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**DYNAMIC OPTIMAL FACTOR DEMAND**

**UNDER FINANCIAL CONSTRAINTS**



\* C I N O O 5 0 0 \*

by

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and

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Jean Frijns and Anton Hempenius<sup>1)</sup>

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1 Introduction.

In the economic literature the influence of financial variables on the investment behaviour of the firm is widely accepted<sup>2)</sup>. In the traditional neo-classical investment literature (e.g. Jorgenson [ 8 ] ) financial variables in the form of a (constant) internal interest rate are one of the determinants of the user cost of capital. Other authors suggest that the adjustment speed of factor inputs depends on financial variables such as cash-flows. (See e.g. Eisner and Strotz [ 2 ], Coen [ 1 ], Hempenius [ 6 ], Gardner and Sheldon [ 4 ] ). Some authors (e.g. Meyer and Kuh [ 12 ], Meyer and Glauber [ 11 ] ) have studied the financing-investment behaviour over the cycle. They conclude that in a boom the interest rate (on external funds) is the important financial variable whereas in a recession cash-flow is the important financial determinant of investment.

- 1) The authors are grateful to S.E. de Jong for helpful comments on an earlier draft of this paper.
- 2) For a survey of the influence of financial variables see also Rowley and Trivedi [ 14 ] .

In the financial literature the determination of the cost of capital is closely connected with the valuation theory one accepts. In the Modigliani and Miller [13] hypothesis the cost of capital is independent of how investments are financed (except for the possible influence of fiscal parameters). An opposite view (the traditional hypothesis, defended by Lintner [9] et al.) is that the value of the firm (in a world of uncertainty) depends on the degree of financial leverage in the firm's capital structure. This leads to a cost of capital function which rises with leverage (excluded the influence of fixed parameters). In a recent study<sup>3)</sup> of Inselbag [7] the influence of a strictly convex cost of borrowing function on optimal investment and optimal financial structure is studied in the context of a dynamic model. Further independent adjustment paths for capital and total borrowing are derived under the assumption that dividends can be adjusted freely in order to balance revenues and outlays at each moment.

In this paper the influence of the interest rate (marginal cost of funds) on optimal factor inputs and the influence of cash-flows and resulting shifts in the financial structure on the adjustment speed of factor inputs are studied in the context of a dynamic model. It is assumed that the expansion of the firm is financed from internal funds or by borrowing, that the marginal cost of external funds depend on the financial structure (e.g. the firm's debt-equity ratio) and that a stable dividend policy is maintained over the planning period. In this paper we will not study the optimal financial structure but analyse the influence of the apriori determined financial policy and resulting shifts in the financial structure on investment and employment decisions. We will show that under these assumptions adjustment of factor inputs depends on the generation of internal funds (cash-flows).

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3) Which came to our attention after completing our study.

2. Optimal firm policy under a shifting financial structure.

2.1. Specification of the object function and the form of the financial constraints.

In most neoclassical models of factor demand (e.g. the model analysed in [3 ]) the influence of the existing financial structure of the firm on (marginal) costs of funds is not explicitly studied. To incorporate the financial structure of the firm explicitly in a model one has to make additional assumptions with respect to the possible financial constraints, e.g. in the form of a financial equilibrium equation, and with respect to the financial policy of the firm.

For the financial equilibrium equation we firstly enumerate all possible receipts and outlays. Receipts (in a wide sense, being all incoming cash streams) in period  $t$  are:

- (i)  $Y_t - \mathbf{Q}(\Delta X_t)$ , being the receipts from selling the product produced:  $Y_t$  the potential receipts if (internal) adjustment services would be zero and  $\mathbf{Q}(\Delta X_t)$  the receipts sacrificed because of expanding the factors from  $X_{t-1}$  to  $X_{t-1} + \Delta X_t = X_t$ , to be called adjustment costs.
- (ii)  $F_t$ , if positive.  $F_t$  is the amount of funds borrowed and used to finance new investments in period  $t$  if  $F_t > 0$  and  $-F_t$  is the amount of excess internal funds (i.e. internal funds not (yet) invested within the firm) in period  $t$  if  $F_t < 0$ . We assume that there are one-year contracts for borrowing ( $F_t > 0$ ) and also one-year contracts for lending the available internal funds ( $F_t < 0$ ). Moreover we assume that  $R(F_t)$  measures the costs of borrowing if  $F_t > 0$  and the receipts from lending if  $F_t < 0$ .

Outlays (in a wide sense) in period  $t$  are:

- (i)  $\tau C_t$ , with  $C_t$  profits and  $\tau$  the tax rate.
- (ii)  $q_{1t} I_t$ , with  $q_{1t}$  the price of capital goods and  $I_t$  gross investments in period  $t$ . We have  $I_t = \Delta X_{1t} + B_t$  where  $B_t$  are replacement investments, which are assumed to be proportional with capital stock  $X_{1t}$ , i.e.  $B_t = q_{1t} \delta X_{1t}$  ( $\delta$  is the replacement parameter).

(iii)  $w'_{2t} X_{2t}$  with  $w_{2t}$  the vector of wage rates for the labour inputs  $X_{2t}$ .

(iv)  $R(F_t)$ , being the (positive or negative) costs of borrowing or lending.

(v)  $R_0$ , being the interest payments on the consolidated debt  $V_0$  at the beginning of the plan period.

(vi)  $D_t$ , being dividends.

(vii)  $F_{t-1}$ , if positive.  $F_{t-1}$  is the amount of borrowed funds used in period  $t-1$  and paid back in period  $t$ , if  $F_{t-1} > 0$ . If  $F_{t-1} < 0$  then the firm receives  $-F_{t-1}$  in period  $t$ .

As we have by assumption described all positive and negative streams in period  $t$ , equilibrium requires the following equality, to be called the financial balance equation:

$$(1) \quad F_t + Y_t - \mathbf{A}(\Delta X_t) = \tau C_t + q_{1t} \Delta X_{1t} + q_{1t} B_t + w'_{2t} X_{2t} + R(F_t) + D_t + F_{t-1} + R_0$$

where profits  $C_t$  are defined as follows <sup>4,5)</sup>

$$(2) \quad C_t = Y_t - \mathbf{A}(\Delta X_t) - q_{1t} B_t - w'_{2t} X_{2t} - R(F_t) - R_0$$

The financial balance equation can now be written in the following form

$$(3) \quad F_t = F_{t-1} - (1-\tau)C_t + q_{1t} \Delta X_{1t} + D_t$$

Given the balance equation, the existing financial structure, the production technology and the expected situation on output and

<sup>4)</sup> For the sake of simplicity it is assumed that taxes are levied on  $C_t$ , an assumption which can easily be replaced for a more realistic one but which would complicate the following derivations unnecessarily.

