Three-Dimensional Efficient Portfolio Frontier: Mean, Variance, and Farm Size

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Abstract

With continuing trends in increasing farm size and declining number, the impact of size of operation on risk may have significant policy implications. Diversification indexes for 288 Kansas farms from 1984 to 2003 indicated that sample farms had become slightly more diversified over time. Previous findings on the impact of farm size on diversification of farms have been controversial. To capture observed size impacts on enterprise-specific risks, a mean-variance model that allows for variance of enterprise returns to be decreasing in assets allocated is conceptualized. Efficient farm enterprise mixes are estimated using farm level data, showing that the optimal levels of diversification differ for farms of different sizes. For example, small farms should diversify more than large farms to achieve the same level of return. Results from a panel regression analysis showed that the relationship between farm size and diversification depended on how farm size was measured, reconciling the past findings.

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Declining numbers of farms and increasing farm size have been decisively clear trends in the U.S. farm structure. Risk has been noted, among others, as a factor shaping these changes in farm structure, and diversification has long been recognized as a strategy that can be used to reduce risk (e.g., Gardner et al.; Hardaker et al.). A tradeoff has been noted as farms grow in size between increases in expected return from scale economies and risk reduction from diversification (Pope and Prescott), but our understanding of the relationship between farm size, diversification, and risk remains limited.

For example, there is no consensus in the literature regarding the relationship between farm size and diversification. Some studies found larger farms to be more diversified (Pope and Prescott), while others find to the contrary (e.g., Mishra and El-Osta). According to the 2001 Agricultural Resource Management Survey, about 65% of large and very large family farms produced more than two commodities, compared to the overall average of 24% (Hoppe and Korb). Aside from differences in farm samples, mixed findings might be reconciled if the relationship among various diversification measures and the relationship between diversification and different size measures are recognized.

More fundamentally, the directly application of portfolio theory (Markowitz) beyond the original context of financial portfolio management calls for reevaluation. For example, the management literature has found conflicting evidence of how diversification impacts corporate performance. The trend in diversification among large U.S. corporations reversed in the 1980s with many firms re-focusing their operations (Markides). Recent findings (e.g., Palich, Cardinal, and Miller) suggest that performance increases as firms diversify to related businesses from a single business but decreases as they further diversify to unrelated businesses.

In farming, portfolio theory is applied regarding enterprises as investment assets with differing rates of return. For financial investments such as stocks, additional shares of a stock of a company are identical in terms of return and risk no matter how many shares of the stock are included in the portfolio. However, additional units of farming enterprises bring in varying levels of returns and impact the risk associated with the enterprise. For example, the yield of one field of corn will differ from that of another field of corn, particularly if they are geographically distant but even if they are adjacent. Because returns from additional units of an enterprise are not perfectly correlated with those from the original units, having more acres or animals will reduce the variability of returns from the particular enterprise (Schurle). If this size impact on risk is incorporated, the optimal degree of diversification might depend on the scale of operation.

The objective of this paper is to examine the impact of size on diversification of farms accounting for the size effect on return variability. An understanding of the relationship between farm size, diversification, and risk will contribute to predicting future changes in farm structure and formulating effective farm policy. For example, subsidy payments that are contingent on the production of specific crops can have significant impacts on a firm's decision to specialize or diversify. First, to address the paucity of documented levels of farm diversification, the trends in observed farm diversification are documented. The analysis uses Kansas Farm Management Associations (KFMA) data, which allow for a cross-section of farms to be studied over time. The second and third specific objectives are to investigate how optimal and observed degrees of farm diversification vary, if any, by farm size. The use of various measures of farm size is explored to possibly reconcile the conflicting past findings.

We find that sample farms had become specialized into fewer enterprises but generated revenues more evenly from these enterprises over the last decade. During the last two decades, variability of enterprise returns was declining in scale of enterprise, and accounting for this relationship, the optimal farm enterprise mix was shown to depend on size. Thus, our findings invalidate a blanket recommendation for diversification as a risk management means. Moreover, the relationship between size and diversification is sensitive to how the variables are measured. Particularly, farms with greater gross revenue are more specialized, while farms with more acreage are more diversified.

The next section begins by measuring the degree of diversification of sample farms and observing trends. Then, Markov's portfolio theory is adapted for farm enterprise analysis, and the theoretical relationship between efficient farm mix and size is illustrated using KFMA data. The relationship between size and the observed degree of diversification is examined in a panel regression. Lastly, implications of our findings are discussed.

Trends in Diversification

There are approximately 2000 farms in the KFMA database which contains financial and production records. These farms are generally large commercial farms with value of farm production averaging $265,629 in 2004. Only a portion of the farms have enterprise records, and some farms do not have continuous records over time. For our analysis, 288 farms were selected because they had enterprise records and continuous records from 1984 to 2003. Farming-related enterprises (or revenue sources) were aggregated into the following thirteen: wheat, corn, sorghum, soybeans, cotton, sunflower, hay, beef, dairy, swine, crop insurance, government payments, and other. The "other" revenues are primarily generated from custom work, minor crops, custom feeding of livestock, and patronage refunds.

