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RESEARCH MEMORANDUM

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THE PROBLEM OF NOT OBSERVING SMALL EXPENDITURES IN A CONSUMER EXPENDITURE SURVEY

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Abstract

In consumer expenditure surveys one often faces the problem that, in practice, full information on small consumption expenditures is hardly available. In this paper we propose a method to correct for the underestimation of consumption resulting from not recording the small expenditures. We apply this method to a Dutch panel which is divided into the subsamples A and B. For subsample B all expenditures are registered, but from subsample A only expenditures above 10 Dutch guilders. The method consists of constructing a model, which explains for each consumption category the sum of expenditures below 10 guilders made during a month, by explicitly taking into account that each expenditure should be below 10 guilders, and estimating this model using data of subsample B. The model can then be used to calculate the expected values of the small expenditures made by households in subsample A.

§ 1. Introduction

In consumer expenditure surveys one often faces the problem that information on all consumption expenditures is not complete. Especially collecting complete information on small expenditures is a difficult task.

In the 1980-1981 Consumer Expenditure Survey of the Netherlands, conducted by the Netherlands Central Bureau of Statistics, for example, information on small expenditures is only gathered during a so-called registration month, once a year. Afterwards, the values of annual expenditures are obtained by inflating the monthly figures. In another Dutch panel, the so-called Expenditure Index, conducted by a private marketing research agency (INTOMART), the sample is divided into two subsamples. The respondents in the first subsample, A, say, is only asked to give information on large expenditures, defined as expenditures in excess of 10 Dutch guilders, whereas the other subsample, B, say, is asked to give also information on small expenditures (expenditures less than or equal to 10 Dutch guilders). The consequence of this procedure is that we do not know the values of the small expenditures of the households in subsample A. Table 1.1 clearly shows that using only expenditures in excess of 10 Dutch guilders to determine the total values of the expenditures would lead to considerable underestimation for several expenditure categories in case of subsample B. It is likely that such underestimation would also occur for subsample A.

Table 1.1: Mean level of the sum of expenditures below 10 Dutch guilders and of expenditures in excess of 10 Dutch guilders by the households of subsample B in April 1984.

Consumption category	Expenditures in excess of Dfl. 10	Expenditures below Dfl. 10
1) Food	470	47
2) Clothing and Footwear	217	4
3) Housing, including rents and interest payments on and redemptions of mortgage payments	630	0
4) Domestic decoration including furniture, expenditures on do-it-yourself articles and on gardening	404	10
5) Recreation, Entertainment including holiday expenditures	183	11
6) Vehicles, including purchases of cars, bicycles etc.	48	1
7) Transportation, including expenditures on fuel and public transportation	96	2
8) Insurance	140	0
9) Appliances, including electric appliances, such as hifi-equipments and washing machines and other personal expenditures	162	7
10) Other expenditures, including medical expenditures, gifts and donations	140	11

In this paper we describe an approach to correct for this underestimation. Our approach is to construct an econometric model that explains for each consumption category, for each period (one month), and for each household the level, Y, of expenditures below 10 Dutch guilders. Once the model has been estimated by using data from subsample B we can use this model to calculate the expected values of the small expenditures, EY, made by the households in subsample A.

The model consists of two parts. The first part which we call the Count Model explains on a household level the number of expenditures, N, below 10 Dutch guilders by using a probability distribution defined on the nonnegative integers, whereas the second part which we call the Amount Model explains the amount of each expenditure that is below Dfl. 10 by using a (conditional) probability distribution defined on the [0;10]-interval. Household characteristics are included by parameterization of the parameters of the probability distribution. This model bears some resemblance with that of Robin (1987).

It is clear that there are alternative ways of modelling the monthly sum of the small expenditures Y. For instance, one can formulate a TOBIT-model, where the latent variable corresponding to the sum of the small expeditures, Y^*, depends on some household characteristics. But this way of modelling neglects the information, that each single expenditure should be below Dfl 10,-. Moreover, there is a second argument for our way of modelling. If we had for all households information about the number of small purchases at our disposal, it would be easy to incorporate this information in our model. In that case, we only have to estimate the Amount Model and we would be able to predict for the households in subsample A the expected values of the small expenditures given the number of these purchases by $EY|N$. We presume that this would lead to a better prediction of the sum of small penditures, than if one uses EY, the unconditional expectation. Therefore, it would be desirable to collect in the future some information about N.

Notice that this way of modelling can be used in other applications as well. For example, instead of small expenditures we can explain gasoline consumption by car owners by first modelling the number of times an individual visits a gas station and then modelling the quantity of petrol the person buys anytime he or she visits a gas station. In the same way other examples can be conceived of.

The structure of the paper is as follows. In the second section we present the model and discuss some estimation strategies. Also, we derive an explicit formula for the expected values of expenditures below 10 Dutch guilders. In Section 3 we present the empirical application of our model to the data of the Expenditure Index. Section 4 concludes.

