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Multistate asymmetric ACD model: an application to order dynamics in the EUR/PLN spot market

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Abstract

This paper examines a process of order submissions and cancellations in the interbank order driven market of the EUR/PLN currency pair. Our contribution to the existing literature is twofold. We generalize the *Asymmetric ACD model (AACD)* of Bauwens & Giot (2003) with respect to more than two competing risks. It results in the flexible multistate econometric model for durations between moments in which order submissions or cancellations take place. Thanks to the Multistate AACD model we are able to examine timing of order submissions/cancellations that (1) take place on different sides of the market and (2) vary according to the level of order aggressiveness. We show how to simulate from the proposed *Multistate Asymmetric ACD model*, which enables us to study the transition probabilities between selected events. We investigate different market microstructure factors that exert an influence on the intraday pattern of order submission or cancellation strategies.

JEL classification: $G10, F30, C30$

Keywords: asymmetric ACD model, order dynamics, intraday liquidity

1

1 Introduction

Order dynamics is an important foundation for the quality of an order-driven market. The various factors that determine whether or not to place an order of a particular type at a particular moment influence the process whereby liquidity is supplied to the market via limit orders or is demanded from the market via market orders. Through detailed insight into the behavior of market participants it is also possible to understand the process of price formation. The disclosure of factors that determine the fluctuation of liquidity is of primary importance for both market clients and for regulators as an undesirable liquidity squeeze can lead to extreme and undesirable price movements as seen during the most recent financial turmoil of 2008-2009.

The paper analyzes the process of order submissions and cancellations in the interbank order driven market of the EUR/PLN currency pair. In order to capture the entirety of the dynamics of the order book, we present a generalized version of the Asymmetric ACD model that we will call the Multistate Asymmetric ACD model (MAACD). Within this wider setup we are able to describe the expected durations between moments when orders, classified according to selected "classes of aggressiveness", will arrive to the market or will be withdrawn from it. Thus, we are able to differentiate between potential factors governing the pace at which liquidity is exhausted or is replenished. Our work has been inspired by the empirical study of Lo & Sapp (2008) where the intraday mixture of best limit orders and market orders was analyzed by means of the standard asymmetric ACD model of Bauwens & Giot (2003). The study tests the impact of different factors not only on whether a trader chooses a particular order type but also on the timing of such a decision. The inclusion of time in the modeling framework allows the user to investigate time-varying preferences with regard to the arrival of particular orders in a continuously changing market environment. The precedence of market orders over limit orders (or vice versa) and the order clustering effects have been investigated from the perspective of time-varying liquidity and information-related market characteristics.

In this paper our goal is to further develop the work of Lo & Sapp (2008) in an effort to contribute to the empirical literature on order dynamics in two main dimensions. First, as opposed to Lo & Sapp (2008), we do not focus only on the best orders submitted within the system (i.e. market orders or best limit orders). Instead, we take into account all of the orders that enter into the trading system and we classify them according to their level of aggressiveness (i.e. the likelihood of being executed). Second, we model both sides of the order book jointly and simultaneously differentiate between the bid and ask sides of the market. Such a modelling strategy allows us to account for the possible endogeneity among different moments where buy and/or sell orders are posted or cancelled. Additionally, we demonstrate how to easily simulate from the MAACD model. Such a simulation algorithm can be helpful if we want to test the properties of the model. For example, we can easily simulate a most probable sequence according to which orders enter the market or are withdrawn from it. The obtained transition probability estimates can be compared with the existing theoretical (see Parlour (1998); Foucault (1999) and Goettler, Parlour & Rajan (2005)) or empirical (see Biais (1995); Hall & Hautsch (2006) and Hall & Hautsch (2007)) results from the literature concerning order book dynamics. With the application of the MAACD model we are also able to investigate how the arrival time of orders will relate to each other when they are (1) characterized by differing levels of aggressiveness and are (2) posted on different market sides. We also enrich our model with a variety of explanatory variables reflecting a continually changing market environment as well as the state of the order book. We then verify their impact on trader decisions with respect to the type of order placed and the time of such a submission or cancellation. This modelling framework allows us to verify selected theoretical microstructure hypotheses in a more explicit manner. Our model is very much related to the studies of Hall & Hautsch (2006) and Hall & Hautsch (2007) where the multidimensional autoregressive conditional intensity (ACI) function has been developed in order to account for the instantaneous arrival rates of different order types. In a close analogy to the multidimensional ACI model, the MAACD model can also account for the complex dynamics of the order book. Both models can describe the arrival rates characterizing particular classes of orders as well as the interdependence between these individual processes. Therefore, our effort is to apply another flexible econometric specification that can adequately describe a complicated intertemporal game of order submissions while at the same time being tractable and easy to estimate and/or to simulate.

The second aim of our analysis is to enrich what is currently a very scarce area of literature concerning the microstructure of currency markets in emerging economies. From the econometric viewpoint, the process of order submissions and cancellations in our modeling approach is reflected as an ordered point process. Accordingly, we model the temporal accelerations and/or decelerations in the pace of dealer activity with the highest resolution possible. In fact, the driving forces for order flow in our study are much different than those for stocks or major currency markets. Clearly, zloty and euro are not treated as substitutes with regards to level of risk undertaken; thus, the willingness to invest in emerging market currencies is closely linked to the changes in global risk aversion versus the risk appetite as well as to the fundamental foundings of the Polish economy as compared to the other CE3 members (i.e. Hungary and the Czech Republic).

The paper is structured as follows. In section 2 we introduce the theoretical argumentation for our model. We present a survey of the theoretical and empirical literature that deals with different aspects of order submission regularities. We also touch on the fact that there are market microstructure hypotheses that refer to selected factors which impact order choice. In the section 3 we present the Reuters Dealing 3000 Spot Matching System and we introduce the data to be used in the empirical study. Section 4 contains the theoretical econometric background for the specifications of the Multistate Asymmetric ACD model. In the sections 5 and 6 we present the results of the empirical analysis. The conclusion sums up the results of the study and comments their importance.

2 Literature Overview and Economic Hypotheses

There is a large strand of literature that underlines the informative content of time as one of the latent factors that influences the behavior of market participants. The theoretical microstructure models that implicitly entwine the concept of time date back to studies of Admati & Pfleiderer (1988), Diamond & Verrecchia (1987) and Easley & O'Hara (1992). In the first model, fluctuations in trading activity signal the arrival of new information. When trading intensity is high it is easier for informed traders to hide their strategic intensions. Simultaneously, liquidity traders (uninformed traders) who are discretionary (i.e. they can choose the periods in which they trade) also prefer to trade in such heavy periods as their activity does not induce undesirable price movements. On the other hand, periods of slow trading where there are no discretionary liquidity traders in the market is an area where there will be a relatively large fraction of informed traders present. This hypothesis is often described in the literature as "slow trading means informed trading". Informational content of trade activity is also underlined in the model of Easley & O'Hara (1992). In their approach, periods of increased trading signal the presence of informed traders; thus, their model is often described as "no trade means no news". In the model of Diamond & Verrecchia (1987), due to the presence of the short selling restrictions in some capital markets, informed traders cannot speculate on bad information. Therefore, if the bad news arrives trading activity declines. The essence of this phenomenon is often described as "no trading means bad information".

The aforementioned models underline the importance of time in explaining various strategic information-motivated intentions of market participants. The theoretical framework refers to a price-driven market with a single market maker. On the other hand, much theoretical research has also been carried out in order to fully understand the order submission strategies in an automated order-driven market (limit order market). Such trading mechanisms have gained in importance over the last decade and today limit order markets dominate trading in stocks and foreign exchange markets[1]. In the automated order-driven markets, dealers willing to trade can enter either a market order which is immediately executed at the prevailing bid or ask quotes or they can post a limit order that waits for a given time period for execution at a more favorable price. Market orders were initially perceived as information-motivated. Since superior information can quickly lose its value, the use of market orders always guarantees an immediate execution. Limit orders are perceived as patient, passive and liquidity-motivated (e.g. Glosten (1994) and Seppi (1997)). Most recently the conviction about sole informational content of market

[1] Examples of limit order books are: the Euronext Paris, the SEAQ or the NASDAQ (stock markets), the Reuters Dealing 3000 and the EBS (currency pairs) and the MTS (bonds).

orders has been challenged and there is a widespread notion that limit orders and the whole process of the liquidity provision can be also initiated by informed traders (e.g. Bloomfeld, O'Hara & Saar (2005); Anand, Chakravarty & Martell (2005) and Hasbrouck & Saar (2009)). The decision to post a limit order is always associated with a trade-off between the potential gain from obtaining a better price and the potential risk of either non-execution or being "picked-off". With a limit order a trader can get a better price; however, there is also the risk that the order may never be executed. On the other hand, as the price of limit orders are fixed, there is an additional risk that the orders may become mispriced when new information arrives. This adverse selection risk may lead to losses if a limit order is executed at an unfavorable price. The dilemma is determining what kind of order to choose and when to submit a given order. All of this provides the basis for numerous theoretical and empirical research studies on the dynamics of a limit order book.

Theoretical dynamic equilibrium models that describe the order choice problem as a sort of a multi-agent bargaining game have been formulated by Parlour (1998), Foucault (1999), Foucault, Kadan & Kandel (2005), Goettler et al. (2005) and Rosu (2009). In the first two studies some testable hypotheses with regard to the process of order submissions have been formulated. In Parlour (1998), the individual order choices depend on the state of the order book and the awaited arrival rate of market orders over the remainder of the day. Traders in this case anticipate the aftereffects of their actions on other market participants. Even if there is no information-based incentive to trade in this model, some regularities in the patterns of order submission can be obtained. Parlour (1998) predicts that after a market buy (sell), the most probable order would be limit sell (buy). It is because the payoff of a limit sell (buy) order (the utility gain) depends on the probability of execution. If the probability rises after a market buy (sell), the order book will be reduced by at least one unit on the ask (bid) side. The limit sell (buy) will gain the priority of execution and thus it will be more profitable. Such a phenomenon has been named "a crowding out effect" because orders of a given type that are submitted on one side of the order book "crowd out" other orders by making them unattractive. Accordingly, market orders submitted on the ask (bid) side of the market are crosscorrelated with limit orders on the bid (ask) side of the market. This regularity is summed up by the following hypothesis:

- H1: The probability of observing a limit sell (limit buy) order is larger if the previous transaction was a market buy (market sell), than if it was a market sell (market buy).

Another hypothesis has been formulated with regard to market orders. Once a market buy (sell) order is observed, a limit sell (buy) order allows for a higher utility gain than a market sell (buy). Traders who want to sell would rather choose limit sells than market sells. Thus, the liquidity provision is more profitable on the ask side of the market. This is another type of a the "crowding out effect" which predicts serial correlation in the order flow. This clustering of market orders on one market side can arise either from pure liquidity dynamics or from informed trading. Thus, a second hypothesis:

- H2: The probability of a market sell (market buy) is larger if the previous transaction was a market sell (market buy) than if it was a market buy (market sell).

The aforementioned hypotheses are based on the assumption that traders optimize their order choice with regard to (1) the state of the order book (which is determined by the past behavior of traders), and (2) the anticipated actions of other rational traders. The "crowding out mechanism" can also be used to formulate hypotheses that relate directly to the depth of the order book (see also in Hall & Hautsch (2006) and Hall & Hautsch (2007). Thus, a third hypothesis:

- H3: Increase in the depth on the ask (bid) side increases the aggressiveness of market trading on the ask (bid) side.

The rationale behind this hypothesis is this: a limit order placed in a book has a lower probability of execution when there are already many orders standing in front of it. Accordingly, the contemporaneous mixture of orders will be shifted toward market orders.

Some regularity with regard to the order sequencing has been discovered from the stochastic sequential model of Goettler et al. (2005). In this very general modeling setup, traders can submit multiple limit orders at different prices and trade different quantities. The generalization of this model comes with a cost and the equilibrium must be solved numerically. The simulation results depict a "diagonal effect", which means that market buys are more probable after market buys then after market sells. This agrees with the second hypothesis (H2). Moreover, market orders are often followed by limit orders on the same side of a market. The logic is that after an increase (decrease) of the consensus value of an asset, sell (buy) orders become mispriced. This induces a flow of buy (sell) market orders that "pick off" these limit orders.

In the dynamic equilibrium model of Foucault (1999), the proportion of limit to market orders depends on the volatility of an asset. An increase in the fundamental risk of an investment magnifies the risk of being picked-off. Thus, the reservation prices of limit orders become less aggressive and the bid-ask spread widens. As a consequence of this, the use of market orders becomes more expensive and traders opt for limit orders. The mixture of orders shifts in favor of limit orders but the trading activity declines. In this case the following hypotheses arise:

- H4: An increase in the bid-ask spread decreases the probability of a market order and increases the probability of an aggressive limit order.