Measuring Diversification

There are two measurement issues with diversification: one is the definition of measure and the other is what variable is used to construct the measure. Regarding the second issue, farm enterprise diversification has been measured using crop acreage (Pope and Prescott), values of production (Mishra and El-Osta), or indices of production enterprises (McNamara and Weiss). Since farms in Kansas frequently have both crops and livestock, we decided to use gross revenue generated per enterprise, adjusted for inventory, which is comparable across enterprises and a better measure of farm size than acres (Hoppe and Korb).[1] Thus, our construction will be similar to studies of the manufacturing sector, where diversification is typically measured by values of shipment, such as those reported in the Census of Manufacturers (e.g., Gollop and Monahan).

Multiple measures of diversification appear in the literature, and several studies report comparisons (e.g., Hackbart and Anderson; Gollop and Monahan). One commonly used is the entropy measure, which can be normalized to be bound between 0 and 1 as:

$$d^E = \sum_{i=1}^{N} s_i \log\left(1/s_i \right) \Big/ \log(N), \quad (1)$$

where s_i is the share of gross revenue generated from the ith enterprise. The index takes the value of one for farms with equal shares in all enterprises.

To examine the sensitivity of our analysis to the choice of the diversification measure, three other indexes were considered, all normalized between zero and one and are increasing in diversification. The Berry index is one minus the Herfindahl index:

$$d^B = 1 - \sum_{i=1}^{N} s_i^2 \, . \quad (2)$$

[1] Gross revenue is adjusted for inventory changes so that it more closely reflects dollars generated from the current year production.

One minus the primary product specialization ratio is sometimes referred to as a modified concentration ratio:

(3)
$$d^P = 1 - \frac{s_{\max}}{\sum_{i=1}^{N} s_i}.$$

Lastly, the normalized Utton's index utilizes the rankings of the enterprises as

(4)
$$d^U = \left(2\sum_{i=1}^{N} q_i s_i - 1 \right) \Big/ N$$

where q_i is the ranking of the *i*th enterprise.

The four diversification measures were computed for 288 farms over the 20 years. The three indexes (d^B, d^P, and d^U) were plotted against the entropy-based index (d^E) in figure 1. The Berry index (d^B) values are greater than or equal to d^E. The correspondence is more variable around the index values 0.4 and 0.6. The normalized Utton index (d^U) values are related to d^E in a monotone, exponential manner with least variability. The PPSR-based index (d^P) has the most variable correspondence with d^E. The correlations among the indexes were over 0.894. The changes in diversification over time were computed using all four indexes. All indexes consistently indicated that the member farms have become slightly more diversified over time. The entropy measure averaged 0.512 from 1989 to 1993, which increased to 0.539 from 1999 to 2003. Judging from high correlations and comparable general trends, the rest of the analysis is based on the normalized entropy measure (d^E).

As an alternate way to examine diversification, the number of enterprises that generated positive gross revenue in a given year was counted for each farm over the sample period. Then, the farms were sorted by the number of enterprises. Table 1 reports three sets of five-year averages (1989-1993 and 1999-2003) of the percentages of farms with respective number of enterprises: the left set of averages considers all enterprises including crop insurance, government payments, and other, which is consistent with the diversity index computed above, the middle set excludes crop insurance, and the right set is based on production enterprises only (seven crops and three livestock). The majority of farms had produced more than two crops, which is more diversified than the averages reported in the 2001 national study (Hoppe and Korb). Considering all enterprises (the left columns), more farms on average were engaged in eight or more enterprises in 1999-2003 than a decade ago, implying diversification. Yet, the early 2000s were poor crop years in Kansas; 170 sample farms obtained indemnity payment from 1999 to 2003 on average, while only 46 did from 1989 to 1993. Indeed, if crop insurance is ignored (the middle columns), then the sample farms on average specialized into fewer enterprises over the decade: more farms were engaged in five or less enterprises and less farms were in six or more enterprises. Consistently, if only production enterprises are considered (the right columns), the proportion of farms with four or less enterprises increased, while that of farms with five or more decreased.

Diversification by Size

To gain an insight on factors underlying the changes in farm mix, the diversity index values and the enterprise numbers were analyzed by farm size. Farm size can be measured by acreage, sales revenue, or assets or capital managed. Thus, the farm observations in a given year were sorted into deciles by total acres managed, total gross revenue, and total capital managed, respectively.[2] In the interest of space, only five-year averages from 1989 to 1993 and from 1999 to 2003 of the diversity index values, sorted by various farm size measures, are reported in Table 2. Additional detail is available from the authors upon request.

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[2] Total capital managed includes assets owned by the farm and assets (typically land) rented from other owners.

