A Quantile Monte Carlo approach to measuring extreme credit risk

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ABSTRACT

We apply a novel Quantile Monte Carlo (QMC) model to measure extreme risk of various European industrial sectors both prior to and during the Global Financial Crisis (GFC). The QMC model involves an application of Monte Carlo Simulation and Quantile Regression techniques to the Merton structural credit model. Two research questions are addressed in this study. The first question is whether there is a significant difference in distance to default (DD) between the 50% and 95% quantiles as measured by the QMC model. A substantial difference in DD between the two quantiles was found. The second research question is whether relative industry risk changes between the pre-GFC and GFC periods at the extreme quantile. Changes were found with the worst deterioration experienced by Energy, Utilities, Consumer Discretionary and Financials; and the strongest improvement shown by Telecommunication, IT and Consumer goods. Overall, we find a significant increase in credit risk for all sectors using this model as compared to the traditional Merton approach. These findings could be important to banks and regulators in measuring and providing for credit risk in extreme circumstances.

Keywords: Quantile Monte Carlo model; Extreme credit risk; Distance to default; European industrial sectors.

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1. Introduction

Many prevailing credit models are based on 'average' or 'median' risk over a specified period, or in the case of Value at Risk (VaR) models, on risks below a selected threshold. The problem with this approach is that it does not capture the most extreme economic circumstances in which firms are most likely to fail. The Merton (1974) model, modified by KMV (Crosbie & Bohn, 2003) measures Distance to Default (DD) based on the standard deviation of market asset value fluctuations. We modify the Merton model using a combination of Monte Carlo Simulation and Quantile Regression to compare DD at different risk quantiles. The revised model is hereafter referred to as the Quantile Monte Carlo (QMC) model.

We apply the QMC model to measure DD of European industries both prior to and during the GFC. We consider Europe a particularly important setting in which to measure extreme credit risk, given the instability experienced by European financial markets, and particularly the European banking sector, during the GFC and subsequent sovereign debt crisis.

The first question addressed by this study is whether asset volatility (and the associated DD) is significantly different between median and extreme levels. Using the QMC model we measure this difference at the 50% and 95% quantiles. Secondly, we use the QMC model to investigate whether relative extreme industry risk is different between the pre-GFC and GFC periods. That is, were the most risky industries pre-GFC also the most risky during the GFC?

The study finds the difference between risk quantiles to be highly significant, suggesting that traditional approaches to credit risk fail to capture extreme events. This could lead to a shortfall in key risk management tools, such as provisions and capital, just when they are most needed. In addition, the study finds that relative industry risk changes during the GFC as compared to pre-GFC. These findings can be important to lenders and regulators in making decisions on credit portfolio mix, provisions and capital allocation.

There have been other studies on sectoral risk in Europe (e.g., Allen, Powell, & Singh, 2011), but using different models to this study. The innovative QMC model is new and unique to the authors. QMC techniques have only previously been applied in a few studies (viz. Allen, Kramadibrata, Powell, & Singh, 2010, 2011, 2011a) and have not been applied to sector comparisons or to any European studies. Thus, this paper makes a unique contribution in this regard.

The literature survey in Section 2 provides a brief overview of the risk climate in Europe brought about by the GFC and sovereign debt crisis, as well as a summary of relevant credit risk literature. Data and methodology are discussed in Section 3, followed by findings and discussion in Section 4 and conclusions in Section 5.

2. Background

A combination of the GFC and the associated sovereign debt crisis has led to a prolonged period of financial instability in European markets. As happened in the US and other global markets, the GFC led to a need in Europe for financial sector support programmes and macroeconomic stimulus to be provided by Central banks and Governments. The 2008 UK Government £500bn financial support package is one example. Another example is the liquidity support measures provided by the Bank of England (BOE) and European Central Bank (ECB), which included extension of maturity terms on refinancing operations and allowing banks to swap illiquid securities for liquid ones. Following an emergency Paris summit in 2008, Euro-area governments provided co-ordinated support measures to their banks such as increasing deposit insurance, providing guarantees on bank bond issues and making capital injections into banks. Asset relief measures were introduced to remove or insure toxic bank assets (European Central Bank, 2009).

In 2010, concern that a number of European countries, including Greece, Portugal, Spain, Italy and Ireland would default on high debt levels led to a sovereign debt crisis. Comprehensive rescue packages by the International Monetary Fund and other Eurozone countries were formulated. The ECB also introduced measures to reduce volatility in financial markets and improve liquidity, such as commencing open market operations by buying government and private debt securities.