- H5: An increase in the volatility decreases the probability of a market order and increases the probability of an aggressive limit order.

In the dynamic equilibrium model of Foucault et al. (2005) the concept of time has been introduced through the cost of waiting. The proportion of market and limit orders depends on a trade-off between the cost of immediacy (the bid-ask spread) and the cost of delayed execution (the waiting costs). Within this framework, waiting costs are proportional to the time required for traders to complete their transactions. Therefore, the increase in waiting cost prompts liquidity suppliers to bid or to ask more aggressively. This logic supports the fifth hypothesis (H5). According to the model, the amount by which traders improve upon the prevailing quotes depends on the size of the bid-ask spread.

In order-driven markets, the risk embedded in limit orders can be limited by an appropriate monitoring market information. Once the submitted orders become mispriced traders can always cancel or revise them in order to mitigate the free-option risk or the risk of no-execution. The literature provides many examples of information-motivated cancellations of limit orders. Hasbrouck & Saar (2009) document that over one-third of nonmarketable limit orders for NASDAQ-listed stocks on the INET are cancelled within two seconds. They show that the use of such "fleeting orders" is a very recent phenomenon that stems from improved technology and the active trading culture. Fong & Liu (2010) show that order revisions generate a net economic benefit for market participants and that the process of order cancellations/revisions is based on the continual monitoring of market conditions. Liu (2009) proposes a formal model where the relationship between order submission risks and cancellation/revision activity has been investigated. He predicts that if the bid-ask spread widens then the cancellation rates will fall. If the bid-ask spread is large the expected cost of execution of an limit order at an undesirable price as well as the expected cost of non-execution is low. The same relationship is predicted by Hasbrouck & Saar (2009). According to their "cost-of-immediacy" hypothesis, if the cost of immediate execution (i.e. the bid-ask spread) decreases, the "gravitational pull" of immediate execution with a market order rises. The limit orders that were placed inside the quotes and were not purely liquidity-motivated are then are changed to market orders. Thus, we have a sixth hypothesis:

- H6: A decrease in the bid-ask spread increases the probability of a limit order cancellation.

In agreement with the model described by Foucault (1999), cancellation activity should be also higher if volatility rises. Large and unpredictable swings of the exchange rate increase the risk that the order will be mispriced (the free option risk of a limit order); thus, we predict that the following hypothesis should be true:

- H7: An increase in the volatility increases the probability of a limit order cancellation.

The availability of high-resolution quote datasets has spurred an interest in empirical verification of the hypotheses stated above. There is a large body of empirical literature that deals with the impact of different market conditions on order choice in limit order markets (Bae, Jang & Park (2003); Ranaldo (2004); Verhoeven, Ching & Ng (2003); Pascual & Veredas (2004); Ellul, Holden, Jain & Jennings (2007); Lo & Sapp (2008); and others). The empirical analysis of Biais (1995) is one of the first studies on this matter. The study investigates order sequences on the Paris Bourse and finds some systematic patterns. For example, it empirically demonstrates "the diagonal effect", which is the clustering of orders of the same type. Biais (1995) proposes following explanations for this. First, traders may split large orders into small ones in an effort to limit a price reaction. Second, traders may imitate other traders' behavior ("piggyback"). Third, traders may indeed react to selected market events in a similar way. Although the findings of Biais (1995) spurred serious interest in confirming such regularities within different markets, the majority of the econometric tools that have been applied were strictly static in nature. The studies of Verhoeven et al. (2003); Ranaldo (2004) and Ellul et al. (2007) use logit regressions. Such specifications can adequately account for numerous exogenous factors that have an impact on the order choice; however, they cannot take into account the role of time variation between order submissions. Trading needs do not occur synchronically among traders and they may be evoked by the actions taken by other market participants[2]. Static models do not account for the casual effects such as possible spill-overs between the arrival rates of various order types exist. Moreover, nearly all of the empirical findings that have been reported depict stock markets; there are only a few exceptions devoted to currency trading (Bloomfeld et al. (2005), Lo & Sapp (2008)).

[2]This is most pronounced in the automated FX markets where, based upon the information obtained from FX dealers, about 80% of trades are purely speculative in their motivation. The remainder are motivated by real trading needs.

3

3 Market and Data

The FX market of the Polish Zloty is the most liquid among all the currency markets of the Central European emerging economies[3]. Spot transactions can be executed on the OTC market using the Reuters Dealing Direct system, via a voice broker or by telephone. About half of all trades is conducted via the Reuters Dealing 3000 Spot Matching System (RDSMS) and the importance of this platform is continually growing over the subsequent years. It is a very liquid and transparent electronic brokerage system, operating as an order-driven market that can automatically match incoming buy and sell orders once their prices agree. FX dealers can submit either limit or market orders; market orders are perceived as liquidity-consuming and aggressive since they are immediately executed against most competitive limit orders in the order book. As far as the transparency of the system is concerned, traders can observe continually changing best ask and bid prices that correspond to the most competitive buy and sell limit orders stored in the order book. They can also see the whole size of best ask and best bid limit orders (the depth at the best ask and the depth at the best bid) but only if this size does not exceed 10 M. EUR. If it is the case, the quantity of the whole depth is hidden and the sign "R" (regular order) appears next to the best ask or the best bid quotation. Such a solution allows for the size of large orders submitted to the system at the best prices to be hidden. Traders can see neither the price nor the quantity of all other (less competitive) orders placed in the system. Unlike the EUR/USD currency pair, traders also cannot see the potential price at which 10 M. EUR could be traded. Nevertheless, traders have the possibility of observing the continually changing price of all of the executed transactions with corresponding indicators "P" or "T". This allows them to deduce whether the trade was "Paid" (initiated by an ask side, hence a market sell) or "Taken" (initiated by a bid side, henceforth a market buy). Although the exact volume of trades is not able to be seen, market participants can gain a certain intuition about the changes in the order flow via the observed quantities of executed orders.

The datasets used in this study comprise market and limit orders submitted to the system or cancelled from the market between 2 Jan 2007 and 31 Jan 2007. The EUR/PLN exchange rate is quoted as a quantity of Zlotys per one Euro. During the period of study, the Zloty followed an appreciating trend towards Euro. Trading of the Polish Zloty takes place within offshore markets (i.e. London banks) as well

[3] According to the survey of FX market activity conducted by the Bank for International Settlements BIS (2007), the average daily turnover in interbank spot transactions amounted to 4,851 million USD in April 2007. This market is, therefore, nearly two times bigger than the spot market for the Hungarian Forint where the average daily turnover amounted to 2,959 million USD in April 2007 and about three times more liquid than the market for the Czech Koruna (1,630 million USD).

as within Poland[4] and the datasets cover both of these trading venues. Every order includes an exact date and time of submission as well as an execution/cancellation indicator, a firm quote, a size and an indicator for the market side of a quote. The detailed structure of the datasets makes it possible to rebuild the entire order book at each moment of market activity. In figure 3 we present the exemplary state of the order book on 18 January, 16:13 CET. In this particular snapshot, the whole depth on the ask market side (sell limit orders) amounts to 22 M. EUR (the depth at the best ask that is visible to traders equals 2 M. EUR) whereas the depth on the bid market side (buy limit orders) equals 15 M. EUR (the depth at the best bid that is visible to traders equals 2 M. EUR).

3

Although trading on the interbank market can take place 24 hours a day and 7 days a week, it is heavily concentrated on working days between 8:00 and 18:00 Central European Time (GMT+1, with Daylight Savings Time). The amount of orders submitted beyond these time frames is quite negligible (see figure 3). In order to limit the undesired impact of periods where trading is particularly thin, we exclude observations registered on weekends and on working days between 18:00 and 8:00 CET[5].

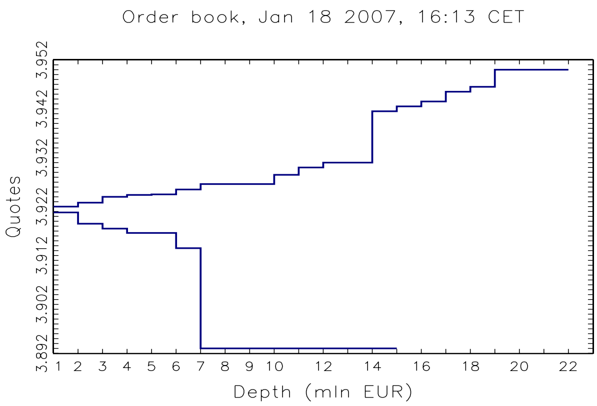

Figure 1: A snapshot of the order book for the EUR/PLN currency pair. The stepwise lines depict the ask (upper line) and bid (lower line) slope curves.

All of the incoming orders have been categorized according to their level of aggressiveness as in the studies of Hall & Hautsch (2006) and Ellul et al. (2007). In the analysis we also included cancellations of best buy and sell limit orders. Our sample covers 92,818 selected events and each of them fall into the one of ten categories:

[4]About 80% of the turnover takes place in the offshore market and the remaining 20% are trades between banks located within Poland.

[5]A similar truncation procedure has also been performed in the study of Lo & Sapp (2008).

- $k = 1$ (MS) – Submission of a market sell order or a marketable limit sell order; in this case the price of the incoming sell order is lower or equal to the most competitive bid price prevailing in the order book. Such orders are immediately executed against the limit orders stored on the bid side of the order book. There are 10,808 MS order submissions in the sample.

- $k = 2$ (IQS)– Submission of an inside-the-quote limit sell order; in this case the price of the incoming sell order is lower than the best ask price but higher than the best bid price. Such orders improve the best ask price. There are 14,395 IQS order submissions in the sample.

3

- $k = 3$ (AQS) – Submission of an at-the-quote limit sell order; in this case the price of the incoming sell order is equal to the best ask price prevailing in the system. These orders increase the depth at the best ask. There are 4,871 AQS order submissions in the sample.

- $k = 4$ (BQS) – Submission of a behind-the-quote limit sell order; in this case the price of the sell order is higher than the lowest (most competitive) ask price in the order book. There are 9,308 BQS order submissions in the sample.

- $k = 5$ (CS) – Cancellation of an inside-the-quote or an at-the-quote limit sell order. There are 7,429 CS events in the sample[6].

- $k = 6$ (MB) – Submission of a market buy order or a marketable limit buy order; in this case the price of the incoming buy order is greater then or equal to the lowest (most competitive) ask price in the order book. Such orders are immediately executed against limit orders stored on the ask side of the order book. There are 11,069 MB order submissions.

- $k = 7$ (IQB) – Submission of an inside-the-quote limit buy order; in this case the price of the incoming buy order is lower than the best ask price but is higher than the best bid price. Such orders improve the best bid price. There are 14,239 IQB order submissions.

- $k = 8$ (AQB) – Submission of an at-the-quote limit buy order; in this case the price of the incoming buy order is equal to the price of the best buy limit order prevailing in the book. Such orders increase the depth at the best bid. There are 4,231 AQB order submissions.

[6]We include cancellations of best limit sell and best limit ask orders since they are believed to have greater informational content then cancellations of behind-the-quote orders. In this way we do not cover cancellations of orders that have "moved away" from the market price and were perhaps left in the system for a very long time. Their removal from the system does not bring much insight into either the process of price formation or liquidity provision in comparison to the behavior of best limit orders.

- $k = 9$ (BQB) – Submission of a behind-the-quote limit buy order; in this case the price of the incoming buy order is lower than the best bid price. There are 9,188 BQB order submissions.

- $k = 10$ (CB) – Cancellation of an inside-the-quote or an at-the-quote limit buy order. There are 7,280 CB events in the sample.

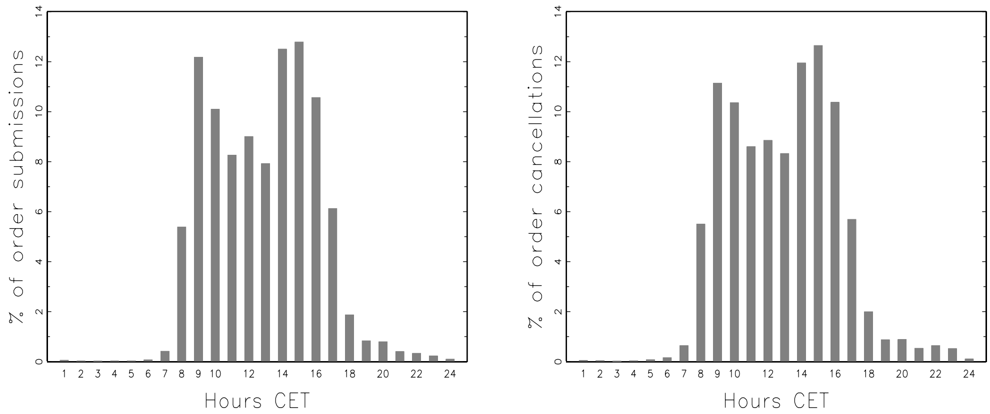

Figure 2: Intraday frequency of order submissions (left panel) and order cancellations (right panel).

