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PRELIMINARY VERSION

**THE REACTION OF THE FIRM ON GOVERNMENTAL POLICY: A GAME-THEORETICAL APPROACH**

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**FEW 250**

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# THE REACTION OF THE FIRM ON GOVERNMENTAL POLICY: A GAME-THEORETICAL APPROACH

by R.H.J.M. Gradus *)

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Abstract:
In this paper we describe the reaction of the firm on governmental policy. We present a model in which the government influences the firm by announcing a certain tax rate and the firms (represented by one) decide about whether to invest its money or pay out dividend. We model the interactions between the government and the firm as an open-loop game, in which different solution concepts (Pareto, Stackelberg and Nash) are possible.

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# The Reaction of the Firm on Governmental Policy: a Game-theoretical Approach

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## 1. Introduction:

A crucial question in economic theory is: "what exactly is the relationship between macroeconomic variables such as employment, growth of production etc., and the micro economic variables such as profit, investment, number of workers, etc.". Formulated more specifically in terms of economic policy the following question could be posed: how can the government by tax policy, wage regulations or monetary measures influence the decisions of the enterprise in such a way that the objectives of the national economy are achieved. A first impulse in carrying out this kind of research was given by Verheyen [10]. In this paper Verheyen described firm behaviour in national economics by means of an optimal control model and analyzed consequences of actions of governmental policy. Verheyen studied two kinds of economic systems: a labour-managed and a market economy. In the first one the government tries to influence the economy by means of monetary policy and in the second one by means of wage policy. In contrast to that paper, we deal with the above problem by using the technique of differential games and give the government the possibility to influence the economy by tax policy.

In section 2 we introduce a simple model in which the government maximizes its consumption and the firms (represented by one) want to pay out a maximum of dividend to their shareholders. In spite of the simplicity of the model, we are able to study some main issues of governmental policy. Section 3 contains a brief discussion of some conceptual problems that arise by using differential games, while in section 4 the open-loop solutions and their economic interpretation are given. In section 5 we discuss what will happen if we extend the model. Finally, in section 6 we make some remarks and suggestions for future research.

## 2. The model:

### 2.1 The firms:

We assume that the firm behaves as if it maximizes the shareholder's value of the firm. This value consists of the sum of the dividend stream over the planning period. Assuming a zero discount rate yields:

$$\max. \int_0^T D(t)dt, \qquad (1)$$

in which t = time

T = planning period

D(t) = dividend

Assume that the amount of capital goods could only be raised by investment and there is no depreciation:

$$\dot{K}(t) = I(t), \qquad (2)$$

in which K(t) = capital good stock

I(t) = investment

We also suppose that profit is a linear function of capital good stock:

$$O(t) = qK(t), \qquad (3)$$

in which O(t): profit (before tax payment)

q : capital productivity

Assuming that profit after taxation could be used for investment or to pay out dividend, we get the next relation:

$$O(t) - TX(t) = D(t) + I(t) \qquad (4),$$

in which TX(t) = tax payment

Furthermore, investment and dividend must be greater than zero:

$$D(t) \geq 0 \qquad (5)$$

$$I(t) \geq 0 \qquad (6)$$

### 2.2 The government:

Also for the government we make some extremely simple assumpties: all the tax payments received will immediately be spent on government consumption (,which is not productive). We may think that the government will spend its money on building bridges and houses, hospital care and military forces. The government is not able to spend more than it receives (i.e. no budgetary deficit):

$$TX(t) = G(t) \qquad (7),$$

in which $G(t)$ = government spendings

As objective for the government we take:

$$\max. \int_0^T U(G(t))dt, \tag{8}$$

where $U(G(t))$ is the utility function for the government, which is defined in terms of government consumption. In this section we assume that:

$$U(G(t)):=G(t), \tag{9}$$

so the government has a linear utility function. Of course, other utility functions are possible, but we have taken the linear one for simplicity. In that case (8) becomes:

$$\max. \int_0^T G(t)dt \tag{10}$$

Furthermore we assume that the tax payments are restricted by:

$$\tau_1 O(t) \leq TX(t) \leq \tau_2 O(t), \tag{11}$$

where $\tau_1$ and $\tau_2$ are determined by social limits.

2.3 The total model:

We can easily rewrite the model as follows

-government

$$\max_{u_1(t)} \int_0^T qK(t)u_1(t)dt \qquad , \tau_1 \leq u_1 \leq \tau_2 \tag{12},$$

where $u_1(t)$:=tax rate=$\frac{TX(t)}{O(t)}$ which can be controlled by the government.

-firm

$$\max_{u_2(t)} \int_0^T qK(t)(1-u_1(t))(1-u_2(t))dt \qquad , 0 \leq u_2 \leq 1 \tag{13},$$

where $u_2(t)$:=investment rate=$\frac{I(t)}{O(t)-T(t)}$ which can be controlled by the firm.

-state equation

$$\dot{K}(t) = qK(t)(1-u_1(t))u_2(t) \tag{14}$$

In this model there are two control variables $u_1(t)$ and $u_2(t)$, one state variable $K(t)$ and two players which can control one variable. In this way we have derived the same mathematical model as Lancaster [2] , but we use it to solve a total other economic problem.

In our model the government has to deal with the following interesting dilemma: the government wishing to maximize its tax receipts should choose the high rate, but if it chooses the high rate, then the firm has less to invest and the future tax income may decrease. If it chooses the low rate it has less to spend at this moment, but perhaps more in the future.

