PUBLIC GOODS AND INPUT–OUTPUT ANALYSIS

by

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Research memorandum

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Summary

The economic structure of the Leontief model, which is characterized by its linearity assumptions, is expanded to include also public goods. The input-output table is correspondingly rearranged. This allows for explicitly introducing public goods in the cost structure of production sectors and for a refinement of the policy-oriented relations in input-output analysis.

Contents

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1. Introduction.

The national accounts can be decomposed into an input-output table in order to analyse the flows of commodities and services within an economy. Without an economic structure behind these data, an input-output table cannot be more than an interesting statistical representation of figures. Leontief [1936] developed the economic structure than suited very well to the assembling of data through an input-output table, and made a strong, policy oriented instrument of it. The Leontief model has a cost- or input-structure consisting of intermediate deliveries of sector products, and of primary inputs. The supply or output-structure is determined by the necessary intermediate deliveries and the final demands. One of the sectors of final demands is the government sector, which is assumed not to influence the input- or cost-structure. The assumption is not harmful if the government sector is relatively small, or if the effects of government activities on the various sectors are hardly distinguishable or observable. In these cases, all government expenditures can be considered as consumption from which only households benefit (and should therefore bear the costs). This point of view is also expressed in the definition of national income, used in the national accounts.

The relative growth of the public sector and the interest of business and government for sector policy, cast doubt on the validity of the assumption mentioned above.
It is the purpose of this paper to enrich the economic structure of the Leontief model such that public goods enter the cost structure of the sectors, and that its linearity characteristics are still present. The effects on the input-output representation will be analysed.

2. The Leontief structure.

Leontief has simplified the general equilibrium model by adapting the following assumptions:

(A.1) The economy consists of m production sectors, k sectors of final demands and h sectors of primary inputs.

(A.2) Every production sector produces one commodity, according to a linear production function in which all inputs (intermediate and primary) are perfect complements.

(A.3) All commodities are private goods and have a uniform price compatible with the production structure.

Let x_{ij} denote the flow of commodity i to sector j, and p_i being the price of commodity i. Further, y_{ij} is the amount of primary input i for sector j, having price w_i.

Let $c_i := \sum_{j=1}^{k} x_{ij}$, the total final demand for commodity i.

Assumption (A.2) implies that for each sector j the production function is defined by:

$$(2.1) \qquad x_j := \min \left\{ \frac{x_{1j}}{a_{1j}}, \ldots, \frac{x_{mj}}{a_{mj}}, \frac{y_{1j}}{b_{1j}}, \ldots, \frac{y_{hj}}{b_{hj}} \right\} ,$$

given $a_{ij}, b_{ij} \geqq 0$. It follows that the input required to produce the commodity bundle $x \in R^m$ is: a commodity bundle $Ax \in R^m$ and a primary input bundle $Bx \in R^h$.

From (2.1) can be derived that (under competitive conditions) the price of commodity j equals:

$$(2.2) \qquad p_j = p_1 a_{1j} + \ldots + p_m a_{mj} + w_1 b_{1j} + \ldots + w_h b_{hj} ,$$

which is equal to the weighted sum of all inputs per unit output j, weighted by their respective prices.

Equilibrium is defined by: (A) quantity supplied equals quantity demanded for each commodity, and (B) its market price equals the cost price, i.e.:

$$(2.3) \qquad \begin{aligned} Ax + x &= x \\ Bx &= y \end{aligned}$$

$$(2.4) \qquad pA + wB = p$$

The matrices A and B can be estimated empirically from input-output tables in which the elements are expressed in terms of money values rather than in quantities (or prices). Let $\hat{P}$, $\hat{W}$, $\hat{X}$ and $\hat{Y}$ denote diagonal matrices with the elements of p, w, x and y respectively in the diagonal. The input-output table is represented by:

$$(2.5) \qquad \begin{array}{ll|l} \hat{P}A\hat{X} =: \bar{\bar{X}}_x & \hat{P}x =: \bar{C} & \hat{P}x =: \bar{X} \\ \hat{W}B\hat{X} =: \bar{\bar{Y}}_x & & \hat{W}y =: \bar{Y} \\ \hline p\hat{X} =: \bar{X}' & pc =: C & \end{array}$$

