A Fully Calibrated Generalized CES Programming Model of Agricultural Supply

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Abstract

The use of prior information on supply elasticities to calibrate programming models of agricultural supply has been advocated repeatedly in the recent literature (Heckelei and Britz 2005). Yet, Mérel and Bucaram (2009) have shown that the dual goal of calibrating such models to a reference allocation while replicating an exogenous set of supply elasticities is not always feasible. This article lays out the methodological foundation to exactly calibrate programming models of agricultural supply using generalized CES production functions. We formally derive the necessary and sufficient conditions under which such models can be calibrated to replicate the reference allocation while displaying crop-specific supply responses that are consistent with prior information. When it exists, the solution to the exact calibration problem is unique. From a microeconomic perspective, the generalized CES model is preferable to quadratic models that have been used extensively in policy analysis since the publication of Howitt's (1995) *Positive Mathematical Programming*. The two types of specifications are also compared on the basis of their flexibility towards calibration, and it is shown that, provided myopic calibration is feasible, the generalized CES model can calibrate larger sets of supply elasticities than its quadratic counterpart. Our calibration criterion has relevance both for calibrated positive mathematical programming models and for "well-posed" models estimated through generalized maximum entropy following Heckelei and Wolff (2003), where it is deemed appropriate to include prior information regarding the value of own-price supply elasticities.

Introduction

Positive mathematical programming (PMP) models of agricultural supply that use CES production functions to specify the farming technology have been popularized by Howitt (1995a). The CES-quadratic model constitutes a natural generalization of the classic Leontief-quadratic model that allows the analyst to account for substitutability between farm inputs, while retaining much of the simplicity of the standard PMP procedure. While the initial purpose of PMP was to calibrate model parameters so that the maximization of aggregate farm returns under resource and policy constraints would replicate the observed base year allocation, more recently analysts have asked of such models that their implied supply responses be consistent with exogenous prior information (Heckelei and Britz 2005; Helming et al. 2001). The idea was to avoid selecting a set of calibrating parameters that would lead to unreasonable magnitudes for the model's implied supply elasticities. Prior information on supply elasticities typically comes from econometric estimates that implicitly take into account limitations faced by farmers, notably the land constraint (Buysse et al. 2007).[1] Thus, a PMP model of agricultural supply that incorporates these constraints should yield supply elasticities that are consistent with such prior information. Yet, Mérel and Bucaram (2009) demonstrated that the dual goal of calibrating against the base year allocation while replicating exogenously given supply elasticities is not always achievable in practice. Despite the fact, as Heckelei and Britz (2005) note, that a single-year observation on activity and input levels does not provide any information on second-order properties of the objective function, not all sets of supply elasticities are compatible with the information contained in the reference allocation. Mérel and Bucaram (2009) derived the necessary and sufficient conditions under which quadratic models, including the CES-quadratic specification of Howitt (1995a), can be calibrated against an exogenous set of supply elasticities. These conditions, referred to by these authors as the "number of crops" and the "no dominant response" rules, ensure that the base year data is compatible with the set of

[1] For instance, Russo et al. (2008) estimate supply elasticities for California commodities using a partial adjustment model, and they do not control for the price of land. As such, their elasticity estimates incorporate the land constraint.

exogenous elasticities, and provide the analyst with a clear-cut, *ex ante* test to determine whether exact calibration of the model is possible.[2]

This article extends Mérel and Bucaram's analysis to a more desirable model that we refer to as the generalized CES model. In this model, the strict concavity in the objective function arises from a decreasing returns to scale production relation rather than the addition of a quadratic adjustment cost, while the possibility of substitution between farm inputs is preserved. The change has at least three main consequences. First, the objective function is directly interpretable as the difference between a production relation and a linear cost term, as required by microeconomic theory. Second, for each activity, there is only one parameter controlling for the supply elasticity, and therefore the under-determinacy of the model is less severe than with the use of a full matrix of quadratic cost coefficients, eliminating the need for arbitrary assumptions—a popular choice is to set all off-diagonal terms to zero—or the use of maximum entropy methods (Paris and Howitt 1998).[3] Third, while the CES-quadratic model singles out one input—typically, land—as the source of decreasing returns in the production of each crop, the generalized CES treats all inputs evenly. This modeling difference has important consequences regarding the implied input allocation response to policy shocks.

The contribution of this article is three-fold. First, we derive a closed-form expression for the implied supply elasticities in the generalized CES model, which means that calibration against supply elasticities can be achieved through the resolution of a simple system with as many equations as activities. This constitutes a significant improvement over the current technique of duplicating the model's entire set of first-order conditions for *ceteris paribus* increments in the price of *each* activity, to indirectly recover the value of the model parameters consistent with the exogenous information on supply elasticities.[4] Our elasticity equations can also be easily incorporated into

[2] These conditions are stringent in practice. We applied the results of Mérel and Bucaram (2009) to Howitt's SWAP model of California agriculture, which uses the CES-quadratic specification. None of the 26 SWAP regions could be calibrated to the initially specified set of elasticities.

[3] An attendant implication is that the generalized CES model does not allow the analyst to control for the magnitude of cross-price elasticities.

"well-posed" models based on more than one observation and estimated through generalized maximum entropy (GME), whenever it is deemed appropriate to include prior information on supply elasticities (Heckelei and Wolff 2003).

Second, the availability of a closed-form elasticity equation allows us to derive the necessary and sufficient conditions under which the model can be simultaneously calibrated against the reference allocation and the exogenous set of supply elasticities. These conditions, which relate the information contained in the observed allocation to the set of supply elasticities, implicitly delineate the range of elasticities that is "compatible" with this allocation and the chosen model specification. They can easily be tested *ex ante* by the analyst to determine whether calibration is feasible. The calibration criterion is also relevant for "well-posed" models estimated through GME that incorporate prior information on supply elasticities (Heckelei and Wolff 2003). This is because such models typically require the analyst to specify a set of supports for the supply elasticities, and it is important that these supports contain elasticity values that are compatible with the "mean allocation" at which the elasticities are to be evaluated.

Third, we compare the generalized CES model and the CES-quadratic model of Howitt (1995a) on the basis of their flexibility with regard to calibration, and conclude that, subject to a caveat, the general CES model can accommodate larger sets of supply elasticities, for a given reference allocation.

The article is organized as follows. First, a "fixed proportion" variant of the generalized CES model is presented. The necessary and sufficient conditions for exact calibration are derived, and it is shown that when they are satisfied the solution to the calibration problem is unique. The relative simplicity of the derived calibration system in this simplified model allows us to interpret the calibrating equations easily, and, with little notational complexity, allows for a basic understanding of the conditions under which the calibration system has a solution. We then generalize these

[4] This method was first proposed by Heckelei (2002) in the context of generalized maximum entropy estimation, but his suggestion can be applied to calibrated models as well.

results to the case of variable proportions. Finally, we provide a comparison of the generalized CES model and the CES-quadratic model on the basis of their flexibility towards calibration and their empirical response to three simple policy experiments. All of our results are derived for the case where one linear constraint is binding, and we will interpret it as a land constraint.[5]

The fixed-proportion case

The letter I denotes the number of non-zero activities in the base year. We denote by x_i the acreage of crop i, p_i the price of crop i per unit, and C_i the per acre cost. The notation $\bar{x}_i$ is used to denote the observed land allocation, and $\bar{q}_i$ the observed output. The value of land in the reference allocation, which is usually obtained from the first-step linear programming model subject to resource and calibration constraints (Howitt 1995b), is denoted by $\bar{\lambda}_1$. The set of exogenous supply elasticities is $\bar{\boldsymbol{\eta}} = (\bar{\eta}_1, ..., \bar{\eta}_I)$, and $\bar{\boldsymbol{\eta}} >> 0$.

The optimization program is written

$$(1) \qquad \max_{x_i \geq 0} \sum_{i=1}^{I} p_i \alpha_i x_i^{\delta_i} - (C_i + \lambda_{2i}) x_i \quad \text{subject to} \quad \sum_{i=1}^{I} x_i = \bar{L}$$

where $\boldsymbol{x} = (x_1, ..., x_I)$ denotes the acreage allocation and $\bar{L}$ the available land.

In model (1), the output of activity i is $\alpha_i x_i^{\delta_i}$. The coefficients δ_i lie within the interval $(0,1)$ and are used to calibrate against the set of elasticities $\bar{\boldsymbol{\eta}}$, while the crop-specific parameters λ_{2i} are introduced to allow the model to exactly calibrate against the base year allocation $(\bar{q}_i, \bar{x}_i, \bar{\lambda}_1)$. For a given set of parameters $\delta_i \in (0,1)$, calibration against $(\bar{q}_i, \bar{x}_i, \bar{\lambda}_1)$ requires the following relationships to

[5] Given the mathematical complexity of the question, we reserve the treatment of the two-constraint case to further research. The one-constraint case is, of course, of primary empirical significance. In Howitt's SWAP model of California agriculture for instance, out of 26 regions, 23 have only one binding resource constraint.

be satisfied for all i:

$$(2) \qquad \begin{cases} p_i \bar{q}_i \delta_i = (C_i + \lambda_{2i} + \bar{\lambda}_1)\bar{x}_i \\ \alpha_i \bar{x}_i^{\delta_i} = \bar{q}_i \end{cases}$$

which determines the parameters α_i and λ_{2i} as functions of the reference allocation and δ_i.

