The Equity Premium and Risk-Free Rate Puzzles in a Turbulent Economy: Evidence from 105 Years of Data from South Africa[1]

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The Equity Premium and Risk-Free Rate Puzzles in a Turbulent Economy: Evidence from 105 Years of Data from South Africa*

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Abstract

This paper presents a detailed empirical examination of the South African equity premium; and a quantitative theoretic exercise to test the canonical inter-temporal consumption-based asset-pricing model under power utility. Over the long run, the South African stock market produced average returns six to eight percentage points above bonds and cash; and at the 20-year horizon, an investor would not have experienced a single negative realised equity premium over the entire 105-year period we examine. Yet, the maximum equity premium rationalised by the consumption-based model is 0.4%. The canonical macro-financial model closely matches the average risk-free rate, using realistic parameters for the coefficient of risk aversion and a positive rate of time preference.

JEL Classifications: G12, E21, N27

Keywords: consumption-based asset pricing; stochastic discount factor; equity risk premium puzzle; risk-free rate; risk aversion coefficient; South Africa

1 Introduction

Existing evidence on the failure of the canonical inter-temporal consumption-based asset pricing model (Lucas (1978), Cochrane (2005)) to account for the magnitude of the equity premium (i.e. the return on a stock market index

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in excess of a proxy for the risk-free rate), commonly known as the equity premium puzzle, since Mehra and Prescott (1985), is drawn predominantly from historically stable advanced economies (Campbell (2003), Dimson, Marsh and Staunton (2008)).[1]

Cochrane (2005), referring to what did not happen in the United States over the second half of the twentieth century, observes that "we had no banking panics, and no depressions; no civil wars, no constitutional crises (...). If any of these things had happened, we might well have seen a calamitous decline in stock values, and I would not be writing about the equity premium puzzle." (Cochrane (2005), page 461.)

Over the second half of the same century, South Africa experienced four substantially distinct constitutional dispensations; a series of official states of emergency; and organised resistance (at times armed, and always met with a violent response) to the Apartheid regime. Political instability was recurrent, and reached particularly high levels in the 1960s, late 1970s, and, especially, the mid 1980s. (Fedderke, de Kadt, and Luiz (2001).) 1994 saw the first democratic elections, after over three decades of oppressive restrictions on political and civil liberties, and the elections were preceded by military civil confrontation in some areas. By some measures, the unemployment rate reached 41 percent (Bhorat and Oosthuizen (2006)), and remains officially at or above 23 percent (Statistics South Africa (2009)) – one percentage point below the rate of unemployment in the US at the nadir of the Great Depression (Romer (1993), page 32). There were currency crises in 1996, 1998, and 2001 (Aron and ElBadawi (1999), Bhundia and Ricci (2005)).[2]

South Africa's economy is also highly capitalised. Its stock market is the world's 14th largest, and its government debt market among the world's ten most liquid.[3] The market value of its stock market is close to (recently above) 100 percent of Gross Domestic Product, making it a more valid proxy for aggregate wealth (or a claim to aggregate consumption) than in some advanced economies (e.g. Italy and Germany – see Campbell (2003), page 811). As explained below, it is also one of few countries, and the only non-advanced economy, for which capital market data is available for a period of over a century (see Dimson, Marsh, and Staunton (2002), (2008)).

It is important to use long sample periods when examining the equity premium due to the wide variation that can be observed in year-to-year market

[1] As common in the literature, we will refer to a nearly default-free short-term rate as the "risk-free" rate.

[2] These aspects illustrate that South Africa experienced *at least* one of the events that Cochrane (2005, page 461) suggests may preclude the finding of an equity premium puzzle. There were of course other elements to the financial history of South Africa, including periods of capital controls, changes in banking regulation and monetary policy. See, for example, Aron and Muellbauer (2005) and Farrell (2001).

[3] By market capitalisation (see www.jse.co.za). The JSE is also the 8th largest equities exchange in the Europe – Africa – Middle East region; and the 6th largest emerging market stock exchange (see World Federation of Exchanges, Annual Report and Statistics, 2008). According to the Bank of International Settlements, South Africa's government debt market is the world's sixth most liquid by turnover (see Bank of International Settlements (2007), page 45).

returns. Dimson, Marsh and Staunton (2008) show that three-quarters of the countries for which a century of data are available experienced intervals of negative stock market returns (in inflation-adjusted terms) lasting more than two decades. Japan, France, and Germany experienced periods of over half a century during which cumulative real equity returns remained negative. (See also Bansal and Yaron (2004).)

This paper extends and complements previous analyses of the long-term South African equity premium in Digby, Firer, and Gilbert (2006) and Dimson, Marsh, and Staunton (2008), by providing a more comprehensive examination of the equity premium in South Africa over a period of 105 years; and by confronting the standard inter-temporal consumption-based asset pricing model with the data. Specifically, we compute returns using arithmetic as well as geometric averages, due to serial correlation in excess returns; examine different sub-periods, determined by differences in data reliability; employ alternative proxies for the risk-free rate; consider alternative investment horizons; document the covariance between the premium and aggregate economic activity; and the evolution of the Sharpe ratio. Using per-capita consumption growth and its variance, and defensible coefficients of risk aversion, we obtain theoretic values for the South African equity premium and the risk-free rate.

