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RESEARCH MEMORANDUM

THE SHAPLEY-ENTRANCE GAME

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THE SHAPLEY-ENTRANCE GAME

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Abstract

In this paper we study a game of timing that is based on the interpretation of the Shapley value as the players enter one by one. It is shown that in this game there exists a unique Nash Equilibrium. This Nash Equilibrium is explicitly given.

KEYWORDS: Shapley value, Nash Equilibrium, Games of timing.

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1 Introduction.

This paper investigates a non-cooperative game that is based on the entrance-interpretation of the Shapley value.

One story that is told in textbooks to introduce the Shapley value is as follows. The players of the grand coalition enter one by one. When a player enters he will get his marginal contribution to the coalition that is already present. If each order of entrance has the same probability then the Shapley value is the expected payoff to the players.

In this paper we study whether the assumption that each order of entrance is equally likely is justified if players choose their entry time strategically. To that end we study the (non-cooperative) Shapley entrance-game that we introduce in Section 2. In this game each player independently selects an entrance time. The payoff of a player is his marginal contribution with regard to the cooperative game (N, v) , discounted over time. It is assumed that the discount parameter is the same for each player. In this paper only the two player model is analyzed. This model defines a game of timing which is called "silent" [see Karlin (1959)]. [1]

In Section 3 a Nash Equilibrium in mixed strategies is explicitly given. It is shown that this Nash Equilibrium is unique. The payoff of the Nash Equilibrium is independent of the discounting parameter and is not efficient with regard to the cooperative game (N, v). In particular, the Nash equilibrium payoff does not coincide with the Shapley-value.

2 The Shapley-Entrance Game

In this section the non-cooperative Shapley-Entrance Game (SEG)is defined. Let (N, v) be a two person cooperative game such that v (1)= α , v (2)= β and v (N)=1 and $\alpha > 0, \beta > 0$ and $\alpha + \beta < 1$. Taking (N, v) fixed, the SEG is defined as follows. The set of pure strategies of player i is $S_i = [0, \infty)$ where $t_i \in S_i$ represents the time player i will enter. If (t_1, t_2) is a strategy pair, then the payoff function of player 1 is

[1] Another type of games of timing are the "noisy" games. In such game the payoff of the follower depends only on when the other player moves. An example of such a game is the War of Attrition [see Hendricks et al. (1988)]

$$\pi_1(t_1,t_2) = \begin{cases} \alpha\delta^{t_1} & t_1 < t_2 \\ \{p\alpha + (1-p)(1-\beta)\}\delta^{t_1} & t_1 = t_2 \\ (1-\beta)\delta^{t_1} & t_1 > t_2 \end{cases}$$

and of player 2

$$\pi_2(t_1,t_2) = \begin{cases} (1-\alpha)\delta^{t_2} & t_1 < t_2 \\ \{p(1-\alpha) + (1-p)\beta\}\delta^{t_2} & t_1 = t_2 \\ \beta\delta^{t_2} & t_1 > t_2 \end{cases}$$

Hence each player receives his discounted marginal contribution, where $p \in [0,1]$ is the probability that player 1 has to enter the room first when the arrival times of both players are identical. Further we suppose that the discounting parameter satisfies $0 < \delta < 1$.

A mixed strategy of player i is a probability distribution. Let X_i be the (arrival-)time of player i with probability distribution function $F_i(x) = P[X_i \leq x]$. For convenience we define $q_i(x) = P[X_i = x]$. The payoff of player 1 playing x when player 2 plays the mixed strategy F_2 is given by

$$\pi_1(x,F_2) = \int_{[0,\infty)} \pi_1(x,y)\,dF_2(y)$$

$$= \int_{x<y} \alpha\delta^x\,dF_2(y) \;+\; \int_{x>y} (1-\beta)\delta^x\,dF_2(y)$$

$$+\;\delta^x\{p\alpha + (1-p)(1-\beta)\}q_2(x)$$

$$= \delta^x\{\alpha + (1-\alpha-\beta)F_2(x)\} - \delta^x p(1-\alpha-\beta)q_2(x) \tag{1}$$