§ 2. The model

In this section we present the model. Each consumption category will be considered separately. The following diagram, which represents the small expenditures made by some arbitrarily chosen household during a given period, is the starting point of our modelling

Y_i: Amount of i-th purchase $\longrightarrow$

N: Number of purchases ↓		1	2	...	n	n+1	
	N=0	$Y_1=0$	$Y_2=0$	...	$Y_n=0$	$Y_{n+1}=0$	...
	N=1	$Y_1 \in [0;10]$	$Y_2=0$	...	$Y_n=0$	$Y_{n+1}=0$	...
	N=2	$Y_1 \in [0;10]$	$Y_2 \in [0;10]$	...	$Y_n=0$	$Y_{n+1}=0$	...
	⋮	⋮	⋮		⋮	⋮	
	N=n	$Y_1 \in [0;10]$	$Y_2 \in [0;10]$	...	$Y_n \in [0;10]$	$Y_{n+1}=0$	...
	⋮	⋮	⋮		⋮	⋮	

N denotes the number of purchases, $N \in \{0,1,2,...\}$. Y_i denotes the amount of the i-th purchase; if $i \leq N$, then $Y_i \in [0;10]$, otherwise $Y_i \equiv 0$, $i \in \{1,2,...\}$.

Define $Z \equiv (Y_1, Y_2, \ldots)$. Then we construct the probability distribution of (N,Z) as follows [1),2)].

$$Pr\{N=n, Z\in\zeta\} = Pr\{N=n\} * Pr\{Z\in\zeta | N=n\} \qquad (2.1)$$

Pr{N=n} is the Count Model, whereas Pr{Z∈ζ|N=n) is the Amount Model.

With regard to Pr{Z∈ζ|N=n} we assume

$$Pr\{Z\in\zeta | N=n\} = \Pi_{i=\{1,2,\ldots,n\}} \int_{\zeta_i} f(v)dv * \Pi_{j\in\{n+1,n+2,\ldots\}} II_{\{0\}}(\zeta_j) \qquad (2.2)$$

where $\zeta = \zeta_1 \times \zeta_2 \times \ldots$ $(= \Pi_{i\in\mathbb{N}}\zeta_i)$ and

$$II_{\{0\}}(\zeta_i) = 1 \quad \text{if } 0 \in \zeta_i$$
$$= 0 \quad \text{otherwise}$$

f(v) stands for a probability density function.
Notice that we assume for $k,\ell \le N$ the amount of the k-th purchase to be stochastically independent of the ℓ-th purchase, conditional upon N = n. This means that we do not take into account possible correlation between Y_k and Y_ℓ with $k \ne \ell$ and $k, \ell < N$.
The second part of the right-hand-side of (2.2) says that the conditional probability is positive only if $Y_j \equiv 0$ for $j > N = n$.

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1) An alternative approach is to take $Y_1, Y_2, \ldots$ to be i.i.d. random variables, distributed on the interval [0;10], and to assume that $Z = (Y_1, Y_2, \ldots)$ and N are independent. The difference between this approach and the one presented in the main text lies in the definition of the total expenditures. Following the main text we have as total expenditures $\Sigma_{j\in\mathbb{N}} Y_j$, whereas the alternative approach results in $\Sigma_{j=1}^{N} Y_j$.

2) To be precise: The probability space with respect to which (N,Z) is defined is $((\mathbb{N}\times\mathbb{R}^{\mathbb{N}}), \mathcal{B}(\mathbb{N}\times\mathbb{R}^{\mathbb{N}}), P)$, with $Z \equiv \Pi_{i\in\mathbb{N}} Z_i \in \mathcal{B}(\mathbb{R}^{\mathbb{N}})$.

The next step consists of choosing families of distribution functions from which one will be chosen by way of estimation. For Pr{N=n}, the Count Model, we first consider the Poisson (λ) model, i.e.,

$$Pr(N=n) = e^{-\lambda} \frac{\lambda^n}{n!}$$

The parameter λ is specified as follows

$$\lambda = \exp(X'\beta) \tag{2.3}$$

where $X :=$ vector of exogenous variables (household characteristics)
$\beta :=$ vector of parameters.

The Poisson distribution is restrictive in several ways (see Cameron and Trivedi (1986), Gourieroux, Montfort and Trognon (1984b)). For instance, the assumption, that the conditional mean and variance of N given X are equal, may be too strong. One way to relax this restriction is to allow for unobserved heterogeneity in λ by replacing (2.3) by the following equation

$$\lambda = \exp(X'\beta) \exp(\varepsilon) \tag{2.4}$$

where $\varepsilon :=$ error term with $E(\exp \varepsilon | X) = 1$.

Equation (2.4) implies, that N given X and ε is Poisson (λ) distributed. Since ε is an unobservable random variable, we must integrate it out to obtain the conditional distribution of N given X. Cameron and Trivedi (1986), among others, show that if $\lambda \sim$ Gamma (φ,ν), with $\varphi = \exp(x'\beta)$, then $N|X=x \sim$ Negative Binomial (φ,ν), i.e.,

$$Pr(N=n|X=x) = \frac{\Gamma(n+\nu)}{\Gamma(n+1)\ \Gamma(\nu)} \left[\frac{\nu}{\nu+\varphi}\right]^{\nu} \left[\frac{\varphi}{\nu+\varphi}\right]^{n} \tag{2.5}$$

Notice that $E(N|X=x) = \varphi = \exp(x'\beta)$ and $\mathrm{var}(N|X=x) = \varphi + \varphi^2/\nu$.