4

4 The Econometric Approach

In a very close analogy to Bauwens & Giot (2003), we consider the model for the marked point process $\{x_i, y_i\}$, where $x_i = t_i - t_{i-1}$ is a duration between the moments in which subsequent orders arrive to the system or are withdrawn from it and y_i is an indicator variable for a particular type of an event $y_i = k$ (where $k = 1, 2, ..., 10$)[7]. At the end of each duration x_i one of ten possible states: MS, IQS, AQS, BQS, CS, MB, IQB, AQB, BQB or CB can be observed. Accordingly, x_i can be treated as an outcome variable of a function $x_i = \min(x_{i,1}, x_{i,2}, ..., x_{i,10})$, where each of variables $x_{i,k}$ (for $k = 1, 2, ..., 10$) corresponds to an order duration that would end up in the state k. As in the standard framework of a competing risks model, only the shortest from ten possible durations is observed (realized), which happens if an order of a corresponding type enters the system or is cancelled from it. For example, if a MS order ($y_i = 1$) were posted, only the MS duration $x_{i,1}$ would be observed at t_i. Other durations that would have ended up in other states (i.e. IQS, AQS, BQS, CS, MB, IQB, AQB, BQB and CB durations) would be therefore unobservable and are treated as censored by the arrival of the MS order at time t_i.

As in the standard framework of a competing risks model, we consider a joint conditional bivariate density for x_i and y_i[8]:

$$f(x_i, y_i|\mathcal{F}_{i-1}) = \prod_{k=1}^{10} h_{x_k}(x_i|\mathcal{F}_{i-1})^{I_i^k} S_{x_k}(x_i|\mathcal{F}_{i-1}) \tag{1}$$

where I^k is a dummy variable ($I_i^k = 1$ if a state $y_i = k$ is observed at time t_i and $I_i^k = 0$ if a state $y_i \neq k$ is observed at time t_i). $\mathcal{F}_{i-1}$ denotes an information set up to a time point $t-1$ that contains past realizations of x_i and y_i, h_{x_k} and S_{x_k} denote a hazard and a survival function for x_k, respectively.

The duration that is realized (observed) contributes to the joint conditional density function given by equation (1) via its density function, whereas other unrealized (censored) durations contribute to it via their survival functions. For example, if a state MS is observed at t_i, the conditional bivariate density of the pair $\{x_i, y_i\}$ is given by:

$$f(x_i, y_i = 1|\mathcal{F}_{i-1}) = h_{x_1}(x_i|\mathcal{F}_{i-1})^{I_i^1} S_{x_1}(x_i|\mathcal{F}_{i-1}) \prod_{k=2}^{10} S_{x_k}(x_i|\mathcal{F}_{i-1}) \tag{2}$$

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[7] Although a subset of natural numbers is applied to define a discrete process $y_i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, the numbers are used as qualitative indicators in order to discriminate selected events. Accordingly, natural ordering of y_k does not have any informative meaning in the context of this model.

[8] The model assumes independence (conditionally on $\mathcal{F}_{i-1}$) between durations $x_{i,1}, x_{i,2}, ..., x_{i,10}$.

$$= \quad f_{x_1}(x_i|\mathcal{F}_{i-1}) \prod_{k=2}^{10} S_{x_k}(x_i|\mathcal{F}_{i-1})$$

Therefore, if a duration x_i ends with an MS order ($y_i = 1$), x_i contributes to the density function via: (1) the conditional density of $x_{i,1}$ evaluated at x_i, i.e. $f_{x_1}(x_i|\mathcal{F}_{i-1})$ and (2) the joint conditional probability that all other unobserved durations $x_{i,k}$ (with end states $k = 2, ..., 10$) are longer than the realized duration x_i: $\prod_{k=2}^{10} S_{x_k}(x_i|\mathcal{F}_{i-1})$.

The conditional hazard and survival functions for each of selected durations $x_{i,k}$ are specified with the Logarithmic ACD models with a Weibull distribution of an error term (see Bauwens & Giot (2000)). The conditional duration expectations are specified in a dynamic fashion, such as previous states and previously observed durations could exert an influence on their length. In the standard framework of the ACD model $x_{i,k}$ is given as:

$$x_{i,k} = \Phi_{i,k}\varepsilon_{i,k} \tag{3}$$

where $\Phi_{i,k} = \Psi_{i,k} \cdot \mu_{\varepsilon_i}^{-1}$, $\Psi_{i,k} = E(x_{i,k}|\mathcal{F}_{i-1})$ and μ_{ε_i} is the mean of the Wiebull distribution[9], $\varepsilon_{i,k}$ is a Weibull-distributed error term ($\varepsilon_{i,k} \sim i.i.d.\ W(\gamma_k, 1)$, where γ_k is a shape parameter and a dispersion parameter is restricted to 1). Conditional (with respect to $\mathcal{F}_{i-1}$) duration expectations are modelled with the Log-ACD model of type I[10]. The logarithms of duration expectations $\psi_{i,k} = \ln(\Psi_{i,k})$ are:

$$\begin{aligned} \psi_{i,k} &= \sum_{l=1}^{10}(\omega_{l,k} + \alpha_{l,k}\ln x_{i-1})I_{i-1}^{l} + \beta_k\psi_{i-1,k} \qquad (4) \\ &= \sum_{l=1}^{10}(\omega_{l,k} + \alpha_{l,k}\ln x_{i-1,l})I_{i-1}^{l} + \beta_k\psi_{i-1,k} \end{aligned}$$

where $l = 1, 2, ..., 10$ and I_i^l is a dummy indicator ($I_i^l = 1$, if a state $y_i = l$ at the end of duration x_i and $I_i^l = 0$, if $y_i \neq l$).

The econometric specifications of duration expectations according to the previously realized state of y_i. Thus, the expected waiting times till particular order submissions or cancellations will take place vary with the type of the previously observed events and the time that had elapsed until they occurred.

Assuming the Weibull distribution for the error terms $\varepsilon_{i,k}$, the joint conditional density function for the pair $\{x_i, y_i\}$ can be derived as:

[9] An exposition of the Weibull distribution and some major properties of the MAACD model were provided in the Appendix.

[10] Detailed properties of the Log-ACD models can be found in Bauwens, Galli & Giot (2008).

$$f(x_i, y_i|\mathcal{F}_{i-1}) = \prod_{k=1}^{10} \left[\frac{\gamma_k}{\Phi_{i,k}} \left[\frac{x_i}{\Phi_{i,k}} \right]^{\gamma_k - 1} \right]^{I_i^k} \cdot e^{-\left[\frac{x_i}{\Phi_{i,k}}\right]^{\gamma_k}} \tag{5}$$

Accordingly, the joint log-likelihood function can be given as the sum of ten log-likelihoods:

$$\begin{aligned} \ln L(\Theta|x_i, y_i, \mathcal{F}_{i-1}) &= \sum_{k=1}^{10} \ln L_k(\Theta_k|x_i, y_i, \mathcal{F}_{i-1}) \\ &= \sum_{k=1}^{10} I_i^k \left[\ln \left[\frac{\gamma_k}{\Phi_{i,k}} \left[\frac{x_i}{\Phi_{i,k}} \right]^{\gamma_k - 1} \right] \right] - \sum_{k=1}^{10} \left[\frac{x_i}{\Phi_{i,k}} \right]^{\gamma_k} \end{aligned} \tag{6}$$

where $\Theta = \bigcup \Theta_k$, $\Theta_k = \{\omega_{l,k}, \alpha_{l,k}, \beta_k, \gamma_k\}$ (for $l = 1, 2, ..., 10$). Because there are no relations between the parameters from the distinct subsets Θ_k (as durations $x_{i,k}$ are independent conditionally on $\mathcal{F}_{i-1}$), we are able to estimate the model by separately maximizing ten log-likelihoods: $\ln L_k(\Theta_k|x_i, y_i, \mathcal{F}_{i-1})$. The proposed specification of the MAACD model is very complex and demands very rich parametrization. In fact, we allow for 220 different parameters (i.e. 22 parameters for each of ten possible states). But this generality does not come at a cost of a burdensome estimation process. The real advantage of the model is that the estimation is very fast and easy if we profit from the decomposition of the likelihood function.

5 The Empirical Application

The intraday activity of the process of order submissions experiences very strong intraday seasonality (see Figure 2). We assume the multiplicative diurnality pattern and then, we model the deseasonalized variable $\bar{x}_i = \frac{x_i}{s_t}$ using the MAACD specification. The intraday seasonality factor s_t is derived by the nonparametric method. We apply kernel regression of durations on a time-of-day variable as has been proposed in Bauwens & Veredas (2004):

$$s(t) = \frac{\sum_{i=1}^{T} K\left[\frac{t-t_i}{h}\right] x_i}{\sum_{i=1}^{T} K\left[\frac{t-t_i}{h}\right]} \tag{7}$$

where K is the quartic kernel function, h is the optimal bandwidth selected as $2.78sN^{-\frac{1}{5}}$ (s is the standard deviation of the data), N is the number of observations and t_i is a time-of-day variable standardized on the [0,1] interval (i.e. a cumulative number of seconds from 8.00 CET until the moment of the order submission and then divided by the number of seconds in each day).

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Using the deseasonalized series $\bar{x}_i$ and the corresponding y_i indicators, we estimated the MAACD model with separate maximizing ten components of the log likelihood function given by the formula 6. We used the BHHH optimization algorithm from the maxlik library of Gauss 10.0. The parameter estimates and their corresponding p-values (corresponding to the standard errors computed with a robust "sandwich" formula) are presented in Table 1. All variables that are responsible for dynamic features of the MAACD model are statistically significant, which confirms strong auto and crosscorrelations among time periods elapsing to different orders. Each of the main columns in the upper and lower panels of the table contain parameter estimates that describe the conditional expectation of a duration that ends as one of the ten possible outcomes. For example, in the first column of the upper panel of the table we show estimates of equation 4 for the duration expectation that elapses with the arrival of the MS order. Analogously, the second column of the lower panel of the table presents estimates of the conditional expected duration that elapses if an IQB order is posted in the system. Different intercepts (i.e. $\hat{\omega}_{l,k}$) correspond to various reactions of the expected duration to the preceding event, whereas $\hat{\alpha}_{l,k}$ and $\hat{\beta}_k$ estimates are responsible for the scale of duration clustering. The clustering effect may also vary according to the previous state because we allow for different $\hat{\alpha}_{l,k}$ coefficients for the past duration, conditional on the type of a preceding order.

I. ASK SIDE OF THE MARKET (SELL ORDERS)										
	MS, k=1		IQS, k=2		AQS, k=3		BQS, k=4		CS, k=5	
parameters	estimate	p - value	estimate	p - value	estimate	p - value	estimate	p - value	estimate	p - value
$\omega_{1,k}$ MS	-0.0437	0.0539	0.6665	0.0000	0.5988	0.0000	0.5437	0.0000	0.4920	0.0000
$\omega_{2,k}$ IQS	0.6044	0.0000	1.2937	0.0000	0.4715	0.0000	0.4570	0.0000	0.4430	0.0000
$\omega_{3,k}$ AQS	0.4858	0.0000	0.9437	0.0000	0.2432	0.0001	0.5662	0.0000	-0.1009	0.0176
$\omega_{4,k}$ BQS	0.8061	0.0000	1.1856	0.0000	0.7460	0.0000	0.2791	0.0000	0.9958	0.0000
$\omega_{5,k}$ CS	0.8260	0.0000	-0.0784	0.0000	0.5894	0.0000	0.5341	0.0000	0.9454	0.0000
$\omega_{6,k}$ MB	0.5240	0.0000	0.1420	0.0000	0.4005	0.0000	0.1479	0.0000	1.0578	0.0000
$\omega_{7,k}$ IQB	0.8512	0.0000	1.1050	0.0000	0.8605	0.0000	0.5094	0.0000	0.9527	0.0000
$\omega_{8,k}$ AQB	0.6912	0.0000	1.2293	0.0000	0.7605	0.0000	0.4827	0.0000	1.0244	0.0000
$\omega_{9,k}$ BQB	0.8512	0.0000	1.1436	0.0000	0.8629	0.0000	0.6585	0.0000	1.0939	0.0000
$\omega_{10,k}$ CB	0.6912	0.0000	0.9714	0.0000	0.9284	0.0000	0.6396	0.0000	1.0208	0.0000
β_k	0.8256	0.0000	0.6862	0.0000	0.8573	0.0000	0.8732	0.0000	0.7859	0.0000
$\alpha_{1,k}$ MS	0.1120	0.0000	0.1199	0.0000	0.1090	0.0000	0.1004	0.0000	0.1007	0.0000
$\alpha_{2,k}$ IQS	0.1002	0.0000	0.1732	0.0000	0.1119	0.0000	0.1474	0.0000	0.1621	0.0000
$\alpha_{3,k}$ AQS	0.0917	0.0000	0.2173	0.0000	0.1424	0.0000	0.1764	0.0000	0.1292	0.0000
$\alpha_{4,k}$ BQS	0.1609	0.0000	0.2239	0.0000	0.1854	0.0000	0.1475	0.0000	0.1509	0.0000
$\alpha_{5,k}$ CS	0.1411	0.0000	0.1429	0.0000	0.1253	0.0000	0.0740	0.0000	0.2012	0.0000
$\alpha_{6,k}$ MB	0.1266	0.0000	0.0779	0.0000	0.0860	0.0000	0.0374	0.0006	0.1010	0.0000
$\alpha_{7,k}$ IQB	0.1394	0.0000	0.2285	0.0000	0.1382	0.0000	0.1091	0.0000	0.1421	0.0000
$\alpha_{8,k}$ AQB	0.1570	0.0000	0.2428	0.0000	0.1235	0.0001	0.0893	0.0000	0.1614	0.0000
$\alpha_{9,k}$ BQB	0.1394	0.0000	0.2285	0.0000	0.1682	0.0000	0.1237	0.0000	0.1974	0.0000
$\alpha_{10,k}$ CB	0.1570	0.0000	0.2585	0.0000	0.1883	0.0000	0.1559	0.0000	0.1044	0.0000
γ_k	0.7533	0.0000	0.7959	0.0000	0.7897	0.0000	0.7878	0.0000	0.7954	0.0000

