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# WALS estimation and forecasting in factor-based dynamic models with an application to Armenia*

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**Abstract:** Two model averaging approaches are used and compared in estimating and forecasting dynamic factor models, the well-known BMA and the recently developed WALS. Both methods propose to combine frequentist estimators using Bayesian weights. We apply our framework to the Armenian economy using quarterly data from 2000–2010, and we estimate and forecast real GDP and inflation dynamics.

**JEL Classification:** C11, C13, C52, C53, E52, E58.

**Keywords:** Dynamic models; Factor analysis; Model averaging; Monte Carlo; Armenia.

# 1 Introduction

In the recent macroeconomic literature, factor-based dynamic models have become popular. The idea underlying these models is that, while there are potentially a very large number of explanatory variables, most of the movement in the dependent variable can be explained by only a few variables or linear combinations thereof. One of the reasons why this happens it that the explanatory variables are often highly correlated.

We mention three recent examples where this approach has been successfully applied. Stock and Watson (2002) performed forecasting experiments for key USA macroeconomic variables using 215 explanatory variables. From this large number of variables they extracted a few factors which were sufficient for their purpose. Then they used the extracted factors to forecast key macroeconomic indicators. Forni et al. (2000, 2003) provided a time-series forecasting method based on spectral analysis, and applied this method to forecast Euro-area industrial production and inflation using 447 explanatory variables. Finally, Bernanke et al. (2005) took a VAR model and augmented it with factors based on 120 macroeconomic variables. All three papers find that the mean squared errors of estimates and forecasts based on factor models are lower than those obtained from vector autoregressive models.

After extracting factors, these models are typically estimated in the traditional econometric way, that is, separating model selection and estimation. Recent advances in econometric theory allow us to combine model selection and estimation into one procedure, thus avoiding the undesirable problem of pretesting. This procedure is called 'Bayesian model averaging'. The purpose of the current paper is to extend the basic model averaging framework to include dynamics and factor extraction, and to apply this extended framework to explain and forecast Armenian real GDP and inflation dynamics.

In addition, we wish to compare in this context the standard Bayesian model averaging (BMA) approach to the 'weighted average least squares' (WALS) approach, recently developed in Magnus et al. (2010). The WALS approach has both theoretical and computational advantages over BMA. Theoretical, because it generates bounded risk and contains an explicit treatment of ignorance; computational, because its computing time increases linearly rather than exponentially with the dimension of the model selection space. In Magnus et al. (2010), WALS was applied to growth empirics, but without dynamics or lagged dependent variables.

Estimation and forecasting in factor-based dynamic models using the

BMA algorithm was first applied by Koop and Potter (2004) to US data. Our current paper follows their general approach, but also reports on experiments where the two model averaging methods (WALS and BMA) are compared.

The paper is organized as follows. In Section 2 we present the WALS and BMA model averaging methods. The factor-based dynamic model is introduced in Section 3. Some characteristics of Armenia are provided in Section 4 and the data are described in Section 5, which also contains a preliminary analysis of the data. The estimation results are given in Section 6. We report on two experiments. First, an estimation simulation in Section 7, then a forecast experiment in Section 8. Section 9 concludes.

# 2 Bayesian combinations of frequentist estimators

The idea behind combining estimators (or forecasts) is to use information from all models within a given family in a continuous fashion. In contrast to standard econometrics — where one first selects a model and then estimates the parameters within the chosen model, a discrete procedure — we combine the estimators from all models considered, where some models get a higher weight than others, based on priors and diagnostics. One advantage of this procedure is that we avoid the well-known pretest problem: our procedure is a joint procedure, where model selection and estimation are combined.

As our framework we choose the linear regression model

$$y = X_1\beta_1 + X_2\beta_2 + \epsilon = X\beta + \epsilon, \qquad \epsilon \sim \mathrm{N}(0, \sigma^2 I_n),$$

where $y$ $(n \times 1)$ is the vector of observations, $X_1$ $(n \times k_1)$ and $X_2$ $(n \times k_2)$ are matrices of nonrandom regressors, $\epsilon$ is a random vector of unobservable disturbances, and $\beta_1$ and $\beta_2$ are unknown parameters which we need to estimate. We assume that $k_1 \geq 1$, $k_2 \geq 0$, $k = k_1 + k_2 \leq n - 1$, that $X = (X_1 : X_2)$ has full column-rank, and that the disturbances are independent and identically distributed.

The reason for distinguishing between $X_1$ and $X_2$ is that $X_1$ contains variables that we want to be in the model (whatever $t$-values or other diagnostics we find), while $X_2$ contains variables that may or may not be in the model. The columns of $X_1$ are called 'focus' regressors, the columns of $X_2$ 'auxiliary' regressors. The uncertainty about each auxiliary regressor, that is whether we should or should not include the regressor in our model, is a very common situation, and the application of model averaging is then a natural procedure. Rather than choosing one model by some preliminary diagnostic tests, we assume that each model tells us something of interest about our focus parameters. We do not, however, trust each model to the same degree, and the resulting weights are determined by priors and data. In this paper we concentrate on two model averaging algorithms, the well-known BMA algorithm and the recently introduced WALS algorithm. We briefly discuss each in turn.