Following the procedure described in Mérel and Bucaram (2009), we can derive the supply elasticity of crop i implied by model (1) as[6]

$$\eta_i = \frac{\delta_i}{1-\delta_i} \left[1 - \frac{\frac{\bar{x}_i^2}{p_i \bar{q}_i \delta_i (1-\delta_i)}}{\sum_{j=1}^{I} \frac{\bar{x}_j^2}{p_j \bar{q}_j \delta_j (1-\delta_j)}} \right]$$

an expression that shows that the implied elasticities depend on the base year allocation and the parameters δ_i, but not on the parameters α_i or λ_{2i}. Calibration against the exogenous supply elasticities may thus be conducted independently of the calibration against the base year allocation. Defining $b_i = \frac{\bar{x}_i^2}{p_i \bar{q}_i}$, the corresponding calibration system can be written

$$(3) \qquad \forall\, i = 1, ..., I \qquad \bar{\eta}_i = \frac{\delta_i}{1-\delta_i} \left[1 - \frac{\frac{b_i}{\delta_i (1-\delta_i)}}{\sum_{j=1}^{I} \frac{b_j}{\delta_j (1-\delta_j)}} \right].$$

In equation (3), the second term in the bracket captures the effect of the change in the shadow value of land induced by the change in the price of crop i. To see why, first note that the "myopic" value of parameter δ_i, that is, the one that obtains if the change in the shadow price of land is ignored, is simply $\delta_i^{\text{myopic}} = \frac{\bar{\eta}_i}{1+\bar{\eta}_i}$, a number that lies automatically between zero and one. As such, the factor $\frac{\delta_i}{1-\delta_i}$ in (3) represents the supply elasticity of crop i, *holding the price of land constant*. The second term in the bracket thus reflects the adjustment to this implied elasticity necessary to take account of the fact that the shadow price of land λ_1 changes with p_i. We show in the appendix that the terms $\frac{b_j}{\delta_j (1-\delta_j)}$ represent the (opposite of the) acreage reactivity of crop j to a rise in the price of land,

[6] See the appendix for the derivation.

keeping all other prices (including output price) constant. The adjustment term in (3) thus involves the ratio of the acreage reactivity of crop i to the sum of the acreage reactivities of all crops. That the adjustment term should be proportional to the acreage reactivity of crop i to the price of land is intuitive, since the term adjusts for the fact that the "myopic" elasticity $\frac{\delta_i}{1-\delta_i}$ ignores the change in λ_1. This acreage reactivity is deflated by the sum of all acreage reactivities, a quantity that we show is inversely related to the magnitude of the change in λ_1. Therefore, the adjustment term can be interpreted as the product of the acreage reactivity of crop i, keeping p_i constant, multiplied by a measure of the change in λ_1 arising from the change in p_i.

Denote $\bar{\omega}_i = b_i\bar{\eta}_i = \frac{\bar{x}_i^2\bar{\eta}_i}{p_i\bar{q}_i}$. We shall now state and prove the first proposition of this article, that identifies the necessary and sufficient condition under which model (1) can be calibrated against the base year allocation $(\bar{q}_i, \bar{x}_i, \bar{\lambda}_1)$ while replicating the exogenous set of supply elasticities $\bar{\boldsymbol{\eta}}$. Since the subsystem (2) has a solution no matter the value of δ_i in $(0,1)$, calibration will be feasible whenever system (3) has an acceptable solution, that is, a solution $\boldsymbol{\delta} = (\delta_1, ..., \delta_I)$ such that $\delta_i \in (0,1)$ for all $i = 1, ..., I$.

Proposition 1 *Suppose that $I \geq 2$. Then, the calibration system (3) has a solution in the acceptable range $(0,1)^I$ if and only if*

$$(4) \qquad \forall\, i = 1, ..., I \quad \bar{\omega}_i < \sum_{j\neq i} \bar{\omega}_j \left(1 + \frac{1}{\bar{\eta}_j}\right)^2.$$

When this condition is satisfied, the set of calibrating parameters $\boldsymbol{\delta}$ is unique and satisfies $\delta_i \geq \delta_i^{myopic}$ for all $i = 1, ..., I$.

<u>Proof</u>. *Necessity*. Suppose that a solution to system (3) exists that satisfies $\delta_i \in (0,1)$ for all i, and denote $S = \sum_{j=1}^{I} \frac{b_i}{\delta_i(1-\delta_i)}$. Then, we have

$$\bar{\omega}_i = \frac{b_i\delta_i}{1-\delta_i}\left[\frac{\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)}}{S}\right]$$

while

$$\sum_{j\neq i}\bar{\omega}_j\left(1+\frac{1}{\bar{\eta}_j}\right)^2 = \frac{1}{S}\sum_{j\neq i}\frac{b_j\delta_j}{1-\delta_j}\left(\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)}\right)\left[\frac{1}{\delta_j}+\left(\frac{1-\delta_j}{\delta_j}\right)\left(\frac{\frac{b_j}{\delta_j(1-\delta_j)}}{\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)}}\right)\right]^2$$
$$= \frac{1}{S}\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)}\left(\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)}\right)\left[1+\left(1-\delta_j\right)\left(\frac{\frac{b_j}{\delta_j(1-\delta_j)}}{\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)}}\right)\right]^2.$$

Since $\delta_i \in (0,1)$, it is apparent from these expressions that condition (4) must hold.

Sufficiency. Suppose that condition (4) is satisfied. Starting from the expression in (3), we can unambiguously solve for δ_i, which yields the following calibrating equation for activity i:

$$(5)\qquad \delta_i = \frac{\bar{\eta}_i}{2(1+\bar{\eta}_i)}\left[1+\sqrt{1+\frac{4b_i\left(1+\frac{1}{\bar{\eta}_i}\right)}{\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)}}}\right]$$

an expression that clearly shows that any acceptable solution $\boldsymbol{\delta}$ to system (3) has to satisfy $\delta_i \geq \frac{\bar{\eta}_i}{1+\bar{\eta}_i}$ for all i. Since the function $\delta_j \mapsto \delta_j(1-\delta_j)$ is bell-shaped on the interval $(0,1)$, with its maximum at $\delta_j = \frac{1}{2}$, for $\delta_j \in [\frac{\bar{\eta}_j}{1+\bar{\eta}_j},1)$ we have that $\frac{b_j}{\delta_j(1-\delta_j)} \geq 4b_j$. This, in turn, implies that when $\delta_j \in [\frac{\bar{\eta}_j}{1+\bar{\eta}_j},1)$ for all $j \neq i$ the right-hand side of equation (5) is always smaller than or equal to the positive number

$$\zeta_i = \frac{\bar{\eta}_i}{2(1+\bar{\eta}_i)}\left[1+\sqrt{1+\frac{4b_i\left(1+\frac{1}{\bar{\eta}_i}\right)}{\sum_{j\neq i}4b_j}}\right].$$

Now denote $\Delta_i = [\frac{\bar{\eta}_i}{1+\bar{\eta}_i},\zeta_i]$, $\Delta = \prod_{i=1}^{I}\Delta_i$ and define the following function:

$$\begin{array}{rcl}\phi_i:\ \Delta & \to & \Delta_i \\ \boldsymbol{\delta}=(\delta_1,\ldots,\delta_I) & \mapsto & \phi_i(\boldsymbol{\delta}) = \begin{cases}\frac{\bar{\eta}_i}{2(1+\bar{\eta}_i)}\left[1+\sqrt{1+\frac{4b_i\left(1+\frac{1}{\bar{\eta}_i}\right)}{\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)}}}\right] & \text{if } \forall j\neq i\ \delta_j<1 \\ \frac{\bar{\eta}_i}{1+\bar{\eta}_i} & \text{otherwise}\end{cases}\end{array}.$$

Clearly, the range of ϕ_i is included in Δ_i. The function ϕ_i is also continuous on its entire domain, including points $\boldsymbol{\delta}$ such that $\exists\ j \neq i$ s.t. $\delta_j = 1$, because $\lim_{\substack{\delta_j \to 1 \\ \delta_j < 1}} \frac{b_j}{\delta_j(1-\delta_j)} = +\infty$. Let

$$\begin{array}{rccc} \boldsymbol{\phi}: & \Delta & \to & \Delta \\ & \boldsymbol{\delta} & \mapsto & \boldsymbol{\phi}(\boldsymbol{\delta}) = (\phi_1(\boldsymbol{\delta}), ..., \phi_I(\boldsymbol{\delta})) \end{array}.$$

The function $\boldsymbol{\phi}$ is continuous on the compact subset Δ, and Δ is stable by $\boldsymbol{\phi}$. By Brouwer's fixed point theorem, $\boldsymbol{\phi}$ has a fixed point.

