that there are economic benefits from adopting an IWM approach to managing wild radish and wild oats. With the exception of the AT scenario, each IWM scenario resulted in higher NPVs over a 20-year simulation period than the PE scenario. Consequently, the technologies of increasing competition between the crop and weed and selective spray-topping herbicides would appear to have economic benefits for managing weeds in the long-term.

An important finding from the DP model was that the same decision rules (data not shown) for a particular weed were obtained for both conditions of single weed (ie. single state) or a mixed population (ie. multiples states). This implies that optimal IWM strategies can be developed from modelling an individual weed seed bank despite the fact that weeds often occur as part of a mixed population of species in the field. To model multiple weed species using a dynamic optimisation framework can become extremely burdensome from a computational perspective.

Under the assumption of a continuous cropping system the selective spray-topping technology was the major factor for reducing wild radish and wild oat seed banks. However, when the analysis was broadened to allow for crop rotations that include a ley pasture the benefits of this technology were dramatically reduced. This was because of the influence of rotational options such as pasture and fallow in reducing seed banks. However, this technology along with increased competition was still selected by the model during the cropping phase of the rotation to manage weed seed banks within a rotational farming system.

An important finding from the SDP model was that although the decision rules were the same as those obtained from the DP model, the economic benefits of adopting IWM were significantly greater than derived from the deterministic simulations. In particular, the IWM scenarios including a seed kill strategy had significantly higher ENPVs. A comparison of the CDFs of a plant kill and seed kill scenario indicated the economic superiority of the latter approach to weed management.

The main conclusion drawn from this study are that there are significant potential economic benefits from the adoption of IWM for the long-term management of weeds such as wild oats and wild radish. The economic benefits of an IWM strategy are substantially enhanced by the inclusion of seed kill technologies such as selective-spray topping. Consequently, the priority for research on the long-term control of weeds should be focussed on technologies and strategies that aim to minimise weed seed banks.

To properly measure the economic benefits of IWM requires the use of dynamic economic frameworks such as optimal control and dynamic programming models. These models were found to be superior to a static model for measuring the benefits of IWM over a 20-year period. Variability in weed population dynamics due to seasonal conditions was determined to be extremely important in the valuation of the benefits of IWM.

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