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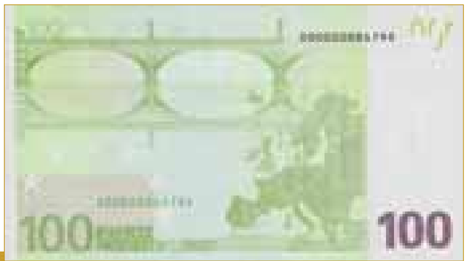

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CONTENTS

Abstract

Identification of structural VARs using sign restrictions has become increasingly popular in the academic literature. This paper (i) argues that identification of shocks can benefit from introducing a global dimension, and (ii) shows that summarising information by the median of the available impulse responses—as commonly done in the literature—has some undesired features that can be avoided by using an alternatively proposed summary measure based on a "scaled median" estimate of the structural impulse response. The paper implements this approach in both a small scale model as originally presented in Uhlig (2005) and a large scale model, introducing the sign restrictions approach to the global VAR (GVAR) literature, that allows to explore the global dimension by adding a large number of sign restrictions. We find that the patterns of impulse responses are qualitatively similar though point estimates tend to be quantitatively much larger in the alternatively proposed approach. In addition, our GVAR application in the context of global oil supply shocks documents that oil supply shocks have a stronger impact on emerging economies' real output as compared to mature economies, a negative impact on real growth in oil-exporting economies as well, and tend to cause an appreciation (depreciation) of oil-exporters' (oil-importers') real exchange rates but also lead to an appreciation of the US dollar. One possible explanation would be the recycling of oil-exporters' increased revenues in US financial markets.

Keywords: Identification of shocks, sign restrictions, VAR, global VAR, oil shocks.
JEL Classification: C32, E17, F37, F41, F47

Non-technical summary

Identification of structural VARs by means of sign restrictions has become increasingly popular in applied econometrics over the recent past. Maybe surprisingly, the performance of identification schemes using sign restriction has only received limited attention and there is indeed little evidence that imposing sign restrictions actually helps identifying structural shocks at all. Sign restrictions do not pin down a unique structural model and it is therefore not surprising that any sign restriction identification procedure is bound to be imperfect. Nevertheless, one would expect that with an increasing number of sign restrictions imposed one should obtain a better understanding of the structural shock in question. The global or cross-section dimension offers an intuitive and straightforward way of imposing a large number of sign restrictions to identify shocks that are global in nature—i.e. shocks that affect many cross-section units at the same time.

The mainstream literature reports the median of the impulse responses, which is sometimes interpreted as a "consensus" view of the magnitudes of the responses, and quantiles are used to give an impression of the distribution of impulse responses. The global dimension offers a new way of dealing with the problem of summarising information. We look at the summary measure as an estimation problem and expect that with an increasing cross-section dimension and an increasing number of restrictions imposed, the summary measures gets closer to the true structural impulse response (consistency property). This allows us to label the summary measure as our "best guess" whereas it is difficult to interpret the traditionally reported median impulse response. Indeed, the median impulse response does not get closer to the structural IR with an increasing number of restrictions imposed. We propose an alternative way of summarising information that satisfies the intuitive consistency property and show by means of Monte Carlo simulations that it outperforms the traditional summary measures.

Finally we implement the sign restriction approach in the context of a Global VAR (GVAR) model of the world economy, which given its global dimension allows for imposing a large number of sign restrictions, and we identify the effect of oil shocks on the global economy. Our results suggest that negative oil supply shocks (i) have a stronger impact on emerging economies' real output as compared to mature economies, (ii) have a negative impact on real growth in oil-exporting economies as well, (iii) tend to cause an appreciation (depreciation) of oil-exporters' (oil-importers') real exchange rates but also lead to an appreciation of the US dollar. One possible explanation would be recycling of oil-exporters' increased revenues in US financial markets.

1 Introduction

Vector autoregressive models (VARs) have become an indispensable tool in macroeconometric modelling given their ability to describe *econometric* reduced form relationships without the need to a priori impose any economic theory. When it comes to unveiling the underlying structural *economic* relationships, however, some way of identification is needed.

In practice, identification has often been based on economic theory—rather than purely statistical considerations. Common identification approaches typically include a recursive ordering of variables (Cholesky decomposition), structural identification by imposing zero-restrictions in the system of linear equations (Blanchard and Watson, 1986; Bernanke 1986) or a decomposition in temporary and permanent components (Blanchard and Quah, 1989). An alternative way to identify shocks that has over the recent past become increasingly popular is to impose restrictions on the *signs* of structural impulse responses for a given number of periods after the shock. This identification approach has been developed inter alia by Faust (1998), Canova and Pina (1999), Canova and de Nicoló (2002), Uhlig (2005) and Mountford and Uhlig (2005). The basic intuition is that structural shocks can be identified by checking whether the signs of the corresponding impulse responses (IR) are in line with economic theory.

While there exists a large body of literature on "traditional" identification schemes, the sign restriction approach has received much less attention. Indeed, there is little evidence that imposing sign restrictions actually helps identifying structural shocks at all.[1] Sign restrictions do not pin down a unique structural model and it is therefore not surprising that any sign restriction identification procedure is bound to be imperfect. Nevertheless, one would expect that with an increasing number of sign restrictions imposed one should obtain a better understanding of the structural shock in question.

The global or cross-section dimension offers an intuitive and straightforward way of imposing a large number of sign restrictions to identify shocks that are global in nature—i.e. shocks that affect many cross-section units at the same time. However, no matter how large the dimension of the model is and how many restrictions are imposed, there will always be an infinite number of structural models that satisfy the sign restrictions and therefore it is necessary to find a way to summarise the available information. How to deal with the multiplicity of models is not straightforward. The mainstream literature reports the median of the impulse responses, which is sometimes interpreted as a "consensus" view of the magnitudes of the responses, and quantiles are used to give an impression of the distribution of impulse responses. Fry and Pagan (2007) propose a different way of summarising available models by selecting the IR that is closest to the median IR in some measure.

[1] Fry and Pagan (2007) provide an analytical example of a two-variable two-shock system in which sign restrictions are not able to pin down the correct sign of the impulse response to the shock under inspection given the weak information embodied in the restrictions. Paustian (2007) shows that sign restrictions are able to pin down the correct sign of the impulse responses generated by a DSGE model only if a sufficiently large number of sign restrictions is imposed and if the volatility of the shock under inspection is sufficiently large relative to that of the remaining shocks in the system.

We argue that the global dimension offers a new way of dealing with this problem. This paper looks at the summary measure as an estimation problem and we expect that with an increasing cross-section dimension and an increasing number of restrictions imposed, the summary measure gets arbitrarily close to the true structural impulse response. We call this feature consistency.[2]

Surprising enough, neither the median IR nor the solution proposed by Fry and Pagan (2007) satisfy this intuitive consistency feature and we show by means of Monte Carlo experiments that the distance between the structural shock and these two measures increases with the increasing number of restrictions imposed. We propose an alternative way of summarising the available models, based on a "scaled median" estimate of the structural impulse response, which performs well in our Monte Carlo experiments and gets closer to the true structural shocks with an increase in the number of restrictions imposed.

The paper implements this approach in both a small scale model as originally presented in Uhlig (2005) and a large scale model, introducing the sign restrictions approach to the global VAR (GVAR) literature,[3] that allows to explore the global dimension by adding a large number of sign restrictions. We find that the patterns of impulse responses are qualitatively similar though point estimates tend to be quantitatively much larger in the alternatively proposed approach.

In addition, our application of the sign restrictions approach in the context of a GVAR model of oil prices and the global economy shows that oil supply shocks have a stronger impact on emerging economies' real output as compared to mature economies, have a negative impact on real growth in oil-exporting economies as well, and tend to cause an appreciation (depreciation) of oil-exporters' (oil-importers') real exchange rates but also lead to an appreciation of the US dollar. One possible explanation would be the recycling of oil-exporters' increased revenues in US financial markets.

The remainder of the paper is as follows. Section 2 outlines the problem of identification, reviews the sign restriction approach, discusses the problems of summarising multiple models and argues that the global dimension can help to identify shocks. Section 3 documents the main issues by means of Monte Carlo experiments. Section 4 implements the sign restriction approach applying our alternative summary measure of impulse responses in the context of a Global VAR (GVAR) model of the world economy, which given its global dimension allows for imposing a large number of sign restrictions for identifying the effect of oil shocks on the global economy. Section 5 analyses how the alternative summary measure of impulse responses fares in a small scale model such as Uhlig's (2005) well-known application to monetary shocks. The final section offers some concluding remarks.