*Bayesian model averaging (BMA).* With the exception of Magnus et al. (2010), the whole literature on Bayesian model averaging considers the case $k_1 = 1$. We summarize the approach of Magnus et al. (2010, Section 2). Since there are $k_2$ auxiliary regressors, we have $2^{k_2}$ models to consider. The posterior probability for model $\mathcal{M}_i$ is given by

$$\lambda_i = p(\mathcal{M}_i|y) = \frac{p(\mathcal{M}_i)p(y|\mathcal{M}_i)}{\sum_j p(\mathcal{M}_j)p(y|\mathcal{M}_j)} \qquad (i = 1, \dots, 2^{k_2}),$$

and if we take $p(\mathcal{M}_i) = 2^{-k_2}$, which is the common assumption, then $p(\mathcal{M}_i)$ does not depend on $i$, and we have simply $\lambda_i \propto p(y|\mathcal{M}_i)$, the marginal likelihood of $y$ in model $\mathcal{M}_i$. If we adopt Zellner's $g$-prior and let $X_{2i}$ denote the $n \times k_{2i}$ matrix containing the $k_{2i}$ auxiliary regressors that appear in model $\mathcal{M}_i$, then

$$\lambda_i \propto \left(\frac{g_i}{1+g_i}\right)^{k_{2i}/2} (y' M_1 A_i M_1 y)^{-(n-k_1)/2},$$

where

$$A_i = \frac{g_i}{1+g_i} M_1 + \frac{1}{1+g_i}(M_1 - M_1 X_{2i}(X_{2i}' M_1 X_{2i})^{-1} X_{2i}' M_1)$$

and

$$M_1 = I_n - X_1(X_1' X_1)^{-1} X_1'.$$

The $\lambda_i$ are the required weights to obtain the BMA estimates and precisions. For example, the BMA estimator of $\beta_1$ is given by $\mathrm{E}(\beta_1|y) = \sum_i \lambda_i \mathrm{E}(\beta_1|y, \mathcal{M}_i)$.

There are several problems with BMA. First, all $2^{k_2}$ models have to be evaluated implying a huge computational effort; second, the priors are based on the normal distribution, leading to unbounded risk; and third, the treatment of 'ignorance' is ad hoc and unsatisfactory. These problems are avoided in an alternative model averaging procedure, called WALS.

*Weighted average least squares (WALS).* In the WALS algorithm, developed in Magnus et al. (2010), we first 'orthogonalize' the columns of $X_2$ such that $P'X_2{}'M_1X_2P = \Lambda$, where $P$ is orthogonal and $\Lambda$ is diagonal. Then we define $X_2^* = X_2P\Lambda^{-1/2}$ and $\beta_2^* = \Lambda^{1/2}P'\beta_2$, so that $X_2^*\beta_2^* = X_2\beta_2$. Our prior will be on $\beta_1$ and $\beta_2^*$ (rather than on $\beta_2$), and this gives us enormous computational advantage, because all models which include $x_{2j}^*$ as a regressor will have the same estimator of $\beta_{2j}^*$, irrespective which other $\beta_2^*$'s are estimated.

The second ingredient is the 'equivalence theorem' (Magnus and Durbin, 1999; Danilov and Magnus, 2004), which tells us that the WALS estimator $b_1$ of $\beta_1$ will be 'good' (in the mean squared error sense) if and only if $W\hat{\beta}_2^*$ is a good estimator of $\beta_2^*$, where $\hat{\beta}_2^*$ denotes the least squares estimator of $\beta_2^*$ in the unrestricted model, and $W$ is a diagonal matrix or order $k_2 \times k_2$. The diagonal elements $w_j$ of $W$ will depend on the weights $\lambda_i$, but while there are $2^{k_2}$ $\lambda$'s, there are only $k_2$ $w$'s. This is where the computation advantage comes from.

The third ingredient is the treatment of ignorance. Based on the fact that a $t$-value of one indicates that including an auxiliary regressor gives us the same mean squared error of the estimated focus parameter as excluding the auxiliary regressor, we define ignorance on an auxiliary parameter $\eta$ by the properties

$$\Pr(\eta > 0) = \Pr(\eta < 0), \qquad \Pr(|\eta| > 1) = \Pr(|\eta| < 1),$$

and we propose the Laplace distribution $\pi(\eta) = (c/2)\exp(-c|\eta|)$ with $c = \log 2$. Details of the procedure can be found in Magnus et al. (2010), Section 3. The WALS estimator is a Bayesian combination of frequentist estimators, and possesses major advantages over standard Bayesian model averaging (BMA) estimators: the WALS estimator has bounded risk, allows a coherent treatment of ignorance, and its computational effort is negligible. The sampling properties of the WALS estimator as compared to BMA estimators have been examined in Magnus et al. (2011), where Monte Carlo evidence shows that the WALS estimator performs significantly better than standard BMA and pretest alternatives. Because of the light computational cost, extensions are possible in many directions. For example, Magnus et al. (2011) extend the WALS theory to allow for nonspherical disturbances.

# 3 Model and assumptions

In the current paper we consider another extension, namely to allow for lagged dependent variables. The $y_t$ will then be correlated with the current and all previous disturbances, but uncorrelated with all future disturbances. Hence, the regressor $y_{t-1}$ will be uncorrelated with the current disturbance and all future disturbances, although it will be correlated with all previous disturbances. The standard OLS assumptions do therefore not hold, and the finite-sample properties of the least squares estimators are not valid. However, as shown by Mann and Wald (1943), these properties will hold asymptotically.

