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RESEARCH MEMORANDUM

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An implementation of an inventory model with stochastic lead time

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1. Introduction.

In this paper we analyse an inventory model having as lead time demand distribution a Schmeiser-Deutsch distribution (S.D. distribution) [4]. We demonstrate that this type of distribution is very suitable as it can have many different shapes.
After developing a new method for estimating the parameters of this distribution, we derive the explicit cost function of the model as function of the order quantity and the reorder level. Properties of this function are given together with a global algorithm to find the optimal order quantity and reorder point.

2. A static inventory model with stochastic lead time demand.

We analyse a model under the following assumptions :

a. The system is of the continuous review type.

b. The order quantity is not restricted.

c. The purchase cost b(q) is a continuous differentiable function of the order quantity q.

d. The lead time of an order, also called the delivery lag, has distribution function L(h).

e. The order quantities are assumed to be delivered in the order in which they are purchased.

f. The demand per unit of time has an arbitrary distribution function. The expectation of the demand per unit of time is r.

g. The holding cost per unit inventory per unit of time is c1.

h. Unfilled demand during the lead time is backlogged. The shortage cost per shortage unit per unit of time is c2.

The criterion used is minimization of the average cost per unit ordered. The cost function looks as follows :

$$(2.1) \quad K(x,q) = (c1/(r.q)).\int_0^q [\int_0^{x+y} (x+y-z)f(z)dz]dy +$$

$$(c2/(r.q)).\int_0^q [\int_{x+y}^{\infty} (z-x-y)f(z)dz]dy + b(q)/q.$$

where :

f(x) : the density function of the demand during the lead time;
x : the order quantity expressed in terms of units of economic inventory.
b(q) : the ordering cost. We assume that b(q) = c0 + q.a(q), with a(q) a two times differentiable function.

The cost function for this model can be derived in the following manner. Let us look at a cycle starting at the moment of delivery of an order and ending just before the next delivery. The inventory at the beginning of this cycle equals x+y-z if we assume that the demand during the lead time is z, the reorder level is x and the order quantity is q. The order quantity at the end of a cycle is x-z. Graphically this looks as follows :

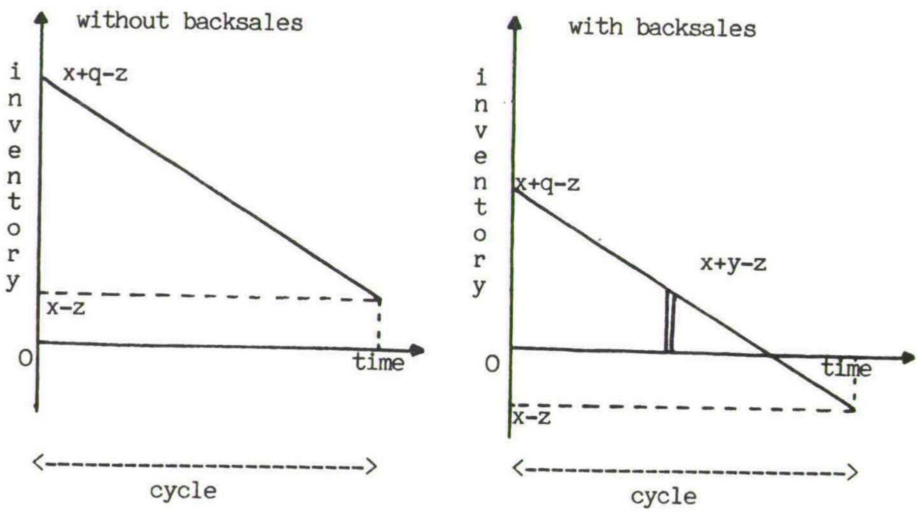

Figure 2.1 : The inventory level during a cycle.