A brief word on notation: All vectors are column vectors, denoted by bold lower case letters. Matrices are denoted by bold capital letters. Column vector $i \in \{1, ..., n\}$ of an $n \times n$ matrix $\mathbf{A}$ is denoted as $\mathbf{a}_i$. $\|\mathbf{A}\|_c \equiv \max_{1 \leq j \leq n} \sum_{i=1}^{n} |a_{ij}|$ denotes the maximum absolute column-sum matrix

[2] The sign restriction approach has typically been implemented in the literature in a Bayesian set-up, but sign restrictions per se are not a Bayesian concept. In fact, this paper completely abstracts from estimation uncertainty in the theoretical exposition below in order to focus on the underlying identification problem in a transparent and easy way.

[3] The GVAR approach has been proposed by Pesaran, Schuermann and Wiener (2004).

norm of $\mathbf{A}$, $\|\mathbf{A}\|_r \equiv \max_{1\leq i\leq n} \sum_{j=1}^{n} |a_{ij}|$ is the maximum absolute row-sum matrix norm of $\mathbf{A}$.[4] $\|\mathbf{A}\| = \sqrt{\varrho(\mathbf{A}'\mathbf{A})}$ is the spectral norm of $\mathbf{A}$, where $\varrho(\mathbf{A})$ is the spectral radius of $\mathbf{A}$.[5]

2 Identification of structural shocks and sign restrictions

2.1 Identification of shocks in a reduced form VAR

Suppose that n endogenous variables collected in a vector $\mathbf{x}_t = (x_{1t}, ..., x_{nt})'$ are generated from the following structural VAR(1) model,

$$\mathbf{A}_0\mathbf{x}_t = \mathbf{A}_1\mathbf{x}_{t-1} + \mathbf{e}_t, \tag{1}$$

where one lag is assumed for simplicity of exposition, $\mathbf{A}_0$ and $\mathbf{A}_1$ are $n \times n$ matrices of unknown structural coefficients, and $\mathbf{e}_t$ is a vector of orthogonal structural innovations with individual elements e_{it}, for $i = 1, .., N$, having unit variance. Assuming the matrix $\mathbf{A}_0$ is invertible, the reduced form of the structural VAR model (1) is:

$$\mathbf{x}_t = \mathbf{\Phi}\mathbf{x}_{t-1} + \mathbf{u}_t, \tag{2}$$

where the vector of reduced form errors is given by the "spatial" model $\mathbf{u}_t = \mathbf{D}\mathbf{e}_t$, $\mathbf{D} = \mathbf{A}_0^{-1}$, and $\mathbf{\Phi} = \mathbf{A}_0^{-1}\mathbf{A}_1$.

There is not much disagreement on how to estimate reduced form VARs, namely the coefficient matrix $\mathbf{\Phi}$ and the covariance matrix of errors, denoted as $\mathbf{\Sigma} = E(\mathbf{u}_t\mathbf{u}_t') = \mathbf{D}\mathbf{D}'$. The identification problem is how to decompose reduced form errors into economically meaningful shocks, i.e. to construct $\mathbf{D}$. The decomposition $\mathbf{\Sigma} = \mathbf{D}\mathbf{D}'$ is not a unique decomposition of the covariance matrix $\mathbf{\Sigma}$. In particular, for any $n \times n$ orthogonal matrix $\mathbf{Q}$ we have $\mathbf{\Sigma} = (\mathbf{D}\mathbf{Q}')(\mathbf{Q}\mathbf{D}')$ and hence an infinite number of candidate matrices $\mathbf{B}$, where $\mathbf{Q}'\mathbf{Q} = \mathbf{I}$ (orthogonality conditions) and $\mathbf{B} = \mathbf{D}\mathbf{Q}'$. Orthogonality of $\mathbf{Q}$ implies $n(n+1)/2$ restrictions on the elements of $\mathbf{Q}$ leaving thus $n(n-1)/2$ free parameters in constructing the matrix $\mathbf{Q}$. Thus $n(n-1)/2$ restrictions need to be imposed for exact identification.

Structural shocks in the structural VAR model (1) are given by the individual column vectors of matrix $\mathbf{D}$. Let $r_{ij\ell}$ denote response of variable x_{jt} to the structural shock i in the period ℓ after the shock, that is:

$$r_{ij\ell} = \mathbf{s}_j'\mathbf{\Phi}^{\ell}\mathbf{d}_i, \text{ for } \ell \geq 0 \text{ and } i, j \in \{1, .., n\}, \tag{3}$$

where $\mathbf{s}_j$ is $n \times 1$ selection vector that selects the j-th element and $\mathbf{d}_i$ is the j-th column of matrix $\mathbf{D}$. For future reference, we denote the IR vector to shock $\mathbf{g}$ as

$$\mathbf{r}_{\ell}(\mathbf{g}) = \mathbf{\Phi}^{\ell}\mathbf{g}, \text{ for } \ell \geq 0,$$

[4] The maximum absolute column sum matrix norm and the maximum absolute row sum matrix norm are sometimes denoted in the literature as $\|\cdot\|_1$ and $\|\cdot\|_\infty$, respectively.

[5] Note that if $\mathbf{x}$ is a vector, then $\|\mathbf{x}\| = \sqrt{\varrho(\mathbf{x}'\mathbf{x})} = \sqrt{\mathbf{x}'\mathbf{x}}$ corresponds to the Euclidean length of vector $\mathbf{x}$.

and the structural IR (SIR) vector to the structural shock $\mathbf{d}_i$, as

$$\mathbf{r}_{i\ell} = \mathbf{\Phi}^{\ell}\mathbf{d}_i, \text{ for } \ell \geq 0 \text{ and } i \in \{1, .., n\}.$$

2.2 Sign restriction approach

In order to focus on the issue of identification in the simplest and the most transparent way, let us abstract from the estimation uncertainty of the reduced form parameters. In particular, matrices $\mathbf{\Phi}$ and $\mathbf{\Sigma}$ are assumed to be known. In addition, we assume the absence of any dynamics, i.e. $\mathbf{A}_1 = \mathbf{0}$, since the number of lagged terms is not pertinent to the identification problem. The focus of the exposition is thus on the simplest case where

$$\mathbf{x}_t = \mathbf{u}_t, \tag{4}$$

and the covariance matrix $\mathbf{\Sigma} = \mathbf{DD}'$ is known, but the matrix of structural shocks $\mathbf{D}$ is unknown. It follows that the contemporaneous IR (in period $\ell = 0$) equals the corresponding element of the structural shock, i.e. $r_{ij0} = d_{ij}$ for any $i, j, \in \{1, .., n\}$, and that variables do not respond to past shocks, i.e. $r_{ij\ell} = 0$ for $\ell > 0$. In this static example, the focus is on contemporaneous responses, $\ell = 0$, and we omit the subscript ℓ where not necessary in the exposition below.[6]

Suppose we are interested in identifying only the first structural shock $\mathbf{d}_1$ and some (or all) signs of the vector of the contemporaneous IR $\mathbf{r}_1 = \mathbf{r}(\mathbf{d}_1) = (r_{11}, r_{21}, ..., r_{N1})'$ are known. For any decomposition of the covariance matrix $\mathbf{\Sigma}$ into the product of matrices $\mathbf{BB}'$, the potential candidates for the structural shock $\mathbf{d}_1$ are fully characterized by the set of vectors $\mathbf{g}_\alpha = \mathbf{B}\boldsymbol{\alpha}$, where $\|\boldsymbol{\alpha}\| = 1$. Typically there are many different shocks $\mathbf{g}_\alpha$ that satisfy the sign restrictions. Denote $\mathcal{S}$ to be a set of $n \times 1$ vectors $\boldsymbol{\alpha}$ of unit length such that the responses $\mathbf{r}(\mathbf{g}_\alpha)$, for $\boldsymbol{\alpha} \in \mathcal{S}$, satisfy the imposed sign restrictions. A measure that somehow summarises the available information is then required as one often wishes to know what is a "consensus" view. This measure could be interpreted as a point estimate of the SIR at each point in time following the shock. A common way to summarize the available models in the literature is to compute the median of the IRs that satisfy the sign restrictions, that is

$$\widehat{\mathbf{r}}_1^{med} = med\left\{\mathbf{r}(\mathbf{g}_\alpha), \boldsymbol{\alpha} \in \mathcal{S}\right\}. \tag{5}$$

2.3 Global dimension to the rescue of weak information

One criticism of the sign restriction approach is that it delivers only a weak (imprecise) identification since it uses only weak information.[7] The fact that weak information delivers weak results is

[6] For the sake of clarity of exposition, we continue to refer to the impulse responses as r_{ij}, whereas we refer to the structural shocks as d_{ij}.