The coefficients which are obtained by dividing each column by the total value $p_j x_j$ produced by each sector, are $\bar{a}_{ij} := p_i a_{ij}/p_j$ and $\bar{b}_{ij} := w_i b_{ij}/p_j$. They indicate the money amount of each input i necessary to produce a unit value of output j. From (2.4) follows that each column adds up to unity:

$$(2.6) \qquad \begin{array}{lll} \bar{A} := \hat{P}A\hat{P}^{-1} & = & \bar{\bar{X}}_x \hat{\bar{X}}^{-1} \\ + \underline{\quad \bar{B} \quad} := \underline{\hat{W}B\hat{P}^{-1}} & = & \underline{\bar{\bar{Y}}_x \hat{\bar{X}}^{-1}} \\ (1,\ldots,1) \quad (1,\ldots,1) & & (1,\ldots,1) \end{array}$$

These coefficients are used to calculate e.g. the multiplier effects of a bundle $\bar{C}$ of exogeneous demand, giving $\bar{X} = (1-\bar{A})^{-1} \bar{C}$.

The input-output table contains usually at least the following sectors of final demand, resp. primary inputs:

N : exports of goods and services
U : gross fixed capital formation (incl. stocks)
C : private final consumption expenditure
G : government final consumption expenditure
M : imports of goods and services
V : consumption of fixed capital (depreciation)
W : compensation of employees
R : operating surplus
T : indirect taxes less subsidies

Table (2.5) is thus expanded to:

(2.7)

$\bar{\bar{X}}_X$	X_N	X_U	X_C	X_G	X
M_X	M_N	M_U	M_C	M_G	M
V_X	-	-	-	V_G	V
W_X	-	-	-	W_G	W
R_X	-	-	-	-	R
T_X	T_N	T_U	T_C	T_G	T
X	N	U	C	G	

Net national income at market prices is defined to be equal to W + R + T, which is equal to G + C + (U-V) + (N-M). Since the last two terms (net investment and net exports are irrelevant to our problem, we will assume that they equal zero and delate the corresponding rows and columns from following tables.

3. Desaggregation of the government sector.

The government sector of final demand, G, has been considered as an aggregate of all expenditures by the government. This sum will be replaced by the public goods categories for which the expenditures have been made, such as defense, police, civil service management, education, health service, transportation and environment. The following assumption will serve that purpose:

(A.4) The government produces n public goods. Each such good is produced according to a linear production function in which all inputs are private goods and perfect complements.

This assumption implies, just like (A.2), that the production of a public good z_j is defined by:

(3.1) $$z_j := \min \{\frac{x_{1j}}{\underline{a}_{1j}}, \ldots, \frac{x_{mj}}{\underline{a}_{mj}}, \frac{y_{1j}}{\underline{b}_{1j}}, \ldots, \frac{y_{hj}}{\underline{b}_{hj}}\},$$

where $\underline{a}_{ij}, \underline{b}_{ij} \geqq 0$. The price r_j of public good j is then equal to

(3.2) $$r_j = p_1\underline{a}_{ij} + \ldots + p_m\underline{a}_{mj} + w_1\underline{b}_{ij} + \ldots + w_h\underline{b}_{hj}.$$

Assume further that the indirect taxes T are levied proportional to the quantity produced by each sector:

(A.5) Taxes paid by each sector are proportional to the quantity produced by that sector: $t_j = p_{Tj}x_j$, and $t_0 = w_T y$.