It remains to be shown that there is one fixed point in the set $\prod_{i=1}^{I}[\frac{\bar{\eta}_i}{1+\bar{\eta}_i}, 1)$. Suppose first that $\tilde{\boldsymbol{\delta}}$ is a fixed point of $\boldsymbol{\phi}$ in Δ with more than one element $\tilde{\delta}_i$ greater than or equal to one. Then, for i_0 such that $\tilde{\delta}_{i_0} \geq 1$, by the definition of ϕ_i it must be that $\phi_{i_0}(\tilde{\boldsymbol{\delta}}) = \frac{\bar{\eta}_{i_0}}{1+\bar{\eta}_{i_0}} < 1$, which contradicts the fact that $\tilde{\boldsymbol{\delta}}$ is a fixed point of $\boldsymbol{\phi}$. Now suppose that one and only one component $\tilde{\delta}_{i_0}$ of a fixed point $\tilde{\boldsymbol{\delta}}$ is greater than or equal to one. By the definition of ϕ_i, the image of $\tilde{\boldsymbol{\delta}}$ by $\boldsymbol{\phi}$ is

$$\boldsymbol{\phi}(\tilde{\boldsymbol{\delta}}) = \left(\frac{\bar{\eta}_1}{1+\bar{\eta}_1}, ..., \frac{\bar{\eta}_{i_0-1}}{1+\bar{\eta}_{i_0-1}}, \frac{\bar{\eta}_{i_0}}{2(1+\bar{\eta}_{i_0})} \left[1 + \sqrt{1 + \frac{4b_{i_0}\left(1+\frac{1}{\bar{\eta}_{i_0}}\right)}{\sum_{j \neq i_0} \frac{b_j}{\tilde{\delta}_j(1-\tilde{\delta}_j)}}} \right], \frac{\bar{\eta}_{i_0+1}}{1+\bar{\eta}_{i_0+1}}, ..., \frac{\bar{\eta}_I}{1+\bar{\eta}_I} \right)$$

and since $\boldsymbol{\phi}(\tilde{\boldsymbol{\delta}}) = \tilde{\boldsymbol{\delta}}$ the component $\tilde{\delta}_{i_0}$ must equal

$$\tilde{\delta}_{i_0} = \frac{\bar{\eta}_{i_0}}{2(1+\bar{\eta}_{i_0})} \left[1 + \sqrt{1 + \frac{4b_{i_0}\left(1+\frac{1}{\bar{\eta}_{i_0}}\right)}{\sum_{j \neq i_0} b_j \bar{\eta}_j \left(1+\frac{1}{\bar{\eta}_j}\right)^2}} \right].$$

But the premise (4) ensures that this last expression is strictly smaller than one, which contradicts the fact that $\tilde{\delta}_{i_0} \geq 1$. Therefore, any fixed point of $\boldsymbol{\phi}$ in Δ has to lie within the set $\prod_{i=1}^{I}[\frac{\bar{\eta}_i}{1+\bar{\eta}_i}, 1)$, which completes the existence proof.

Uniqueness. We here provide a proof for the case $I = 2$. The case $I = 3$ is treated in the appendix.[7] When $I = 2$, system (3) can be solved analytically to obtain the following two candidate solutions[8]

$$\begin{cases} \delta_1^+ = \frac{\bar{\eta}_1 + \sqrt{\frac{b_1}{b_2}\bar{\eta}_1\bar{\eta}_2}}{1+\bar{\eta}_1+\bar{\eta}_2} \\ \delta_2^+ = \frac{\bar{\eta}_2 + \sqrt{\frac{b_2}{b_1}\bar{\eta}_1\bar{\eta}_2}}{1+\bar{\eta}_1+\bar{\eta}_2} \end{cases} \quad \text{and} \quad \begin{cases} \delta_1^- = \frac{\bar{\eta}_1 - \sqrt{\frac{b_1}{b_2}\bar{\eta}_1\bar{\eta}_2}}{1+\bar{\eta}_1+\bar{\eta}_2} \\ \delta_2^- = \frac{\bar{\eta}_2 - \sqrt{\frac{b_2}{b_1}\bar{\eta}_1\bar{\eta}_2}}{1+\bar{\eta}_1+\bar{\eta}_2} \end{cases}.$$

Since $\delta_1^- > 0 \Rightarrow \delta_2^- < 0$, it is clear that (δ_1^-, δ_2^-) lies outside the acceptable set. Therefore, system (3) has at most one acceptable solution.

Proposition 1 establishes the condition under which an exogenous set of elasticities $\bar{\boldsymbol{\eta}}$ is compatible with a given observed allocation. Condition (4) implicitly delineates a subregion of $\mathbb{R}^I_{++}$ within which the vector $\bar{\boldsymbol{\eta}}$ should lie for calibration to be possible. Such a region is depicted in Figure 1 for the case $I = 2$ and $b_1 = b_2 = 1$. If $\bar{\boldsymbol{\eta}}$ lies outside of this region, exact calibration is not technically feasible. Yet, depending on the "extent" of the violation and the degree of confidence the analyst has in the set of elasticities, it may be possible to modify the elasticities so as to meet the criterion. Suppose, for instance, that the prior on elasticities consists of a set of confidence intervals $E_i = [\bar{\eta}_i^{\min}, \bar{\eta}_i^{\max}]$, such that the calibration criterion (4) is violated for $\bar{\boldsymbol{\eta}} = (\frac{\bar{\eta}_1^{\min}+\bar{\eta}_1^{\max}}{2}, ..., \frac{\bar{\eta}_I^{\min}+\bar{\eta}_I^{\max}}{2})$. Assume further that the set of vectors $\boldsymbol{\eta} \in \prod_i E_i$ for which (4) is satisfied is nonempty. A reasonable way of calibrating the model would be to first solve the GME program

$$\max_{0 \le w_{ik} \le 1} -\sum_{i=1}^{I}\sum_{k=1}^{2} w_{ik} \ln w_{ik} \quad \text{subj. to} \quad \forall i = 1, ..., I \begin{cases} w_{i1} + w_{i2} = 1 \\ \eta_i = w_{i1}\bar{\eta}_i^{\min} + w_{i2}\bar{\eta}_i^{\max} \\ b_i\eta_i \le \sum_{j \ne i} b_j\eta_j\left(1 + \frac{1}{\eta_j}\right)^2 \end{cases}.$$

Then, denoting the corresponding solution $(\tilde{\eta}_1, ..., \tilde{\eta}_I)$, one could choose as the set of exogenous elasticities the vector $(\tilde{\eta}_1, ..., \tilde{\eta}_{i_0-1}, \tilde{\eta}_{i_0} - \varepsilon, \tilde{\eta}_{i_0+1}, ..., \tilde{\eta}_I)$, where i_0 denotes the index of the crop for

[7] We were not able to formally establish uniqueness in the general case. We confirmed through numerical simulations in MATLAB that the uniqueness result very likely holds for larger values of I. More specifically, for each value of $I \in \{4, ..., 12\}$, we calculated the determinant of the calibration system for 10,000 draws of parameters values and showed that its sign was constant on the acceptable range, which by the Index theorem implies that there is at most one solution to the system.

[8] These solutions can be obtained, for instance, using the algebraic capabilities of MATHEMATICA 6.

which the condition $b_i\tilde{\eta}_i \leq \sum_{j\neq i} b_j\tilde{\eta}_j\left(1+\frac{1}{\tilde{\eta}_j}\right)^2$ is satisfied with equality, for an arbitrarily small $\varepsilon > 0$. Therefore, Proposition 1 should not be construed as a rigid sentence that seals the fate of the model, but rather as a guide to making calibration possible at the lowest cost in terms of deviation from prior information.

The generalized CES model

Here, we still consider the case where one constraint (say, land) is binding, but there is substitution between land and other farming inputs. There are L inputs, and we denote by x_{il} the quantity of input l allocated to the production of crop i. Land is the first input and is thus denoted x_{i1}. The market price of input l is denoted c_l.[9] The reference allocation is denoted $(\bar{q}_i, \bar{x}_{il}, \bar{\lambda}_1)$.

The allocation program is

$$\max_{x_{il}\geq 0} \quad \sum_{i=1}^{I} p_i\alpha_i\left(\sum_l \beta_{il}x_{il}^{\rho_i}\right)^{\frac{\delta_i}{\rho_i}} - (c_1+\lambda_{2i})x_{i1} - \sum_{l\geq 2} c_l x_{il}$$

$$\text{subject to} \quad \sum_{i=1}^{I} x_{i1} = \bar{L} \tag{6}$$

where $\rho_i = \frac{\sigma_i - 1}{\sigma_i}$, σ_i denoting the elasticity of substitution between inputs in the production of crop i . In program (6), this elasticity of substitution is given, while the parameters $(\alpha_i, \beta_{il}, \lambda_{2i}, \delta_i)$ are chosen by the analyst to replicate the observed base year allocation $(\bar{q}_i, \bar{x}_{il}, \bar{\lambda}_1)$ and the set of exogenous supply elasticities $\bar{\boldsymbol{\eta}}$. Following the practice initiated by Howitt (1995a), we introduced the calibrating parameter λ_{2i} as a crop-specific increment to the price of land, rather than other inputs.[10]

[9] With this notation, the variable c_1 represents the observed land rent.

[10] This choice is somewhat arbitrary and constitutes the only element of under-determinacy left in this fully calibrated generalized CES model. We can, however, heuristically defend this choice by noting that in this model with one resource constraint, the shadow value of land $\bar{\lambda}_1$ is the element of the reference allocation that is the most subject to criticism, because it is typically not observed, but obtained from the first stage of the PMP procedure. Adding the parameters λ_{2i} to the land cost implies that if one changes the value of $\bar{\lambda}_1$, the values of the parameters λ_{2i} will adjust accordingly, so that the sum $\bar{\lambda}_1 + \lambda_{2i}$ will remain the same, and the other model parameters (δ_i, α_i and β_{il}) will be unaffected by this change. For completeness, we have derived conditions for calibration in the case where the parameter λ_{2i} appears as an increment to the price of an input other than land. The conditions are available upon request to the authors.