[7] It is also true, on the other hand, that weak information could rather be seen as an advantage than a disadvantage, because there are limits to how much we really know about structural shocks, in the sense that there is significant uncertainty regarding the underlying structural model that generated the data.

a powerful wisdom and we do not object this view. Instead, we argue that introducing a "global dimension" can considerably improve the character of information embodied in the sign restrictions in a way that the resulting identification procedure is arbitrarily good as the number of variables (and the number of sign restrictions) tends to infinity. The following simple example suffices to make this point clear.

Suppose that the structural shocks are given by the following matrix

$$\underset{n\times n}{\mathbf{D}} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & 0 & & 0 \\ 1 & 0 & 1 & 0 & & 0 \\ 1 & 0 & 0 & 1 & & 0 \\ \vdots & & & & \ddots & \\ 1 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}, \tag{6}$$

and the objective is to estimate a "global" shock $\mathbf{d}_1$, using the knowledge about the signs of $\mathbf{d}_1$. As before, suppose that the matrix $\mathbf{\Sigma}$ is known and consider any decomposition $\mathbf{\Sigma} = \mathbf{B}\mathbf{B}'$. For $\boldsymbol{\alpha}$ uniformly distributed on a unit sphere, the distribution of random vectors $\mathbf{B}\boldsymbol{\alpha}$ and $\mathbf{D}\boldsymbol{\alpha}$ are the same. For fixed n, information embodied in the sign restrictions is imperfect and rather weak if n is relatively small. In particular, the probability that a linear combination of structural shocks $\{\mathbf{d}_2, ..., \mathbf{d}_n\}$ will yield the same signs as the last $n-1$ elements of the structural shock $\mathbf{d}_1$ is:

$$p\left\{\sum_{j=2}^{n} \alpha_j \mathbf{d}_j > 0\right\} = \frac{1}{2^{n-1}}. \tag{7}$$

Clearly there are many models that satisfy the sign restrictions and equation (7) suggests that chances that structural shocks $\{\mathbf{d}_2, ..., \mathbf{d}_n\}$ distort the estimate of $\mathbf{d}_1$ are good for small values of n. As $n \to \infty$, this probability goes to zero at an exponential rate. This provides the basic intuition for the "global dimension" to be able to help with the estimation of global structural shocks or—to be more precise—structural shocks which have unbounded maximum absolute column-sum norm in n.

2.4 The problem of summarising available models

As Fry and Pagan (2007) argue, the median IR (denoted as $\widehat{\mathbf{r}}_1^{med}$) does not necessarily belong to the space of impulse responses. To illustrate the problem of using the median IR, suppose we have two draws of impulse responses $\{\mathbf{r}(\mathbf{g}_{\alpha 1}), \mathbf{r}(\mathbf{g}_{\alpha 2})\}$ which satisfy the sign restrictions. There is no guarantee that there exists a model, where $\widehat{\mathbf{r}}_1^{med} = med\{\mathbf{r}(\mathbf{g}_{\alpha 1}), \mathbf{r}(\mathbf{g}_{\alpha 2})\}$ is a structural impulse response. In particular, the solution for $\boldsymbol{\alpha}$ in system $\widehat{\mathbf{r}}_1^{med} = \mathbf{B}\boldsymbol{\alpha}$ need not lie on the unit circle and therefore $\widehat{\mathbf{r}}_1^{med}$ need not even belong to the space of IRs.

Since the median is not a monotonic transformation, i.e. $med\{\mathbf{g}_{\alpha 1}, ..., \mathbf{g}_{\alpha s}\} \neq \mathbf{B}\cdot med\{\boldsymbol{\alpha}_1, ..., \boldsymbol{\alpha}_s\}$, let us for the moment and for the sake of ease of exposition consider arithmetic averages instead. Averaging is a monotonic transformation and the average of the candidate IR (at the time of impact), denoted as $\widehat{\mathbf{r}}_1^{ave} = [\mathbf{r}(\mathbf{g}_{\alpha 1}) + ... + \mathbf{r}(\mathbf{g}_{\alpha s})]/s$, can be written as $\widehat{\mathbf{r}}_1^{ave} = \mathbf{B}\overline{\boldsymbol{\alpha}} = \mathbf{B}(\boldsymbol{\alpha}_1 + ... + \boldsymbol{\alpha}_s)/s$, where $\overline{\boldsymbol{\alpha}} = s^{-1}\sum_{i=1}^{s}\boldsymbol{\alpha}_i$. Note that triangle inequality implies that $\|\overline{\boldsymbol{\alpha}}\| < 1$ (unless $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2 = ... = \boldsymbol{\alpha}_s$). Therefore, as long as the set of draws $\mathcal{S}$ has different elements, $\widehat{\mathbf{r}}_1^{ave}$ (the average of the candidate IR) *never* belongs to the space of IRs. The more candidate draws for the structural shock one obtains and the higher dimensionality of the model, the smaller $\|\overline{\boldsymbol{\alpha}}\|$ could get. Similar arguments also apply to the median as opposed to averaging, although the vector $\boldsymbol{\alpha}$ that would correspond to the median response could lie on either side of the unit circle in general. The Monte Carlo experiments in Section 3 show that the median behaves similarly to the averaging.

This paper looks at the same problem of multiplicity of models from a slightly different perspective than Fry and Pagan (2007). As opposed to summarizing the available information by means of quantiles and medians of the impulse responses, we suggest to find an IR vector, which would be as close as possible to the true (unknown) IR. Ideally we would like to find $\widehat{\boldsymbol{\alpha}}$ on the unit circle that would satisfy

$$\widehat{\boldsymbol{\alpha}} = \arg \min_{\boldsymbol{\alpha}'\boldsymbol{\alpha}=1} \|\widehat{\mathbf{r}}_1 - \mathbf{r}_1\|, \tag{8}$$

where $\widehat{\mathbf{r}}_1 = \mathbf{B}\boldsymbol{\alpha}$. However, it is not possible to solve (8) without knowing the matrix $\mathbf{D}$ and for this reason, we opt for the following weaker consistency requirement,

$$\lim_{n\to\infty} \|\widehat{\mathbf{r}}_1 - \mathbf{r}_1\| = 0, \tag{9}$$

that is the error in estimating IRs vanishes as $n \to \infty$. General treatment of the problem (9) goes beyond the analysis presented here; instead we propose to compute the impulse response corresponding to the scaled median of draws $\boldsymbol{\alpha} \in \mathcal{S}$, that is

$$\widehat{\mathbf{r}}_1^{scaled} = \mathbf{B}\widehat{\boldsymbol{\alpha}}, \tag{10}$$

where

$$\widehat{\boldsymbol{\alpha}} = \frac{\boldsymbol{\alpha}_{med}}{\|\boldsymbol{\alpha}_{med}\|}, \text{ and } \boldsymbol{\alpha}_{med} = med\{\mathcal{S}\}.$$

In the remainder of this paper, we refer to $\widehat{\mathbf{r}}_1^{scaled}$ as a "scaled" estimate of the SIR.

3 Monte Carlo experiments

3.1 Monte Carlo design

In this section, we illustrate the main ideas by means of Monte Carlo experiments. In particular, we show that introducing a global dimension turns weak information strong: As the number of variables—and hence the leeway to introduce sign restrictions increases—our scaled estimate $\widehat{\mathbf{r}}_1^{scaled}$ does converge to the "true" SIR, while the traditionally used measures do not.

