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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

IDENTIFICATION AND ESTIMATION OF
HOUSEHOLD PRODUCTION MODELS

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Identification and Estimation of Household Production Models

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Abstract

A convenient framework for an empirical model of time allocation data was derived in Gronau (1977). The fundamental dichotomy property of this model can be employed to derive a behavioural model for home production decisions without having to specify a household utility function. Even if the model is extended to allow for specific types of direct utility from household production activities (so called joint production) the dichotomy property prevails. We discuss the identification properties of this model for one and two adult households. Next, we turn to a specific formulation, that generalizes the Cobb-Douglas model used in Graham & Green (1984). It is argued that the original model is not well-specified and that estimates of the level of home production from that model are determined by arbitrary normalising assumptions, implicit in the model specification. In fact it is shown, that in the more general model, estimates of the level of output of household production cannot be calculated from data on the time inputs only. The modified model is applied to time-allocation data from the Swedish HUS-data. The estimation results provide further insights into the limitations of the specification used and of the use of Gronau's dichotomy property in empirical applications.

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1 Introduction

For several of the issues that motivate the analysis of the allocation of time within households, a purely descriptive, reduced form analysis of time allocation figures does not suffice. One such issue is the evaluation of the output of home production in an economy. This quantity can be estimated by the total value of the production factors that were used. Normally, however, we do not have information about all of these inputs (e.g., auxiliary goods). But even if we could measure all the quantities of the factors that were employed, this approach would systematically ignore any returns to scale and therefore the value added in home production. Furthermore, we would have to know the value of an individual's time. For those individuals, that have a job, their value of time can be set equal to their net marginal wage rate. But for individuals that chose not to participate in the labour force, no wage rate is observed and although a potential wage rate could be estimated, this would only constitute a lowerbound for the value of their time.

Determining an individual's value of time is also important for different types of questions, for example in studying labour supply or job search. Knowledge of the household production structure provides us with a method for estimating the value of time for everyone. Estimation of the value of time of economic agents is therefore another issue for which structural models of household production decisions may be required.

A micro-economic framework within which this kind of decisions can be formalized, was developed by Gary Becker (1965). Together with Lancaster (1966) this article served as a starting-point for what came to be known as the *household production approach.* In these models the preferences of a household are not defined in terms of quantities of goods and non-labour time, but rather in terms of activities or household products that are produced with the aid of these goods and time endowments. As a flexible model for the micro-economic decision process this approach has proved to be of great value, in particular in theoretical analyses. The implicit need for a detailed description of activities and the introduction of production functions, about the outputs of which typically no information is available, has prohibited empirical applications of this framework. With only slightly more detailed information about the time uses, the researcher now has to identify a two-layered utility factory. More specifically, we only know that some observed combination of inputs (time and goods) was used in producing a set of (latent) activities, that was preferred over all other sets of activities that could have been produced by feasible combinations of inputs. For an empirical analysis one is forced to cut down on the level of detail in which the activities are defined and one is bound to impose further restrictions in order to be able to disentangle the roles of the production structure and the preferences.

An elegant way of doing so was proposed by Gronau (1977, 1980). The central assumption in these articles is that home produced commodities and market goods are perfect substitutes. This assumption and the high degree of aggregation of activities considerably simplify an empirical analysis and lead to explicit testable predictions. In the first article Gronau still used a tobit-model to test for these properties, but in Gronau (1980) the framework was put to use in deriving a structural econometric specification.

Pollak & Wachter (1975) argued that people don't always spend their time exclusively on one activity at a time. Even in the highly aggregated Gronau model it is likely to be the case that the time people spend on home production activities is also partially considered as leisure. Graham & Green (1984) estimated a version of the Gronau model in which they allowed for this type of *joint production.* This extension raises the question of identifiability once again. Moreover, the estimates Graham & Green presented for American time allocation data were not very promising. Estimates of this model for Dutch and Swedish data were even less satisfactory.

In the next section we will discuss the Gronau model and joint production in more detail and present some results on the (non-parametric) identifiability whithin this class of models. In section 3 we will consider a specific formulation. This model slightly generalizes the model that was used by Graham & Green. It will be argued that the differences between these models can partially explain the poor record of the Graham & Green specification. These differences also lead to different assessments as to whether the complete household production function can be identified from time allocation data only. An empirical application to the Swedish HUS-data will be presented in section 4. Section 5 contains some concluding remarks.

2 The Gronau Model

In a neo-classical analysis of time allocation decisions, the marginal value of time for each of the activities the individual decides to partake in, will be set equal. In principle, these relationships can serve as a point of departure for an empirical analysis. For an individual with a paid job the monetary (marginal) value of time is given by his or her net marginal wage rate. A comparison to the monetary marginal value of time in household activities, however, still involves the household's marginal utility of money, the marginal utilities of these activities and the marginal productivities of time used in producing them. Moreover, as the output of home production and some of the inputs are unobserved the effect of the utility functions can in general not be distinguished from that of the production functions. Identification in that general a setting can only be forced by the choice of specific functional forms.

Gronau (1978, 1980) noted, that the analysis will be substantially simplified by the assumption that output of the home production process is a perfect substitute for goods that can be bought in the market. For the employed, the trade-off between spending one more hour on home production and selling this hour in the labour market, will thereby be reduced to a comparison of the wage rate and the marginal product value of household production time. In this way the role of the utility function is limited to the decision to allocate the time that is not used for home production activities to leisure and work.

In the Gronau model the general framework of Becker (1965) is simplified in the following sense. In the utility function three types of arguments are distinguished (cf. the activities in Becker's model): consumption goods, commodities produced at home and leisure of each of the household members[1]. The consumption goods enter the utility function as a Hicksian composite good. This commodity can be considered as *total consumption expenditures* and will be denoted by X_m. The output of home production will also be modelled as a composite good. This commodity, denoted by Z, corresponds closest to Becker's concept of an activity: it is produced by combining an amount of auxiliary goods (X_z) with home production time (H_m and H_f, for the male and the female partner respectively). The time people do not spend on working, at home or in a job, is considered to be (pure) leisure. The crucial assumption in this model is that consumption goods X_m and the output of household production Z are perfect substitutes. We can therefore write the utility function as follows

$$U(X_m + Z, L_m, L_f), \tag{1}$$

where L_m and L_f are leisure enjoyed by the male and female partner respectively. U is assumed to be strictly increasing, differentiable and quasi-concave. Efficient production is characterized by the household production function

[1] We will only model the behaviour of the head of the household and – if present – his or her partner. We can therefore distinguish two types of households: with one and with two adults. The presence of children or any other persons in the household will be treated as exogenous in the model. For this analysis we will take the two adult family as the standard case.

$$Z = Z(H_m, H_f, X_z). \tag{2}$$

Z is assumed to be a monotonically increasing and twice differentiable function of the production factors. The optimal choice of purchases and time uses will be subject to the household budget constraint:

$$X_m + X_z = V + W_m N_m + W_f N_f, \tag{3}$$

stating that total expenditures[2] are equal to the sum of non-labour income (V) and labour income, where W_i is the net wage rate of partner i and N_i is the number of hours he or she works in a paid job. Finally, a feasible allocation must satisfy the following time constraints

$$H_i + L_i + N_i = T, \qquad i = m, f; \tag{4}$$

T being the (daily) time endowment.

The household's decision problem can now be formalized as follows: the household members maximize their joint utility as defined in (1), subject to (2), (3), (4) and non-negativity constraints on X_m, X_z and L_i, N_i, and H_i ($i = m, f$). We will assume that by the choice of the utility function and the production function none of the inequalities will be binding in an optimal allocation, except those of N_m and N_f. This assumption is made for every combination of wage rates, non-labour income and other exogenous variables, that we consider of interest. Moreover, it will be assumed that for each of these combinations the optimum is unique. The solution to the household's decision problem then has to satisfy the following Kuhn-Tucker conditions (ξ_m and ξ_f are the shadow prices of the inequality constraints on labour time; restrictions (2), (3) and (4) have been substituted into the utility function):

$$\frac{\partial Z}{\partial X_z} = 1 \tag{5}$$

$$\frac{\partial U}{\partial Z}\,\frac{\partial Z}{\partial H_m} = \frac{\partial U}{\partial L_m} = \frac{\partial U}{\partial Z}\,\frac{\partial Z}{\partial X_z} W_m + \xi_m \tag{6}$$

$$\frac{\partial U}{\partial Z}\,\frac{\partial Z}{\partial H_f} = \frac{\partial U}{\partial L_f} = \frac{\partial U}{\partial Z}\,\frac{\partial Z}{\partial X_z} W_f + \xi_f \tag{7}$$

$$\xi_m N_m = \xi_f N_f = 0 \tag{8}$$

$$\xi_m \geq 0,\ \xi_f \geq 0,\ N_m \geq 0,\ N_f \geq 0.$$

If in the optimum both partners participate in the labour force ($N_m > 0$, $N_f > 0$ and $\xi_m = \xi_f = 0$) equations (5) to (8) imply:

[2]The price level of the composite good is normalized at one.

$$\frac{\partial Z}{\partial H_i}(H_m, H_f, X_z) = W_i, \qquad i = m, f. \tag{9}$$

Equations (9) and (5) mirror the fundamental dichotomy in the Gronau model. Provided that both partners have a job, the quantities of goods and time devoted to household production are chosen without recourse to the utility function. In deriving a sub-model that describes the household production decision we therefore only need to specify the production function.[3]

For someone with a paid job, the existence of a unique interior optimum implies that the production function has to be strictly concave in terms of his or her time input and in X_z, at the optimal allocation. As a result, an increase in one's wage rate will lead to a reduction of the number of hours spent on home production activities. If the time inputs are complements in home production, the partner's home production time will also be reduced. The reverse holds if the time inputs are substitutes. With regard to changes in non-labour income, this model predicts the absence of an income effect in the home production decisions of working couples. An increase of the non-labour income will – if leisure is a normal good – reduce both partners' number of hours worked in a job. But, provided both stay in the labour force, H_m, H_f and X_z will remain unchanged. An overall increase in household productivity leads to an incipient increase in the value of time. For individuals with a job, this will lead to a reduction in labour time and an increase in both leisure and home production time. When it is no longer optimal to participate in the labour force, a further rise in productivity might even lead to a decrease in household production time, provided the income effect is sufficiently strong. Productivity rises will therefore only increase the marginal value of time for individuals that do not participate in the labour force.