Thus motivated, we consider the dynamic regression model

$$y_t = \alpha(L)y_{t-1} + \beta(L)x_{t-1} + \xi_t \qquad (t = 1, \dots, T), \tag{1}$$

where $y_t$ is a scalar dependent variable, $x_t$ is a $k \times 1$ vector of nonrandom explanatory variables, $\alpha(L)$ and $\beta(L)$ are polynomials in the lag operator of dimensions $p_1$ and $p_2$, respectively, and $\xi_t$ is a random vector of unobservable disturbances, independently and identically distributed as $\mathrm{N}(0, \sigma^2)$.

We have $p_1 + kp_2$ explanatory variables, which may be a large number. Moreover, many of the parameters may be close to zero. These two factors make it difficult to apply standard estimation methods (Koop and Potter, 2004). It is then common in the macro-econometric literature to replace the $k$ explanatory variables with a much smaller number of variables. This can be achieved by using principal component or factor analysis algorithms. Then, after extracting the principal components, Model (1) can be rewritten as

$$y_t = \alpha(L)y_{t-1} + \gamma(L)f_{t-1} + \epsilon_t \qquad (t = 1, \dots, T), \tag{2}$$

where $f_t$ $(m \times 1)$ is the vector of extracted principal components (Stock and Watson, 2002). We assume that $m < k$ and $m < T$.

Koop and Potter (2004) were the first to show how Bayesian model averaging can be applied to estimation and forecasting using dynamic factor models. Their study applies BMA to the problem of forecasting GDP and inflation using quarterly US data on 162 time series. Our paper follows their approach, but also compares two competing estimation procedures: BMA and WALS. This will not only tell us something about the power of the two algorithms, but will also provide information about the robustness of our results.

We shall assume that the lagged dependent variables are always focus regressors. But the extracted principal components can be either focus or auxiliary. Thus we write

$$y = X_1\beta_1 + X_2\beta_2 + \epsilon, \tag{3}$$

where $X_1$ contains the lagged dependent variables and a subset (possible empty) of the principal components, and $X_2$ contains the remainder of the principal components. In this form we can apply BMA and WALS to this system.

# 4 Characteristics of Armenia

Armenia is a small country in the Southern Caucasus, about 65% the size of Moscow region. It is bordered by Georgia to the North and East, Azerbaijan to the West, and Turkey and Iran to the South. Armenia was the first nation to adopt Christianity as a state religion, in 301 AD. The population of Armenia, close to three million people, is homogeneous: about 98% is ethnic Armenian with some small minorities, mostly Yazidis (1.3%) and Russians (0.5%).

Until 1991 Armenia was a republic of the former Soviet Union. During the Soviet period Armenia was transformed from an agricultural to an industrial society, and produced machine tools, electronic products, synthetic rubber, and textiles to trade with other Soviet republics in exchange for raw materials and energy. But the regional conflict with Azerbaijan over Nagorno-Karabakh and the break-up of the Soviet Union contributed to a severe economic decline in the early 1990s. As a result, GDP in 1992/93 was only about 40% of the level in 1989.

In 1994 the Armenian Government launched an ambitious IMF-sponsored economic program, which has resulted in positive growth since 1995. Today, Armenia's economy is stable with a high growth rate and low inflation. From 2000–2009 the economy grew at an annual average rate of 8.8%, while the inflation rate was 3.0%. The reason for this rapid growth lies mainly in the expanding construction and service sectors; according to Armenia's National Statistical Agency, the construction sector accounted for about 27% of GDP in 2008. Cash remittances from migrant workers (of which 95% are employed in Russia) are another important factor.

Despite marked progress, Armenia still suffers from a large trade imbalance which is an impediment to economic growth. Armenia is still largely dependent upon foreign aid and remittances from Armenian nationals working abroad. The economy was hit hard by the recent global economic crisis as worker remittances fell and exports of key mineral products (copper, aluminum, molybdenum, and processed diamonds) dropped. The total value of foreign debt is high: the ratio between the GDP and foreign debt has reached 46%. The unemployment rate is nearly 30%, and a huge gap exists between actual and potential GDP.

# 5 Data description and preliminary analysis

Our data consist of quarterly time series of 42 macroeconomic variables from 2000 (second quarter) to 2010 (third quarter), in total 42 observations for each variable. This set comprises information on national accounts data (9 variables) and consumer prices and exchange rate data (13 variables), listed in Table 1; and on financial and monetary policy indicators (13 variables) and international macroeconomic indicators (7 variables), listed in Table 2. All variables in Table 1 are in logarithmic form, in first differences. The

Table 1: National accounts, consumer prices, and exchange rates

| National accounts | Price indices | Price indices and exchange rates |
|---|---|---|
| GDP | Consumer price index | Wheat price index |
| Consumption | Food price index | Fuel price index |
| Investment | Nonfood price index | Imported food price index |
| Exports | Services price index | Imported nonfood price index |
| Imports | Home food price index | Administrative price index |
| Industrial output | | AMD/USD exchange rate |
| Agricultural output | | AMD/EURO exchange rate |
| Construction | | AMD/RR exchange rate |
| Services | | |

variables in column 1 are all real. The variables in columns 1 and 2 are seasonally adjusted.