According to the diversity index values (Table 2), 10% of farms with the smallest acreage were the most specialized, while 10% of farms with the highest gross revenue were the most specialized on average; these farms were not necessarily the same farms specializing in livestock. Nonetheless, the bottom decile, according to all three size measures, was engaged in the fewest number of enterprises. In terms of total acreage, farms of all sizes appear to have become more diversified from 1989-93 to 1999-2003, with the largest average increases among farms between the 40th and 60th percentiles. According to gross revenue, smallest 30% of farms became more specialized on average, analogous to farms between the 10th and 20th percentiles of capital managed. The largest farms by any measurement were not the most diversified and had not diversified at the fastest rates.

Generalized Portfolio Selection Model

In the next two sections, relationship between farm mix and size is conceptualized. The mean-variance model of portfolio selection has been used extensively in both the finance and agricultural economics literature, since it was developed by Markowitz. In the model, a decision-maker allocates total assets across *N* different sources of revenue stream, or investment assets. The rates of return on these investment assets are assumed to be linearly independent and have a non-singular variance-covariance matrix, **V**. Denoting an *N*-vector of shares as **w**, $\mathbf{w}'\mathbf{V}\mathbf{w}$ is a portfolio variance. An efficient portfolio *p* has a share vector that solves the following constrained optimization problem:

$$\min_{\mathbf{w}} \mathbf{w}'\mathbf{V}\mathbf{w} \tag{5}$$

subject to

$$\mathbf{w}'\mathbf{r} = \tilde{r}_p \quad \text{and} \quad \mathbf{w}'\mathbf{1} = 1,$$

where **r** denotes an *N*-vector of expected rates of return on the *N* investment assets, $\tilde{r}$ denotes the expected rate of return on portfolio *p*, and **1** is an *N*-vector of ones. That is, efficient portfolios have the smallest variance among portfolios that have the same expected rate of return. The mean-variance model is consistent with expected utility maximization under two cases: when the assumed utility function is quadratic, and when the rates of return are multivariate normally distributed (Huang and Litzenberger, p. 61).

The mean-variance model in (5) assumes that distributions of rates of return remain constant regardless of how total assets are allocated. The assumption is appropriate for financial portfolios, where most investors cannot influence the distributions of stocks' prices through purchases or sales. That is, the rates of return for various units of the same investment are identical. The assumption needs to be re-evaluated for applications beyond financial portfolios. In enterprise diversification, the investment assets are enterprises with respective production activities funded by the allocated assets. Unless constant returns to scale is assumed, production technologies can be increasing or decreasing to scale. This invalidates the assumption of constant expected rates of return. Thus, in problem (5),

(6) $$\mathbf{r} = f_r\left(\mathbf{A}\right)$$

That is, the expected rates of return is a function of the *N*-vector of assets allocated to each enterprise, **A**, which can also be written as $A\mathbf{w}$, where *A* is the total assets allocated.

Moreover, additional units of the investment assets are not identical. For example, a new plant cannot be identical by definition—the plant will be operated and managed by different individuals, and its equipment is physically distinct from those of other plants. In agriculture, a crop field cannot be identical to another crop field due to difference in location and differences in physical characteristics that are associated with location (soil quality, weather, etc.) Thus, the rates of returns from various units of the same investment asset may be highly correlated but not perfectly correlated. In fact, as Schurle found for crop yields, it is likely that due to imperfect correlations among returns, it is reasonable to expect that variance of returns from an enterprise will decline with the magnitude of the asset allocated.

While the main emphasis of this paper will be on the potential decrease in variance of returns as more assets are allocated to an enterprise, covariances may also be influenced by relative and absolute amounts of assets invested in various farm enterprises. To put this in the context of problem (5), it is possible that for each element in **V**, σ_{ij}^2 is a function of **A**, implying:

(7) $$\sigma_{ij} = f_{ij}\left(\mathbf{A}\right) = f_{ij}\left(A\mathbf{w}\right)$$

The following empirical illustration will focus on the generalized assumption (7), and assume that expected rates of return are constant. While it is generally recognized that there are economies of scale, these farms generally operate at a size where many of these economies are already achieved. Thus, the generalized portfolio problem, applicable to enterprise diversification is:

(8) $$\min_{\mathbf{w}} \mathbf{w}'\mathbf{V}\left(\mathbf{A}\right)\mathbf{w}$$

subject to

$$\mathbf{w}'\mathbf{r} = \tilde{r}_p, \quad \mathbf{w}'\mathbf{1} = 1, \quad \text{and} \quad A\mathbf{w} = \mathbf{A}.$$

The important issue problem (8) raises is that efficient portfolios will depend on the total assets available to be allocated to the enterprises.