For two-earner households this model implies a two-stage decision structure, in which the production decisions constitute the first stage and the remaining decisions – the allocation of non-production time and the purchase of consumption goods – are made in the second stage. The utility function only figures in the second stage. It must be noted, however, that the labour force participation decisions do also involve the utility function. This is important for econometric applications of the first-stage sub-model ((5) and (9)). This model only applies for the non-random sub-sample of working couples. For a structural model that would incorporate this endogenous sample stratification, the utility function has to be specified. The first stage decisions for the working couple can be stated by the following (partial) optimization problem:

$$\max_{H_m, H_f, X_z} Z(H_m, H_f, X_z) - W_m H_m - W_f H_f - X_z. \tag{10}$$

Pollak & Wachter stressed that the presence of joint production poses a problem in modelling household production. In this relatively aggregated setting possibilities for joint production between activities are mostly contained in the production function.

[3]Note that the assumption that the net marginal wage rate is (considered to be) constant is vital for the dichotomy property to hold. Otherwise, N_m or N_f will enter the right hand sides of (9).

One important type of joint production is still excluded in this model: household production time that is also perceived as leisure. An extension that fits into the Gronau framework was suggested by Graham & Green (1984). Their specification can be interpreted as follows: of the H_i hours individual i spends on home production, he or she considers $g_i(H_i)$ as a perfect substitute for leisure. The *jointness function* $g_i(.)$ will be assumed to be increasing, twice differentiable and concave in H_i. Furthermore it seems reasonable to require $g_i'(.) \leq 1$ and $\lim_{H\uparrow T} g_i'(H) = 0$.[4] The direct utility function (1) is thus replaced by:

$$U(X_m + Z, L_m + g_m(H_m), L_f + g_f(H_f)). \tag{11}$$

It is easy to see that the dichotomy property survives. The partial optimization problem for the household production decisions of working couples now reads:

$$\max_{0\leq H_m, H_f \leq T; X_z \geq 0} Z(H_m, H_f, X_z) + W_m g_m(H_m) + W_f g_f(H_f) \\ -W_m H_m - W_f H_f - X_z. \tag{12}$$

The first order conditions are:

$$\frac{\partial Z}{\partial X_z}(H_m, H_f, X_z) = 1 \tag{13}$$

$$\frac{\partial Z}{\partial H_m}(H_m, H_f, X_z) = W_m(1 - g_m'(H_m)) \tag{14}$$

$$\frac{\partial Z}{\partial H_f}(H_m, H_f, X_z) = W_f(1 - g_f'(H_f)). \tag{15}$$

For a solution of these equations to be a local maximum, strict concavity of Z is still sufficient, but it is no longer necessary. In fact, some increasing returns to scale of household production time is allowed, provided that the extent to which this time is perceived leisure falls more. In line with the assumptions we made for the model without joint production, we assume Z, g_m and g_f are such that for each pair of wage rates that we consider, a unique solution to maximization problem (12) exists. This solution is assumed to be an interior point of $[0, T] \times [0, T] \times \mathbb{R}_+$. Clearly, therefore, the objective function in (12) has to be (locally) strictly concave in the optimal allocation.

An empirical model for the subsample of households in which both partners have a job, can now be based on equations (13), (14) and (15). In Gronau (1980) this is done by specifying a semi-log functional form for the marginal production function of women. There is no joint production in his model and the role of the male partner's time and auxiliary goods are ignored. In Graham & Green (1984) the production function itself is specified. The Cobb-Douglas specification and the corresponding

[4]Graham & Green also assume $\lim_{H\downarrow 0} g_i'(H) = 1$. Moreover, they require g_i to be strictly concave.

form of the jointness functions they employed will be discussed at length in the next section. They solved the system of first order equations and used the resulting reduced form equation for H_f to estimate the parameters of Z, g_m and g_f.

These applications raise the question whether the equations that were estimated are only identified by the specific functional forms that were used or that – given sufficiently many observations – we can distinguish the correct specification from any other. Put differently, the question is whether or not the class of extended Gronau models contains sub-classes of models that are observationally equivalent, if estimation is based on (13), (14) and (15) or on reduced form equations derived from these. In a model for household production in which household products and market goods are no perfect substitutes, we are unable to distinguish between the utility function and the production function, without further assumptions about the functional forms. The assumption that both types of commodities are perfect substitutes eliminates this ambiguity by removing the utility function from the sub-model for the home production decisions of the employed. By introducing jointness functions into the model, we also reintroduce elements of the utility function and with it, potential identification problems. In order to know what significance can be attributed to estimated household production levels and whether a methodology based on choosing more flexible functional forms makes sense, we have to investigate what parts of this model can be identified non-parametrically on the basis of equations (13), (14) and (15), using data on H_m, H_f and – possibly – X_z.

The first thing to notice is that we only observe allocations, that are optimal for some input vector (W_m, W_f, V). The relationships we want to estimate may thus be unobservable on parts of the choice set. To make this point more precise, we have to start by specifying what (W_m, W_f, V) can be observed. Input vectors may be unobservable for several reasons. There might be institutional restrictions, like minimum wage legislation. Wage rates are likely to be bounded above by a person's productivity. Furthermore, as we restrict our attention to households in which both partners have a paid job, extremely low values of W_m or W_f and high values of V will not be present in the sample. Define the set of input vectors (W_m, W_f, V) that are observable to be Ω.[5] Non-labour income influences the labour force participation decision, but given that both partners are employed, V has no further effect on the household production decisions. We will therefore usually refer to pairs of wage rates rather than to elements of Ω. For that reason we define the set of observable wage rates as $\mathcal{W} = \{(W_m, W_f) \in \mathbf{R}^2_+ \mid (W_m, W_f, V) \in \Omega$, for some $V \in \mathbf{R}\}$. By the assumptions made before, we can attribute a unique optimal allocation (H^*_m, H^*_f, X^*_z) to each input vector in $\mathcal{W}$. Now $\mathcal{H}$ is defined as the set of pairs (H_m, H_f) that are optimal for some input vector from Ω. This step may involve a further reduction in what we are able to observe, among other things due to non-concavities in Z. We will assume that $\mathcal{W}$ is a connected subset of $\mathbf{R}^2_+$. Given the assumptions about Z,

[5]Starting from a sampling distribution for (W_m, W_f, V) on $\mathbf{R}^3_+$, Ω can be defined as the support of this distribution.

g_m and g_f, made before, $\mathcal{H}$ is a connected subset of $[0,T] \times [0,T]$.

If we could observe the output of home production, the identification problem would be that observations of the production function are limited to $\mathcal{H}$. This type of identification problem is not specific for the household production model, but is in fact common to almost every structural micro-econometric model. In practice, however, we do not have information about the output of home production, so that we can only 'observe' the first order conditions. Even in the absence of joint production we cannot observe the relationship between production factors and output directly, but we can only scan the curvature of the production function by varying the input vectors over $\mathcal{W}$. Moreover, as the price of auxiliary goods is normalized at one, the information we have, can at most describe this curvature along a two-dimensional non-linear variety in (H_m, H_f, X_z)-space.

For each (H_m, H_f), the values for X_z that could make this an optimal choice at some $W \in \mathcal{W}$, have to satisfy (13). This means that X_z will be chosen such as to maximize the net production given H_m and H_f:

$$\tilde{Z}(H_m, H_f) \stackrel{\text{def}}{=} \max_{X_z \geq 0} Z(H_m, H_f, X_z) - X_z. \tag{16}$$

It will be assumed that for all elements of the closure of $\mathcal{H}$ this maximization problem has a unique finite, non-zero solution.[6] This choice of X_z will be denoted by $\psi_Z(H_m, H_f)$. In order to ensure that ψ_Z is differentiable on the closure of $\mathcal{H}$, we will also assume that $\partial Z / \partial X_z$ is continuously differentiable.[7]

Time allocation surveys usually contain information on H_m and H_f, but hardly ever on the amount of auxiliary goods that is used in home production activities. If no information about X_z is available , it is clear that we can at most identify the net product value function $(\tilde{Z})$ on $\mathcal{H}$. However, even if we do observe X_z, the previous reasoning indicates, that we still cannot identify Z non-parametrically. But the additional information will improve the identifiability of the production function if we restrict our choice to some parametric class of models. An example of this is the Cobb-Douglas model that will be discussed in the next section. Without data on X_z we cannot identify all parameters of the production function, because the net product value function $\tilde{Z}$ is in that case also of the Cobb-Douglas type. If, on the other hand, we observe X_z, all parameters are identified.

In this discussion we will ignore small-sample considerations and treat the relationships about which we have observations as perfectly known on the set of observable combinations of W_m, W_f, V, H_m, H_f and X_z. For any given household production

[6] A sufficient condition is that the marginal product of auxiliary goods is a strictly decreasing function of X_z, is greater than one for $X_z = 0$ and eventually falls below one when X_z is increased (for every $(H_m, H_f) \in \mathcal{H}$).