Let the data generating process be $\mathbf{x}_t = \mathbf{u}_t$, where errors in vector $\mathbf{u}_t$ are generated by the following spatial model,

$$\mathbf{u}_t = \mathbf{D}\mathbf{e}_t,$$

and the vector of structural innovations is $\mathbf{e}_t \sim IIDN\left(\mathbf{0}, \mathbf{I}_n\right)$. The matrix of structural shocks $\mathbf{D}$ is generated as follows. The "global" shock, which is to be estimated, is the first structural shock given by

$$\underset{n\times 1}{\mathbf{d}_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix},$$

and all signs of vector $\mathbf{d}_1$ are assumed to be known in the identification procedure. The lower right $(n-1)\times(n-1)$-dimensional submatrix of matrix $\mathbf{D}$, denoted as $\mathbf{D}_{22}$ is generated as

$$\mathbf{D}_{22} = \left(\mathbf{I}_{n-1} - \rho\mathbf{S}\right)^{-1},$$

where

$$\mathbf{S} = \begin{pmatrix} 0 & 1 & 0 & & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & & \\ 0 & \frac{1}{2} & 0 & \ddots & 0 \\ & & \ddots & \ddots & \frac{1}{2} \\ 0 & & 0 & 1 & 0 \end{pmatrix},$$

that is the process $\mathbf{D}_{22}\left(e_{2t}, ..., e_{nt}\right)'$ is a bilateral spatial autoregressive process. Dependence of the first reduced error u_{1t} on the remaining structural shock is generated as $d_{1j} \sim U\left(-\rho/N, \rho/N\right)$. Note that for $\rho = 0$, matrix $\mathbf{D}$ becomes (6), whereas as $\rho \to 1$, the degree of cross-section dependence increases. Two experiments are considered: $\rho = 0$ (benchmark case) and $\rho = 0.4$ (stronger cross-section dependence). Matrix $\mathbf{D}$ is generated at the beginning of the experiment and $R = 2000$ replications are carried out. The structural IR vector at the time of impact, is estimated in the following three different ways:

1. As a median IR, denoted as $\widehat{\mathbf{r}}_1^{med} = med\left\{\mathbf{B}\boldsymbol{\alpha}, \boldsymbol{\alpha} \in \mathcal{S}\right\}$, where $\mathbf{BB}'$ is the Cholesky decomposition of the known covariance matrix $\mathbf{\Sigma}$.[8]

2. As an IR that is closest to the median IR (as proposed by Fry and Pagan, 2007), denoted as $\widehat{\mathbf{r}}_1^{FP} = \mathbf{B}\boldsymbol{\alpha}_m$, where $\boldsymbol{\alpha}_m = \arg\min_{\boldsymbol{\alpha}\in\mathcal{S}} \left\|\mathbf{B}\boldsymbol{\alpha} - \widehat{\mathbf{r}}_1^{med}\right\|$.

3. As a scaled estimate $\widehat{\mathbf{r}}_1^{scaled} = \mathbf{B}\widehat{\boldsymbol{\alpha}}$, where $\widehat{\boldsymbol{\alpha}} = \frac{med(\mathcal{S})}{\|med(\mathcal{S})\|}$.

[8] The decomposition of $\mathbf{\Sigma}$ is arbitrary and other decompositions should yield similar results since no other information, besides the signs of responses are imposed (see, for instance, Uhlig, 2005).

The covariance matrix $\mathbf{\Sigma}$ is taken to be known. Matrix $\mathbf{D}$ is treated as unknown, except for the signs of $\mathbf{d}_1$. The vector $\boldsymbol{\alpha}$ is generated randomly as

$$\boldsymbol{\alpha} = \frac{\boldsymbol{\eta}}{\|\boldsymbol{\eta}\|},$$

where $\boldsymbol{\eta} \sim N(0, \mathbf{I}_n)$. The number of draws that satisfy the sign restrictions are chosen to be $s \in \{100, 500, 1000, 5000\}$.

3.2 Monte Carlo findings

In order to provide a first impression of the performance of the summary measures under inspection, Figures 1–2 plot the histograms of the errors of the estimated structural vector $\mathbf{r}_1$ for 2000 repetitions of the benchmark experiment ($\rho = 0$) and two choices for the dimension of the model, $(n, s) \in \{(5, 100); (20, 1000)\}$. Two different metrics for computing the error as the distance between estimated SIRs and true SIRs are considered: the maximum absolute row sum norm, denoted as $\|\widehat{\mathbf{r}}_1 - \mathbf{r}_1\|_r$, and the euclidean norm, denoted as $\|\widehat{\mathbf{r}}_1 - \mathbf{r}_1\|$.

As can be inferred from Figure 1, which plots the absolute frequency of the errors computed as euclidian norm, both the median impulse response as well as the measure suggested by Fry and Pagan (2007) perform less well than the scaled response as proposed here.

Furthermore, comparing the errors obtained in the relatively smaller system as shown in Figure 1 with the errors obtained in a relatively larger system embedding more information on the true signs of the IRs as shown in Figure 2 shows that the error in estimating the SIRs actually increases in n and hence the number of restrictions imposed when using the conventional summary measures. The scaled response estimator $\widehat{\mathbf{r}}_1^{scaled}$ however improves with rising n, and hence fulfills the intuitive consistency property outlined in Section 1. The latter result is robust to the degree of cross-section dependence as evidenced by Figure 3.

The preliminary findings from inspection of the error histograms are confirmed when considering a wide range of alternative values for the model dimension. Tables 1–2 report the average error of the individual estimates of the structural impulse response vector $\mathbf{d}_1$ for $(n, s) \in \{(5, 10, 20, 50, 100); (100, 500, 1000, 5000)\}$ for both the benchmark experiment ($\rho = 0$) as well as the case of higher cross-section dependence ($\rho = 0.4$). It can be inferred from Table 1 that the median response ($\widehat{\mathbf{r}}_1^{med}$) as well as the response which is closest to median response ($\widehat{\mathbf{r}}_1^{FP}$) do not provide a good estimate of the structural impulse response. First, performance does not improve in the number of models that satisfy sign restrictions. Second, the estimated responses become even worse as the number of variables and hence the numbers of signs restrictions increases. These results confirm that even if a large amount of sign restrictions is imposed, the conventional summary measures $\widehat{\mathbf{r}}_1^{med}$ and $\widehat{\mathbf{r}}_1^{FP}$ can be highly misleading when interpreted as a "consensus" view. In particular, both $\left\|\widehat{\mathbf{r}}_1^{FP} - \mathbf{r}_1\right\|$ and $\left\|\widehat{\mathbf{r}}_1^{med} - \mathbf{r}_1\right\|$ seem to diverge to infinity with n and therefore the estimation error is arbitrarily large for n large enough. On the other hand, the scaled response $\widehat{\mathbf{r}}_1^{scaled}$, clearly gets closer to the true response as n and the number of draws increase. Finally, the results also show that introducing a "global dimension" helps gaining accuracy: for a system of some tens of variables, $\widehat{\mathbf{r}}_1^{scaled}$ quickly approaches the "true" impulse response keeping the number of successful draws large enough.

4 Identification of a global shock: oil supply shocks in a GVAR model

4.1 The basic GVAR setup

In this section, we implement the sign restriction approach in the context of a large scale model of the global economy that allows us to impose a large number of sign restrictions. The application illustrates how our scaled median approach can be implemented in a straightforward way in the context of a GVAR and compares results obtained from both the traditional summary measure of impulse responses as well as the proposed scaled median summary measure. Overall, the observed patterns of impulse responses tend to be qualitatively similar, although the scaled median approach systematically yields quantitatively larger point estimates of impulse responses, as we will explain below.

We follow Pesaran, Schuermann and Weiner (2004) and estimate the following country-specific VARX* (p, q) models:

$$\mathbf{\Phi}_{ii}(L, p_i)\,\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \mathbf{\Lambda}_i(L, q_i)\,\mathbf{x}_{it}^* + \mathbf{u}_{it}, \tag{11}$$

where $\mathbf{x}_{it}$ denotes a $k_i \times 1$ vector of macroeconomic variables belonging to country $i \in \{1, .., N\}$ with N being the number of countries, $\mathbf{x}_{it}^*$ denotes its cross-section averages, and error terms, collected in a $k \times 1$ vector $\mathbf{u}_t = (\mathbf{u}_{1t}', \mathbf{u}_{2t}', ..., \mathbf{u}_{Nt}')'$ with $k = \sum_{i=1}^N k_i$ are assumed to be cross-sectionally weakly dependent.[9]

Once estimated on a country by country basis, individual VARX* models for $i = 1, .., N$, can be stacked together and solved as one system:

$$\mathbf{G}(L, p)\,\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{u}_t, \tag{12}$$

where $\mathbf{a}_\ell = (\mathbf{a}_{\ell 1}', ..., \mathbf{a}_{\ell N}')'$ for $\ell = 0, 1$, and

$$\mathbf{G}(L, p) = \begin{pmatrix} \mathbf{B}_1(L, p) \\ \vdots \\ \mathbf{B}_N(L, p) \end{pmatrix}.$$

GVAR model (12) can be used for impulse response or persistence profile analysis in the usual manner. See Pesaran, Schuermann and Wiener (2004) for detailed exposition of GVARs.