Empirical Illustration of a Three-Dimensional Efficient Portfolio Frontier

Enterprise Returns

Since farm portfolios in Kansas need to consider both crops and livestock enterprises among investment alternatives, enterprise returns were computed as dollars of gross revenue generated per dollar of asset allocated to the enterprise, as a common measure of return. For the computation, we used asset turnover ratios from Kansas State University enterprise budgets to obtain the amounts of assets allocated to each enterprise by each individual farm. Enterprise budgets represent typical farms, and the asset turnover ratio for an enterprise reflects the gross revenue expected for each dollar of asset in the enterprise. Asset turnover ratios were collected from 1984 through 2003 for the seven crop and three livestock enterprises. Missing asset turnover ratios were generated using state average prices, yields, land values, and expenses reported in the enterprise budgets.

Three-year moving averages of asset turnover ratios were computed for the enterprises from 1984 to 2003. Then, for individual farms, similar three-year moving averages were computed for gross revenue generated from each enterprise. Moving averages of gross revenue were divided by moving averages of the asset turnover ratio of the corresponding enterprise to approximate the assets allocated to the enterprises on a given farm. These amounts of assets were adjusted proportionally so that they summed to the yearly total capital managed by each farm. In this way, rates of gross return were computed for every enterprise and every farm.

The asset turnover ratios are only valid for farms that maintained the same mix of enterprises during the three-year period over which the averages were computed. Thus, enterprise returns were excluded from year t of farms that dropped or added enterprises during years t-2 to t. For example, suppose a farm grossed revenue from wheat and corn in 1984 and 1985. In 1986, the farm grossed from wheat, corn, and hay. Then, the computed returns from this farm for 1986 were considered invalid. Thus, only returns from farms that generated positive revenue from the same set of enterprises over the three-year period were retained for the analysis. Due to the lack of observations, cotton, sunflower, and dairy were dropped from further analysis in this section. The useable observations of enterprise returns are summarized in table 3. The averages will be regarded as the (constant) expected rates of gross return, **r**, in (8).

The Variance-Covariance Matrix of Enterprise Returns

Next, the variance and covariance matrix of the enterprise returns were computed. As in equation (7), it is possible that every element in the variance-covariance matrix depends on the total assets and the allocation vector. Here, a simplifying assumption is adopted as an initial attempt to investigate the relationship between size of a diversified operation and variability of overall returns: covariances are assumed to be constant $\sigma_{ij}\left(\mathbf{A}\right)=\sigma_{ij}$ for $i \neq j$, and the ith variance is assumed to depend only on the asset allocated to the ith enterprise, i.e., $\sigma_{ii} = f_i\left(A_i\right)$. The variance-covariance matrix used in the analysis is found in table 4. Covariances were computed between two enterprise returns that were recorded for the same farm and year.

The relationship between variances and the assets allocated to the enterprises was empirically investigated. For each enterprise, returns were sorted by assets allocated to the enterprise, and standard deviations were computed for each decile (quintile for swine) by allocated assets. Then, simple relationships were estimated between the standard deviations and the decile- (quintile-) averages of assets. Linear and piece-wise linear functions were specified for wheat, corn, soybean, beef, and swine. Double-log and exponential functions were used for sorghum and hay, respectively. The estimated relationships appear at the bottom of table 4. For all enterprises, return variability was declining in assets allocated.

Efficient Portfolios

Using the expected enterprise returns (means in table 3) and the variance-covariance matrix (table 4), efficient portfolios were solved for according to (8) for various values of target portfolio returns $\left(\tilde{r}\right)$ and total assets $\left(A\right)$. The values of the two parameters were chosen to encompass the levels in the data. Specifically, 50 values of portfolio returns between 5 and 54% were considered, and 70 values of assets were considered ranging from $100,000 to $7 million. The optimization used MATLAB's fmincon function. For some combinations of target return and assets, there were no feasible solutions. Figure 2 depicts the efficient portfolio surface against the total assets. If efficient portfolios were solutions from (5), the figure would have looked cylindrical. Instead, shape and location of the portfolio frontier vary with total assets.

For each efficient portfolio, normalized entropy was computed to measure diversification according to (1). The entropy measure is graphed against the amounts of assets for selected levels of target returns in Figure 3. The graph illustrates that the relationship between the optimal levels of diversification and total assets vary by target returns, and that at least for some target levels, the degree of diversification is non-linearly related with assets allocated. That is, the optimal levels of diversification differ for farms of different size. For example, based on the target return of 24%, farms smaller than $800,000 and between $2.2 and 2.7 million in total capital managed should diversify to minimize the variability of the overall gross revenue. Farms managing more than $2.7 million in total capital should specialize in fewer enterprises to meet the same objective. Thus, diversification is not beneficial for all farms.

Panel Regression Analysis

To examine the impact of farm size on the observed farm mix, a regression model similar to Pope and Prescott and Mishra and El-Osta was specified. The dependent variable was the entropy-based diversity index for farm *k* in year *t* $\left(d_{kt}^{E}\right)$. The farm characteristics considered as explanatory variables were the organizational form, the age of the operator, location of the farm, and farm size. The organizational form was captured by dummy variables for corporations and partnerships relative to the sole proprietors. Location was represented by dummy variables for the KFMA areas. Three different variables were considered for farm size: total acres operated, total gross farm revenue, and total capital managed. The correlations among these size variables are reported in table 5.