[7] Given the assumptions we made before, differentiability of ψ_Z follows from the Implicit Function Theorem.

model (Z, g_m, g_f) observational equivalence with another specification (F, k_m, k_f) requires the first order conditions to be identical, i.e.[8]

$$\frac{\partial Z}{\partial H_m}(H_m, H_f, \psi_Z(H_m, H_f)) = \frac{1 - g_m'(H_m)}{1 - k_m'(H_m)} \frac{\partial F}{\partial H_m}(H_m, H_f, \psi_F(H_m, H_f)) \quad (17)$$

$$\frac{\partial Z}{\partial H_f}(H_m, H_f, \psi_Z(H_m, H_f)) = \frac{1 - g_f'(H_f)}{1 - k_f'(H_f)} \frac{\partial F}{\partial H_f}(H_m, H_f, \psi_F(H_m, H_f)), \quad (18)$$

for all $(H_m, H_f) \in \mathcal{H}$. If the values of X_z are also observed we must also have:

$$\psi_Z(H_m, H_f) = \psi_F(H_m, H_f), \quad (19)$$

for all $(H_m, H_f) \in \mathcal{H}$. These equations guarantee that for each observable pair of wage rates, the same values for H_m, H_f and – if observed – X_z satisfy the respective first order conditions. From (17) and (18) it can be seen that the capability to separate the effects of the jointness functions and the production function in the observed relationships, draws heavily upon the fact that the jointness functions are individual-specific, while the production function also contains cross-effects. Accordingly, it is important that one partner's optimal choice of H_i $(i = m, f)$ is influenced by a change in the wage rate of the other partner, except possibly on a subset of $\mathcal{W}$ which has Lebesgue measure zero. We will therefore assume that:

$$\frac{\partial^2 Z}{\partial H_m \partial X_z}(H_m, H_f, \psi_Z(H_m, H_f)) \frac{\partial \psi_Z}{\partial H_f}(H_m, H_f) + \frac{\partial^2 Z}{\partial H_m \partial H_f}(H_m, H_f, \psi_Z(H_m, H_f)) \neq 0 \text{ a.e. on } \mathcal{H}. \quad (20)$$

This assumption establishes that the subset of $\mathcal{H}$, on which the derivative of the left hand side of (17) with respect to H_f vanishes, has an empty interior.

The next proposition states that it is in general possible to identify the presence of joint production in a model for two adult households. Only if the jointness functions for both partners are identical and linear, an observationally equivalent model without joint production exists. On the other hand, the proposition implies that the specific functional form of the joint production is not completely identified non-parametrically; even if X_z is observed. In general there exist one dimensional equivalence classes of observationally equivalent specifications. These implications will be elaborated in the four points following the proposition. It stands to reason that we restrict our attention to models for household production in which the production function and jointness functions satisfy the regularity conditions (monotonicity, differentiability etc.) we imposed above. We will refer to these models as being *admissible*.

[8] By the assumption that for every $W \in \mathcal{W}$ the household's utility maximization problem has a unique interior optimum, the slopes of the jointness functions are less than one in optimal allocations.

Proposition 1 *Given an admissible household production model characterized by a production function Z and jointness functions g_m and g_f, there exists an admissible model characterized by F, k_m and k_f that satisfies (17), (18) and (19) on $\mathcal{H}$, only if for some non-negative constant ϑ*

$$\frac{1 - g'_m(H_m)}{1 - k'_m(H_m)} = \frac{1 - g'_f(H_f)}{1 - k'_f(H_f)} = \vartheta, \tag{21}$$

for every $(H_m, H_f) \in \breve{\mathcal{H}}$, where[9]

$$\breve{\mathcal{H}} \stackrel{\text{def}}{=} \{h \in \mathcal{H} \mid \text{span}\,(\cap_{n=1}^{\infty} \overline{\text{cone}}\,\{x \in \mathbf{R}^2 \mid h + x \in \mathcal{H} \text{ and } \|x\| < n^{-1}\}) = \mathbf{R}^2\}.$$

Proof: In the Appendix.

The implications of this proposition for identification in the extended Gronau model will be discussed in the following four points.

1. What does this proposition tell us about the ability to determine whether joint production is present? Suppose we cannot distinguish a given model from some alternative specification without joint production. In that case (21) becomes: $1 - g'_m(H_m) = 1 - g'_f(H_f) = \vartheta$ on $\breve{\mathcal{H}}$. This implies that the only case in which we cannot conclude whether or not there is joint production, is when the jointness functions of both partners are linear and have identical slopes on $\breve{\mathcal{H}}$. Generically, therefore, the *presence* of joint production is identified.

2. Although identification of the presence of joint production is generically possible, the specific functional forms are in general not completely identified. For a given household production model (Z, g_m, g_f), Proposition 1 can be used to construct a range of observationally equivalent models, indexed by ϑ. From (21) we can derive

$$k_i(H_i) = \frac{\vartheta - 1}{\vartheta} H_i + \frac{g_i(H_i)}{\vartheta}, \qquad i = m, f. \tag{22}$$

Provided $\vartheta \geq 1 - \min\{\inf_{\breve{\mathcal{H}}} g'_m(H_m), \inf_{\breve{\mathcal{H}}} g'_f(H_f)\}$, these functions satisfy all the conditions we imposed on jointness functions, except for the boundary condition $\lim_{H \uparrow T} k'(H) = 0$. As long as $\breve{\mathcal{H}}$ does not contain combinations (H_m, H_f), for which either of the two variables takes on values arbitrarily close to T, the jointness functions are not identified in the neighbourhood of T, and the definitions above can be adapted arbitrarily so as to satisfy the boundary condition. A specification of F that would then satisfy the equivalence relationships (17), (18) and (19) is:

$$F(H_m, H_f, X_z) \stackrel{\text{def}}{=} Z(H_m, H_f, X_z) + \frac{1 - \vartheta}{\vartheta} \{Z(H_m, H_f, \psi_Z(H_m, H_f)) - \psi_Z(H_m, H_f)\}. \tag{23}$$

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[9]In words: $\breve{\mathcal{H}}$ is the set of elements of $\mathcal{H}$, from which we can move along at least two paths in $\mathcal{H}$, with linearly independent directional derivatives in the starting point. This local property is necessary to be able to differentiate equivalences on $\mathcal{H}$.

This specification is well-defined and admissible on $\mathcal{H} \times \mathbf{R}_+$ and can in general be extended to an admissible specification on $[0,T] \times [0,T] \times \mathbf{R}_+$.[10] If X_z is not observed, the choice of F is much easier. Take for example:

$$F(H_m, H_f, X_z) \stackrel{\text{def}}{=} \frac{1}{\vartheta} Z(H_m, H_f, \vartheta X_z). \tag{23'}$$

The existence of observationally equivalent specifications may seem to be slightly disappointing, but in fact the identification is far stronger than might have been expected. Having a sufficiently large amount of data at our disposal, we will be able to identify (g_m, g_f) non-parametrically within the class of admissible pairs of jointness functions on $\mathcal{H}$, up to one degree of freedom. An instance in which we may even get rid of this last degree of freedom is discussed in the next point.

3. If the closure of $\breve{\mathcal{H}}$ contains any points for which $H_m = T$ or $H_f = T$, i.e. if we can observe values for H_m or H_f that are arbitrarily close to T, (non-parametric) identification on $\breve{\mathcal{H}}$ is established, because

$$\vartheta = \lim_{H \uparrow T} \frac{1 - g_i'(H)}{1 - k_i'(H)} = 1, \qquad \text{for } i = m \text{ or } i = f; \tag{24}$$

implying $g_m'(H_m) = k_m'(H_m)$ and $g_f'(H_f) = k_f'(H_f)$, for $(H_m, H_f) \in \breve{\mathcal{H}}$.

4. If we restrict the jointness functions to be elements of some parametric class of functions, they are identified if no other functions (k_m, k_f) in that class satisfy (21) for some $\vartheta \neq 1$. As an application of this result it is easy to verify that the jointness functions, that are used in the next section are identified within that parametric class of specifications.

Identification of joint production in models for one adult households is much weaker. In the following proposition it will be shown that the presence of joint production is not (non-parametrically) identified, even if observations on X_z are available. Define $\mathcal{W}$ and $\mathcal{H}$ by analogy to the two adult case. Necessary conditions for a model with home production function Z and jointness function g to be equivalent with a model with production function F, but *no* joint production, is that the following conditions are satisfied on $\mathcal{H}$

$$\frac{\partial Z}{\partial H}(H, \psi_Z(H)) = (1 - g'(H)) \frac{\partial F}{\partial H}(H, \psi_F(H)) \tag{25}$$

$$\psi_Z(H) = \psi_F(H). \tag{26}$$

$\mathcal{H}$ now is an interval. Define $H_0 = \inf \mathcal{H}$ and $H^* = \sup \mathcal{H}$. We can then derive the following result.

[10]If we would restrict our attention to globally strictly concave production functions, it is easy to show that whenever Z is strictly concave, so is F (on $\mathcal{H} \times \mathbf{R}_+$).

Proposition 2 *Let an admissible one adult household production model be characterized by a production function Z and a jointness function g. If the set of observable wage rates $\mathcal{W}$ is bounded above, the production function F defined by*

$$F(H,X) = Z(H,X) + \frac{g'(H_0)}{1-g'(H_0)}\,\frac{\partial Z}{\partial H}(H_0,\psi_Z(H_0))\min(H_0,H)$$
$$+\int_{\min(H_0,H)}^{\min(H,H^*)}\frac{g'(\tilde H)}{1-g'(\tilde H)}\,\frac{\partial Z}{\partial H}(\tilde H,\psi_Z(\tilde H))\,d\tilde H \qquad (27)$$
$$+\frac{g'(H^*)}{1-g'(H^*)}\,\frac{\partial Z}{\partial H}(H^*,\psi_Z(H^*))\max(0,H-H^*)$$

satisfies (25) and (26) and is admissible. If Z is strictly concave on $[0,T]\times[0,T]\times\mathbf{R}_+$, so is F.