<sup>5)</sup> In this specification  $\mathbf{A}(\Delta X_t)$  is treated as costs in period  $t$ . A possible alternative is to treat the adjustment expenditures as investment expenditures, analogous to the treatment of  $q_{1t} \Delta X_t$ .

factor markets and on the capital market it is, in principle, possible to determine simultaneously for a given criterium function the optimal production and factor input levels and the optimal financial structure. However since we are primarily interested in the influence of a given financial structure and given financial policy (optimal or not optimal) on factor input decisions we will assume that both criterium function and dividend policy are exogenously determined. Further we assume that changes in production and input levels are financed from internal funds (retained earnings) or by borrowing. Only in exceptional cases will firms finance their planned expansion by stock issuing. We will therefore exclude the possibility of equity financing by stock issuing. Finally we make the important assumption that increasing debt-financing relative to equity financing implies rising marginal costs of funds (e.g. reflecting increasing risk for a more leveraged firm).

As to the specification of the objective function one usually assumes that the firm tries to maximize its market value. The specification of the market value depends on the point of view with respect to the importance of the dividend policy for the (determination of the) value of the firm.<sup>6)</sup> For the dividend theorists the dividend policy is a critical factor in determining the market value, defined as present value of (future) dividends with a discount factor which depends on the dividend policy. An increase of the retained earnings ratio shifts dividends to more distant periods. Since, under conditions of uncertainty distant dividends are less preferred than near dividends, investors will require a higher discount rate if the payout ratio is lower. For the earnings theorists the valuation of the market value of the firm depends on future earnings discounted by a factor which does not depend on the dividend policy, thus the objective function is specified as present value of (future) earnings.

6) See Mao [10, Ch 12] for a survey of the theoretical dispute around this topic.



We will now make the following general assumptions with respect to the dividend policy and (corresponding) form of the objective function. The specification of the exogenously determined dividend policy is <sup>7)</sup>

$$(.4) \quad D_t = d((1-\tau)C_t - D_0) + D_0, \quad 0 < d < 1, \quad t = 1, \dots, T$$
$$D_t = (1-\tau)C_t, \quad t \geq T+1$$

For the first T periods of the infinite planning horizon we can write the dividend equation as

$$D_t = d(1-\tau)C_t + (1-d)D_0$$

so that the dividend policy is a mixed one of a pure dividend stabilisation policy (where  $D_0$  are normal dividends which can be seen as a target percentage of equity capital) and a dividend policy based on actual earnings. Beyond period T investment activities are assumed to be nil (see relation (6)) so that further accumulation of retained earnings is no longer necessary.

Since dividends are determined as an apriori determined function of net profits the specification of the objective function as present value of (expected) net profits seems appropriate both for the dividends theorists' and the earnings theorists' point of view.

We thus propose as objective the maximization of

$$(.5) \quad \sum_{t=1}^{\infty} \beta^t (1-\tau)C_t$$

In the view of the dividend theorists the discount rate  $-\frac{1-\beta}{\beta}$  will depend on the dividend policy. For the earnings theorists the rate of return required by the stock holders is a constant. In both cases the discount rate  $(1-\beta)/\beta$  is, in the context of this model, exogenously determined.

7) The dividend equation is only well defined if  $D_t \geq 0$ , i.e.

$$(1-\tau)C_t \geq -\frac{1-d}{d} D_0$$

where  $D_0 \geq 0$ . We assume that this condition is always satisfied.

Further we assume

$$(6) \quad X_t = X_{t-1} + \Delta X_t, \quad (t = 1, 2, \dots)$$

$$\Delta X_t = 0, \quad (t \geq T+1)$$

where  $X_0$  and  $F_0$  are given amounts at the start of period 1, so that after period  $T$  no further changes in factor input levels are planned.

The model defined in (6) is an infinite horizon model. However, due to the specification of  $D_t$  and  $\Delta X_t$  for  $t \geq T+1$ , the analytical treatment of this model is greatly simplified. A consequence of the assumptions  $D_t = (1-\tau)C_t$  and  $\Delta X_t = 0$  for  $t \geq T+1$  is that  $F_t = F_T$  for  $t \geq T+1$ . It is assumed that after period  $T$ ,  $F_T$  is consolidated in the form of long term debt, if  $F_T > 0$ , or long-term lendings, if  $F_T < 0$ . Equations (3) and (5) determine changes in the financial structure of the firm as function of future investments and (expected) profits. The development of the financial structure is highly dependent on the apriori determined dividend equation.

It was assumed above that the financial structure of the firm is an important determinant of the marginal costs of external funds,  $\partial R / \partial F$  (here we are talking of course of the positive part of the  $R$ -function). It seems reasonable to use the Debt-Equity-ratio (D.E.-ratio) as the important determinant of the m.c.f.-function  $\partial R / \partial F$ <sup>8)</sup>: below a certain critical value of the D.E.-ratio  $\partial R / \partial F$  is assumed to be constant. If the D.E.-ratio exceeds this critical value we assume that  $\partial R / \partial F$  is a monotonously increasing function of the D.E.-ratio. However the introduction of the D.E.-ratio as an explicit argument in the m.c.f.-function greatly complicates the analytical

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8) In general  $\partial R / \partial F$  may depend on the size of the firm, e.g. on an average size measured as  $q' \bar{X}$ , on the market conditions, its previous rentability, etc. The critical D.E.-ratio may depend on tradition, economic outlook, "etc."

derivation of the optimal factor demand in the next sections. Therefore some slight modifications are desirable. Assuming that changes in the D.E.-ratio are mainly caused by changes in the debt term  $F_t$  we specify a critical value of  $F_t$ , say  $\tilde{F}$ , so that

$$(7) \quad \frac{\partial R}{\partial F} = r_0 \quad , \quad F \leq \tilde{F}$$

$$\frac{\partial R}{\partial F} > r_0, \quad \frac{\partial^2 R}{\partial F^2} > 0 \quad , \quad F > \tilde{F}$$

Further we assume that  $\partial R/\partial F$  is a continuous and differentiable function of  $F$ .