This implies that the payoff of player 1 playing F_1 when player 2 plays F_2 is given by

$$\pi_1(F_1,F_2)$$

$$= \int_{[0,\infty)} \delta^x\{\alpha + (1-\alpha-\beta)F_2(x)\} - \delta^x p(1-\alpha-\beta)q_2(x)\,dF_1(x)$$

$$= \int_{[0,\infty)} \delta^x\{\alpha + (1-\alpha-\beta)F_2(x)\}\,dF_1(x)$$

$$-\sum \delta^x p(1-\alpha-\beta)q_2(x)q_1(x) \tag{2}$$

Analogously we find for player 2

$$\pi_2(F_1, y) = \delta^y\{\beta + (1-\alpha-\beta)F_1(y)\} - \delta^y(1-p)(1-\alpha-\beta)q_1(y) \quad (3)$$

and

$$\begin{aligned}&\pi_2(F_1, F_2)\\ &= \int_{[0,\infty)} \delta^y\{\beta + (1-\alpha-\beta)F_1(y)\}\, dF_2(y)\\ &-\sum \delta^y(1-p)(1-\alpha-\beta)q_1(y)q_2(y) \qquad (4)\end{aligned}$$

In this paper we only present the results for player 1. The results for player 2 are obtained if the set of ordered symbols $(\alpha, \beta, 1, 2, x, y, p)$ is replaced by $(\beta, \alpha, 2, 1, y, x, 1-p)$.

Assume there exists a NE (P_1, P_2) in mixed strategies with payoff (η_1, η_2). The first lemma shows a general property of a NE. When player 1 has a masspoint in t and player 2 plays the equilibrium strategy P_2 then player 1 is indifferent between playing the pure strategy t or his equilibrium strategy P_1.

Lemma 1 *If (P_1, P_2) is a NE with $q_1(t) > 0$ then $\pi_1(P_1, P_2) = \pi_1(t, P_2)$.*

PROOF: Suppose player 1 has a masspoint in t and $\pi_1(t, P_2) < \pi_1(P_1, P_2)$. From the definition of a NE follows that $\pi_1(t^*, P_2) \le \pi_1(P_1, P_2) \;\; \forall t^*$. This implies that $\eta_1 = \pi_1(P_1, P_2) \le (1-q_1(t))\eta_1 + q_1(t)\pi_1(t, P_2) < \eta_1$. Contradiction. □

3 NE of Shapley-entrance game

In this section we first discuss the existence of a NE and secondly we prove uniqueness of the NE.

It is obvious that the SEG has no pure NE. Suppose there exists a pure NE (t_1, t_2) . When $t_i < t_j$ then player j is better of with playing $t^* \in (t_i, t_j)$. This means that $t_1 = t_2$. But then at least one player has the incentive to arrive later. This proves:

Proposition 1 *The Shapley-entrance game has no pure NE.*

From Proposition 1 it follows that a NE, if it exists, is one in mixed strategies. Before proving our theorems we derive some conditions upon NE (P_1, P_2) with payoff (η_1, η_2) .

The second lemma gives an inequality that follows immediately from the definition of a NE.

Lemma 2 *Let (P_1, P_2) be a NE with payoff (η_1, η_2). Then*

$$P_1(x) - q_1(x)(1-p) \leq \frac{\eta_2 - \beta\delta^x}{(1-\alpha-\beta)\delta^x} \qquad \forall\, x \in [0, \infty)$$

PROOF: Follows by substitution of $\eta_2 = \pi_2(P_1, P_2) \geq \pi_2(P_1, x)$ in (3) □

In the next lemma we show that it is not possible that both players have a masspoint at time t . Intuitively this is clear. If player 1 enters the room at time t it is better for player 2 to wait a little longer.