II. BID SIDE OF THE MARKET (BUY ORDERS)										
	MB, k=6		IQB, k=7		AQB, k=8		BQB, k=9		CB, k=10	
parameters	estimate	p - value	estimate	p - value	estimate	p - value	estimate	p - value	estimate	p - value
$\omega_{1,k}$ MS	1.0270	0.0000	0.1547	0.0000	0.5206	0.0000	0.0657	0.0145	1.1219	0.0000
$\omega_{2,k}$ IQS	0.4451	0.0000	1.2617	0.0000	1.0773	0.0000	0.3939	0.0000	1.0776	0.0000
$\omega_{3,k}$ AQS	0.8078	0.0000	1.3036	0.0000	0.9076	0.0000	0.3966	0.0000	1.1653	0.0000
$\omega_{4,k}$ BQS	0.7997	0.0000	1.3170	0.0000	1.1397	0.0000	0.5467	0.0000	1.2018	0.0000
$\omega_{5,k}$ CS	1.3696	0.0000	1.0206	0.0000	1.1696	0.0000	0.6053	0.0000	1.1347	0.0000
$\omega_{6,k}$ MB	-0.0040	0.8710	0.7478	0.0000	0.8637	0.0000	0.5319	0.0000	0.6670	0.0000
$\omega_{7,k}$ IQB	0.6199	0.0000	1.4317	0.0000	0.6794	0.0000	0.3993	0.0000	0.5209	0.0000
$\omega_{8,k}$ AQB	0.5822	0.0000	1.1176	0.0000	0.5541	0.0000	0.4958	0.0000	0.0350	0.4910
$\omega_{9,k}$ BQB	0.9516	0.0000	1.3174	0.0000	1.0480	0.0000	0.2266	0.0000	1.2221	0.0000
$\omega_{10,k}$ CB	1.0118	0.0000	-0.1800	0.0000	0.8500	0.0000	0.4308	0.0000	1.1967	0.0000
β_k	0.8024	0.0000	0.6544	0.0000	0.8048	0.0000	0.8919	0.0000	0.7519	0.0000
$\alpha_{1,k}$ MS	0.2099	0.0000	0.0591	0.0000	0.0424	0.0072	0.0282	0.0033	0.1037	0.0000
$\alpha_{2,k}$ IQS	0.1393	0.0000	0.2168	0.0000	0.1669	0.0000	0.0917	0.0000	0.1782	0.0000
$\alpha_{3,k}$ AQS	0.1661	0.0000	0.2385	0.0000	0.1425	0.0000	0.0937	0.0000	0.2091	0.0000
$\alpha_{4,k}$ BQS	0.2034	0.0000	0.2358	0.0000	0.1759	0.0000	0.0961	0.0000	0.1985	0.0000
$\alpha_{5,k}$ CS	0.1812	0.0000	0.2173	0.0000	0.1699	0.0000	0.1408	0.0000	0.1519	0.0000
$\alpha_{6,k}$ MB	0.1141	0.0000	0.1336	0.0000	0.1157	0.0000	0.1092	0.0000	0.0916	0.0000
$\alpha_{7,k}$ IQB	0.0696	0.0000	0.1473	0.0000	0.1474	0.0000	0.0948	0.0000	0.2148	0.0000
$\alpha_{8,k}$ AQB	0.1279	0.0000	0.2240	0.0000	0.1701	0.0000	0.1658	0.0000	0.1270	0.0000
$\alpha_{9,k}$ BQB	0.2077	0.0000	0.2587	0.0000	0.2022	0.0000	0.1230	0.0000	0.1735	0.0000
$\alpha_{10,k}$ CB	0.1913	0.0000	0.1765	0.0000	0.1960	0.0000	0.0471	0.0025	0.2448	0.0000
γ_k	0.7428	0.0000	0.7787	0.0000	0.7833	0.0000	0.7940	0.0000	0.7861	0.0000

Table 1: Estimation results for the MAACD model.

The obtained results allow for the following interpretation:

- High autoregressive coefficients ($\hat{\beta}_k$) prove the quite strong persistence of the duration process, which is especially pronounced for submissions of the least aggressive limit orders. The highest coefficients correspond to the BQS and BQB orders. We could risk saying at this point that these orders are not as much information-motivated as the market and inside-the-quotes orders and that the decision to submit them is not so vulnerable to the flow of short-lived information that may quickly lose its value. Accordingly, the clustering of behind-the-quote orders is most persistent, which means that the forecasted duration has a longer memory with respect to the past history of order submissions. On the other hand, the smallest $\hat{\beta}$ coefficients can be observed for orders that are placed inside of the best quotes, which means that they do not rely as much on the information from the distant past, especially in comparison to behind-the-quote orders.

- The coefficients $\hat{\omega}_{l,k}$ represent "regime-switching" intercepts that change with the type of the preceding order. A closer look at the various values of these estimates for the MS and MB durations (the first column in Table 1) allows for the following conclusion: the coefficient $\hat{\omega}_{1,1} = -0.0437$, corresponding to the previous MS order, is the smallest among all other $\hat{\omega}_{l,1}$ estimates and thus means that the expected duration to a market sell will be most considerably shortened in result of a market sell. Analogously, the same observation emerges for the bid side of the market. From the contents within first column of the lower panel of the Table 1 we see that the obtained coefficient $\hat{\omega}_{6,6} = -0.0040$ is the smallest among all other intercepts $\hat{\omega}_{l,6}$, for $l = 1, 2, ..., 10$. Therefore, realization of a market/marketable buy order exerts the strongest influence on decreasing the time until another market/marketable buy order occurs. This observation agrees with Biais (1995) showing that market orders cluster together as traders split large orders into small parts in an effort to limit the undesired price impact of a huge transaction. Interestingly, for the expected duration at the end of which an IQS order is observed (second column of the table 1), $\hat{\omega}_{5,2} = -0.0784$ is the smallest intercept. This coefficient refers to a previous cancellation of an order at the best ask (best sell order). Quite naturally then, such an action results in a new submission of a best limit sell order. The coefficient $\hat{\omega}_{6,2} = 0.1420$ is the second smallest among $\hat{\omega}_{l,2}$, which means that transactions at the ask (buys) also accelerate the submissions of best limit sell orders that establish the new most competitive ask price. This results from a price-reverting behavior, a kind of a micro cycle build from phases of liquidity consumption and replenishment as predicted by Parlour (1998) (see Hypothesis H1). Execution of a MB order exhausts liquidity on the ask side of a market and dealers can compete for an ask price that is at least one tick (pip) better than the current one. Analogously, a symmetrical pattern can be observed for the arrival rates of the IQB orders (the second column in lower panel of Table 1). The expected duration to posting an inside-the-quote buy shrinks most considerably after a cancellation on the same market side because $\hat{\omega}_{10,7} = -0.1800$ is the smallest intercept among all other $\hat{\omega}_{l,7}$ estimates. The second smallest coefficient is $\hat{\omega}_{1,7} = 0.1547$; this corresponds to a previous market sell.

As a general rule the expected waiting times to market/marketable and inside-the-quotes order submissions intercepts $\hat{\omega}_{4,1}$, $\hat{\omega}_{4,2}$ and $\hat{\omega}_{9,6}$, $\hat{\omega}_{9,7}$ are quite large in value. This can mean that these most aggressive and price-improving or-

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ders are (at least to some extent) information-motivated and do not have the tendency to follow any behind-the-quote orders that are hidden in the depth of the order book. The MS and MB durations will also rise in value if a cancellation occurs. Thus, if the bid-ask spread widens after a cancellation on any side of a market, the transaction costs of the MB or MS orders will be higher. In this case it will be more profitable to place an inside-the-quote limit buy or sell order than a market order.

The expected AQS duration shrinks most considerably in the presence of the AQS order ($\hat{\omega}_{3,3} = 0.2432$ is the smallest coefficient among $\hat{\omega}_{l,3}$). For the bid side of the market $\hat{\omega}_{1,8} = 0.5206$ is the smallest in value and thus the expected AQB duration shrinks most considerably in the presence of the MS order. Watching the dynamics of the market sell may enhance FX dealers to submit more orders at the best bid as the probability of their execution rises. The arrival rate of behind-the-quote orders is strongly linked to the arrival of market orders posted on the opposite side of the market. In terms of BQS orders, the smallest intercept, $\hat{\omega}_{6,4} = 0.1479$, corresponds to the previous MB order. Symmetrically, for the BQB orders, $\hat{\omega}_{1,9} = 0.0657$ is the smallest in value thus pointing to the former MS order. This confirms that limit orders are perceived by market participants as a kind of a bet on the level of the FX rate. Once there is a chance that the price will move upward as a result of a large buy trade (induced by a MB order), BQS orders become more profitable and so they increase significantly. At the same time if there is an opportunity for the price to move downward as a result of a market sell (induced by a MS order), traders bet on the scale of this movement and place the BQB orders more frequently. It is also possible that if traders anticipate a rise (a drop) in the FX rate they will post a MB (a MS) order and later a BQS (a BQB) order just to close their currency position and to realize their gains more quickly.

A series of unexpected results were obtained for the best order cancellations. Interestingly, the expected time to sell (buy) best limit order cancellations shrinks most considerably after an AQS (AQB) order ($\hat{\omega}_{3,5} = -0.1009$ is the smallest among $\hat{\omega}_{l,5}$ and $\hat{\omega}_{8,10} = 0.0350$ is the smallest among $\hat{\omega}_{l,10}$ coefficients). This means that many orders, once entered into an order book at the prevailing best quotes, are quickly withdrawn from the market before any other dealer can react. One explanation for this may be a type of "spoofing" practice. Traders insert very short-lived "fleeting" orders and have no intention of executing a trade at the submitted price. Such actions are supposed to move the exchange rate down or up. Once an order appears and then subsequently disappears the illusion of excess demand or excess supply at the best quotes can influence the actions of other traders and move the rate in a desirable direction.

- The $\alpha_{l,k}$ coefficients inform how the duration observed between two events that directly precede the current one urge or delay future trader actions. These coefficients are responsible for a strong clustering of order durations. However, it should be mentioned, that the comparison of this impact among durations attributed to different classes cannot be performed with the obtained coefficients values. Such a reasoning stems from the fact that the unconditional (with respect to $\mathcal{F}_{i-1}$) expected durations that end up in particular states are also different in value. Some classes of durations (i.e the MB durations or the BQS durations) can be on average much shorter or much longer than the others. This means that we should not compare the coefficients $\alpha_{l,k}$ for a selected duration ending with a state y_k. For example, if we look at $\alpha_{l,1}$ estimates for the expected MS duration we see that they differ in value (for $l = 1, 2, ..., 10$). However, each of these coefficients relates to the realization of a different duration $x_{i-1,l}$ if the state y_l was observed at the time $t-1$ (see Equation 4). Because each of the durations $x_{i-1,l}$ (for $l = 1, 2, ..., 10$) has a different distribution that is characterized by its distinct conditional expectation $\Psi_{i-1,l}$ and shape parameter δ_l, the corresponding coefficient values $\alpha_{l,1}$ cannot be directly compared. We should not compare the coefficients values across the current state k (the coefficients in the same column of Table 1), although we could do this for the state l (the coefficients in the same row of Table 1).