Using the procedure in Mérel and Bucaram (2009), the implied model elasticity for activity i can be derived as[11]

$$\eta_i = \frac{\delta_i}{1-\delta_i}\left[1 - \frac{\frac{b_i}{\delta_i(1-\delta_i)}}{\sum_{j=1}^{I}\frac{b_j}{\delta_j(1-\delta_j)} - \frac{\sigma_j b_j}{\delta_j(1-\frac{p_j\bar{q}_j}{\sum_{l\geq 2} c_l \bar{x}_{jl}}\delta_j)}}\right]$$

where as before $b_i = \frac{\bar{x}_{i1}^2}{p_i\bar{q}_i}$. The second term in the bracket represents the effect of the induced change in the shadow value of the land constraint when the price of crop i increases. The myopic value of δ_i in the generalized CES model (6) is thus the same as in the fixed proportion model (1), $\delta_i^{\text{myopic}} = \frac{\bar{\eta}_i}{1+\bar{\eta}_i}$. However, successful calibration against the observed allocation now requires that for all $i = 1,...,I$, $p_i\bar{q}_i\delta_i > \sum_{l\geq 2} c_l\bar{x}_{il}$. To see why, consider the following calibration conditions for program (6), conditional on the choice of δ_i:

$$(7)\quad \begin{cases} p_i\alpha_i\delta_i\left(\sum_l \beta_{il}\bar{x}_{il}^{\rho_i}\right)^{\frac{\delta_i}{\rho_i}-1}\beta_{i1}\bar{x}_{i1}^{\rho_i-1} = c_1 + \bar{\lambda}_1 + \lambda_{2i} \\ p_i\alpha_i\delta_i\left(\sum_l \beta_{il}\bar{x}_{il}^{\rho_i}\right)^{\frac{\delta_i}{\rho_i}-1}\beta_{il}\bar{x}_{il}^{\rho_i-1} = c_l \quad l = 2,...,L \\ \bar{q}_i = \alpha_i\left(\sum_l \beta_{il}\bar{x}_{il}^{\rho_i}\right)^{\frac{\delta_i}{\rho_i}} \\ \sum_l \beta_{il} = 1 \\ \beta_{il} > 0,\ l = 1,...,L \\ \alpha_i > 0 \end{cases}.$$

The first three conditions in (7) represent the optimality conditions of program (6), evaluated at the reference allocation $(\bar{q}_i, \bar{x}_{il}, \bar{\lambda}_1)$, while the last three conditions reflect standard parameter restrictions. Together, they implicitly define the value of the parameters α_i, β_{il} and λ_{2i} that are consistent with the reference allocation, conditional on the choice of δ_i. Rearranging, one can express the value of parameter λ_{2i} as a sole function of the base year allocation and the value of δ_i,

$$\lambda_{2i} = \frac{1}{\bar{x}_{i1}}\left[p_i\bar{q}_i\delta_i - \sum_{l\geq 2} c_l\bar{x}_{il} - (c_1 + \bar{\lambda}_1)\bar{x}_{i1}\right]$$

[11] The derivation mirrors that of the elasticity equation for model (1), and is available upon request to the authors.

so that $c_1 + \bar{\lambda}_1 + \lambda_{2i} = \frac{1}{\bar{x}_{i1}} \left(p_i \bar{q}_i \delta_i - \sum_{l \geq 2} c_l \bar{x}_{il} \right)$. Considering the first, fifth and sixth conditions in system (7), it is then clear that the condition $p_i \bar{q}_i \delta_i > \sum_{l \geq 2} c_l \bar{x}_{il}$ must be satisfied in order for the model to replicate the reference allocation. Defining $\theta_i = \frac{p_i \bar{q}_i}{\sum_{l \geq 2} c_l \bar{x}_{il}} > 1$, an acceptable set of calibrating parameters $\boldsymbol{\delta}$ must therefore satisfy

$$\forall\, i = 1, \ldots, I \quad \delta_i \in (\frac{1}{\theta_i}, 1).$$

In particular, if the myopic parameter δ_i^{myopic} is used, we must have $\frac{\bar{\eta}_i}{1+\bar{\eta}_i} > \frac{1}{\theta_i}$, a condition equivalent to

$$\text{(8)} \quad \forall\, i = 1, \ldots, I \quad \bar{\eta}_i > \frac{1}{\theta_i - 1}.$$

Using the definition of θ_i, the exact calibration system can be written

$$\text{(9)} \quad \forall i = 1, \ldots, I \quad \bar{\eta}_i = \frac{\delta_i}{1 - \delta_i} \left[1 - \frac{\frac{b_i}{\delta_i (1-\delta_i)}}{\sum_{j=1}^{I} \frac{b_j}{\delta_j (1-\delta_j)} + \frac{\sigma_j b_j}{\delta_j (\theta_j \delta_j - 1)}} \right].$$

We now state and prove the main proposition of this article, which defines the conditions under which exact calibration of the generalized CES model (6) is feasible.[12]

Proposition 2 *Let $I \geq 2$, and suppose that condition (8) holds. The calibration system (9) has a solution in the acceptable range $\prod_i \left(\frac{1}{\theta_i}, 1 \right)$ if and only if, for all $i = 1, \ldots, I$, the following condition is satisfied*

$$\text{(10)} \quad \bar{\omega}_i \left(1 - \frac{\sigma_i}{\bar{\eta}_i (\theta_i - 1)} \right) < \sum_{j \neq i} \bar{\omega}_j \left(1 + \frac{1}{\bar{\eta}_j} \right)^2 \left(1 + \frac{\sigma_j}{\bar{\eta}_j (\theta_j - 1) - 1} \right).$$

When condition (10) is satisfied, the set of calibrating parameters $\boldsymbol{\delta}$ is unique and satisfies $\delta_i \geq \delta_i^{myopic}$ for all $i = 1, \ldots, I$.

[12] We did not attempt to prove uniqueness for the generalized CES specification.

Proof. *Necessity*. Suppose that condition (8) holds, and that there exists a set of parameters $\delta_i \in (\frac{1}{\theta_i}, 1)$ that solve system (9). Denoting $S_\sigma = \sum_{k=1}^{I} \frac{b_k}{\delta_k(1-\delta_k)} + \frac{b_k\sigma_k}{\delta_k(\theta_k\delta_k-1)}$ and using the equality in (9), we have

$$\bar{\omega}_i \left(1 - \frac{\sigma_i}{\bar{\eta}_i(\theta_i - 1)}\right) = \frac{1}{S_\sigma}\frac{b_i\delta_i}{1-\delta_i}\left[\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)} + \sum_j \frac{b_j\sigma_j}{\delta_j(\theta_j\delta_j - 1)} - \left(\frac{\sigma_i}{\theta_i - 1}\right)\left(\frac{1-\delta_i}{\delta_i}\right)S_\sigma\right].$$

The condition that $\bar{\eta}_i > \frac{1}{\theta_i - 1}$ implies that $\frac{1}{S_\sigma}\frac{b_i}{\delta_i(1-\delta_i)} < 1 - \frac{1-\delta_i}{\delta_i(\theta_i-1)}$, which in turn implies that $\frac{b_i\sigma_i}{\delta_i(\theta_i\delta_i-1)} < \frac{\sigma_i S_\sigma}{\theta_i-1}\frac{1-\delta_i}{\delta_i}$. Therefore, we have that

$$(11) \quad \bar{\omega}_i \left(1 - \frac{\sigma_i}{\bar{\eta}_i(\theta_i - 1)}\right) < \frac{1}{S_\sigma}\frac{b_i\delta_i}{1-\delta_i}\left[\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)} + \sum_{j\neq i} \frac{b_j\sigma_j}{\delta_j(\theta_j\delta_j - 1)}\right].$$

Using (9), we also have

$$\begin{aligned}
\bar{\omega}_j\left(1+\frac{1}{\bar{\eta}_j}\right)^2 &= \frac{1}{S_\sigma}\frac{b_j\delta_j}{1-\delta_j}\left[\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)} + \sum_k \frac{b_k\sigma_k}{\delta_k(\theta_k\delta_k-1)}\right] \\
&\quad \times \left[\frac{1}{\delta_j} + \frac{\frac{b_j}{\delta_j{}^2}}{\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)} + \sum_k \frac{b_k\sigma_k}{\delta_k(\theta_k\delta_k-1)}}\right]^2 \\
&= \frac{1}{S_\sigma}\frac{b_j}{\delta_j(1-\delta_j)}\left[\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)} + \sum_k \frac{b_k\sigma_k}{\delta_k(\theta_k\delta_k-1)}\right]\left[1+R_j\right]^2
\end{aligned}$$

with $R_j > 0$ and thus

$$\begin{aligned}
\bar{\omega}_j\left(1+\frac{1}{\bar{\eta}_j}\right)^2\left(1+\frac{\sigma_j}{\bar{\eta}_j(\theta_j-1)-1}\right) &= \frac{1}{S_\sigma}\frac{b_j}{\delta_j(1-\delta_j)}\left[\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)} + \sum_k \frac{b_k\sigma_k}{\delta_k(\theta_k\delta_k-1)}\right] \\
&\quad \times \left[1+R_j\right]^2\left[1+T_j\right]
\end{aligned}$$

where $T_j = \frac{\sigma_j}{\bar{\eta}_j(\theta_j - 1) - 1} > 0$. It is clear that the term $\frac{1}{S_\sigma}\frac{b_i\delta_i}{1-\delta_i}\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)}$ in (11) is dominated by the sum $\sum_{j\neq i}\frac{1}{S_\sigma}\frac{b_j}{\delta_j(1-\delta_j)}\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)}$ since $\delta_i \in (0,1)$. In addition, since $\theta_j > 1$, we can write

$$\begin{aligned} T_j &= \frac{\sigma_j(1-\delta_j)S_\sigma}{\theta_j\delta_j\left(\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)} + \sum_k \frac{b_k\sigma_k}{\delta_k(\theta_k\delta_k - 1)}\right) + \frac{b_j}{1-\delta_j} - S_\sigma} \\ &> \frac{\sigma_j(1-\delta_j)}{\theta_j\delta_j - 1} \end{aligned}$$

so that the term $\frac{1}{S_\sigma}\frac{b_i\delta_i}{1-\delta_i}\sum_{j\neq i}\frac{b_j\sigma_j}{\delta_j(\theta_j\delta_j - 1)}$ in (11) is dominated by the term $\sum_{j\neq i}\frac{1}{S_\sigma}\frac{b_jT_j}{\delta_j(1-\delta_j)}\sum_{k\neq j}\frac{b_k}{\delta_k(1-\delta_k)}$. Q.E.D.