## 3. A differential game:

In the problem we have described above, the government and the firm do not have the same interests. But both players have a direct influence on the state variable. We can say it is a dynamic game. In (12)-(14) we have described the objectives, the dynamics and the admissible strategies of the game. If we want to solve this game we have to make some assumptions about the information structure (open-loop, feedback, closed-loop) and the solution concept (Nash, Stackelberg and Pareto). It is also important in our game that it has a non-zero property (the sum of all 'players'criteria is not constant). Another important question is which solution concept we choose. The Nash solution provides a reasonable noncooperative solution for this game when neither government nor firm dominates the decision process. But if one of the players is in a position where he can impose his strategy on the other player, then the relevant concept may be Stackelberg. There is also a possibility that the government and firm work together and cooperate. In that case we can use the Pareto solution concept. For the information structure we can choose between several possibilities: the following three are most commonly used:

$$u_i = u_i(K, K_0, t) \qquad \text{(closed-loop no memory)} \qquad (15)$$

$$u_i = u_i(K, t) \qquad \text{(feedback)} \qquad (16)$$

$$u_i = u_i(K_0, t) \qquad \text{(open-loop)} \qquad (17)$$

For the model we have specified in (12)-(14) we refer to a series of papers, which has been written about the Lancastermodel. Lancaster derived the open-loop Nash-solution and concerning the Stackelberg open-loop problem, solutions have been given by Pohjola [7]. Recently Basar et al. [2]

have developed the Feedback-Nash and -Stackelberg solutions. Another possibility is to introduce and model the threats and bargaining between government and firm as in Pohjola [5].

## 4. The open-loop solution:

In this paper we only deal with the open-loop case. This is the case in which each player has to stick to predetermined plan. In practice this is only realistic, when there is a binding contract between the government and the firm. In the Stackelberg game we assume that the government is the leader.

table 1: The Nash-solution if $\tau_2 \geq \frac{1}{2}$

| | $t \in [0,\bar{t})$ | $t \in [\bar{t},T]$ |
|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_2$ |
| $u_2(t)$ | 1 | 0 |
| $K(t)$ | $K_0 e^{q(1-\tau_1)t}$ | $K^*$ |
| $G(t)$ | $q\tau_1 K(t)$ | $q\tau_2 K^*$ |
| $D(t)$ | 0 | $q(1-\tau_2)K^*$ |
| $I(t)$ | $q(1-\tau_1)K(t)$ | 0 |

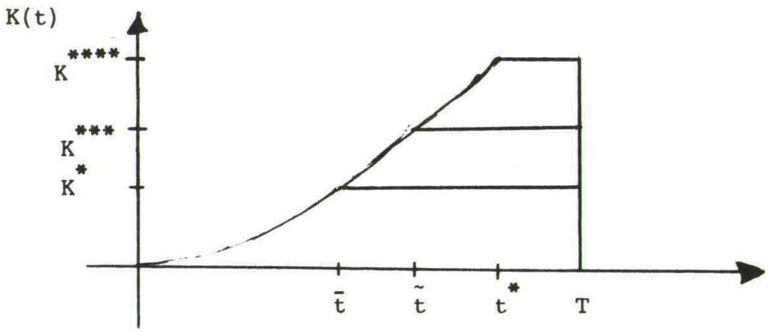


Figure 1: Capital good stock by different solutions concepts $(\tau_2 > \frac{1}{2})$

table 2: The Nash-solution if $\tau_2 < \frac{1}{2}$

| | $t \in [0,\hat{t})$ | $t \in [\hat{t},\bar{t})$ | $t \in [\bar{t},T]$ |
|---|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_2$ | $\tau_2$ |
| $u_2(t)$ | 1 | 1 | 0 |
| $K(t)$ | $K_0 e^{q(1-\tau_1)t}$ | $K(\hat{t})e^{q(1-\tau_2)(t-\hat{t})}$ | $K^{**}$ |
| $G(t)$ | $q\tau_1 K(t)$ | $q\tau_2 K(t)$ | $q\tau_2 K^{**}$ |
| $D(t)$ | 0 | 0 | $q(1-\tau_2)K^{**}$ |
| $I(t)$ | $q(1-\tau_1)K(t)$ | $q(1-\tau_2)K(t)$ | 0 |

table 3: The Stackelberg-solution if $\tau_2 > \frac{1}{2}$

| | $t \in [0,\tilde{t})$ | $t \in [\tilde{t},\tilde{\tilde{t}})$ | $t \in [\tilde{\tilde{t}},T]$ |
|---|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_1$ | $\tau_2$ |
| $u_2(t)$ | 1 | 0 | 0 |
| $K(t)$ | $K_0 e^{q(1-\tau_1)t}$ | $K^{***}$ | $K^{***}$ |
| $G(t)$ | $q\tau_1 K(t)$ | $q\tau_1 K^{***}$ | $q\tau_2 K^{***}$ |
| $D(t)$ | 0 | $q(1-\tau_1)K^{***}$ | $q(1-\tau_2)K^{***}$ |
| $I(t)$ | $q(1-\tau_1)K(t)$ | 0 | 0 |

table 4: The Pareto-solution

| | $t \in [0,t^*)$ | $t \in [t^*,T]$ |
|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau \in [\tau_1,\tau_2]$ |
| $u_2(t)$ | 1 | 0 |
| $K(t)$ | $K_0 e^{q(1-\tau_1)t}$ | $K^{****}$ |
| $G(t)$ | $q\tau_1 K(t)$ | $q\tau K^{****}$ |
| $D(t)$ | 0 | $q(1-\tau)K^{****}$ |
| $I(t)$ | $q(1-\tau_1)K(t)$ | 0 |