This implies that the equilibrium conditions (2.3) and (2.4) must be replaced by (p_H denotes market prices and w_F are wages paid by firms):

$$(3.3)\qquad \begin{array}{l} Ax + c + \underline{A}z = x \\ Bx + \underline{B}z = y \\ p_H A + w_F B + p_T = p_H \\ p_H \underline{A} + w_F \underline{B} = r \end{array}$$

The input-output table (2.5) has to be replaced by:

$$(3.5)\qquad \begin{array}{lll|l} \hat{P}_H A\hat{X} =: \bar{\bar{X}}_x & \hat{P}_H c =: \bar{C} & \hat{P}_H \underline{A}\hat{Z} =: \bar{X}_G & \hat{P}_H x =: \bar{X} \\ \hat{W}_F B\hat{S} =: \bar{\bar{Y}}_x & & \hat{W}_F \underline{B}\hat{Z} =: \bar{Y}_G & \hat{W}_F y =: \bar{Y} \\ p_T\hat{X} =: \bar{T}_x & & & p_T x =: T \\ \hline p_H\hat{X} =: \bar{X}' & p_H c =: C & r\hat{Z} =: \bar{G} & \end{array}$$

From (3.3) and (3.4) follows that:

$$(3.6)\qquad \begin{array}{l} p_H c + rz = w_F y + p_T x \quad , \quad \text{or:} \\ C + G = Y + T \quad . \end{array}$$

This is again equal to national income at market prices, with $Y := W+R$, as defined in the previous section. It endorses the point of view that all government expenditure is non-private final consumption by the households. The expenditures are financed partly directly by the households ($w_T y := w_F y - w_H y$, where w_H denotes the after tax income), and partly through indirect taxes ($p_T x := p_H - p_F x$). Although this choice about financing has welfare implications (see below), it is not considered as a matter of principle within the economic framework. The only problem in this view is that the market prices p_H are distorted by the taxes p_T and are not giving information any more about the correct scarcety and production relations. This is only done by pre-tax, or factor-prices p_F, which should therefore be considered as correct export prices.

Before introducing assumptions which allow for a more general approach, and a different point of view, I will indicate that the desaggregation of government activities contributes to the Leontief model as an instrument of economic analysis.

From (3.3) can be deduced the distribution of primary inputs for private demand and public demand:

$$(3.7) \qquad y = B(I-A)^{-1} c + [B(I-A)^{-1} \underline{A} + \underline{B}] z.$$

The effect of an autonomous change in the provision of some public good on the demand for primary inputs can thus be calculated. From (3.4) follows:

$$(3.8) \qquad p_H = w_F B(I-A)^{-1} + p_T(I-A)^{-1}$$

$$(3.9) \qquad r = w_F[B(I-A)^{-1}\underline{A} + \underline{B}] + p_T(I-A)^{-1}\underline{A}.$$

It is evident that the choice between indirect taxes ($p_T > 0$) and direct taxes ($w_T > 0$) has influence on the price structure.
Since $(I-A)^{-1}$ is a nonnegative matrix, it is highly improbable that the effects of p_T are proportional to the factor prices p_F or the market prices p_H. In this case substitution and income effects can be induced through consumers' demand. The same is true for the prices of public goods r.

If some price vector and all quantities are kept constant, a charge in the ratio direct/indirect taxes will also change the value of national income. It is easily checked from (3.6, 3.8 and 3.9) that, given factor remunerations w_F, indirect taxation ($p_T > 0$) increases the value of national income. This increase is of course compensated by an increase in disposable income ($w_H y$) and market prices p_H, as long as the equilibrium conditions remain volid. The same is true

if p_H is given: let $p_T =: p_T^B B$ and next abolish indirect taxes; then wages increase with p_T^B and prices of public goods r increase with $p_T^B \underline{B}$. From (3.6) follows that national income increases, p_H, C and z being constant.
Finally, estimates of the matrices $\underline{A}$ and $\underline{B}$ can be obtained empirically by dividing each column in table (3.5) by the value of its sum $\overline{G}$:

$$(3.10) \quad \frac{\begin{matrix} \overline{A} \\ \overline{B} \end{matrix}}{(1...1)} \begin{matrix} := \\ := \end{matrix} \frac{\begin{matrix} \hat{P}\underline{A}\hat{R}^{-1} \\ \hat{W}\underline{B}\hat{R}^{-1} \end{matrix}}{(1...1)} \begin{matrix} = \\ = \end{matrix} \frac{\begin{matrix} \overline{X}_G \hat{\overline{G}}^{-1} \\ \overline{Y}_G \hat{\overline{G}}^{-1} \end{matrix}}{(1...1)}$$

Although the cost structures of private goods (2.6) and of public goods (3.10) are in fact quite similar, the matrices A and $\overline{A}$ do not play a similar role in the economic system because only A refers to intermediate deliveries and is therefor necessarily a square matrix.
The intermediate role of public goods is introduced in the next section.

4. Public goods as inputs in production.

It is an established fact that market prices p_H (and the cost structures of an input-output model) do not generally give correct information about scarcety and productivity relations, if indirect taxes (p_T) are levied. If these taxes, however, are related to the benefits which sectors receive from the provision of public goods, the situation is changed. In this section we will assume that public goods increase the productivity of sectors, and derive the benefit (and tax) for the respective sectors.
Since public goods are supplied in equal amount to all agents in the economy, but their valuation in terms of money will probably be different for each agent, the following table will be valid, where r_{ij} denotes the money price of one unit public good i to be paid for by sector j, and r_{i0} the

money price paid by the households:

(4.1)

z_1r_{11}	... z_1r_{1m}	z_1r_{10}	z_1r_1
⋮	⋮	⋮	⋮
+ z_nr_{n1}	... z_nr_{nm}	z_nr_{n0}	z_nr_n
$p_{T1}x_1$	... $p_{Tm}x_m$	w_Ty	

In table (4.1) has been expressed the equilibrium conditions that the cost of each public good (z_ir_i) should be equal to its benefits, and that the sum of these benefits for some sector i should be equal to the taxes ($p_{Ti}x_i$) paid by that sector. See also (4.6) below.

The contributions r_{ij} can only be determined if the production technology is adequately defined. In order to maintain the linear structure of the model, the public goods are assumed to be perfect substitutable inputs in the production function of each sector. This assumption is implied by the dual structure of private and public goods, and is not based on empirical investigations. Presumably, it is feasible to suppose more flexibility regarding to changes in government policy than to inputs in private goods, necessary for production. Assumptions (A.1-3) are thus replaced by:

(A.6) The economy consists of m production sectors of private goods, 1 sector of final demand, h sectors of primary inputs and 1 government sector producing n public goods.

(A.7) Every production sector produces one private good according to a linear production function in which the private goods (i.e. intermediate products and primary factors) are perfect complements, and the public goods are perfect substitutes. All public goods together are complements of private goods.

The production function (2.1) is replaced by:

$$(4.2)\quad x_j =: \min\{\frac{x_{1j}}{a_{1j}},\dots,\frac{x_{mj}}{a_{mj}},\frac{y_{1j}}{b_{1j}},\dots,\frac{y_{hj}}{b_{hj}}\ ,\ (z_1 d_{1j}+\dots+z_n d_{nj})\},$$

where a_{ij}, b_{ij}, $d_{ij} \geqq 0$.

It follows that the input necessary to produce a commodity bundle $x \in R^m$ is equal to: a commodity bundle $Ax \in R^m$, a bundle of primary inputs $Bx \in R^h$, and a bundle of public goods $z \in R^n$ such that $zD = x$.
From (3.2) can be derived that the costprice of commodity j is equal to:

$$(4.3)\quad p_j = (p_1 a_{1j}+\dots+p_m a_{mj}+w_1 b_{1j}+\dots+w_h b_{hj}+\min\{\frac{r_{1j}}{d_{1j}},\dots,\frac{r_{nj}}{d_{nj}}\}),$$

with $p_{Tj} = \frac{r_{1j}}{d_{1j}} = \dots = \frac{r_{nj}}{d_{nj}}$, if the r_{ij} are equilibrium values.