We find that the equity premium in South Africa is large, and among the highest from the set of economies for which there are equally long data sets – but in the same order of magnitude, despite its significantly more turbulent history. Additionally, we find that the observed premium is too high to be explained by the standard consumption-based asset pricing model – evidence of an equity premium puzzle from South Africa, of comparable magnitude to the evidence from advanced economies. However, we also find that the quantitative prediction of the South African risk-free rate from the consumption-based model closely matches our sample estimates of the average real risk-free return. Interestingly, this is achieved using values for the coefficient of risk aversion which are both theoretically and empirically defensible; and without resorting to implausible negative rates of time preference.

The remainder of the paper proceeds as follows. Section 2 presents a compact literature review. Section 3 discusses the data and computation of returns. Section 4 compares the sample estimates of average returns from long-term investments in equity, bonds and cash in South Africa, and presents the evidence on the magnitude of the equity premium over different sample periods. Section 5 documents the cyclicality of the premium; excess stock market returns and standard deviations over different investment horizons; and the evolution of the Sharpe ratio. Section 6 presents a compact treatment of the canonical consumption-based asset pricing model with power utility. It also estimates the theoretic premium for South Africa; analyses the extent of an equity premium puzzle in South Africa; reports on the coefficients of risk aversion implied by the data (if the consumption-based model were correct); and turns to the implications of the model for the average risk-free rate. Section 7 contains concluding remarks.

2 Related literature

The literature on the equity premium puzzle in advanced economies is extensive, and reliably reviewed in, for example, Campbell (2003), Mehra and Prescott (2003), and Mehra and Prescott (2008). There is less published evidence from emerging markets and, to our knowledge, no evidence based on long-term data.[4]

Salomons and Grootveld (2003) compare the equity premium in a number of developed and emerging economies (including South Africa) over a 25-year period, and find it to be higher in emerging markets. No attempt is made at determining whether the observed premia in emerging economies are consistent with asset pricing theory. Erbaš and Mirakhor (2007) compute an average equity premium across a range of emerging markets (including South Africa), and find it higher than the premium justified by risk aversion – an equity premium puzzle. But the positive average premium they found could mask low or negative premia in individual countries (which are not reported); and, above all, their sample period, 1996-2005, is inappropriately brief for examining the equity premium (see Dimson, Marsh, and Staunton (2008)).

Country-specific emerging market studies, other than for South Africa, include Penha Cysne (2006), examining Brazilian data from 1992 to 2004, and Mehra (2006), studying Indian data from 1984 to 2004. Both studies find the observed equity premium to be too high to be explained by the basic consumption-based model; but their findings are somewhat limited by the short sample periods.

Firer and Staunton (2002) document the size of the premium for South Africa, over a century of data, and explain in detail the construction of the South African data set (also used here). Digby, Firer, and Gilbert (2006) update the same data set, provide an extensive review of previous estimates of the South African premium, and concentrate on the use of discounted cash flow models (based on accounting ratios) to estimate the premium. Dimson, Marsh, and Staunton (2008) compare the magnitude of the equity premium in the 17 countries for which a century of equity market data are available, which includes South Africa as the only emerging economy. However, none of these studies confronts the canonical consumption-based asset pricing model with the data, leaving unanswered whether a theoretically defensible measure of non-diversifiable risk can explain the long-term return on the South African stock market in excess of a risk-free rate. Thus, none of the studies establishes whether an equity premium puzzle, in the sense identified by Mehra and Prescott (1985) and at the source of a large literature in macroeconomics and finance (Campbell (2003)), applies to South Africa. Moreover, none of the previous studies of the South African premium examine the related risk-free rate puzzle (Weil (1989)).

[4] Note a distinction between merely documenting the equity premium, and confronting the size of the premium with the prediction from asset pricing theory.

3 Data and computation of returns

3.1 Data

Systematically collected capital markets data for South Africa are only available from the 1960s. A number of papers attempt to remedy this. Firer and McLeod (1999) draw on a number of different sources and employ the methods in Ibbotson and Sinquefield (1989) to construct series on South African equities, bonds and cash from 1925 to 1998. This data set was extended (and the equity index broadened, to include resources) back to 1900, by Dimson, Marsh and Staunton (2002), resulting in South Africa being one of only 17 countries with reasonably reliable capital markets data stretching over (now more than) a century. (See also Dimson, Marsh and Staunton (2008).) Firer and Staunton (2002) pool these data and construct a combined equity index for South Africa for the period from 1900 to 2001, updated to 2004 in Digby, Evan and Firer (2006).[5] We use an update of the Firer and Staunton (2002) equity index as our proxy for the aggregate stock market, extended to 2005.[6]

For the proxy of the risk-free rate, we use a short-term money market rate constructed from negotiable certificates of deposit (see Firer and Staunton (2002) for the details and motivation for the NCD rate rather than the more common 90-day Treasury bill rate for the short rate). To test the role of liquidity in generating an equity premium (Bansal and Coleman (1996)), we also compute and report stock market returns over a long-term government bond index (based on the JSE Actuaries All Bond Index for the post-1986 period). The inflation rate is the rate of change in the Consumer Price Index, and the consumption proxy is per-capita consumption of non-durables and services (Campbell (2003)), from Datastream.

Due to evident differences in the reliability of the pre- and post-1960 data, we report separate results for the full sample, and the post-1960 sub-period. In addition, we consider a post-1975 sub-period, for which we use a published long-term government bond index unavailable for earlier periods, as the proxy for the long rate (permitting the examination of a liquidity premium). For this sub-period, we also use the Treasury bill rate as an internationally more common proxy for the short rate.[7]

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[5] Firer and McLeod (1999) use a 1948 Bureau of Economic Research study on commercial and industrial share-price industries in South Africa (1910-1947); the Rand Daily Mail 100 Industrial Index (1949 to 1959); and the JSE Actuaries index for the post 1960 period. Dimson, Marsh, and Staunton (2002) create a market capitalisation index using a weighted basket of different sectors, including resources. The Firer and Staunton (2002) set effectively consists of the Dimson, Marsh, and Staunton (2002) series pre-1960; and the ("cleaner") JSE Actuaries Index post-1960.