In addition, the previous section illustrated that the relationship between size and efficient farm mix depended on rates of return of the portfolios. However, rates of return could only be calculated for a limited number of farms (see Table 3). As a proxy, the economic total expense ratio, a ratio of variable expenses to value of farm production, was included as interaction terms with the size variables.[3] A ratio below 1.00 indicates that the farm is earning an economic profit. The summary statistics of the variables used in the regression appear in table 6.

The model was estimated using the panel estimator in LIMDEP. Based on various tests, a two-way fixed effects model was preferred, and location variables were incorporated into farm-specific effects.[4] Table 7 reports results using the three farm size variables separately (models 1-3) and jointly (model 4). The directional impacts of the explanatory variables were robust across different models. For the organizational characteristics, the degrees of diversity on average were

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[3] The numerator of the economic total expense ratio is the sum of total cash expenses (except feed and cash interest paid), management depreciation, opportunity charge on unpaid labor, and an opportunity charge on owned assets. This numerator is divided by value of farm production, which is crop income and net livestock income less feed purchased. All income items are inventory adjusted or accrual.

[4] The Hausman test statistic for the null that the covariances of the ordinary least squares and generalized least squares are equivalent was 40.75, rejecting the null in favor of the fixed effects model with a p-value of 0.000. Adding the time dummy variables to the farm dummy variables was statistically significant at the 1% level with an F-test statistic of 13.57.

higher for corporate farms than farms owned by a sole proprietor, holding everything else constant. The difference between partnership farms and sole-proprietor farms was not statistically significant. The more experienced the operator was, the more specialized the farm was, on average, holding everything else constant. In other words, younger operators diversified their operations more on average. The sign on the age variable is consistent with past results using the same entropy-based index (Pope and Prescott; Mishra and El-Osta).

The interesting aspect of the results was the varying size effects on diversification. In particular, farms with more total acres and total capital managed were more diversified at a decreasing rate. This positive association is consistent with Pope and Prescott using crop acreage as their farm size variable. In contrast, farms that generated more gross revenue were less diversified, and the relationship is likely non-linear, although in the full model, statistical significance was lost. This negative association between value of production as farm size and diversification is consistent with Mishra and El-Osta's finding. Thus, our analysis clearly implies that the definition of farm size matters. Particularly in Kansas, the largest farms based on acres are in western Kansas, but since their yields are typically low, they do not generate much gross revenue. Also, the farms in western Kansas have fewer options to diversify due to highly arid climate.

Statistical significance of the squared size terms and the interaction terms confirm the three-dimensional portfolio frontier illustrated above. That is, the degree of diversification is non-linearly related to size, and the relationship depends on the farms' overall rates of return, proxied by the expense ratio. The interaction terms using expense ratio and size suggest that for any given size, if the expense ratio is smaller, the farm is more diversified. Expense ratio is often used as a measure of managerial ability with lower expense rations indicating better management. This suggests that better managed farms may recognize their capability and choose to be more diversified.

Concluding Remarks

The overall objective of the paper was to examine impact of size on diversification of farms. To document the actual diversification among farms, four diversification measures were computed and the number of enterprises was counted for 288 farms over 20 years. All indexes consistently indicated that the sample farms had become slightly more diversified over time. Since the farms engaged in fewer enterprises, revenues from the fewer enterprises had become more evenly distributed.

To explore the relationship between optimal levels of diversification and farm size, a mean-variance model that allows for variance of enterprise returns to be decreasing in assets allocated was conceptualized to capture size impacts on enterprise-specific risks. Then, efficient farm enterprise mixes were solved for using farm level data for various farm sizes to construct a three-dimensional efficient portfolio frontier. Results showed that the optimal levels of diversification were related to farm size. For example, small farms should diversify more than large farms to achieve the same level of return.

Panel regression addressed the relationship between observed farm mix and size. Size was related to diversification, regardless of measurement, but the particular measure of size made a difference in terms of the relationship that was found. Farms with more total acres or total capital managed were more diversified, while farms that generated more gross revenue were less diversified. Our results reconcile the conflicting findings in the past studies on farm size and diversification.

Clearly, the nature of the relationship between farm mix and size is specific to our sample. Yet, our finding that optimal degree of diversification depends on farm size is likely universal and provides rich policy implications. Programs designed to reduce risk need to recognize that size of operation matters and that optimal diversification depends on size of operation. If the policy objective is to reduce risk or provide a safety net, policy design needs to be based on an understanding of risk and size. Another valuable contribution of our findings is that the size measure matters. The effectiveness of a farm policy will depend on whether the target farms are defined by acreage or assets versus revenue generated.