Proof: In the Appendix.

As it is this simple to find an observationally equivalent model without joint production, it will be even easier to find arbitrary equivalent models with joint production. In fact, given a model (Z,g), for every jointness function k with $k' < g'$ on $[0,T]$, an admissible production function that makes (F,k) observationally equivalent to (Z,g), is given by (27) with $g'/(1-g')$ replaced by $(g'-k')/(1-g')$. It may be noted that for some specifications – among which the one discussed in the next section – the integral in (27) still exists when the interval of integration is replaced by $[0,T]$. In such cases $\mathcal{W}$ does not have to be bounded and H_0 and H^* can be set equal to zero and T, respectively. If we cannot observe X_z, we get even more freedom in choosing observationally equivalent specifications. We must therefore conclude that this class of models has limited power for the analysis of time allocation data of one adult households in the presence of joint production. Estimation results from these models have to be considered with great care.

3 A Specific Formulation

In this section we will focus on a particular specification of the extended Gronau model, taking Z to be a *Cobb-Douglas* production function. This specification slightly generalizes the model of Graham & Green (1984). To ease comparison, we will stay as close as possible to the notation employed in the Graham & Green article.

$$Z = A(M_m H_m)^{\gamma_m}(M_f H_f)^{\gamma_f} X_z^{\beta} \tag{28}$$

with:

$$\begin{aligned}
\log A &= \alpha_0 + \sum_{k=1}^{N_A} \alpha_k \log A_k \\
\log M_m &= \sum_{k=1}^{N_m} a_k \log M_{m,k} \\
\log M_f &= \sum_{k=1}^{N_f} b_k \log M_{f,k}
\end{aligned}$$

$$0 < \beta < 1; \quad \gamma_m, \gamma_f > 0.$$

The A_k terms are household characteristics such as family size or the number of young children. $M_{m,k}$ and $M_{f,k}$ are characteristics that specifically influence the productivity of the male and the female partner, respectively (like age and education). Clearly, in the Cobb-Douglas specification there is no real difference between the roles of the general and the sex-specific exogenous variables.[11] A matching pair of jointness functions is given by:

$$g_i(H_i) = H_i\left(1 - \frac{1}{1+\delta_i}\left(\frac{H_i}{T}\right)^{\delta_i}\right), \qquad i = m, f \tag{29}$$

with δ_m, $\delta_f \geq 0$. If δ_m=δ_f=0, there is no joint production. Increasing δ_m and δ_f raises the amount of joint production, until eventually – at infinitely large values for δ_m and δ_f – all home production time is perceived as leisure. Solving the first order conditions (13), (14) and (15) for this specification, we get [12]

$$\begin{aligned}
\log H_m = {} & D^{-1}\left\{\frac{\gamma_f}{1-\beta}\log\left(\frac{\gamma_f}{\gamma_m}\right) + (1+\delta_f)\left(\log\gamma_m + \frac{\beta}{1-\beta}\log\beta\right)\right. \\
& \left. + \left(\frac{\delta_f - \delta_m}{1-\beta}\gamma_f + \delta_m(1+\delta_f)\right)\log T\right\}
\end{aligned}$$

[11] For that reason, Graham & Green's interpretation of M_i as partner i's embedded human capital is vacuous. Equally meaningless is their use of $\sum_{k=1}^{N_m} a_k$ and $\sum_{k=1}^{N_f} b_k$ as measures of the relative productivity at home as compared to his or her market productivity.

[12] As we have no data on auxiliary goods that are used in home production, the equation for X_z cannot be used.

$$+D^{-1}\left(\frac{\gamma_f}{1-\beta}-1-\delta_f\right)\log W_m - D^{-1}\frac{\gamma_f}{1-\beta}\log W_f \tag{30}$$

$$+D^{-1}\frac{1+\delta_f}{1-\beta}\log A + D^{-1}\frac{1+\delta_f}{1-\beta}\gamma_m \log M_m$$

$$+D^{-1}\frac{1+\delta_f}{1-\beta}\gamma_f \log M_f$$

$$\log H_f = D^{-1}\left\{\frac{\gamma_m}{1-\beta}\log\left(\frac{\gamma_m}{\gamma_f}\right) + (1+\delta_m)\left(\log\gamma_f + \frac{\beta}{1-\beta}\log\beta\right)\right.$$

$$\left. + \left(\frac{\delta_m-\delta_f}{1-\beta}\gamma_m + \delta_f(1+\delta_m)\right)\log T\right\}$$

$$-D^{-1}\frac{\gamma_m}{1-\beta}\log W_m + D^{-1}\left(\frac{\gamma_m}{1-\beta}-1-\delta_m\right)\log W_f \tag{31}$$

$$+D^{-1}\frac{1+\delta_m}{1-\beta}\log A + D^{-1}\frac{1+\delta_m}{1-\beta}\gamma_m \log M_m$$

$$+D^{-1}\frac{1+\delta_m}{1-\beta}\gamma_f \log M_f.$$

D is the determinant of the subsystem of first order conditions in terms of H_m and H_f alone:

$$D = (1+\delta_m)(1+\delta_f) - (1+\delta_m)\frac{\gamma_f}{1-\beta} - (1+\delta_f)\frac{\gamma_m}{1-\beta}.$$

For this to be a well-defined structural household production model, the solutions to the first order conditions have to satisfy the second order conditions for local optimality. It can be shown that this is achieved by adding one more inequality to the above mentioned sign conventions:

$$\frac{\gamma_m}{1+\delta_m} + \frac{\gamma_f}{1+\delta_f} + \beta < 1. \tag{32}$$

This inequality generalizes the requirement that there be decreasing returns to scale. It is equivalent to the additional restriction $D > 0$.

This is a convenient specification, that – provided identification is guaranteed – can be estimated by means of the maximum likelihood method, but that could also be estimated using OLS and imposing the implied parameter restrictions in an additional ALS-step. The analytical tractability is acquired at the cost of using a simple and relatively inflexible functional form for the production function. Firstly, there are no person-specific productivity effects. A characteristic that increases one partner's marginal productivity, increases that of the other partner in the same proportion. Secondly, H_m and H_f are restricted to be complements in home production.

Application of Proposition 1 of the previous section shows that if two jointness functions of this type – with $\delta_i^{(1)}$ and $\delta_i^{(2)}$, say – are observationally equivalent, we must have

$$H_i^{(\delta_i^{(1)}-\delta_i^{(2)})} = \vartheta, \quad \text{for some } \vartheta \geq 0.$$

As a consequence the jointness functions are identified. On the other hand, we have no data on auxiliary goods X_z. Accordingly we can at most (non-parametically) identify the net product value function. For this model, the optimal amount of auxiliary goods and the net product value for given (H_m, H_f) are

$$\psi_Z(H_m, H_f) = \beta^{1/(1-\beta)} A^{1/(1-\beta)} (M_m H_m)^{\gamma_m/(1-\beta)} (M_f H_f)^{\gamma_f/(1-\beta)}$$

$$\tilde{Z}(H_m, H_f) = (1-\beta)\beta^{\beta/(1-\beta)} A^{1/(1-\beta)} (M_m H_m)^{\gamma_m/(1-\beta)} (M_f H_f)^{\gamma_f/(1-\beta)}.$$

Clearly, $\tilde{Z}$ is itself a Cobb-Douglas production function, where the factor elasticities have been redefined. The person-specific scale factors M_m and M_f are identical to the ones in Z and the general scale coefficient has the same functional form as A. This implies that we cannot identify all parameters of the household production function. In particular, the technical coefficient of X_z, β, is underidentified.

More formally: the reduced form coefficients of W_m and W_f in (30) and (31) allow us to identify $\gamma_m/(1-\beta)$, $\gamma_f/(1-\beta)$, δ_m and δ_f. Furthermore, a_k, b_k and $\alpha_k/(1-\beta)$ $(k \neq 0)$ can be identified from the reduced form coefficients of M_m, M_f and A. That leaves the constant terms to identify α_0 and β. Substitution of the identified coefficients into the expressions for the constant terms, gives two equations of the form:

$$\frac{\alpha_0}{1-\beta} + \frac{\beta}{1-\beta}\log\beta + \log(1-\beta) = C_i, \qquad i = m, f;$$

where C_m and C_f are known expressions in terms of the reduced form parameters. Clearly, we cannot identify both α_0 and β. Therefore we will re-parameterize the model by defining:[13]

$$\begin{aligned}
\tilde{\gamma}_i &= \frac{\gamma_i}{1-\beta}, \qquad i = m, f \\
\tilde{\alpha}_0 &= \frac{\alpha_0}{1-\beta} + \frac{\beta}{1-\beta}\log\beta + \log(1-\beta) \\
\tilde{\alpha}_k &= \frac{\alpha_k}{1-\beta}, \qquad k = 1, \ldots, N_A \\
\log\tilde{A} &= \tilde{\alpha}_0 + \sum_{k=1}^{N_A} \tilde{\alpha}_k \log A_k.
\end{aligned}$$

These parameters are all identified and represent everything that can be identified in this setting. In terms of these parameters the level of output of home production as a function of the observed production factors is given by

[13] The a_k and b_k parameters are not changed. As the discussion above indicates, these parameters are (over-)identified. Additional knowledge of β would identify all parameters in (30) and (31).