Sufficiency. Suppose that condition (10) is satisfied. Multiplying both sides of the equation in (9) by the quantity $(1-\delta_i)\sum_{j=1}^{I}\frac{b_j}{\delta_j(1-\delta_j)} + \frac{\sigma_j b_j}{\delta_j(\theta_j\delta_j - 1)}$ and rearranging, the calibrating equation can be expressed as

$$\begin{aligned} &\delta_i^2\left[(1+\bar{\eta}_i)\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)} + \frac{\sigma_j b_j}{\delta_j(\theta_j\delta_j - 1)}\right] \\ &-\delta_i\bar{\eta}_i\left[\sum_{j\neq i}\left\{\frac{b_j}{\delta_j(1-\delta_j)} + \frac{\sigma_j b_j}{\delta_j(\theta_j\delta_j - 1)}\right\} - \frac{1+\bar{\eta}_i}{\bar{\eta}_i}\frac{\sigma_i b_i}{\theta_i\delta_i - 1}\right] - b_i\bar{\eta}_i\left(1 + \frac{\sigma_i}{\theta_i\delta_i - 1}\right) \quad = 0 \end{aligned}$$

which can be viewed as a polynomial equation of degree two in δ_i. (Note that δ_i still appears in the coefficients of this polynomial.) An acceptable solution to (9) must be such that $\delta_i \in (\frac{1}{\theta_i}, 1)$, and therefore the only acceptable root of this polynomial is

$$\begin{aligned} \delta_i = \quad &\frac{\bar{\eta}_i}{2(1+\bar{\eta}_i)}\left[1 - \frac{\frac{\sigma_i b_i\left(1+\frac{1}{\bar{\eta}_i}\right)}{\theta_i\delta_i - 1}}{\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)} + \frac{\sigma_j b_j}{\delta_j(\theta_j\delta_j - 1)}}\right. \\ &\left. + \sqrt{\left(1 + \frac{\frac{\sigma_i b_i\left(1+\frac{1}{\bar{\eta}_i}\right)}{\theta_i\delta_i - 1}}{\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)} + \frac{\sigma_j b_j}{\delta_j(\theta_j\delta_j - 1)}}\right)^2 + \frac{4b_i\left(1+\frac{1}{\bar{\eta}_i}\right)}{\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)} + \frac{\sigma_j b_j}{\delta_j(\theta_j\delta_j - 1)}}}\,\right]. \end{aligned} \tag{12}$$

Denote $\psi_i(\boldsymbol{\delta})$ the right-hand side of equation (12). Solving system (9) over the acceptable range $\prod_i(\frac{1}{\theta_i},1)$ is therefore equivalent to solving a system of equations of the form

(13) $\forall\, i=1,..,I \quad \delta_i=\psi_i(\boldsymbol{\delta}).$

First note that any solution to system (13) that lies in the acceptable range $\prod_i(\frac{1}{\theta_i},1)$ has to also satisfy $\delta_i \geq \frac{\bar{\eta}_i}{1+\bar{\eta}_i}$ for all i. This is obvious from the expression of $\psi_i(\boldsymbol{\delta})$. Second, note that for $\delta_j \in [\frac{\bar{\eta}_j}{1+\bar{\eta}_j},1)$, the following inequalities apply: $\frac{b_j}{\delta_j(1-\delta_j)} \geq 4b_j$ and $\frac{\sigma_j b_j}{\delta_j(\theta_j\delta_j-1)} \geq \frac{\sigma_j b_j}{\theta_j-1}$. (The function $\delta_j \mapsto \delta_j(\theta_j\delta_j-1)$ is increasing on $[\frac{\bar{\eta}_j}{1+\bar{\eta}_j},1)$ given (8).) Further, define

$$\xi_i=\frac{\bar{\eta}_i}{2(1+\bar{\eta}_i)}\left[1+\sqrt{\left(1+\frac{\frac{\sigma_i b_i\left(1+\frac{1}{\bar{\eta}_i}\right)}{\theta_i\left(\frac{\bar{\eta}_i}{1+\bar{\eta}_i}\right)-1}}{\sum_{j\neq i}4b_j+\frac{\sigma_j b_j}{\theta_j-1}}\right)^2+\frac{4b_i\left(1+\frac{1}{\bar{\eta}_i}\right)}{\sum_{j\neq i}4b_j+\frac{\sigma_j b_j}{\theta_j-1}}}\right].$$

and denote $\Lambda_i=[\frac{\bar{\eta}_j}{1+\bar{\eta}_j},\xi_i]$ and $\Lambda=\prod_{i=1}^{I}\Lambda_i$. Now consider the following function:

$$\begin{array}{rccl}\tilde{\psi}_i: & \Lambda & \rightarrow & \Lambda_i \\ & \boldsymbol{\delta} & \mapsto & \tilde{\psi}_i(\boldsymbol{\delta})=\begin{cases}\psi_i(\boldsymbol{\delta}) & \text{if } \forall j=1,...,I\ \delta_j<1 \\ \psi_i(\delta_1,...,\delta_{i-1},1,\delta_{i+1},...,\delta_I) & \text{if } \forall j\neq i\ \delta_j<1 \text{ and } \delta_i\geq 1 \\ \frac{\bar{\eta}_i}{1+\bar{\eta}_i} & \text{otherwise}\end{cases}\end{array}.$$

It is clear from the above that the range of $\tilde{\psi}_i$ is indeed included in Λ_i. The function $\tilde{\psi}_i$ is also continuous on its entire domain. In particular, it is continuous at points $\boldsymbol{\delta}$ such that $\delta_j<1$ for all $j\neq i$ and $\delta_i=1$ because the function ψ_i itself is continuous at such points. In addition, it is continuous at points $\boldsymbol{\delta}$ such that $\delta_j = 1$ for at least one $j \neq i$ because $\lim_{\substack{\delta_j \to 1 \\ \delta_j < 1}} \frac{b_j}{\delta_j(1-\delta_j)} = +\infty$. Let

$$\begin{array}{rccc} \tilde{\boldsymbol{\psi}}: & \Lambda & \to & \Lambda \\ & \boldsymbol{\delta} & \mapsto & \tilde{\boldsymbol{\psi}}(\boldsymbol{\delta}) = (\tilde{\psi}_1(\boldsymbol{\delta}), ..., \tilde{\psi}_I(\boldsymbol{\delta})) \end{array}.$$

The function $\tilde{\boldsymbol{\psi}}$ is continuous on the compact subset Λ, and Λ is stable by $\tilde{\boldsymbol{\psi}}$. By Brouwer's fixed point theorem, $\tilde{\boldsymbol{\psi}}$ has a fixed point in Λ.

It remains to be shown that given the premice (10), any fixed point of $\tilde{\boldsymbol{\psi}}$ in Λ has to lie in the acceptable set $\prod_i(\frac{1}{\theta_i}, 1)$. Given (8), it is clear that any fixed point $\tilde{\boldsymbol{\delta}}$ in Λ must satisfy $\tilde{\delta}_i > \frac{1}{\theta_i}$. Given the definition of $\tilde{\boldsymbol{\psi}}$, it is also clear that at most one component of a fixed point $\tilde{\boldsymbol{\delta}}$, say $\tilde{\delta}_{i_0}$, can be greater than or equal to one. Let us assume that this is the case. Then, given the definition of $\tilde{\psi}_i$ and the fact that $\tilde{\boldsymbol{\delta}}$ is a fixed point, we must have

$$\tilde{\psi}_{i_0}(\tilde{\boldsymbol{\delta}}) = \psi_{i_0}\left(\frac{\bar{\eta}_1}{1+\bar{\eta}_1}, ..., \frac{\bar{\eta}_{i_0-1}}{1+\bar{\eta}_{i_0-1}}, 1, \frac{\bar{\eta}_{i_0+1}}{1+\bar{\eta}_{i_0+1}}, ..., \frac{\bar{\eta}_I}{1+\bar{\eta}_I}\right).$$

But a straightforward calculation shows that the premice (10) implies that this expression must be strictly smaller than one, which contradicts the facts that $\tilde{\delta}_{i_0} \geq 1$ and $\tilde{\boldsymbol{\delta}}$ is a fixed point of $\tilde{\boldsymbol{\psi}}$. Q.E.D.

For a given reference allocation, Proposition 2 implicitly delineates a subset of $\mathbb{R}^I_{++}$ within which the vector $\bar{\boldsymbol{\eta}}$ must lie for calibration to be feasible. The discussion following Proposition 1 is relevant here, too: if the analyst has priors $E_i = [\bar{\eta}_i^{\min}, \bar{\eta}_i^{\max}]$, it may be possible to deviate from the midpoint of the intervals E_i so as to satisfy the calibration criterion, in a way that minimizes the total "information cost".

Condition (8), which we have taken as a premise to derive the necessary and sufficient conditions for the exact calibration of model (6), represents the necessary and sufficient condition under which *myopic* calibration of this model is feasible. This seems to be a reasonable prerequisite in and of itself.[13] More importantly, as shown in the following proposition, when condition (8) is violated, even though there may technically exist a solution to the calibration problem, this solution is not guaranteed to be unique. Deriving conditions for calibration under which uniqueness of the set of calibrating parameters is not guaranteed seems to have limited practical relevance, because there is no objective way of choosing among multiple sets of calibrating parameters. Therefore, we believe condition (8) ought to be satisfied for the calibration to be meaningful.

Proposition 3 *Let $I \geq 2$. If condition (8) is violated, there may be more than one acceptable solution to system (9).*

Proof. Consider the special case where $I = 3$ and $b_i = 1$, $\sigma = 0.5$ and $\theta_i = 5$ for all i. Acceptable solutions must satisfy $\delta_i \in (0.2, 1)$. When $\bar{\eta}_1 = \bar{\eta}_3 = 1$ and $\bar{\eta}_2 = .24$, condition (8) is violated and the sets $(0.517419, 0.210727, 0.517419)$ and $(0.534521, 0.22708, 0.534521)$ both solve system (9).[14]

Flexibility of the generalized CES model

We argued in the introduction that the generalized CES model, unlike the CES-quadratic specification, is fully consistent with microeconomic theory, because the objective function is directly interpretable as the difference between a well-specified revenue function and a well-specified (direct) cost function. This, alone, should constitute a sufficient reason for preferring the generalized CES model.