, where $\bar{t} = T - \frac{1}{q(1-\tau_2)}$, (18)

$$\hat{t} = \bar{t} + \frac{\ln(2\tau_2)}{q(1-\tau_2)}, \quad (19)$$

$$\tilde{\tilde{t}} = \min\{T, T - \frac{1-2\tau_1}{q(\tau_2-\tau_1)}\}, \quad (20)$$

$$\tilde{t} = \min\{T - \frac{2}{q}, T - \frac{1}{q(1-\tau_1)}\}, \quad (21)$$

$$t^* = T - \frac{1}{q}, \quad (22)$$

$$K^* = K_0 e^{q(1-\tau_1)\bar{t}}, \quad (23)$$

$$K^{**} = K_0 e^{q(1-\tau_1)\hat{t}} e^{q(1-\tau_1)(\bar{t}-\hat{t})}, \quad (24)$$

$$K^{***} = K_0 e^{q(1-\tau_1)\tilde{t}}, \quad (25)$$

$$K^{****} = K_0 e^{q(1-\tau_1)t^*} \quad (26)$$

In the Nash solution the government starts to tax at a low rate and the firm invests at its maximum rate. At a certain moment, $\bar{t}$, the shareholders do not want the firm to invest. They are more interested in collecting dividend because the end of the planning horizon comes nearer. So they decide to invest no more. The government immediately reacts by introducing a high tax rate. Even if the government asks a low rate, the firm will not invest. In spite of the fact that the government wants more investment, it cannot force the firm to do so. In the situation where $\tau_2 < \frac{1}{2}$ the firm is more interested in investment than the government. Before the moment that the firm has stopped the government chooses the high rate.

If $\tau_2 \leq \frac{1}{2}$, the Stackelberg solution is equal to the Nash solution in that case. In the Stackelberg solution for the case that $\tau_2 > \frac{1}{2}$ the firm's investment period is longer than in the Nash-solution ($\tilde{t} > \bar{t}$). As compensation the government will postpone the application of the high rate. So there is a period in which the firm pays out dividend and the government asks a low rate. In a Nash game such a period could not exist. In a Stackelberg game the government knows the reaction of the firm on every possible strategy. It is easy to see that the leader of the game alsways became better, when Stackelberg is played. In our situation both players become better when Stackelberg has been chosen. Moreover, if the government had the possibility, at a moment between $\tilde{t}$ and $\tilde{\tilde{t}}$, to make a new initial plan the high rate is the plan, i.e. the open-loop solution is not time-consistent.

It is also possible to derive the Pareto solution. In principle there are many Pareto solutions (see Hoel [3]). But we are only interested in what Lancaster called the social optimum. In that case the government and the firm want to maximize the sum of government spendings and dividend. The time $t^*$, when investment stops, is later than in a noncooperative game. So there will be more capital in this economy. Also the value of both objectives is greater than in a noncooperative case. After the time $t^*$ we can say nothing about the way that $S(t)$ is divided between government consumption and dividend. The Pareto solution will only give an answer to the question what is the total of both.

## 5. A more sophisticated model:

In section 2 we have presented a very easy model. Admittedly, the economic model has many unrealistic features. However, it gives a framework for an analysis of governmental policy, although it could be generalized in several ways. In this section we will change a number of assumptions in the basic model and ask what will happen with the main conclusions. We make the model of section 2 more realistic by incorporating the following extensions:

- a discount rate (5.1)
- a concave profit function (5.2)
- a salvage value for the firm at the end of the planning period (5.3)
- a logarithmic utility function (5.4)
- investment grants (5.5)

- depreciation (5.6)
- unemployment payments (5.7)

We will compare the results with those of the basic model presented in section 2. We are especially interested in the switching time $\bar{t}$, when the firm changes its policy and pays out dividend, and the final value of the capital good stock. We confine our interest to the Nash game.

5.1 A discount rate:

Assume that the government discount the future at a rate i and the shareholders at a rate j. In that case our model becomes:

$$\int_0^T qK(t)u_1(t)e^{-it}dt \tag{27}$$

$$\int_0^T qK(t)(1-u_1(t))(1-u_2(t))e^{-jt}dt \tag{28}$$

$$\dot{K}(t) = qK(t)(1-u_1(t))u_2(t) \tag{29}$$

In appendix 1 we used Pontryagin's maximumprinciple to derive the solution for this model. The switch from investment to dividend takes place at:

$$\bar{t} = T + \frac{1}{i}\ln\left(1 - \frac{i}{q(1-\tau_2)}\right), \tag{30}$$

where we assume that $i<q(1-\tau_2)$. From (18) and (30) we can conclude that there will be an earlier switch and therefore the final capital stock will be less. In the case that i is close to zero (30) becomes

$$\bar{t} = T - \frac{1}{q(1-\tau_2)}, \tag{31}$$

which is the same as in the basic model. As we did in section 4 we can represent the solution in a table. However, nothing changes in table 1 and 2, except the criteria for the different solutions:

Table 1: the Nash-solution if $\frac{q\tau_2}{i}(e^{-i\bar{t}} - e^{-iT}) \geq e^{-i\bar{t}}$

Table 2: the Nash-solution if $\frac{q\tau_2}{i}(e^{-i\bar{t}} - e^{-iT}) < e^{-i\bar{t}}$

5.2 A concave profit function:

One of the strong assumptions of the basic model is that profit is a linear function of capital. In this subsection we will assume that there is a concave relation between profit O(K) and the capital good stock:

$$\frac{dO}{dK} > 0, \ \frac{d^2O}{dK^2} < 0. \tag{34}$$

In the model we replace qK by O(K)

$$\int_0^T O(K(t))u_1(t)dt \tag{35}$$

$$\int_0^T O(K(t))(1-u_1(t))(1-u_2(t))dt \tag{36}$$

$$\dot{K}(t) = O(K(t))(1-u_1(t))u_2(t) \tag{37}$$

In this situation the so-called reaction functions will not change (see appendix 1). The introduction of this concave profit function has only consequences for the development of the shadow prices of capital. For the switching time we get the following expression:

$$\bar{t} = T - \frac{1}{O'(K^*)(1-\tau_2)}, \tag{38}$$

where $K^*$ is the final value of the capital good stock and

$O'(K^*) = \left.\frac{dO}{dK}\right|_{K=K^*}$. Before we can compare $\bar{t}$ and $K^*$ to the result of the basic model we have to say something about the value of $O'(K_0)$ and $O'(K^*)$.