The average time that there is a certain inventory level is equal to the average time between two successive demands, 1/r. If the demand at a specific moment is more then one unit this is considered as demands with time between successive demands of zero. The cost of an inventory level of x+y-z is equal to :

$$(c1/r).(x+y-z), \text{ if } x+y-z \geq 0$$

and

$$(c2/r).(z-x-y), \text{ if } x+y-z < 0.$$

When z, the demand during the lead time, is a stochastic variable and the order quantity is considered as a continuous variable, then the total costs per cycle are :

$$\int_0^q [(c1/r) \int_0^{x+y} (x+y-z)f(z)dz + (c2/r) \int_{x+y}^{\infty} (z-x-y)f(z)dz]dy + b(q) .$$

The average costs per unit ordered are then equal to (2.1).
Kriens and de Leve [1], have studied a model with the same cost function. However, the assumptions they made are more severe : they assume a Poisson distribution for the demand during a fixed period of time, with an average demand per unit of time of r units. In our concept this is superfluous. The end equations they have to solve to determine the optimal order quantity and reorder level are :

$$(2.2) \qquad (1/q^*) \int_0^{q^*} F(x^* + y)dy = c1/(c1+c2)$$

$$(2.3) \qquad c1/(2.r) - [(c1+c2) / (r.(q^*)^2)]. \int_0^{q^*} y.(1-F(x^* + y))dy$$

$$- c0/(q^*)^2 + \left[\partial b(q)/ \partial q \right]_{q=q^*} = 0,$$

where

$$(2.4) \qquad F(x^* + y) = \int_0^{x^* + y} f(z)dz.$$

In solving the equations (2.2) and (2.3) two causes can lead to difficulties :

1. In choosing a specific form for the density function f(z), it is advisable to choose one that has an explicit expression for its distribution function F(z), because this simplifies numerical matters considerably.

2. The total cost function K(x,q) should behave well otherwise the solution of the equations is not necessarily the solution of the problem. In other words, only in the situation that K(x,q) fulfils the normal conditions for the second derivatives, the equations (2.2) and (2.3) give, sufficient and necessary conditions for the minimizing problem.

Distribution functions of the demand during the lead time that fulfils these conditions should also fit the real world distributions of demand reasonably well.
We have chosen the four parameter S.D. distribution [4], whose proporties are summarized in the next section. This distribution satisfies in many ways the above remarks. The S.D. distribution is can assume many shapes. Two families of distributions which do also include a complete range of shapes are due to Pearson [3] and Johnson [2]. However, those systems are not well suited because in general an explicit form of their conditional expectations is missing.

3. Properties of the Schmeiser-Deutsch distribution.

Schmeiser and Deutsch [4] have recently developed a versatile system of four parameter distributions. The distribution's versatility in assuming a wide variety of shapes makes it a reasonable model for a wide range of processes. Shapes ranging from U-shaped distributions to the uniform distribution, also heavier tailed and skewed distributions are attainable.
The density is defined as follows

$$(3.1)\quad f(x) = 1/(12.13)\ \left|(11-x)/12\right|^{(1-13)/13}$$

where, $t = 11-12.14^{13}$, $p = 11+12.(1-14)^{13}$, $x \in [\ t,p\ ]$,

$12\ ,\ 13 > 0\ ;\ -\infty < 11 < \infty\ ;\ 0 \leq 14 \leq 1$.

The distribution function looks as follows :

$$(3.2)\qquad F(x) = \begin{cases} 14 - [\ (11-x)/12\ ]^{1/13}, & t \leq x \leq 11, \\ 14 + [\ (x-11)/12\ ]^{1/13}, & 11 \leq x \leq p. \end{cases}$$

The inverse distribution function is :

$$(3.3)\qquad x = F^{-1}(w) = \begin{cases} 11 - 12.(14-w)^{13}, & w < 14, \\ 11 + 12.(w-14)^{13}, & w > 14. \end{cases}$$

In figure 3.1 some characteristic shapes are illustrated.