Looking at the parameter estimates presented in Tables (1), it is not possible to establish *ad hoc* the most probable sequence that describes when orders of a given type would most probably enter into the trading system. The difficulty arises from the considerable size of the model, a huge number of estimated coefficients and the construction of the model for the the outcome variable $x_i = \min(x_{i,1}, x_{i,2}, ..., x_{i,10})$. The expected time until a particular state depends not only on the type of preceding order but also on the length of time prior to this order. Analytical formulas for the conditional (with respect to $\mathcal{F}_{i-1}$ and not the current duration x_i) transition probabilities do not have a closed analytic form. Moreover, the comparison of conditional expectations of durations that end up in selected states are not enough to make predictions about the future precedence of events. In our competing risk framework, ten competing durations $x_{i,k}$ that construct the process of a realized duration $x_i = \min\{x_{1,i}, ..., x_{10,i}\}$ are characterized by different Weibull distributions. These distributions have not only different conditional expectations $\Psi_{k,i}$ but also different shape parameters γ_k. The shape parameter has an impact on the concentration of the probability mass near zero. Therefore, the answer to the question of which duration, $x_{i,k}$, wins the competition at time point t_i by being the shortest depends on both factors: the conditional expected values of their distributions (hence $\Psi_{i,k}$) and the different skewness of these distributions (hence γ_k).

In order to obtain the most probable chronological order according to which selected events occur, we can use an easy way of simulating data from the MAACD model. The simulation algorithm can be outlined as following:

1. For $i = 1$, set initial values for $\hat{\Psi}_{i,k}$ as mean estimates of (observable) series $x_{i,k}$ (for $k = 1, 2, ..., 10$).
2. For $k = 1, 2, ..., 10$, draw n values of $\varepsilon_{j,k}$ ($j = 1, 2, ..., n$) from independent Weibull distributions. Each distribution is characterized by the corresponding shape parameter $\hat{\gamma}_k$.
3. Compute $x_{i,k} = \hat{\Phi}_{i,k}\varepsilon_{i,k}$, where $\hat{\Phi}_{i,k} = \hat{\Psi}_{i,k} \cdot \mu_{\varepsilon_k}^{-1} = \hat{\Psi}_{i,k}(\Gamma(1 + \hat{\gamma_k}^{-1}))^{-1}$.
4. Set $y_i = l$ and $x_i = x_{i,l}$ if a duration $x_{i,l}$ is the shortest (for $l = 1, 2, ..., 10$), i.e. $x_{i,l} = \min\{x_{i,1}, ..., x_{i,l}, ..., x_{i,10}\}$.
5. Compute $\hat{\Psi}_{i+1,k}$ with obtained y_i and x_i.
6. Iterate from (3) where i=i+1.

With the application of the algorithm we can obtain one simulated series of artificial data generated from the MAACD model. With this simulated time series, we are able to compute marginal expectations for all latent durations, hence $\hat{E}(x_{i,k}) = \frac{1}{N}\sum_{i=1}^{N} \hat{x}_{i,k}$. These values could be treated as proper initial values for $\hat{\Psi}_{1,k}$ in the first step of the second run of the simulation.

We performed three runs of the simulations each time having sampled 1,000,000 realizations of the pair (x_i, y_i). The accuracy of the last run of the performed simulation has been checked, because we reestimated the MAACD model with the simulated data series and obtained estimates that were very similar to initial parameter values (with an accuracy equal to a second or third digit right of the decimal point).

The simulation output can be of importance if one wants to compute the frequencies that proxy the unconditional transition probabilities between the ten selected classes of orders. The simulation results are presented in Table (2). In the first column of the table we indicate the class of an order submitted or cancelled at time t_{i-1} (i.e. the type of the preceding order). The further ten columns correspond to obtained frequencies of events that took place at t_i. For example, in the first row of Table (2), we show the estimate of the transition probability that after a MS order another MS order (column 2), IQS order (column 3), AQS order (column 4), BQS order (column 5), CS order (column 6), MB order (column 7), IQB order (column 8), AQB order (column 9), BQB order (column 10) or CB order (column 11) is observed. Thus, the elements in each row of the table must sum to one. The obtained results can be summarized as this:

- We confirm the Hypotheses 1. After the occurrence of a MS order an IQB order is the most likely (19.4 % cases). However, the following MS orders are nearly as frequent as the IBQ (18.5 % cases). Symmetrically, after a MB order an IQS order takes place 19.7 % of the time or another MB order 19.5 % of the time. As predicted by Parlour (1998), market sell (market buy) orders absorb liquidity from the market. As a result of that the bid-ask spread widens and it becomes more costly to cross the market and initiate an immediate trade. Thus, it is more profitable for traders who want to buy (sell) to use limit orders and to compete for the bid (ask) price that is at least one pip (tick) better than the current quote. The obtained regularity agrees with results of Lo & Sapp (2008), who document a kind of a book rebalancing scenario. Our result expand upon their findings since we differentiate between the sides of the market and prove that after a market sell (buy) the arrival of the best sell (buy) limit order is the most probable. On the other hand, we also document the follow-on strategy outlined by Biais (1995) which shows that market buy (sell) orders arrive in clusters. The events with the smallest probability are the AQB and AQS orders (about 4-5 % of cases); this is simply due to the small number of these orders in our data. We also confirm the Hypothesis 2 as the probability of observing a MS after a MB order or vice versa is very small (about 8 % of cases). This confirms the "crowding out" effect outlined by Parlour (1998).

- After the arrival of an IQS order the most frequent event is a MB order (about 14 % of cases). Symmetrically, after an IQB order a MS order is expected to arrive. Submission of the inside-the-quote order decreases the bid-ask spread and creates a kind of enhancement for the opposite side of the market to execute a profitable transaction. We also see that orders placed inside-the-quotes tend to follow each other. This proves the "diagonal effect" that is demonstrated also in Goettler et al. (2005). Yeo (2002) predicts that if a new limit order changes the best price then the former best order will be cancelled and resubmitted at the new and better price. Hasbrouck & Saar (2009) report that this kind of "price-chasing" scenario is a possible explanation for existence of fleeting orders.

- As expected, AQS and AQB orders are most probably followed by CS and CB events (about 16 % of cases). This can be explained by the possible spoofing practice. AQS and AQB orders tend also to be followed by IQS and the IQB orders (about 13 % of cases), respectively. This finding relates closely to the

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depth of the order book and supports the Hypothesis 4. The submission of an at-the-quote order lengthens the queue of orders that wait for realization at the best prices available. These dealers who want to buy or to sell would rather "jump-the-queue" and submit an inside-the-quote order to get the priority of execution (about 13 % of cases) or even submit a market order (about 11 % of cases).

- BQS and BQB orders are unobservable thus traders cannot react to them directly. Nevertheless, their arrival rates reflect certain market conditions which can influence trader behavior indirectly. The most striking observation is their clustering effect. The placement of the BQS (BQB) order induces more BQS (BQB) orders, which means that the process of liquidity supply is characterized by clustering.

- Cancellations of best orders tends to be followed by submission of the best inside-the-quote orders on the same market side. This observation is striking since about 28 % of events that succeed CS order are IQS orders and about 31 % of the events that follow CB orders are IQB orders. After a cancellation of the best order there will be a free space (of at least one unit) for another best order placed on the same market side.

	MS	IQS	AQS	BQS	CS	MB	IQB	AQB	BQB	CB
MS	0.185 (2)	0.131 (3)	0.043 (10)	0.077 (6)	0.081 (5)	0.075 (7)	0.194 (1)	0.046 (9)	0.114 (4)	0.049 (8)
IQS	0.110 (5)	0.122 (3)	0.065 (9)	0.112 (4)	0.105 (6)	0.142 (1)	0.126 (2)	0.040 (10)	0.103 (7)	0.070 (8)
AQS	0.111 (3)	0.133 (2)	0.079 (8)	0.107 (5)	0.156 (1)	0.111 (4)	0.106 (6)	0.038 (10)	0.094 (7)	0.061 (9)
BQS	0.103 (5)	0.130 (3)	0.063 (9)	0.158 (1)	0.072 (8)	0.139 (2)	0.122 (4)	0.041 (10)	0.098 (6)	0.072 (7)
CS	0.104 (3)	0.281 (1)	0.061 (8)	0.093 (4)	0.081 (6)	0.067 (7)	0.131 (2)	0.035 (10)	0.087 (5)	0.059 (9)
MB	0.080 (5)	0.197 (1)	0.055 (8)	0.114 (4)	0.048 (9)	0.195 (2)	0.127 (3)	0.037 (10)	0.074 (6)	0.073 (7)
IQB	0.137 (1)	0.136 (2)	0.046 (10)	0.100 (7)	0.071 (8)	0.114 (4)	0.120 (3)	0.058 (9)	0.106 (6)	0.113 (5)
AQB	0.103 (6)	0.113 (4)	0.050 (10)	0.094 (7)	0.061 (9)	0.118 (3)	0.133 (2)	0.065 (8)	0.105 (5)	0.157 (1)
BQB	0.134 (2)	0.131 (3)	0.054 (9)	0.099 (6)	0.072 (7)	0.109 (5)	0.130 (4)	0.048 (10)	0.158 (1)	0.065 (8)
CB	0.062 (7)	0.134 (2)	0.042 (10)	0.089 (4)	0.053 (8)	0.095 (3)	0.312 (1)	0.051 (9)	0.089 (5)	0.074 (6)

Table 2: Transition probabilities – simulation results. Column (1) contains a type of a directly preceding event. Numbers in brackets sort the transition probabilities in a descending order.

6 Impact of the Explanatory Variables

In order to examine potential microstructure factors that may have an impact on order choice and the pace of order submissions/cancellations, we enriched the MAACD model with the following explanatory variables:

- Bid-ask spread (*spr*), defined as the difference between the best ask price and the best bid price in the system before the moment of order submission/cancellation.

- Ask depth (*adep*) and bid depth (*bdep*), defined as the cumulated sizes of limit orders offered to sell (buy) at the best ask and at the best bid price, respectively.

- Ask depth and bid depth dummies (*adepd*, *bdepd*). The indicators are equal to one if the ask depth or the bid depth is larger than 10 M. EUR and if the actual amount of depth is hidden.

- Ask and bid quote slopes (*askQS*, *bidQS*). Ask quote slope is defined as the ratio of the difference between the worst (the highest) ask price prevailing in the system (P_{ask}^{worst}) and the best ask price (P_{ask}^{best}) to the cumulated (whole) ask depth ($\sum_{i=1}^{m} Size_{ask,i}$):
$$askQS = \frac{P_{ask}^{worst} - P_{ask}^{best}}{\sum_{i=1}^{m} Size_{ask,i}} \tag{8}$$
Bid quote slope is defined as the ratio of the difference between the best bid price (P_{bid}^{best}) and the worst (the lowest) bid price prevailing in the system (P_{bid}^{worst}) to the cumulated (whole) bid depth ($\sum_{i=1}^{n} Size_{bid,i}$):
$$bidQS = \frac{P_{bid}^{best} - P_{bid}^{worst}}{\sum_{i=1}^{n} Size_{bid,i}} \tag{9}$$

- EUR/PLN rate volatility (*vol*), constructed as a realized volatility estimate for the past 10 minutes prior to the moment of order submission. In order to calculate realized volatility estimate, log returns of all observable mid prices have been used.

- EUR/PLN return (*ret*), as the log return of the EUR/PLN mid price during past 10 minutes prior to the moment of a given event.

- EUR/USD return (*EURUSDret*), as the log return of the EUR/USD mid price during past 10 minutes prior to the moment of a given event.

- Two time-of-day dummies that describe possible fluctuations in the disproportion of selected order types: (1) the morning effect (*begin*) (d1 dummy is equal to 1 if an order is submitted between 8:00 and 8:30 and is equal to zero otherwise), (2) the afternoon effect (*end*) (d2 dummy is equal to 1 if an order is placed between 17:30 and 18:00 and is equal to zero otherwise).

These explanatory variables that were characterized by a cyclical behaviors (the bid-ask spread, the ask and bid quote slopes, the realized volatility estimate) have been initially deseasonalized. Diurnality factors were estimated with a formula (7). After such a transformation the series depicts a kind of "innovation" that is independent from the repetitive day-by-day seasonal pattern.