There are five possible situations:

table 5: comparison with basic model

| | $O'(K_0) < q$ | $O'(K_0) = q$ | $O'(K_0) > q$ |
|---|---|---|---|
| $O'(K^*) < q$ | $(\bar{t})$ - $(K^*)$ - | - - | - ? |
| $O'(K^*) = q$ | | | 0 + |
| $O'(K^*) > q$ | | | + + |

In this table a "+" expresses that this variable is greater than the same variable in the basic model. Here we can use table 1 and 2 to describe the way in which the economy develops.

5.3 A salvage value for the firm at the end of the planning period:

Also in this case nothing changes in the main conclusions. The paths we have derived in tables 1 and 2 still apply. In this case the model becomes :

$$\int_0^T qK(t)u_1(t)dt + aK(T) \tag{39}$$

$$\int_0^T qK(t)(1-u_1(t))(1-u_2(t))dt + bK(T) \tag{40}$$

$$\dot{K}(t) = qK(t)(1-u_1(t))u_2(t) \tag{41}$$

It is realistic to assume that $0 \leq a < 1$ and $0 \leq b < 1$. If, for example, $b = 1$ then the firm does not pay out during the whole planning period. It has a greater affinity to capital in the period $[T,\infty)$ than in the period we discuss. We can calculate the switching time as follows:

$$\bar{t} = T - \frac{1 - b}{q(1-\tau_2)} \tag{42}$$

In the case that b is greater than zero the period of capital accumulation will be greater. So there is more capital at the end of the period.

5.4 A logarithmic utility function:

An interesting case is the situation in which we incorporate in a logarithmic utility function for the government. In that case the problem may lose his bang-bang structure. We take as objective for the government:

$$\max. \int_0^T \ln(G(t))dt \tag{43}$$

The objective of the firm is unchanged.

For the tax policy of the government we can say:

- if the firm does not invest the high tax rate will be asked
- if the firm invests the government asks the following rate:

(with $t' = T - \frac{1}{q\tau_1}$ and $t'' = T - \frac{1}{q\tau_2}$)

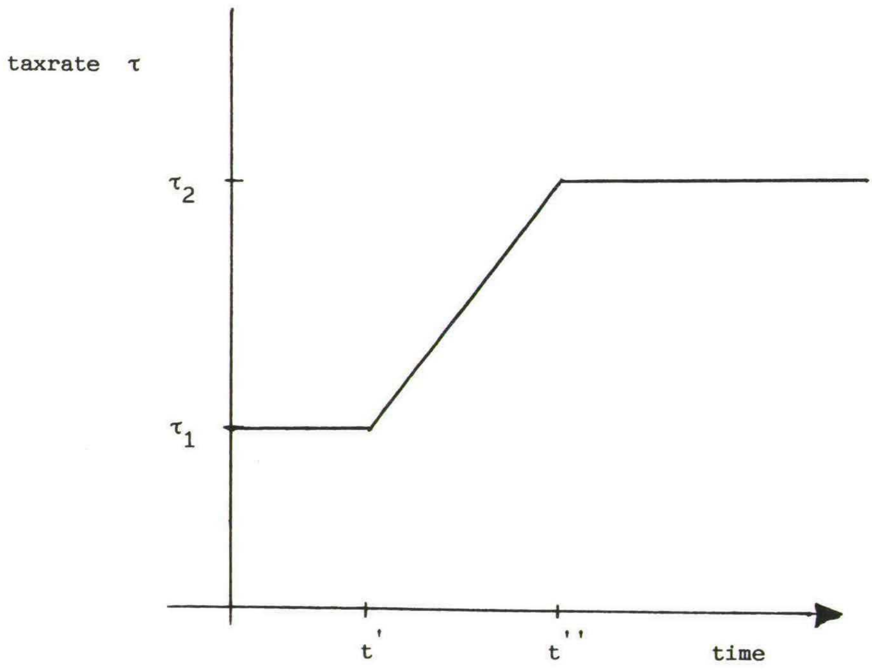


figure 2: tax-rate by logarithmic utility function

So there are three possible situations:

i) $\bar{t} \leq t' \leq t''$ if $\tau_2 \geq \frac{1}{2} \wedge \tau_1 + \tau_2 \geq 1$ $(K' = K^*)$

ii) $t' < \bar{t} \leq t''$ if $\tau_2 \geq \frac{1}{2} \wedge \tau_1 + \tau_2 < 1$ $(K' < K^*)$

iii) $t' \leq t'' \leq \bar{t}$ if $\tau_2 < \frac{1}{2}$, $(K' < K^*)$

where $K'$ is the final value of capital good stock. Notice that the switching time is exactly the same as in the basic model. In situation i) the switching time lies before the time, that the government wants to change its tax policy. In situations ii) and iii) there is a progressive move of $\tau$ until the moment that the firm pays out dividend or until the time that the high rate has been reached. From above we can conclude that in situation ii) and iii) the bang-bang structure of the tax policy (i.e. the tax rate jumps at once from its lower- to its upperbound) disappears.