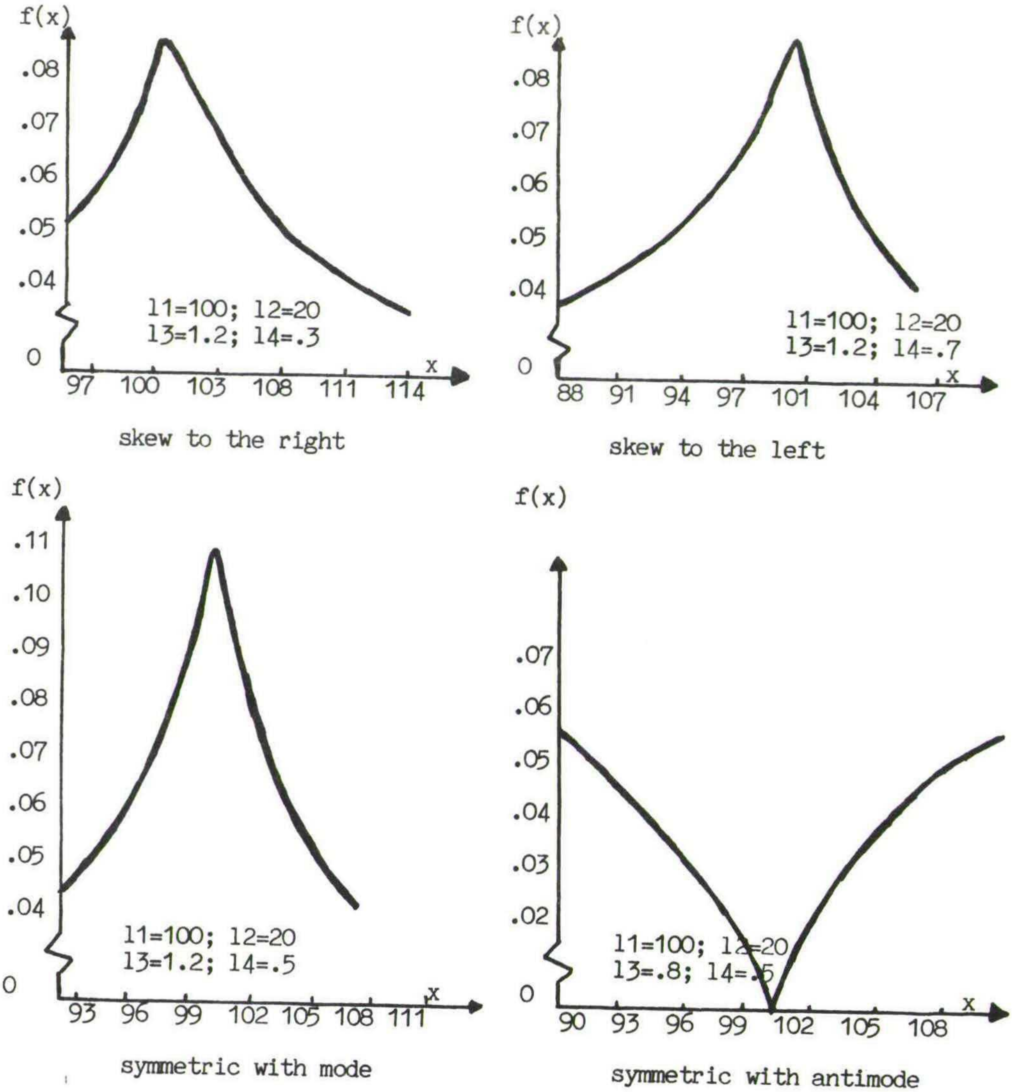

Figure 3.1 Some characteristic shapes of
the Schmeiser-Deutsch distribution.

The central moments of $\underline{x}$ are as follows:

$$\mu = l1 + E(\underline{x} \mid l1=0) ,$$

$$\sigma^2 = E[\underline{x}^2 \mid l1=0] - E[\underline{x} \mid l1=0]^2 ,$$

$$\mu_3 = E[\underline{x}^3 \mid l1=0] - 3.E[\underline{x}^2 \mid l1=0].E[\underline{x} \mid l1=0] + 2.E[\underline{x} \mid l1=0]^3 ,$$

$$\mu_4 = E[\underline{x}^4 \mid l1=0] - 4.E[\underline{x}^3 \mid l1=0].E[\underline{x} \mid l1=0] + $$
$$+ 6.E[\underline{x}^2 \mid l1=0].E[\underline{x} \mid l1=0]^2 - 3.E[\underline{x} \mid l1=0]^4 ,$$

where,

$$E[\underline{x}^k \mid l1=0] = (l2^k)/(k.l3 + 1).[(-1)^k .l4^{k.l3+1} + (1 - l4)^{k.l3+1}] .$$

The third and fourth standardized moments are:

(3.4) $\alpha_3 := \mu_3 / \sigma^3$ and $\alpha_4 = \mu_4 / \sigma^4$,

which are measures of skewness and kurtosis respectively.

