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ABSTRACT.

Economic interdependency of wildlife or fish stocks is usually attributed to ecological interdependency, such as predator – prey and competitive relationships, or to density dependent migration of species between different areas. This paper provides another channel for economic interdependency of wildlife where density *in*dependent migration and market price interaction affect the management strategies among different landowners. Management is studied under three market conditions for selling hunting licences: price taking behaviour, monopoly market and duopoly market. Harvesting of the Scandinavian moose is used as an example. The paper provides several results on how economic interdependency works through the migration pattern. When a duopoly market is introduced, hunting license price interaction among the landowners plays an additional role in determining the optimal harvesting strategy.

KEY WORDS: wildlife management, moose, migration, grazing damage, market structure

## 1. Introduction

In most models with *ecological* interdependency, such as predator-prey models and models of competition, there is also *economic* interdependency; that is, the (optimal) exploitation of one species, e.g., a fish stock, influences the exploitation of the other species, and *vice versa* (e.g., Hannesson [1983] and Clark [1990] ). If these species are managed by different agents, there will also be economic interdependency between the agents. We find the same situation if wildlife or fish stocks disperse in a density dependent manner between different areas, or patches, managed by different owners (e.g., Conrad [1999], Sanchirico and Wilen [2001], Armstrong and Skonhoft [2005]). Economic interdependency also can occur even without any direct biological, or ecological, connections. This may happen when, for example, a migratory fish stock is exploited sequentially over the year cycle by different agents (e.g., Charles and Reed [1985]), or in fisheries with imperfect selectivity (Clark [1990], Ch. 10). Another example is when two agents are supplying fish or wildlife products to a common market where the harvested products interact in the market (e.g., Halsema and Withagen [2008]). The same may happen in various payment for environmental service (PES) situations, for example, when the payment for conserving a wildlife species is related to the stock abundance and where the resource owner uses the size of the PES strategically (e.g., Staahler [1996], Bulte and van Kooten [2002]).

In this paper, we show that economic interdependency also may occur even when animal species migrate in a density *in*dependent manner between different areas. The particular case under study is Scandinavian moose hunting, where the moose frequently migrate between different areas over the annual cycle, depending on snow and forage conditions, and are subject to hunting by different landowners. The most common migratory pattern among moose is density *in*dependent; that is, the migration, or dispersal, is not contingent upon species density in the various areas (Saether *et al*. [1992]). Moose are valuable in the hunting season, which takes place in the fall, but cause browsing damage to forest products during the winter. Therefore, the moose provides value but is also a pest. It is the browsing damage that may create an economic interdependency between different landowners. The moose is one example of a species migratory pattern that occurs seasonally over the year and is more or less density independent. The wildebeest migration in the Serengeti-Mara ecosystem is a famous example of such a pattern (e.g., Sinclair and Arcese [1995]).

The scope of this paper is to study the degree and strength of such economic interdependencies under different market and damage cost assumptions. Two landowners and two moose subpopulations are considered. The core of the biological model, describing harvest, natural growth and migration, draws on Skonhoft and Olaussen [2005], and a continuous time version of their model is considered. The model is also extended to a dynamic framework where a dynamic open loop game is generated when the market for hunting licences is interconnected (e.g., Dockner *et al.* [2000]). Various economic interdependency situations among the landowners are analysed. In these situations, we assume that both landowners aim to find hunting quotas that maximize present value net benefit, which equals the hunting value minus the forestry damage. Optimal harvest strategies in steady state as well as in transitional dynamics will be studied. The various types of economic interdependency considered have important management implications, as both unidirectional migration and market price interaction play an important role in the harvesting decision. Unified management schemes (the social planner solution) are discussed in Skonhoft and Olaussen [2005] and hence are not covered in this paper. Our model and analysis are closest to that of Halsema and Withagen [2008]. Harvest cost functions in their model and browsing damage cost functions in our model are both density dependent; that is, both models are contingent upon the number of species. The important difference is that we include an additional link through the density independent migration of the animals.

The paper is organised as follows. In section two, the ecological model and the cost and benefit functions of the landowners are presented. We first study two extreme market conditions. The perfect competitive market for selling moose hunting licenses is considered in section three, while monopoly power is analyzed in section four. We find that the classification of the moose as a 'value' or a 'pest' has crucial importance for the dynamics as well as for the steady state analysis in the monopoly case. The moose is considered valuable if the marginal moose harvest revenue is positive, while it is a pest when the harvest revenue is negative. The intermediate situation, duopoly market, which is far more complicated than the two extremes, is analyzed in section five. In section six, we illustrate the various characteristics in these models numerically and consider both transitional dynamics and equilibrium. As the Scandinavian moose typically will be valuable and not a pest, we focus basically on the value case. Finally, section seven concludes the paper.

## 2. Model

The moose (*Alces alces*) is the world's largest member of the deer family and is found in the northern forests of North America, Europe and Russia. It is the most important game species in the Scandinavian countries. In Norway and Sweden about 40,000 and 100,000 animals, respectively, are shot every year (Statistics Norway [2011], Svenska Jägerforbundet [2009]). However, the moose population also causes various costs. There is a high incidence of moose-vehicle collisions, as well as browsing damage during the winter, when young pine trees are an important food source (e.g., Solstad [2007]). The browsing damage may be considerable in some areas. Because of large spatial variation in densities during the winter season, browsing damage is usually unevenly distributed between different areas. Migration and concentration are two important factors explaining these differences, as some subpopulations tend to leave their summer ranges and browse in specific winter ranges due to snow and forage conditions (Saether *et al.* [1992], Ball *et al.* [2001]). Hence, as hunting takes place in the fall, before yearly migration, there is often an asymmetry between areas obtaining benefit from harvesting and areas with heavy browsing damage.

Two areas, areas 1 and 2, with two different landowners, owners 1 and 2, and two subpopulations of moose, subpopulation 1 and 2, are considered. Both landowners are allowed to issue and sell licenses for hunting on their own land, and the licence fee is paid to the landowners. For subpopulation 1, we use the most common of three migration patterns (distinguished by Saether *et al.* [1992]); this is a distinct and more or less fixed yearly migration pattern between a summer range and a winter range. The migration may be of a rather short distance, possibly between two adjoining areas, or it may be a long distance migration, possibly of several hundred kilometres. Snow, topographical and forage conditions during the winter are of particular importance (Saether *et al.* [1992]). This migration pattern is modelled by letting a fixed (exogenous) fraction of one of the subpopulations migrate during the winter. By convention, we assume that the dispersal runs from area 1 to area 2. These two areas are considered as a closed system in which, after the winter, all the migratory moose return to their summer range. The migration system is the same as that in Skonhoft and Olaussen [2005] where a real life example is also provided. The hunting season is in September/October, before the yearly migration. Harvesting income is therefore directly related to the summer range of the two subpopulations. The migrating fraction of subpopulation 1 causes forestry damage in area 2 during the winter season, but subpopulation 2 does not cause damage in area 1, because it is non-migratory. See Figure 1.

Figure 1 about here

Neglecting any stochastic variations in environment and biology, the equations

(1) $dX_1 / dt = F(X_1) - h_1$

and

(2) $dX_2 / dt = G(X_2) - h_2$

represent the populations dynamics, where $X_i\,(i = 1,2)$ is size of subpopulation $i$, measured in biomass (or number of animals) at time $t$ (time index is omitted), $h_i$ is harvest, and $F(X_1)$ and $G(X_2)$ are density dependent natural growth functions. Natural growth is assumed to be logistic, where $F(X_1) = rX_1(1 - X_1 / K_1)$ is for subpopulation 1 and $G(X_2) = rX_2(1 - X_2 / K_2)$ is for subpopulation 2. $r > 0$ is the identical intrinsic growth rate and $K_i > 0$ is the carrying capacity ($i = 1,2$), typically depending on the size of the land and habitat productivity. Notice that there is no ecological interdependency between the two subpopulations, as there is no density dependent growth process, due to, e.g., forage competition, during the winter when part of the subpopulations are located within the same area. Generally, there is no evidence of density dependent mortality of the Scandinavian moose (see, e.g., Nilsen at al. [2005]).

The fraction of the population migrating from area 1 to area 2 after hunting season, depending on snow and food conditions, as well as topography and size of the areas, is fixed as $0 \le \alpha \le 1$. The migratory population out of area 1 is therefore $\alpha X_1$ so that the remaining stock browsing in its home range during the winter becomes $Z_1 = (1 - \alpha)X_1$. $Z_2 = X_2 + \alpha X_1$ is the stock browsing in area 2 during the winter season. As already indicated, the forest browsing damage on pine occurs during the winter when other food sources are restricted. The damage is directly related to the number of animals (Skonhoft and Olaussen [2005]; Wam and Hofstad [2007]). The damage cost function may vary between areas due to different quality of the timber stands, or simply because of different forest productivity. Both a linear and a strictly convex function are considered, and the damage function is hence defined as $D_i(Z_i)$ with $D_i(0) = 0$, $\partial D_i / \partial Z_i = D_i' > 0$ and $D_i'' \ge 0$ ($i = 1,2$).