$$Z(H_m, H_f, \psi_Z(H_m, H_f)) = \frac{1}{1-\beta}\tilde{A}(M_m H_m)^{\tilde{\gamma}_m}(M_f H_f)^{\tilde{\gamma}_f}.$$

In order to obtain estimates of the volume of home production, β has to be known. Information about X_z or Z is required to estimate the last unidentified parameter, β. Estimates of the structural parameters of this model will therefore not enable us to derive estimates of the output of home production! We can, however, estimate the net product value function $\tilde{Z}$ and the jointness functions. Furthermore, even though we cannot estimate the production function itself, we have enough information to analyze the type of returns to scale, as:

$$1 - \tilde{\gamma}_m - \tilde{\gamma}_f = \frac{1 - \beta - \gamma_m - \gamma_f}{1 - \beta}.$$

In terms of the identified parameters the equations for H_m and H_f become

$$\begin{aligned} \log H_m \;=\; & \tilde{D}^{-1}\left\{\tilde{\gamma}_f \log(\tilde{\gamma}_f/\tilde{\gamma}_m) + (1+\delta_f)(\tilde{\alpha}_0 + \log\tilde{\gamma}_m) \right. \\ & \left. \quad + ((\delta_f - \delta_m)\tilde{\gamma}_f + \delta_m(1+\delta_f))\log T\right\} \\ & + \tilde{D}^{-1}(\tilde{\gamma}_f - 1 - \delta_f)\log W_m - \tilde{D}^{-1}\tilde{\gamma}_f \log W_f \\ & + \tilde{D}^{-1}(1+\delta_f)\sum_{k=1}^{N_A}\tilde{\alpha}_k \log A_k + \tilde{D}^{-1}(1+\delta_f)\tilde{\gamma}_m \sum_{k=1}^{N_m} a_k \log M_{m,k} \\ & + \tilde{D}^{-1}(1+\delta_f)\tilde{\gamma}_f \sum_{k=1}^{N_f} b_k \log M_{f,k}, \end{aligned} \tag{33}$$

$$\begin{aligned} \log H_f \;=\; & \tilde{D}^{-1}\left\{\tilde{\gamma}_m \log(\tilde{\gamma}_m/\tilde{\gamma}_f) + (1+\delta_m)(\tilde{\alpha}_0 + \log\tilde{\gamma}_f) \right. \\ & \left. \quad + ((\delta_m - \delta_f)\tilde{\gamma}_m + \delta_f(1+\delta_m))\log T\right\} \\ & - \tilde{D}^{-1}\tilde{\gamma}_m \log W_m + \tilde{D}^{-1}(\tilde{\gamma}_m - 1 - \delta_m)\log W_f \\ & + \tilde{D}^{-1}(1+\delta_m)\sum_{k=1}^{N_A}\tilde{\alpha}_k \log A_k + \tilde{D}^{-1}(1+\delta_m)\tilde{\gamma}_m \sum_{k=1}^{N_m} a_k \log M_{m,k} \\ & + \tilde{D}^{-1}(1+\delta_m)\tilde{\gamma}_f \sum_{k=1}^{N_f} b_k \log M_{f,k}, \end{aligned} \tag{34}$$

with

$$\tilde{D} = (1+\delta_m)(1+\delta_f) - (1+\delta_m)\tilde{\gamma}_f - (1+\delta_f)\tilde{\gamma}_m.$$

Estimation of this model differs in some respects from the application in Graham & Green (1984). The most evident difference is that they only use the equation for $\log H_f$. As a result, their model is underidentified by two parameters. Another difference is that they do not use the structural restrictions on the intercept of that equation. Even if they would have done so, their model would still have been underidentified by one parameter. Surprisingly enough, the only parameter that is identified in their model, is β. In Kooreman & Kapteyn (1987) it was demonstrated that – even without imposing the structural restrictions on the intercepts – the specification of Graham & Green is identified if both the log H_f-equation and the log H_m-equation are used. These results deviate sharply from the identification properties of the model described above. This difference is caused by two additional assumption underlying the Graham & Green model. In terms of model (28) the assumptions Graham & Green implicitly – and as it seems unwittingly – made are $\alpha_0 = 0$ and $\sum_{k=1}^{N_A} \alpha_i = 1$. These additional assumptions clearly reduce the identification requirements, but they lack any theoretical foundation or meaningful interpretation. In fact, these restrictions are sensitive to changes in the dimension of the exogenous and endogenous variables. For example, the time uses may be defined in hours, minutes or even in seconds. We do not have a natural unit in which time is measured in this model. Due to the assumption α_0=0, the estimated production function is restricted to be in different – and even disjunct – sub-classes of models, for each unit of time measurement. In the general model, discussed above, α_0 adjusts in such a way that, whatever the unit in which time is measured, the model we estimate is the same. These arbitrary assumptions may thus also be the cause of the unsatisfactory results in applications of the Graham & Green model. More importantly, as a result of these restrictions, the fact that it is not possible to estimate β and Z, from data on time allocations only, remained unnoticed. The β and Z Graham & Green calculated can be related to the true but unknown parameters (denoted by *) and the true, but unknown output level Z^* by the following expressions:

$$\text{plim } \hat{\beta}_{G\&G} = \frac{\sum_{k=1}^{N_A} \alpha_k^* - 1 + \beta^*}{\sum_{k=1}^{N_A} \alpha_k^*}$$

$$\text{plim } \hat{Z}_{G\&G} = \left(Z^* e^{-\alpha_0^*} X_z^{\sum_{k=1}^{N_A} \alpha_k^* - 1} \right)^{1/\sum_{k=1}^{N_A} \alpha_k^*} .$$

Rescaling and redefining the exogenous variables, we can make these estimates as high or as low as we want. In general, therefore, estimates from the Graham & Green approach are grossly misleading.[14]

In the previous section we concluded that the identification of the jointness function in the one adult household model, has to be forced by the choice of functional form. Although this makes it difficult to interpret estimates from a model of that type, it is interesting to see how the results on non-parametric identification relate to the identification of the parameters in a Cobb-Douglas specification. If we have no data on X_z, the model is underidentified by two: from the coefficient of W in the reduced form equation of $\log H$, we have to identify γ, β and δ. If, on the other hand, X_z is observed, we can jointly estimate the reduced form equations of $\log H$ and $\log X_z$. In that case all parameters are identified.

[14]Likewise, it can be seen that the type of returns to scale Graham & Green derive, is correct if and only if $\sum_{k=1}^{N_A} \alpha_k^* > 0$. The validity of that condition essentially depends on the definition of the exogenous variables. Consider for example $(1/A_k)$ instead of A_k!

4 Estimation Results

The data that were used to estimate (33) and (34) are from the Swedish HUS-data (Klevmarken & Olovsson (1986)). The first wave of this panel dataset contained detailed information on home production activities. The respondents were asked about their activities on the day preceding the interview. As was indicated in the previous sections we will restrict our attention to two adult households. The sample that was used consisted of 517 households in which both partners were employed and 302 in which one or both of the partners did not have a job.

A, M_m and M_f are defined as:

$$\begin{aligned}
\log A_1 &= 1, \text{ if the family owns a house, 0 otherwise} && (\tilde{\alpha}_1);\\
\log A_2 &= \log \text{familysize} && (\tilde{\alpha}_2);\\
\log A_3 &= \log(1 + \text{number of children younger than 7}) && (\tilde{\alpha}_3);\\
\log A_4 &= \log(1 + \text{number of cars}) && (\tilde{\alpha}_4);\\
\log M_{m,1} &= \log \text{age of male partner} && (a_1);\\
\log M_{m,2} &= \log \text{years of education of male} && (a_2);\\
\log M_{f,1} &= \log \text{age of female partner} && (b_1);\\
\log M_{f,2} &= \log \text{years of education of female} && (b_2).
\end{aligned}$$

An important assumption in deriving the model is that individuals assume that their (net) wage rates are constant. Use of the reported average net wage rates thus seems most appropriate.[15] The econometric model is specified as follows:

$$\begin{aligned}
\log H_{m,j} &= f_m(X_j;\theta) + u_{m,j}\\
\log H_{f,j} &= f_f(X_j;\theta) + u_{f,j}
\end{aligned} \tag{35}$$

with:

$$\begin{pmatrix} u_{m,j} \\ u_{f,j} \end{pmatrix} \mid X_j \overset{i.i.d.}{\sim} \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{1,1} & \sigma_{2,1} \\ \sigma_{2,1} & \sigma_{2,2} \end{pmatrix}\right),$$

and f_m and f_f as defined in (33) and (34). The nuisance terms $u_{m,j}$ and $u_{f,j}$ may contain unobserved heterogeneity in the production function, modelling $\tilde{\alpha}_0$ as a random variable. Furthermore, u accounts for optimization errors and for measurement errors (independent and multiplicative) in the reported time allocations, in the wage rates or in other exogenous variables. Finally, there may be excess variation in daily time expenditures relative to the optimal daily-average levels.

The maximum likelihood estimates of this model are presented in table 4.1. One is tempted to compare these to the estimates Graham & Green presented. For each combination of identifying restrictions they considered, at least one of the γ and δ coefficients was negative. Other applications of that model – using only the $\log H_f$-equation – were reported in Homan, Hagenaars & van Praag (1987,1988). In the first

[15] Estimates based on the *marginal* net wage rates are very similar to the estimates in this section.