5.5 Investment grants:

What we can easily incorporate in our model are investment grants. We model investment grants as follows: the government has the possibility to pay back to the firm a certain amount of the tax payments, if it continues investment:

$$TX(t) = G(t) + u_3(t)I_n(t), \tag{44}$$

in which $I_n(t)$ = investment financed by the firm

$u_3(t)I_n(t)$ = investment financed by the government

$u_3(t)$ = investment grants rate

The state-equation becomes:

$$\begin{aligned} \dot{K}(t) = I(t) &= I_n(t)(1+u_3(t)) \\ &= qK(t)(1-u_1(t))u_2(t)(1+u_3(t)) \end{aligned} \tag{45}$$

The objectives get the following form:

$$\max_{u_1,u_3} \int_0^T qK(t)(u_1(t)-u_3(t)(1-u_1(t))u_2(t))dt \tag{46}$$

$$\max_{u_2} \int_0^T qK(t)(1-u_1(t))(1-u_2(t))dt \tag{47}$$

The government has two control variables. For $u_3(t) = 0$, $0 \le t \le T$, we have the basic model. We assume for $u_3$: $0 \le u_3 \le g$ (48)

It is realistic to assume that $g < \frac{\tau_1}{1-\tau_1}$, (49)

because then it always hold that $u_1 - u_3(1-u_1)u_2 > 0$, which implies that there is no budgetary deficit.

Also now we have two situations:

table 6: The Nash-solution if $\tau_2 > \frac{1}{2}$ and $g < \frac{2\tau_2 - 1}{1-\tau_2}$

| | $t \in [0,\bar{t})$ | $t \in [\bar{t},T]$ |
|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_2$ |
| $u_2(t)$ | 1 | 0 |
| $u_3(t)$ | g | 0 |
| $K(t)$ | $K_0 e^{q(1-\tau_1)(1+g)t}$ | $\tilde{K}^*$ |

table 7: The Nash-solution if ($\tau_2 > \frac{1}{2}$ and $g > \frac{2\tau_2 - 1}{1-\tau_2}$) or $\tau_2 \le \frac{1}{2}$

| | $t \in [0,\bar{\bar{t}})$ | $t \in [\bar{\bar{t}},\bar{t})$ | $t \in [\bar{t},T]$ |
|---|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_2$ | $\tau_2$ |
| $u_2(t)$ | 1 | 1 | 0 |
| $u_3(t)$ | g | g | 0 |
| $K(t)$ | $K_0 e^{q(1-\tau_1)(1+g)t}$ | $K(\bar{\bar{t}}) e^{q(1-\tau_2)(1+g)(t-\hat{t})}$ | $\tilde{K}^{**}$ |

, with $\tilde{K}^{*} = K_0 e^{q(1-\tau_1)(1+g)\bar{t}}$

$$\tilde{K}^{**} = K_0 e^{q(1-\tau_1)(1+g)\hat{t}} e^{q(1-\tau_2)(1+g)(\bar{t}-\hat{t})}$$

In this case with investment grants the firm will go on longer with investment: $\bar{t} = T - \dfrac{1}{q(1-\tau_2)(1+g)}$ (50)

So the final value of capital good stock will be greater. Also the optimal payoff to both players will be greater.

5.6 Depreciation of capital good stock:

If we incorporate depreciation capital good stock will increase by:

$$\dot{K}(t) = I(t) - aK(t), \qquad (51)$$

in which a:depreciation rate. Following Van Loon [6], we write down the financial position of the firm as follows:

Balance Sheet

| | |
|---|---|
| K | X |

Income Statement

| | |
|---|---|
| S = qK | aK |
| | $u_1(q-a)K$ |
| | D |
| | $\dot{X}$ |

Cash account

| | |
|---|---|
| S = qK | I |
| | TX |
| | D |

In which X: equity

S: earnings

We can rewrite the model and get the following expressions:

$$\int_0^T (q-a)K(t)u_1(t)dt \tag{52}$$

$$\int_0^T (qK(t)(1-u_1(t))(1-u_2(t))+u_1aK)dt \tag{53}$$

$$\dot{K}(t) = qK(t)(1-u_1(t))u_2(t) - aK(t), \tag{54}$$

where we assume that $q(1-\tau_2) > a$. This is also a very interesting case because there are two different switching times (from investment to dividend). The switching time is dependent on the fiscal regime at that moment:

$$\bar{t} = T - \frac{1}{q(1-\tau_2)}(1 - \frac{\tau_1}{1-\tau_1}\frac{a}{q}) \quad \text{if} \quad \frac{\tau_2}{1-\tau_2}(1 - \frac{\tau_1}{1-\tau_1}\frac{a}{q}) > 1 \quad \text{(i)} \tag{55}$$

$$\bar{\bar{t}} = T - \frac{1}{q(1-\tau_2)}(1 - \frac{\tau_2}{1-\tau_2}\frac{a}{q}) \quad \text{if} \quad \frac{\tau_2}{1-\tau_2}(1 - \frac{\tau_1}{1-\tau_1}\frac{a}{q}) \leq 1 \quad \text{(ii)} \tag{56}$$

For the paths we have two possible situations:

table 8: The Nash-solution (i)

| | $t \in [0,\bar{t})$ | $t \in [\bar{t},T]$ |
|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_2$ |
| $u_2(t)$ | 1 | 0 |
| $K(t)$ | $K_0e^{(q(1-\tau_1)-a)t}$ | $K(\bar{t})e^{a(\bar{t}-t)}$ |

table 9: The Nash-solution (ii)

| | $t \in [0,\hat{t})$ | $t \in [\hat{t},\bar{\bar{t}})$ | $t \in [\bar{\bar{t}},T]$ |
|---|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_2$ | $\tau_2$ |
| $u_2(t)$ | 1 | 1 | 0 |
| $K(t)$ | $K_0e^{(q(1-\tau_1)-a)t}$ | $K(\hat{t})e^{(q(1-\tau_2)-a)(t-\hat{t})}$ | $K(\bar{\bar{t}})e^{a(\bar{\bar{t}}-t)}$ |

, with $K(\bar{t}) = K_0e^{(q(1-\tau_1)-a)\bar{t}}$

$$K(\hat{t}) = K_0e^{(q(1-\tau_1)-a)\hat{t}}$$

$$K(\bar{\bar{t}}) = K_0e^{(q(1-\tau_2)-a)(\bar{\bar{t}}-\hat{t}} e^{(q(1-\tau_1)-a)\hat{t})}$$

In both situations there will be a later switch. Notice that in this case no stationary stage of capital good stock arises (it would arise again when we combine depreciation with a final value of the firm). The value of capital good stock will have a lower value at each point of time and that of the basic model.