Before deriving in the next sections the optimal growth path of the firm we make some additional assumptions. Price expectations for the factor markets and capital markets are constant over the (infinite) planning horizon. Further we assume a stable long-run demandcurve, a strictly concave revenue function  $Y(X_t)$  and a strictly convex (internal) adjustment costs function  $\mathbf{a}(X_t - X_{t-1})$  for all  $t = 1, 2, \dots$

### 3. The model

In this Section we will analyse the model defined in ( 3 ) - ( 7 ) under the just mentioned assumption of stable expectations. We can write the Lagrange-function as

$$\begin{aligned}
 (8) \quad \phi = & \sum_{t=1}^T \beta^t (1-\tau) (Y_t - w'X_t - \mathbf{A}(X_t - X_{t-1}) - R(F_t)) \\
 & + \frac{\beta^{T+1}}{1-\beta} (1-\tau) [Y_T - w'X_T - R(F_T)] \\
 & - \sum_{t=1}^T \lambda_t [F_t - F_{t-1} - q'(X_t - X_{t-1}) + (1-d)(1-\tau) \cdot \\
 & \quad (Y_t - w'X_t - \mathbf{A}(X_t - X_{t-1}) - R(F_t)) - (1-d)D_0]
 \end{aligned}$$

where  $w = (q_1 \delta, w_2)$  and  $q = (q_1, 0)'$ .

Assuming that this model satisfies the necessary concavity restrictions for a uniquely determined maximum and that this maximum lies in the economically relevant region, i.e.  $\forall_t: X_t > 0$ , this maximum can be found by solving the first order conditions. These first order conditions can be written as, writing  $\mathbf{Q}_t$  for  $\mathbf{A}(X_t - X_{t-1})$  and  $R_t$  for  $R(F_t)$ :

$$\begin{aligned}
 (9) \quad & \beta^t (1-\tau) \left( \frac{\partial Y_t}{\partial X_t} - w - \frac{\partial \mathbf{a}_t}{\partial X_t} \right) - \beta^{t+1} (1-\tau) \frac{\partial \mathbf{a}_{t+1}}{\partial X_t} \\
 & - \lambda_t (-q + (1-d)(1-\tau) \left( \frac{\partial Y_t}{\partial X_t} - w - \frac{\partial \mathbf{a}_t}{\partial X_t} \right)) - \\
 & - \lambda_{t+1} (q - (1-d)(1-\tau) \frac{\partial \mathbf{a}_{t+1}}{\partial X_t}) = 0, (t=1, \dots, T-1) \\
 & \beta^T (1-\tau) \left( \frac{\partial Y_T}{\partial X_T} - w - \frac{\partial \mathbf{a}_T}{\partial X_T} \right) + \frac{\beta^{T+1}}{1-\beta} (1-\tau) \left( \frac{\partial Y_T}{\partial X_T} - w \right) \\
 & - \lambda_T (-q + (1-d)(1-\tau) \left( \frac{\partial Y_T}{\partial X_T} - w - \frac{\partial \mathbf{a}_T}{\partial X_T} \right)) = 0
 \end{aligned}$$

$$-\beta^t(1-\tau) \frac{\partial R_t}{\partial F_t} - \lambda_t(1-(1-d)(1-\tau)) \frac{\partial R_t}{\partial F_t} + \lambda_{t+1} = 0, \quad (t=1, \dots, T-1)$$

$$-\beta^T(1-\tau) \frac{\partial R_T}{\partial F_T} - \frac{\beta^{T+1}}{1-\beta} (1-\tau) \frac{\partial R_T}{\partial F_T} - \lambda_T(1-(1-d)(1-\tau)) \frac{\partial R_T}{\partial F_T} = 0.$$

For the Lagrange parameters  $\lambda_t$  we find from (9)

$$\lambda_{t+1} = \lambda_t + (\beta^t - \lambda_t(1-d))(1-\tau) \frac{\partial R_t}{\partial F_t}, \quad (t=1, \dots, T-1)$$

(10)

$$\lambda_T = - \left( \frac{\beta^T}{1-\beta} - \lambda_T(1-d) \right) (1-\tau) \frac{\partial R_T}{\partial F_T}$$

so that

$$\lambda_T = \frac{\frac{-\beta^T}{1-\beta} (1-\tau) \frac{\partial R_T}{\partial F_T}}{1-(1-d)(1-\tau) \frac{\partial R_T}{\partial F_T}}$$

(11)

$$\lambda_t = \frac{-\beta^t(1-\tau) \frac{\partial R_t}{\partial F_t} + \lambda_{t+1}}{1-(1-d)(1-\tau) \frac{\partial R_t}{\partial F_t}}, \quad (t = 1, \dots, T-1)$$

Writing  $c_i = 1-(1-d)(1-\tau) \frac{\partial R_i}{\partial F_i}$  we obtain for  $t = 1, \dots, T$

$$(12) \quad \lambda_t = - (1-\tau) \beta^{t-1} \left[ \sum_{j=t}^{T-1} \left( \prod_{i=t}^j \left( \frac{\beta}{c_i} \right) \right) \frac{\partial R_j}{\partial F_j} + \left( \prod_{i=t}^T \left( \frac{\beta}{c_i} \right) \right) \frac{\partial R_T / \partial F_T}{1-\beta} \right]$$

so that  $(-\lambda_t)$  can be interpreted as the (after tax) discounted marginal interest expenses from period  $t$  onwards.  $\lambda_t$  is computed with a variable discount rate which is the ratio of  $\beta$  and  $c_i$  (the marginal increase of disposable funds in period  $i$ ). The parameter  $\lambda_t$  can thus be interpreted as a marginal costs of funds index.

The term  $(1-(1-d)(1-\tau) \frac{\partial R_t}{\partial F_t})$  measures the marginal increase

of disposable funds. The case

$$1-(1-d)(1-\tau)\partial R_t/\partial F_t < 0$$

corresponds to an inferior firm policy (increasing the external funds  $F_t$  implies a decrease in net-disposable funds) and can therefore be ignored in our analysis. Thus we may safely assume that

$$1-(1-d)(1-\tau)\partial R_t/\partial F_t > 0, \quad \forall t$$

so that since  $\partial R_t/\partial F_t \geq r_0 > 0$  we obtain from (11)

$$(13) \quad \left\{ \begin{array}{l} \lambda_T < 0 \\ \text{and using the "complete induction theorem"} \\ \lambda_1 < \lambda_2 < \dots < \lambda_T < 0, \quad (t = 1, \dots, T) \end{array} \right.$$

For further analysis it is interesting to analyse  $\lambda_t$  if  $\partial R_t/\partial F_t = r_0$  for  $t = 1, \dots, T$ . Introducing the symbols

$$(14) \quad \left\{ \begin{array}{l} r_1 = (1-\tau)r_0 \\ r_2 = (1-d)(1-\tau)r_0 \\ \delta = \left(\frac{\beta}{1-r_2}\right)^T \cdot \frac{r_1}{1-r_2} \cdot \frac{\beta r_2}{(1-\beta)(1-r_2-\beta)}, \quad (1-r_2 \neq \beta) \end{array} \right.$$

we obtain, if  $1-r_2 \neq \beta$ , after some manipulations

$$(15) \quad \lambda_t = \delta(1-r_2)^t - \frac{\beta^t r_1}{1-r_2-\beta}, \quad (t = 1, \dots, T)$$