The Negative Binomial distribution will be abbreviated as NEGBIN distribution. Concerning the parametrization of ν, we consider three possibilities:

a) $$\nu = \alpha^{-1}\varphi = \alpha^{-1} \exp(x'\beta) \qquad \varphi = \exp(x'\beta) \tag{2.6}$$

b) $$\nu = \alpha^{-1}\varphi^2 = \alpha^{-1} (\exp(x'\beta))^2 \qquad \varphi = \exp(x'\beta) \tag{2.7}$$

c) $$\nu = \exp(\gamma_0 + x'\gamma) \qquad \varphi = \exp(x'\beta) \tag{2.8}$$

With regard to (2.6) and (2.7), the parameterization of the NEGBIN distribution coincides with the NEGBINI- and NEGBINII-parameterization of Cameron and Trivedi (1986), respectively. The NEGBINI and NEGBINII distribution reduce to the Poisson-distribution if $\alpha \downarrow 0$. In the sequel, the NEGBIN distribution with ν parameterized as in (2.8) will be called the NEGBINA-distribution. Notice that the NEGBINI and NEGBINII distributions are special cases of the NEGBINA distribution.[3)]

The densities f(v) of formula (2.2.) are modelled in two ways:
The uniform distribution:

$$f(v) = \frac{1}{10} v \, I_{[0;10]}(v) \tag{2.9}$$

The Beta-distribution:

$$f(v) = \frac{1}{10} \frac{1}{B(p,q)} \left(\frac{v}{10}\right)^{p-1} \left(1-\frac{v}{10}\right)^{q-1} I_{[0,10]}(v) \tag{2.10}$$

$I_{[0,10]}(v)$ is an indicator function defined by

$$I_{[0;10]}(v) = 1 \text{ if } v \in [0;10]$$
$$= 0 \text{ otherwise.}$$

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3) To get from the NEGBINA distribution the NEGBINI distribution, just define $\alpha^{-1} = \exp(\gamma_0)$ and set $\gamma = \beta$, to get the NEGBINII distribution also define $\alpha^{-1} = \exp(\gamma_0)$ and set $\gamma = 2\beta$.

In the empirical application of our model we do not further parameterize the parameters p and q.

Given the above modelling we can derive for each consumption category a likelihood function of the observations. We assume independence across households and across time. So, for a particular consumption category, the likelihood function is

$$L = \Pi_{t\in T}\, \Pi_{h\in H}\, Pr_{ht}\{N_{ht} = n_{ht}\} \qquad \text{if } n_{ht} = 0$$

$$\qquad\qquad (2.11)$$

$$= \Pi_{t\in T}\, \Pi_{h\in H}\, Pr_{ht}\{N_{ht} = n_{ht}\}\, \Pi_{i\in\{1,2,\ldots,n_{ht}\}}\, f_t(y_{iht}) \text{ if } n_{ht}>0$$

H stands for the set of households, T is the set of periods, n_{ht} is the number of purchases by household h in period t, and y_{iht} is the (positive) amount of the i-th purchase by household h in period t, $i = 1,\ldots,n_{ht}$. We add the subscripts ht and t because in the Count Model we assume the exogenous variables to be dependent on household characteristics and the parameters to be time dependent whereas in the Amount Model in case of the Beta distribution we assume the parameters p and q to be dependent on time. Let us now consider the log of the likelihood:

$$\log L = \Sigma_{t\in T}\, \Sigma_{h\in H} \log Pr_{ht}\{N_{ht} = n_{ht}\} +$$

$$+ \Sigma_{t\in T}\, \Sigma_{h\in H}\, \Sigma_{i\in\{1,\ldots,n_{ht}\}} \log f_t(y_{iht}) \qquad (2.12)$$

where the second part of the right-hand side does not appear if $n_{ht} = 0$. As long as the sets of parameters of the first and of the second part of the right-hand-side are disjoint, maximizing the log L as function of the parameters can be done by maximizing each part separately. In our case we assume the parameter sets of the Count and the Amount Model to be disjoint. But even if the parameters of the Amount Model depend on some household characteristics or on the number of purchases, N, we would still be able to estimate both models independently of one another. E.g., in formula (2.10) we could specify $p = p(n,x)$, $q = q(n,x)$. As a consequence, in formula (2.2) we should replace $f(v)$ by $f_n(v)$.