Table 4.1 Maximum Likelihood Estimates of model (35)

$\tilde{\gamma}_m$	1.1165**	(4.57)	3.4088	(1.15)
$\tilde{\gamma}_f$	2.4912**	(4.55)	8.6474	(1.19)
δ_m			6.4943	(0.86)
δ_f			8.9808	(0.85)
a_1	-0.0315	(-0.04)	-0.0448	(-0.17)
a_2	-0.2032	(-0.80)	-0.0899	(-0.67)
b_1	-0.0213	(-0.07)	-0.0020	(-0.02)
b_2	0.4986**	(4.08)	0.1479	(1.13)
$\tilde{\alpha}_0$	0.4029	(0.24)	-26.2935	(-0.84)
$\tilde{\alpha}_1$	-0.5485**	(-2.15)	-0.5999**	(-1.97)
$\tilde{\alpha}_2$	-0.3798	(-1.40)	-0.3966	(-1.27)
$\tilde{\alpha}_3$	-1.0132**	(-2.56)	-1.1777**	(-2.35)
$\tilde{\alpha}_4$	-0.3258	(-1.21)	-0.4072	(-1.29)
$\sigma_{1,1}$	1.3259**	(15.94)	1.2464**	(15.99)
$\sigma_{2,1}$	0.0672*	(1.86)	0.0959**	(2.77)
$\sigma_{2,2}$	0.5008**	(15.96)	0.4906**	(16.00)
Log-likelihood	−1359.5940		−1336.1404	

t-statistics between parentheses.
* = significant at 10% -level.
** = significant at 5% -level.

of these papers, they use, among other methods, the Graham & Green model to calculate the monetary value of home production for the Netherlands. They found only one set of identifying restrictions – out of nine – for which the γ and δ parameters had the right sign. In the second paper they just state: "(...) the empirical performance of this model was not well enough to present the estimation results here.". Furthermore, joint estimation of the equations for $\log H_m$ and $\log H_f$ of the Graham & Green specification, using the HUS-data, invariably turned up with large negative values for γ_m and γ_f. After the discussion in the previous section this doesn't come as a surprise.

Turning to the estimation results in table 4.1, we see that the estimates of the $\tilde{\gamma}$ and δ parameters all have the right sign. The estimates in the first column are for the model without jointness (δ_m=δ_f=0). The restrictions δ_m=δ_f=0 are strongly rejected by the likelihood ratio test. Including joint production leads to higher, but – due to correlation with $\hat{\delta}_m$ and $\hat{\delta}_f$ – insignificant estimates for $\tilde{\gamma}_m$ and $\tilde{\gamma}_f$. The effect of the presence of young children on their parents' household productivity ($\tilde{\alpha}_3$), is estimated to be significantly negative. A similar discording result applies to the coefficient of the house-owner dummy ($\tilde{\alpha}_1$). However, we can calculate from these estimates, that owning a house increases the amount of time in home production by 28.3% for the male, and by 20.6% for the female. The elasticities of home production time with respect to the number of young children in the household, are also significantly positive, as would be expected (0.4891 for men and 0.3673 for women). The first child increases the number of hours devoted to home productive activities by 40% and 29%, for males and females respectively. Similarly, the level of education of women (b_2) tends to raise the productivity, but has a negative effect on home production time.

The apparent anomaly of opposite effects on the productivity and the amount of time spent on home production, signals that still something must be wrong. Closer inspection reveals that the estimates do not satisfy the second order conditions (32). Therefore, $D<0$, leading to the sign reversal of the estimated effects on productivity. The overidentifying restrictions, that the structural model imposes on the reduced form coefficients are not rejected by a likelihood ratio test at the 5% level (the test statistic is 11.5 and $\chi^2_9(16.9)=0.95$). The violation of the second order conditions implies, on the other hand, that we cannot interpret these restrictions as being derived from a well-defined behavioural model of time allocation.

In estimating model (35), we ignored the problem of endogenous sample stratification. A next step, therefore, is to include the labour force participation decisions in the analysis. This could be achieved by modelling the participation decision in the context of the original utility maximization problem. This would, however, force us to specify the utility function. We would have to make further parametric assumptions and the analytical tractability would most likely be lost. This approach also ignores the presence of involuntary unemployment. We therefore extended model (35) with a bivariate probit model, that describes the employment probabilities.

$$\begin{aligned}
&\left.\begin{aligned}\log H_{m,j} &= f_m(X_j,\theta) + u_{m,j}\\ \log H_{f,j} &= f_f(X_j,\theta) + u_{f,j}\end{aligned}\right\} \quad \text{if} \quad S^*_{m,j}>0 \ \text{ and } \ S^*_{f,j}>0,\\
&S^*_{m,j} = R'_{m,j}\beta_m + v_{m,j}\\
&S^*_{f,j} = R'_{f,j}\beta_f + v_{f,j}
\end{aligned} \tag{36}$$

where

$$\begin{pmatrix} u_{m,j}\\ u_{f,j}\\ v_{m,j}\\ v_{f,j}\end{pmatrix} \mid X_j, R_{m,j}, R_{f,j} \overset{\text{i.i.d.}}{\sim} \mathcal{N}\left(\begin{pmatrix}0\\0\\0\\0\end{pmatrix}, \begin{pmatrix} \sigma_{1,1} & \sigma_{2,1} & \sigma_{3,1} & \sigma_{4,1}\\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{3,2} & \sigma_{4,2}\\ \sigma_{3,1} & \sigma_{3,2} & 1 & \sigma_{4,3}\\ \sigma_{4,1} & \sigma_{4,2} & \sigma_{4,3} & 1\end{pmatrix}\right).$$

In the employment probits we use age, education level, non-labour income, familysize and the number of young children in the household as exogenous variables. The

Table 4.2 Maximum Likelihood Estimates of model (36)

$\tilde{\gamma}_m$	1.1307**	(4.43)	2.2987*	(1.70)
$\tilde{\gamma}_f$	2.5239**	(4.47)	6.9718*	(1.83)
δ_m			3.9897	(1.24)
δ_f			5.3548	(1.22)
a_1	-0.4401	(-0.57)	-0.3530	(-0.67)
a_2	-0.1586	(-0.61)	-0.1324	(-0.67)
b_1	0.0878	(0.28)	0.0449	(0.32)
b_2	0.4970**	(3.91)	0.2045*	(1.73)
$\tilde{\alpha}_0$	0.9805	(0.56)	-15.178	(-1.13)
$\tilde{\alpha}_1$	-0.5715**	(-2.18)	-0.6979*	(-1.90)
$\tilde{\alpha}_2$	-0.3650	(-1.29)	-0.4358	(-1.19)
$\tilde{\alpha}_3$	-0.9223**	(-2.33)	-1.1548**	(-2.03)
$\tilde{\alpha}_4$	-0.3109	(-1.15)	-0.4472	(-1.24)
$\sigma_{1,1}$	1.5545**	(12.04)	1.4425**	(12.29)
$\sigma_{2,1}$	0.0935	(1.37)	0.1280**	(2.30)
$\sigma_{2,2}$	0.5042**	(14.17)	0.4980**	(14.29)
$\sigma_{3,1}$	-0.9530**	(-6.94)	-0.8931**	(-6.19)
$\sigma_{3,2}$	-0.0240	(-0.14)	-0.0262	(-0.19)
$\sigma_{3,3}$	1.		1.	
$\sigma_{4,1}$	0.5153**	(3.54)	0.4716**	(3.42)
$\sigma_{4,2}$	0.1007	(0.54)	0.1401	(1.03)
$\sigma_{4,3}$	-0.0537	(-0.45)	-0.0499	(-0.41)
$\sigma_{4,4}$	1.		1.	

bivariate probit model is a reduced form model, in the sense that its coefficients are not related to the structural parameters. As opposed to the model of home production time, the employment probit is marginal with respect to the wage rate; i.e. the wage rate has been eliminated from the probit equations, for example by substituting wage equations.

The ML-estimates of this model are in table 4.2. Once more, the restrictions $\delta_m=\delta_f=0$ are firmly rejected. The estimates of the probit part of the model are very satisfactory. The parameters have the signs and relative magnitudes that one expects them to have. The correlation of the disturbances are well below 0.5, except for the correlation between u_m and v_m, which equals -0.74. This means that unexplained variation in the male participation probit and in the $\log H_m$-equation works in opposite directions.

The overidentifying restrictions on the equations of $\log H_m$ and $\log H_f$ cannot be

Table 4.2 Continued

S_h^*-equation				
Constant	-38.916**	(-3.13)	-39.852**	(-3.24)
log(age$_m$)	15.595	(1.67)	16.961*	(1.85)
log(age$_m$)^2	-2.3345*	(-1.86)	-2.5187**	(-2.04)
log(education$_m$)	0.8616	(0.47)	0.8093	(0.46)
log(education$_m$)^2	-0.1180	(-0.29)	-0.1284	(-0.32)
log(age$_f$)	5.8627	(0.70)	5.2208	(0.63)
log(age$_f$)^2	-0.7229	(-0.63)	-0.6346	(-0.57)
log(education$_f$)	1.5900	(0.80)	1.3789	(0.76)
log(education$_f$)^2	-0.2064	(-0.45)	-0.1565	(-0.37)
Non-labour income	-2.0793**	(-4.75)	-2.1204**	(-4.86)
(Non-labour income)^2	0.3198	(1.14)	0.3593	(1.30)
log(familysize)	-0.2394	(-0.92)	-0.2677	(-1.01)
log(1+# young children)	-0.2132	(-0.81)	-0.1977	(-0.76)
S_w^*-equation				
Constant	-40.525**	(-4.18)	-40.942**	(-4.20)
log(age$_m$)	3.0056	(0.42)	2.9326	(0.41)
log(age$_m$)^2	-0.3906	(-0.41)	-0.3771	(-0.39)
log(education$_m$)	4.0709**	(2.34)	4.2891**	(2.44)
log(education$_m$)^2	-0.8647**	(-2.33)	-0.9047**	(-2.42)
log(age$_f$)	19.750**	(3.08)	19.783**	(3.08)
log(age$_f$)^2	-2.8381**	(-3.21)	-2.8416**	(-3.21)
log(education$_f$)	-2.6087	(-1.23)	-2.5078	(-1.20)
log(education$_f$)^2	0.6360	(1.38)	0.6166	(1.35)
Non-labour income	-1.6187**	(-4.77)	-1.6109**	(-4.74)
(Non-labour income)^2	0.7296**	(3.39)	0.7226**	(3.35)
log(familysize)	-0.0737	(-0.35)	-0.0663	(-0.31)
log(1+# young children)	-0.8046**	(-4.50)	-0.7988**	(-4.44)
Log-likelihood	-1929.7609		-1908.0945	

t-statistics between parentheses.
* = significant at 10% -level.
** = significant at 5% -level.

rejected by a likelihood ratio test at the 5% level: the likelihood ratio is 13.3, the critical value 16.9. The parameter estimates of these equations are very similar to those in table 4.1. The estimates of the $\tilde{\gamma}$'s and δ's are lower, but gained in significance. In the reduced form of the $\log H_m$-equation, $\log W_m$ now has the theoretically required negative sign, albeit insignificant. But the other three wage coefficients in the reduced form equations have the wrong sign. Indeed, condition (32) is still violated.