When the cost is assumed to account for a fixed fraction of the licence price, the yearly hunting income writes $P_i h_i$ such that $P_i\ (i = 1, 2)$ is the 'net' hunting licence price. The hunting licence price may be equal among the landowners, or it may be different due to different market and demand conditions. Two extremes, the market with perfect competition and the market with monopoly power, are considered, with the former situation possibly being the most common market structure in Scandinavia[1]. In addition, market interaction through duopoly is studied. When any further cost and benefit components are neglected[2], the yearly net benefit for landowner 1 is:

(3) $$\pi_1 = P_1 h_1 - D_1(Z_1) = P_1 h_1 - D_1((1-\alpha)X_1)$$

while

(4) $$\pi_2 = P_2 h_2 - D_2(Z_2) = P_2 h_2 - D_2(X_2 + \alpha X_1)$$

is for landowner 2.

Because of the dispersal, and also because the moose is not only a value but also a pest, there will be economic interdependency between the two landowners. We start by analysing interdependency under the perfect competition assumption.

## 3. Price taking behaviour

With perfect competition, both owners are price takers and sell hunting licenses at the given market price $P_1 = P_2 = P$. When the owners aim to maximize present-value profit, the management problem of owner 1 is to maximize

$PV_1 = \int_0^\infty \pi_1 e^{-\delta_1 t} dt = \int_0^\infty [Ph_1 - D_1((1-\alpha)X_1)] e^{-\delta_1 t} dt$ subject to the animal growth constraint (1) .

In a similar manner, the management problem of landowner 2 is to maximize

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[1] In this paper, we are considering situations where hunting for meat is the main motive. For a more general discussion of institutional arrangements and factors affecting the hunting price, where hunting for trophy is also considered, see Naevdal et al. [2011].[2] As indicated, such costs may include the cost of moose-vehicle collisions, while the intrinsic value of moose could have been included as a possible benefit component.

[2] As indicated, such costs may include the cost of moose-vehicle collisions, while the intrinsic value of moose could have been included as a possible benefit component.

$PV_2 = \int_0^\infty \pi_2 e^{-\delta_2 t} dt = \int_0^\infty [Ph_2 - D_2(X_2 + \alpha X_1)]e^{-\delta_2 t} dt$ subject to the animal growth constraint (2). $\delta_1 \geq 0$ and $\delta_2 \geq 0$ are the discount rates for landowner 1 and 2, respectively. Notice that subpopulation 1 is included in landowners 2's objective function, but not *vice versa*.

For landowner 1, the current value Hamiltonian reads $H_1 = Ph_1 - D_1((1-\alpha)X_1) + \lambda_1(F(X_1) - h_1)$, where $\lambda_1$ is the subpopulation 1 shadow value. The first order necessary conditions for this maximum problem read $\partial H_1 / \partial h_1 = P - \lambda_1 = 0$ and $d\lambda_1 / dt = \delta_1 \lambda_1 - \partial H / X_1 = \delta_1 \lambda_1 + (1-\alpha) D_1'((1-\alpha)X_1) - \lambda_1 F_1'(X_1)$ when assuming an interior solution (harvesting takes place at the steady state). As the current value Hamiltonian is linear in the control variable, optimal harvest strategy is a combination of a singular solution and the Most Rapid Approach type (MRAP). MRAP will be adopted before the steady state is reached while the singular solution will be applied once the steady state arrives. Accordingly, it is beneficial for the landowner to use the harvesting capacity up to its maximum if the initial stock level ( $t = 0$) is above that of steady state, $X_1(0) > X_1^*$, and postpone harvest until steady state is reached when $X_1(0) < X_1^*$ (superscript '*' denotes the optimal steady state value). The sufficient condition of the above problem is that the Hamiltonian is jointly concave in the control and stock variables. With a strictly concave natural growth function, convex damage cost function and concave (linear) hunting revenue function, we find this condition satisfied (see also Appendix).

The singular harvest solution, or sustainable harvest, follows as $h_1^* = F(X_1^*)$, while the optimal steady state, or 'golden rule', condition can be derived by combining the first order conditions above. We then find:

$$(5) \qquad F'(X_1^*) = \delta_1 + \frac{(1-\alpha)D_1'((1-\alpha)X_1^*)}{P}.$$

This condition says that the stock should be maintained such that the marginal natural growth is equal to the marginal grazing damage, evaluated at hunting license price, plus the discount rent. Multiplying with $P$ and rearranging, equation (5) also indicates that stock should be kept at the point where marginal net benefit, namely, marginal harvesting value minus marginal browsing damage, should be equal to the marginal opportunity cost of keeping the stock. In other words, the net marginal value of the moose 'in the forest' should be equal to the marginal value of the moose 'in the bank'. The equilibrium $X_1^*$ will be unique as the damage function is convex, the natural growth function strictly concave and the hunting license price is fixed. It is seen that $X_1^* < X_1^{msy}$ (msy = maximum sustainable yield).

In a similar manner, the current value Hamiltonian of landowner 2 is $H_2 = Ph_2 - D_2(X_2 + \alpha X_1) + \lambda_2(G(X_2) - h_2)$. The optimal steady state stock here satisfies:

$$(6) \qquad G'(X_2^*) = \delta_2 + \frac{D_2'(\alpha X_1^* + X_2^*)}{P},$$

while the sustainable harvest reads $h_2^* = G(X_2^*)$. Condition (6) has the same interpretation as equation (5) of landowner 1. However, due to dispersal, subpopulation 1 is included here as this subpopulation contributes to browsing damage also in area 2. There is hence present a unidirectional externality through equations (5) and (6)[3]. Because the adjustment of subpopulation 2 is contingent upon the growth pattern of subpopulation 1, this externality is also present through the transitional phase, before steady state is reached. For this reason, the dynamic path of subpopulation 2 may also be different from that of subpopulation 1. The economic interaction will be analysed under two damage cost function assumptions, namely constant marginal damage and increasing marginal damage.

### *Case 1: Constant marginal damage*

When the marginal browsing damage of each area is constant, with $a_i > 0$ ($i = 1,2$), the browsing damage function is defined as $D_i(Z_i) = a_i Z_i$; that is, $D_1 = a_1(1-\alpha)X_1$ for area 1 and $D_2 = a_2(X_2 + \alpha X_1)$ for area 2. When inserting the damage functions into the golden rule conditions (5) and (6), we find $F'(X_1^*) = \delta_1 + (1-\alpha)a_1 / P$ and $G'(X_2^*) = \delta_2 + a_2 / P$, respectively.

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[3] In the present context, landowner 2 will typically argue that landowner 1 profits at her expense, but this argument is unconvincing, because harvesting of subpopulation 1 always *reduces* the browsing damage taking place in area 2. Notice, however, that a unified management scheme (social planner) will yield a smaller number of subpopulation 1 and hence less browsing damage in area 2 (Skonhoft and Olaussen [2005]).

These conditions represent two independent equations, and hence, when the damage functions are linear, there is no management interaction among the landowners at the steady state. Because MRAP will be adopted by landowner 1, and because marginal damage in area 2 is constant for all sizes of the migratory population in the transitional phase, landowner 2 will also find it beneficial to adopt MRAP. Therefore, there will not be any management interaction among the landowners before reaching steady state. This is stated as:

*Result 1: With price taking behaviour and constant marginal damage costs, there is no economic interdependency between the landowners.*

However, not surprisingly, the harvest decision of landowner 1 will influence the profitability of landowner 2, because the steady state profit reads $\pi_2^* = PG(X_2^*) - a_2(\alpha X_1^* + X_2^*)$. The profitability effect is channelled directly through the dispersal parameter, as well as indirectly through the size of migratory stock. Because increased dispersal means less browsing damage in area 1, we find $\partial X_1^* / \partial \alpha > 0$. Therefore, a higher dispersal rate and a higher area 1 optimal stock work in the same direction and reduce the landowner 2 profit. On the contrary, the landowner 1 steady state profit, $\pi_1^* = PF(X_1^*) - a_1(1-\alpha)X_1^*$, increases with more dispersal. This is due not only to reduction in browsing damage, but also to higher harvest income when $\alpha$ shifts up. We also find the effects $\partial \pi_2^* / \partial \delta_1 > 0$ and $\partial \pi_2^* / \partial a_1 > 0$ channelled through the size of the migratory stock.

By inserting the logistic growth functions, explicit expressions for profit and stock size for both areas can be derived. The steady state stock sizes read as $X_1^* = (K_1/2)\left[1 - \delta_1/r - (1-\alpha)a_1/rP\right]$ and $X_2^* = (K_2/2)(1 - \delta_2/r - a_2/rP)$ while we find landowner 1 profit after some rearrangements as $\pi_1^* = (K_1 P/4r)\{r - [(1-\alpha)a_1/P]^2 - \delta_1^2\}$. The profit expression for landowner 2 is complicated, but is available upon request from the authors.