Table 2: Financial, monetary, and international indicators

| Financial policy indicators | Interest rates | International indicators |
|---|---|---|
| Cash money | AMD deposits | USA real GDP |
| Money aggregate, M0 | USD deposits | EU real GDP |
| Money aggregate, M1 | AMD loans | USA consumer price index |
| Money aggregate, M2X | USD loans | EU consumer price index |
| Total deposits | Central Bank interbank | Gasoline price index |
| Loans to economy | | Petroleum price index |
| Loans to enterprizes | | Wheat price index |
| Loans to households | | |

The variables in Table 2 are also in logarithmic form, in first differences, and the variables in columns 1 and 3 are seasonally adjusted. The international indicators in column 3 are taken from the International Financial Statistics published by the IMF and are already seasonally adjusted by the IMF.

In this paper we estimate and forecast factor-based dynamic models using principal components. These principal components are based on the

Table 3: Characteristics of the extracted principal components

| Principal components | Rotated eigenvalue | % of total variance | Cumulative % | Correlation with real GDP | Correlation with inflation |
|---|---|---|---|---|---|
| *Int_rate* | 5.17 | 12.94 | 12.94 | 0.04 | −0.17 |
| *Ex_rate* | 5.13 | 12.82 | 25.75 | −0.04 | 0.32 |
| *Invest* | 3.85 | 9.63 | 35.38 | 0.66 | 0.09 |
| *Mon_agg* | 3.47 | 8.67 | 44.05 | 0.39 | −0.05 |
| *Credit* | 3.21 | 8.02 | 52.07 | 0.02 | 0.05 |
| *ImpExp* | 2.97 | 7.41 | 59.49 | 0.27 | −0.06 |
| *Pr_index* | 2.94 | 7.35 | 66.84 | 0.27 | 0.70 |
| *Nat_acc* | 2.15 | 5.39 | 72.22 | 0.27 | −0.21 |
| *GDP_star* | 2.04 | 5.11 | 77.34 | 0.28 | −0.13 |
| *Hfood_pr* | 1.42 | 3.55 | 80.89 | 0.04 | 0.38 |

underlying data set of 42 variables. The extracted principal components have been given names, based on the correlation coefficients between the extracted principal components and the underlying time series. Some important characteristics of the extracted principal components are presented in the Table 3. The first principal component is *Int_rate* and its contribution to the total variance of the underlying variables is 12.94%. The second principal component is *Ex_rate* with a contribution of 12.82%, and the third is *Invest* with a contribution of 9.63%. The ten most important principal components (those with a rotated eigenvalue larger than 1) explain more than 80% of the variance of the underlying variables, which we consider to be sufficient.

Table 4: Focus and auxiliary variables ($j = 1, \ldots, 4$)

| Real GDP | | | Inflation | | |
|---|---|---|---|---|---|
| Regressor | Model 1.1 | Model 1.2 | Regressor | Model 2.1 | Model 2.2 |
| *Intercept* | focus | focus | *Intercept* | focus | focus |
| $GDP_{t-j}$ | focus | focus | $INF_{t-j}$ | focus | focus |
| $Invest_{t-j}$ | auxiliary | focus | $Int\_rate_{t-j}$ | auxiliary | auxiliary |
| $Mon\_agg_{t-j}$ | auxiliary | auxiliary | $Ex\_rate_{t-j}$ | auxiliary | focus |
| $ImpExp_{t-j}$ | auxiliary | auxiliary | $Credit_{t-j}$ | auxiliary | auxiliary |
| $Nat\_acc_{t-j}$ | auxiliary | focus | $Pr\_index_{t-j}$ | auxiliary | focus |
| $GDP\_star_{t-j}$ | auxiliary | auxiliary | $Hfood\_pr_{t-j}$ | auxiliary | auxiliary |

Each of the extracted principal components could be used for estimation in our factor-based dynamic models. However, we use our knowledge of economic theory and Armenian practice to include only those principal components which contain important information about real GDP and inflation dynamics. Regarding real GDP, the three highest correlations are obtained by *Invest*, *Mon_agg*, and *GDP_star*, to which we add *ImpExp* and *Nat_acc*, but not *Pr_index*. Regarding inflation, the three highest correlations are obtained by *Pr_index*, *Hfood_pr*, and *Ex_rate*, to which we add *Int_rate* and *Credit*, but not *Nat_acc* and *GDP_star*. These choices then lead to the four models in Table 4. Model 1 refers to real GDP and Model 2 to inflation. Each model has two variants. In variant 1 (Models 1.1 and 2.1) we take as our focus variables only the lagged values of the dependent variable (and the intercept), while all other variables are auxiliary, that is, we are uncertain whether they should be in the model or not. In variant 2 (Models 1.2 and 2.2) we have more focus variables, namely *Invest* and *Nat_acc* in the Model 1.2 and *Ex_rate* and *Pr_index* in Model 2.2. These new focus variables are always in the model; we do not question whether they should be or not. Having specified the four models, we can now estimate and forecast these models using the WALS and BMA algorithms, and compare the two algorithms.

# 6 Estimation results

We have two models, one for GDP and one for inflation. Each models has two variants, one with only the intercept and lagged dependent variable as focus regressors, the other with additional focus regressors. For each of these four cases we can consider one lag, two lags, three lags, or four lags. In addition, we have two different model averaging algorithms: WALS and BMA. All WALS and BMA results are obtained using Matlab algorithms, which are freely available from http://center.uvt.nl/staff/magnus/wals. The WALS estimates for the GDP equation are presented in Tables 5 and 6.