Table 1 Percentages of Farms by the Number of Enterprises

	All Enterprises[a] N=288		All Enterprises Less Crop Insurance N=288		Production Enterprises[b] N=288	
Number of Enterprises	Average 89-93	Average 99-03	Average 89-93	Average 99-03	Average 89-93	Average 99-03
1	0.0	0.0	0.0	0.0	1.0	1.9
2	0.2	0.1	0.2	0.1	6.5	8.5
3	1.2	1.8	1.3	2.6	18.1	19.1
4	6.3	5.1	7.3	9.0	30.3	30.9
5	16.2	13.8	18.1	19.7	27.8	25.3
6	29.1	23.1	31.3	30.7	13.2	12.6
7	28.3	27.2	27.0	24.2	3.0	1.7
8	14.1	19.2	11.9	12.1	0.1	0.0
9	4.2	8.8	2.8	1.7	0.0	0.0
10	0.5	1.0	0.1	0.0	0.0	0.0
11	0.1	0.0	0.0	0.0	-	-

[a] Including wheat, corn, sorghum, soybeans, cotton, sunflower, hay, beef, dairy, swine, crop insurance, government payments, and other.

[b] Only seven crop and three livestock enterprises from *a* are considered.

Table 2 Diversification Index Values for Different Size Operations, Size Measured by Total Acreage, Gross Revenue, and Capital Managed

Total Acreage:	89-93 Average	99-03 Average	Total Gross Revenue:	89-93 Average	99-03 Average	Total Capital Managed:	89-93 Average	99-03 Average
Largest 10%	0.487	0.523	Highest 10%	0.457	0.480	Highest 10%	0.514	0.533
10-20%	0.541	0.580	10-20%	0.511	0.562	10-20%	0.527	0.574
20-30%	0.544	0.562	20-30%	0.490	0.559	20-30%	0.519	0.555
30-40%	0.526	0.546	30-40%	0.505	0.572	30-40%	0.491	0.532
40-50%	0.520	0.577	40-50%	0.525	0.560	40-50%	0.505	0.576
50-60%	0.517	0.559	50-60%	0.515	0.558	50-60%	0.550	0.556
60-70%	0.507	0.537	60-70%	0.527	0.541	60-70%	0.490	0.533
70-80%	0.504	0.515	70-80%	0.527	0.519	70-80%	0.498	0.533
80-90%	0.499	0.515	80-90%	0.523	0.523	80-90%	0.525	0.486
Smallest 10%	0.472	0.475	Lowest 10%	0.538	0.513	Lowest 10%	0.500	0.510

Table 3 Summary Statistics of Enterprise Gross Returns

	No.of Obs.	Mean	Std Dev.	Min.	Max.
Wheat	655	9.69%	6.32%	0.00%	55.60%
Corn	335	23.72%	15.05%	1.56%	116.62%
Sorghum	555	11.35%	7.85%	0.02%	69.06%
Soybeans	514	15.13%	10.82%	0.00%	104.35%
Hay	448	26.61%	20.78%	0.15%	125.01%
Beef	375	19.27%	11.74%	0.55%	94.80%
Swine	54	33.29%	22.31%	0.13%	111.95%

Table 4 Variance-Covariance Matrix of Enterprise Gross Returns[a]

	Wheat	Corn	Sorghum	Soybeans	Hay	Beef	Swine
Wheat	σ^2_{Wheat}	48.183	18.166	31.711	55.987	39.132	-13.927
		(288)	(544)	(461)	(428)	(355)	(52)
Corn	48.183	σ^2_{Corn}	18.224	68.266	92.655	67.688	-45.977
	(288)		(210)	(302)	(206)	(149)	(30)
Sorghum	18.166	18.224	$\sigma^2_{\text{Sorghum}}$	42.974	71.020	28.122	20.970
	(544)	(210)		(402)	(371)	(315)	(44)
Soybeans	31.711	68.266	42.974	σ^2_{Soy}	98.811	52.163	66.791
	(461)	(302)	(402)		(332)	(266)	(47)
Hay	55.987	92.655	71.020	98.811	σ^2_{Hay}	81.834	174.205
	(428)	(206)	(371)	(332)		(313)	(32)
Beef	39.132	67.688	28.122	52.163	81.834	σ^2_{Beef}	-1.810
	(355)	(149)	(315)	(266)	(313)		(23)
Swine	-13.927	-45.977	20.970	66.791	174.205	-1.810	σ^2_{Swine}
	(52)	(30)	(44)	(47)	(32)	(23)	

$$\sigma_{Wheat} = \begin{cases} 11.190 - 0.022 A_{Wheat} & \text{if } A_{Wheat} < 268.159 \\ 5.189 & \text{if } A_{Wheat} \geq 268.159 \end{cases}$$

$$\ln \sigma_{Hay} = 3.116 - 0.003 A_{Hay}$$

$$\sigma_{Corn} = \begin{cases} 16.401 - 0.007 A_{Corn} & \text{if } A_{Corn} < 1140.022 \\ 8.382 & \text{if } A_{Corn} \geq 1140.022 \end{cases}$$