To analyse the behaviour of  $\lambda_t$  if  $T \rightarrow \infty$  we distinguish three cases :

(i)  $1 - r_2 > \beta$ . If  $1-r_2 > \beta$  we obtain

$$\lim_{T \rightarrow \infty} \delta = 0$$

(16) and

$$\lim_{T \rightarrow \infty} \lambda_t = \frac{-\beta^t r_1}{1-r_2-\beta}$$

(ii)  $1-r_2 < \beta$ . If  $1-r_2 < \beta$  the limit of  $\delta$  does not exist, i.e.

$$\lim_{T \rightarrow \infty} \delta = -\infty$$

(iii) If  $1-r_2 = \beta$  we obtain from (11)

$$(17) \quad \lambda_t = \frac{-\beta^{t-1} r_1}{1-\beta}, \quad (t = 1, \dots, T)$$

In the next section we will use the expression

$$(18) \quad \frac{\beta^{t+1} - (1-d)\lambda_{t+1}}{\beta^t - (1-d)\lambda_t}$$

Expression (18) can be analysed under the assumptions that  $\partial R_t / \partial F_t = r_0$  ( $t = 1, 2, \dots, T$ ) and that  $T \rightarrow \infty$ . Distinguishing the three cases  $(1-r_2) > \beta$ ,  $(1-r_2) = \beta$  and  $(1-r_2) < \beta$  we obtain

a)  $(1-r_2) > \beta$  with

$$(19) \quad \lim_{T \rightarrow \infty} \lambda_t = \frac{-\beta^t r_1}{1-r_2-\beta}$$

and

$$(20) \quad \lim_{T \rightarrow \infty} \frac{\beta^{t+1} - (1-d)\lambda_{t+1}}{\beta^t - (1-d)\lambda_t} = \frac{\beta^{t+1}(1+(1-d)r_1/(1-r_2-\beta))}{\beta^t(1+(1-d)r_1/(1-r_2-\beta))} = \beta$$

b)  $1-r_2 = \beta$  with .

$$\lambda_t = \frac{-r_1 \beta^{t-1}}{1-\beta}$$

and

$$(21) \quad \lim_{T \rightarrow \infty} \frac{\beta^{t+1} - (1-d)\lambda_{t+1}}{\beta^t - (1-d)\lambda_t} = \frac{\beta^t (\beta + (1-d)r_1 / (1-\beta))}{\beta^{t-1} (\beta + (1-d)r_1 / (1-\beta))} = \beta$$

c)  $1-r_2 < \beta$  with .  $\lim_{T \rightarrow \infty} \delta = \infty$  and

$$(22) \quad \lim_{T \rightarrow \infty} \frac{\beta^{t+1} - (1-d)\lambda_{t+1}}{\beta^t - (1-d)\lambda_t} = \lim_{T \rightarrow \infty} \frac{\beta^{t+1} / \delta - (1-d)(1-r_2)^{t+1} + (1-d) \frac{\beta^{t+1} r_1}{\delta(1-r_2-\beta)}}{\beta^t / \delta - (1-d)(1-r_2)^t + (1-d) \frac{\beta^t r_1}{\delta(1-r_2-\beta)}} =$$

$$= \frac{(1-r_2)^{t+1}}{(1-r_2)^t} = (1-r_2)$$

Results (14) - (22) will be particularly useful in the analysis of the adjustment process of  $X_t$ , following from the first order conditions (9). Before deriving this adjustment process we will study a special case, viz. the case with marginal costs of adjustment,  $\partial Q / \partial X$ , equal to zero.



4. The adjustment path of optimal factor inputs if marginal adjustment costs are zero.

If the marginal costs of adjustment are zero, the first order conditions (10) can be simplified. Since by assumption

$$\frac{\partial Q_t}{\partial X_t} = 0 \quad (t = 1, \dots, T)$$

(23)

$$\frac{\partial Q_t}{\partial X_{t-1}} = 0 \quad (t=1, \dots, T-1)$$

we obtain, combining (9) and (23), after some manipulations

$$(\beta^t - \lambda_t(1-d)) \left( \frac{\partial Y_t}{\partial X_t} - w - \frac{\partial R_t}{\partial F_t} \cdot q \right) = 0 \quad (t = 1, \dots, T-1)$$

(24)

$$\left( \frac{\beta^T}{1-\beta} - \lambda_T(1-d) \right) \left( \frac{\partial Y_T}{\partial X_T} - w - \frac{\partial R_T}{\partial F_T} \cdot q \right) = 0$$

From (13) follows that in the optimum  $(\beta^t - \lambda_t(1-d)) > 0$  ( $t=1, \dots, T-1$ ) and  $(\beta^T/(1-\beta) - (1-d)\lambda_T) > 0$  so that (24) implies

$$\frac{\partial Y_t}{\partial X_t} = w + \frac{\partial R_t}{\partial F_t} \cdot q \quad (t=1, \dots, T-1)$$

(25)

$$\frac{\partial Y_T}{\partial X_T} = w + \frac{\partial R_T}{\partial F_T} \cdot q$$

The conditions in (25) are the well known equilibrium conditions in a static (or quasi-dynamic) profit maximizing model. The conditions in (25) differ from the conditions in a model without explicit financial constraints with respect to the presence of the marginal costs of funds function  $\partial R_t/\partial F_t$  in stead of the discount rate. This is so

because funds have explicit costs in our approach.

Let us write the solutions of (25) as  $(X_t^*, F_t^*)$ ,  $t = 1, \dots, T$ , then from (3) it follows

$$F_1^* = F_0 + q'(X_1^* - X_0) - (1-d)(1-\tau)C_1 + (1-d)D_0, \quad (t = 1)$$

$$F_t^* = F_{t-1}^* + q'(X_t^* - X_{t-1}^*) - (1-d)(1-\tau)C_t + (1-d)D_0, \quad (t=2, \dots, T)$$

or

$$(26) \quad \Delta F_t^* = q' \Delta X_t^* - (1-d)(1-\tau)C_t + (1-d)D_0, \quad (t=2, \dots, T)$$

Further follows from (25)

$$(27) \quad \frac{\partial Y}{\partial X}(X_t^*) - \frac{\partial Y}{\partial X}(X_{t-1}^*) = \left( \frac{\partial R}{\partial F}(F_t^*) - \frac{\partial R}{\partial F}(F_{t-1}^*) \right) q, \quad (t=2, \dots, T)$$

so that, using the mean value theorem

$$(28) \quad \Gamma_t \Delta X_t^* = b_t \Delta F_t^* q, \quad (t=2, \dots, T)$$

where  $\Gamma_t$  and  $b_t$  are second derivatives, evaluated in a point between  $X_t^*$  and  $X_{t-1}^*$  respectively between  $F_t^*$  and  $F_{t-1}^*$ .