[6] We are grateful to Colin Firer for kindly sending us his data set. Note that the JSE performed exceptionally well between 2006 and 2008, with the All Share Index exceeding 30 000 before falling sharply as a consequence of the US sub-prime crisis. The sharp decline in the market during 2008/2009 brought the JSE back to 2005 levels. The index closed at 18 640 on 6 March, 2009 (and reached a low of 17 953 in the same week). The closing value on 31 December 2005 was 18 096.

[7] The long bond index is the yield on government bonds with maturity of ten years or

3.2 Arithmetic and geometric averages

The expected future value of an initial unit rand investment is obtained by compounding the average return. The most commonly used method for computing average returns in equity premium studies is the arithmetic average. This is a reliable statistic for computing the mean terminal value of the investment, provided returns are serially uncorrelated. If returns are correlated over time, arithmetic averages may overstate the terminal value. When this is the case, the geometric average, which is the correct statistic for computing the median terminal value of the investment, is more reliable (Mehra and Prescott (2003)).

With one exception, previous research on the magnitude of the South African equity premium uses only arithmetic averages. (See Digby, Evan and Firer (2006), page 5.) If the log-normal distribution is a valid approximation for aggregate South African equity market returns, arithmetic average returns will exceed geometric average returns by one-half the variance of returns. (Mehra and Prescott (2008).) The extent to which the South African equity premium was overestimated (when returns are serially correlated) in previous studies will therefore depend on the variability of returns. We show that over the 1900 – 2005 period the variance of the annual South African equity premium was 0.04, corresponding to a difference in average arithmetic and geometric returns of two percentage points per annum. For illustration, consider a one rand initial investment, compounded annually over 105 years. If the average annual return is five percent, the investment's terminal value will be 168; if the average return is instead seven percent annually, the terminal value will be 1.217 – seven times larger.

The autocorrelation coefficient for South African real equity returns over the entire sample period is *circa* 0.04 – higher than the serially uncorrelated US returns over the same period, but lower than the 0.07 average serial correlation reported in Dimson, Marsh and Staunton (2008, page 15). However, excess returns over the post 1960 sub-period are more auto-correlated, with a coefficient of 0.15 for the premium over the long-term bond rate – not particularly high, but twice the average rate in Dimson, Marsh and Staunton (2008). (South African interest rates are highly serially correlated, with coefficients close to 0.9.) Thus, we report the results for the equity premium (excess real returns) using both arithmetic and geometric means.

4 Estimating the equity premium

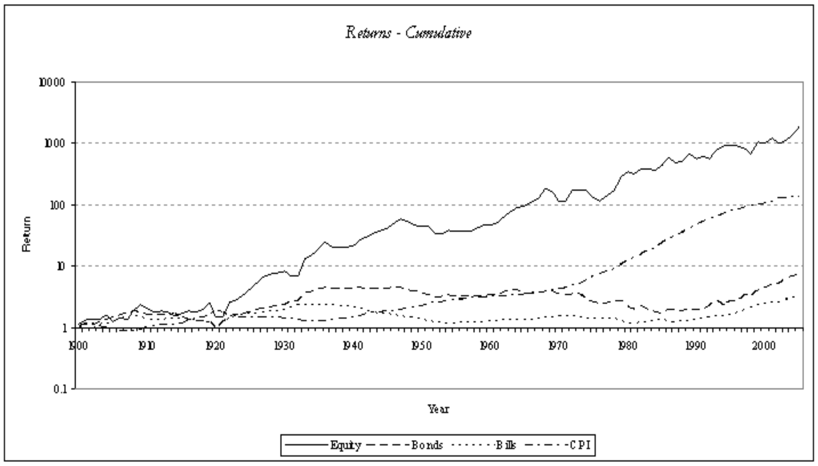
Figure 1 plots a cumulative history of equity, bonds, bills and the consumer-price inflation index, with an asset's cumulative return (from year 0 to year t), $\Gamma_{i,t}$,computed as

$$\Gamma_{i,t} = \prod_{k=0}^{t} R_{i,k}, \tag{1}$$

longer, traded on the bond exchange (series code KBP2003M).

where $R_{i,k}$ is the gross return (i.e. one plus net return) on asset i in year k. The graph illustrates vividly the higher real (i.e. inflation-adjusted) cumulative returns offered by the stock market, producing a large equity premium.

Figure 1: Comparison of cumulative returns



Over long horizons, the variability in the equity premium is almost exclusively due to variability in returns from equity, as the risk-free instruments display relatively smooth series. (Of course, this need not be the case over short horizons.)