5.8 Unemployment payments:

Until now we have said nothing about labour. We cannot conclude that the firm does not need labour for production. If we assume that there is a Leontief technology and capital is the most restricted factor, we get:

$X = \min(q^{'}K,q^{''}L) = q^{'}K$ ( see Varian [9] page 5), (57)

where $q^{''}/q^{'}$:slope of the Leontief technology. At this level of capital good stock, a rational firm will choose $q^{'}/q^{''}K(t)$ as the level of labour.

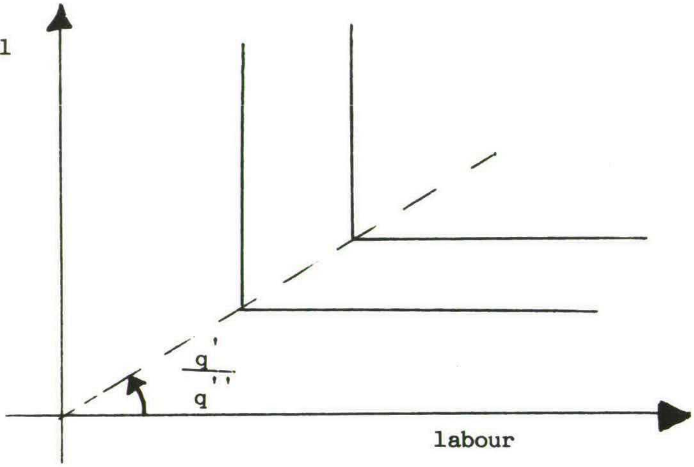


Figure 3: Isoquants of a Leontief technology

In that case is

$$
\begin{aligned}
O(t) &= pX(t) - wL(t) \\
&= (pq' - w\frac{q'}{q''})K(t) \\
&= qK(t) , \qquad (58)
\end{aligned}
$$

in which w is wage. This is the same expression as (3).

The situation will change if we introduce unemployment payments. Let $L^*$ denote the supply of labour. So the unemployment will be:

$$L_u = L^* - L \qquad (59)$$

The government has to pay $w'L_u$ (=U) (, where $w'<w$) for unemployment payments and there remains $G = T - w'L_u$ for government spendings. G is greater than zero if

$$u_1 > \frac{w'(L^* - (q'/q'')K_0)}{qK_0} \qquad (60)$$

It is quite reasonable that in these situation the government has a greater impulse for capital accumulation and there may be a longer period of a low tax rate.

table 10: The Nash-solution if $\frac{\tau_2 q + w'}{q(1-\tau_2)} > 1$.

| | $t \in [0,\bar{t})$ | $t \in [\bar{t},T]$ |
|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_2$ |
| $u_2(t)$ | $1$ | $0$ |
| $K(t)$ | $K_0 e^{q(1-\tau_1)t}$ | $K^*$ |
| $T(t)$ | $q\tau_1 K(t)$ | $q\tau_2 K^*$ |
| $U(t)$ | $w'(\frac{q'}{q''}K(t) - L^*)$ | $w'(\frac{q'}{q''}K^* - L^*)$ |
| $G(t)$ | $(q\tau_1 - w'\frac{q'}{q''})K(t) + w'L^*$ | $(q\tau_1 - w'\frac{q'}{q''})K^* + w'L^*$ |

table 11: The Nash-solution if $\frac{\tau_2 q + w'}{q(1-\tau_2)} \leq 1$

| | $t \in [0,\hat{\hat{t}})$ | $t \in [\hat{\hat{t}},\bar{t})$ | $t \in [\bar{t},T]$ |
|---|---|---|---|
| $u_1(t)$ | $\tau_1$ | $\tau_2$ | $\tau_2$ |
| $u_2(t)$ | $1$ | $1$ | $0$ |
| $K(t)$ | $K_0 e^{q(1-\tau_1)t}$ | $K(\hat{\hat{t}}) e^{q(1-\tau_2)(t-\hat{t})}$ | $K^{**}$ |
| $T(t)$ | $q\tau_1 K(t)$ | $q\tau_2 K(t)$ | $q\tau_2 K^{**}$ |
| $U(t)$ | $w'(\frac{q'}{q''}K(t) - L^*)$ | $w'(\frac{q'}{q''}K(t) - L^*)$ | $w'(\frac{q'}{q''}K^{**} - L^*)$ |
| $G(t)$ | $(q\tau_1 - w'\frac{q'}{q''})K(t)+w'L^*$ | $(q\tau_1 - w'\frac{q'}{q''})K(t)+w'L^*$ | $(q\tau_1 - w'\frac{q'}{q''})K^{**} + w'L^*$ |

$\hat{\hat{t}}$ will be on a later point of time than in the basic model.

### 5.8 Summary:

We can summarize sections 5.1 through 5.7 in the following table, where

$\bar{t}$ = switching time from investment to dividend

$K^*$ = final value of capital good stock at t=T

| table 12: | $\bar{t}$ | $K^*$ |
|---|---|---|
| 5.1 a discounting rate | - | - |
| 5.2 concave earnings function | ? | ? |
| 5.3 a endvalue | + | + |
| 5.4 a log. utility function | 0 | -/0 |
| 5.5 investments grants | + | + |
| 5.6 depreciation | + | - |
| 5.7 unemployment payments | 0 | 0/+ |

In this table a "+" expresses that this variable is greater than the same variable in the basic model.