Assuming that  $\Gamma_t$  can for all  $X_t^*$  ( $t = 1, \dots, T$ ) be approximated by a negative definite matrix  $\Gamma$ , that  $b_t$  can be approximated for all  $F_{t-1}^*, F_t^* > \tilde{F}$  by the positive constant  $b$  and that  $b_t = 0$  for all  $F_{t-1}^*, F_t^* \leq \tilde{F}$ , we obtain that (28) can be written as <sup>9)</sup>

$$(29) \quad \Gamma \Delta X_t^* = (b \Delta F_t^*) q \quad \text{if } F_{t-1}^*, F_t^* > \tilde{F}, \quad (t = 2, \dots, T)$$

$$= 0 \quad \text{if } F_{t-1}^*, F_t^* \leq \tilde{F}$$

9) The case  $F_t^* > \tilde{F}$  and  $F_{t-1}^* \leq \tilde{F}$  (or v.v.) requires special treatment.

See also footnote 8.

If  $F_{t-1}^*, F_t^* > \tilde{F}$  for some  $t \in (2, \dots, T)$  we obtain after combining (26) and (29)

$$(30) \quad \Delta X_t^* = (b q q' - \Gamma)^{-1} b q [(1-d)(1-\tau)C_t - (1-d)D_0]$$

and

$$\Delta F_t^* = (-1 + q'(-\Gamma + b q q')^{-1} q \cdot b)((1-d)(1-\tau)C_t - (1-d)D_0)$$

If  $F_{t-1}^*, F_t^* \leq \tilde{F}$  for some  $t \in (2, \dots, T)$  we obtain, combining (26) and (29)

$$(31) \quad \Delta X_t^* = 0$$

$$\Delta F_t^* = -(1-d)(1-\tau)C_t + (1-d)D_0$$

From (30) follows that, if  $F_{t-1}^*, F_t^* > \tilde{F}$  for  $2 \leq t \leq T_1$  (with  $T_1 \leq T$ ), the generation of internal funds implies changes in the optimal factor inputs and changes in the amount of external funds. The initial change in  $X$ ,  $X_1^* - X_0$ , can be interpreted as the market induced adjustment in  $X$ ; the changes  $\Delta X_t^*$ ,  $t \geq 2$ , can be interpreted as financially induced adjustments in  $X$ . To study the changes in optimal factor inputs,  $\Delta X_t^*$ , as a function of generated internal funds, we assume that there is a period  $t_0$  so that from period  $t_0$  onwards retained earnings are positive:

$$(32) \quad \forall t \geq t_0 : (1-d)(1-\tau)C_t - (1-d)D_0 > M > 0$$

Combining (30) and (32) we obtain, since

$$q'(b q q' - \Gamma)^{-1} q > 0$$

and

$$-1 + q'(-\Gamma + b q q')^{-1} q \cdot b = -1 + \text{tr}(-\Gamma + b q q')^{-1} (b q q') < 0$$

10)  
that

$$(33) \quad F_{t_0}^* > F_{t_0+1}^* > \dots > F_{T_1}^*$$

and

$$q' X_{t_0}^* \leq q' X_{t_0+1}^* \leq \dots \leq q' X_{T_1}^*$$

where  $q'X$  is the value of capital inputs.

If  $T \rightarrow \infty$  and (32) holds then  $F_t^*$  will in some period (the transition period) cross the critical value  $\tilde{F}$ . From (29) follows that the transition from the regime  $F_t^* > \tilde{F}$  to the regime  $F_t^* < \tilde{F}$  brings about some intricate mathematical problems. Without analysing these

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10) The matrix  $b q q' = b \begin{bmatrix} q_1^2 & 0 \\ 0 & 0 \end{bmatrix}$ ,  $b > 0$ , and  $-\Gamma$  is a positive definite matrix so that

$$\text{tr}(-\Gamma + b q q')^{-1} (b q q') < \frac{b q_1^2}{b q_1^2} = 1$$

problems in detail <sup>11)</sup> we limit our attention to the after-transition period where both  $\tilde{F}_{t-1}$  and  $\tilde{F}_t \leq \tilde{F}$ . Thus we can postulate:

$$(34) \quad \exists t_1 > t_0, \forall t \geq t_1: F_{t-1}^* > \tilde{F}, F_t^* \leq \tilde{F}$$

so that  $\forall t \geq t_1$ :

$$(35) \quad \frac{\partial R_t}{\partial F_t} = r_0 \quad ; \quad \frac{\partial^2 R_t}{\partial F_t^2} = 0$$

and from (31)

$$\Delta X_t^* = 0$$

11) Let period  $(T_1+1)$  be the period where the critical value  $\tilde{F}$  is exceeded, so that  $F_{T_1}^* > \tilde{F}$  and  $F_{T_1+1}^* \leq \tilde{F}$ , then we propose, analogous to (29), as linearisation of (27) for  $t = T_1+1$ :

$$\Gamma \Delta X_t^* = \delta b (\Delta F_t^*) q \quad , \quad 0 \leq \delta \leq 1$$

so that in combination with (26) we obtain analogous to (30)

$$\Delta X_t^* = (\delta b q q' - \Gamma)^{-1} \delta b q [(1-d)(1-\tau)C_t - (1-d)D_0]$$

(30A) and

$$\Delta F_t^* = (-1 + q'(-\Gamma + \delta b q q')^{-1} q \cdot \delta b) [(1-d)(1-\tau)C_t - (1-d)D_0]$$

For  $\delta = 1$  we obtain (30) and for  $\delta = 0$  we obtain (31) as special cases; for the intermediate cases  $0 < \delta < 1$  we expect from (30.A) a smaller change in optimal factor inputs than if  $\delta = 1$  and a larger change in  $F^*$  than if  $\delta = 0$ . Thus using (30) instead of (30.A) in the transition period we overestimate  $\Delta X_t^*$  but underestimate  $\Delta F_t^*$ .

For  $0 \leq F_t^* \leq \tilde{F}$  the parameter  $r_0$  measures the marginal costs of external funds. If  $F_t^* < 0$ ,  $r_0$  is equal to the marginal revenue on available internal funds, invested outside the firm, and can thus be interpreted as the marginal opportunity costs of (internal) funds.

If (32) and (34) hold we obtain from (35) that

$$(36) \quad X_t^* = X^*, \quad t \geq t_1$$

where  $X^*$  satisfies (25), i.e.

$$(37) \quad \frac{\partial Y}{\partial X}(X^*) = w + r_0 q$$

which is the well known equilibrium condition from static analysis.