Figure 2: Stock market excess returns

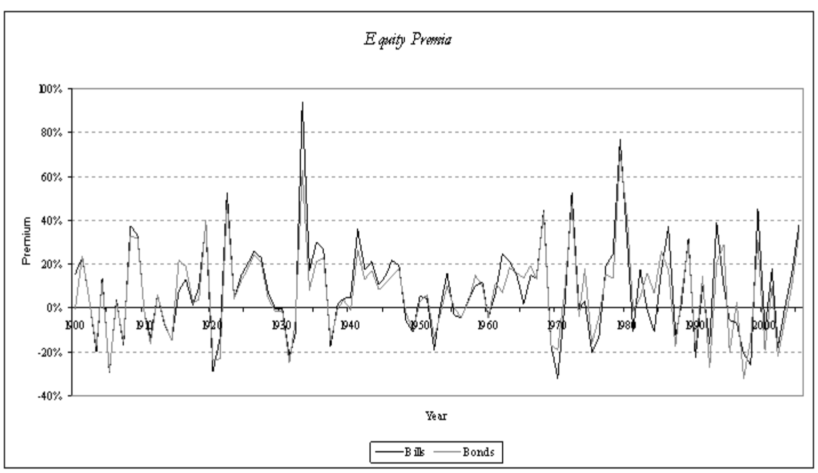


Figure 2 shows that equity returns over bills were higher than over bonds; but the two series move closely together. Averages for the entire period are presented in Table 1 (part A). For this long sample, the premium over bonds is 5.49%. The premium over bills is 6.30%. Arithmetic results produce estimates of 7.08% and 8.22% for the premium over bonds and bills, respectively – among the highest of the list of countries with similarly long data sets, but comparable to Australia and Sweden over the same period, using arithmetic means (see Dimson, Marsh, and Staunton (2008)).

Table 1: Equity premia
A. Sample period: 1900-2005

Return from Equity		**Return from Bond/Bill**		**Equity Premium**		
Geom	Arith	Geom	Arith	Geom	Arith	
7.38%	9.46%	1.89%	2.38%	5.49%	7.08%	**Bond**
7.38%	9.46%	1.08%	1.24%	**6.30**%	8.22%	**Bill**

B. Sample period: 1960-2005

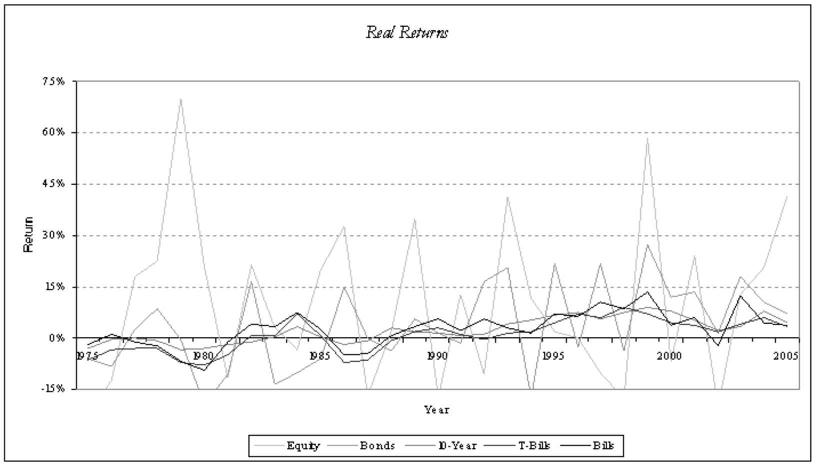
Return from Equity		**Return from Bond/Bill**		**Equity Premium**		
Geom	Arith	Geom	Arith	Geom	Arith	
8.36%	10.56%	1.78%	2.39%	6.59%	8.17%	**Bond**
8.36%	10.56%	2.05%	2.15%	**6.32**%	8.42%	**Bill**

C. Sample period: 1975-2005

Return from Equity		**Return from Bond/Bill**		**Equity Premium**		
Geom	Arith	Geom	Arith	Geom	Arith	
8.07%	10.39%	2.31%	2.38%	5.76%	8.01%	**10-Year**
8.07%	10.39%	0.73%	0.84%	7.34%	9.55%	**T-Bill**

The post-1960 sub-period offers more reliable stock market data. Part B of Table 1 shows that over this period the observed premium (geometric returns, over cash/bills) was only marginally higher than the average for the whole period, at 6.32%. The post-1975 sub-period offers more reliable risk-free proxy data: the long-term bond (the 10-year government bond index) and Treasury-bill rate returns data. Figure 3 illustrates how the returns from each of the different instruments vary. Clearly, equity is the most volatile, and the 10-year government bond index the least.

Figure 3: Variation in real returns across asset classes



The performance of long-term bonds improved over the post-1975 period; but so did the stock market. The equity premium over bonds remains relatively consistent at 5.76% (8.01%, arithmetic);[8] the premium over t-bills increased to 7.34% (9.55% arithmetic). Appendix A illustrates the wealth creation implications of the magnitude of the South African equity premium, over the full sample and each of the two sub-samples.

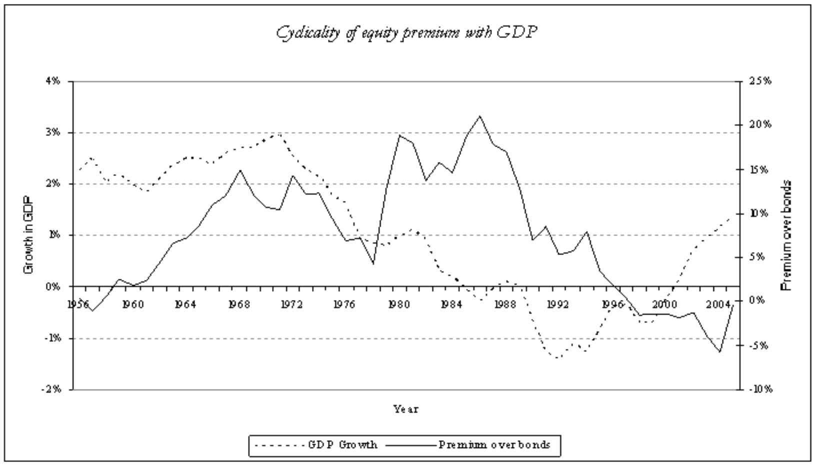
5 Variation over time

5.1 Cyclicality

The equity premium has varied significantly over time, as Figure 4 shows. The 10-year average realised equity premium over bonds is plotted against growth in seasonally adjusted GDP.