## 6. Conclusions:

In the previous sections we have described in what way the government can influence the growth of the economy. A general conclusion is that a government, which spends a lot and needs a lot of tax payments, will have a negative influence on the growth of the economy. This is one of the things we see today in western society. Of course this conclusion is only realistic in the framework of the model, but we believe it has its impact on modern society. From this we cannot conclude that the situation of less government spendings is better. Perhaps in terms of welfare it would be better to have more government spendings and less capital accumulation. This is a political choice.

In the basic model we have shown the difference between the Nash- and Stackelberg solutions. In the Stackelberg game, where the government has insight in the way the firm will react to every possible strategy, both players are better off. For the government it is important to know how the firm will react. We have not derived the Stackelberg solutions of the models described in section 5. Since these soltions involve some serious analytical problems. This is a topic of future research. We have only studied the open-loop information structure in this paper. More general patterns, i.e. closed-loop strategies with memory and feedback, are more desirable, because they are time consistent. This is also a topic of future research.

The model, we have presented, even with the extensions has still many unrealistic features. For example we could replace the fixed-coefficient production function by a neoclassical one. In that case the firm has the possibility to choose the production technique. For the government we can build in more instruments, such as wage control. We believe, certainly for the Dutch case, that the government in practice does not have a great influence on the real wages. In this paper we have assumed that government consumption is not productive. In practice some government spendings (like those for a new electricity plant) will raise the investment. Of course, society is more complex than we described. In spite of this we believe that dynamic game theory is a fruitful way to model the interaction between government and firm. Both have their own interests and their own controls, and neither of them can force the other to do exactly what he wants.

Appendix 1: the derivation of the Nash-solution in section 5:

The necessary conditions for a Nash-solution are (see Basar&Olsder [1]):

$$H_1(\hat{K}(t), \hat{u}_1(t), \hat{u}_2(t), \lambda_1(t), t) \geq H_1(\hat{K}(t), u_1(t), \hat{u}_2(t), \lambda_1(t), t),$$

$$\forall\ u_1 \in U_1(=[\tau_1, \tau_2]) \qquad (0.1)$$

$$H_2(\hat{K}(t), \hat{u}_1(t), \hat{u}_2(t), \lambda_2(t), t) \geq H_2(\hat{K}(t), \hat{u}_1(t), u_2(t), \lambda_2(t), t),$$

$$\forall\ u_2 \in U_2(=[0,1]) \qquad (0.2)$$

$$\dot{\lambda}_1(t) = \frac{\partial H_1}{\partial K} \qquad (0.3)$$

$$\dot{\lambda}_2(t) = \frac{\partial H_2}{\partial K} \qquad (0.4)$$

$$\lambda_1(T)=0 \qquad (0.5)$$

$$\lambda_2(T)=0 \qquad (0.6)$$

$$K(0)=K_0 \qquad (0.7)$$

$$\dot{K}(t) = f(K(t), u_1(t), u_2(t), t) \qquad (0.8)$$

For the basic model (12)-(14) (0.1),...(0.8) can be written as:

$$H_1 = qKu_1 + \lambda_1 qK(1-u_1)u_2 \qquad (1.1)$$

$$H_2 = qK(1-u_1)(1-u_2) + \lambda_2 qK(1-u_1)u_2 \qquad (1.2)$$

$$\dot{\lambda}_1 = -qu_1 - \lambda_1 q(1-u_1)u_2 \qquad (1.3)$$

$$\dot{\lambda}_2 = -q(1-u_1)(1-u_2) - \lambda_2 q(1-u_1)u_2 \qquad (1.4)$$

$$\lambda_1(T) = 0 \qquad (1.5)$$

$$\lambda_2(T) = 0 \qquad (1.6)$$

$$\dot{K} = qK(1-u_1)u_2 \qquad (1.7)$$

$$K(0)=K_0 \qquad (1.8)$$

(1.1),(1.2) together with (0.1),(0.2) gives us the so-called reaction functions:

player one: $u_1 = \tau_2$ if $1-\lambda_1 u_2 > 0$ (1.9)

$$u_1 = \tau_1 \quad \text{if} \quad 1-\lambda_1 u_2 \leq 0 \tag{1.10}$$

player two: $$u_2 = 1 \quad \text{if} \quad \lambda_2 \geq 1 \tag{1.11}$$

$$u_2 = 0 \quad \text{if} \quad \lambda_2 < 1 \tag{1.12}$$

These conditions are not only necessary but also sufficient because of the fact that the Hamiltonian is linear in the state variable.
Using (1.1) through (1.12) we can easily calculate the optimal solution. For each of the models presented we will write down the optimal conditions (1.1) through (1.12) in case something changes.
-section 5.1:

$$H_1 = qKu_1e^{-it} + \lambda_1 qK(1-u_1)u_2 \tag{2.1}$$

$$H_2 = qK(1-u_1)(1-u_2)e^{-jt} + \lambda_2 qK(1-u_1)u_2 \tag{2.2}$$

$$\dot{\lambda}_1 = -qu_1e^{-it} - \lambda_1 q(1-u_1)u_2 \tag{2.3}$$

$$\dot{\lambda}_2 = -q(1-u_1)(1-u_2)e^{-jt} - \lambda_2 q(1-u_1)u_2 \tag{2.4}$$

The reaction function:

player one: $$u_1 = \tau_2 \quad \text{if} \quad e^{-it}-\lambda_1 u_2 > 0 \tag{2.9}$$

$$u_1 = \tau_1 \quad \text{if} \quad e^{-it}-\lambda_1 u_2 \leq 0 \tag{2.10}$$

player two: $$u_2 = 1 \quad \text{if} \quad \lambda_2 \geq e^{-jt} \tag{2.11}$$

$$u_2 = 0 \quad \text{if} \quad \lambda_2 < e^{-jt} \tag{2.12}$$

When $\lambda_2$ will fall below $e^{-jt}$, $u_2$ will become zero and $u_1$ $\tau_2$ (if not already).