The model is determined by the assumptions (A.6),(A.7),(A.4) and (A.5).
Let equilibrium be defined by the conditions:

$$(4.4)\quad \begin{aligned} Ax + c + \underline{A}z &= x \\ Bx + \underline{B}z &= y \end{aligned}$$

$$(4.5)\quad \begin{aligned} pA + wB + p_T &= p \\ p\underline{A} + w\underline{B} &= r \end{aligned}$$

$$(4.6)\quad \begin{aligned} Dp_T' + r_0' &= r' \\ z'D &= x' \end{aligned}$$

From these conditions follows that:

$$(4.7)\quad \begin{aligned} pc + r_0 z &= wy \quad , \quad \text{or:} \\ C + Z_c &= Y \quad , \quad \text{and} \end{aligned}$$

$$(4.8)\quad \begin{aligned} p_T x + r_0 z &= rz \quad , \quad \text{or:} \\ Z_x + Z_c &= Z \quad . \end{aligned}$$

Equilibrium now implies that the household sector (final consumption of private and public goods equals factor-income), and the government sector (benefits of public goods equal costs) have both to be in equilibrium. This was not required in the provious section: see (3.6).
It also has implications on the definition of national income. The input-output table can be written as:

$$
(4.9)\qquad
\begin{array}{ccc|c}
\hat{P}A\hat{X} & \hat{P}c & \hat{P}\underline{A}\hat{Z} & \hat{P}x \\
\hat{W}B\hat{X} & & \hat{W}\underline{B}\hat{Z} & \hat{W}y \\
\hat{Z}D\hat{P}_T & \hat{Z}r'_0 & & \hat{Z}r' \\
\hline
p\hat{X} & wy & r\hat{Z} &
\end{array}
\quad +
$$

or equivalently:

$$
\begin{array}{ccc|c}
\bar{\bar{X}}_x & \bar{X}_c & \bar{\bar{X}}_G & \bar{X} \\
\bar{\bar{Y}}_x & - & \bar{\bar{Y}}_G & \bar{Y} \\
\bar{\bar{Z}}_x & \bar{Z}_c & & \bar{Z} \\
\hline
\bar{X}' & Y & \bar{Z}' &
\end{array}
\quad +
$$

National income at market-prices is redefined to be equal to all factor income which can be spend on private and public consumption according to the preferences of the households:

$$(4.10)\qquad Y := X_c + Z_c = Y_x + Y_G .$$

According to the former definition, national income was equal to $X_c + Z = X_c + Z_c + Z_x$. In our point of view, those benefit of public goods which can be imputed to the production sectors, should be withdrawn from national income.

5. Observations about policy and specification.

From equation systems (4.4) - (4.6) can be deduced the impact of a bundle of public goods on final consumption and on primary inputs (apart from the still valid equations (3.7)-(3.9)):

$$(5.1)\qquad c = [\,(1-A)D'-\underline{A}]\,z$$

$$(5.2)\qquad y = [\,BD'+\underline{B}]\,z$$

The equilibrium conditions also imply that the net benefits of a unit bundle of public goods to production sectors and to households (r_0) equal the value of primary input spend per unit bundle:

$$(5.3)\qquad p[\,(1-A)D'-\underline{A}] + r_0 = w[\,BD'+\underline{B}]\,.$$

Problems arise however in connection with the rank of the matrices. The system (4.4)-(4.6) counts 3m + 2n + h equations and 4m + 3n + 2h variables. Given (m+n+h) variables, e.g. c, r_0 and y, the other variables can be determined, if no inconsistencies occur in the model. This is the case if, for example, the supply of public goods is equal to the equilibrium supply for all sectors except for some sector who needs more or less of it. In the first case, public goods are a bottle in production for that sector, which is willing to pay more than equilibrium taxes; in the second case, public goods are considered free by that sector[1].
If the model were formulated in terms of inequalities, these economic problems would not have been translated in