*Case 2: Increasing marginal damage*

The damage function is now specified as $D_i = (b_i / 2)Z_i^2$ with $b_i > 0$ ($i = 1, 2$). The golden rule conditions become $F'(X_1^*) = \delta_1 + b_1(1-\alpha)^2 X_1^* / P$ and $G'(X_2^*) = \delta_2 + b_2(\alpha X_1^* + X_2^*) / P$, and are hence no longer independent equations. Therefore, landowner 1's harvest decision influences the harvest decision of landowner 2. The subpopulation 1 dynamics are still of the MRAP type. Following, e.g., Wilen and Brown [1986], the subpopulation 2 dynamics will also be of the MRAP type[4]. With this harvest strategy adopted in the transitional phase, there will also be a unidirectional management interaction among landowners before steady state is reached. We find a similar effect whenever the landowner 2 marginal cost is not constant (which suggests that the sufficient conditions hold). This is stated as:

*Result 2. With price taking behaviour and changing marginal damage cost, there is a unidirectional economic interdependency between the landowners in the transitional phase as well as in the steady state.*

It can easily be shown that more dispersal through a higher $\alpha$ unambiguously increases subpopulation 1 and reduces subpopulation 2. Just as in the constant marginal cost case, more migration also means lower profit for landowner 2 and higher profit for landowner 1. We still find that a more myopic harvest strategy of landowner 1 yields $\partial \pi_2^* / \partial \delta_1 > 0$, but now $\partial X_2^* / \partial \delta_1 > 0$. The steady state stock sizes can be found explicitly when applying the logistic natural growth functions as $X_1^* = K_1 P(r - \delta_1) / [2rP + K_1 b_1 (1-\alpha)]$ and $X_2^* = K_2 P\{(r-\delta_2) - b_2 \alpha (r - \delta_1) K_1 / [b_1 (1-a)^2 K_1 + 2rP]\} / (2rP + b_2 K_2)$. Both profit expressions are complicated and are available upon request from the authors. Profit interactions between the landowners will also be demonstrated in the numerical section six.

## 4. Monopoly market

So far, the market for hunting licenses is assumed to be competitive, with equal hunting price facing both landowners. We now turn to monopoly power as the other extreme, with the inverse demand functions given as $P_i = P_i(h_i)$ and where $P_i' < 0$ ($i = 1, 2$). It is further assumed

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[4] Wilen and Brown [1986] studied a one-way tropic interaction ecological system where the prey abundance (lower tropical level) influences the predator natural growth, but not *vice versa*. In their model, only the predator is harvested and the profit is linear in the harvest. The management of the predator harvest in this system is therefore, in principle, the same as the management of subpopulation 2.

that the hunting revenue function $P_i(h_i)h_i$ of both landowners is strictly concave, i.e., $(P_i''h_i + 2P_i') < 0$. As indicated in section two, the two areas can be neighbouring areas, or they can be located rather far away from each other if there is long-distance migration. Obviously, the possibility of monopoly pricing fits the last case best. The current Hamiltonian function of landowner 1 reads now $H_1 = P_1(h_1)h_1 - D_1((1-\alpha)X_1) + \lambda_1(F(X_1) - h_1)$ with control condition $\partial H_1 / \partial h_1 = P_1(h_1) + P_1'(h_1)h_1 - \lambda_1 = 0$. The portfolio condition is the same as that in the competitive case. Combining these two equations gives:

(7) $dh_1/dt = [(P_1(h_1) + P_1'(h_1)h_1)(\delta_1 - F'(X_1)) + (1-\alpha)D_1'((1-\alpha)X_1)]/(P_1''(h_1)h_1 + 2P_1'(h_1))$.

Therefore, this equation together with equation (1) yields the dynamics of landowner 1's management problem. The golden rule stock condition now satisfies:

$$(8) \qquad F'(X_1^*) = \delta_1 + \frac{(1-\alpha)D_1'((1-\alpha)X_1^*)}{P_1(F(X_1^*)) + P_1'(F(X_1^*))F(X_1^*)}.$$

This condition conveys similar a message as that in the competitive case, except that the marginal revenue term $P_1 + P_1'F$ now is included to value the moose. As long as the marginal revenue is positive, that is, $P_1 + P_1'F > 0$ in the optimal solution, $X_1^* < X_1^{msy}$ still holds. On the other hand, if $P_1 + P_1'F < 0$, the solution can be located at the right hand side of $X_1^{msy}$ (more details below).

The $h_1$-isocline is defined through equation (7) and reads $(P_1 + P_1'h_1)(\delta_1 - F') + (1-\alpha)D_1' = 0$. When taking the total differential, we find $dh_1/dX_1 = [(P_1 + P_1'h_1)F'' - (1-\alpha)^2 D_1'']/[(P_1''h_1 + 2P_1')(\delta_1 - F')]$ after a small rearrangement. With $\alpha = 1$ and hence no browsing damage in area 1, the isocline is simply fixed by $(\delta_1 - F') = 0$. Otherwise, when $0 \le \alpha < 1$, it is negatively sloped and defined for all stock values $[0, K_1]$ except when $(\delta_1 - F') = 0$. Figure 2 illustrates. It is assumed that the part of the isocline to the right of the asymptote $(\delta_1 - F') = 0$ intersects twice with the $X_1$-isocline $h_1 = F(X_1)$.

Therefore, the situation depicted in this figure indicates three equilibria with different characteristics. Equilibrium point I, where $(\delta - F') < 0$, and hence also $X_1^* < X_1^{msy}$, is accompanied by a positive marginal hunting revenue, $(P_1 + P_1'h_1) > 0$. This follows directly from the above definition of the $h_1$-isocline. Positive marginal hunting revenue also implies a positive shadow price, $\lambda_1 > 0$, such that this subpopulation represents a *value*. On the other hand, at the two other equilibria, II and III, where $(\delta - F') > 0$, we have $(P_1 + P_1'h_1) < 0$. The shadow price is then negative, $\lambda_1 < 0$, and the moose may be regarded as a nuisance, or *pest* [5]. Therefore, in the pest case, the equilibrium harvest II or III must be larger and the equilibrium price lower than in the value case where marginal harvesting revenue is positive.

From the definition of the $h_1$-isocline, we find that equilibrium point I always will exist, while the occurrence of II and III depends upon circumstances, such as the demand for hunting licenses and the severity of browsing damage. For instance, equilibria II and III will exist only when the demand for hunting licenses is 'low'**.** Otherwise, this part of the $h_1$-isocline will not intersect with the $X_1$-isocline, and point I will be the only equilibrium. Equilibrium I is saddle point stable, while the other ones, if existing, will either be saddle point stable or unstable. See Appendix for a formal proof. In the Appendix, we also show that the current value Hamiltonian is jointly concave in the control and stock variable at equilibrium point I, and hence this point is a (local) maximum. On the other hand, characteristics of points II and III depend on the parameter values of the damage function and the population growth function. When either point II or point III is a saddle point or when both points are saddle points, or when the optimality of the point(s) cannot be determined, equilibrium point I will be the global maximum equilibrium, and there could be a control rule that spans the entire control-state space that leads to point I, even if point II and III exist. See Figure 2, and also Figure 4 numerical section below. See also the discussion in Wirl and Feichtinger [2005] and Brock and Starrett [2003]. In what follows, we will examine the value case with positive marginal revenue of the harvest, because this case fits the reality of Scandinavian moose hunting (e.g., Solstad [2007]). More precisely, the value case here means

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[5] Similar classifications, but in other settings, can be found in Schulz and Skonhoft [1996], Zivin *et al.* [2000] and Horan and Bulte [2004].

the demand for hunting licences is ‘high’ such that equilibria II and III do not exist (see also below).

Figure 2 about here

For landowner 2, the control condition reads $\partial H_2 / \partial h_2 = P_2(h_2) + P_2{}'(h_2)h_2 - \lambda_2 = 0$, while the portfolio condition is the same as that in competitive case. The golden rule satisfies:

$$(9) \qquad G'(X_2^*) = \delta_2 + \frac{D_2'(\alpha X_1^* + X_2^*)}{P_2(G(X_2^*)) + P_2'(G(X_2^*))G(X_2^*)}.$$

The $h_2$-isocline is defined by $(P_2 + P_2{}'h_2)(\delta_2 - G') + D_2{}' = 0$. As for landowner 1, there can either be one equilibrium or three equilibria. The difference is that these equilibria may be contingent upon the size of subpopulation 1; in this case, only the equilibrium where the moose represents a value, corresponding to point I in Figure 2 for landowner 1, is a global maximum steady state. With a constant marginal damage cost, however, there will be no interaction among the stocks and hence no management interdependency (see below). The dynamics of landowner 2’s management problem will then not differ from that of landowner 1. On the other hand, with increasing marginal damage, there will be economic interdependency. This interaction can be complex, as landowner 1’s management may influence the speed and circumstances under which equilibrium I of landowner 2 will be reached. This is stated as:

*Result 3: With monopolistic hunting licence pricing, there may be a complex economic unidirectional management interdependency among the landowners.*

Economic interaction will now be analysed in detail under both constant and increasing marginal damage cost. In both cases, the linear inverse demand curve, $P_i = \gamma_i - \beta_i h_i$ with $\gamma_i > 0$ and $\beta_i > 0$ ($i = 1, 2$), is applied. Again, we start with the constant marginal case.