[8] This shows that the puzzle cannot be explained by the liquidity of short-term instruments, as discussed further below.

Figure 4: Co-variation of equity premium with growth

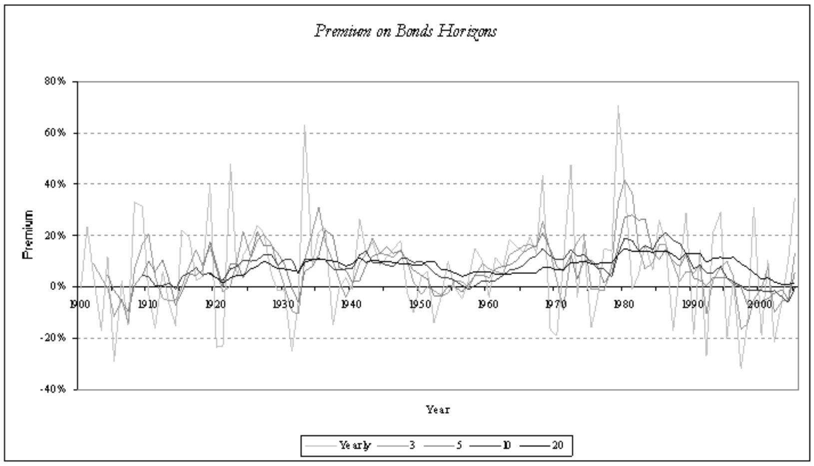


The correlation between the premium and GDP growth for the entire sample is weakly positive at 0.074. If the sample is changed to 1985 to 2005, the correlation is -0.255. At the low frequency there seems to be no evident, consistent counter- or pro-cyclicality of the equity premium in South Africa. But, over the last couple of decades, increases (respectively, decreases) in GDP growth seem to precede an increase (resp., decrease) in the premium.

5.2 Time horizons

We compute three-, five-, ten- and twenty-year moving averages of the yearly realised premium over bonds. Figure 5 shows that the volatility in the equity premium decreases as we increase the time horizon over which it is measured; and it can easily be negative, at relatively short investment horizons. Interestingly, at the 20-year investment horizon an investor would not have experienced a single negative realised equity-premium over the entire 105-year period – i.e. over 20-year horizons, the stock market always outperforms the bond market.

Figure 5: Equity premium over different investment horizons



Moreover, as we extend the horizon, the standard deviation diminishes (while the average realised premium increases steadily), as reported in Table 2.

Table 2: Equity premium horizons and standard deviations

		Bonds	Bills
Yearly	Geom	5.37%	6.17%
	Arith	7.08%	8.22%
	S.D.	19.36%	21.76%
3-Year	Geom	6.36%	7.24%
	Arith	6.89%	7.92%
	S.D.	10.79%	12.26%
5-Year	Geom	6.68%	7.62%
	Arith	6.97%	7.96%
	S.D.	7.95%	8.66%
10-Year	Geom	7.29%	8.15%
	Arith	7.43%	8.30%
	S.D.	5.57%	5.65%
20-Year	Geom	8.14%	8.94%
	Arith	8.19%	9.00%
	S.D.	3.36%	3.49%

5.3 Sharpe ratios

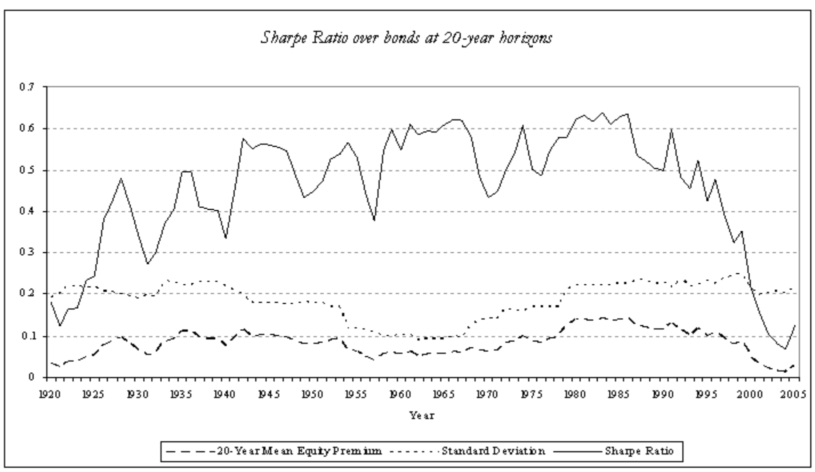
We examine the commonly used Sharpe ratio – the ratio of the average return to its standard deviation. Table 3 shows the Sharpe ratios for the different instruments. A value of the Sharpe ratio of approximately 0.3 is comparable to the ratios calculated for advanced economies where the equity premium puzzle has been identified (Mehra & Prescott (2003)).

Table 3: Sharpe ratios

	Bonds	Bills	Bonds (1960)	Bills (1960)	10-Year Bond Index (1975)	T-Bills (1975)
Equity Premium	5.49%	6.30%	6.59%	6.32%	5.76%	7.34%
Standard Deviation	22.07%	22.07%	22.66%	22.66%	23.55%	23.55%
Sharpe Ratio	0.249	0.285	0.29	0.279	0.245	0.31

Figure 6 examines the evolution of the Sharpe ratio for the South African stock market, using a twenty-year investment horizon, of the premium over bonds. This reveals that although the average was close to 0.3, the ratio was in fact above 0.3 from the early 1930s to the end of the 20th century, with a marked downward trend starting in the mid to late 1980s, reversed from 2005.