Concerning the Count Model two estimation methods are considered:

1. Maximum Likelihood (ML)
2. Pseudo Maximum Likelihood (PML)
   (See Gourieroux, Montfort and Trognon ((1984a),(1984b))

The PML estimators are obtained by maximizing a likelihood function associated with some family of probability distributions, which does not necessarily contain the true distribution. Gourieroux, Montfort and Trognon (1984a) show that under certain assumptions the PML-method will in the case of a linear exponential family give consistent estimators of the parameters appearing in the first order moment of the true distribution (cf. the parameters β in the equations (2.4) and (2.5))
Since, in that case, the PML-method only assumes a correctly specified mean, the ML-method will give more efficient estimates than the PML-method, if the distribution of $N|X$ is correctly specified. However, the PML-method is more robust. In our empirical application, we shall consider the PML-estimators associated with the Poisson family. A consistent estimate of the variance-covariance matrix of the PML-estimates is calculated by using the results of Gourieroux, Montfort, and Trognon (1984a). See also Cameron and Trivedi (1986, page 37). Note, that PML estimates can also be derived from the NEGBINA-distribution with the parameter-vector γ being given (see Gourieroux Montfort and Trognon (1984b)).

Once the model has been estimated by using data of the subsample B, we can use the model to predict the values of the small expenditures made by the households in subsample A. This prediction can be made by taking the expectation of the total value of the small expenditures Y, where $Y \equiv \Sigma_{j \in N} Y_j$. We have

$$E(Y) = E(E(Y|N)) = E(E(\Sigma_{j \in N} Y_j|N)) = E(\Sigma_{j \in N} E(Y_j|N)) =$$

$$= E(\Sigma_{j \in N} E(V)\ I_{\{j,j+1,\ldots\}}(N)) \qquad (2.13)$$

where $I_{\{j,j+1,...\}}(N) = 1$ if $N \in \{j,j+1,...\}$
$= 0$ otherwise

and where E(V) denotes the expectation of a stochastic variable V with density function f(v).

E(V) does not depend on j. So

$$E(Y) = E(V)\ E(\Sigma_{j \in \mathbb{N}}\ I_{\{j,j+1,...\}}(N)) = E(V)\ E(N) \tag{2.14}$$

§ 3. Data, empirical model and results

Data

The data stem from a panel survey conducted in the Netherlands between april 1984 and december 1986. In this study we only use data of the period april 1984-december 1984. Information about expenditures on different categories is collected on a monthly basis, while data on background variables such as net household income and family size are gathered once a year.

The data set consists of about 800 households per month, of which approximately 10 per cent is in the B-subsample. This group reports all expenditures distinguished on the basis of a detailed goods classification. The other group (the A-subsample), however, records only purchases in excess of 10 guilders in a broader goods classification. In this study only data of the B-subsample are used to estimate the model formulated in Section 2.

From a first inspection of the data of subsample B it appears, that purchases below 10 guilders occur most frequently in the following consumption categories:

1) Food
2) Clothing and Footwear
3) Domestic Decoration
4) Recreation, Entertainment
5) Appliances
6) Other Expenditures

In the case of the other consumption categories (Housing, Vehicles, Transportation, and Insurance), such purchases are rarely found in the B-subsample. Therefore, we do not estimate our model for these consumption categories.

In the Count Model (see the equations (2.4) and (2.6)) the following regressors appear for each consumption category

1. C := Constant term
2. URB := Degree of Urbanization
   1 = rural municipality
   2 = commuter towns and small towns
   3 = medium sized cities (30,000 -100,000 inhabitants)
   4 = large cities (over 100,000 inhabitants)
3. CHILD = 1 if household contains children younger than five years of age
   = 0 otherwise
4. FS := Family size
5. WORKH = 1 if head of household doesn't have a paid job
   = 0 otherwise
6. AGEP := Age of the housewife (age of the head of the household if there is no housewife.)
7. SOC := Social group
   1. upper class
   2. upper middle class
   3. middle class
   4. lower middle class
   5. lower class
8. SINGLE = 1 if head of household lives alone
   = 0 otherwise

The NEGBIN models (see the formulae (2.4) and (2.6)) are chosen as a starting point of our analysis. To begin with, we assume that the parameter vectors of the NEGBIN-models do not change over time (the sample period). The resulting ML and PML estimates of the NEGBINI, NEGBINII and NEGBINA-models are presented in table 3.1.

In case of the NEGBINA-model only the estimates of the parameters appearing in the first order moment of the negative binomial distribution are presented. In order to calculate the expected value of the small purchases (cf. formula 2.14) we only need these parameters. A comparison of the ML and PML-estimates of the parameters, which appear in the first order moment of the NEGBINA-model, shows, that these are similar in sign and magnitude. This observation especially applies to the significant parameters. This is an encouraging result, because both the ML and PML-estimates of these parameters are consistent, if the first order moments are correctly specified. The ML and PML-estimates could, however, differ considerably, if the first order moment of the Negative Binomial distribution is not correctly specified.

Next we turn to the topic of model selection. In section 2 we noted, that the NEGBINI- and NEGBINII-models follow from the NEGBINA model by imposing suitable restrictions. By means of a likelihood ratio test we check whether we may impose such restrictions. This test statistic, which under the null hypothesis, is asymptotically distributed as $\chi^2(7)$, is significant for both the NEGBINI- and NEGBINII-model in case of Food, Recreation and Other Expenditures (c.f. row "LR" in table 3.1). For these consumption categories we select the NEGBINA-model and for the other categories we choose the NEGBINI-model, because from the values of Akaike Information Criterion (c.f. row AIC or Table 3.1) we can conclude that, in terms of this criterion, the NEGBINI model shows a better performance than the NEGBINII model for Clothing, Footwear and a similar performance for Domestic decoration and Appliances.