*Case 1: Constant marginal damage*

With constant marginal damage, and the linear demand function, the $h_1$-isocline reads

$(\gamma_1 - 2\beta_1 h_1)[\delta_1 - F'(X_1)] + a_1(1-\alpha) = 0$ which may also be written as $h_1 = (1/2\beta_1)[\gamma_1 + a_1(1-\alpha)/(\delta_1 - F'(X_1))]$. Differentiation yields $dh_1/dX_1 = (1/2\beta_1)a_1(1-\alpha)F''/(\delta_1 - F')^2 < 0$. $h_1$ unambiguously shifts down with shrinking market demand conditions through smaller $\gamma_1$ and/or higher $\beta_1$. In addition, it shifts down with less damage and a smaller $a_1$ when $(\delta_1 - F') > 0$, and does the opposite when $(\delta_1 - F') < 0$. This confirms the above discussion about potential forces making the moose a pest, with negative marginal hunting revenue $(\gamma_1 - 2\beta_1 h_1^*) < 0$ and two intersections between the $h_1$-isocline and the $X_1$-isocline when $(\delta_1 - F') > 0$. We also see that more dispersal shifts up the $h_1$-isocline in the value case and hence the steady state stock becomes higher. The golden rule condition now satisfies $F'(X_1^*) = \delta_1 + (1-\alpha)a_1/[\gamma_1 - 2\beta_1 F(X_1^*)]$.

For landowner 2, the golden rule condition reads $G'(X_2^*) = \delta_2 + a_2/[\gamma_2 - 2\beta_2 G(X_2^*)]$. Just as under the perfect competition assumption, there is no stock interaction, and landowner 2's management is independent of landowner 1's management at the steady state. This will also be so during the transitional phase before steady state is reached. The conditions for obtaining one, or three equilibria, will be of a similar type. Also, for landowner 2, the equilibrium conveying the value case with $\gamma_2 - 2\beta_2 h_2^* > 0$ will be the maximum, and there could be a control rule that spans the entire control-state space that leads to point I, even if point II and III exist. We may then state:

Result 4: *With monopoly pricing and constant marginal damage, there is still no management interaction among landowners. In the value case, more dispersal means a larger steady state size of subpopulation 1, while the size of subpopulation 2 is not affected.*

We may also compare the optimal steady state stocks and harvest under the monopoly market with that under the competitive market in the situation where the moose represents a value. In order to make the market price of a hunting license comparable under these two market structures, we introduce the same downward sloping linear demand schedule in both cases. The harvest dynamics of landowner 1 with price taking may then be written as

$dh_1 / dt = [(\gamma_i - \beta_i h_i)(\delta_1 - F'(X_1)) + (1-\alpha)D_1'((1-\alpha)X_1)] / (-\beta_i)$, and we find the $h_1$-isocline as $h_1 = (1/\beta_1)[\gamma_1 + a_1(1-\alpha)/(\delta_1 - F'(X_1))]$. It can easily be shown that the intersection of the $h_1$-isocline with the $X_1$-axis is similar in the price taking case and monopoly case, while the intersection with the $h_1$-axis takes place at a higher value in the price taking case. Therefore, the $h_1$-isocline intersects with the $X_1$-isocline at a point with more animals hunted in the competitive case than in the monopoly case. That is, in the competitive case, we find a higher number of animals hunted and hence, as suspected, a lower price. At the same time, this means that the equilibrium stock is higher in the competitive case than in the monopoly case. This result is stated as:

*Result 5: When moose is considered valuable, with a linear demand function and a linear browsing damage function, monopoly leads to a less stock-conserving harvest policy than that in the competitive case.*

This result contrasts with what is often found in resource economic models, but we reach similar results as those in Staahler [1996] and Bulte and van Kooten [2002]. The above reasoning could also be related to the analysis of Wirl and Feichtinger [2005] and Clark [1990, Ch. 6.3]. Clark also studies a situation with a falling marginal revenue curve, but no damage, and ends up with three equilibria, just as in Figure 2.

*Case 2: Increasing marginal damage*

With increasing marginal browsing damage and the cost functions written as $D_i = (b_i/2)Z_i^2$ ($i = 1, 2$), we find the $h_1$-isocline to satisfy

$(\gamma_1 - 2\beta_1 h_1)(\delta_1 - F(X_1)) + b_1(1-\alpha)^2 X_1 = 0$ while the golden rule condition becomes

$F'(X_1^*) = \delta_1 + \dfrac{(1-\alpha)^2 b_1 X_1^*}{P_1(F(X_1^*)) + P_1'(F(X_1^*))F(X_1^*)}$. In a similar manner, the $h_2$-isocline reads

$(\gamma_2 - 2\beta_2 h_2)(\delta_2 - G(X_2)) + b_2(\alpha X_1 + X_2) = 0$ while

$G'(X_2^*) = \delta_2 + \dfrac{b_2(\alpha X_1^* + X_2^*)}{P_2(G(X_2^*)) + P_2'(G(X_2^*))G(X_2^*)}$ states the golden rule condition for landowner 2.

Therefore, with increasing marginal cost, just as in the price taking market situation, there is a unidirectional management interaction at the steady state as well as during the transitional phase before reaching the steady state. In the value case where demand and cost conditions ensure a unique equilibrium, the dynamics for both subpopulations are of the saddle point type. The impact of dispersal rate change on the steady state with increasing marginal damage is illustrated in the numerical section six.

## 5. Duopoly market

In sections three and four, two extreme market situations have been considered. We now proceed to analyse an intermediate case with market interaction among the agents structured as a duopoly. Both landowners then face the same inverse demand function $P = P(h_1 + h_2)$ with $P' < 0$. With an infinite planning horizon, the two landowners play a dynamic Cournot game. Only the value case with positive marginal revenue of the harvest is examined.

We consider the open loop strategy in the Cournot game. That is, the landowners commit their optimal harvest (number of hunting licenses) to each other at time $t = 0$ over the infinite planning horizon, given the expectation of the entire optimal harvest path of the other player (Dockner *et al.* [2000]). The closed loop Nash equilibrium, which conditions next period harvest strategy on the current state (i.e., Markovian strategies), is thus left out of our analysis. The main reason is that it is too complex to identify proper value functions for nonlinear Hamiltonians in a closed loop game. Although we may expect different harvesting levels under Markovian strategies, our results with open loop Nash equilibrium is sufficient to demonstrate that not only unidirectional migration but also the interaction of hunting licence prices affects harvesting strategy. Therefore, the management problem of landowner 1 under the open loop strategy is to maximize present-value profit

$PV_1 = \int_0^\infty [P(h_1 + h_2)h_1 - D_1((1-\alpha)X_1)]e^{-\delta_1 t}dt$ subject to the animal growth constraint (1) and the expected harvest $h_2$ of landowner 2. In a similar manner, the management problem of landowner 2 is to maximize $PV_2 = \int_0^\infty [P(h_1 + h_2)h_2 - D_2(\alpha X_1 + X_2)]e^{-\delta_2 t}dt$ subject to growth constraints (2) and the expected harvest $h_1$, but also subject to $X_1$.

The current value Hamilton of landowner 1 reads

$H_1 = P(h_1 + h_2)h_1 - D_1((1-\alpha)X_1) + \lambda_1(F(X_1) - h_1)$ with the control condition $\partial H_1 / \partial h_1 = P(h_1 + h_2) + P'(h_1 + h_2)h_1 - \lambda_1 = 0$ . The portfolio condition is the same as in the previous market cases. When combining these equations, we find the harvest dynamics for landowner 1as:

(10) $$2P'(h_1 + h_2)dh_1 / dt + P'(h_1 + h_2)dh_2 / dt = (\delta_1 - F'(X_1))(P'(h_1 + h_2)h_1 + P(h_1 + h_2)) + (1-\alpha)D_1'((1-\alpha)X_1) .$$

In a similar way, the harvest dynamics for landowner 2 reads:

(11) $$2P'(h_1 + h_2)dh_2 / dt + P'(h_1 + h_2)dh_1 / dt = (\delta_2 - G'(X_2))(P'(h_1 + h_2)h_2 + P(h_1 + h_2)) + D_2'(\alpha X_1 + X_2) .$$

Equations (10) and (11), together with the population dynamics constraints (1) and (2), define the dynamics of the open loop game. In contrast to the competitive market and the monopoly market, the interaction among the agents is no longer unidirectional. $dh_2 / dt$ as well as $h_2$ are included in condition (10), while $dh_1 / dt$ and $h_1$ are included in condition (11). This is stated as:

*Result 6: Because of the market interaction, there is a reciprocal economic interaction among the agents. The interaction channels through the hunting market as well as through migration.*

The $h_1$-isocline found through (10) is $(\delta_1 - F')(P + P'h_1) + (1-\alpha)D_1' = 0$ . For a fixed value of $h_2$, it has a similar shape to the isocline in the monopoly case (Figure 2). When the moose is considered valuable, which is the case here, the $h_1$-isocline intersects once with the $X_1$-isocline and ensures a unique solution. The $h_2$-isocline having a similar shape which, for a given value of $h_1$, also indicates a unique intersection between this isocline and the $X_2$-isocline when the moose is considered valuable. The golden rule stock condition for subpopulation 1 reads:

$$(12)\qquad F'(X_1^*)=\delta_1+\frac{D_1'((1-\alpha)X_1^*)}{P(F(X_1^*)+G(X_2^*))+P'(F(X_1^*)+G(X_2^*))F(X_1^*)},$$

and the golden rule condition for subpopulation 2 satisfies:

$$(13)\qquad G'(X_2^*)=\delta_2+\frac{D_2'(\alpha X_1^*+X_2^*)}{P(F(X_1^*)+G(X_2^*))+P'(F(X_1^*)+G(X_2^*))G(X_2^*)}.$$

The golden rule conditions represented by these two equations also include marginal revenue terms to value whether the moose should be kept in the forest or in the bank.

The impact of dispersal rate $\alpha$ on the steady state stocks and harvests in the duopoly market is different from that of the competitive and monopoly case. With fixed marginal damage cost, landowner 2's harvest decision is unaffected by the amount of dispersal in the competitive and monopoly market situations. Under the duopoly market, on the other hand, we find that a higher dispersal rate will lower the steady stock $X_2^*$ and harvest $h_2^*$ through the market price interaction. In a first step, higher dispersal rate $\alpha$ increases $X_1^*$ and $h_1^*$. Higher $h_1^*$ next lowers marginal harvest revenue of landowner 2 and reduces the steady stock $X_2^*$ and harvest $h_2^*$. When browsing damage is nonlinear, dispersal rate change affects landowner 2's decision through changes in both market price and marginal browsing damage. These effects are similar to those in the monopoly case. The numerical simulations (next section) confirm these analytical findings.

## 6. Numerical Illustration

### *6.1 Data and specific functional forms*

The above theoretical reasoning will now be illustrated numerically. As already specified, animal growth is described by the standard logistic growth function. The damage functions also follow previous specifications, and results with both linear and strictly convex functional forms are demonstrated. The same linear demand function is assumed for both monopoly and duopoly market. The slope and the choke price are assumed to be similar for both landowners, i.e., $\beta_1=\beta_2=\beta$ and $\gamma_1=\gamma_2=\gamma$. The choke price is fixed as $\gamma=10{,}000$ (NOK/animal) while the slope is given as $\beta_1=\beta_2=6.77$ (NOK/animal$^2$). The same discount rate is also assumed for both landowners. The value of the intrinsic growth rate of moose is based on Skonhoft and

Olaussen [2005], while the carrying capacity, typically depending on the size of the areas (section two), is assumed similar $K_1 = K_2 = 4,550$ (number of animals) for both subpopulations. The maximum sustainable yield stock level is therefore $X_i^{msy} = 2,275$ ($i = 1,2$). The baseline dispersal rate is assumed to be 20%, that is, $\alpha = 0.20$ (Skonhoft and Olaussen [2005]). Browsing damage, cost data and price data are adopted from Solstad [2007]. Table 1 summarizes the baseline data used in the simulations. For these baseline data, the harvest isoclines intersect with the natural growth functions only once in the monopoly and duopoly market situations (again, see Figure 2). Hence these data convey information only about the case in which moose are regarded as valuable. We examine the robustness of other results by changing some of the key parameter values such as the dispersal rate. We also examine what happens in the monopoly case when the demand for hunting licence is ‘low’, such that the pest case and three steady states are included. The effect of the changed initial situation in the monopoly case is also studied to confirm whether there is a control rule that spans the entire control-state space leading to point I, even if point II and III exist.

Table 1 about here

### *6.2 Results*

Tables 2-4 report optimal *steady state* stock levels and harvest under the three different market assumptions. Results with different dispersal rates are included to illustrate different migration patterns. Price taking behaviour is first considered in Table 2. The results confirm that the degree of dispersal has no effect on the size of subpopulation 2 when marginal browsing damage cost is constant (Result 1). The opposite happens for landowner 1, who finds it beneficial to keep a larger subpopulation and harvest more with more dispersal. It is also seen that more dispersal means higher profit for landowner 1 and less profit for landowner 2. With increasing marginal damage, more dispersal means a larger subpopulation 1, which spills over to a lower subpopulation 2 (Result 2). As expected, the population sizes are always below that of $X_i^{msy}$ ($i = 1,2$).

Table 2 about here

Table 3 demonstrates what occurs under monopoly pricing. We find that $F'(X_1) - \delta_1 = 0$ and $G'(X_2) - \delta_2 = 0$ yield $X_1 = X_2 = 2,033$. Therefore, for the baseline demand conditions, all optimal stock values in Table 3 (and Table 4, see below) are below the critical number, indicating that we have the value case. In the baseline case with $\alpha = 0.2$, landowner 1 charges the monopoly price $P_1^* = \gamma_1 - \beta_1 h_1^* = 10,000 - 6.77 \cdot 499 = 6,622$ (NOK/animal) while landowner 2 charges 6,683 (NOK/animal). The marginal revenues are $\gamma_1 - 2\beta_1 h_1^* = 3,243$ and $\gamma_2 - 2\beta_2 h_2^* = 3,367$ (NOK/animal), respectively. With constant marginal damage, there is still no management interaction among landowners (Result 4). However, just as in the competitive market price case, the profit for landowner 1 increases with a higher dispersal rate, while it decreases for landowner 2. With a linear demand function and a linear browsing damage function, we found that monopoly leads to a less stock-conserving harvest policy than in the competitive case when moose is considered as a value (Result 5). However, as the price is fixed as $P = 5,000$ (Table 1), and hence no downward sloping demand function is introduced in the competitive case reported in Table 2, stock values as well as harvest are not comparable in that respect here.

Table 3 about here

The steady state results of the duopoly market are reported in Table 4, where only the value case is considered. Steady state stock of subpopulation 2 decreases with higher dispersal rate even under the constant marginal damage cost assumption (Result 6). With baseline dispersal rate $\alpha = 0.2$ and linear damage functions, market price for the hunting licence equals $P^* = \gamma_1 - \beta_1(h_1^* + h_2^*) = 10,000 - 6.77(428 + 406) = 4353$ (NOK/animal). The marginal revenue for the landowner 1 and 2 are 1,453 NOK and 1,605 NOK, respectively. Impacts of dispersal rate change on the profit for the two landowners work in the same manner as those in the competitive and the monopoly market cases.

Table 4 about here

We then look at the dynamics. The dynamics under the competitive market are of the MRAP type. Accordingly, if the initial stock $X_1(0)$ is lower than the steady state stock, no harvest will take place. The dynamic path of subpopulation 1 then simply follows the solution of $dX_1/dt = rX_1(1-X_1/K_1)$, or $X_1 = K_1/\{e^{-rt}[(K_1 - X_1(0))/X_1(0)]+1\}$. The subpopulation 2 dynamic path has a similar pattern.

Figure 3 shows the dynamics for the two subpopulations under the monopoly market and the duopoly market for baseline value of $\alpha = 0.2$, and for $\alpha = 0.4$ when the marginal damage is constant[6]. We again only consider the value case with a 'high' demand and $\gamma = 10{,}000$ (NOK/animal) and hence with only one steady state equilibrium and with saddle path dynamics under both market forms. In the linear damage case, the possibility of a complex unidirectional management interdependency among the landowners is not present, unlike the general monopoly situation (Result 3). Figure 3 indicates that a higher dispersal rate yields a higher stock size of subpopulation 1 at every point of time along the transitional phases as well as in the steady state (see also Tables 3 and 4) under the monopoly market and the duopoly market. For subpopulation 2, there are no changes with increasing dispersal in the monopoly case (Result 4), while the steady states as well as the transitional paths shift down with more dispersal in the duopoly market.

Figure 3 about here

Although the following results are not presented in the paper, but are available upon request from the authors, we find that, with increasing marginal browsing damage, the dynamic paths of subpopulation 1 will exhibit similar pattern as those under constant marginal damage. For subpopulation 2, the transitional dynamics will be similar to that of the duopoly market under constant marginal damage; that is, the stock will be affected by dispersal rate in both markets with nonlinear browsing damage. In the monopoly market, the impact channels only through increased marginal damage. A higher migration rate will raise marginal damage both directly

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[6] The dynamic optimization was performed with the Boundary Value Problem (BVP) of Sumlink tool box in Matlab 7.0.

and indirectly by increasing subpopulation 1 and by reducing the stock of subpopulation 2. In the duopoly market, however, the impact operates through both market factors and marginal browsing damage.