In Table 5 the focus variables are the intercept and lagged values of real GDP, while in Table 6 we add lagged values of *Invest* and *Nat_acc* to the focus variables. The first lag of the real GDP is positively correlated with current GDP, and the parameter appears to be close to one in both models, and in each of the four lag structures. This suggests that this dynamic relationship is quite robust. Current GDP is negatively correlated with lagged values of *Invest*, and positively correlated with *Nat_acc*. This is to be expected, because one of the main ingredients of the *Nat_acc* is final consumption, which in turn is one of the basic components of GDP. Thus, final consumption should be positively correlated with GDP. Also, current consumption is positively correlated with previous-period consumption, and hence consumption of the previous period and GDP of the current period should be positively

Table 5: WALS estimates for Model 1 (GDP), Version 1

| | Est (Std) | Est (Std) | Est (Std) | Est (Std) |
|---|---|---|---|---|
| Focus regressors | | | | |
| *Intercept* | 0.30 (0.42) | 0.72 (0.47) | 1.23 (0.60) | 0.95 (0.87) |
| $GDP_{t-1}$ | 0.82 (0.18) | 1.03 (0.22) | 0.95 (0.27) | 0.95 (0.33) |
| $GDP_{t-2}$ | — | −0.43 (0.20) | −0.70 (0.30) | −0.62 (0.36) |
| $GDP_{t-3}$ | — | — | 0.05 (0.23) | −0.05 (0.42) |
| $GDP_{t-4}$ | — | — | — | 0.08 (0.37) |
| | | | | |
| Auxiliary regressors | | | | |
| $Invest_{t-1}$ | −0.33 (0.34) | −0.70 (0.34) | −0.66 (0.36) | −0.55 (0.40) |
| $Mon\_agg_{t-1}$ | 0.15 (0.27) | 0.19 (0.25) | 0.34 (0.27) | 0.32 (0.32) |
| $ImpExp_{t-1}$ | −0.21 (0.23) | −0.11 (0.21) | 0.02 (0.21) | 0.05 (0.28) |
| $Nat\_acc_{t-1}$ | 0.46 (0.24) | 0.20 (0.22) | 0.13 (0.26) | 0.26 (0.37) |
| $GDP\_star_{t-1}$ | −0.25 (0.24) | −0.14 (0.25) | −0.08 (0.27) | −0.14 (0.44) |
| $Invest_{t-2}$ | — | 0.09 (0.33) | 0.19 (0.39) | 0.10 (0.43) |
| $Mon\_agg_{t-2}$ | — | 0.16 (0.23) | 0.27 (0.26) | 0.33 (0.37) |
| $ImpExp_{t-2}$ | — | 0.25 (0.21) | 0.42 (0.22) | 0.26 (0.32) |
| $Nat\_acc_{t-2}$ | — | 0.80 (0.24) | 0.91 (0.25) | 0.63 (0.29) |
| $GDP\_star_{t-2}$ | — | 0.09 (0.23) | 0.06 (0.31) | 0.20 (0.36) |
| $Invest_{t-3}$ | — | — | 0.12 (0.35) | 0.06 (0.63) |
| $Mon\_agg_{t-3}$ | — | — | 0.34 (0.24) | 0.27 (0.29) |
| $ImpExp_{t-3}$ | — | — | 0.24 (0.21) | 0.18 (0.28) |
| $Nat\_acc_{t-3}$ | — | — | 0.44 (0.29) | 0.28 (0.37) |
| $GDP\_star_{t-3}$ | — | — | 0.18 (0.31) | −0.05 (0.48) |
| $Invest_{t-4}$ | — | — | — | 0.04 (0.53) |
| $Mon\_agg_{t-4}$ | — | — | — | −0.38 (0.31) |
| $ImpExp_{t-4}$ | — | — | — | 0.02 (0.27) |
| $Nat\_acc_{t-4}$ | — | — | — | 0.08 (0.44) |
| $GDP\_star_{t-4}$ | — | — | — | 0.11 (0.45) |

correlated.

Concerning *Invest* we see that the first lag is negatively correlated with current GDP, but that higher lags are positively correlated. Apparently, investments have a short-term (one quarter) negative impact, but an medium-term (2–4 quarters) positive impact on economic activity (and therefore on the level of the current real GDP). Many of the auxiliary parameters are not statistically significant.