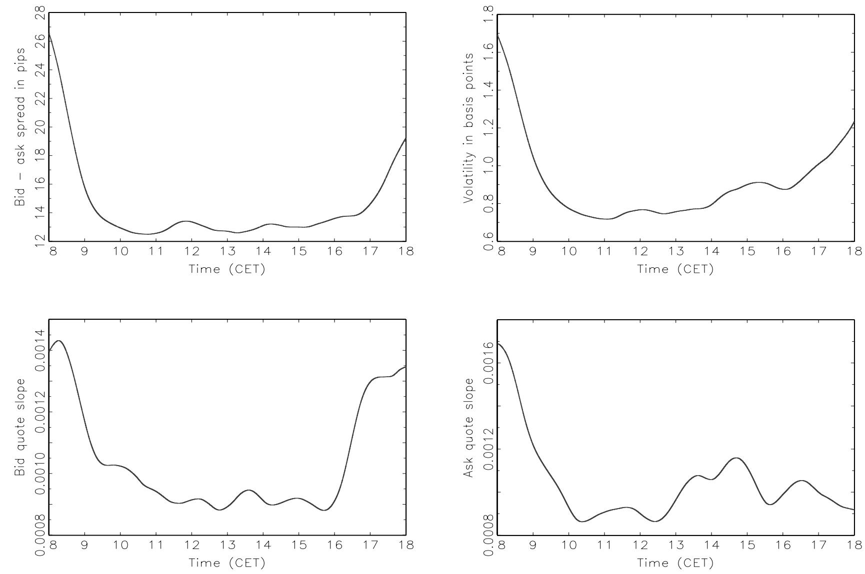

Figure 3: Diurnality pattern for the bid-ask spread, the realized volatility, the bid quote slope and the ask quote slope.

The estimation results are presented in Table 4. Interestingly, after the inclusion of the explanatory variables in the MAACD specification the parameters responsible for the persistence of MB and MS order submissions changed. Parameter estimates for the market order durations, $\hat{\beta}_1 = 0.3976$ and $\hat{\beta}_6 = 0.4475$, are much smaller than in the initial specification without explanatory variables. This means that the additional factors took over much of the duration variation. It further confirms our hypothesis that in the event of the most aggressive orders, very prompt information flows reflected by changes in market microstructure features (i.e. the bid-ask spread, depth, volatility, past returns) are the major driving force behind the pace of their submissions. As the persistence of the process decreased the impact of the information from the distant past has a smaller value. In the next chapter we will present the impact of the individual factors.

6.1 Bid-Ask Spread

As suggested by the theoretical literature (Foucault (1999)), we document the significant negative impact[11] (at a 1 % significance level) of the bid-ask spread on the expected time to a MB or a MS order. Thus, the increase in the bid-ask spread deflates the probability of market/marketable order submissions. Simultaneously, it significantly increases the probability that an IQB or IQS order is placed in the system. This finding is consistent with the results of a number of empirical studies (i.e. Biais (1995); Harris (1998); Bae et al. (2003); Ranaldo (2004); Ellul et al. (2007)). Hitting or taking the quotes is much more expensive if the bid-ask spread is large and traders prefer to consume liquidity when it is cheap. On the other hand, it is much more profitable for market participants to provide liquidity in the form of IQB or IQS limit orders. As the coefficients $\hat{\delta}_{spr,2}$ and $\hat{\delta}_{spr,7}$ are significant and negative, the wide bid-ask spread encourages the submission of the most competitive limit orders. If the bid-ask spread is wide it is easier to compete for the price that is at least one tick (pip) better then the current one, as it would improve the likelihood of an order execution at a very small expense. Hence, we confirm the Hypothesis 4. The rise in the bid-ask spread also prompts the submission of at-the-quotes and behind-the-quotes orders on both sides of the market. A possible explanation for this is that when the bid-ask spread rises as a result of an adverse selection risk the traders prefer to place orders that are far away from the midquote in order to avoid the risk of being "picked-off". Although the coefficients corresponding to the bid-ask spread in the case of at-the-quote and behind-the-quote orders are negative and significant (at a 5 % significance level), they are very small in value when compared to market or inside-the-quote orders. Accordingly, the bid-ask spread has a much greater impact on the arrival rate of the most aggressive orders. The impact of the bid-ask spread is rather symmetric on the ask and bid market side although slight disproportions can be observed. The expected duration to a MS order (i.e. sell EUR and buy PLN) reacts more strongly to the increase of the bid-ask spread than the expected waiting time to a MB order (buy EUR and sell PLN). Maybe this is because a fraction of the bid-ask spread could be information-motivated and an increase in the adverse selection costs results in a different valuation of the open currency positions in euros versus in the Polish zloty. Investments in zloty are treated as more risky and so the probability of aggressive selling deflates in a more considerable way. We also report that the increase in the bid-ask spread induces more cancellations on the both market sides. The coefficients $\hat{\delta}_{spr,5}$ and $\hat{\delta}_{spr,10}$ are significant (at a 5 %

[11] In the sample there are 92,818 observations. We rest on the assumption that the number of observations is large enough to base our inference on the asymptotic normality of the maximum likelihood estimator $\hat{\Theta} \rightarrow N(\Theta, A^{-1}BA^{-1})$. A and B have been estimated as: $\hat{A} = \frac{1}{N}\sum_i^N \frac{\partial^2 LnL_i}{\partial\Theta\partial\Theta'}$ $\hat{B} = \frac{1}{N}\sum_i^N (\frac{\partial LnL_i}{\partial\Theta})'(\frac{\partial LnL_i}{\partial\Theta})$.

level) and negative but quite small when compared with the impact of the bid-ask spread on the MS or MB durations. Therefore, we cannot support the hypothesis H6 that small bid-ask spread induces more cancellations while best limit orders tend to be cancelled and replaced by market orders. On the contrary, if the bid-ask spread widens, traders tend to cancel the best orders and replace them by more competitive ones.

6.2 Depth

The second important microstructure variables are the ask and the bid depths. An increase in the size of orders that are offered at the best prices on both sides of the market inflates the probability that a MS order will arrive as predicted by the hypothesis H3. The negative coefficients $\hat{\delta}_{adep,1}$ and $\hat{\delta}_{bdep,1}$ prove that a rise in these observable liquidity measures shortens MS durations. A large depth at the best bid encourages aggressive sells. Therefore, if a trader wants to execute a high-volume sell order, sufficient depth at the best bid guarantees a prompt execution without the risk of hitting an unfavorable price. This "enhancement effect" is additionally pronounced by the significant (at a 1 % level) coefficient $\hat{\delta}_{bdepd,1} = -0.4484$, indicating that a depth of more than 10 M. EUR is hidden at the best bid. Such a signal could be interpreted as additional encouragement for submitting aggressive sell orders. On the other hand, we obtained a negative and significant (at a 1 % level) coefficient $\hat{\delta}_{adep,1} = -0.048$ thus the increase in the size of best limit sell orders prevailing in the system induces more sell trades. The large depth at the best ask can have an informative meaning for the context of a quasi-technical analysis performed by market participants (see Osler (2003); Ellul et al. (2007)). An increase in the ask depth may indicate a near future drop in the exchange rate. Ellul et al. (2007) found a similar result for the NYSE stocks and called it the "short-term forecasting hypothesis".

Similar results as for MS durations can be found for MB durations. An increase in the market depth, either at the best ask or at the best bid, exerts a significant positive influence on the probability of submitting an aggressive market order. Although, the corresponding depth dummies are insignificant. Thus, the results support the hypothesis H3. It is interesting to note that in the case of MB durations the impact of the ask depth is slightly larger than the impact of the bid depth as we have $|\hat{\delta}_{adep,6}| > |\hat{\delta}_{bdep,6}|$. Analogously, for MS orders the following equation holds true $|\hat{\delta}_{adep,1}| < |\hat{\delta}_{bdep,1}|$. The inequalities evidence that the "enhancement effect" is more pronounced than the effect of the "short-term forecasting effect" of Ellul et al. (2007).

As far as IQB and IQS orders are concerned, we find perfect evidence for the "jump the queue" hypothesis as stated in Ellul et al. (2007). The large depth at the best ask prompts the submission of best limit sell orders that improve on the current price ($\hat{\delta}_{adep,2} < 0$). Accordingly, the large depth at the best bid encourages traders to post a limit buy order with a price at least one tick better in order to gain execution priority ($\hat{\delta}_{bdep,7} < 0$ and $\hat{\delta}_{bdepd,7} < 0$). On the other hand, an increase in the visible depth on the bid (ask) side hinders the pace of the IQS (IQB) orders as forecasted by the "short-term forecasting hypothesis".

The impact of depth on AQS and AQB durations is significantly negative so that the large queue at the best quotes does not discourage traders to offer/bid more quantities at these prices. Such a result is difficult to explain. The submission of AQB and AQS orders rules out the priority of execution; nevertheless, at-the-quote orders have a tendency to arrive in clusters. The reaction of BQS and BQB orders to the changes in depth is much smaller in value but also lead to some interesting conclusions. The BQS durations shrink as a result of an increase in the depth at the best bid but lengthen with the depth at the best ask. Symmetric observation refers to BQB orders. The BQB duration shrinks as a result of an increase in the depth of the best ask. In agreement with the "short-term forecasting hypothesis", the increase in quantity offered to sell at the best ask signals a short-term price drop. Consequently, it increases the probability that a BQB order will be executed. Similarly, if there is a large order placed at the best bid, traders anticipate that the EUR/PLN rate will rise and thus a BQS order will be executed with greater probability. Conversely, large depth at the best ask deflates the probability of BQS order execution.

6.3 Quote Slopes

Quote slopes are totally unobservable for market participants and thus they cannot be taken into consideration when determining their order decisions. On the other hand, fluctuations in overall liquidity reflected in quote slopes may influence trader behavior indirectly. Periodic liquidity shocks with respect to bid or ask sides of the market induce certain trading choices. We observe that the quote slope of the ask side of the order book has a significant and positive impact on the expected IQS and AQS durations. An increase in the slope indicates a deterioration of the liquidity supply. If there is a liquidity squeeze on the ask side of an order book, the placement of the IQS and AQS orders slows down. The state of the order book is also closely bound to the value of the technical analysis. A lesser liquidity on the ask side signals an upward price pressure and thus traders restrain from submitting aggressive limit orders to avoid the risk of being picked off. Instead, traders submit more market buy orders as can be observed from the significant negative coefficient $\hat{\delta}_{askQS,6}$.

The steeper slope on the bid side of the market is correlated with the accelerated submission of MS, IQS and AQS orders. This can also be predicted from the "short-term forecasting hypothesis" and can be explained by the fact that the state of the order book can be partially anticipated by traders on the basis of orders left by their non-bank clients. Interestingly, the deterioration of the liquidity supply on the bid side of the market is linked with accelerated submission of MB and IQB orders, hence an increase in the overall trading activity.