Figure 4 presents the $X_1 - h_1$ phase plane and dynamic paths for subpopulation 1 with different initial stock values under the monopoly market. The upper panel shows the baseline case situation when the demand for hunting licence is 'high' with $\gamma_1 = 10,000$ (NOK/animal), the moose is valuable, and point I is the only steady state. The parabola describes the saddle path of an initial point starting from the left side of the steady state with $X_1(0) = 800$. See also Figure 3. The lower panel demonstrates the situation when demand for hunting license is 'low' with $\gamma_1 = 4,000$ (NOK/animal) and there exist three equilibra (see also Figure 2). The saddle path demonstrates that when subpopulation 1 initially has a relatively high value as given by $X_1(0) = 3,500$, the stock value may decrease over time and reach equilibrium I even if point II and III exist.

Figure 4 about here

## 7. Concluding remarks

Using Scandinavian moose as an example, we have analyzed the economic interdependency of exploitation of two subpopulations of wildlife located in two areas with two landowners, and have considered three market situations with both linear and nonlinear damage cost assumptions. Just as in models with ecological interconnections, and models where wildlife or fish populations disperse in a density dependent manner between different areas, we find in this paper that damage associated with density independent dispersal and market price interaction can create economic interdependency between different agents.

Table 5 about here

Our main findings may be summarized as follows (Table 5). Under price taking behaviour, the combination of MRAP and singular path at equilibrium is the optimal harvest strategy for both landowners. Harvest along the singular path at equilibrium is the optimal strategy in both monopoly and duopoly markets when moose are regarded as valuable. The stock and the harvest of the subpopulation that migrates in our model increases with dispersal rate in all the three markets, regardless of cost specification. Under price taking as well as monopoly pricing, when a linear damage function is assumed, optimal stock and its dynamic path for the non-migratory subpopulation whose landowner suffers from dispersal-associated browsing damage is not affected by the dispersal rate change of the other migratory subpopulation. The non-migratory subpopulation will decrease with a higher dispersal rate when the damage function is nonlinear. In the duopoly market, the stock and dynamic path of this subpopulation shift down under both linear and nonlinear cost functions, due to the effect of price interaction in addition to change in browsing damage. The result is robust in both equilibrium and dynamic states.  We reach similar results as those in Staahler [1996] and Bulte and van Kooten [2002], showing that the market with perfect competition works in a more stock-conservative manner than does a monopoly market. Nevertheless, our results are different from the findings in Halsema and Withagen [2008] due to the different type of stock-dependent cost assumed in our model.

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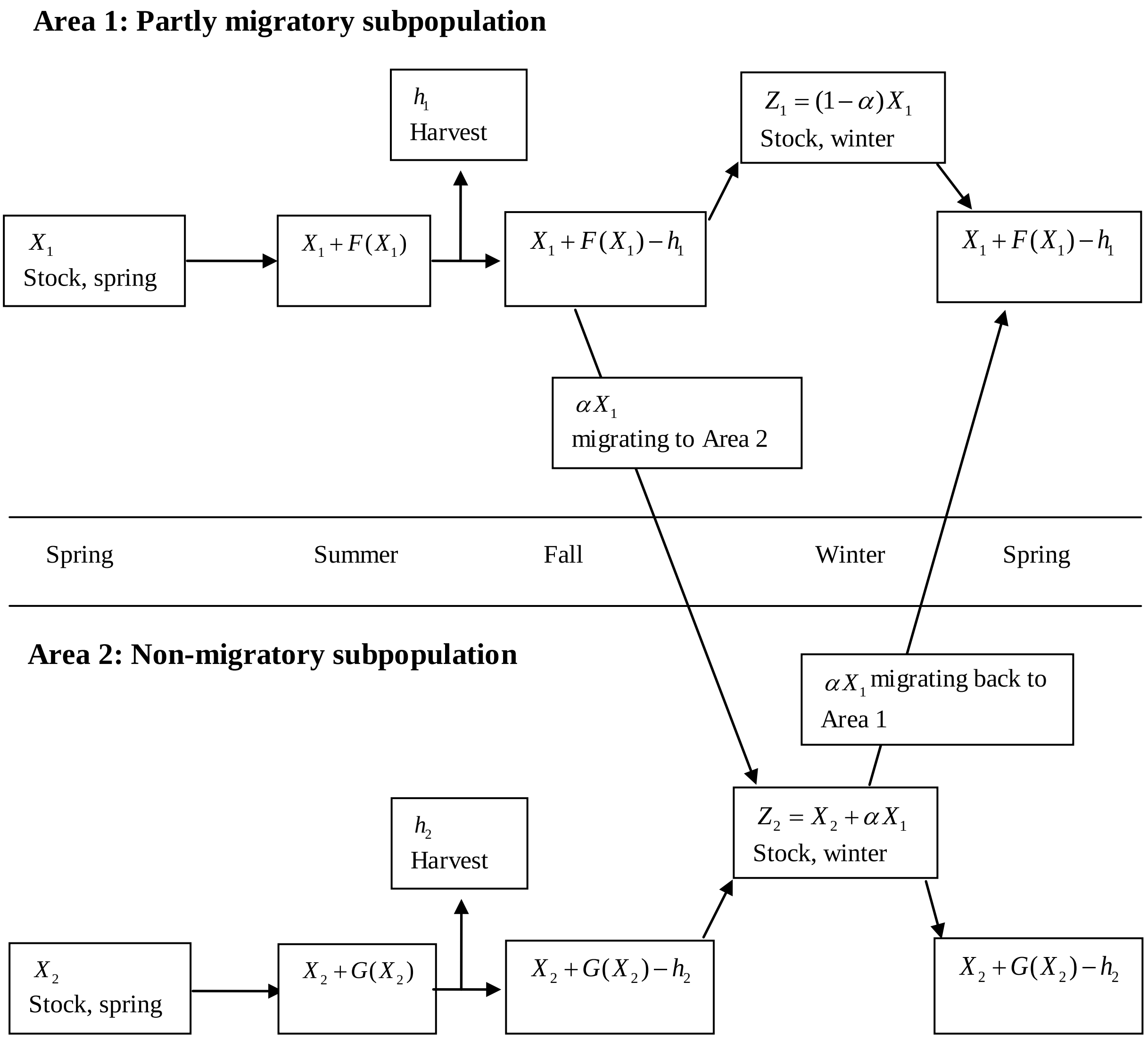


FIGURE 1: The moose migration pattern.

Figure note: Notice that our model (Equations (1)-( 4)) is formulated in continuous time indicating that all events over the year cycle take place simultaneously.