The location and the spread of the distribution are determined by l1 and l2 respectively. The shape of the distribution is determined by :

$$(3.5) \qquad g(l3,l4) = \begin{cases} -(l4 - p)^{l3} , & \text{if } p \leq l4, \\ (p - l4)^{l3} , & \text{if } p > l4. \end{cases}$$

Symmetric distributions correspond to l4 = .5. For l3 > 1 and l4 < .5 the distribution is skew to the right; for l3 > 1 and l4 > .5 it is skew to the left. For l3 < 1 the direction of skewness is reversed. For l3 > 1 the unique mode is at x = l1, l3 = 1 gives a uniform distribution and for l3 $\in$ (0,1) the unique antimode is at x = l1.
An important property of the S.D. distribution is :

Given k = $\bar{k}$ and f(z) is a S.D. density function , x can be determined from

$$\int_0^x f(z)dz = \bar{k} , \text{ for } l1 - l2.l4^{l3} = 0,$$

as follows

$$x = F^{-1}(\bar{k}) = \begin{cases} l1 - l2.(l4 - \bar{k})^{l3} & , \ \bar{k} \leq l4, \\ l1 + l2.(\bar{k} - l4)^{l3} & , \ \bar{k} > l4. \end{cases}$$

4. Fitting a Schmeiser-Deutsch distribution function.

Schmeiser and Deutsch [4] propose a modified method of moments to estimate the parameters of the distribution function. They use a two stage procedure where in the first stage the parameters l3 and l4 are estimated via a least squares fit on the standardized third and fourth moment . In the second stage the parameters l1 and l2 are calculated from equalization of the sample mean and variance to their population counterparts expressions in terms of the parameters.
This method has two severe disadvantages :

first, this criterion is based on a good fit of the first four moments and not on a fit of the total distribution;
second, the proposed procedure is a graphically one and so not very suitable for a computer decision model.

Therefore we propose another way to fit the distribution function to the actual data.
Suppose we have a set of observations x1,....,xn, which are ordered as an increasing sequence : x(1),....,x(n).We want to fit the distribution function to the observations so we can use a least squares method or a chi-square method.
At first sight the l.s. method is not as complex as the chi-square one, so we develop a method to solve the first problem. This leads to the following problem:

minimize

$$z = \sum_{i=1}^{n} (F[x(i)] - i/n)^2$$

subject to the constraints on the parameters.

F[x(i)] is the S.D. distribution function in the ordered point x(i). The main difficulty of this problem is caused by the fact that the S.D. distribution function is not continuous differentiable, so the parameters are difficult to calculate. The precise problem to solve is:

minimize

$$z = \sum_{i=1}^{m} \left(l4 - \left[(l1-x(i))/l2 \right]^{1/l3} - i/n \right)^{2} + \sum_{i=m+1}^{n} \left(l4 + \left[(x(i)-l1)/l2 \right]^{1/l3} - i/n \right)^{2}$$

subject to

$$l2 > 0 \;,\; l3 > 0 \;,\; 0 \le l4 \le 1,$$

and

$$m = \max_{i} \left[i \mid x(i) < l1 \right],$$

$$x(1) \ge l1 - l2.l4^{l3} ,$$

$$x(n) \le l1 + l2.l4^{l3} .$$

The methods to solve this kind of problems are mostly based on Newton-like algorithms.The difficulty arise through the occurence of parameters in the summation borders. In fact m is a function of l1.
A method to solve the above problem, is the following two stage iterative procedure :

1. Start with an estimate for l1, $\hat{l1}$, e.g. the sample mode; and thus

$$m = \max_{i} \left[i \mid x(i) < \hat{l1} \right].$$

2. Solve the problem : minimize z under the given constraints for the given value of m. This gives an approximation of the optimal values of l1, l2, l3 and l4 :

$$\hat{l1},\ \hat{l2},\ \hat{l3} \text{ en } \hat{l4}.$$

3. If $x(m) < \hat{l1} < x(m+1)$, then the solution is optimal and the minimum is reached . If not, then if :

$\hat{l1} = x(m)$, then m := m-1, and return to step two;

$\hat{l1} = x(m+1)$, then m = m+1, and return to step two.