In Tables 7 and 8 we report the result for inflation dynamics. Lagged

Table 6: WALS estimates for Model 1 (GDP), Version 2

| | Est (Std) | Est (Std) | Est (Std) | Est (Std) |
|---|---|---|---|---|
| Focus regressors | | | | |
| *Intercept* | 0.27 (0.44) | 0.63 (0.48) | 1.21 (0.61) | 1.09 (0.90) |
| $GDP_{t-1}$ | 0.83 (0.19) | 1.10 (0.22) | 1.00 (0.29) | 0.98 (0.35) |
| $Invest_{t-1}$ | −0.49 (0.39) | −0.98 (0.36) | −0.95 (0.39) | −0.87 (0.45) |
| $Nat\_acc_{t-1}$ | 0.64 (0.26) | 0.22 (0.24) | 0.16 (0.29) | 0.23 (0.41) |
| $GDP_{t-2}$ | — | −0.46 (0.21) | −0.71 (0.32) | −0.65 (0.40) |
| $Invest_{t-2}$ | — | 0.13 (0.35) | 0.20 (0.43) | 0.07 (0.51) |
| $Nat\_acc_{t-2}$ | — | 0.97 (0.26) | 1.09 (0.27) | 0.86 (0.33) |
| $GDP_{t-3}$ | — | — | 0.01 (0.24) | 0.05 (0.43) |
| $Invest_{t-3}$ | — | — | 0.22 (0.39) | 0.15 (0.68) |
| $Nat\_acc_{t-3}$ | — | — | 0.49 (0.33) | 0.40 (0.41) |
| $GDP_{t-4}$ | — | — | — | 0.01 (0.38) |
| $Invest_{t-4}$ | — | — | — | 0.24 (0.59) |
| $Nat\_acc_{t-4}$ | — | — | — | 0.16 (0.48) |
| Auxiliary regressors | | | | |
| $Mon\_agg_{t-1}$ | 0.13 (0.26) | 0.15 (0.23) | 0.26 (0.27) | 0.29 (0.33) |
| $ImpExp_{t-1}$ | −0.16 (0.22) | −0.12 (0.20) | 0.03 (0.21) | 0.04 (0.29) |
| $GDP\_star_{t-1}$ | −0.21 (0.23) | −0.15 (0.23) | −0.08 (0.27) | −0.14 (0.45) |
| $Mon\_agg_{t-2}$ | — | 0.17 (0.22) | 0.28 (0.26) | 0.31 (0.37) |
| $ImpExp_{t-2}$ | — | 0.20 (0.19) | 0.35 (0.22) | 0.28 (0.31) |
| $GDP\_star_{t-2}$ | — | 0.10 (0.23) | 0.12 (0.31) | 0.20 (0.36) |
| $Mon\_agg_{t-3}$ | — | — | 0.35 (0.23) | 0.23 (0.27) |
| $ImpExp_{t-3}$ | — | — | 0.26 (0.21) | 0.19 (0.27) |
| $GDP\_star_{t-3}$ | — | — | 0.18 (0.30) | −0.07 (0.48) |
| $Mon\_agg_{t-4}$ | — | — | — | −0.32 (0.31) |
| $ImpExp_{t-4}$ | — | — | — | 0.01 (0.27) |
| $GDP\_star_{t-4}$ | — | — | — | 0.06 (0.46) |

values of inflation are positively correlated with current inflation, but comparing with the real GDP estimates we see that inflation in Armenia is less backward-looking than real GDP. The first lags of *Pr_index* and *Ex_rate* are positively correlated with current inflation, which is again reasonable. The positive correlation between *Pr_index* and inflation tells us that price fluctuations in Armenia are autocorrelated. It appears that *Ex_rate* dynamics are positively correlated with inflation, due to the fact that Armenia is a small open economy with an imports-to-GDP ratio of about 40%. The home price

Table 7: WALS estimates for Model 2 (Inflation), Version 1

| | Est (Std) | Est (Std) | Est (Std) | Est (Std) |
|---|---|---|---|---|
| Focus regressors | | | | |
| *Intercept* | 1.09 (0.32) | 0.81 (0.49) | 0.86 (0.71) | 0.84 (1.32) |
| $INF_{t-1}$ | 0.05 (0.27) | 0.16 (0.32) | 0.00 (0.45) | −0.09 (0.74) |
| $INF_{t-2}$ | — | 0.14 (0.27) | 0.13 (0.36) | 0.21 (0.50) |
| $INF_{t-3}$ | — | — | 0.14 (0.34) | 0.03 (0.47) |
| $INF_{t-4}$ | — | — | — | 0.08 (0.40) |
| | | | | |
| Auxiliary regressors | | | | |
| $Pr\_index_{t-1}$ | 0.41 (0.25) | 0.42 (0.28) | 0.47 (0.36) | 0.48 (0.53) |
| $Ex\_rate_{t-1}$ | 0.20 (0.17) | 0.19 (0.21) | 0.26 (0.26) | 0.19 (0.30) |
| $Int\_rate_{t-1}$ | −0.09 (0.15) | 0.34 (0.53) | 0.26 (0.61) | 0.31 (0.87) |
| $Credit_{t-1}$ | 0.10 (0.14) | −0.14 (0.20) | −0.14 (0.26) | −0.05 (0.39) |
| $Hfood\_pr_{t-1}$ | −0.10 (0.18) | −0.22 (0.21) | −0.13 (0.26) | −0.11 (0.38) |
| $Pr\_index_{t-2}$ | — | −0.21 (0.27) | 0.00 (0.34) | −0.10 (0.45) |
| $Ex\_rate_{t-2}$ | — | −0.01 (0.19) | −0.07 (0.26) | 0.04 (0.37) |
| $Int\_rate_{t-2}$ | — | −0.44 (0.50) | 0.01 (0.69) | −0.33 (0.87) |
| $Credit_{t-2}$ | — | 0.25 (0.19) | 0.08 (0.30) | 0.29 (0.40) |
| $Hfood\_pr_{t-2}$ | — | 0.03 (0.21) | 0.03 (0.26) | −0.01 (0.33) |
| $Pr\_index_{t-3}$ | — | — | −0.35 (0.34) | −0.07 (0.47) |
| $Ex\_rate_{t-3}$ | — | — | 0.04 (0.22) | −0.19 (0.29) |
| $Int\_rate_{t-3}$ | — | — | −0.39 (0.64) | −0.72 (0.84) |
| $Credit_{t-3}$ | — | — | 0.10 (0.24) | 0.02 (0.35) |
| $Hfood\_pr_{t-3}$ | — | — | −0.25 (0.24) | −0.17 (0.32) |
| $Pr\_index_{t-4}$ | — | — | — | −0.34 (0.40) |
| $Ex\_rate_{t-4}$ | — | — | — | 0.16 (0.25) |
| $Int\_rate_{t-4}$ | — | — | — | 0.64 (1.05) |
| $Credit_{t-4}$ | — | — | — | −0.04 (0.28) |
| $Hfood\_pr_{t-4}$ | — | — | — | −0.25 (0.32) |

index therefore depends strongly on the international price index level.