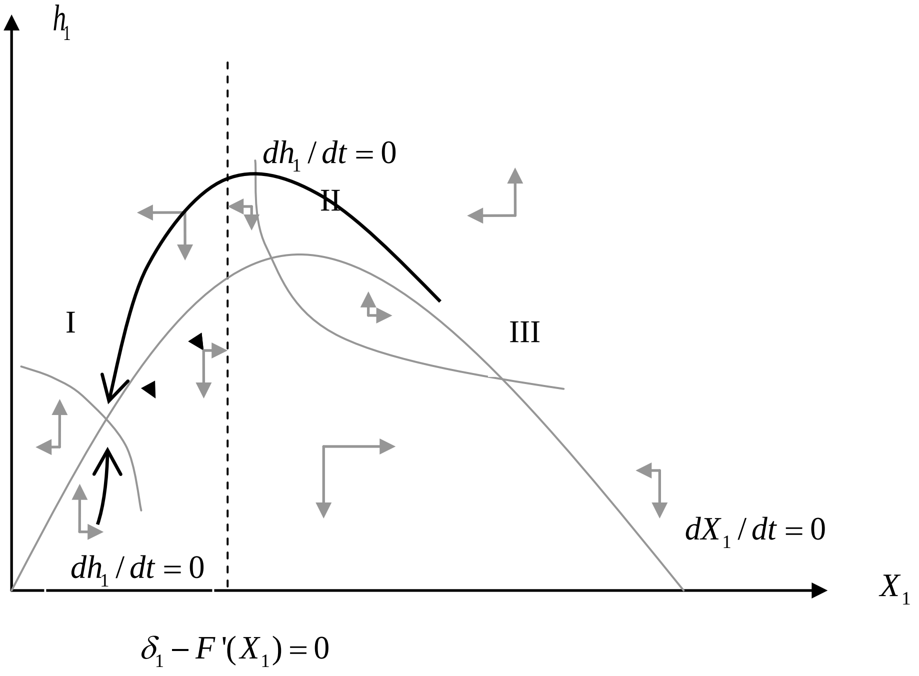


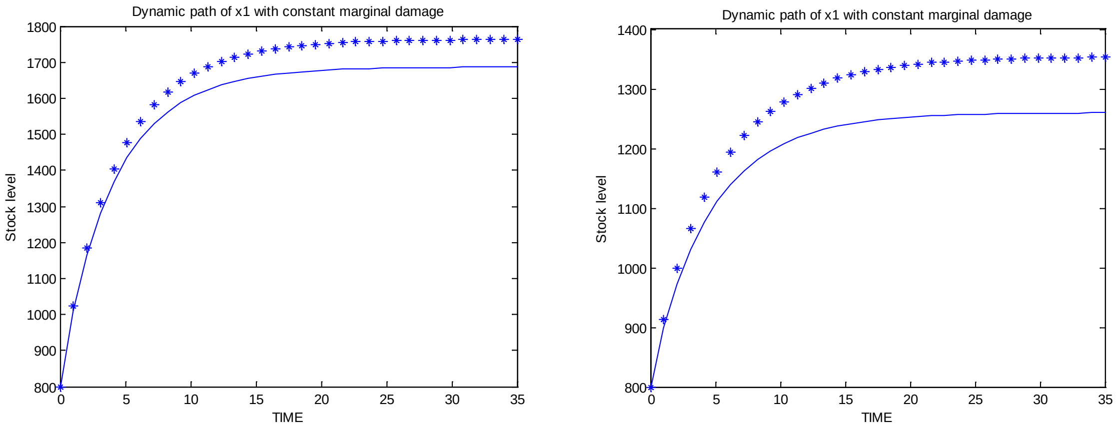
FIGURE 2. Isoclines of landowner 1’s management problem under monopoly pricing. Solid lines with arrows indicate saddle paths.

Monopoly market

Subpopulation $X_1$

Duopoly market

Subpopulation $X_1$



Monopoly market

Subpopulation $X_2$

Duopoly market

Subpopulation $X_2$

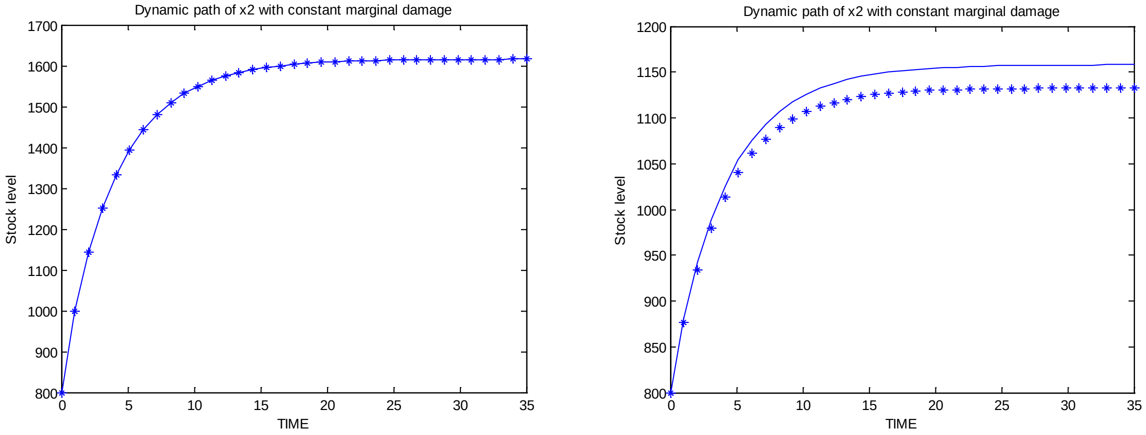


FIGURE 3. Transitional stock dynamics. Monopoly market and duopoly market. Constant marginal costs and different dispersal rates. Initial stock size $X_1(0) = X_2(0) = 800$. Straight lines: baseline value with dispersal rate, $\alpha = 0.2$. Lines with asterisk: high dispersal rate, $\alpha = 0.4$.

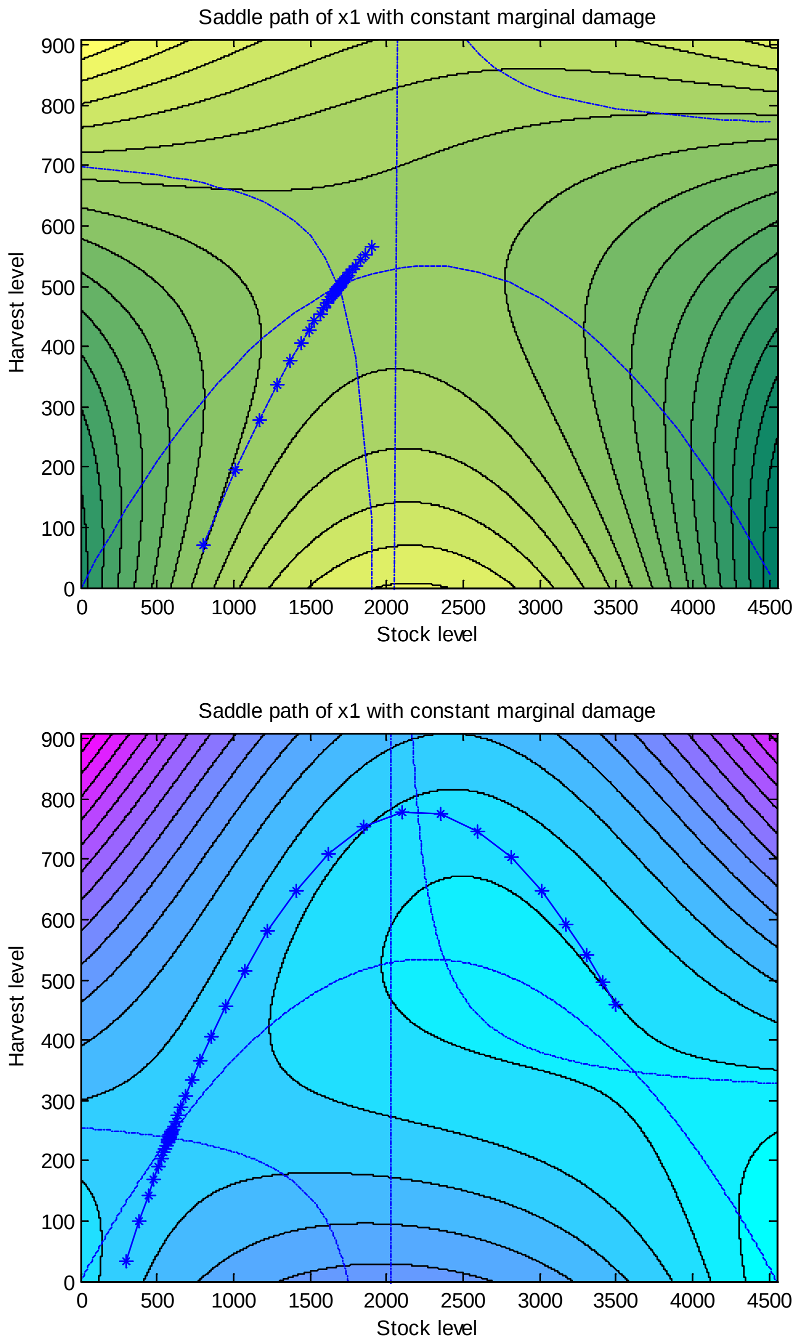


FIGURE 4. The $X_1 - h_1$ phase planes with and saddle paths for subpopulation 1 with different initial points under monopoly market. Linear damage function is assumed. Upper panel: Moose regarded as valuable and demand for hunting licenses is 'high': $\gamma_1 = 10,000$ (NOK/animal). Lower panel: Moose regarded as a pest and demand for hunting licenses is 'low': $\gamma_1 = 4,000$ (NOK/animal). The straight dash-dot line: asymptote $(\delta_1 - F') = 0$. Curving dash-dot lines: $h_1$-isoclines. The dashed line: $X_1$-isocline. Lines with asterisk: saddle paths. Contours with solid line: current value Hamilton.