Figure 6: Sharpe ratio at twenty-year horizons



Significantly, the post 1980s downward trend in the Sharpe ratio was due to a decline in the mean equity premium rather than an increase in the standard deviation.

6 Is the premium compensation for non-diversifiable risk?

6.1 The canonical consumption-based asset pricing model[9]

Let $R_{i,t+1}$ denote the gross return (i.e. one plus net return) from a (any) risky asset, at time $t+1$, and E_t the expectation conditional on information at time t. In rate of return form, we can write the first-order condition for the standard inter-temporal investment-consumption problem (Campbell (2003), Cochrane (2005)) as

$$1 = E_t \left[R_{i,t+1} M_{t+1} \right], \tag{2}$$

where M_{t+1} is the stochastic discount factor (also known as the pricing kernel or inter-temporal marginal rate of substitution), given by

$$M_{t+1} = \frac{\delta U'(C_{t+1})}{U'(C_t)}. \tag{3}$$

U is the (increasing, concave) utility function, and δ is the subjective time discount factor. Consider a representative agent with time-separable power utility (so that the risk premium does not change with the scale of the economy), defined over aggregate consumption,

$$U(C_t) = \frac{C^{1-\gamma} - 1}{1-\gamma}, \tag{4}$$

where γ is the coefficient of relative risk aversion. The stochastic discount factor becomes, after taking logs, $m_{t+1} = \log \delta - \gamma \Delta c_{t+1}$, where $m_{t+1} = \log M_{t+1}$ and $\Delta c_{t+1} = \log C_{t+1} - \log C_t$.Let a portfolio of JSE-listed shares be the risky asset in question. Following Hansen and Singleton (1983), assume that stock market returns and the stochastic discount factor (or aggregate consumption) are jointly log-normal and homoskedastic. Re-arranging equation 2; using 4; the fact that for a risk-free asset $R_{f,t+1} = 1/E_t M_{t+1}$; and the log-normality and homoskedasticicty assumptions, we have

$$r_{f,t+1} = -\log \delta + \gamma E_t \Delta c_{t+1} - \left(\gamma^2 \sigma_c^2 \right)/2, \tag{5}$$

and

$$E_t \left[r_{i,t+1} - r_{f,t+1} \right] + \sigma_i^2/2 = \gamma \sigma_{ic}, \tag{6}$$

where $r_{f,t+1} = \log R_{f,t+1}$, $r_{i,t+1} = \log R_{i,t+1}$, and $\sigma_{ic} = cov(r_{i,t+1}, c_{t+1})$. In words, the risk-free rate is determined by the rate of time preference; the coefficient of risk aversion times expected future consumption; and a factor of risk

[9] A popular alternative to the theoretic asset pricing approach used here is to simply exploit the implications of the breakdown of stock returns into a dividend yield plus capital gain. See Fama and French (2002). In the general equilibrium settings of Lucas (1978) and Mehra and Prescott (1985), the stock market, as a proxy for all wealth, is priced so as to pay consumption as its dividend. Consumption growth drives the expected dividend growth rate. (Dividends need not equal measured consumption though. See also Campbell (2005), page 808 and 840-841.)

aversion and consumption variability. The first term reflects preference for current rather than future consumption; the second reflects the desire to borrow today if future consumption is expected to be high (consumption smoothing over time); the third term reflects demand for precautionary saving, or hedging against adverse consumption realisations in the future (consumption smoothing over states).

The risk premium is determined by the coefficient of risk aversion times the covariance of stock market returns with consumption growth. (The $\sigma_i^2/2$ term is a Jensen's inequality adjustment, translating arithmetic to geometric averages.) Intuitively, high covariance with consumption means low returns when consumption is low, and therefore marginal utility of consumption is high. Such an asset is risky in that it pays poorly when times are bad and additional wealth is valued highly; but pays well when times are good and marginal utility is low (due to concave utility). To compensate, investors require a higher return than they would for an asset with low or negative covariance with consumption.

6.2 Quantifying the equity premium puzzle in South Africa

The average rate of growth in real per-capita consumption in South Africa is 1.4 percent; the standard deviation of consumption growth is 2 percent. Start by setting the coefficient of relative risk aversion to 10, considered its maximum plausible level by Mehra and Prescott (1985); assume perfect correlation between stock market returns and consumption growth; and for the subjective discount factor, let $\delta = 0.99$. Using the formulas in 5 and 6, we obtain the following estimates:

Table 4: Theoretic equity premium estimate

Risk-free Rate	Return on Equity	Equity Premium
13.01%	13.41%	0.4%

Comparing to Tables 1-2 it is evident that, although the signs are correct, from a quantitative viewpoint the model cannot remotely account for any of the observed average equity premium estimates, which range between 5.4 and 7.3 percent (geometric), or 7.0 and 9.5 percent (arithmetic). Decreasing the coefficient of risk aversion, or assuming lower covariance between stock returns and consumption growth, only increases the distance between the observed and theoretic premium (by reducing the predicted risk-free rate, and excess returns).

Table 5 illustrates further the extent of the equity premium puzzle in South Africa, by using the equity premium equation 6 to obtain the coefficients of risk aversion implied by the data.