Here, we compare the two CES models on the basis of their flexibility with regard to calibration against exogenous sets of elasticities, in the context where there is one binding constraint (land). In other words, we ask the question: "Given a reference allocation, does one model always accommodate larger sets of elasticities than the other?" The answer is not clear-cut, but, overall,

[13] For the CES-quadratic model, Mérel and Bucaram (2009) show that exact calibration is *necessarily* infeasible if myopic calibration is infeasible. Although the generalized CES model seems to be less rigid in this respect, their finding provides a heuristic justification to focusing on the case where condition (8) is satisfied.

[14] These solutions were obtained numerically using MATHEMATICA 6.

the generalized CES model appears to be more flexible than its quadratic counterpart. The CES-quadratic specification we consider is the one described in Howitt (1995a) and Mérel and Bucaram (2009), where all off-diagonal terms are set to zero and the land input is used as the quadratic term in the nonlinear cost function. More specifically, the CES-quadratic program is written

$$\max_{x_{il}\geq 0} \quad \sum_{i=1}^{I} p_i \alpha_i \left(\sum_l \beta_{il} x_{il}^{\rho_i}\right)^{\frac{1}{\rho_i}} - (c_1 + \lambda_{2i} - \gamma_i \bar{x}_{i1}) x_{i1} - \sum_{l\geq 2} c_l x_{il} - \frac{1}{2}\gamma_i x_{i1}^2$$
$$\text{subject to} \quad \sum_{i=1}^{I} x_{i1} = \bar{L}.$$

A first advantage of the generalized CES model is that it can calibrate systems with as little as two activities, while the CES-quadratic model requires at least three positive activities (Mérel and Bucaram 2009). The following proposition establishes another advantage of the generalized CES model.

Proposition 4 *For a given base year allocation* $(\bar{q}_i, \bar{x}_{il}, \bar{\lambda}_1)$, *assuming that the following conditions hold for all* $i = 1, ..., I$ *:*

(i) $\bar{\eta}_i > \frac{1}{\theta_i - 1}$ *and*

(ii) $\frac{\bar{\eta}_i}{\sigma_i} > \frac{1}{\theta_i - 1}$,

the generalized CES model can calibrate a larger set of supply elasticities than the CES-quadratic model.

The proof follows directly from examination of the necessary and sufficient conditions in Proposition 2 of this article and in Proposition 9 of Mérel and Bucaram (2009).

Although Proposition 4 seems to establish the greater flexibility of the generalized CES specification, it relies on caveats (i) and (ii). These caveats imply that the generalized CES and CES-quadratic models, respectively, are able to accommodate *myopic* calibration.[15] Therefore, conditional on myopic calibration being feasible in both models, the generalized CES model is more flexible than its quadratic counterpart.

Empirical implementation

In this section, we calibrate a generalized CES model and a CES-quadratic model against the same reference allocation and the same set of supply elasticities, and compare their responses to three policy experiments: (i) an incremental output price increase, (ii) a non-incremental output price increase, and (iii) an input price increase.

The agricultural region we consider corresponds to the region labeled "Rest of the US" in Howitt (1995a), that is, all the US but California. The land constraint is the only binding constraint. There are three crops: cotton (C), wheat (W) and rice (R). The reference allocation is published in Howitt (1995a). We use the following set of supply elasticities: $\bar{\eta}_C = 0.47$, $\bar{\eta}_W = 0.4$ and $\bar{\eta}_R = 0.8$. There are four inputs: land (1), water (2), capital (3) and chemical inputs (4).

Calibration against the reference allocation and the set of supply elasticities yields the parameter values reported in Table 1. The results of the various policy experiments are reported in Tables 2, 3 and 4.

Tables 2 and 3 show that calibration against the set of supply elasticities is successful: the observed output responses are fully consistent with the assumed elasticity of supply for cotton. The output cross-price effects are not as consistent between the two models, particularly for rice where the reduction in output is more than twice as large in the generalized CES model as in the CES-quadratic model. Output effects when the price of chemical inputs increases (Table 3) are fairly consistent between the two models, for all three crops.

[15] Caveat (ii) corresponds to the condition $\bar{\omega}_i' > 0$ in Mérel and Bucaram (2009).

Regarding input allocations, apart from the own-price effect of the increase in c_4 on the use of chemicals, the two models yield inconsistent results. The discrepancy is most acute for the land use effect, across all policy experiments. This outcome is in fact expected: while the generalized CES specification treats all inputs equally, the CES-quadratic model singles out the land input through the quadratic land term $\gamma_i x_{i1}^2$. In fact, the concavity of the objective function in the variables x_{ik} arises solely from this quadratic land term, and therefore the onus is exclusively put on land to generate the required decreasing returns to scale. The choice of land as the source of decreasing returns has been justified heuristically in the literature by the supposed heterogeneity of land quality. This makes it difficult to give a meaningful interpretation to land use effects in this model, because "not all acres are treated equal". In contrast, the generalized CES specification exploits the concavity of the production function itself, and does not single out any particular input. It is therefore not surprising that the two models yield different results regarding input use, and particularly land use. This observation should constitute yet another reason to prefer the generalized CES model over the CES-quadratic model in future applications.

Conclusion

Although the use of exogenous supply elasticities in PMP models of agricultural supply has been advocated repeatedly in the recent literature, exact calibration of CES models against elasticities represents a challenging modeling task, because the analytical relationship between the model implied elasticities and the calibrating parameters is difficult to elucidate, and the conditions under which the set of calibrating equations has a (unique) solution are not trivial. An early answer to such difficulties was to use parameters obtained from myopic calibration, where the change in the shadow price of constrained resources is ignored (Helming et al. 2001). Heckelei (2002) later suggested duplicating the entire set of first-order conditions for an incremental change in the price of each activity and choosing the value of the calibrating parameters that force the supply response to coincide with that implied by the prior information. In addition to being demanding in terms of programming effort, this method does not enable the analyst to determine *ex ante* whether a solution to the calibration problem exists.

In this article, we provided the methodological foundation for exactly calibrating constrained generalized CES models of agricultural supply against a reference allocation and a set of exogenous supply elasticities. Using the methodology introduced by Mérel and Bucaram (2009), we derived a closed-form expression for the supply elasticity equation. We then showed that a generalized CES model can be calibrated for systems with as little as two activities, and we provided the exact calibration conditions. The conditions we derived further ensure that the set of calibrating parameters is unique.

Another contribution of this article was to compare the generalized CES specification to the CES-quadratic specification. Despite their popularity, quadratic models are not consistent with microeconomic theory and lead to conceptual issues when interpreting acreage responses (if, as is often the case, land is used as the source of decreasing returns). Derivation of the calibrating conditions for the generalized CES model showed that this latter model is more flexible than its quadratic counterpart, provided that myopic calibration is feasible in both models. Overall, our results provide support for the use of generalized CES models as a preferred alternative to quadratic specifications.

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Appendix

Elasticity equation for model (1)

Consider the increase in p_i. For activities $j \neq i$, the first-order conditions to program (1) imply that $x_j = \left(\frac{C_j+\lambda_{2j}+\lambda_1}{p_j\alpha_j\delta_j}\right)^{\frac{1}{\delta_j-1}} = X_j(\lambda_1)$. We then have

$$\begin{aligned} \frac{dX_j}{d\lambda_1} &= -\frac{1}{1-\delta_j}\left(C_j+\lambda_{2j}+\bar{\lambda}_1\right)^{\frac{2-\delta_j}{\delta_j-1}}\left(p_j\alpha_j\delta_j\right)^{\frac{1}{1-\delta_j}} \\ &= -\frac{1}{1-\delta_j}(p_j\alpha_j\delta_j)^{-1}\bar{x}_j^{2-\delta_j} \\ &= -\frac{\bar{x}_j^2}{p_j\bar{q}_j\delta_j(1-\delta_j)} \\ &= -\frac{b_j}{\delta_j(1-\delta_j)} \end{aligned}$$

where we have used the definition $b_j = \frac{\bar{x}_j^2}{p_j\bar{q}_j}$. Since the land constraint is binding, we can write x_i itself as a function X_i of λ_1, with derivative

$$(14) \quad \frac{dX_i}{d\lambda_1} = \sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)}.$$

The FOC with respect to x_i is

$$p_i\alpha_i\delta_i x_i^{\delta_i-1} - C_i - \lambda_{2i} - \lambda_1 = 0.$$

Totally differentiating with respect to λ_1 and p_i, we obtain

$$\frac{d\lambda_1}{dp_i} = \frac{\alpha_i\delta_i\bar{x}_i^{\delta_i-1}}{1-p_i\alpha_i\delta_i(\delta_i-1)\bar{x}_i^{\delta_i-2}\frac{dX_i}{d\lambda_1}} = \frac{\delta_i\bar{q}_i\bar{x}_i^{-1}}{1+p_i\bar{q}_i\bar{x}_i^{-2}\delta_i(1-\delta_i)\frac{dX_i}{d\lambda_1}} = \frac{\frac{b_i\bar{q}_i\bar{x}_i^{-1}}{1-\delta_i}}{\sum_j\frac{b_j}{\delta_j(1-\delta_j)}}$$

where we have used (14). Now, we can derive the total effect of the increase in p_i on x_i as

$$\begin{aligned} \frac{dx_i}{dp_i} &= \frac{dX_i}{d\lambda_1}\frac{d\lambda_1}{dp_i} \\ &= \left[\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)}\right]\left[\frac{\frac{b_i\bar{q}_i\bar{x}_i^{-1}}{1-\delta_i}}{\sum_j \frac{b_j}{\delta_j(1-\delta_j)}}\right]. \end{aligned}$$