In step two we have to solve a convex optimization problem. This is possible with well-known methods, using first and second derivatives. In a next paper we will describe an algorithm as well as a computer implementation to solve this problem.

5. Analysing the total cost function and the derived functional equations.

5.1 Introduction.

Kriens and de Leve give in their study as result two functional equations (see (2.2) and (2.3)). Solving these equations leads to the optimal order quantity and reorder level. However, by using the S.D. distribution as lead time demand distribution, we had to solve non-linear functional equations under constraints on the decision variables x and q. This causes a lot of numerical problems in which evaluation of the total cost function on certain boundary points was neccesary. Then we decided to consider the possibility of solving the problem directly by minimizing the total cost function. As we shall see in the following sections, this leads to a usable method.

5.2 Evaluation of the total cost function with a S.D. distribution function.

In the total cost function (2.1) an integral of the following type plays an important role:

$$(5.2.1) \qquad \int_0^{x+y} (x+y-z)f(z) = (x+y)F(x+y) - \int_0^{x+y} zf(z)dz \; .$$

In evaluating this integral the boundaries of the S.D. distribution are important as well as the mode. This is illustrated in figure 5.2.1, where

$$(5.2.2) \qquad t = l1 - l2.l4^{l3} \quad \text{and} \quad p = l1 + l2(1-l4)^{l3} \; .$$

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t                   l1                                  p
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Figure 5.2.1. The boundaries of the S.D. distribution.

Given this property of the S.D. distribution the following intervals will be distinguished to evaluate the above integral:

I : $x+y \in (0,t]$; II : $x+y \in (t,l1]$;
III : $x+y \in (l1,p]$; IV : $x+y \in (p,\infty)$.

Defining the integral (5.2.1) as g(i,x,y) for the four intervals, thus i = I,II,III,IV and evaluating this functions with the S.D. distribution the results are:

$$(5.2.3)\quad \begin{cases} g(I,x,y) = 0 \\ g(II,x,y) = 14(x+y) + b3((11-x-y)/12)^{b4} - b1 \\ g(III,x,y) = 14(x+y) + b3((x+y-11)/12)^{b4} - b1 \\ g(IV,x,y) = x + y - b2 \end{cases}$$

where

$$b1 = 11.14 - (12.14)^{13+1} / (13+1),$$

$$b2 = 11 + (12/(13+1)).(\ (1-14)^{13+1} - 14^{13+1}\ ),$$

$$b3 = (12.13)/(13+1),$$

$$b4 = (1+13)/13.$$

The next step in the evaluation of the total cost function is the evaluation of the integrals:

$$h(i,x,q) = \int_0^q g(j(i),x,y)dy,$$

where the indices i and j(i) are determined by the definition regions of the functions g(i,x,y) and the integration regions of y. We distinguish ten intervals, see table 5.2.1..

region	i	j(i)
$x+q \in (0,t]$ and $x \in (0,t]$	1	I
$x+q \in (t,11]$ and $x \in (0,t]$	2	II
$x+q \in (t,11]$ and $x \in (t,11]$	3	II
$x+q \in (11,p]$ and $x \in (0,t]$	4	III
$x+q \in (11,p]$ and $x \in (t,11]$	5	III
$x+q \in (11,p]$ and $x \in (11,p]$	6	III
$x+q \in (p,\infty)$ and $x \in (0,t]$	7	IV
$x+q \in (p,\infty)$ and $x \in (t,11]$	8	IV
$x+q \in (p,\infty)$ and $x \in (11,p]$	9	IV
$x+q \in (p,\infty)$ and $x \in (p,\infty]$	10	IV

Table 5.2.1 The integration regions of the cost function.