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1) Weddepohl suggested to me to consider in A.7 the public goods as substitutes to all private goods, instead of complements. This would give some relief in the quantity space, but replace the problems to the price space.

mathematical inconsistencies, such as a situation in which there does not exist a z such that $D'z = x$, for a given x. These difficulties are got round if we restrict the choice of exogeneous variables such that:

a) in case $n \leqq m$ (not more public goods than sectors): the rank of matrix D' is equal to the rank of the matrix $[D',x]$, where x is the column of exogeneous variables;
b) in case $n \geqq m$: the rank of D is equal to the rank of $[D,r']$, with r' the column of exogeneous variables in the price space.

This restriction is less active as the rank of D is higher. It must be remarked that this restriction also has to be applied to the well known matrix B, and to the less known matrices $\underline{A}$ and $\underline{B}$. The matrix A is supposed to be regular, as is (1-A).

The choice of the number of public goods (and of production sectors) is a problem related to the theory of index numbers. Criteria are aspects such as: the relative weight of the different categories; the similarity of characteristics within each category. Since the matrix D receives its structure from the differences between categories <u>and</u> from the differences in valuation of a category between the sectors, this aspect should also be taken into account. The information about D has to be received from the production sectors, which are not necessarily inclined to give correct information even if they can give it. This is related with the incentive problem to reveal correct preferences about public goods, and forms another problem to be solved. If, for example, every expansion of public goods that a sector demands must be withdrawn from its factor income, benefits from public goods will be underestimated unless a shortage in supply hurts all concerned still more. But if public goods are supplied freely, i.e. by the households, benefits will be overestimated by the production sectors.

Whatever procedure to determine the coefficients of the matrix D will be chosen, presumably it will depend on situations out of equilibrium.
In Leontief models, however, not so much the matrices A and B are estimated as well the matrices $\overline{A}$ and $\overline{B}$, which are expressed in terms of money values (see 2.6). Using the notation of table (4.9), the corresponding matrices are defined by:

$$
(5.4)\qquad
\begin{array}{rclcl}
\overline{A} & := & \overline{\overline{X}}_x . \hat{\overline{X}}^{-1} & = & \hat{P}A\hat{P}^{-1} \\
\overline{B} & := & \overline{\overline{Y}}_x . \hat{\overline{X}}^{-1} & = & \hat{W}B\hat{P}^{-1} \\
+\ \overline{D} & := & \overline{\overline{Z}}_x . \hat{\overline{X}}^{-1} & = & (\hat{Z}D\hat{\mathbf{P}}_T)(\hat{P}\hat{X})^{-1} \\
\hline
e & := & (1,\dots,1) & & (1,\dots,1)
\end{array}
$$

and

$$
(5.5)\qquad
\begin{array}{rclcl}
\overline{\underline{A}} & := & \overline{\overline{X}}_G . \hat{\overline{Z}}^{-1} & = & \hat{P}\underline{A}\hat{R}^{-1} \\
+\ \overline{\underline{B}} & := & \overline{\overline{Y}}_G . \hat{\overline{Z}}^{-1} & = & \hat{W}\underline{B}\hat{R}^{-1} \\
\hline
e & := & (1,\dots,1) & & (1,\dots,1)
\end{array}
$$

The problems related with getting correct information about the matrix $\overline{\overline{Z}}_x$ are not changed, of course. But the advantage of this approach is that, bu using prices as weights in aggregation, the cost structure of production can be made clear through (5.4) and (5.5), replacing (4.5), and that (4.4) and (4.5) can be replaced by:

$$
(5.6)\qquad
\begin{array}{lllll}
\overline{AX} + \overline{X}_c & + \overline{\underline{A}Z} & = & \overline{X} \\
\overline{BX} & + \overline{\underline{B}Z} & = & \overline{Y} \\
\overline{DX} + \overline{Z}_c & & = & \overline{Z}
\end{array}
$$