Analogously we can show that if

$$\forall t \geq t_0: (1-d)(1-\tau)C_t - (1-d)D_0 < 0$$

the optimal value of  $F_t^*$  increases and the optimal value of  $q'X_t^*$  decreases. Eventually such a policy implies negative optimal values of  $q'X_t^*$  which is at variance with our basic assumptions ( $X_t^*$  lies in the economically relevant region for all  $t$ ).

Further we can show that sign-alterations of the term  $(1-d)(1-\tau)C_t - (1-d)D_0$  are impossible for all  $t \geq 1$ , given the constant price expecta-

tions, product demand function and technical structure.<sup>12)</sup>

Thus we can limit our attention to the case that

$$(38) \quad (1-d)(1-\tau)C_t - (1-d)D_0 > 0, \quad (t = 1, 2, \dots)$$

Thus if there are no adjustment costs and  $F_t > \tilde{F}$  for  $1 < t \leq T_1$ , we can distinguish a market induced adjustment in  $X$  in the first period and financially induced adjustments in  $X$  for  $t \geq 2$ . The adjustment of optimal factor inputs for  $t \geq 2$ , to an equilibrium value  $X^*$ , depends on the generation of internal funds. If the retained earnings are positive and  $T$  is large enough the equilibrium  $X^*$  is reached for some  $t > T_1$ . If  $(-F_0)$  is large enough so that  $F_t \leq \tilde{F}$  for all  $t \geq 1$  an immediate adjustment of  $X_t$  to  $X^*$  takes place.

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12) This can be understood as follows:

If  $\exists X_t > 0: (1-d)(1-\tau)C_t - D_0 > 0$  then the starting position in period  $t+1$  is at least as favourable as in period  $t$  so that

$$\bar{\exists} X_{t+1} > 0: (1-d)(1-\tau)C_{t+1} - D_0 > 0$$

for  $t = 1, 2, \dots$

However if  $\forall X_t > 0: (1-d)(1-\tau)C_t - D_0 < 0$  then the starting position in period  $(t+1)$  is less favourable than in period  $t$  so that

$$\nexists X_{t+1} > 0: (1-d)(1-\tau)C_{t+1} - D_0 > 0.$$

for  $t = 1, 2, \dots$

5. The analysis of the optimal adjustment path under positive marginal adjustment costs.

The analysis of the adjustment path if the marginal costs of adjustment are not zero is more complicated. We can rewrite the first-order conditions (9) as

$$(39) \quad \frac{\partial Y_t}{\partial X_t} = w + \frac{\partial R_t}{\partial F_t} \cdot q + \frac{\partial a_t}{\partial X_t} - \gamma_t \frac{\partial a_{t+1}}{\partial X_t}, \quad (t = 1, \dots, T-1)$$

$$\frac{\partial Y_T}{\partial X_T} = w + \frac{\partial R_T}{\partial F_T} \cdot q + \gamma_T \frac{\partial a_T}{\partial X_T}$$

where  $\gamma_t, \gamma_T$  are defined as

$$(40) \quad \gamma_t = \frac{\beta^{t+1} - \lambda_{t+1}(1-d)}{\beta^t - \lambda_t(1-d)}, \quad (t=1, \dots, T-1)$$

$$\gamma_T = \frac{\beta^T - \lambda_T(1-d)}{\frac{\beta^T}{1-\beta} - \lambda_T(1-d)}$$

We can write  $\gamma_t$  for  $t = 1, \dots, T-1$  as

$$\gamma_t = \frac{\beta^t - \frac{\lambda_{t+1}}{\beta}(1-d)}{\beta^t - \lambda_t(1-d)} = \beta \cdot \frac{1 - \frac{\lambda_{t+1}}{\beta^{t+1}}(1-d)}{1 - \frac{\lambda_t}{\beta^t}(1-d)}$$

Thus  $\gamma_t$  can be interpreted as the discount factor  $\beta$  times a measure of discounted marginal costs of funds from period  $(t+1)$  onwards relative to the discounted marginal costs of funds from period  $t$  onwards; see (12). For  $\gamma_T$  we obtain after some rearrangements

$$\gamma_T = (1-\beta) \left( 1 + \frac{\beta}{1-\beta} \frac{\partial R_T}{\partial F_T} \right)$$



so that  $\gamma_T$  is  $(1-\beta)$  times a measure of discounted marginal costs of funds from period  $T+1$  onwards.

Equation (39) can be simplified further if we linearize  $\partial Y_t / \partial X_t$ ,  $\partial Q_t / \partial X_t$  and  $\partial Q_{t+1} / \partial X_t$  around  $X^*$ , so that

$$Q(\Delta X_t) = \frac{1}{2}(\Delta X_t)' A \Delta X_t$$

and

$$\partial Q_t / \partial X_t = A \Delta X_t$$

$$\partial Q_{t+1} / \partial X_t = -A \Delta X_{t+1}$$

where  $A$  is evaluated in the point  $X^*$ . Using this linearization we can rewrite (39) as

$$(41) \quad \Gamma(X_t - X^*) = \left( \frac{\partial R_t}{\partial F_t} - r_0 \right) q + A \Delta X_t - \gamma_t A \Delta X_{t+1}, \quad (t=1, \dots, T-1)$$

$$\Gamma(X_T - X^*) = \left( \frac{\partial R_T}{\partial F_T} - r_0 \right) q + \gamma_T A \Delta X_T$$

The system (41) can be rewritten as a system of non-linear difference-equations in  $X_t$  with forcing function  $(\partial R_t / \partial F_t - r_0)q + \Gamma X^*$  and boundary conditions for  $t = 0$  and  $t = T$ . This nonlinear system of difference equations determines the optimal growth path of the firm.