ASK SIDE OF THE MARKET (SELL ORDERS)										
	MS, k=1		IQS, k=2		AQS, k=3		BQS, k=4		CS, k=5	
parameters	estimate	p - value	estimate	p - value	estimate	p - value	estimate	p - value	estimate	p - value
$\omega_{1,k}$ MS	0.5120	0.0000	0.9070	0.0000	0.6950	0.0000	0.5686	0.0000	0.5411	0.0000
$\omega_{2,k}$ IQS	1.4404	0.0000	1.6483	0.0000	0.5352	0.0000	0.4882	0.0000	0.5061	0.0000
$\omega_{3,k}$ AQS	1.5189	0.0000	1.2003	0.0000	0.3803	0.0000	0.6025	0.0000	-0.0612	0.3029
$\omega_{4,k}$ BQS	1.9256	0.0000	1.4814	0.0000	0.8493	0.0000	0.3198	0.0000	1.0612	0.0000
$\omega_{5,k}$ CS	1.0452	0.0000	0.2923	0.0000	0.6849	0.0000	0.5752	0.0000	1.0402	0.0000
$\omega_{6,k}$ MB	1.8821	0.0000	0.3869	0.0000	0.5006	0.0000	0.1816	0.0000	1.1055	0.0000
$\omega_{7,k}$ IQB	0.9158	0.0000	1.4603	0.0000	0.9434	0.0000	0.5402	0.0000	1.0317	0.0000
$\omega_{8,k}$ AQB	1.9606	0.0000	1.5049	0.0000	0.8947	0.0000	0.5451	0.0000	1.0942	0.0000
$\omega_{9,k}$ BQB	1.6969	0.0000	1.4421	0.0000	0.9573	0.0000	0.6806	0.0000	1.1762	0.0000
$\omega_{10,k}$ CB	2.3439	0.0000	1.3623	0.0000	1.0425	0.0000	0.6832	0.0000	1.1176	0.0000
β_k	0.3976	0.0000	0.6523	0.0000	0.8441	0.0000	0.8686	0.0000	0.7715	0.0000
$\alpha_{1,k}$ MS	0.2193	0.0000	0.1191	0.0000	0.1132	0.0000	0.1026	0.0000	0.1057	0.0000
$\alpha_{2,k}$ IQS	0.1739	0.0000	0.1770	0.0000	0.1240	0.0000	0.1537	0.0000	0.1701	0.0000
$\alpha_{3,k}$ AQS	0.2065	0.0000	0.2083	0.0000	0.1486	0.0000	0.1772	0.0000	0.1366	0.0000
$\alpha_{4,k}$ BQS	0.2279	0.0000	0.2387	0.0000	0.1874	0.0000	0.1457	0.0000	0.1620	0.0000
$\alpha_{5,k}$ CS	0.3252	0.0000	0.1690	0.0000	0.1331	0.0000	0.0796	0.0000	0.2112	0.0000
$\alpha_{6,k}$ MB	0.3377	0.0000	0.0697	0.0000	0.0884	0.0000	0.0364	0.0017	0.1042	0.0000
$\alpha_{7,k}$ IQB	0.2558	0.0000	0.2329	0.0000	0.1441	0.0000	0.1123	0.0000	0.1512	0.0000
$\alpha_{8,k}$ AQB	0.3006	0.0000	0.2602	0.0000	0.1344	0.0000	0.0971	0.0000	0.1707	0.0000
$\alpha_{9,k}$ BQB	0.3887	0.0000	0.2438	0.0000	0.1738	0.0000	0.1253	0.0000	0.2045	0.0000
$\alpha_{10,k}$CB	0.3604	0.0000	0.2565	0.0000	0.1970	0.0000	0.1573	0.0000	0.1137	0.0000
γ_k	0.7360	0.0000	0.8012	0.0000	0.7941	0.0000	0.7902	0.0000	0.7963	0.0000
$\delta_{spr,k}$	1.2086	0.0000	-0.2029	0.0000	-0.0219	0.0024	-0.0161	0.0018	-0.0317	0.0064
$\delta_{adep,k}$	-0.0480	0.0000	-0.0230	0.0000	-0.0088	0.0200	0.0047	0.0665	-0.0046	0.2758
$\delta_{bdep,k}$	-0.0329	0.0000	0.0104	0.0069	-0.0104	0.0005	-0.0088	0.0000	0.0036	0.3294
$\delta_{adepd,k}$	0.2272	0.0859	0.0361	0.6052	0.1380	0.0712	0.0642	0.2625	-0.1506	0.0289
$\delta_{bdepd,k}$	-0.4484	0.0016	0.0275	0.7962	0.0430	0.6148	0.0473	0.4115	0.0323	0.7571
$\delta_{askQS,k}$	0.0217	0.1074	0.0141	0.0585	0.0131	0.0097	-0.0082	0.0065	0.0016	0.7880
$\delta_{bidQS,k}$	-0.1136	0.0000	-0.0132	0.0300	-0.0106	0.0191	0.0033	0.2596	-0.0013	0.7938
$\delta_{vol,k}$	-0.2640	0.0000	-0.0029	0.7819	0.0059	0.4540	0.0070	0.1592	0.0147	0.1107
$\delta_{ret,k}$	0.0061	0.0000	-0.0021	0.0004	-0.0017	0.0007	-0.0013	0.0005	0.0018	0.0018
$\delta_{EURUSDret,k}$	-0.0005	0.7596	-0.0001	0.9083	0.0014	0.0077	-0.0004	0.2760	0.0007	0.2283
$\delta_{begin,k}$	0.0044	0.9444	0.0541	0.1007	0.0471	0.0698	0.0609	0.0004	0.0369	0.1781
$\delta_{end,k}$	0.1604	0.0086	-0.0679	0.0085	0.1835	0.0000	0.0666	0.0000	0.0121	0.6100

BID SIDE OF THE MARKET (BUY ORDERS)										
	MB, k=6		IQB, k=7		AQB, k=8		BQB, k=9		CB, k=10	
parameters	estimate	p - value	estimate	p - value	estimate	p - value	estimate	p - value	estimate	p - value
$\omega_{1,k}$ MS	1.8377	0.0000	0.4381	0.0000	0.6530	0.0000	0.1089	0.0026	1.2560	0.0000
$\omega_{2,k}$ IQS	0.6959	0.0000	1.6300	0.0000	1.1895	0.0000	0.4382	0.0000	1.2476	0.0000
$\omega_{3,k}$ AQS	1.6917	0.0000	1.5650	0.0000	1.0634	0.0000	0.4908	0.0000	1.3391	0.0000
$\omega_{4,k}$ BQS	1.4718	0.0000	1.6438	0.0000	1.2553	0.0000	0.5919	0.0000	1.3905	0.0000
$\omega_{5,k}$ CS	2.0186	0.0000	1.4878	0.0000	1.3046	0.0000	0.6718	0.0000	1.3524	0.0000
$\omega_{6,k}$ MB	0.3217	0.0000	1.0124	0.0000	0.9868	0.0000	0.5735	0.0000	0.8154	0.0000
$\omega_{7,k}$ IQB	1.1890	0.0000	1.8083	0.0000	0.7642	0.0000	0.4357	0.0000	0.6647	0.0000
$\omega_{8,k}$ AQB	1.3654	0.0000	1.4361	0.0000	0.7222	0.0000	0.5415	0.0000	0.1565	0.1066
$\omega_{9,k}$ BQB	1.8564	0.0000	1.6380	0.0000	1.1848	0.0000	0.2807	0.0000	1.3916	0.0000
$\omega_{10,k}$CB	1.1032	0.0000	0.1960	0.0001	0.9632	0.0000	0.4874	0.0000	1.4084	0.0000
β_k	0.4475	0.0000	0.6181	0.0000	0.7909	0.0000	0.8824	0.0000	0.7170	0.0000
$\alpha_{1,k}$ MS	0.3136	0.0000	0.0539	0.0000	0.0439	0.0241	0.0295	0.0040	0.1128	0.0000
$\alpha_{2,k}$ IQS	0.2359	0.0000	0.2218	0.0000	0.1771	0.0000	0.0995	0.0000	0.1961	0.0000
$\alpha_{3,k}$ AQS	0.3633	0.0000	0.2383	0.0000	0.1495	0.0000	0.1039	0.0000	0.2218	0.0000
$\alpha_{4,k}$ BQS	0.3326	0.0000	0.2548	0.0000	0.1811	0.0000	0.1012	0.0000	0.2155	0.0000
$\alpha_{5,k}$ CS	0.3517	0.0000	0.2410	0.0000	0.1745	0.0000	0.1511	0.0000	0.1753	0.0000
$\alpha_{6,k}$ MB	0.2049	0.0000	0.1274	0.0000	0.1216	0.0000	0.1151	0.0000	0.1020	0.0000
$\alpha_{7,k}$ IQB	0.1250	0.0000	0.1493	0.0000	0.1537	0.0000	0.1024	0.0000	0.2324	0.0000
$\alpha_{8,k}$ AQB	0.2076	0.0000	0.2365	0.0000	0.1831	0.0000	0.1726	0.0000	0.1406	0.0000
$\alpha_{9,k}$ BQB	0.3402	0.0000	0.2716	0.0000	0.2079	0.0000	0.1271	0.0000	0.1916	0.0000
$\alpha_{10,k}$CB	0.3335	0.0000	0.1803	0.0000	0.2014	0.0000	0.0554	0.0008	0.2590	0.0000
γ_k	0.7324	0.0000	0.7839	0.0000	0.7855	0.0000	0.7970	0.0000	0.7875	0.0000
$\delta_{spr,k}$	1.1108	0.0000	-0.2401	0.0000	-0.0112	0.3121	-0.0143	0.0022	-0.0450	0.0195
$\delta_{adep,k}$	-0.0489	0.0000	0.0105	0.0293	-0.0137	0.0060	-0.0109	0.0000	-0.0034	0.5070
$\delta_{bdep,k}$	-0.0349	0.0000	-0.0229	0.0000	-0.0112	0.0146	0.0024	0.2276	-0.0080	0.0754
$\delta_{adepd,k}$	0.0551	0.6419	-0.0250	0.7924	0.1125	0.2491	0.0837	0.0490	-0.1036	0.2198
$\delta_{bdepd,k}$	0.0029	0.9852	-0.2029	0.0141	0.0290	0.7940	0.0254	0.6555	-0.0228	0.8348
$\delta_{askQS,k}$	-0.0249	0.0377	0.0175	0.0423	-0.0096	0.1614	0.0116	0.0003	-0.0109	0.1011
$\delta_{bidQS,k}$	-0.0419	0.0003	-0.0203	0.0021	-0.0038	0.5423	-0.0094	0.0009	0.0140	0.0469
$\delta_{vol,k}$	-0.1693	0.0000	0.0276	0.0191	-0.0007	0.9516	0.0058	0.2113	0.0194	0.1183
$\delta_{ret,k}$	-0.0139	0.0000	0.0022	0.0014	0.0022	0.0028	0.0009	0.0041	-0.0026	0.0007
$\delta_{EURUSDret,k}$	0.0072	0.0000	-0.0004	0.6406	0.0007	0.3593	-0.0005	0.1159	-0.0004	0.6584
$\delta_{begin,k}$	0.2118	0.0009	-0.0492	0.1118	0.0337	0.2909	0.0294	0.0270	0.0947	0.0069
$\delta_{end,k}$	0.0550	0.2649	-0.0408	0.1660	0.1854	0.0001	0.0638	0.0000	0.0006	0.9819

Table 3: Estimation results for the MAACD model with explanatory variables.

6

6.4 Volatility

We find a significant impact of the return volatility on the submissions of market orders (for both sides of the market) and inside-the-quotes limit orders (only for the bid side of the market). The impact is negative for MS and MB durations and positive for the IQB durations, contrary to predictions of Foucault (1999). Our results also do not confirm the findings of Lo & Sapp (2008) who report an overall trading decline in the presence of increased volatility. Thus, we do not find a support for the Hypothesis 5. The FX rate volatility is a traditional measure of market uncertainty. An increase in volatility increases the risk of being picked off and so the frequency of submitting best limit orders decreases. Handa & Schwartz (1996) report on the rationale of placing limit orders for informed or liquidity traders; they find that if volatility increases due to informed trading, the risk of being bagged by informed traders rises. This finding generally agrees with results of Fong & Liu (2010) who report that if traders observe a large price swing they will withdraw or relocate their orders away from the market price. On the other hand, in volatile periods traders close their currency positions that may be associated with the more intense use of market orders. Interestingly enough, we obtained the relation $\hat{\delta}_{vol,1} < \hat{\delta}_{vol,6}$, which means that traders accelerate buy transactions more considerably if they want to buy zloty (sell euro) than if they want to sell zloty (buy euro). As far as orders cancellations are concerned, volatility does not seem to significantly influence their arrival rate and thus we cannot support Hypothesis 7.

6.5 Momentum

Our results clearly confirm that the direction of the FX rate in the past 10 minutes has a significant impact on the order choice. Upward movement of the EUR/PLN rate, hence a Polish zloty depreciation, hinders placement of MS orders (market orders to sell euro and buy zloty) and enhances MB orders (market orders to buy euro and sell zloty). This means that traders act according to a momentum, or a trend-following strategy. The acceleration of MB order submissions is particularly pronounced since the $\hat{\delta}_{ret,6}$ is quite large in value. The selling pressure of the zloty makes IQB, AQB and BQB orders unprofitable since once the trend continues their probability of execution declines ($\hat{\delta}_{ret,7} > 0$, $\hat{\delta}_{ret,8} > 0$, $\hat{\delta}_{ret,9} > 0$). Similarly, those traders who want to sell would rather use AQS or even BQB orders as there is a chance that they will be matched when the FX rate reaches a certain level. A zloty depreciation hinders cancellation of best limit sell orders whereas it prompts cancellation of best limit buys as predicted by a trend-chasing practice.

We also observe a significant impact concerning EUR/USD changes on the submission rate of MB orders. If during the past 10 minutes the EUR/USD rate moves in the upward direction (dollar depreciates towards euro), the submission of market orders to buy euro and sell zloty will be deterred. In the period under study, the dollar depreciation was closely linked to a decrease in the global risk aversion and an increased interest in currencies of emerging markets. Therefore, traders refrained from selling Polish zloty.

6.6 Time of day

The begin-of-day dummy is significantly positive and large in value for market orders to buy euro and sell Polish zloty. The coefficient is also positive for BQS and BQB (at the 5 % significance level) durations. This means that once trading begins, the provision of liquidity is deteriorated in comparison to the rest of the day. The submission of behind-the-quotes orders becomes unprofitable because of their small execution probability due to a slow trading intensity. The impact of the afternoon effect is quite similar; submissions of AQS, BQS, AQB and BQB orders are also less frequent. Additionally, traders refrain from aggressive buying of euro (selling zloty) in the morning and from aggressive selling of euro (buying zloty) at the end of the day. We were not able to confirm the conviction that market orders dominate trading in the beginning of a day whereas limit orders cluster at the end of a day (see Bloomfeld et al. (2005)).