Now we will discuss the estimation results. For the selected models we have carried out a Likelihood ratio test of the hypothesis, that the parameters corresponding to all explanatory variables except the constant term are equal to zero. The test statistic is asymptotically distributed as $\chi^2(14)$ for Food, Recreation, and Other Expenditures and $\chi^2(7)$ for the other consumption categories. In all cases we must reject the null hypothesis (see row LC in table 3.1).

Table 3.1: ML- and PML-estimates of β for the NEGBIN-models (Asymptotic t- values in parentheses)

	FOOD				CLOTHING. FOOT WEAR			
	ML NEGBINI	ML NEGBINII	ML NEGBINA	PML	ML NEGBINI	ML NEGBINII	ML NEGBINA	PML
Constant	2.234	2.115	2.926	2.810	-0.710	-0.555	-0.712	-0.896
	(9.68)	(13.02)	(6.252)	(10.87)	(-1.68)	(-1.69)	(-1.41)	(-1.91)
URB	-0.145	-0.108	-0.136	-0.104	-0.121	-0.109	-0.093	-0.081
	(-3.86)	(-4.43)	(-2.05)	(-2.96)	(-1.83)	(-2.15)	(-1.13)	(-1.17)
Child	-0.104	-0.055	-0.310	-0.234	-0.075	-0.164	-0.002	0.043
	(-1.05)	(-0.87)	(-1.46)	(-2.15)	(-0.44)	(-1.13)	(-0.007)	(0.22)
FG	0.1145	0.089	0.073	0.082	0.218	0.176	0.239	0.229
	(3.22)	(4.02)	(0.99)	(2.36)	(3.63)	(3.65)	(3.29)	(4.18)
WORKH	0.020	-0.010	0.0032	0.069	-0.086	-0.052	-0.002	-0.096
	(0.24)	(-0.17)	(0.02)	(0.69)	(-0.54)	(-0.425)	(-0.01)	(-0.60)
AGEP	0.0088	0.008	-0.002	0.0003	0.012	0.009	0.007	0.012
	(3.35)	(4.56)	(-0.290)	(0.009)	(2.11)	(2.235)	(0.86)	(2.27)
SOC	-0.075	-0.030	-0.084	-0.115	-0.171	-0.146	-0.139	-0.146
	(-2.40)	(-1.31)	(-1.70)	(-2.55)	(-2.98)	(-3.25)	(-2.07)	(-2.26)
SINGLE	0.182	0.168	0.052	0.042	0.034	0.066	-0.0167	-0.003
	(1.61)	(2.26)	(0.29)	(0.33)	(0.16)	(0.42)	(-0.065)	(-1.51)
α	19.448	0.169	-	-	1.070	0.00076	-	-
	(11.12)	(7.34)	-	-	(6.41)	(5.30)	-	-
Log L.	15446.19	15451.11	15458.70	-	-593.94	-594.77	-589.24	-
LR	25.02	15.18	-	-	9.40	11.06	-	-
LRP	6121.64	6131.48	6146.66	-	184.58	182.92	193.88	-
LC	-	-	71.94	-	49.44	-	-	-
AIC	-30874.38	-30884.22	-30885.4	-	1205.88	1207.54	1210.48	-

Log L = Log Likelihood

LR = Likelihood ratio test of the hypothesis, that the NEGBINA-distribution reduces to the NEGBINI or the NEGBINII distribution (depending on the model selection).

LRP = Likelihood ratio test of the hypothesis, that the number of small purchases is Poisson distributed.

LC = Likelihood ratio test of the hypothesis, that the parameters corresponding to all explanatory variables except the constant term are equal to zero