# 7 An estimation simulation experiment

While the previous results are of practical and theoretical interest, a proper comparison between WALS and BMA can only be done through a simulation experiment, where we know the true data-generating process and can therefore relate the estimates with the truth. The data-generating process

Table 8: WALS estimates for Model 2 (Inflation), Version 2

| | Est (Std) | Est (Std) | Est (Std) | Est (Std) |
|---|---|---|---|---|
| Focus regressors | | | | |
| *Intercept* | 1.27 (0.32) | 0.89 (0.54) | 0.83 (0.76) | 0.87 (1.37) |
| $INF_{t-1}$ | −0.12 (0.26) | 0.01 (0.34) | −0.18 (0.46) | −0.31 (0.76) |
| $Pr\_index_{t-1}$ | 0.64 (0.27) | 0.68 (0.32) | 0.72 (0.40) | 0.76 (0.58) |
| $Ex\_rate_{t-1}$ | 0.31 (0.19) | 0.28 (0.24) | 0.39 (0.28) | 0.32 (0.33) |
| $INF_{t-2}$ | — | 0.22 (0.29) | 0.22 (0.38) | 0.27 (0.52) |
| $Pr\_index_{t-2}$ | — | −0.37 (0.32) | −0.07 (0.40) | −0.20 (0.50) |
| $Ex\_rate_{t-2}$ | — | −0.02 (0.21) | −0.12 (0.30) | 0.03 (0.40) |
| $INF_{t-3}$ | — | — | 0.27 (0.37) | 0.11 (0.49) |
| $Pr\_index_{t-3}$ | — | — | −0.50 (0.38) | −0.11 (0.51) |
| $Ex\_rate_{t-3}$ | — | — | 0.03 (0.26) | −0.19 (0.33) |
| $INF_{t-4}$ | — | — | — | 0.10 (0.42) |
| $Pr\_index_{t-4}$ | — | — | — | −0.43 (0.45) |
| $Ex\_rate_{t-4}$ | — | — | — | 0.23 (0.29) |
| Auxiliary regressors | | | | |
| $Int\_rate_{t-1}$ | −0.09 (0.14) | 0.33 (0.52) | 0.25 (0.61) | 0.29 (0.88) |
| $Credit_{t-1}$ | 0.09 (0.13) | −0.14 (0.20) | −0.15 (0.25) | −0.07 (0.39) |
| $Hfood\_pr_{t-1}$ | −0.09 (0.16) | −0.20 (0.21) | −0.12 (0.25) | −0.09 (0.38) |
| $Int\_rate_{t-2}$ | — | −0.44 (0.50) | 0.01 (0.69) | −0.35 (0.87) |
| $Credit_{t-2}$ | — | 0.23 (0.19) | 0.08 (0.28) | 0.28 (0.40) |
| $Hfood\_pr_{t-2}$ | — | 0.00 (0.21) | 0.02 (0.25) | −0.01 (0.32) |
| $Int\_rate_{t-3}$ | — | — | −0.39 (0.64) | −0.72 (0.84) |
| $Credit_{t-3}$ | — | — | 0.10 (0.23) | 0.02 (0.34) |
| $Hfood\_pr_{t-3}$ | — | — | −0.28 (0.25) | −0.17 (0.32) |
| $Int\_rate_{t-4}$ | — | — | — | 0.63 (1.06) |
| $Credit_{t-4}$ | — | — | — | −0.03 (0.27) |
| $Hfood\_pr_{t-4}$ | — | — | — | −0.24 (0.31) |

follows closely that models that we have estimated before. Regarding GDP, we assume one of the following two processes to be true:

$$GDP_t = 0.2 + 0.85\, GDP_{t-1} + 1.5\, u_t,$$
$$GDP_t = 0.2 + 0.85\, GDP_{t-1} - 0.7\, Invest_{t-1} + 0.3\, Nat\_acc_{t-1} + 1.5\, u_t,$$

which we label (1.1) and (1.2), respectively. Regarding inflation we assume:

$$INF_t = 0.5 + 0.1\, INF_{t-1} + u_t,$$
$$INF_t = 0.5 + 0.1\, INF_{t-1} + 0.4\, Pr\_index_{t-1} + 0.2\, Ex\_rate_{t-1} + u_t,$$

labeled (2.1) and (2.2). The values of the parameters resemble the estimates and the error variances are set to $\sigma^2 = 2.25$ for the GDP equations and $\sigma^2 = 1$ for inflation. Given the data-generating process and the values of the regressors, we randomly draw the $\{u_t\}$ from a standard-normal distribution. Then, we generate the time series for real GDP or inflation from the data-generating process. Now that we have all the data, we estimate the parameters using the models and the estimation algorithms of Section 6. This gives us parameter estimates.

Next we draw new errors $\{u_t\}$, obtain new values for the dependent variable, and hence new parameter estimates. We repeat this 1000 times, and compute the simulation root mean squared errors:

$$\text{RMSE}_k^{wals} = \sqrt{\frac{1}{1000}\sum_{l=1}^{1000}(\beta_k^{wals_l} - \beta_k^{true})^2},$$
$$\text{RMSE}_k^{bma} = \sqrt{\frac{1}{1000}\sum_{l=1}^{1000}(\beta_k^{bma_l} - \beta_k^{true})^2},$$

where $\beta_k^{true}$ denotes the true value of $\beta_k$, and $\beta_k^{wals_l}$ and $\beta_k^{bma_l}$ are the corresponding WALS and BMA estimates, respectively, for the $l$-th iteration.