For further analysis we will make some additional assumptions with respect to the flow of internal funds. We assume that from period  $t_0$  ( $t_0 \geq 1$ ) onwards the retained earnings  $(1-d)(1-\tau)C_t - (1-d)D_0$  are larger than the net investment expenses,  $q' \Delta X_t$ , so that for  $T$  large enough

$$(42) \quad \exists t_1, t_0 \leq t_1 \leq T, \forall t \geq t_1: F_t \leq \tilde{F}$$

whereas for  $t < t_1: F_t > \tilde{F}$ . From these assumptions and the balance

equation (3) follows that

$$\Delta F_t < 0 \quad , \quad (t_0 \leq t \leq T)$$

so that

$$(43) \quad \frac{\partial R_t}{\partial F_t} - \frac{\partial R_{t+1}}{\partial F_{t+1}} > 0 \quad , \quad (t_0 \leq t \leq t_1)$$

and

$$\frac{\partial R_t}{\partial F_t} = r_0 \quad , \quad (t \geq t_1)$$

Further follows from (18) - (22) for  $\gamma_t$ , for  $t_1 \leq t < T$  and  $T \rightarrow \infty$

$$(44) \quad \gamma_t = \begin{cases} \beta & \text{if } (1-r_2) \geq \beta & , (t_1 \leq t < T) \\ 1-r_2 & \text{if } (1-r_2) < \beta & , (t_1 \leq t < T) \end{cases}$$

and

$$\gamma_T = 1-\beta$$

Assuming that shareholders require a return on invested capital larger than the interest rate on borrowed funds we have for the discount rate

$$\frac{1-\beta}{\beta} > r_0$$

so that, if  $(1-d)(1-\tau) \leq \beta$ , we obtain

$$1-r_2-\beta = 1-\beta-(1-d)(1-\tau)r_0 > 0$$

so that, in combination with (44), we find

$$\gamma_t = \beta, \quad t_1 \leq t < T$$

Thus the effective discount rate to evaluate current and future cash-flows is  $(1-\beta)/\beta$ .<sup>13)</sup>

Under the assumptions (42) with corresponding results (43) and (44) we can greatly simplify the adjustment path generating conditions (41) for  $t \geq t_1$ . Substituting (43) and (44) in (41) we obtain

$$(45) \quad \begin{aligned} \Gamma(X_t - X^*) &= A \Delta X_t - \beta A \Delta X_{t+1}, \quad (t=t_1, t_{1+1}, \dots, T-1) \\ \Gamma(X_T - X^*) &= (1-\beta)A \Delta X_T \end{aligned}$$

for large T.

Thus from period  $t_1$  onwards the adjustment of  $X_t$  is similar to the adjustment path analysed in [ 3 ], which results in the well-known multivariate accelerator model. This implies that

$$(46) \quad \Delta X_{t_1} = B(X^* - X_{t_1-1})$$

where B is the adjustment matrix defined in [ 3, Section 4.2 ]<sup>14)</sup>

13) If  $1-r_2 < \beta$  or  $\frac{(1-\tau)(1-d)}{\beta} r_0 > \frac{1-\beta}{\beta}$  the effective discount rate would be based on the interest rate on borrowed funds.

14) B can be specified as

$$B = C(I - \Lambda)C^{-1}$$

where C is the matrix of characteristic vectors of the matrix  $A^{-1} \Gamma$  and  $\Lambda$  is a diagonal matrix with positive elements smaller than one.

and  $X^*$  is defined in (31).

Using (46) we can obtain from (41) a subset of first order conditions in  $X_1, X_2, \dots, X_{t_1-1}$  which is independent of  $X_t, t \geq t_1$ . We shall use the special case  $t_1 = 2$  in order to demonstrate the properties of the adjustment path. For the special case  $t_1 = 2$  (this implies  $F_1^* > \tilde{F}$ ) we obtain as subset

$$(47) \quad \Gamma(X_1 - X^*) = \left( \frac{\partial R_1}{\partial F_1} - r_0 \right) \cdot q + A \Delta X_1 - \gamma_1 A B (X^* - X_1)$$

or

$$(48) \quad [\Gamma - A - \gamma_1 A B] [X_1 - X_0] = [\Gamma - \gamma_1 A B] (X^* - X_0) + \left( \frac{\partial R_1}{\partial F_1} - r_0 \right) q$$

which is an equation in  $X_1$  and  $F_1$  with

$$(49) \quad \left\{ \begin{array}{l} \frac{\partial R_1}{\partial F_1} - r_0 > 0 \\ \lambda_2 = -\beta^2 r_1 / (1 - r_2 - \beta) \\ \text{so that} \\ \lambda_1 = \frac{\lambda_2 - \beta(1-\tau)\partial R_1/\partial F_1}{1 - (1-d)(1-\tau)\partial R_1/\partial F_1} < \frac{\lambda_2 - \beta r_1}{1 - r_2} = \frac{\lambda_2}{\beta} \\ \text{so that} \\ 0 < \gamma_1 = \frac{\beta^2(1-d)\lambda_2}{\beta - (1-d)\lambda_1} = \beta \frac{\beta - (1-d)\lambda_2/\beta}{\beta - (1-d)\lambda_2} < \beta \end{array} \right.$$

Lower values of  $\gamma_1$  slow down the adjustment speed. This can be easily understood if we rearrange (47) as

$$\left( \frac{\partial R_1}{\partial F_1} - r_0 \right) q + A \Delta X_1 = (\gamma_1 A B - \Gamma) (X^* - X_1)$$

The terms on the left hand side can be interpreted as the marginal

adjustment costs in period 1 whereas the right hand side measures the marginal opportunity costs of not being in the unconstrained optimum. A fall of  $\gamma_1$  decreases the opportunity costs and thus implies decreased adjustment<sup>15)</sup>.

15) We can write

$$\begin{aligned} \Delta X_1 = & -(I + \gamma_1 B - A^{-1} \Gamma)^{-1} A^{-1} \left( \frac{\partial R_1}{\partial F_1} - r_0 \right) q + \\ & + (I + \gamma_1 B - A^{-1} \Gamma)^{-1} (\gamma_1 B - A^{-1} \Gamma) (X^* - X_0) \end{aligned}$$

From the specification of the adjustment matrix B in footnote 8 follows that

$$\gamma_1 B - A^{-1} \Gamma = C(\gamma_1(I - \Lambda) + M)C^{-1}$$

where M is the diagonal matrix of (positive) eigenvalues of  $(-A^{-1} \Gamma)$ , so that

$$(I + \gamma_1 B - A^{-1} \Gamma)^{-1} = C(I + \gamma_1(I - \Lambda) + M)^{-1} C^{-1}$$

and

$$\begin{aligned} (I + \gamma_1 B - A^{-1} \Gamma)^{-1} (\gamma_1 B - A^{-1} \Gamma) = & C(I + \gamma_1(I - \Lambda) + M)^{-1} \cdot \\ & \cdot (\gamma_1(I - \Lambda) + M)C^{-1} \end{aligned}$$

A fall of  $\gamma_1$  implies larger values of the diagonal matrix  $(I + \gamma_1(I - \Lambda) + M)^{-1}$  and smaller values of the diagonal matrix  $(I + \gamma_1(I - \Lambda) + M)^{-1} (\gamma_1(I - \Lambda) + M)$  and thus smaller values of  $\Delta X_1$ .