3. Literature Review

These events all provide a climate of extreme risk for investors and banks, leading to the need for accurate measurement of extreme risk. Techniques such as VaR, which measures risk below a predetermined threshold, have come under criticism. VaR, for instance, has undesirable mathematical properties; such as lack of sub-additivity (Artzner et al., 1999; 1997). But perhaps the biggest shortcoming of VaR is that it is focused on risks below a specified threshold and says nothing of the risks beyond VaR. The measurement has also been criticized by Standard and Poor's analysts (Samanta, Azarchs, & Hill, 2005) due to VaR being applied inconsistently across institutions, as well as lack of tail risk assessment. This study provides a mechanism for measuring that tail risk.

The QMC model applied here modifies the Distance to Default (DD) structural approach of Merton (1974) and KMV (Crosbie & Bohn, 2003). The Merton / KMV model measures DD based on a combination of fluctuating asset values and the debt / equity (leverage structure) of the borrower. Examples of studies using structural methodology for varying aspects of credit risk include asset correlation (Cespedes, 2002; Kealhofer & Bohn, 1993; Lopez, 2004; Vasicek, 1987; Zeng & Zhang, 2001), predictive value and validation (Bharath & Shumway, 2008; Stein, 2007), fixed income modelling (D'Vari, Yalamanchili, & Bai, 2003), and effect of default risk on equity returns (Chan, Faff, & Kofman, 2008; Gharghori, Chan, & Faff, 2007; Vassalou & Xing, 2004). Besides fluctuating assets, the other key component of structural modelling is the borrower's leverage ratio. Leverage ratios of banks have come under close scrutiny during the GFC, with many requiring additional capitalisation. The leverage ratios in this study range from 5.3% for

Financials to 53% for Health Care. The importance of fluctuating asset values in measuring credit risk has been raised by the Bank of England (BOE, 2008), who report that during the GFC "system-wide vulnerabilities were exposed…rooted in uncertainties about the value of banks assets…amplified by excessive leverage". As probabilities of default increase, there is greater likelihood of assets needing to be liquidated at market prices. BOE express a need for market participants to revalue their assets with greater weight placed on mark-to-market values. This gives rise to reduced asset values and a need for increased capital. Our model addresses the issue of measuring these extreme asset value fluctuations, with detailed methodology provided in the following section.

4. Methodology

Under Merton structural methodology, the firm defaults when asset values fall below debt levels. Moody's KMV model (Crosbie & Bohn, 2003) is based on the Merton model, and is widely used by banks to measure Distance to Default (DD) from which associated Probability of Default (PD) can be calculated using a normal distribution assumption. Based on the thousands of defaulted firms in their worldwide database, KMV find that DD is most accurately measured when debt is taken as the value of all short-term liabilities (one year and under) plus half the book value of all long term debt outstanding. This is the approach used in this study. DD and PD are calculated as follows:

$$DD = \frac{\ln(V/F) + (\mu - 0.5\sigma_v^2)T}{\sigma_v \sqrt{T}} \tag{1}$$

$$PD = N(-DD) \tag{2}$$

Where V is the market value of the firm, F = face value of firm's debt, μ = an estimate of the annual return (drift) of the firm's assets, σ_V = standard deviation of the asset returns and T is usually set as 1 year.

A key component of determining DD is the calculation of the asset returns. Our data includes 10 years of daily returns for all S&P Euro stocks. This data, together with balance sheet items required to calculate DD, is obtained from Datastream. Data is split into GFC (2007-2009) and pre-GFC (2000-2006) periods. The pre-GFC period aligns with a Basel requirement that 7 years of historical data is used in advanced credit risk models. Following Merton and KMV methodology, using equity returns and the relationship between equity and assets, we estimate an initial asset return for each company in our data set. Crosbie & Bohn (2003) and Bharath & Shumway (2008) provide a detailed explanation of this methodology. Daily log return is calculated and new asset values estimated for every day. This is repeated until asset returns converge. Industry returns are calculated as the asset-weighted average of company returns in that industry. We do not calculate correlations as we are not calculating returns for investment purposes.

It is at this stage that we depart from the Merton model. In measuring these returns, we could just use historical returns as used by Merton and KMV. However, our extreme quantile (as explained below) is only based on a small portion of returns (the 5% tail), and Monte Carlo is a particularly useful tool in enriching the database when dealing with smaller numbers of returns. We generate 20,000 scenarios of returns, a similar number to that suggested by other prominent Monte Carlo Simulation studies (E.g., Uryasev & Rockafellar, 2000). These simulations are obtained by creating random numbers based on the distribution (mean and standard deviation) of historical returns. The daily asset weighted average of returns is calculated for each industry.