The results of the Monte-Carlo simulation are presented in the Table 9. We see that the RMSE values calculated for WALS are generally lower than for BMA, although the difference is not large. Based on these simulations we suggest that WALS gives more accurate estimates than BMA.

# 8 A forecast experiment

We conduct a second experiment, this time in forecasting rather than estimation. Suppose we use $T_1 < T = 42$ quarters on which we base our estimates. This leaves us $T_2 = T - T_1 > 0$ quarters for forecast experiments. The $h$-period forecast is given by

$$\hat{y}_{T_1+h} = \hat{\alpha}(L)y_{T_1+h-1} + \hat{\gamma}(L)f_{T_1+h-1} \qquad (h = 1, \ldots, T_2),$$

Table 9: RMSE for the estimation simulations

| | Version 1 | | Version 2 | |
|---|---|---|---|---|
| | WALS | BMA | WALS | BMA |
| Model 1 (GDP) | | | | |
| *Intercept* | 0.0150 | 0.0173 | 0.0146 | 0.0155 |
| $GDP_{t-1}$ | 0.0075 | 0.0089 | 0.0074 | 0.0077 |
| $Invest_{t-1}$ | — | — | 0.0107 | 0.0101 |
| $Nat\_acc_{t-1}$ | — | — | 0.0090 | 0.0090 |
| Model 2 (Inflation) | | | | |
| *Intercept* | 0.0094 | 0.0089 | 0.0118 | 0.0127 |
| $INF_{t-1}$ | 0.0072 | 0.0075 | 0.0096 | 0.0101 |
| $Pr\_index_{t-1}$ | — | — | 0.0088 | 0.0088 |
| $Ex\_rate_{t-1}$ | — | — | 0.0082 | 0.0079 |

where $y$ denotes either GDP or inflation. In a practical situation we would not know $f_{T_1+h-1}$ and $y_{T_1+h-1}$, when $h \geq 2$. So we would have to forecast these as well. In the experiment we use the observed values of $f_{T_1+h-1}$ and $y_{T_1+h-1}$, hence not the forecasted value $\hat{y}_{T_1+h-1}$ when $h \geq 2$. Then we compute

$$\text{RMSE}_{T_1} = \sqrt{\frac{1}{T-T_1}\sum_{h=1}^{T-T_1}(\hat{y}_{T_1+h} - y_{T_1+h})^2},$$

which depends on the estimation period $T_1$, the model, and the method (BMA or WALS). The results are presented in Table 10.

In general, the smaller is the estimation period $T_1$, the less accurate are the estimates and the forecasts, that is, the RMSE increases as $T_1$ decreases. This is to be expected, but it does not always happen. In particular the behavior for $T_1 = 35$ deviates. The explanation lies in the global financial crisis, which affected Armenia heavily. From the third quarter of 2008 (quarter 34 in our data set) to the second quarter of 2009 (quarter 37) Armenia's real growth of GDP decreased by 18%. The largest decrease (around 9.0%) in real GDP took place in the fourth quarter of 2008 (quarter 35). This large decrease in real GDP causes a large deviation of real GDP from its long-term trend, and this may explain (in part) why the RMSE values calculated for $T_1 = 35$ are relatively large, and for $T_1 = 36$ somewhat smaller. On the whole, the WALS algorithm gives slightly more accurate forecast results than BMA, but the difference is small.

Table 10: RMSE for ex-post forecast accuracy

| | | $T_1$ | | | | |
|---|---|---|---|---|---|---|
| Version | Method | 38 | 37 | 36 | 35 | 34 |
| Model 1 (GDP) | | | | | | |
| 1 | WALS | 0.2997 | 0.3395 | 0.7021 | 2.3682 | 1.3780 |
| | BMA | 0.2845 | 0.3397 | 0.7439 | 2.4683 | 1.3545 |
| 2 | WALS | 0.2352 | 0.3011 | 0.7749 | 2.2081 | 1.3869 |
| | BMA | 0.2315 | 0.3077 | 0.7756 | 2.2264 | 1.3977 |
| Model 2 (Inflation) | | | | | | |
| 1 | WALS | 0.6092 | 0.5020 | 0.5356 | 0.4574 | 0.4321 |
| | BMA | 0.6103 | 0.5625 | 0.6175 | 0.5252 | 0.4649 |
| 2 | WALS | 0.6468 | 0.5212 | 0.5653 | 0.4819 | 0.4377 |
| | BMA | 0.6381 | 0.5201 | 0.5716 | 0.4876 | 0.4412 |

# 9 Concluding remarks

In this paper we have applied two alternative model averaging algorithms (WALS and BMA) to the problem of estimating factor-based dynamic models. The estimated models have also been used to forecast two key macroeconomic variables, namely real GDP and inflation in Armenia. The advantage of using model averaging is that it allows all models to play a role in the estimation and forecasting, thus avoiding the problem of pretesting. A comparison of the performance of the WALS algorithm to BMA shows that the WALS algorithm is to be preferred. Not only does it give more accurate results in the practical application and simulation experiments, but it also has theoretical and computational advantages over BMA.

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