The effect of a change in final consumption on total production (and on the demand on primary inputs and public goods) can be deduced from (5.6):

$$\begin{aligned} \overline{X} &= (I-\overline{A}-\overline{\underline{A}D})^{-1}\ (\overline{X}_c+\overline{\underline{A}Z}_c) \\ \overline{Y} &= (\overline{B}+\overline{\underline{B}D})(I-\overline{A}-\overline{\underline{A}D})^{-1}\ (\overline{X}_c+\overline{\underline{A}Z}_c) + \overline{\underline{B}Z}_c \\ \overline{Z} &= \overline{D}(I-\overline{A}-\overline{\underline{A}D})^{-1}\ (\overline{X}_c+\overline{\underline{A}Z}_c) + \overline{Z}_c \end{aligned} \tag{5.7}$$

The matrix $(I-\overline{A}-\overline{\underline{A}D})^{-1}$ can be considered as a generalization of the multiplier $(I-\overline{A})^{-1}$ of the original Leontief model. The equations (5.7) and (5.8) are generalizations of the Leontief relations:

$$\begin{aligned} \overline{X} &= (I-\overline{A})^{-1}\ \overline{X}_c \\ \overline{Y} &= \overline{B}(I-\overline{A})^{-1}\overline{X}_c \end{aligned} \tag{5.8}$$

According as public goods get more important in the economic structure, it will be necessary to replace the system (5.8) by the system (5.7). The policy recommendations are then based upon a richer economic structure.

In order to clarify these results, I have made an attempt to apply this theory on data from the Dutch economy in 1972. It must be remarked that insufficient information is avaible to get correct results, so the input-output table (2) obtained must be considered as an example.
The cost structure $\overline{\overline{X}}_G$ and $\overline{\overline{Y}}_G$ (or $\overline{\underline{A}}$ and $\overline{\underline{B}}$) can be estimated from publications of the Central Bureau of Statistics. More problems are caused by the benefit structure $\overline{\overline{Z}}_x$ and $\overline{\overline{Z}}_c$ (or $\overline{D}$). The following assumption is made:

(A.8) The benefits of all public goods for some (production) sector are equal to the indirect taxes (minus subsidies) paid by that sector. The difference between cost and the sum of these benefits of public goods are benefits for the households.

Presumably, it would be more adequate to consider also direct taxes paid by the sectors as indicators of the benefits of public goods. But since much better information is required to give correct estimates, and since the purpose here is mainly to give an example, assumption 8 will do. The distribution of the benefits within a sector is guessed, subject to the constraint that for each public good the costs are equal to the sum of the benefits over the sectors. The result is given in table 2, which can be compared with the traditional input-output model given in table 1.

The vector of the costs of public goods is equal to the vector of benefits of public goods, due to assumption 8. This equality follows in an equilibrium situation from the conditions (4.5) and (4.6), giving:

$$p\underline{A} + w\underline{B} = r = p_T D' + r_o \ ,$$

which results in (after postmultiplying by $\hat{Z}$ and transposing):

$$(e\overline{\underline{A}\hat{Z}} + e\overline{\underline{B}\hat{Z}})' = \overline{Z} = \overline{DX} + \overline{Z}_c$$

These equilibrium conditions correspond to the conditions defined in more general economic models with public goods (see Milleron [1972] and Ruys [1974]).

Finally, redistribution aspects are not treated in this paper, because the emphasis is put on the use of commocities rather than the financing or the decision aspects (see e.g. Keller [1976]). Further specification of the benefits of public goods used in production is necessary not only to get correct information about the cost structure of an economy (and to get correct prices), but also to support any sector approach in economic policy.