The second order conditions - or equivalently the sign restrictions on the structural parameters and the condition $D > 0$ - imply that the reduced form coefficients of the wage rates in the equations of home production time are all negative. Unrestricted estimation of the reduced form equations indicates that this claim is not supported by the data. In the $\log H_f$-equation both wage coefficients are significantly positive. As can be seen from equations (33) and (34), this tends to push the value of D below zero and reverses the signs of the parameters of the other exogenous variables in the production function (the a's, b's and α's). Furthermore, this result seems to be robust for the Cobb-Douglas specification. We estimated various alternative specifications of the probit equations, M_m, M_f and A and - among other things - allowed $\tilde{\gamma}_m$ and $\tilde{\gamma}_f$ to be functions of age and education. Invariably, D was estimated to be negative.

We must conclude from this that the causes of the problem are more fundamental, possibly contravening the Cobb-Douglas specification or even the general framework. The positive correlations between $\log W_m$ and $\log H_f$ and between $\log W_f$ and $\log H_m$ (the latter one is often not significant) indicate that home production time of the two partners are substitutes, rather than complements. For this reason the Cobb-Douglas specification might be too restrictive and it seems worthwhile to investigate more flexible specifications.

The Gronau model does not predict anything about the cross-wage effects, but the own-wage effects must be negative. The positive correlation between $\log H_f$ and $\log W_f$ is therefore not only a problem for the specification of section 3, but is at variance with the Gronau model itself.[16] Similar adverse correlations were reported in Flood (1988), for roughly the same dataset as we have used.[17] We could follow Flood in concluding that this discredits the general framework, but it seems more appropriate that the positive correlation between home production time and the wage rate reflects a spurious correlation across the sample, rather than the response of individuals to a change in their wage rate. This spurious correlation could for instance be caused by the endogeneity of the average net wage rate.

Suppose for example, that gross wage rates are equal, but taxation is progressive. In that case the (average) net wage rate will be negatively related to the number of

[16]The same correlation for the male partner was always close to zero and insignificant.

[17]Flood reports positive correlations between time spent on maintenance and repair activities and the net wage rate. This correlation was not significant for males, but strongly significant for females: "A 1 percent increase in the marginal tax rate implies on average that females spend about half an hour less time on repair and maintenance activities each week.".

hours worked in a paid job. If, furthermore, differences in the productivity at home are the predominant source of heterogeneity in the population,[18] H and N tend to move in opposite directions, individuals allocating their non-leisure time mainly to the type of work – at home or in the labour market – that they perform best. This would induce a positive correlation between the net wage rate and the time devoted to household production. Stating this argument in a slightly different way: divide the sample into subsamples of people with the same gross wage rates. Between these subsamples, we might expect to find the negative correlation between the (gross) wage rate and the number of hours devoted to home production, but within these subsamples, the people that allocate little time to home production typically work more hours in a paid job and have a lower marginal or average net wage rate.

The spurious correlation will thus be stronger if the endogeneity of the wage rates is more important; i.e. if the tax system is more progressive. The more equal the wage distribution, the more likely it is that this correlation dominates the negative correlation between H and W, caused by behavioural responses to wage variation. Sweden, with its relatively equal wage distribution and a progressive tax system fits these requirements very well. The positive correlation between H_f and W_f in the HUS-data does therefore not imply that we have to abandon the Gronau-type of model as our maintained hypothesis. But it indicates that the assumption of constant wage rates is troublesome and the endogeneity of the wage rate should be modelled explicitly.[19]

Reverting our explanation of the positive correlation between H_f and W_f in the HUS-data, we would expect that in American data – with larger wage differences and a less progressive tax system – the negative correlation predicted by the theory will dominate. In fact, in the reduced form estimates Graham & Green present for the American PSID-data, the coefficient of $\log W_f$ in the equation for $\log H_f$ is significantly *negative*. This indicates that the model set out in section 3 might perform much better on American time allocation data.

[18] If the population is only heterogeneous with respect to the preference for leisure, time spent on home production and in a paid job move in the same direction.

[19] See Kooreman (1987) for a possible way to extend the Gronau-framework with progressive taxation. Allowing for the fact that individuals take the progressive tax system into account, we have to specify the utility function. In general this will render the derivation of reduced form equations difficult, if not impossible.

5 Concluding Remarks

The analysis in this paper illustrates that the Gronau model, extended with the possibility of joint production, provides a powerful and convenient framework for analysing time allocation data. The ability to identify the household production function from the preferences, is attained by the assumption that the output of home production activities is a perfect substitute for goods that can be bought in the market. The resulting dichotomy property for the decisions in households in which both partners have a job, simplifies an otherwise complicated analysis considerably. It was shown, that in a model for two adult households, it is possible to identify joint production non-parametrically on the basis of time allocation data only. For one adult households that property is lost, implying that the analysis is in fact restricted to the Gronau model without joint production.

A specification similar to that proposed by Graham & Green was discussed in section 3. It was shown that their formulation is not well-specified. In the more general specification we proposed, it is not possible to identify the complete household production function from time allocation data only. For that purpose we also need observations on the auxiliary goods (X_z) or on the output level (Z).

The application of this model to the HUS-data demonstrates the central role of the assumption that wage rates are constant. For countries in which the endogeneity of the wage rate is of limited importance, the Gronau framework may be applicable. But for countries with progressive taxation or other significant non-linearities in the budget sets of the agents, the resulting endogeneity of the wage rate should be incorporated in the model. As far as this means that we have to drop the assumption that individuals perceive their wage rate as being constant, the relative simplicity of the Gronau model will most definitely be lost.

Appendix

Proof of Proposition 1

For notational convenience the arguments H_m and H_f will be suppressed. We will abbreviate expressions like

$$\frac{\partial^2 Z}{\partial H_m \partial H_f}(H_m, H_f, \psi_Z(H_m, H_f)) \quad \text{as} \quad \frac{\partial^2 Z}{\partial H_m \partial H_f}(\psi_Z).$$

Differentiating (17) with respect to H_f and (18) with respect to H_m gives

$$\begin{aligned}&\frac{\partial^2 Z}{\partial H_m \partial H_f}(\psi_Z) + \frac{\partial^2 Z}{\partial H_m \partial X_z}(\psi_Z)\,\frac{\partial \psi_Z}{\partial H_f} = \\ &\qquad \frac{1-g'_m}{1-k'_m}\left(\frac{\partial^2 F}{\partial H_m \partial H_f}(\psi_F) + \frac{\partial^2 F}{\partial H_m \partial X_z}(\psi_F)\,\frac{\partial \psi_F}{\partial H_f}\right),\end{aligned} \tag{37}$$

$$\begin{aligned}&\frac{\partial^2 Z}{\partial H_f \partial H_m}(\psi_Z) + \frac{\partial^2 Z}{\partial H_f \partial X_z}(\psi_Z)\,\frac{\partial \psi_Z}{\partial H_m} = \\ &\qquad \frac{1-g'_f}{1-k'_f}\left(\frac{\partial^2 F}{\partial H_f \partial H_m}(\psi_F) + \frac{\partial^2 F}{\partial H_f \partial X_z}(\psi_F)\,\frac{\partial \psi_F}{\partial H_m}\right),\end{aligned} \tag{38}$$

for $(H_m, H_f) \in \breve{\mathcal{H}}$. From the definition of ψ_Z and ψ_F we have

$$\frac{\partial Z}{\partial X_z}(\psi_Z) = 1 = \frac{\partial F}{\partial X_z}(\psi_F), \tag{39}$$

on $\mathcal{H}$. Differentiating these two equalities with respect to H_m and H_f, respectively, gives for $(H_m, H_f) \in \breve{\mathcal{H}}$

$$\frac{\partial^2 Z}{\partial X_z \partial H_i}(\psi_Z) + \frac{\partial^2 Z}{(\partial X_z)^2}(\psi_Z)\,\frac{\partial \psi_Z}{\partial H_i} = 0, \qquad i = m, f;$$

$$\frac{\partial^2 F}{\partial X_z \partial H_i}(\psi_F) + \frac{\partial^2 F}{(\partial X_z)^2}(\psi_F)\,\frac{\partial \psi_F}{\partial H_i} = 0, \qquad i = m, f.$$