AIC = Akaike's Information criterion

Table 3.1: continued

	APPLIANCES				OTHER EXPENDITURES			
	ML NEGBINI	ML NEGBINII	ML NEGBINA	PML	ML NEGBINI	ML NEGBINII	ML NEGBINA	PML
Constant	0.236	0.284	0.571	0.489	0.559	0.623	0.467	0.397
	(0.768)	(1.26)	(1.48)	(1.27)	(1.46)	(2.38)	(0.80)	(1.23)
URB	-0.148	-0.118	-0.132	-0.119	-0.499	-0.316	-0.541	-0.559
	(-3.29)	(-3.50)	(-2.52)	(-2.13)	(-9.50)	(-7.38)	(-8.41)	(-8.07)
Child	-0.183	-0.103	-0.342	0.335	-0.104	-0.002	-0.244	-0.266
	(-1.345)	(-1.08)	(-2.08)	(-2.44)	(-0.656)	(-1.98)	(-1.34)	(-1.50)
FG	0.205	0.174	0.176	0.172	0.133	0.083	0.183	0.167
	(4.56)	(5.36)	(2.88)	(3.87)	(2.551)	(2.07)	(2.23)	(3.53)
WORK II	-0.320	-0.010	-0.065	-0.049	0.222	0.112	0.356	0.353
	(-0.29)	(-0.12)	(-0.42)	(-0.43)	(1.699)	(1.16)	(1.98)	(2.78)
AGEP	0.010	0.008	0.007	0.007	0.015	0.011	0.011	0.015
	(2.57)	(2.93)	(1.19)	(1.63)	(3.321)	(0.326)	(1.59)	(3.47)
SOC	(-0.045)	-0.034	-0.072	-0.049	-0.120	-0.117	-0.073	-0.081
	(-1.073)	(-1.14)	(-0.102)	(-1.32)	(-2.69)	(-3.35)	(-1.23)	(-1.65)
SINGLE	0.075	0.151	-0.048	-0.125	0.159	0.153	0.119	0.071
	(0.59)	(1.42)	(0.250)	(-0.79)	(1.019)	(1.29)	(0.55)	(0.42)
α	2.72	0.006	-	-	1.966	0.003	-	-
	(10.69)	(7.81)	-	-	(7.794)	(6.61)	-	-
Log L.	119.49	119.45	125.2	-	-305.80	-319.60	-296.64	-
LR	11.42	11.50	-	-	18.32	45.92	-	-
LRP	848.68	848.60	860.1	-	447.84	420.29	466.16	-
LC	60.92	-	-	-	-	-	146.98	-
AIC	-220.98	-220.90	-218.9	-	629.60	657.20	625.28	-

Log L = Log Likelihood

LR = Likelihood ratio test of the hypothesis, that the NEGBINA-distribution reduces to the NEGBINI or the NEGBINII distribution (depending on the model selection).

LRP = Likelihood ratio test of the hypothesis, that the number of small purchases is Poisson distributed.

LC = Likelihood ratio test of the hypothesis, that the parameters corresponding to all explanatory variables except the constant term are equal to zero

AIC = Akaike's Information criterion

Table 3.1: continued

	DOMESTIC DECORATION				RECREATION, ENTERTAINMENT			
	ML NEGBINI	ML NEGBINII	ML NEGBINA	PML	ML NEGBINI	ML NEGBINII	ML NEGBINA	PML
Constant	0.587	0.514	0.879	0.754	1.363	1.197	1.191	1.091
	(1.94)	(2.201)	(2.11)	(2.50)	(4.25)	(5.49)	(2.98)	(2.36)
URB	-0.199	-0.152	-0.198	-0.206	-0.290	-0.194	-0.294	-0.232
	(-4.30)	(-4.23)	(-3.69)	(-4.52)	(-5.60)	(-5.47)	(-4.52)	(-2.75)
Child	0.131	0.135	0.069	0.060	0.424	-0.258	-0.492	-0.598
	(0.94)	(1.26)	(0.38)	(0.42)	(-3.01)	(-2.62)	(-2.61)	(-3.28)
FG	0.030	0.027	0.033	0.017	0.257	0.158	0.327	0.343
	(0.65)	(0.74)	(0.53)	(0.38)	(5.17)	(4.72)	(5.21)	(6.06)
WORKH	0.042	0.010	0.229	0.094	-0.125	-0.203	0.187	0.268
	(0.37)	(0.11)	(1.59)	(0.80)	(-1.07)	(-2.36)	(1.33)	(1.43)
AGEP	0.015	0.013	0.008	0.012	-0.0025	0.0015	-0.013	-0.012
	(3.80)	(4.37)	(1.26)	(3.73)	(-0.68)	(0.60)	(-2.79)	(-2.65)
SOC	-0.185	-0.152	-0.181	-0.174	-0.188	-0.134	-0.076	-0.111
	(-4.39)	(-4.64)	(-3.27)	(-4.54)	(-4.05)	(-4.29)	(-1.44)	(-1.84)
SINGLE	0.061	0.102	0.022	-0.049	0.922	0.629	-0.928	0.818
	(0.43)	(0.94)	(0.124)	(-0.36)	(5.66)	(5.67)	(4.50)	(4.34)
α	1.543	0.00023	-	-	4.754	0.012	-	-
	(7.56)	(6.06)	-	-	11.85	(8.26)	-	-
Log L.	-458.73	-458.49	-454.57	-	757.45	755.55	776.34	-
LR	8.32	7.84	-	-	37.78	41.58	-	-
LRP	329.22	329.7	337.54	-	1647.60	1643.80	1685.38	-
LC	63.0	-	-	-	-	-	143.42	-
AIC	935.46	934.88	941.14	-	-1496.90	-1493.10	-1520.68	-

Log L = Log Likelihood
LR = Likelihood ratio test of the hypothesis, that the NEGBINA-distribution reduces to the NEGBINI or the NEGBINII distribution (depending on the model selection).
LRP = Likelihood ratio test of the hypothesis, that the number of small purchases is Poisson distributed.
LC = Likelihood ratio test of the hypothesis, that the parameters corresponding to all explanatory variables except the constant term are equal to zero
AIC = Akaike's Information criterion

The age of the housewife, AGEP, has a significant[4] positive influence on the expected number of purchases below Dfl. 10 (see equation (2.6)) except for Recreation, Entertainment and Food. In the Netherlands partners of lower age more frequently have a paid job and consequently have less leisure, than partners of relative higher age. Therefore, our intuition is that most households with partners of low age spend relatively little time on shopping, but buy these goods in large quantities in supermarkets and department stores. The signs of the parameters, corresponding to AGEP, conform to this intuition.