[9] See Chudik, Pesaran and Tosetti (2011) for a definition of strong and weak cross section dependence.

4.2 Data and sign restrictions

We set $p_i = p = 2$ and $q_i = q = 1$ and estimate a GVAR in first differences with quarterly data over the period 1979:Q3–2003Q3 for $N = 26$ countries.[10] We include four country-specific variables —real output, inflation, a short-term interest rate, and the real exchange rate—as well as real oil prices as an observable global factor.

In order to identify oil supply shocks we rely on a simple identification scheme that allows us to discriminate oil supply shocks from a large set of alternative shocks. In particular, we require *negative* oil supply shocks to be contemporaneously associated with (i) a decrease in real output across all oil-importers and (ii) an increase in real oil prices. In all, we impose 21 contemporaneous sign restrictions. We do not impose any restrictions on real output in countries that have been significant oil-exporters over the sample period.[11] To the extent that no other economically meaningful shocks are able to produce a negative correlation between real output and real oil prices across *all* oil-importing economies, this identification scheme uniquely identifies oil supply shocks.

4.3 Empirical results

In the following we analyse the effect of negative oil supply shocks on (i) real output in different regions of the global economy including oil-exporting economies themselves; as well as (ii) on global exchange rate configurations.

Figure 4 gives a general overview of the reaction of output to a one-standard deviation shock to oil supply. On average, mature economies—including the United States and the euro area—tend to record a decline in real output by between 0.5 and 0.75% cumulated over the four quarters following a one-standard deviation oil supply shock (that causes oil prices to increase by 2.4%). Emerging economies—both in Latin America and Asia—record somewhat higher declines in growth of on average between 1 and 1.5%. The stronger effect on emerging economies could be a reflection of higher energy intensity of production in these countries, on the one hand, and dependence on external demand from mature economies, on the other hand. In this respect, China stands out as a notable exception with a relatively modest reaction of output to the oil supply shock, possibly reflecting the fact that despite high energy intensity a large part of energy demand is met domestically. Interestingly though not largely unexpectedly, real output also declines across all oil-exporting economies in our sample, including Saudi Arabia and Norway, although to a lesser extent, despite the reaction of real output being unrestricted in the identification procedure.

Figure 5 provides further detail and compares the impulse response based on the scaled median approach to impulse responses computed as in Uhlig (2005). A main finding from this exercise is that—while the patterns of impulse responses are qualitatively comparable—our scaled median impulse response tends to systemically yield larger point estimates of the shocks under inspection.

[10]The sample includes Argentina, Australia, Brazil, Canada, Chile, China, Euro area, India, Indonesia, Japan, Korea, Malaysia, Mexico, New Zealand, Norway, Peru, Philippines, Saudi Arabia, Singapore, South Africa, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United states.

[11]We take these countries to be Saudi Arabia, Norway, Indonesia, Mexico and the United Kingdom.

The quantitative difference is sizeable as impulse responses based on the scaled median tend to be about 3 times larger than those based on the traditional measure. This is because the traditional measure is based on median values of $\boldsymbol{\alpha}$ that—at least in the case of a single shock—is bound to lie *within* the unit circle (and therefore does not belong to the space of impulse responses), while our scaling approach avoids this caveat (and $\widehat{\mathbf{r}}_1^{scaled}$ belongs to the space of IRs).

Finally, results for the real exchange rate are summarised in Figures 6 and 7. As expected, oil-importers' real exchange rates mostly tend to depreciate in response to the negative terms of trade shock by on average between 0.5 and 1.5%, whereas the large oil exporters' appreciate by up to 1%. Maybe surprisingly, the United States appreciates by 1.3%. One rationalisation for this unexpected result could be recycling of oil-exporters' increased revenues in US financial markets. As a final note, Figure 7 shows that as in the case of real output before, our measure yields similar but systematically larger point estimates of impulse responses confirming our findings above.

5 Identification in a small scale model: US monetary policy shocks

In the following, this section assess how our alternative summary measure of impulse responses compares to the traditionally used median impulse response measure in the context of a small scale single country model.

In order to allow for a meaningful comparison, we use as a benchmark the application presented in Uhlig (2005), which employs sign restrictions to identify the effects of a contractionary monetary policy shock. In particular, we compute impulse responses to a contractionary monetary shock identified via the same sign restrictions as in Uhlig (2005) using both the traditional median impulse response and the approach based on scaled median draws from the unit circle, as proposed in this paper.

We use the data and code provided by Uhlig (2005) with monthly data (partly generated from interpolation) for the period 1965:01–2003:12 on real GDP, GDP deflator, commodity prices, federal funds rate, non-borrowed reserves, and total reserves. Following Uhlig (2005), we estimate a VAR in levels using 12 lags. Contractionary monetary policy is identified via sign restrictions implemented as follows. The GDP deflator, the commodity price index and non-borrowed reserves are restricted not to be positive and the federal funds rate is restricted not to be negative for $k = 5$ months after the shock. We take 1,000 draws from the posterior for each of which we compute 40,000 draws from the unit circle. [12]

[12] Uhligs code is based on 200 draws from each the posterior and the unit circle, resulting in overall 40,000 draws, but stops after having generated 1,000 draws that satisfy the sign restrictions. From these 1,000 draws impulse responses are generated and summarised in terms of their median. In order to compute the alternative summary measure proposed in this paper, we need to compute the median of draws of $\boldsymbol{\alpha}$ (for which the corresponding impulse responses satisfy the sign restrictions) for each draw from the posterior. Hence, an overall larger amount of draws from both the posterior and the unit circle is need in order to compute meaningful medians at both stages. We therefore increase the number of draws from the posterior to 1,000 draws for each of which we compute 40,000 draws from the unit circle. This leaves us with on average around 1,100 (and in no case fewer than 1,000) accepted draws from the unit circle for each of the 1,000 draws from the posterior distribution. From these roughly 1,100 draws we compute the scaled median and corresponding impulse responses, for which we then report the median across all 1,000 draws from the posterior.

Impulse responses are presented in Figure 8. First, we note that the code exactly replicates the impulse responses depicted in Uhlig (2005). Secondly, as in the case of the large scale model presented before, impulse responses are qualitatively not very different. However, impulse responses based on the *scaled* median $\boldsymbol{\alpha}$ also in the small model tend to be quantitatively larger, by between one quarter and one half, confirming the findings from the previous analysis.

6 Concluding remarks

Identification of structural VARs by means of sign restrictions has become increasingly popular in applied econometrics over the recent past. Maybe surprisingly, the performance of identification schemes using sign restriction has only received limited attention and there is indeed little evidence that imposing sign restrictions actually helps identifying structural shocks at all. Sign restrictions do not pin down a unique structural model and it is therefore not surprising that any sign restriction identification procedure is bound to be imperfect. Nevertheless, one would expect that with an increasing number of sign restrictions imposed one should obtain a better understanding of the structural shock in question. The global or cross-section dimension offers an intuitive and straightforward way of imposing a large number of sign restrictions to identify shocks that are global in nature—i.e. shocks that affect many cross-section units at the same time.

The mainstream literature reports the median of the impulse responses, which is sometimes interpreted as a "consensus" view of the magnitudes of the responses, and quantiles are used to give an impression of the distribution of impulse responses. The global dimension offers a new way of dealing with the problem of summarising information. We look at the summary measure as an estimation problem and expect that with an increasing cross-section dimension and an increasing number of restrictions imposed, the summary measures gets closer to the true structural impulse response (consistency property). This allows us to label the summary measure as our "best guess" whereas it is difficult to interpret the traditionally reported median impulse response. Indeed, the median impulse response does not get closer to the structural IR with an increasing number of restrictions imposed. We propose an alternative way of summarising information and show by means of Monte Carlo simulations that it seems to satisfy the intuitive consistency property and it outperforms the traditional summary measures.