Lemma 3 *In a NE holds $q_1(t).q_2(t) = 0 \quad \forall t \in [0, \infty)$.*

PROOF: Suppose that both $q_1(t) > 0$ and $q_2(t) > 0$.
If $p > 0$ then there exists an $\epsilon > 0$ [P_2 is a non-decreasing function] such that $q_2(t+\epsilon) = 0$ and $\delta^t\{\alpha + (1-\alpha-\beta)P_2(t) - p(1-\alpha-\beta)q_2(t)\} < \delta^{t+\epsilon}(\alpha + (1-\alpha-\beta)P_2(t+\epsilon)) = \pi_1(t+\epsilon, P_2)$. But then lemma 1 implies $\pi_1(P_1, P_2) = \pi_1(t, P_2) < \pi_1(t+\epsilon, P_2)$. Contradiction. If $p = 0$ the proof is similar. □

The following lemma shows us that the supports of the equilibrium strategies are defined on an interval with the same finite upperbound.

Lemma 4 *The upper endpoints of the supports of the strategies of a NE in the SEG coincide.*

PROOF: Let $c \in \mathbf{R}$ be such that $\delta^c(1-\beta) = \eta_1$. Then $P_1(c) = 1$.
Let $d \in \mathbf{R}$ such that $\delta^d(1-\alpha) = \eta_2$. Then $P_2(d) = 1$.
Suppose that $c > d$ then player 1 has an incentive to play a strategy with support (d, c) because on the interval (d, c) he gets $\delta^x(1-\beta), x \in (d, c)$.

This yields a greater payoff than the equilibrium payoff. Contradiction. So $c \leq d$. On account of symmetry holds $d \leq c$. Analogously can be shown that $c = \inf\{x \mid P_i(x) = 1\}$. □

Lemma 4 immediately implies

Corollary 1 *In the SEG the payoff* (η_1, η_2) *of a NE satisfies*

$$\frac{\eta_1}{\eta_2} = \frac{1-\beta}{1-\alpha}$$

The payoff of player 1 is bounded above by $1-\beta$. This payoff is the supremum of player 1's payoff. This can be shown by choosing the pure strategy $t = 0$ for player 2 and the strategy $t = \epsilon$ for some small $\epsilon > 0$ for player 1. Analogously $1-\alpha$ is the supremum of player 2's payoff. The corollary shows now that the payoff in any equilibrium is equal to the proportion of these upperbounds.

In the next theorem we show the existence of a NE in the SEG. We restrict attention to the case $\alpha \leq \beta$. The case $\beta \leq \alpha$ follows by a similar arguments.

Theorem 1 *If* $\alpha \leq \beta$ *then the Shapley-entrance game has a Nash Equilibrium given by the strategy- pair :*

$$P_1^*(x) = \begin{cases} \frac{\beta - \beta\delta^x}{(1-\alpha-\beta)\delta^x} & 0 \leq x \leq c \\ 1 & x > c \end{cases}$$

and

$$P_2^*(x) = \begin{cases} \frac{\beta(1-\beta) - \alpha(1-\alpha)\delta^x}{(1-\alpha)(1-\alpha-\beta)\delta^x} & 0 \leq x \leq c \\ 1 & x > c \end{cases}$$

with c such that $\delta^c(1-\alpha) = \beta$. *Moreover, the payoff is given by* $\pi(P_1^*, P_2^*) = \frac{\beta}{1-\alpha}(1-\beta, 1-\alpha)$.

PROOF: Note that P_1^* and P_2^* are distribution functions.
It suffices to show that
$\pi_1(x, P_2^*) = \pi_1(P_1^*, P_2^*) \;\; \forall x \in (0, c]$,
$\pi_1(x, P_2^*) < \pi_1(P_1^*, P_2^*) \;\; \forall x \in \{0\} \cup (c, \infty)$ and
$\pi_2(P_1^*, x) = \pi_2(P_1^*, P_2^*) \;\; \forall x \in [0, c]$; $\pi_2(P_1^*, x) < \pi_2(P_1^*, P_2^*) \;\; \forall x \in (c, \infty)$
The proof of these inequalities follows by straightforward calculations (use (1) and (3) and the definition of c). □

We conclude that when the discounting parameter δ become larger the supports of the equilibrium strategies of the last theorem becomes larger. Surprisingly the equilibrium payoff is independent of the discounting parameter. So , when the players play accordingly the above NE, the discounting parameter δ can only affect the duration of the game, but not the payoff of the players. In other words this means that both players become more patient when the discount factor becomes larger.