The expected number of expenditures below Dfl. 10 on Recreation, Entertainment decreases significantly if small children are present in the household, and increases, if the household consists of a single person. The last result would conform to a life style of singles, especially those of young age, who spend a fair amount of time away from the home. Social class, SOC, - a variable which is strongly correlated with income - plays a significant role in explaining the number of purchases below Dfl. 10 for Clothing, Footwear and Domestic Decoration. The higher the social class the higher the expected number of small purchases will be.

The degree of urbanisation, URB, has a significant negative, and family size, FS, a significant positive influence on the expected number of purchases below Dfl. 10 for all consumption categories except for Clothing and Footwear in the case of URB and except for Domestic decoration in the case of FS.

Finally, the results suggest that the dummy variable WORKH, which indicates whether the head of household works, has no explanatory power for the number of purchases below Dfl. 10 for any category, except for Other Expenditures.

In the preceding section it was noticed that the Poisson-model is nested in the NEGBIN-model. A likelihood ratio test indicates that the Poisson-model must be rejected for all consumption categories (see row LRP in table 3.1). This is not a surprising result, because the Poisson model is very restrictive in several ways. For instance, the assumption underlying the Poisson model that the conditional mean and variance of the count, N, given the regressors are equal, may be too strong.

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4) At the 5%-level.

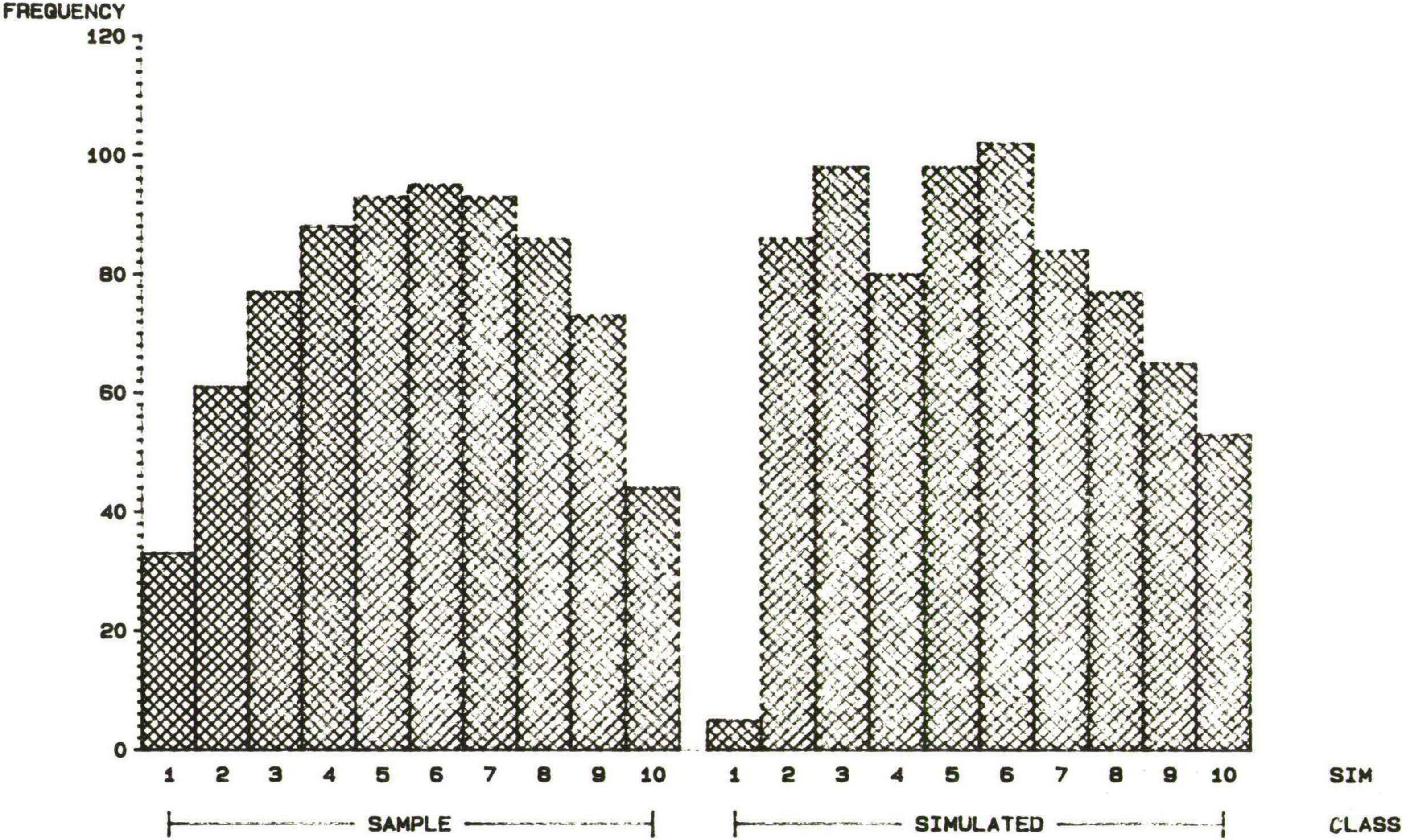
Table 3.2: Comparison of the sample mean and sample variances of the number of purchases below 10 guilders.