$$\sigma_{Beef} = \begin{cases} 21.022 - 0.084 A_{Beef} & \text{if } A_{Beef} < 134.210 \\ 9.713 & \text{if } A_{Beef} \geq 134.210 \end{cases}$$

$$\ln \sigma_{Sorghum} = \begin{cases} 4.088 - 0.419 \ln A_{Sorghum} & \text{if } A_{Sorghum} > 33.075 \\ 2.736 & \text{if } A_{Sorghum} \leq 33.075 \end{cases}$$

$$\sigma_{Swine} = 28.811 - 0.334 A_{Swine}$$

$$\sigma_{Soy} = \begin{cases} 16.573 - 0.006 A_{Soy} & \text{if } A_{Soy} > 875.522 \\ 0.520 + 0.012 A_{Soy} & \text{if } 424.041 < A_{Soy} \leq 875.522 \\ 15.224 - 0.023 A_{Soy} & \text{if } A_{Soy} \leq 424.041 \end{cases}$$

[a] Numbers in parentheses are numbers of observations used in the estimation. The variance term functions are estimated from decile-averages (quintile averages for swine) of standard deviations in percentages (σ) and assets allocated to enterprises in 1,000 dollars (A).

Table 5 Correlation Among Farm Size Variables

	Total Crop Acres	Gross Revenue, Inventory Adjusted	Total Capital Managed
Total Crop Acres	1	0.4420	0.7152
Gross Revenue, Inventory Adjusted		1	0.7498
Total Capital Managed			1

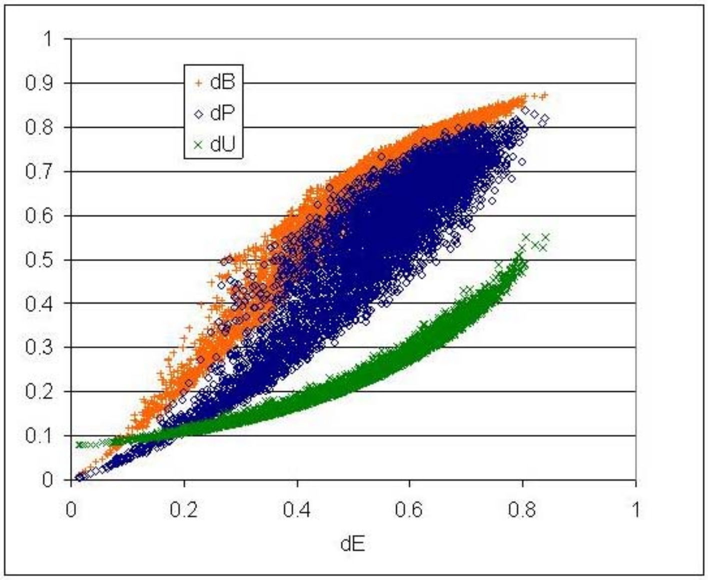
Table 6 Summary Statistics of Variables Used in the Panel Regression (Num. of Obs. = 5760)

	Mean	Std. Dev.	Min.	Max.
Diversification Index	0.522	0.142	0.015	0.840
Corporation	0.086	0.281	0	1
Partnership	0.093	0.290	0	1
Age	52.36	10.76	23	87
Area1 (North Central)	0.163	0.370	0	1
Area2 (South Central)	0.201	0.401	0	1
Area3 (Southwest)	0.073	0.260	0	1
Area4 (Northeast)	0.170	0.376	0	1
Area5 (Northwest)	0.042	0.200	0	1
Area6 (Southeast)	0.351	0.477	0	1
Total Acres (1,000 acres)	1.578	1.033	0.067	9.048
Total Revenue (million $)	0.212	0.190	0.005	2.298
Total Capital Managed (million $)	1.364	0.828	0.122	6.528
Economic Total Expense Ratio	1.080	0.376	0.427	7.810

Table 7 Two-Way Fixed Effects Regression Results (Dependent Variable = d^{E})[a]

Variable	Model 1	Model 2	Model 3	Model 4
Constant	0.5644 *	0.6056 *	0.5725 *	0.5720 *
	(0.018)	(0.017)	(0.018)	(0.018)
Corporation	0.0140	0.0281 *	0.0190	0.0300 *
	(0.013)	(0.013)	(0.013)	(0.013)
Partnership	-0.0044	-0.0065	-0.0061	-0.0028
	(0.007)	(0.007)	(0.007)	(0.007)
Age	-0.0013 *	-0.0011 *	-0.0013 *	-0.0011 *
	(0.000)	(0.000)	(0.000)	(0.000)
Total Acres	0.0289 *			0.0277 *
	(0.006)			(0.009)
Total Acres^2	-0.0030 *			-0.0029 *
	(0.001)			(0.001)
Total Revenue		-0.0016		-0.2203 *
		(0.031)		(0.048)
Total Revenue^2		-0.0396 *		0.0046
		(0.016)		(0.018)
Total Capital Managed			0.0294 *	0.0403 *
			(0.008)	(0.013)
Total Capital Managed^2			-0.0044 *	-0.0020
			(0.001)	(0.002)
Expense Ratio × Total Acres	-0.0075 *			-0.0028
	(0.002)			(0.005)
Expense Ratio × Total Revenue		-0.1070 *		0.0320
		(0.028)		(0.042)
Expense Ratio × Total Capital Managed			-0.0093 *	-0.0228 *
			(0.003)	(0.009)
Number of Observations	5760	5760	5760	5760
R-squared	0.694	0.697	0.694	0.701
Adjusted R-squared	0.676	0.680	0.676	0.684