The question arises if there are other NE's for the SEG. The next theorem shows that all NE's must have the same payoff as the NE in Theorem 1. Thus we have uniqueness in payoffs.

Theorem 2 *In the Shapley-entrance game there does not exist a NE (G_1, G_2) with payoff $(\mu_1, \mu_2) \neq (\eta_1, \eta_2)$, where (η_1, η_2) is the payoff of the NE given in Theorem 1.*

PROOF: Assume there exists an equilibrium (G_1, G_2) with payoff (μ_1, μ_2) such that $(\mu_1, \mu_2) \neq (\eta_1, \eta_2)$.
Since (G_1, G_2) is an equilibrium $\frac{\mu_1}{\mu_2} = \frac{1-\beta}{1-\alpha}$ (Corollary 1). Hence $(\mu_1, \mu_2) < (\eta_1, \eta_2)$ or $(\mu_1, \mu_2) > (\eta_1, \eta_2)$ [componentwise ordering].

The first case, $(\mu_1, \mu_2) < (\eta_1, \eta_2)$, is not possible because of Theorem 1 and since player two can guarantee himself β by switching to the pure strategy $t = 0$.(Note if $\beta < \alpha$ then player one is better of by switching to the pure strategy $t = 0$).

The case $(\mu_1, \mu_2) > (\eta_1, \eta_2)$ needs more explanation.
Lemma 4 implies that there exists a $c^* > 0$ such that $\delta^{c^*}(1 - \beta) = \mu_1$ and $\delta^{c^*}(1 - \alpha) = \mu_2$ and $G_1(c^*) = G_2(c^*) = 1$. Now we can show by straightfor-

ward calculation [2] that the following inequalities hold:

$$[-\mu_1 + \delta^t\{\alpha + (1-\alpha-\beta)G_2(t)\}]q_1(t) \geq 0 \quad \forall t \in [0, c^*] \tag{5}$$

$$[-\mu_2 + \delta^t\{\beta + (1-\alpha-\beta)G_1(t)\}]q_2(t) \geq 0 \quad \forall t \in [0, c^*] \tag{6}$$

Let $r^* = \inf\{x \in [0, c^*] \mid G_1(x) > 0\}$ and $s^* = \inf\{x \in [0, c^*] \mid G_2(x) > 0\}$.
(i) Suppose $r^* < s^*$.
From (5) and $(\mu_1, \mu_2) > (\eta_1, \eta_2) \geq (\alpha, \beta)$ follows that $q_1(r^*) = 0$. This implies that G_1 is continuous in r^*.
For any $\epsilon > 0$ we define $z_\epsilon^* = \inf\{x \in [0, c^*] \mid G_1(x) > \epsilon\}$. Let ϵ^* be such that $z_{\epsilon^*}^* < s^*$. Then analogous to the calculation of (5),(Note that $G_2(z_{\epsilon^*}^*) = 0$), follows:

$$\mu_1 = \int_{[r^*, c^*]} \delta^x\{\alpha + (1-\alpha-\beta)G_2(x)\}\, dG_1(x)$$

$$\leq [\delta^{r^*}\alpha - \mu_1]G_1(z_{\epsilon^*}^*) + \mu_1$$

This last inequality shows that $G_1(z_{\epsilon^*}^*) = 0$. This contradicts the definition of r^*, since $z_{\epsilon^*}^* > r^*$.
(ii) Suppose $r^* > s^*$.
Analogous arguments as used in (i) give a contradiction to this assumption.
(iii) Suppose $r^* = s^*$.
From Lemma 3 , (5) , (6) , and $(\mu_1, \mu_2) > (\eta_1, \eta_2) \geq (\alpha, \beta)$ follows $q_1(r^*) = 0$ and $q_2(r^*) = 0$. So G_1 and G_2 are continuous in $x = r^*$. So for any $\epsilon^* > 0$ there exists a $\delta^* > 0$ such that $G_2(x) < \epsilon^*\ \forall x \in [r^*, r^* + \delta^*]$. Then holds