We then apply quantile regression, which as per Koenker & Basset (1978) and Koenker and Hallock (2001), is a technique for dividing a dataset into parts. Minimising the sum of symmetrically weighted absolute residuals yields the median where 50% of observations fall either side. Similarly, other quantile functions are yielded by minimising the sum of asymmetrically weighted residuals, where the weights are functions of the quantile in question per Equation 3. This makes quantile regression robust to the presence of outliers.

$$\min_{\varepsilon \in R} \sum p_r(y_i - \varepsilon) \qquad (3)$$

where $p_r(.)$ is the absolute value function, providing the r^{th} sample quantile with its solution. β is calculated at each quantile for the regression variables (for example returns of an individual entity against market returns).

We regress the 20,000 Monte Carlo simulated returns for each industry (split into our two time periods) against the Monte Carlo returns for all industries (the portfolio 'benchmark') for the 10 year period, and derive β at the 50% quantile (which yields similar results to the standard Merton model) and 95% quantile (our extreme returns). From this β we are able to calculate DD. Our regression is benchmarked on portfolio returns, and therefore β at the median (50% quantile) of portfolio returns is 1. The denominator of the DD calculation in Equation 1 is based on asset value fluctuations, and so is the β generated by the quantile regression. A change in β will, therefore, result in a proportionately equal change in DD. For example, β of 2 for an industry (i) at a particular quantile (q) and time period (t) yields a DD equal to half the benchmark (b) DD:

$$DD_{iqt} = DD_b / \beta_{igt} \qquad (4)$$

5. Findings and Discussion

Table 1 shows DD and β for each period and quantile.

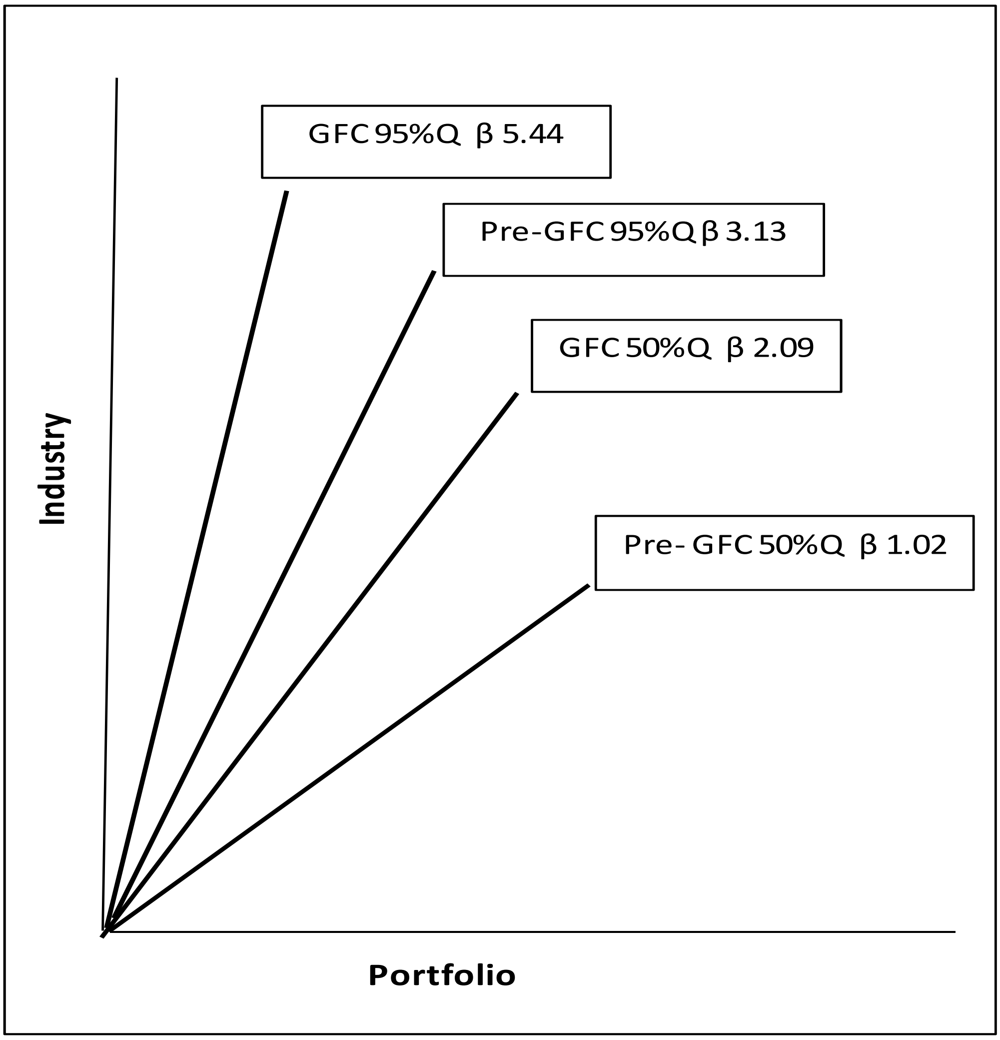
Table 1. Summary of Results

DD (measured by number of standard deviations) is calculated at the 50% quantile (50%Q) and 95% quantile (95%Q) using Equation 1. Pre-GFC incorporates the 7 years to 2006. The GFC period is 2007-2009. Beta measures risk at each quantile relative to the 50%Q total period quantile (shown in bold) as per Equation 3.