Assuming that  $\tilde{F} < F_1 \leq (1+\delta)\tilde{F}$ , where  $\delta$  is small, we may linearize  $\partial R_1/\partial F_1$  analogous to (29), i.e.

$$(50) \quad \partial R_1/\partial F_1 = r_0 + b(F_1 - \tilde{F})$$

where  $F_1$  follows from the financial balance equation (3)

$$F_1 = F_0 + q'(X_1 - X_0) + D_1 - (1-\tau)C_1$$

Substituting (50) in (48) we obtain

$$(51) \quad [\Gamma - A - \gamma_1 A B] [X_1 - X_0] = [\Gamma - \gamma_1 A B] [X^* - X_0] + bq(F_1 - \tilde{F})$$

Combining (3) and (51) we obtain

$$(52) \quad [\Gamma - A - \gamma_1 A B - b q q'] [X_1 - X_0] = \\ = [\Gamma - \gamma_1 A B] [X^* - X_0] - bq [(1-\tau)C_1 - D_1 - F_0] - bq \tilde{F}$$

where  $((1-\tau)C_1 - D_1 - F_0)$  is the sum of retained earnings,  $(1-\tau)C_1 - D_1$ , in period 1 and available internal funds,  $-F_0$ , at the start of period 1.

From (52) follows, defining the positive definite matrix

$$E = [-\Gamma + A + \gamma_1 A B + b q q']$$

$$(53) \quad [X_1 - X_0] = E^{-1} [\gamma_1 A B - \Gamma] [X^* - X_0] + E^{-1} b \cdot q [(1-\tau)C_1 - D_1 - F_0] \\ + E^{-1} bq \cdot \tilde{F}$$

so that the first period change in factor inputs is a "weighted" sum of the unconstrained change in factor inputs  $[X^* - X_0]$  and total internal funds  $[(1-\tau)C_1 - D_1 + F_0]$  except for the constant term  $E^{-1} bq \cdot \tilde{F}$ . We can rewrite (53) as

$$(54) \quad [X_1 - X_0] = [X^* - X_0] - E^{-1} A [X^* - X_0] \\ - E^{-1} b q [q'(X^* - X_0) - (1-\tau)C_1 - D_1 - F_0] + E^{-1} b q \tilde{F}$$

where the third term on the right hand side measures the direct impact of financial constraints. Since, due to high adjustment costs in the first period  $(1-\tau)C_1 - D_1$  will probably be low, the main source of available funds is  $F_0$ , so the financing of current investments is directly linked to cumulated retained earnings in the past.

Since by assumption  $F_1 > \tilde{F}$ , or

$$(55) \quad \tilde{F} + (-F_0 + (1-\tau)C_1 - D_1) \leq q'(X_1 - X_0)$$

we obtain after combining (52) and (55)

$$(56) \quad [-\Gamma + A + \gamma_1 A B] [X_1 - X_0] \leq [-\Gamma + \gamma_1 A B] [X^* - X_0]$$

or

$$(57) \quad [X_1 - X_0] \leq [-\Gamma + A + \gamma_1 A B]^{-1} [-\Gamma + \gamma_1 A B] [X^* - X_0]$$

Result (57) can be used to compare the adjustment of factor inputs under financial constraints with the adjustment of factor inputs if the financial constraints are not effective in the sense that

$$(58) \quad \frac{\partial R_1}{\partial F_1} = r_0$$

From (47) and (44) follows that if (58) holds, we obtain for the factor adjustment under constant marginal costs of funds,  $(X_1 - X_0)^c$ ,

$$(59) \quad (X_1 - X_0)^c = [-\Gamma + A + \beta A B]^{-1} [-\Gamma + \beta A B] [X^* - X_0]$$

Comparing (57) and (59) we obtain that the difference between  $(X_1 - X_0)^c$

and actual adjustment  $(X_1 - X_0)$  is at least equal to

$$(60) \quad \{[-\Gamma + A + \beta A B]^{-1} [-\Gamma + \beta A B] - [-\Gamma + A + \gamma_1 A B]^{-1} \\ \cdot [-\Gamma + \gamma_1 A B]\}(X^* - X_0)$$

so that, since  $\gamma_1 < \beta$ <sup>16)</sup>, if  $(X_1 - X_0)^c > 0$

$$(X_1 - X_0) < (X_1 - X_0)^c$$

Thus we conclude that, if  $(X_1 - X_0)^c > 0$ , increasing marginal costs of funds cause a slower adjustment of factor inputs.<sup>17)</sup>

16) We can rewrite expression (60) as

$$\{[I - A^{-1}\Gamma + \beta B]^{-1} [-A^{-1}\Gamma + \beta B] - [I - A^{-1}\Gamma + \gamma_1 B]^{-1} [-A^{-1}\Gamma + \gamma_1 B]\}(X^* - X_0)$$

and using the notation of footnote 12 we can write for the term between brackets

$$C[(I + M + \beta(I - A))^{-1}(M + \beta(I - A)) - (I + M + \gamma_1(I - A))^{-1}(M + \gamma_1(I - A))]C^{-1}$$

which implies that for  $\gamma_1 < \beta$  this matrix is positive definite.

17) If only  $(X_{11} - X_{10})^c > 0$  we obtain from (60) and  $\gamma_1 < \beta$

$$(X_{11} - X_{10}) < (X_{11} - X_{10})^c$$

If  $(X_1 - X_0)^c < 0$  or  $(X_{11} - X_{10})^c < 0$ , (57) and (60) give contrary results so that the difference between  $(X_1 - X_0)$  and  $(X_1 - X_0)^c$  (or  $(X_{11} - X_{10})$  and  $(X_{11} - X_{10})^c$ ) is indeterminate.



We conclude that the adjustment of factor inputs in period 1 depends on the difference  $X^* - X_0$ , the availability of internal funds, the level of marginal costs of funds in the first period relative to the level of marginal costs of funds in subsequent periods, measured through the parameter  $\gamma_1$ , and on the adjustment matrix A. These results only hold in the special case that  $t_1 = 2$  (i.e. from period 2 onwards:  $\partial R_t / \partial F_t = r_0$ ). If  $t_1 > 2$  the solution is more complicated but basically analogous to the solution obtained in this section. Thus we may expect a similar adjustment path: in the first periods financial constraints will slow down the adjustment of factor inputs. If the cumulation of retained earnings relaxes the financial constraints a further adjustment to the new optimal values of factor inputs is possible.

From this analysis follows that if internal funds are abundant, financing of investments from internal funds and/or external funds will be possible without exceeding the critical D/E ratio. Thus the marginal costs of funds will be constant and the "classical" adjustment model applies. However if, e.g. in a recession, internal funds are small, the financing of new investments requires external funds in such amounts that the critical D/E ratio will be exceeded. Consequently the marginal costs of external funds will depend on the generation of cash flows so that both the cash flows and the interest on external funds are important financial determinants of investments. These theoretical results correspond with the empirical findings of Meyer and Kuh [ 12] and Meyer and Glauber [ 11] .

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