$$\mu_1 \leq \int_{[r^*, r^*+\delta^*]} \delta^x\{\alpha + (1-\alpha-\beta)G_2(x)\}\, dG_1(x) + \{1 - G_1(r^* + \delta^*)\}\mu_1$$

$$\leq [\delta^{r^*}\{\alpha + (1-\alpha-\beta)\epsilon^*\} - \mu_1]G_1(r^* + \delta^*) + \mu_1$$

So for sufficient small ϵ^* holds $G_1(r^* + \delta^*) = 0$. This contradict the definition of r^* .

Hence (i), (ii) and (iii) do not hold by the assumption that $(\mu_1, \mu_2) > (\eta_1, \eta_2)$.

[2]for equation (5) use (2), split the integration-interval in a point and the remaining interval, then use lemma 2

This completes the proof. $\square$

Theorem 2 enables us to prove the uniqueness of the NE of Theorem 1. Again we restrict our attention to the case $\alpha \leq \beta$.

Theorem 3 *If $\alpha \leq \beta$ then (P_1^*, P_2^*) is the unique NE of the Shapley-entrance game.*

PROOF: Suppose that (G_1, G_2) is a NE. The proof is given in four steps:

(i) We first prove: $G_i \leq P_i^*$.
From Theorem 2 it follows that $\pi(G_1, G_2) = (\frac{\beta(1-\beta)}{1-\alpha}, \beta)$. Then Lemma 4 implies that the right endpoint of the supports of P_i^* and G_i coincide. Hence $P_i^*(x) = G_i(x) \ \ \forall x \in (c, \infty)$ with c defined as in Lemma 4.
From Lemma 2 it follows that

$$G_1(x) - q_1(x)(1-p) \leq P_1^*(x) \quad \forall x \in [0, c] \tag{7}$$

Let $A := \{x \mid q_1(x) > 0\}$. Then (7) implies that

$$G_1(x) \leq P_1^*(x) \quad \forall x \in [0, c] \backslash A \tag{8}$$

Suppose there exists a $x^* \in [0, c)$ such that $G_1(x^*) > P_1^*(x^*)$. Hence there exists an $\epsilon > 0$ such that

$$G_1(x^*) = P_1^*(x^*) + \epsilon \tag{9}$$

Since P_1^* is continuous , there exists a $\delta > 0$ such that $\forall x \in [x^*, x^* + \delta)$ holds

$$P_1^*(x) - P_1^*(x^*) < \frac{1}{2}\epsilon \tag{10}$$

Let $z \in (x^*, x^* + \delta) \backslash A$ then

$$G_1(z) \leq P_1^*(z) < P_1^*(x^*) + \frac{1}{2}\epsilon < G_1(x^*)$$

Note that the first inequality holds by (8), the second by (10), the third by (9). Since G_1 is non-decreasing we have a contradition. Analogously we can prove that $G_2 \leq P_2^*$.

(ii) We show here: $G_i = P_i^*$ *a.e.* G_j , with $i \neq j$.
Easy calculations show that

$$\int_{[0,c]} \delta^x\{\beta + (1-\alpha-\beta)G_1(x)\}\, dG_2(x)$$

$$= \int_{[0,c]} \delta^x\{\beta + (1-\alpha-\beta)P_1^*(x)\}\, dG_2(x)$$

This integral-equality and (i) yields (ii) for $i = 1$ and $j = 2$. When $i = 2$ and $j = 1$ the prove is analogously (Note that $G_1(0) = 0$).