6

6.7 Symmetry Restrictions

As the last step of our empirical analysis we tested some symmetry restrictions with respect to the impact of selected explanatory variables on order dynamics on an ask and bid side of a market. With a help of the likelihood ratio test we verified several null hypotheses about the equal impact of: 1.) the bid-ask spread, 2.) market depth, 3.) volatility and 4.) the bid/ask quote slopes on the pace of order submissions when traders intend to buy zloty (sell euro) versus when they intend to sell zloty (buy euro). Interestingly, in case of nearly all order submissions, the null of equal impact of selected microstructure factors should be rejected (at a 95 % confidence level). Only in case of order cancellations, the impact of all selected explanatory variables can be treated as equal for both market sides.

Especially for market orders, the significant disproportions in coefficient values for the bid-ask spread, depth dummies, volatility and the quote slopes can be observed. The parameter estimates for the ask side of a market are much larger in value, which means that the immediate decisions to buy zloty (sell euro) are significantly more sensitive to liquidity or information motivated factors. From this viewpoint, our general and richly parameterized duration model can be fully justified.

H_0 :	$\delta_{spr,1} = \delta_{spr,6}$	$\delta_{spr,2} = \delta_{spr,7}$	$\delta_{spr,3} = \delta_{spr,8}$	$\delta_{spr,4} = \delta_{spr,9}$	$\delta_{spr,5} = \delta_{spr,10}$
$p - val$:	0.0031	0.0001	0.0108	0.0133	0.1730
H_0 :	$\delta_{adep,1} = \delta_{bdep,6}$ $\delta_{adepd,1} = \delta_{bdepd,6}$ $\delta_{bdep,1} = \delta_{adep,6}$ $\delta_{bdepd,1} = \delta_{adepd,6}$	$\delta_{adep,2} = \delta_{bdep,7}$ $\delta_{adepd,2} = \delta_{bdepd,7}$ $\delta_{bdep,2} = \delta_{adep,7}$ $\delta_{bdepd,2} = \delta_{adepd,7}$	$\delta_{adep,3} = \delta_{bdep,8}$ $\delta_{adepd,3} = \delta_{bdep,8}$ $\delta_{bdep,3} = \delta_{adep,8}$ $\delta_{bdepd,3} = \delta_{adep,8}$	$\delta_{adep,4} = \delta_{bdep,9}$ $\delta_{adepd,4} = \delta_{bdepd,9}$ $\delta_{bdep,4} = \delta_{adep,9}$ $\delta_{bdepd,4} = \delta_{adepd,9}$	$\delta_{adep,5} = \delta_{bdep,10}$ $\delta_{adepd,5} = \delta_{bdepd,10}$ $\delta_{bdep,5} = \delta_{adep,10}$ $\delta_{bdepd,5} = \delta_{adepd,10}$
$p - val$:	0.0012	0.1150	0.0794	0.0920	0.2337
H_0 :	$\delta_{vol,1} = \delta_{vol,6}$	$\delta_{vol,2} = \delta_{vol,7}$	$\delta_{vol,3} = \delta_{vol,8}$	$\delta_{vol,4} = \delta_{vol,9}$	$\delta_{vol,5} = \delta_{vol,10}$
$p - val$:	0.0000	0.0064	0.0108	0.0133	0.999
H_0 :	$\delta_{askQS,1} = \delta_{bidQS,6}$ $\delta_{bidQS,1} = \delta_{askQS,6}$	$\delta_{askQS,2} = \delta_{bidQS,7}$ $\delta_{bidQS,2} = \delta_{askQS,7}$	$\delta_{askQS,3} = \delta_{bidQS,8}$ $\delta_{bidQS,3} = \delta_{askQS,8}$	$\delta_{askQS,4} = \delta_{bidQS,9}$ $\delta_{bidQS,4} = \delta_{askQS,9}$	$\delta_{askQS,5} = \delta_{bidQS,10}$ $\delta_{bidQS,5} = \delta_{askQS,10}$
$p - val$:	0.0000	0.0060	0.0370	0.0028	0.1562

Table 4: Symmetry restrictions – results from the likelihood ratio test.

6

7 Conclusions

This paper contributes to the literature on order dynamics in two main aspects. We generalize the asymmetric ACD model of Bauwens & Giot (2003) to the case when there are more than two competing risks. The obtained multistate ACD model can serve as a flexible tool for description of expected durations, at the end of which particular events (i.e. states defined on a "micro scale" by appropriate thinning the data) can take place. In our model the selected events correspond to submissions or cancellations of orders attributed to particular order classes. The model describes the bivariate density for durations (time intervals between selected orders) and the corresponding event classes (discrete variables indicating the type of an event). Thus, it can account for the very complex dynamics inherent to the order-driven market. As the time variable plays first fiddle, the model describes the pace of dealer activities when confronted with actions taken by other market participants. We also show how to simulate data from the multistate ACD specification thus enabling a more detailed insight into the data generating process. In the empirical analysis we use data for the EUR/PLN currency pair from a very popular interbank trading venue: the Reuters Dealing 3000 Spot Matching System. To our knowledge this is the first study that investigates trader preference with regard to order choices within the emerging currency markets.

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We establish the most probable sequence according to which orders classified to selected classes arrive to the market or are withdrawn from it. We identify market microstructure factors that exert an influence on the expected time until a given event takes place. Our results verify selected hypotheses on determinants of order choice and their timing in limit order markets. As the results we obtained are quite extensive, we will restrict ourselves to those that refer to the hypotheses stated in the section 2. The hypotheses (1-4) have been supported. The limit sells (buys) are more probable after market buys (sells) than after market sells (buys). Market orders submitted on the same side cluster together thus the sequence buy-buy (sell-sell) is more probable than buy-sell or sell-buy. Large depth visible on one side of the market enhances more inside-the-quote orders placed on the same side of the order book as is predicted by Hall & Hautsch (2006) and Anand et al. (2005). The same also accounts for aggressive market orders on both market sides, which can be explained by the "enhancement effect" or the "short-term forecasting effect". The bid-ask spread decreases the pace of market order submissions and increases the arrival rate of limit orders (see Biais (1995); Harris (1998); Bae et al. (2003); Ranaldo (2004); Ellul et al. (2007) and Lo & Sapp (2008)). The increase in market uncertainty, proxied by the realized volatility, prompts market orders (especially theses to buy euro and to sell zloty) and decreases the number of best limit orders. Order cancellations take place mostly after the submission of best orders on the same market side, which may be perceived as a signal of spoofing. We do not find support for hypotheses (6-7). A decrease in the bid-ask spread does not initiate submission of best limit orders. On the contrary, they are cancelled if the bid-ask spread is large, although – possibly – they will be resubmitted at a new and better price. An increase in volatility does not have a statistically significant impact on order cancellations.

There are many possible extensions for our empirical study. The model can be easily extended to more than ten competing risks and the multistate ACD model can also be widely used in order to study other market events defined on the "micro-scale" (trades of different volume, price changes of different size, submissions of hidden orders called "iceberg orders", or the use of algo trading). If the events can be reflected as an ordered point process, there is a large spectrum of possible economic enquiries that can be investigated in this competing risk framework.

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A Appendix

I. **Properties of the Weibull distribution.**
The Weibull density for the variable ε_i, with parameters $\gamma > 0$, $c = 1$ (i.e. under the restriction that the dispersion parameter is equal to 1) is:

$$f_{\varepsilon_i}(\varepsilon_i) = \gamma(\varepsilon_i)^{\gamma-1}\exp(-\varepsilon_i^{\gamma}) \tag{10}$$

and the survival function is:

$$S_{\varepsilon_i}(\varepsilon_i) = \exp(-\varepsilon_i^{\gamma}) \tag{11}$$

i.e. $\varepsilon_i \sim W(1,\gamma)$. The expectation of the Weibull distribution is:

$$\mu_{\varepsilon_i} = \Gamma(1+\gamma^{-1}) \tag{12}$$

II. **Main properties of the MAACD model.**
Under the framework of the standard ACD model of (Engle & Russell (1998)), a duration $x_{i,k}$ can be depicted as: $x_{i,k} = \Phi_{i,k}\varepsilon_{i,k}$, where $\Phi_{i,k} = \Psi_{i,k}\cdot\{\Gamma(1+\gamma_k^{-1})\}^{-1}$, $\Psi_{i,k} = E(x_{i,k}|\mathcal{F}_{i-1})$ and $E(\varepsilon_{i,k}|\mathcal{F}_{i-1}) = 0$.

The density and survival functions of $x_{i,k}$ can be given as:

$$\begin{aligned} f_{x_{i,k}}(x_{i,k}) &= \frac{1}{\Phi_{i,k}} f_{\varepsilon_{i,k}}\left(\frac{x_{i,k}}{\Phi_{i,k}}\right) \\ &= \frac{\gamma_k}{\Phi_{i,k}}\left(\frac{x_{i,k}}{\Phi_{i,k}}\right)^{\gamma_k-1}\exp\left[-\left(\frac{x_{i,k}}{\Phi_{i,k}}\right)^{\gamma_k}\right] \end{aligned} \tag{13}$$

$$\begin{aligned} S_{x_{i,k}}(x_{i,k}) &= S_{\varepsilon_{i,k}}\left(\frac{x_{i,k}}{\Phi_{i,k}}\right) \\ &= \exp\left[-\left(\frac{x_{i,k}}{\Phi_{i,k}}\right)^{\gamma_k}\right] \end{aligned} \tag{14}$$

The conditional cumulative density function (CDF) for x_i defined as $x_i = \min\{x_{1,i}, ...,$ can be derived as:

$$\begin{aligned} F(x_i|\mathcal{F}_{i-1}) &= 1 - P(X_i > x_i|\mathcal{F}_{i-1}) \\ &= 1 - \prod_{k=1}^{10} P(x_{i,k} > x_i|\mathcal{F}_{i-1}) \\ &= 1 - \prod_{k=1}^{10} S_{x_{i,k}}\left(\frac{x_i}{\Psi_{i,k}}\right) \end{aligned} \tag{15}$$

Therefore, after inserting the survival given in formula (14) into (15), we have:

$$\begin{aligned} F(x_i|\mathcal{F}_{i-1}) &= 1-\prod_{k=1}^{10}\exp\left[-\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k}\right] \\ &= 1-\exp\left[-\sum_{k=1}^{10}\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k}\right] \end{aligned} \tag{16}$$

The density function for x_i can be obtained from the formula (16):

$$f(x_i|\mathcal{F}_{i-1}) = \frac{\partial F(x_i)}{\partial x_i} = \left[\sum_{k=1}^{10}\frac{\gamma_k}{\Phi_{i,k}}\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k-1}\right]\cdot\exp\left[-\sum_{k=1}^{10}\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k}\right] \tag{17}$$

The conditional (with respect to $\mathcal{F}_{i-1}$ and current duration x_i) transition probabilities between the selected events are given as:

$$\begin{aligned} f(y_i|x_i,\mathcal{F}_{i-1}) &= \frac{f(x_i,y_i|\mathcal{F}_{i-1})}{f(x_i|\mathcal{F}_{i-1})} \\ &= \frac{\prod_{k=1}^{10}\left[\frac{\gamma_k}{\Phi_{i,k}}\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k-1}\right]^{I_i^k}\cdot\exp\left[-\left(\frac{x_i}{\Phi_{i,k}}\right)\right]^{\gamma_k}}{\left[\sum_{k=1}^{10}\frac{\gamma_k}{\Phi_{i,k}}\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k-1}\right]\cdot\prod_{k=1}^{10}\exp\left[-\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k}\right]} \\ &= \frac{\prod_{k=1}^{10}\left[\frac{\gamma_k}{\Phi_{i,k}}\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k-1}\right]^{I_i^k}}{\sum_{k=1}^{10}\frac{\gamma_k}{\Phi_{i,k}}\left(\frac{x_i}{\Phi_{i,k}}\right)^{\gamma_k-1}} \end{aligned} \tag{18}$$

Obviously, as in the case of the asymmetric ACD model, the y_i and x_i are not independent.

The conditional (only with respect to $\mathcal{F}_{i-1}$ and not the current duration x_i) transition probabilities do not have a closed analytic form and are equal to:

$$P(y_i=k|\mathcal{F}_{i-1}) = \int_0^{\infty}\left[\frac{\gamma_k}{\Phi_{i,1}}\left[\frac{x_i}{\Phi_{i,1}}\right]^{\gamma_k-1}\right]\cdot\prod_{l=1}^{10}e^{-\left[\frac{x_i}{\Phi_{i,l}}\right]^{\gamma_l}}\,dx_i \tag{19}$$

Under the restriction $\gamma_k=\gamma$ for $k=1,2,...,10$, the formula 18 boils down to:

$$f(y_i|\mathcal{F}_{i-1}) = \frac{\prod_{k=1}^{10}\left[\Phi_{i,k}^{-\gamma}\right]^{I_i^k}}{\sum_{k=1}^{10}\Phi_{i,k}^{-\gamma}} \tag{20}$$

Thus, y_i and x_i are independent.