	april		may		june	
	mean	variance	mean	variance	mean	variance
1. Food	9.350	79.825	12.145	167.096	10.969	169.312
2. Clothing, footwear	0.713	1.575	0.957	2.101	0.508	1.035
3. Domestic Decoration	1.613	3.734	1.623	4.444	1.631	3.268
4. Recreation, Entertainment	2.463	19.594	2.261	15.225	2.000	18.656
5. Other expenditures	1.550	5.922	1.493	6.342	1.123	3.328
6. Appliances	2.300	10.694	2.043	9.366	2.185	10.684

Table 3.2 suggests, that for all consumption categories the sample variance exceeds the sample mean considerably. By using the negative binomial distribution, one can allow for the inequality of mean and variance.

Next we have tested by means of a likelihood ratio test, whether the parameter vector is varying with time (months). The results, summarized in table 3.3, suggest that the same model may be valid in all months.

FIGURE 1 SIMULATED AND SAMPLE FREQUENCY DISTRIBUTIONS
OF FOOD EXPENDITURES BELOW DFL. 10 IN APRIL 1984



10 CLASSES OF 1 DUTCH GUILDER EACH

Table 3.3: Likelihood ratio test of the hypothesis, that the parameters of the NEGBIN-model do not vary with time (months) (The test statistic is asymptotically χ^2_{72}-distributed in case of the categories Food, Recreation and Other Expenditures and χ^2_{128} for the other categories).

1. Food	76.38
2. Clothing and Footwear	59.98
3. Domestic Decoration	54.24
4. Recreation and Entertainment	118.12
5. Other Expenditures	101.10
6. Appliances	47.26

In Section 2 we have assumed that the amount of the i-th purchase Y_i, follows a Beta ([0,10],p,q) distribution (see equation (2.9)), where the parameters p and q vary over the nine months and across consumption categories. The 108 parameters are estimated by means of maximum likelihood. The following remarks can be made about the results:

- It is well known that, if p = q = 1, the Beta ([0,10],p,q)-distribution reduces to the uniform distribution over [0,10]. A likelihood ratio test indicates that for each month and for each consumption category except Appliances the hypothesis p = q = 1 has to be rejected. From the sample distribution of the small purchases of food in april 1984 (cf. figure 1) it is clear, that one cannot describe this distribution by a uniform distribution. The same remark applies to the other sample distributions.
- By means of a Wald-test we have also tested the hypothesis that the parameters p and q are constant over time (but different across consumption categories). The results are presented in table 3.4. The hypothesis must be rejected for all consumption categories except for Food.

Table 3.4: Test of the null-hypothesis, that the parameters p and q are constant over time. (The Wald test statistic is asymptotically χ^2_{16}-distributed).

1. Food	26.12
2. Clothing and Footwear	46.53
3. Domestic Decoration	82.77
4. Recreation	46.19
5. Supplementary Family costs	50.01
6. Appliances	34.56

- In order to check whether Y_i follows a Beta ([0,10],p,q) distribution, we chose to carry out a Pearson's Chi-Square Goodness of Fit test (see Mood, Graybill and Boes (1974), p. 446). For this purpose we have divided the interval [0,10] in ten equal parts $A_1,\ldots,A_{10}$ and have then calculated the predicted probabilities $P(A_i)$, $i = 1,\ldots,10$, to be used in the test. The results indicate, that, given $\alpha = 0.05$, for all months the purchases of Food and Other Expenditures cannot be described by a Beta-distribution. These results are confirmed in figure 1, in which, for the category Food in the month april, the Beta distribution predicts a too high frequency for the expenditures between zero and one Dutch guilder. The same remark applies to the category Domestic Decoration in all months except May, June and October, to Recreation, Entertainment in the months September, November and December, and to Appliances in the months July, August and September. In all other cases the null-hypothesis cannot be rejected. From these results we can conclude that it may be advisable to make the parameters p and q dependent on some family characteristics. This is a topic of future research.

§ 4. Summary and conclusions

In this paper we presented a two-stage model explaining the sum of small expenditures in one subsample (subsample B) and that can be used to predict the sum of small expenditures in the other subsample (subsample A). As noted in the introduction this kind of model can be used in other applications as well.

Concerning the first-stage model, the Count Model, the empirical results are satisfactory for all consumption categories. Especially, the similarity of the PML- and ML-estimates of the selected NEGBIN models is rather striking. Yet, it may be interesting to investigate other propability distributions for the number of small purchases, like the Double Hurdle model and With Zero (WZ) model proposed by Mullahy (1986).

The results for the second-stage model, the Amount Model, show that the Beta distribution does not fit well for all consumption categories. This may be remedied by a further parameterization of the parameters p and q of the Beta distribution. But other distributions could be tried as well.

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