Table 5: CRRA estimates

	Bills	Bonds	10-Year	T-Bills
Standard Deviation of Log Equity Premium	21.09%	19.69%	21.09%	21.61%
Standard Deviation of Log Consumption Growth	1.96%	1.96%	2.18%	2.18%
Correlation between Consumption Growth and Equity Premium	-0.0344	0.0933	-0.0581	-0.0602
Covariance	-0.0001	0.0004	-0.0003	-0.0003
Covariance*	0.0041	0.0039	0.0046	0.0047
Equity Premium	8.4%	8.4%	7.7%	9.3%
CRRA	-593	233	-288	-330
CRRA*	20	22	17	20

Covariance is the product of the correlation coefficient with the standard deviations, so the product of the first three rows in Table 5 yields the covariance of the observed equity premium with consumption growth. The covariance is in each case very close to zero; but in three columns it is negative. The cases where covariance is negative lead to negative coefficients of risk aversion (of very large magnitudes), shown in the penultimate row, entitled CRRA. We follow Campbell (2003) and set the correlation of stock returns and consumption growth to one, to obtain the adjusted covariances in the row entitled Covariance*.[10] These covariances give the adjusted coefficients of relative risk aversion CRRA*, reported in the last row, and obtained using the observed equity premium for the left-hand side of the premium equation 6. Since we restricted correlation to one, these are the lowest possible coefficients of relative risk aversion given the observed premium – i.e. lower bounds on the coefficients implied by the data, if the model were correct. Yet, for the premium over short-term debt, they are still twice the risk aversion value of 10, commonly regarded as an upper bound

[10] Although a counterfactual exercise, as noted by Campbell (2003, page 822), this indicates the extent to which the equity premium puzzle is due to the smoothness of consumption, rather than low correlation; and can be justified by difficulties in accurately measuring correlation, due to short-term measurement errors in consumption.

in the literature (Mehra and Prescott (1985), (2008)). If we do not restrict the correlation between consumption growth and stock returns, and consider the only case where covariance is positive (the estimate in the third column), we obtain an implied coefficient of 233 – egregiously irreconcilable with both theory and evidence, as discussed in section 6.4.

6.3 Liquidity effect on short-term interest rates

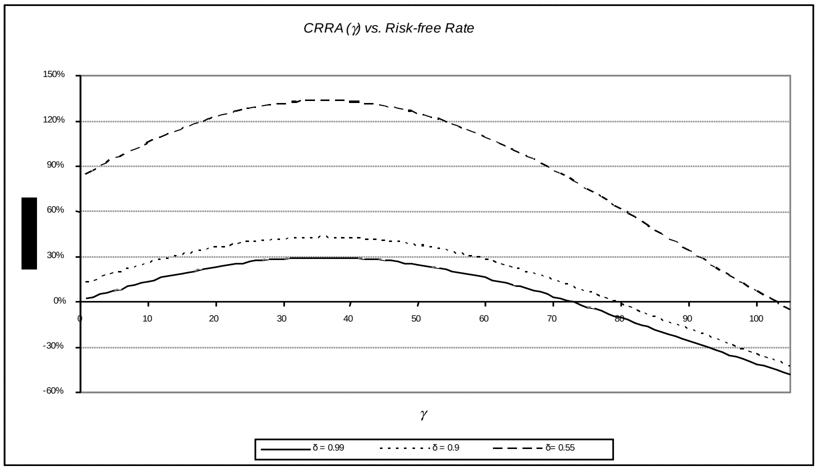
Our discussion has concentrated on the premium computed as the excess return over the short-term rate. Short-term debt instruments are "moneylike." This liquidity advantage reduces the required return on short-term debt and can overstate the equity premium. (Bansal and Coleman (1996).) If the equity premium is due to the liquidity advantage of short term debt, it should be easier to account for the equity-bond premium. Throughout this paper we have reported the excess returns over the risk-free rate as well as the long bond rate. (See tables and figures above.) There is clearly a liquidity effect, since the average equity-bond premium is smaller than the equity premium over the short-term rate. However, the premium remains too large to be reconciled with equation 6 and a parameter of risk aversion equal to or smaller than 10.

6.4 The risk-free rate

Figure 7 plots the equation for $r_{f,t+1}$ in 5, calibrated to South African historical average consumption growth of 1.4 percent, with standard deviation of 2 percent, for three different values of δ. It shows that the equation may match the observed average real default-free rate, which ranges between *circa* 0.7 and 2 percent (see Table 1), with either very low or very large values for γ - the latter case reflecting the effect of the negative quadratic term $-\left(\gamma^2\sigma_c^2\right)/2$ in equation 5, which captures the precautionary motive to save.

As we saw in the previous section, the consumption-based model can only explain the equity premium under very high risk aversion. For any value of δ, a value of 233 for γ results in a negative theoretic risk-free rate. At the implied lower bound for γ of 20, the theoretic real risk-free rate is above 20 percent. Thus, if explaining both the South African equity premium and risk-free rate is at all possible through the consumption-based model, this would require unrealistically high coefficients of risk aversion, as well as a very low time discount factor. Put differently, unlesss we resort to negative rates of time preference, the coefficients of risk aversion required to reconcile the consumption-based model with the observed equity premium lead to the "risk-free rate puzzle" (Weil (1989)): the consumption-based model becomes incompatible with the observed risk-free rate. (Specifically, the observed real interest rate is "too low".)

Figure 7: The risk-free rate puzzle



These observations are consistent with US and international evidence. Examining data from 12 advanced economies, Campbell (2003, page 825) finds, using different values of the risk-aversion parameter, that a realistically low risk-free rate with positive consumption growth can only be reconciled with the consumption-based model under negative rates of time preference. This highly implausible implication applies to all countries in his sample, except one (Switzerland).