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Table 1. Input-output table of the Netherlands, 1972.
(source: Nationale rekeningen 1974, CBS, den Haag)

	(1) agric.	(2) ind.	(3) serv.	(1-3)	(4) exports	(5) conx.	(6) investm.	(7) gov.	(4-7)	totals	
1. agriculture*)	18,7	0,5	1,0	20,2	12,5	13,6	0,5	0,2	26,8	47,0	
2. industry	2,7	28,2	6,1	37,0	36,8	12,3	16,6	7,8	73,5	110,5	
3. services	2,7	6,6	10,7	20,0	16,7	36,8	2,5	1,5	57,5	77,5	
(1-3)	24,1	35,3	17,8	77,2	66,0	62,7	19,6	9,5	157,8	235,0	X
4. imports	7,1	26,9	6,3	40,3	3,1	13,7	7,3	0,8	24,9	65,2	M
5. depreciation	1,5	4,6	5,9	12,0	-	-	-	0,9	0,9	12,9	V
6. factor income	13,4	41,4	44,9	99,7	-	-	-	18,3	18,3	118,0	Y
7. net indirect taxes	1,0	2,3	2,6	5,8	0,3	5,9	2,5	1,3	10,0	15,8	T
(4-7)	22,9	75,2	59,7	157,8	3,4	19,5	9,8	21,3	54,1	211,9	
8. totals	47,0	110,5	77,5	235,0	69,5	82,3	29,4	30,7	211,9		
				X	N	C	U	G			

Domestic income (in market prices) $= Y+T = G+C+ (U-Y) + (N-M) = 133,8$

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*) aggregation: agriculture: sectors 1-6
industry: sectors 7-22
services: sectors 23-35.

Table 2. Input-output table with public goods:
conjectures based on data of the Netherlands, 1972
and on the assumptions 4-8 in the paper.

	(1) agric	(2) ind	(3) serv	(1-3)	(4) exp	(5) cons	(6) inv	(4-6)	(A) civil	(B) def	(C) educ	(D) mise	(A-D)	totals	
1. agriculture	18,7	0,5	1,0	20,2	12,5	13,6	0,5	26,6	0,2	-	-	-	0,2	47,0	
2. industry	2,7	28,2	6,1	37,0	36,8	12,3	16,6	65,7	3,8	1,6	2,0	0,4	7,8	110,5	
3. services	2,7	6,6	10,7	20,0	16,7	36,8	2,5	56,0	0,9	0,2	0,2	0,2	1,5	77,5	
(1-3)	24,1	35,3	17,8	77,2	66,0	62,7	19,6	148,3	4,9	1,8	2,2	0,6	9,5	235,0	X
4. imports	7,1	26,8	6,3	40,3	3,1	13,7	7,3	24,1	0,2	0,6	-	-	0,8	65,2	M
5. depreciation	1,5	4,6	5,9	12,0	-	-	-	-	0,4	0,3	0,2	-	0,9	12,9	V
6. factor income	13,4	41,4	44,9	99,7	-	-	-	-	7,9	3,3	7,1	-	18,3	118,0	Y
(4-6)	22,0	72,8	57,1	152,0	3,1	13,7	7,3	24,1	8,5	4,2	7,3		20,0	196,1	
A. civil task	0,8	1,0	1,0	2,8	0,1	9,9	0,5	10,6						13,3	
B. defense	-	0,3	0,6	0,9	0,1	4,5	0,5	5,1						6,0	
C. education	0,1	1,0	1,0	2,1	0,1	5,8	1,5	7,4						9,5	
D. miscalaneous	-	-	-	-	-	0,6	-	0,6						0,6	
(A-D)	0,9	2,3	2,6	5,8	0,3	20,9	2,5	23,6						29,5	Z
totals	47,0	110,5	77,5	235,0	69,4	97,3	29,4	196,1	13,4	6,0	9,5	0,6	29,5		
				X	N	C	U						Z		

Domestic income = Y = C + (U-V) + (N-M) = 118,0
(given ass. 4-8)

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