This implies that the supply elasticity for activity i is

$$\eta_i = \delta_i x_i^{-1} p_i \frac{dx_i}{dp_i} = \frac{\delta_i}{1-\delta_i}\left[\frac{\sum_{j\neq i}\frac{b_j}{\delta_j(1-\delta_j)}}{\sum_j \frac{b_j}{\delta_j(1-\delta_j)}}\right] = \frac{\delta_i}{1-\delta_i}\left[1-\frac{\frac{b_i}{\delta_i(1-\delta_i)}}{\sum_j \frac{b_j}{\delta_j(1-\delta_j)}}\right].$$

Proof of uniqueness in Proposition 1 for $I = 3$

For $\boldsymbol{\delta} \in (0,1)^3$, let $\mathbf{f}(\boldsymbol{\delta}) = (f_i(\delta_i))_{i=1}^3$, where $f_i(\delta_i) = \frac{\delta_i}{1-\delta_i}\left(1-\frac{\frac{b_i}{\delta_i(1-\delta_i)}}{\sum_{j=1}^I \frac{b_j}{\delta_j(1-\delta_j)}}\right) - \eta_i$. To streamline notation, we define $y_i(\delta_i) = \frac{b_i}{\delta_i(1-\delta_i)}$ and $Y(\boldsymbol{\delta}) = \sum_{j=1}^I y_j(\delta_j)$, and now write $f_i(\boldsymbol{\delta}) = \frac{\delta_i}{1-\delta_i}\left(1-\frac{y_i(\delta_i)}{Y(\boldsymbol{\delta})}\right) - \eta_i$. Henceforth, we will suppress the arguments of $\mathbf{f}$, y_i and Y.

Our task is to show that there is a unique vector $\boldsymbol{\delta} \in (0,1)^3$ such that $\mathbf{f}(\boldsymbol{\delta}) = 0$. We will accomplish this by showing that the determinant of the Jacobian of $\mathbf{f}$, $|J\mathbf{f}(\cdot)|$, is positive on $(0,1)^3$. Uniqueness will then follow from the index theorem (Mas-Colell et al. 1995).[16]

[16] Mas-Colell et al. (1995) apply the index theorem to prove that there is a unique (normalized) price equilibrium of a regular economy with L goods. A normalized price equilibrium is a vector $p \gg 0$, with $p_L = 1$ at which the excess demand functions $z_l(p)$ of the economy are zero. A consequence of the index theorem is that if one can attach a definite sign to the Jacobian of the system of equations $(z_1(p),\ldots,z_{L-1}(p)) = 0$ at any solution point, then the equilibrium is unique (Mas-Colell et al. 1995, Prop. 17.D.2 & pg. 615). In our model, the range of $\boldsymbol{\delta}$ is $(0,1)^3$, whereas Prop. 17.D.2 is stated for equilibrium price vectors on the range $\mathbb{R}_{++}^{L-1}$. It is easy to see that we can rewrite our system of calibrating equations so that the calibrating parameters are defined on $\mathbb{R}_{++}^3$. Simply consider the change of variables $d_i = \frac{\delta_i}{1-\delta_i}$. The Jacobian of the system of calibrating equations with respect to the d_i variables then has the same sign as the Jacobian of $\mathbf{f}$.

Letting $Y_{-i} = Y - y_i$, the Jacobian of $\mathbf{f}$ can be written as

$$J\mathbf{f}(\boldsymbol{\delta}) = \begin{bmatrix} \left(\frac{b_1}{((1-\delta_1)Y)^2}\right)\left\{Y_{-1}\left(\frac{Y_{-1}}{b_1}+\frac{2}{\delta_1}\right), & \frac{dy_2}{d\delta_2}, & \frac{dy_3}{d\delta_3}\right\} \\ \left(\frac{b_2}{((1-\delta_2)Y)^2}\right)\left\{\frac{dy_1}{d\delta_1}, & Y_{-2}\left(\frac{Y_{-2}}{b_2}+\frac{2}{\delta_2}\right), & \frac{dy_3}{d\delta_3}\right\} \\ \left(\frac{b_3}{((1-\delta_3)Y)^2}\right)\left\{\frac{dy_1}{d\delta_1}, & \frac{dy_2}{d\delta_2}, & Y_{-3}\left(\frac{Y_{-3}}{b_3}+\frac{2}{\delta_3}\right)\right\} \end{bmatrix}$$

Noting that $\frac{dy_j}{d\delta_j} = \frac{(2\delta_j-1)y_j^2}{b_j}$ and expanding by cofactors along the first column, we obtain:

$$\begin{aligned} |J\mathbf{f}(\boldsymbol{\delta})| = {} & \frac{\prod_{j=1}^{I} b_j}{Y^6\prod_{j=1}^{I}(1-\delta_j)^2} \times \Bigg\{ \\ & \frac{Y_{-1}\left(2(1-\delta_1)y_1+Y_{-1}\right)}{b_1}\left(\prod_{j=2}^{3}\frac{Y_{-j}\left(2(1-\delta_j)y_j+Y_{-j}\right)}{b_j} - \prod_{j=2}^{3}\frac{(2\delta_j-1)y_j^2}{b_j}\right) \\ - {} & \frac{(2\delta_1-1)y_1^2}{b_1}\left(\frac{(2\delta_2-1)y_2^2}{b_2}\,\frac{Y_{-3}\left(2(1-\delta_3)y_3+Y_{-3}\right)}{b_3} - \prod_{j=2}^{3}\frac{(2\delta_j-1)y_j^2}{b_j}\right) \\ + {} & \frac{(2\delta_1-1)y_1^2}{b_1}\left(\prod_{j=2}^{3}\frac{(2\delta_j-1)y_j^2}{b_j} - \frac{(2\delta_3-1)y_3^2}{b_3}\,\frac{Y_{-2}\left(2(1-\delta_2)y_2+Y_{-2}\right)}{b_2}\right)\Bigg\} \end{aligned}$$

which, after cancelling out the b_j's, and collecting terms

$$\begin{aligned} (15) \quad = {} & Y^{-6}\prod_{j=1}^{I}(1-\delta_j)^{-2} \times \\ & \Bigg\{\underbrace{\prod_{j=1}^{I}\left(Y_{-j}\left(2(1-\delta_j)y_j+Y_{-j}\right)\right)}_{A'} - \underbrace{\left(\prod_{j=1}^{I}(1-2\delta_j)y_j^2\right)\left(\sum_{j=1}^{I}\frac{Y_{-j}\left(2(1-\delta_j)y_j+Y_{-j}\right)}{(1-2\delta_j)y_j^2}+2\right)}_{B}\Bigg\} \end{aligned}$$

Since $Y^{-6}\prod_{j=1}^{I}(1-\delta_j)^{-2}$ is positive, our task is to show that the term in curly brackets in (15) is positive also. Letting $\phi_i = \frac{Y_{-i}}{y_i}\left(\prod_{j=1}^{I} y_j\right)$ and noting that $\prod_{j=1}^{I} Y_{-j} = 2\prod_{j=1}^{I} y_j + \sum_{j=1}^{I}\phi_j$, Term A' can be expanded to

$$\text{Term } A' = \prod_{j=1}^{I} Y_{-j}\left\{\prod_{j=1}^{I} Y_{-j} + \underbrace{\left(2\prod_{j=1}^{I} Y_{-j}\right)\sum_{j=1}^{I}\frac{(1-\delta_j)y_j}{Y_{-j}}}_{C} + 4\left(\prod_{j=1}^{I}(1-\delta_j)y_j\right)\sum_{j=1}^{I}\frac{Y_{-j}}{(1-\delta_j)y_j}\right\}$$

$$+ \left(2\prod_{j=1}^{I} y_j + \sum_{j=1}^{I} \phi_j\right) \underbrace{8\prod_{j=1}^{I}(1-\delta_j)y_j}_{D}$$

Since ($\prod_{j=1}^{I} Y_{-j}$ times Term C) and ($\sum_{j=1}^{I} \phi_j$ times Term D) are unambiguously positive, a sufficient condition for $|J\mathbf{f}(\boldsymbol{\delta})|$ to be positive is that (Term A - Term B) is positive, where A is obtained below from A' by omitting Terms C and D, with their coefficients:

$$\text{Term } A = \underbrace{\left(\prod_{j=1}^{I} Y_{-j}\right)^2}_{A_1} + \underbrace{4\left(\prod_{j=1}^{I}(1-\delta_j)y_jY_{-j}\right)\sum_{j=1}^{I}\frac{Y_{-j}}{(1-\delta_j)y_j}}_{A_2} + \left(\prod_{j=1}^{I} y_j^2\right)\underbrace{16\prod_{j=1}^{I}(1-\delta_j)}_{A_3}$$

Term B in expression (15) can be expanded to

$$\begin{aligned}\text{Term } B = & \left(\prod_{j=1}^{I} y_j^2\right)\Bigg\{\underbrace{\left(-2\prod_{j=1}^{I}(1-2\delta_j)\right)\sum_{j=1}^{I}\frac{\delta_jY_{-j}}{(1-2\delta_j)y_j}}_{B_1} \\ & + \underbrace{\left(\prod_{j=1}^{I}(1-2\delta_j)\right)\sum_{j=1}^{I}\frac{2y_jY_{-j}+(Y_{-j})^2}{(1-2\delta_j)y_j^2}}_{B_2} + \underbrace{2\prod_{j=1}^{I}(1-2\delta_j)}_{B_3}\Bigg\}\end{aligned}$$