In that case: $\dot{\lambda}_2 = -q(1-\tau_2)e^{-jt} \rightarrow \lambda_2(t) = (q(1-\tau_2)/j)(e^{-jt}-e^{-jT}), \; t \geq \bar{t}$

So because $\lambda_2(\bar{t}) = 1 \rightarrow \; 1 = (q(1-\tau_2)/j)(e^{-j\bar{t}}-e^{-jT})$

$$\rightarrow \; \bar{t} = T + (1/j)\ln\left(1 - \frac{j}{q(1-\tau_2)}\right)$$

We have two possibilities:

sit. I:
$$\begin{array}{c|c} \tau_1 & \tau_2 \\ \hline 1 & 0 \end{array}$$

sit. II:
$$\begin{array}{c|c|c} \tau_1 & \tau_2 & \tau_2 \\ \hline 1 & 1 & 0 \end{array}$$

sit. I if $\lambda_2(\bar{t}) > 1 \rightarrow (q\tau_2)/j)(e^{-j\bar{t}}-e^{-jT}) \geq e^{-j\bar{t}}$

sit. II if $\lambda_2(\bar{t}) < 1 \rightarrow (q\tau_2)/j)(e^{-j\bar{t}}-e^{-jT}) < e^{-j\bar{t}}$

-section 5.2:

$$H_1 = O(K)u_1 + \lambda_1 O(K)(1-u_1)u_2 \tag{3.1}$$

$$H_2 = O(K)(1-u_1)(1-u_2) + \lambda_2 O(K)(1-u_1)u_2 \tag{3.2}$$

$$\dot{\lambda}_1 = -\frac{dO}{dK}u_1 - \lambda_1\frac{dO}{dK}(1-u_1)u_2 \tag{3.3}$$

$$\dot{\lambda}_2 = -\frac{dO}{dK}(1-u_1)(1-u_2) - \lambda_2\frac{dO}{dK}(1-u_1)u_2 \tag{3.4}$$

$$\dot{K} = O(K)(1-u_1)u_2 \tag{3.7}$$

-section 5.3:

$$\lambda_1(T) = a \tag{4.5}$$

$$\lambda_2(T) = b \tag{4.6}$$

-section 5.4:

$$H_1 = \ln q + \ln K + \ln u_1 + \lambda_1 qK(1-u_1)u_2 \tag{5.1}$$

$$\dot{\lambda}_1 = -\frac{1}{K} - \lambda_1 q(1-u_1)u_2 \tag{5.3}$$

Let $\psi_1 = \lambda_1 K$ then we can write the reaction function as follows:

player one: $u_1 = \tau_2$ if $\psi_1 < \frac{1}{q\tau_2}$ or $u_2 = 0$ (5.9a)

$u_1 = \frac{1}{q\psi_1}$ if $\frac{1}{q\tau_2} \leq \psi_1 \leq \frac{1}{q\tau_1}$ (5.9b)

$u_1 = \tau_1$ if $\psi_1 > \frac{1}{q\tau_1}$ (5.9c)

-section 5.5:

$$H_1 = qK(u_1-u_3(1-u_1)u_2) + \lambda_1 qK(1-u_1)u_2(1+u_3) \tag{6.1}$$

$$H_2 = qK(1-u_1)(1-u_2) + \lambda_2 qK(1-u_1)u_2(1+u_3) \tag{6.2}$$

$$\dot{\lambda}_1 = -q(u_1-u_3(1-u_1)u_2) - \lambda_1 q(1-u_1)u_2(1+u_3) \tag{6.3}$$

$$\dot{\lambda}_2 = -q(1-u_1)(1-u_2) - \lambda_2 q(1-u_1)u_2(1+u_3) \tag{6.4}$$

$$\dot{K} = qK(1-u_1)u_2(1+u_3) \tag{6.7}$$

The reaction function:

player one: $u_1 = \tau_2$ if $u_2 = 0$ or $\lambda_1 \leq \dfrac{1+u_3u_2}{u_2(1+u_3)}$ (6.9)

$u_1 = \tau_1$ if $\lambda_1 > \dfrac{1+u_3u_2}{u_2(1+u_3)}$ (6.10)

$u_3 = g$ if $\lambda_1 > 0$ (6.9a)

$u_3 = 0$ if $\lambda_1 \leq 0$ (6.10a)

player two: $u_2 = 1$ if $\lambda_2 \geq 1 + u_3$ (6.11)

$u_2 = 0$ if $\lambda_2 < 1 + u_3$ (6.12)

-section 5.6:

$$H_1 = (q-a)Ku_1 + \lambda_1(qK(1-u_1)u_2-aK) \tag{7.1}$$

$$H_2 = qK(1-u_1)(1-u_2) + u_1aK + \lambda_2(qK(1-u_1)u_2-aK) \tag{7.2}$$

$$\dot{\lambda}_1 = -(q-a)u_1 - \lambda_1(q(1-u_1)u_2-a) \tag{7.3}$$

$$\dot{\lambda}_2 = -q(1-u_1)(1-u_2) - u_1a - \lambda_2(q(1-u_1)u_2-a) \tag{7.4}$$

$$\dot{K} = qK(1-u_1)u_2-aK \tag{7.7}$$

The reaction functions are:

player two: $u_2 = 1$ if $\lambda_2 \geq 1 - \dfrac{u_1a}{(1-u_1)q}$ (7.11)

$$u_2 = 0 \quad \text{if} \quad \lambda_2 < 1 - \frac{u_1 a}{(1-u_1)q} \tag{7.12}$$

-section 5.7:

$$H_1 = (qu_1 + w'(q'/q''))K + \lambda_1 qK(1-u_1)u_2 \tag{8.1}$$

$$\dot{\lambda}_1 = -qu_1 - w'(q'/q'') - \lambda_1 q(1-u_1)u_2 \tag{8.3}$$

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