(iii) Now we show: G_1 is continuous on $[0, c]$ and G_2 is continuous on $(0, c]$. Suppose G_1 is not continuous. Then there exists a $z \in (0, c]$ such that $q_1(z) = \epsilon > 0$. Since P_1^* is continuous on $[0, c]$, there exists a $\delta > 0$ such that $\forall x \in (z - \delta, z]$ holds

$$P_1^*(z) - P_1^*(x) < \frac{1}{2}\epsilon$$

Then for all $x \in (z - \delta, z)$ holds

$$G_1(x) \le G_1(z^-) = G_1(z) - q_1(z) = G_1(z) - \epsilon$$

$$\le P_1^*(z) - \epsilon < P_1^*(z) - \frac{1}{2}\epsilon = P_1^*(x)$$

From (ii) and the last inequality follows that $G_2(\{(z-\delta, z)\}) = 0$. Hence G_2 is constant on $(z-\delta, z)$. Since $q_1(z) > 0$ Lemma 3 implies that $q_2(z) = 0$. Hence, G_2 is constant on $(z - \delta, z]$. This implies that $G_2(x) < P_2^*(x)\ \forall x \in (z - \delta, z]$. From (ii) follows that $G_1(\{(z - \delta, z]\}) = 0$. This implies that $G_1(\{z\}) = 0$. Since $G_1(\{z\}) = q_1(z)$ we have a contradiction.
Analogously we can show that G_2 is continuous on $(0, c]$.

(iv) Finally we prove: $G_i = P_i^*$.
Suppose there exists a $z \in (0, c]$ such that $G_1(z) < P_1^*(z)$. Since G_1 is continuous there exists an interval $[a, b]$ that contains z and satisfies $G_1(a) = P_1^*(a), G_1(b) = P_1^*(b)$ and $G_1(x) < P_1^*(x)\ \forall x \in (a, b)$. Then (ii) implies that $G_2(\{(a, b)\}) = 0$. Since G_2 is continuous on $(0, c]$ it follows that G_2 is constant on (a,b] and $G_2(x) < P_2^*(x)\ \forall x \in (a, b]$. Hence $G_1(\{(a, b]\}) = 0$. This implies that G_1 is constant on (a,b] and $G_1(x) < P_1^*(x)\ \forall x \in (a, b]$. Contradiction. Analogously is shown that $G_2 = P_2^*$. □

The equilibrium payoff (η_1, η_2) is not efficient in view of the cooperative game. So the Shapley-entrance game does not yield the Shapley-value as the equilibrium outcome. This means that when players can choose their entrance time strategically, they cannot choose an equilibrium strategy that supports the Shapley value.

4 Conclusion.

In this paper we analyzed the Shapley-entrance game, a game based on the entrance-interpretation of the Shapley value. We explicitly gave a NE and proved that it is unique. Further we showed that the equilibrium payoff was independent of the discounting parameter and inefficient with regard to the cooperative game (N, v). This result did not support the Shapley value.

Note that the Shapley value is implemented by the SEG if $\alpha + \beta = 1$.

Proposition 2 *If $\alpha + \beta = 1$ then $(0,0)$ is the unique NE of the Shapley-entrance game. The payoff of the NE equals the Shapley value.*

PROOF: From (1) and (3) follows that $\pi_1(x, F_2) = \alpha\delta^x$ and $\pi_2(F_1, x) = \beta\delta^x$. Easy calculations show that $(0,0)$ is the unique NE with payoff $(\alpha, \beta) = \phi(v)$. $\square$

Since the Shapley value is independent of a discounting parameter, the reader may be curious about the SEG with $\delta = 1$. This curiosity is provided in the next proposition.

Proposition 3 *If $\delta = 1$ then there does not exist a NE in the Shapley-entrance game.*

PROOF: Suppose player two plays F_2. Then player one can guarantee himself a payoff of $1 - \beta - \epsilon$ by playing the pure strategy x such that $F_2(x) = 1 - \epsilon$. Analogously player two obtains $1 - \alpha - \epsilon$. This is a contradiction since $\epsilon > 0$ is arbitrary. $\square$

Hence the SEG with no discounting does not implement the Shapley-value.

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