None of the components of Term B can be signed unambiguously. To prove that $|J\mathbf{f}(\boldsymbol{\delta})|$ is positive, we will show that each of these components is dominated by a combination of subcomponents of Term A. To establish this, we begin by decomposing terms A_1 and A_2. Each term identified with an underbrace as $A_{r,q}$ will be used to offset some component of Term B.

$$\begin{aligned}\text{Term } A_1 = & \ 2\left(\prod_{j=1}^{I} y_j\right)\sum_{j=1}^{I} y_j^3 + \prod_{j=1}^{I} y_j^2\left(\sum_{j=1}^{I}\left(\frac{Y_{-j}}{y_j}\right)^2 + 6\sum_{j=1}^{I}\frac{Y_{-j}}{y_j} + 10\right) \\ > & \left(\prod_{j=1}^{I} y_j^2\right)\Bigg\{\underbrace{2\sum_{j=1}^{I}\frac{Y_{-j}}{y_j}}_{A_{1,1}} + \underbrace{\left(\sum_{j=1}^{I}\frac{2y_jY_{-j}+(Y_{-j})^2}{y_j^2}\right)}_{A_{1,2}} + \underbrace{2}_{A_{1,3}}\Bigg\}\end{aligned}$$

$$\text{Term } A_2 = \left(\prod_{j=1}^{I} y_j^2\right)\Bigg\{\underbrace{4\left(\prod_{j=1}^{I}(1-\delta_j)\right)\left(\sum_{j=1}^{I}\frac{2y_jY_{-j}+(Y_{-j})^2}{(1-\delta_j)y_j^2}\right)}_{A_{2,2}}$$

$$+\underbrace{8\left(3-2\sum_{j=1}^{I}\delta_j+\left(\prod_{j=1}^{I}\delta_j\right)\sum_{j=1}^{I}\frac{1}{\delta^j}\right)}_{A_{2,3}}\Bigg\}+\underbrace{4\left(\sum_{j=1}^{I}y_j\right)\left(\prod_{j=1}^{I}(1-\delta_j)y_j\right)\sum_{j=1}^{I}\frac{\sum_{k\neq j}y_k^2}{1-\delta_j}}_{>0}$$

We now combine these expressions, to obtain terms $(E_i)_{i=1}^3$ below. To complete the proof of the theorem, it clearly suffices to show that each of these terms is nonnegative.

$$\begin{aligned}
\text{Term } E_1 &= \text{Term } A_{1,1} - \text{Term } B_1 \\
&= 2\sum_{j=1}^{I}\frac{Y_{-j}}{y_j} - \left(-2\prod_{j=1}^{I}(1-2\delta_j)\right)\sum_{j=1}^{I}\frac{\delta_jY_{-j}}{(1-2\delta_j)y_j} \\
&= 2\sum_{j=1}^{I}\frac{Y_{-j}\left(1+\delta_j\prod_{k\neq j}(1-2\delta_k)\right)}{y_j} \\
\text{Term } E_2 &= \text{Term } A_{1,2} + \text{Term } A_{2,2} - \text{Term } B_2 \\
&= \left(\sum_{j=1}^{I}\frac{2y_jY_{-j}+(Y_{-j})^2}{y_j^2}\right) + 4\left(\prod_{j=1}^{I}(1-\delta_j)\right)\sum_{j=1}^{I}\frac{2y_jY_{-j}+(Y_{-j})^2}{(1-\delta_j)y_j^2} \\
&\qquad - \left(\prod_{j=1}^{I}(1-2\delta_j)\right)\sum_{j=1}^{I}\frac{2y_jY_{-j}+(Y_{-j})^2}{(1-2\delta_j)y_j^2} \\
&= 2\sum_{j=1}^{I}\left(2-\sum_{k\neq j}\delta_k\right)\frac{2y_jY_{-j}+(Y_{-j})^2}{y_j^2} \\
\text{Term } E_3 &= \text{Term } A_{1,3} + \text{Term } A_{2,3} + \text{Term } A_3 - \text{Term } B_3 \\
&= 2+8\left(3-2\sum_{j=1}^{I}\delta_j+\left(\prod_{j=1}^{I}\delta_j\right)\sum_{j=1}^{I}\frac{1}{\delta^j}\right)+16\prod_{j=1}^{I}(1-\delta_j)-2\prod_{j=1}^{I}(1-2\delta_j) \\
&= 4\left(10 - 7\sum_{j=1}^{I}\delta_j+4\sum_{j=1}^{I}\prod_{k\neq j}\delta_k\right)
\end{aligned}$$

Term E_1 is nonnegative since for each j, $\delta_j\prod_{k\neq j}(2\delta_k-1)\geq -1$. Term E_2 is nonnegative since for each j, $\left(-\sum_{k\neq j}\delta_k\right)\geq -2$. Term E_3 is nonnegative since the expression

$\left(4\sum_{j=1}^{I}\prod_{k\neq j}\delta_k - 7\sum_{j=1}^{I}\delta_j\right)$ attains a minimum of -10 on $[0,1]^3$ when two of the δ_j's are 1 and the remaining one is 0. ■

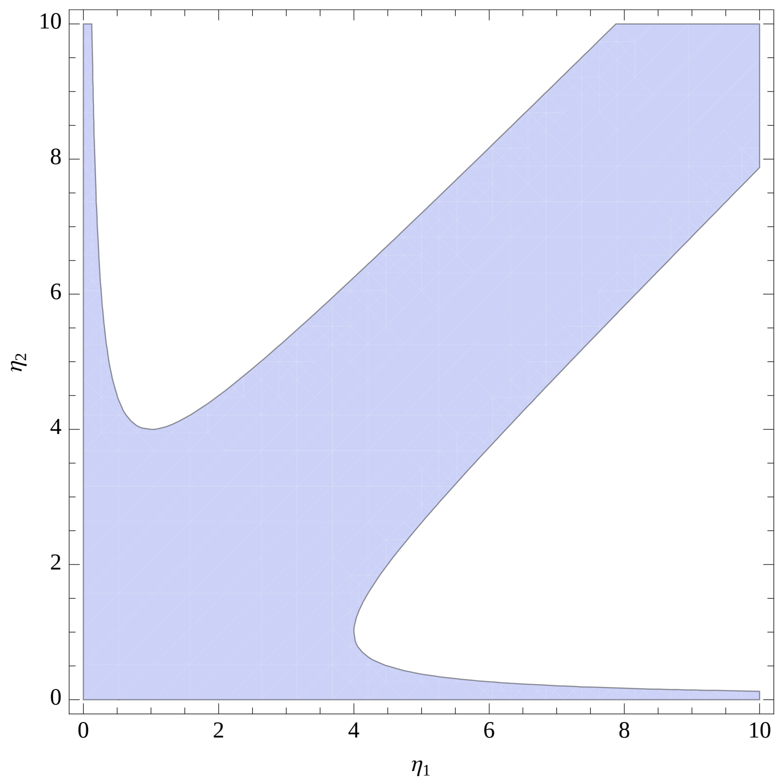

Figure 1. Calibration region for $b_1 = b_2 = 1$.

	Generalized CES			CES-quadratic		
	Cotton	Wheat	Rice	Cotton	Wheat	Rice
β_{i1}	0.830	0.647	0.316	0.937	0.847	0.632
β_{i2}	0.154	0.346	0.624	0.057	0.150	0.336
β_{i3}	0.011	0.004	0.038	0.004	0.002	0.021
β_{i4}	0.006	0.002	0.021	0.002	0.001	0.012
α_i	460.365	199.389	61.842	153.591	69.264	35.826
λ_{2i}	15.651	-77.958	-51.295	315.219	0.000	137.790
δ_i	0.373	0.435	0.476	-	-	-
γ_i	-	-	-	162.382	23.583	217.051

Table 1. Calibrated parameter values for the Generalized CES and CES-quadratic models. The value of the the elasticity of substitution $\sigma_i = \frac{1}{1-\rho_i}$ is set to 0.7 in both models.

	Generalized CES			CES-quadratic		
	Cotton	Wheat	Rice	Cotton	Wheat	Rice
x_{i1}	1.226	-0.824	-0.619	0.418	-0.334	-0.086
x_{i2}	1.409	-0.410	-0.255	1.172	-0.334	-0.086
x_{i3}	1.409	-0.410	-0.255	1.172	-0.334	-0.086
x_{i4}	1.409	-0.410	-0.255	1.172	-0.334	-0.086
q_i	0.468	-0.295	-0.192	0.470	-0.334	-0.086

Table 2. Percentage change in allocated inputs and output after a 1% increase in p_C.

	Generalized CES			CES-quadratic		
	Cotton	Wheat	Rice	Cotton	Wheat	Rice
x_{i1}	12.073	-8.085	-6.155	4.187	-3.349	-0.861
x_{i2}	14.176	-4.105	-2.589	11.924	-3.349	-0.861
x_{i3}	14.176	-4.105	-2.589	11.924	-3.349	-0.861
x_{i4}	14.176	-4.105	-2.589	11.924	-3.349	-0.861
q_i	4.472	-2.972	-1.953	4.700	-3.349	-0.861

Table 3. Percentage change in allocated inputs and output after a 10% increase in p_C.

	Generalized CES			CES-quadratic		
	Cotton	Wheat	Rice	Cotton	Wheat	Rice
x_{i1}	-0.014	0.122	-0.259	-0.007	0.037	-0.072
x_{i2}	-0.057	0.021	-0.347	-0.039	0.010	-0.229
x_{i3}	-0.057	0.021	-0.347	-0.039	0.010	-0.229
x_{i4}	-6.508	-6.434	-6.779	-6.491	-6.444	-6.669
q_i	-0.038	0.015	-0.261	-0.039	0.010	-0.229

Table 4. Percentage change in allocated inputs and output after a 10% increase in c_4.