TABLE 1. Baseline ecological and economic parameter values

| Parameter description | Value |
|---|---|
| Intrinsic growth rate | $r = 0.47$ |
| Carrying capacity | $K_1 = K_2 = 4,550$ (number of animals) |
| Discount rent | $\delta_1 = \delta_2 = 0.05$ |
| Market price (price taking behaviour) | $P = 5,000$ (NOK/animal) |
| Slope parameter demand curve | $\beta_1 = \beta_2 = 6.77$ (NOK/animal$^2$) |
| Choke price demand curve | $\gamma_1 = \gamma_2 = 10,000$ (NOK/animal) |
| Constant marginal damage coefficient | $a_1 = a_2 = 290$ (NOK/animal) |
| Increasing marginal damage coefficient | $b_1 = b_2 = 1.6$ (NOK/animal$^2$) |
| Dispersal parameter | $\alpha = 0.2$ |

TABLE 2. Steady states under price taking behaviour. Optimal harvest $h_i^*$ (number of animals), stock level $X_i^*$ (number of animals) and profit $\pi_i^*$ (in 1,000 NOK)

| | Constant marginal damage | | | Increasing marginal damage | | |
|---|---|---|---|---|---|---|
| | $\alpha = 0$ | $\alpha = 0.2$ | $\alpha = 0.4$ | $\alpha = 0$ | $\alpha = 0.2$ | $\alpha = 0.4$ |
| $X_1^*$ | 1,752 | 1,808 | 1,865 | 798 | 1,021 | 1,305 |
| $X_2^*$ | 1,752 | 1,752 | 1,752 | 798 | 674 | 480 |
| $h_1^*$ | 506 | 512 | 517 | 309 | 372 | 437 |
| $h_2^*$ | 506 | 506 | 506 | 309 | 270 | 202 |
| $\pi_1^*$ | 2,024 | 2,141 | 2,262 | 1,037 | 1,327 | 1,697 |
| $\pi_2^*$ | 2,024 | 1,919 | 1,808 | 1,037 | 732 | 206 |

TABLE 3. Steady states monopoly market when moose are regarded as valuable. Optimal harvest $h_i^*$ (number of animals), stock level $X_i^*$ (number of animals) and profit $\pi_i^*$ (in 1,000 NOK)

| | Constant marginal damage | | | Increasing marginal damage | | |
|---|---|---|---|---|---|---|
| | $\alpha = 0$ | $\alpha = 0.2$ | $\alpha = 0.4$ | $\alpha = 0$ | $\alpha = 0.2$ | $\alpha = 0.4$ |
| $X_1^*$ | 1,616 | 1,687 | 1,764 | 852 | 1,018 | 1,231 |
| $X_2^*$ | 1,616 | 1,616 | 1,616 | 852 | 767 | 654 |
| $h_1^*$ | 490 | 499 | 508 | 326 | 371 | 422 |
| $h_2^*$ | 490 | 490 | 490 | 326 | 300 | 263 |
| $\pi_1^*$ | 2,806 | 2,913 | 3,026 | 1,960 | 2,248 | 2,578 |
| $\pi_2^*$ | 2,806 | 2,708 | 2,601 | 1,960 | 1,637 | 1,110 |

TABLE 4. Steady statesduopoly market when moose are regarded as valuable. Optimal harvest $h_i^*$ (number of animals), stock level $X_i^*$ (number of animals) and profit $\pi_i^*$ (in 1,000 NOK)

| | Constant marginal damage | | | Increasing marginal damage | | |
|---|---|---|---|---|---|---|
| | $\alpha = 0$ | $\alpha = 0.2$ | $\alpha = 0.4$ | $\alpha = 0$ | $\alpha = 0.2$ | $\alpha = 0.4$ |
| $X_1^*$ | 1,181 | 1,260 | 1,354 | 718 | 880 | 1,104 |
| $X_2^*$ | 1,181 | 1,158 | 1,133 | 718 | 618 | 481 |
| $h_1^*$ | 411 | 428 | 447 | 284 | 334 | 393 |
| $h_2^*$ | 411 | 406 | 400 | 284 | 251 | 202 |
| $\pi_1^*$ | 1,480 | 1,572 | 1,672 | 1,336 | 1,619 | 1,995 |
| $\pi_2^*$ | 1,480 | 1,358 | 1,221 | 1,336 | 1,012 | 527 |

TABLE 5.Summary of optimal harvest strategies and impacts on optimal steady state stocks. Moose are considered as valuable in the monopoly as well as duopoly market situation.

| | Optimal harvest strategies | The impact of migration rate on optimal stocks for subpopulation 1 and subpopulation 2 | | | |
|---|---|---|---|---|---|
| | | Constant marginal damage | | Increasing marginal damage | |
| | | $X_1^*$ | $X_2^*$ | $X_1^*$ | $X_2^*$ |
| Market with fixed price | Combination of MRAP and singular path | + | Not affected | + | - |
| Monopoly market | The singular path | + | Not affected | + | - |
| Duopoly market | The singular path | + | - | + | - |

Table note: + indicates positive impact; - indicates negative impact.

## Appendix

In this appendix we look at the properties of the Hamiltonian and stability conditions in the monopoly case for landowner 1 (area 1). We first find $\partial^2 H_1 / \partial h_1^2 = P_1'(h_1) + P_1''(h_1)h_1 + P_1'(h_1) = 2P_1'(h_1) + P_1''(h_1)h_1 < 0$ due to concavity of the revenue function. With linear demand function, it simplifies further to $\partial^2 H_1 / \partial h_1^2 = 2P_1'(h_1) < 0$. We next find $\partial^2 H_1 / \partial X_1{}^2 = -(1-\alpha)^2 D_1''((1-\alpha)X_1) + \lambda_1 F''(X_1)$.

When the moose are regarded valuable and $\lambda_1 = P_1(h_1) + P_1'(h_1)h_1 > 0$, we have $\partial^2 H_1 / \partial X_1{}^2 < 0$ as the damage function is convex and the natural growth function is concave. Because of $\partial^2 H_1 / \partial h_1 \partial X_1 = 0$, the determinant of the Hessian matrix is positive and the Hamiltonian is concave in the stock and control variables, and hence equilibrium point I represent a (local) maximum point.

In the pest case with $\lambda_1 = P_1(h_1) + P_1'(h_1)h_1 < 0$, the sign of $\partial^2 H_1 / \partial X_1^2$ is general unclear. However, under the assumption of linear damage function, i.e. $D_1''((1-\alpha)X_1) = 0$, we find $\partial^2 H_1 / \partial X_1{}^2 = \lambda_1 F''(X_1) > 0$. Therefore, the determinant of the Hessian is negative and the extremes II and III representing the pest case are not maximums, but of the saddle type. When damage is nonlinear, i.e. $D_1''((1-\alpha)X_1) > 0$, the sign of $\partial^2 H_1 / \partial X_1{}^2$ depends on the parameter values of the damage function and the population growth function. If the damage function is weakly convex, we also reach the conclusion that the Hamiltonian is not concave but has saddle point properties. If the damage function is strongly convex, the Hamiltonian could still be concave in the stock and control variables. Similar reasoning applies to landowner 2 and area 2.

We now analyze the stability properties of the dynamic system in the neighbourhood of the three equilibrium points, I, II and III. The Jacobian matrix of the stock and harvest dynamic system reads

$$J_1(X_1,h_1)=\begin{bmatrix} F'(X_1) & -1 \\ \dfrac{-(P_1(h_1)+P_1'(h_1)h_1)F''(X_1)+(1-\alpha)D_1''\left((1-\alpha)X_1\right)}{2P_1'(h_1)} & \delta_1-F'(X_1) \end{bmatrix}$$

. The eigenvalues are found through

$$\left[F'(X_1)-\lambda\right]\left[\delta_1-F'(X_1)-\lambda\right]+\frac{-(P_1(h_1)+P_1'(h_1)h_1)F''(X_1)+(1-\alpha)D_1''\left((1-\alpha)X_1\right)}{2P_1'(h_1)}=0$$

, or

$$\lambda^2-\delta_1\lambda+F'(X_1)\left[\delta_1-F'(X_1)\right]+\frac{-(P_1(h_1)+P_1'(h_1)h_1)F''(X_1)+(1-\alpha)D_1''\left((1-\alpha)X_1\right)}{2P_1'(h_1)}=0$$

When moose are regarded as valuable, $F'(X_1)>0$, $\delta_1-F'(X_1)<0$ and the marginal revenue satisfies $P_1(h_1)+P_1'(h_1)h_1>0$. Since the three inequalities hold, i.e. $F''(X_1)<0$, $D_1''\left((1-\alpha)X_1\right)\geq 0$ and $P_1'(h_1)<0$, we reach

$$Det(J)=F'(X_1)\left[\delta_1-F'(X_1)\right]+\frac{-(P_1(h_1)+P_1'(h_1)h_1)F''(X_1)+(1-\alpha)D_1''\left((1-\alpha)X_1\right)}{2P_1'(h_1)}<0$$

This implies that one eigenvalue is positive and the other is negative. Therefore, the system in the neighbourhood of equilibrium point I shows saddle path dynamics.

When moose are regarded as nuisance, $\delta_1-F'(X_1)>0$ and the marginal harvesting revenue satisfies $P_1(h_1)+P_1'(h_1)h_1<0$. For equilibrium II, when $F'(X_1)>0$ and the damage function $D_1''((1-\alpha)X_1)$ is linear or just weakly convex, $Det(J)>0$. Since $\lambda_1+\lambda_2=\delta_1>0$, both eigenvalues are positive. The system near point II will then be unstable. If and only if the damage function is strongly convex, the system near point II will exhibit saddle path dynamics.

When $F'(X_1)<0$, the dynamics near equilibrium II and equilibrium III will share similar characteristics With linear damage function, $Det(J)>0$ if the marginal revenue has large negative value or the demand has a gentle negative slope. The system near point II and III will then be unstable. When damage function $D_1''((1-\alpha)X_1)$ is strongly convex or marginal revenue is slightly negative, $Det(J)<0$. This implies that both points are saddle point stable.