	DD						Beta (β)					
	50%Q pre-GFC	50%Q GFC	95%Q pre-GFC	95%Q GFC	50%Q Total Period	95%Q Total Period	50%Q pre-GFC	50%Q GFC	95%Q pre-GFC	95%Q GFC	50%Q Total Period	95%Q Total Period
Cons. Disc.	4.80	2.82	1.42	0.87	4.06	1.97	0.92	1.56	3.09	5.06	1.08	2.24
Cons. Stap.	5.15	4.12	1.43	1.24	4.75	2.96	0.86	1.07	3.09	3.55	0.93	1.49
Energy	5.11	2.97	1.70	0.95	4.54	2.12	0.86	1.48	2.59	4.64	0.97	2.08
Financials	4.30	2.10	1.41	0.81	3.60	1.45	1.02	2.09	3.13	5.44	1.22	3.04
Health Care	4.88	4.07	1.45	1.20	4.59	2.90	0.90	1.08	3.04	3.67	0.96	1.52
Industrials	5.32	3.33	1.60	1.08	4.79	2.32	0.83	1.32	2.75	4.09	0.92	1.90
IT	3.59	2.94	1.17	0.99	3.46	2.11	1.23	1.50	3.76	4.45	1.27	2.09
Materials	4.79	3.22	1.27	0.98	4.25	2.26	0.92	1.37	3.48	4.50	1.04	1.95
Telecomm.	4.23	3.99	1.15	1.69	4.30	2.75	1.04	1.10	3.82	2.61	1.02	1.60
Utilities	6.88	3.25	1.95	1.02	5.73	2.18	0.64	1.35	2.26	4.32	0.77	2.02
Total	4.90	3.28	1.45	1.08	**4.41**	2.30	0.90	1.34	3.03	4.07	**1.00**	1.91

The first question addressed by this study is whether there is a significant difference in DD between the 50% and 95% quantiles as measured by the QMC model. We note from Table 1 that, across the board for all industries, there is substantial difference in DD between the two quantiles. Using an F test, we find these differences between the quantiles to be significant at the 99% level for all industries. As an example, GFC DD for the financial industry was 2.09 at the 50% quantile, which has a PD of 2% per Equation 2. But during the worst 5% of asset value fluctuations (95% quantile), DD reduced to 0.81 (due to a combination of high volatility and high leverage). This is precariously close to default with a PD of 21%. The difference between the quantiles for the financial industry is illustrated in Figure 1. We see that at the 95% quantile, the financial industry has a β of 5.44 (based on a DD of 0.81 being 5.44x higher than the portfolio benchmark DD of 4.41) as compared to a β of 2.09 at the 50% GFC level. This means that at the period of the worst asset value fluctuations of the GFC, default risk was substantially higher than indicated by the traditional DD measure.

Figure 1. Financial Industry Betas



Our second research question is whether relative industry risk changes between the pre-GFC and GFC periods at the extreme quantile. From Table 1 we note several changes. The worst deterioration was experienced by Energy, Utilities, Consumer Discretionary and Financials. The strongest improvement was shown by Telecommunication, IT and Consumer goods. These results intuitively make sense. Telecommunications and IT experienced extreme volatility in the early pre-GFC period with the bursting of the dotcom bubble. Discretionary products are in less demand in a crisis than staples. The volatility experienced by financial markets and energy prices is well known. We used a Spearman rank correlation coefficient to test for ranking association, and found no significant association between pre-GFC and GFC 95% rankings. This means that extreme risk changes between industries in different economic circumstances; and that those industries that were riskiest pre-GFC are not the same industries that were riskiest during the GFC.

6. Conclusions and Implications

Applying a combination of Monte Carlo Simulation and Quantile Regression to the Merton structural credit model, the study shows significant DD differences between quantiles for all industries studied. Relative industry risk is also shown to change significantly as a result of the different economic circumstances of the GFC period as compared to the pre-GFC period. This has important implications for regulators and lenders. Traditional DD measures may not identify extreme risk, and capital and provisioning decisions based on these measures may leave banks short during a downturn.

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