Finally we implement the sign restriction approach in the context of a Global VAR (GVAR) model of the world economy, which given its global dimension allows for imposing a large number of sign restrictions, and we identify the effect of oil shocks on the global economy. Our results suggest that negative oil supply shocks (i) have a stronger impact on emerging economies' real output as compared to mature economies, (ii) have a negative impact on real growth in oil-exporting economies as well, (iii) tend to cause an appreciation (depreciation) of oil-exporters' (oil-importers') real exchange rates but also lead to an appreciation of the US dollar. One possible explanation would be recycling of oil-exporters' increased revenues in US financial markets.

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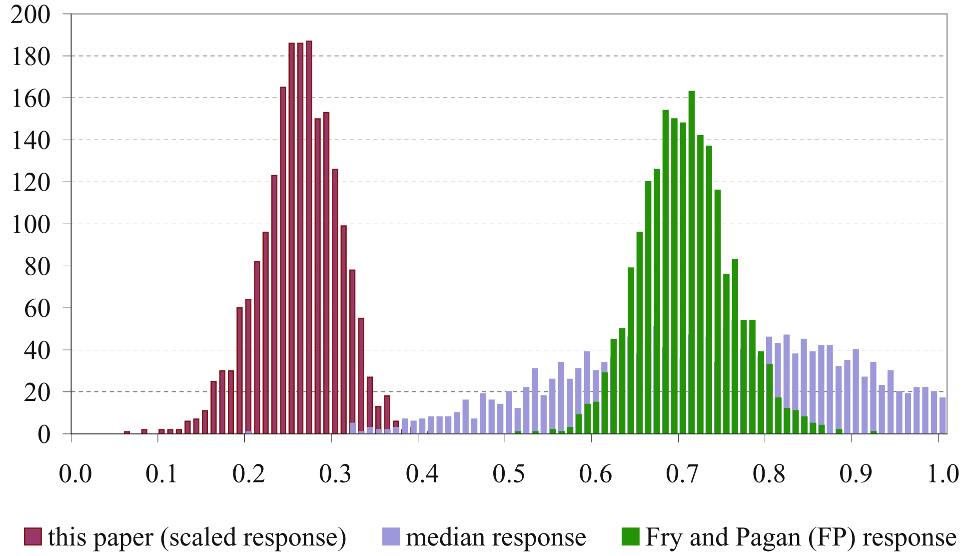
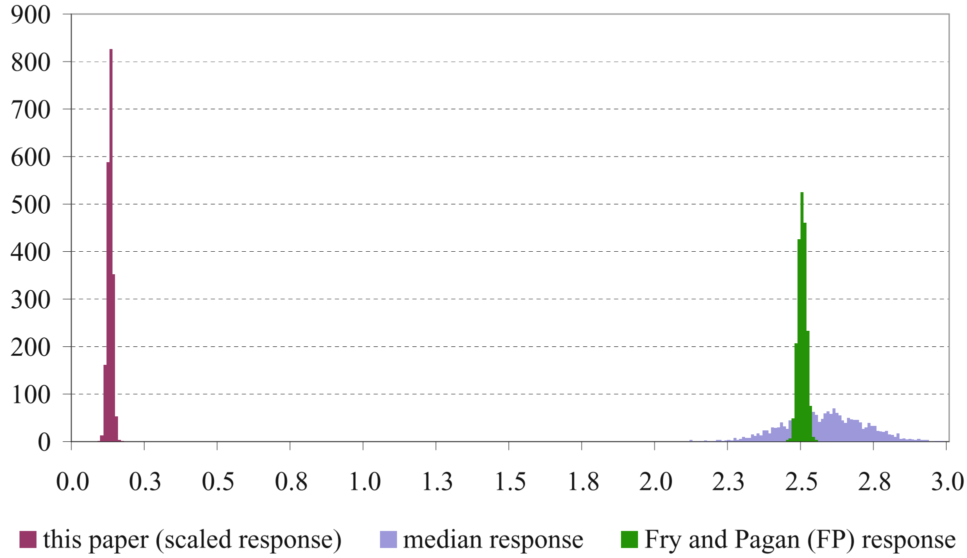
Figure 1: Error histograms of estimated impulse responses for small benchmark model ($\rho = 0, n = 5, s = 100$)



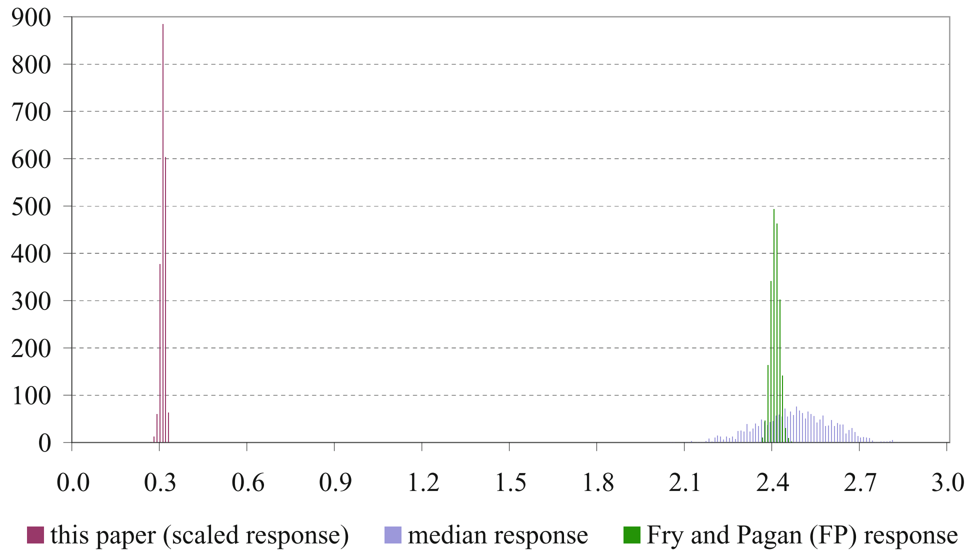
Note: Figure plots absolute frequency of errors computed as euclidean norm distance between true SIRs and estimated SIRs measured as scaled response, median response, and as proposed in Fry and Pagan (2007).

Figure 2: Error histograms of estimated impulse responses for large benchmark model ($\rho = 0, n = 20, s = 1000$)



Note: Figure plots absolute frequency of errors computed as euclidean norm distance between true SIRs and estimated SIRs measured as scaled response, median response, and as proposed in Fry and Pagan (2007).

Figure 3: Error histograms of estimated impulse responses for high cross-section dependence ($\rho = 0.4, n = 20, s = 1000$)



Note: Figure plots absolute frequency of errors computed as euclidean norm distance between true SIRs and estimated SIRs measured as scaled response, median response, and as proposed in Fry and Pagan (2007).

Table 1: Monte Carlo results for benchmark case ($\rho = 0$): error in estimating structural vector $\mathbf{r}_1$

Metric:	Row norm, $\|\widehat{\mathbf{r}}_1 - \mathbf{r}_1\|_r$				Euclidean norm, $\|\widehat{\mathbf{r}}_1 - \mathbf{r}_1\|$			
				Median response $\widehat{\mathbf{r}}_1^{med}$				
n/s	**100**	**500**	**1000**	**5000**	**100**	**500**	**1000**	**5000**
5	0.42	0.41	0.41	0.41	0.70	0.69	0.69	0.69
10	0.52	0.51	0.51	0.51	1.43	1.43	1.43	1.43
20	0.61	0.59	0.59	0.59	2.51	2.50	2.50	2.50
50	0.72	0.69	0.69	0.69	4.78	4.78	4.78	4.78
100	0.78	0.76	0.76	0.75	7.47	7.47	7.47	7.47
				Response closest to median response $\widehat{\mathbf{r}}_1^{FP}$				
5	0.57	0.49	0.45	0.38	0.74	0.68	0.66	0.62
10	0.77	0.76	0.75	0.73	1.52	1.50	1.49	1.47
20	0.87	0.85	0.84	0.83	2.59	2.58	2.58	2.57
50	0.94	0.94	0.94	0.93	4.84	4.84	4.84	4.84
100	0.97	0.96	0.96	0.96	7.52	7.52	7.52	7.52
				Response corresponding to the scaled $med\,(\mathcal{S})$, $\widehat{\mathbf{r}}_1^{sc}$				
5	0.17	0.14	0.13	0.13	0.26	0.24	0.24	0.23
10	0.13	0.10	0.10	0.10	0.24	0.18	0.17	0.17
20	0.10	0.06	0.06	0.05	0.28	0.15	0.13	0.11
50	0.12	0.05	0.04	0.02	0.56	0.18	0.12	0.08
100	0.15	0.05	0.03	0.02	1.11	0.30	0.18	0.07