[a] 288 farms, 20 years. * signifies significance at the 5% level.



Note: dE = entropy-based (equation (1), dB = the Berry's index (equation (2)), dP = PPSR-based (equation (3)), dU = the Utton's index (equation (4)).

Figure 1 Comparison of Diversity indexes

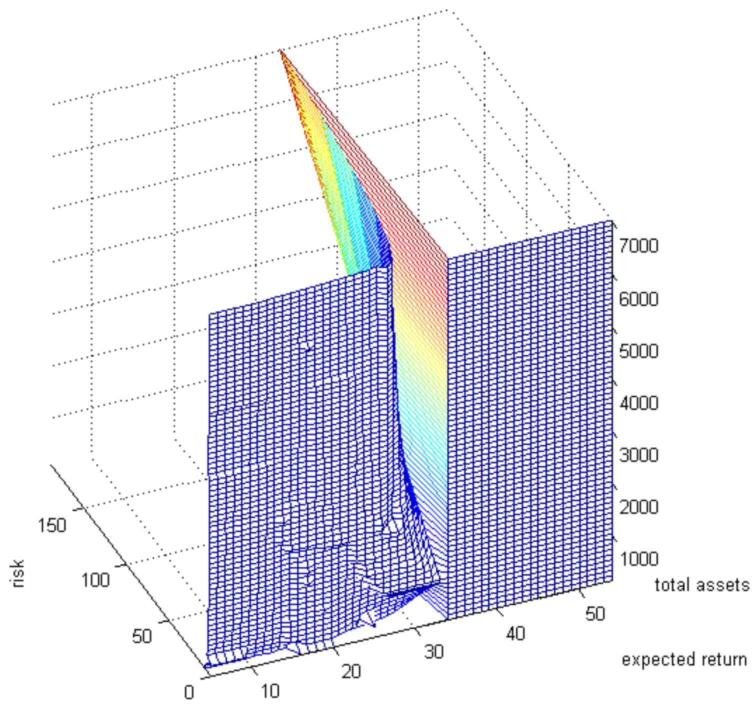


Figure 2 Efficient Farm Portfolios and Total Assets Allocated

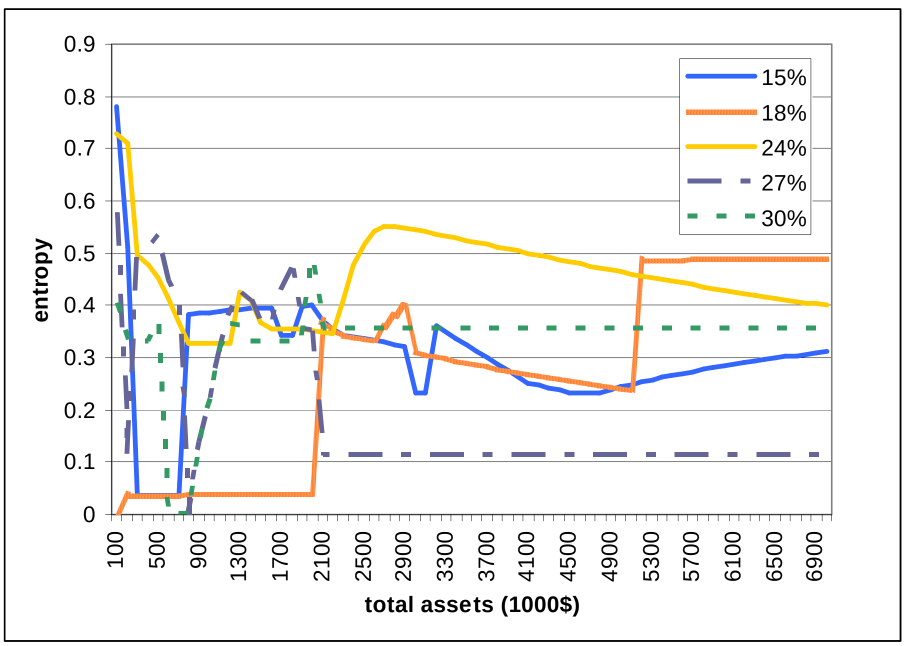


Figure 3 Normalized Entropy of Efficient Portfolios by Total Assets and Portfolio Returns

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