Note, however, that the risk-free rate puzzle is a consequence of constraining the coefficient of risk aversion to values which permit the canonical model, in its basic form, to account quantitatively for the equity premium. It is not a refutation of the basic model's ability to match the real risk-free rate. Discussions of the risk-free rate puzzle (e.g. Weil (1989), Campbell (2003), Cochrane (2005), Mehra and Prescott (2008)) abstract from the fact that the choice of 10 as the maximum plausible level for the coefficient of relative risk aversion, although standard, is somewhat arbitrary. The literature review in Mehra and Prescott (1985, page 154) indicates that any coefficient of relative risk aversion above zero and below 2 is empirically defensible. On theoretical grounds, Arrow (1971) proposes a value for γ of one, which is consistent with most of the evidence reviewed in Mehra and Prescott (1985, page 154) as well as Altug (1989).

It is straightforward to verify that the consumption-based asset pricing model can match the observed average levels of the South African risk-free rate using theoretically and empirically defensible values for the coefficient of risk aversion, and without resorting to a negative rate of time preference. Using the historical averages for consumption growth and standard deviation of consumption growth in South Africa, and a value of 0.99 for δ, the subjective discount factor, we have the following consumption-based model estimates of the risk-free rate, for values of γ between 0.5 and 1.5:

Table 6: Theoretic risk-free rate

γ	r_f
0.5	0.0170
1.0	0.0238
1.5	0.0306

The sample estimates of the risk-free rate were 1.08, 2.05, and 0.73 (geometric, percent); and 1.24, 2.15 and 0.84 (arithmetic, percent) – see Table 1. Take the largest sample point estimate of the geometric average rate, namely 2.05%. The basic consumption-based model matches this rate exactly, with a perfectly plausible coefficient of risk aversion of 0.7 – see Dunn and Singleton (1986) and Altug (1989, page 907), on the plausibility of this value for γ. In appendix B, we show that the basic consumption-based model approximates the average real risk-free rate in most advanced economies in the Campbell (2003) sample, while retaining a plausible rate of time preference, provided we use a realistic value for the coefficient of risk aversion.

7 Concluding remarks

South Africa is a natural complement to the existing long-run evidence on the equity premium (and risk-free rate) puzzle. It is a country for which, for much of its turbulent history, it seems plausible to use the upper bound of the range of "sensible values" for the coefficient of risk aversion; and where the possibility of economic disaster must have appeared non-trivial in the minds of investors on more than once occasion in the country's history. Yet, we find that the canonical consumption-based asset pricing model applies as poorly as it does to advanced economies over equally long sample periods. The magnitude of the equity premium is comparable to the US and other advanced economies; and so is its distance to the consumption-based theoretic premium.

However, we also note that the canonical consumption-based asset pricing model gives a reasonably accurate estimate of the South African real risk-free rate, if we use realistically low values for the coefficient of risk aversion. This modest observation may be worth noting for the following reason. Among the most popular extensions of the basic model, designed to explain the equity premium, are habit-persistence models (see Cochrane (2005)). Some of these models, particularly external habit models, where habit depends on aggregate consumption rather than the investor's own consumption (e.g. Abel (1990, 1999)), result in the same specification for the equity premium adjustment as in the canonical power utility model used here (see equation 6, and Campbell (2003, page 867)). What changes is the theoretic formulation of the risk-free rate (equation 5 here). If the basic model does not perform poorly in matching the risk-free rate when we use realistic values for the coefficient of risk aversion, improvements in quantitatively matching observed equity premia with the predictions of external habit models may be due entirely to a poorer fit to the risk-free rate.

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9 Appendix A: Wealth Creation Implications

The equity premium puzzle has clear implications for the allocation of social security funds and other long-term investment allocation decisions. The next table illustrates the long-term wealth creation implications of the equity premium in South Africa. The second and third columns give the inflation-adjusted terminal value of a R1 initial investment, in the stock market and in Treasury bills, respectively. The fourth column gives the ratio of the former to the latter.

Investment period	Stocks	T-Bills/NCDs	Ratio
1900-2005	1.644,5	3,06	538
1960-2005	34,22	2,44	14
1975-2005	9,49	1,23	8

10 Appendix B: The risk-free rate

This table shows the average real risk-free rate implied by the basic consumption-based asset pricing model, with the coefficient of risk aversion set to one, and a subjective discount factor of 0.99, i.e. $\gamma = 1$ and $\delta = 0.99$. The data on consumption growth, variance of consumption, and risk-free rates for the economies other than South Africa are from Campbell (2003). From equation 5, $r_{f,t+1} = -\log\delta + \gamma E_t \Delta c_{t+1} - \left(\gamma^2 \sigma_c^2\right)/2$; r_f^a is the sample average risk-free rate.

Country	Δc	$\sigma(\Delta c)$	r_f	r_f^a	$r_f - r_f^a$
USA	0.020	0.011	0.030	0.009	-0.021
Australia	0.021	0.021	0.031	0.021	-0.010
Canada	0.021	0.020	0.031	0.027	-0.004
France	0.012	0.029	0.022	0.027	0.005
Germany	0.017	0.024	0.027	0.032	0.006
Italy	0.022	0.017	0.032	0.024	-0.008
Japan	0.032	0.026	0.042	0.014	-0.028
Netherlands	0.018	0.026	0.028	0.034	0.006
Sweden	0.010	0.019	0.019	0.020	0.000
Switzerland	0.005	0.021	0.015	0.014	-0.001
South Africa	0.014	0.020	0.024	0.021	-0.003