Table 2: Monte Carlo results for high cross-section dependence ($\rho = 0.4$): error in estimating structural vector $\mathbf{r}_1$

Metric:	Row norm, $\|\widehat{\mathbf{r}}_1 - \mathbf{r}_1\|_r$				Euclidean norm, $\|\widehat{\mathbf{r}}_1 - \mathbf{r}_1\|$			
	Median response $\widehat{\mathbf{r}}_1^{med}$							
n/s	**100**	**500**	**1000**	**5000**	**100**	**500**	**1000**	**5000**
5	0.49	0.49	0.49	0.49	0.62	0.62	0.61	0.61
10	0.56	0.56	0.56	0.56	1.34	1.34	1.34	1.34
20	0.61	0.60	0.60	0.60	2.41	2.40	2.40	2.40
50	0.70	0.68	0.68	0.68	4.64	4.64	4.64	4.64
100	0.76	0.74	0.74	0.74	7.29	7.29	7.29	7.29
	Response closest to median response $\widehat{\mathbf{r}}_1^{FP}$							
5	0.48	0.40	0.38	0.35	0.64	0.55	0.52	0.49
10	0.76	0.74	0.73	0.69	1.43	1.39	1.38	1.35
20	0.86	0.85	0.84	0.84	2.50	2.48	2.48	2.47
50	0.93	0.93	0.92	0.92	4.72	4.71	4.70	4.70
100	0.96	0.96	0.96	0.95	7.35	7.35	7.35	7.34
	Response corresponding to the scaled $med(\mathcal{S})$, $\widehat{\mathbf{r}}_1^{sc}$							
5	0.33	0.31	0.31	0.30	0.56	0.57	0.57	0.57
10	0.22	0.19	0.18	0.17	0.44	0.44	0.44	0.44
20	0.15	0.12	0.11	0.11	0.34	0.31	0.31	0.31
50	0.11	0.06	0.06	0.04	0.47	0.21	0.19	0.19
100	0.14	0.05	0.04	0.03	0.94	0.25	0.17	0.13

Figure 4: Output decline in response to one standard-deviation negative oil supply shock (four quarters cumulated in percent)

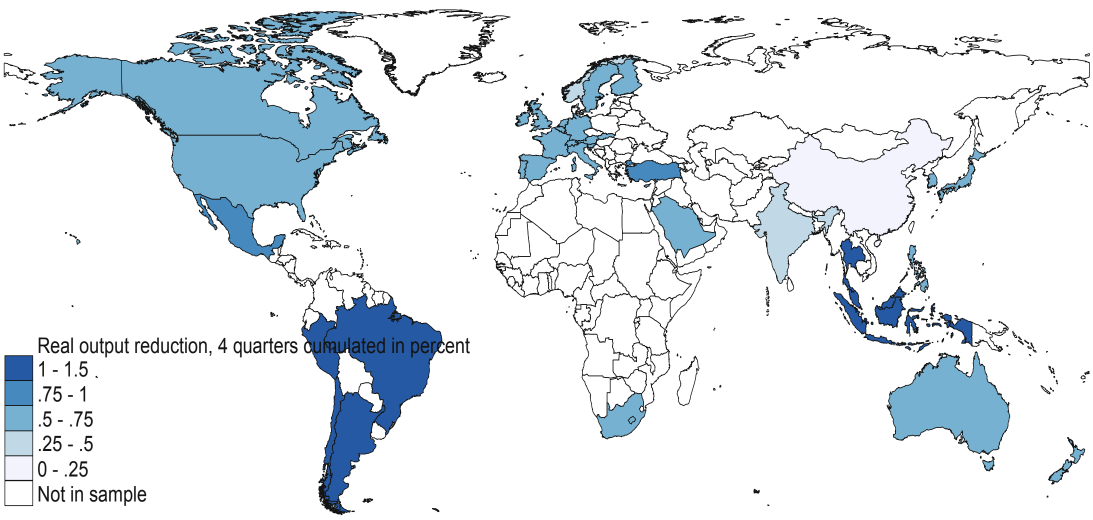
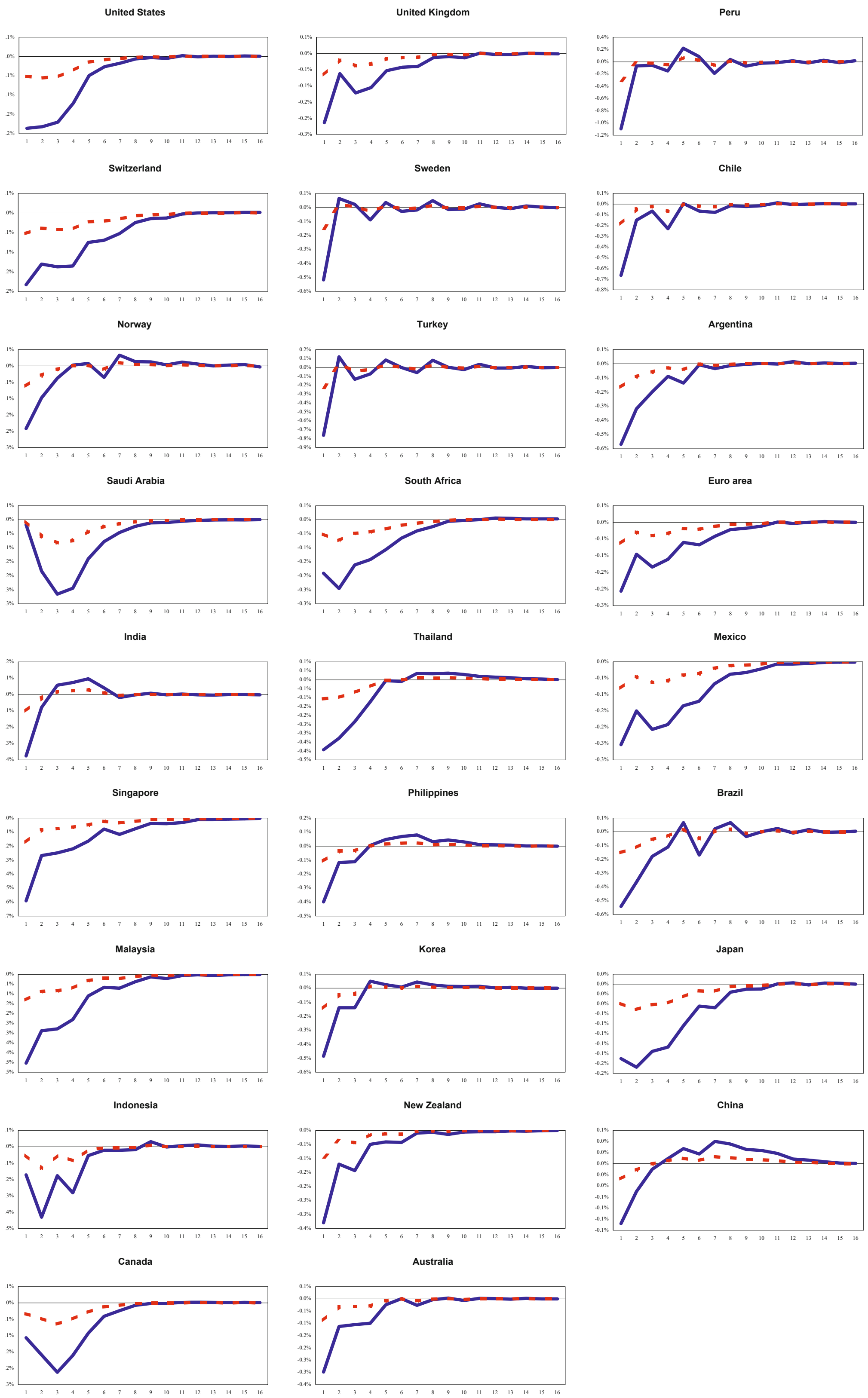


Figure 5: Impulse response of GDP



Note: Solid lines refer to the impulse response based on the scaled median approach. Dashed lines refer to the median impulse response computed as in Uhlig (2005).