These equations imply that the left hand sides of (37) and (38) are equal on $\breve{\mathcal{H}}$. The same goes for the terms in parentheses on the right hand sides of (37) and (38). By assumption (20), this implies that for almost every $(H_m, H_f) \in \breve{\mathcal{H}}$:

$$\frac{1-g'_m(H_m)}{1-k'_m(H_m)} = \frac{1-g'_f(H_f)}{1-k'_f(H_f)} = \vartheta, \tag{40}$$

for some non-negative constant ϑ. By the continuity of the first order derivatives of the jointness functions this property holds for every $(H_m, H_f) \in \breve{\mathcal{H}}$. □

Proof of Proposition 2

Under the assumptions we made about g, we have that for $H \in [H_0, H^*]$:

$$0 \leq \frac{g'(H)}{1-g'(H)} \frac{\partial Z}{\partial H}(H, \psi_Z(H)) \leq \frac{1}{1-g'(H)} \frac{\partial Z}{\partial H}(H, \psi_Z(H)). \tag{41}$$

The term on the right is equal to the wage rate at which $(H, \psi_Z(H))$ is an optimal choice. Therefore, the integrand is bounded on $\mathcal{H}$ and the integral exists. Having established that definition (27) is sound, we have to show that this function satisfies (25) and (26) and is admissible. It is straightforward to verify that F satisfies (25). Equality of ψ_Z and ψ_F on $\mathcal{H}$ follows directly from:

$$\frac{\partial Z}{\partial X_z}(H, \psi_Z(H)) = 1 = \frac{\partial F}{\partial X_z}(H, \psi_F(H)). \tag{42}$$

Clearly, F is twice differentiable and $\partial F / \partial X_z$ is continuously differentiable. By the first inequality in (41), F is also strictly increasing. The household production decisions for a one adult household, where the adult has a paid job, can be described by the following optimization problem

$$\max_{H, X_z} Z(H, X_z) + Wg(H) - X_z - WH. \tag{43}$$

By assumption, this problem has a unique interior solution for each $W \in \mathcal{W}$. Therefore the following second order conditions have to hold:

$$\frac{\partial^2 Z}{(\partial X_z)^2}(\hat{H}, \psi_Z(\hat{H})) < 0, \tag{44}$$

$$\frac{\partial^2 Z}{(\partial H)^2}(\hat{H}, \psi_Z(\hat{H})) + g''(\hat{H})W < 0, \tag{45}$$

$$\frac{\partial^2 Z}{(\partial X_z)^2}(\hat{H}, \psi_Z(\hat{H})) \left\{ \frac{\partial^2 Z}{(\partial H)^2}(\hat{H}, \psi_Z(\hat{H})) + g''(\hat{H})W \right\} - \left[\frac{\partial^2 Z}{\partial X_z \partial H}(\hat{H}, \psi_Z(\hat{H})) \right]^2 > 0, \tag{46}$$

where $\hat{H}$ is the optimal number of hours spent on home production at wage rate W. Differentiating the left hand side of (42) with respect to H we get

$$\frac{\partial^2 Z}{\partial X_z \partial H}(H, \psi_Z(H)) + \frac{\partial^2 Z}{(\partial X_z)^2}(H, \psi_Z(H)) \frac{\partial \psi_Z}{\partial H}(H) = 0 \quad \text{on } \mathcal{H}.$$

Together with (46) and (44) this gives

$$\frac{\partial^2 Z}{(\partial H)^2}(\hat{H}, \psi_Z(\hat{H})) + g''(\hat{H})W + \frac{\partial^2 Z}{\partial X_z \partial H}(\hat{H}, \psi_Z(\hat{H})) \frac{\partial \psi_Z}{\partial H}(\hat{H}) < 0. \tag{47}$$

The partial maximization problem of an employed individual with production function F and no joint production is:

$$\max_{H\in\mathcal{H};X_z\geq 0} Z(H,X_z)+\int_{H_0}^{H}\frac{g'(\tilde{H})}{1-g'(\tilde{H})}\,\frac{\partial Z}{\partial H}(\tilde{H},\psi_Z(\tilde{H}))\,d\tilde{H}-WH-X_z. \tag{48}$$

The conditions for local strict concavity of the maximand in $(\hat{H},\psi_Z(\hat{H}))$ are that

$$\frac{\partial^2 Z}{(\partial X_z)^2}(\hat{H},\psi_Z(\hat{H})) \quad \text{and}$$

$$\frac{\partial^2 Z}{(\partial X_z)^2}(\hat{H},\psi_Z(\hat{H}))\left\{\frac{g''(\hat{H})}{1-g'(\hat{H})}W+\frac{1}{1-g'(\hat{H})}\frac{\partial^2 Z}{(\partial H)^2}(\hat{H},\psi_Z(\hat{H}))\right.$$
$$\left.+\frac{g'(\hat{H})}{1-g'(\hat{H})}\frac{\partial^2 Z}{\partial H\,\partial X_z}(\hat{H},\psi_Z(\hat{H}))\,\frac{\partial\psi_Z}{\partial H}(\hat{H})\right\}-\left[\frac{\partial^2 Z}{\partial H\,\partial X_z}(\hat{H},\psi_Z(\hat{H}))\right]^2$$

should be negative and positive, respectively. The first requirement is met by (44). The second expression can be rewritten as:

$$\frac{\partial^2 Z}{(\partial X_z)^2}(\hat{H},\psi_Z(\hat{H}))\frac{g'(\hat{H})}{1-g'(\hat{H})}\left\{g''(\hat{H})W+\frac{\partial^2 Z}{(\partial H)^2}(\hat{H},\psi_Z(\hat{H}))\right.$$
$$\left.+\frac{\partial^2 Z}{\partial H\,\partial X_z}(\hat{H},\psi_Z(\hat{H}))\,\frac{\partial\psi_Z}{\partial H}(\hat{H})\right\}+$$
$$\frac{\partial^2 Z}{(\partial X_z)^2}(\hat{H},\psi_Z(\hat{H}))\left\{\frac{\partial^2 Z}{(\partial H)^2}(\hat{H},\psi_Z(\hat{H}))+g''(\hat{H})W\right\}-\left[\frac{\partial^2 Z}{\partial H\,\partial X_z}(\hat{H},\psi_Z(\hat{H}))\right]^2.$$

From (44), (47) and (46) this expression is seen to be positive. We now only have to verify that $\hat{H}$ is also a global optimum for (48).

Condition (47) guarantees that the right hand term in (41) is strictly decreasing on $\mathcal{H}$. From this it is easy to see that the integrand in (27) is strictly decreasing on (H_0,H^*), i.e.

$$\frac{g'(H^*)}{1-g'(H^*)}\,\frac{\partial Z}{\partial H}(H^*,\psi_Z(H^*))\leq Wg'(\hat{H})\leq\frac{g'(H_0)}{1-g'(H_0)}\,\frac{\partial Z}{\partial H}(H_0,\psi_Z(H_0)). \tag{49}$$

The objective function of (48) differs from that in (43) by a term $Q(H)-Wg(H)$, where $Q(H)\stackrel{\text{def}}{=}F(H,X_z)-Z(H,X_z)$. From (49) it follows that this term is monotonically increasing on $[0,H_0]$ and monotonically decreasing on $[H^*,T]$. On (H_0,H^*):

$$Q'(H)-Wg'(H)=g'(H)\left\{\frac{1}{1-g'(H)}\,\frac{\partial Z}{\partial H}(H,\psi_Z(H))-W\right\},$$

which is monotonically decreasing, with value zero in $\hat{H}$. Therefore, $Q(H)$ attains its maximum value in $\hat{H}$ and the maximization problem (48) has its unique global maximum in $\hat{H}$. The household production model with production function F and no joint production is admissible.

$Q(H)$ is linear on $[0,H_0]\cup[H^*,T]$ and – as we just learned – concave on (H_0,H^*). As a consequence, F is strictly concave on $[0,T]\times\mathbb{R}_+$ if Z is. □

References

Becker, Gary S. (1965): "A Theory of the Allocation of Time," *The Economic Journal*, 75, 493–517.

Flood, Lennart (1988): "Effects of Taxes on Non-Market Work: the Swedish Case," *Journal of Public Economics*, 36, 259–267.

Graham, John W. & Carole A. Green (1984): "Estimating the Parameters of a Household Production Function with Joint Products," *The Review of Economics and Statistics*, 66, 277–282.

Gronau, Reuben (1977): "Leisure, Home Production and Work – the Theory of the Allocation of Time Revisited," *Journal of Political Economy*, 85, 1099–1123.

Gronau, Reuben (1980): "Home Production – A Forgotten Industry," *The Review of Economics and Statistics*, 62, 408–416.

Homan, M.E., A.J.M. Hagenaars & B.M.S. van Praag (1987): "A Comparison of Six Methods to Estimate the Monetary Value of Home Production," Report Econometric Institute, Erasmus University Rotterdam.

Homan, M.E., A.J.M. Hagenaars & B.M.S. van Praag (1988): "The Distribution of Income and Home Production in One-Earner and Two-Earner Families," Report Econometric Institute, Erasmus University Rotterdam.

Klevmarken, N.A. & P. Olovsson (1986): "Hushållens Ekonomiska Levnadsförhållanden (HUS), Teknisk Beskrivning och Kodbok," Department of Economics, University of Gothenburg.

Kooreman, Peter (1987): "The Effects of Taxes on the Choice between Market and Non-Market Work," Working Paper, Tilburg University.

Kooreman, Peter & Arie Kapteyn (1987): "On the Identifiability of Household Production Functions with Joint Products: a Comment," Research Memorandum, Tilburg University.

Lancaster, K.J. (1966): "A New Approach to Consumer Theory," *Journal of Political Economy*, 74, 132–157.

Pollak, Robert A. & Michael L. Wachter (1975): "The Relevance of the Household Production Function and Its Implications for the Allocation of Time," *Journal of Political Economy*, 83, 255–277.

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