The results of this calculus are:

$h(1,x,q) = 0,$

$h(2,x,q) = (14.x-b1).(q+x-t) + (1/2).14.(q^2 - (t-x)^2) + b5.b7,$

$$h(3,x,q) = (14.x-b1).q + (1/2).14.q^{2} + b5.b8,$$

$$h(4,x,q) = (14.x-b1).(q+x-t) + (1/2).14.(q^{2} - (t-x)^{2}) + $$
$$+ b5.[(11-t)^{b6} + (x+q-11)^{b6}],$$

$$h(5,x,q) = (14.x-b1).q + (1/2).14.q^{2} + b5.[(11-x)^{b6} + (x+q-11)^{b6}],$$

$$h(6,x,q) = (14.x-b1).q + (1/2).14.q^{2} + b5.[-(x-11)^{b6} + (x+q-11)^{b6}],$$

$$h(7,x,q) = (14.x-b1).(p-t) + (1/2).14.[(p-x)^{2} - (t-x)^{2}] + $$
$$+ b5.[(11-t)^{b6} + (p-11)^{b6}] + (x-b2).(q+x-p) + $$
$$+ (1/2).[q^{2} - (p-x)^{2}],$$

$$h(8,x,q) = (14.x-b1).(p-x) + (1/2).14.(p-x)^{2} +b5.[(11-x)^{b6} +(p-11)^{b6}]$$
$$+ (x-b2).(q+x-p) + (1/2).[q^{2} - (p-x)^{2}],$$

$$h(9,x,q) = (14.x-b1).(p-x) + (1/2).14.(p-x)^{2} +b5.[-(x-11)^{b6} +(p-11)^{b6}]$$
$$+ (x-b2).(q+x-p) + (1/2).[q^{2} - (p-x)^{2}],$$

$$h(10,x,q) = (x-b2).q + (1/2).q^{2},$$

where

$$b5 = (12^{-(1/13)} .13^{2}) / (1+13).(1+2.13),$$

$$b6 = (1+2.13) / 13,$$

$$b7 = (11-t)^{b6} - (11-x-q)^{b6},$$

$$b8 = (11-x)^{b6} - (11-x-q)^{b6}.$$

The double integral in the second term in the total cost function (2.1) is evaluated analoguously to the evaluation of $h(i,x,q)$ and is defined over the same regions. The results can be summarized as follows:

$$H(1,x,q) = (b2-x).q - (1/2).q^{2},$$

$$H(2,x,q) = (b2-x).(t-x) - (1/2).(t-x)^{2} + (b9-(1-14).x).(q+x-t) - (1/2).(1-14).(q^{2}-(t-x)^{2}) + b5.b7 ,$$

$$H(3,x,q) = (b9-(1-14).x).q - (1/2).(1-14).q^{2} + b5.b8 ,$$

$$H(4,x,q) = (b2-x).(t-x) -(1/2).(t-x)^{2} + (b9-(1-14).x).(q+x-t) - (1/2).(1-14).(q^{2}-(t-x)^{2}) + b5.[(11-t)^{b6} + (x+q-11)^{b6}] ,$$

$$H(5,x,q) = (b9-(1-14).x).q-(1/2).(1-14).q^{2} +b5.[(11-x)^{b6} +(x+q-11)^{b6}],$$

$$H(6,x,q) = (b9-(1-14).x)q-(1/2).(1-14).q^{2} +b5.[-(x-11)^{b6} +(x+q-11)^{b6}],$$

$$H(7,x,q) = (b2-x).(t-x) - (1/2).(t-x)^{2} + (b9-(1-14).x).(p-t) - (1/2).(1-14).[(p-x)^{2} -(t-x)^{2}] + b5.[(11-t)^{b6} + (p-11)^{b6}],$$

$$H(8,x,q) = (b9-(1-14).x).(p-x) - (1/2).(1-14).(p-x)^{2} + b5.[(11-x)^{b6} + (p-11)^{b6}] ,$$

$$H(9,x,q) = (b9-(1-14).x).(p-x) - (1/2).(1-14).(p-x)^{2} + b5.[-(x-11)^{b6} + (p-11)^{b6}] ,$$

$$H(10,x,q) = 0 ,$$

where

$$b9 = 11.(1-14) + [12.(1-14)^{13+1}] / (13+1).$$

Let's call $h(x,q) = \bigcup_{i=1}^{10} h(i,x,q)$ and $H(x,q) = \bigcup_{i=1}^{10} H(i,x,q)$.

The evaluation of the total cost function $K(x,q)$ is very easy now, because:

$$(5.2.4) \qquad K(x,q) = (1/(r.q)).[c1.h(x,q) + c2.H(x,q)] + b(q)/q .$$

5.3. Properties of the total cost function K(x,q).

After some calculations it can be shown that the functions h(x,q) and H(x,q) are two times continuous differentiable. In the appendix it is shown by way of example that the functions h(x,q) and H(x,q) are two times continuous differentiable on the border of the regions 3 and 5. Generalisation of this proof over the total region is not difficult. If the function b(q) is also two times continuous differentiable it is easy to see that the total cost function K(x,q) is also a two times contiuous differentiable function.