Figure 6: Appreciation of real exchange rate (four quarters cumulated in percent)

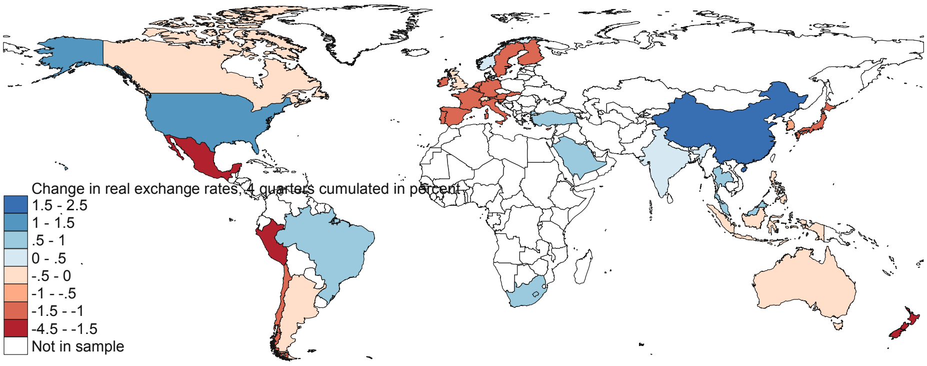
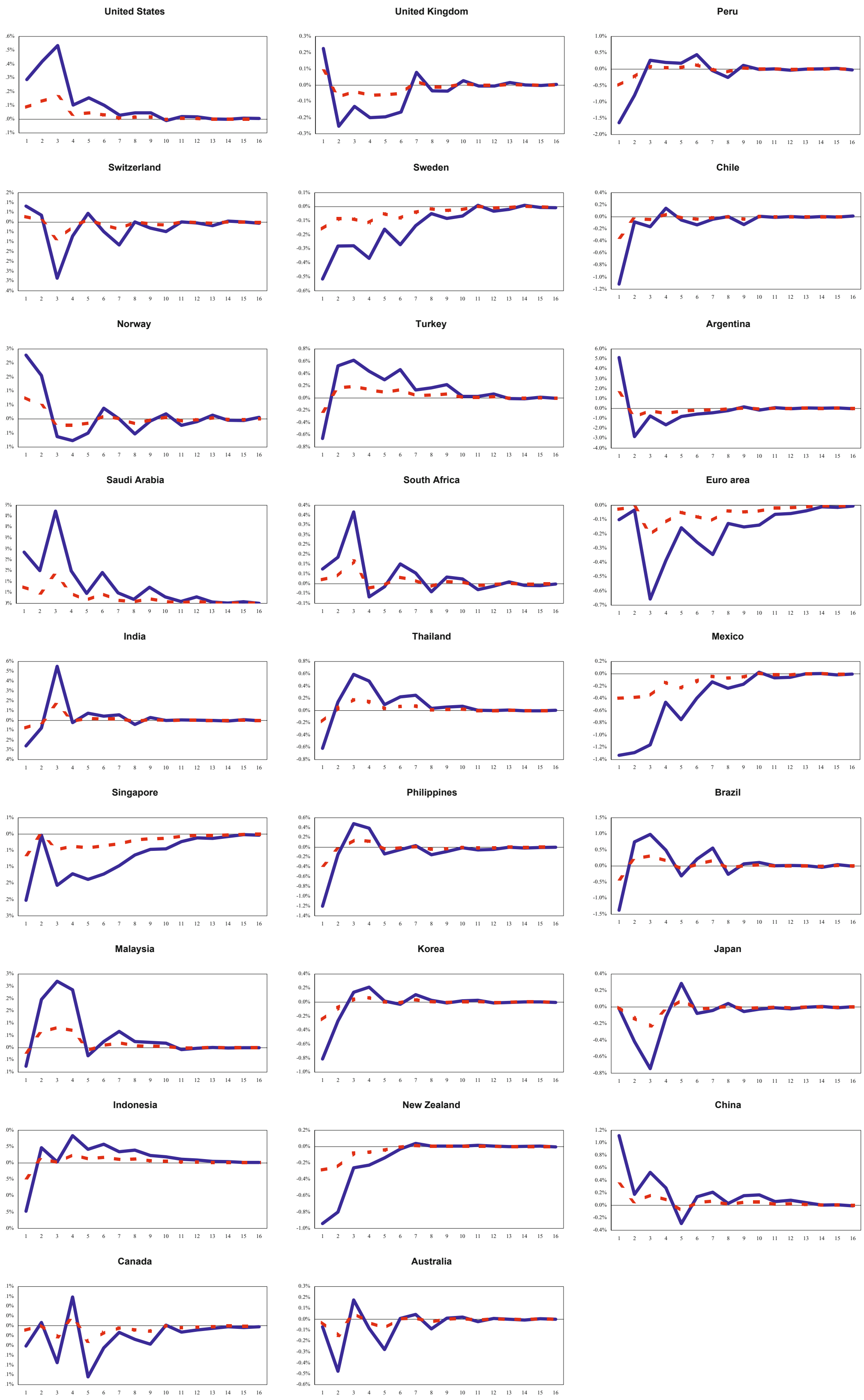
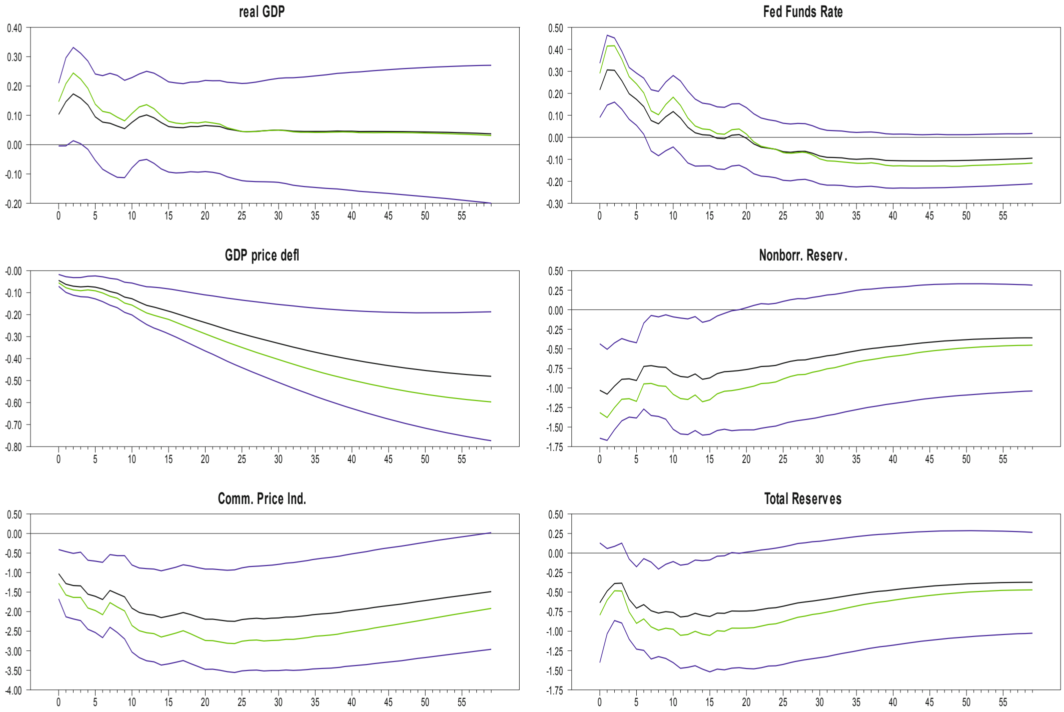


Figure 7: Impulse response of real effective exchange rates



Note: Solid lines refer to the impulse response based on the scaled median approach. Dashed lines refer to the median impulse response computed as in Uhlig (2005).

Figure 8: Impulse response to Uhlig's (2005) contractionary monetary policy shock



Note: Black (and blue) lines refer to the median (and 16th as well as 84th percentile) of impulse responses as in Uhlig (2005); green lines refer to median impulse response of scaled median $\boldsymbol{\alpha}$ as proposed in this paper.