5.4. Some global ideas for an algorithm.

In this section we give some general ideas of an algorithm to find the optimal x and q. In a next paper these ideas will be worked out in a computer algorithm.
Globally this algorithm works as follows:

1. Initialise the cost parameters as well as the parameters of the lead time distribution.

2. Determine starting values of x and q.

3. Determine the necessary function evaluations of K(x,q), ∇(K(x,q)) and $\mathcal{H}$(K(x,q)).

4. Use an iterative procedure, e.g. a Newton-like one, to improve the values of x and q.

6. Summary remarks.

In this paper the following results are reached:

- the cost function of a specific inventory model is derived under more simple assumptions then elsewhere;

- a new parameter estimation procedure for the S.D. distribution is derived;

- the total cost function of the inventory problem considered is evaluated using a S.D. distribution as lead time demand distribution;

- properties of the total cost function are analysed;

- a global algorithm for solving the optimal reorder point and order quantity is considered.

In the next paper the following topics will be analysed further:

- the new parameter estimation technique for the S.D. distribution will be implemented and evaluated;

- an implementation of an algorithm to find the optimal order quantity and reorder point will be given as well as a comparison with other methods.

Appendix. Example of the calculations necessary to prove the two times continuous differentiability of K(x,q).

To prove that the function K(x,q) is two times continuous differentiable it is sufficient to prove that the functions h(x,q), H(x,q) and b(q) are two times continuous differentiable. We have assumed that the function b(q) is continuous differentiable. From the definitions of the other functions it is directly clear that inside the regions all functions are continuous differentiable. So it is sufficient to prove continuity of h(x,q), ∇(h(x,q)), $\mathfrak{H}$ (h(x,q)), H(x,q), ∇(H(x,q)) and $\mathfrak{H}$ (H(x,q)) on the borders of the regions. To illustrate the tedious and boring arithmatic we only give as example the proof for the border of the regions 3 and 5. This border is the plane:

$$x + q = 11 \quad \text{for} \quad x \in (t, 11].$$

As we can see from the definition of the functions :

$$h(3,x,11-x) - h(5,x,11-x) = b5.[b8-(11-x)^{b6} - (x+q-11)^{b6}]_{(x+q=11)} = 0,$$

so the function h(x,q) is continuous on the border of the regions 3 and 5.

The gradients are:

$$\nabla(h(3,x,q)) = \begin{bmatrix} 14.q + b5.b6.((11-x-q)^{b6-1} - (11-x)^{b6-1}) \\ 14.x + 14.q - b1 + b5.b6.(11-x-q)^{b6-1} \end{bmatrix},$$

$$\nabla(h(5,x,q)) = \begin{bmatrix} 14.q + b5.b6.((x+q-11)^{b6-1} - (11-x)^{b6-1}) \\ 14.x + 14.q - b1 + b5.b6.(x+q-11)^{b6-1} \end{bmatrix}.$$

For x+q=11 these both gradients are equal, so there only rests to prove that this is also true for the Hessians. The Hessian of the function h(3,x,q) equals:

$$\begin{bmatrix} ((11-x)/12)^{1/13} - ((11-x-q)/12)^{1/13} & 14-((11-x-q)/12)^{1/13} \\ 14-((11-x-q)/12)^{1/13} & 14-((11-x-q)/12)^{1/13} \end{bmatrix}.$$

The Hessian of the function h(5,x,q) equals:

$$\begin{bmatrix} ((11-x)/12)^{1/13} + ((x+q-11)/12)^{1/13} & 14+((x+q-11)/12)^{1/13} \\ 14+((x+q-11)/12)^{1/13} & 14+((x+q-11)/12)^{1/13} \end{bmatrix}.$$

For x+q=11 both these Hessians are equal and so we have proved that on the border of the regions 3 and 5 the function h(x,q) is two times continuous differentiable. The proof over the total definition region for the functions h(x,q) and H(x,q) is analoguous to the above .

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