Spatial competition with intermediated matching *†

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Abstract

This paper analyzes the spatial competition in commission fees between two match makers. These match makers serve as middlemen between buyers and sellers who are located uniformly on a circle. The profits of the match makers are determined by their respective market sizes. A limited willingness to pay is incorporated by means of reservation prices. If the fraction of buyers and sellers is unequal, the match makers are willing to subsidize the short side of the market, while the long side is exploited completely, provided reservation prices are sufficiently high. Competition is then concentrated entirely on the short side. When reservation prices are low, two local monopolies will emerge.

Keywords: Matching, middlemen, spatial price competition.

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1 Introduction

In many markets, intermediation plays an important role. In this paper, intermediation in bilateral matching markets is studied. In this type of markets, there are two types of agents, each of which seeks to trade with an agent of the other type.

The existence of an intermediating institution may reduce search costs associated with finding a suitable trading partner (see, e.g., Diamond (1984) for a survey of search literature). Intermediation in search economies may take place for instance through money (Kiyotaki and Wright (1989)). In this paper, intermediation by *middlemen* is studied. The role of middlemen in search markets is analyzed by, e.g., Rubinstein and Wolinsky (1987), who derive coexistence of direct and intermediated trade, Bhattacharya and Yavas (1993), who model middlemen as 'traders of last resort', and Yavas (1995).

Essentially, we can distinguish two different types of middlemen, namely *market makers* and *match makers* (see Yavas (1993) for a comparison). Market makers are actually involved in the trade process, in the sense that they buy commodities from sellers, and resell them to buyers. Match makers are not involved in the trading process; they just make trade possible by bringing buyers and sellers together. This paper studies a market organized by match makers.

We analyze a model of spatial competition in commission fees between two match makers. We develop a Salop (1979) type model of competition on a market for one commodity. In our model, there are continuum populations of buyers and sellers, uniformly distributed over a circular city (see also Webers (1994)). Each seller owns one unit of an indivisible commodity, which he desires to sell to one of the buyers[1]. Moreover, each buyer desires to buy one unit. The valuations of buyers and sellers for the commodity are identical.

Buyers and sellers have to make use of the services of one of the two match makers in order to trade. If a buyer or seller goes to a match maker, he pays a commission fee to the match maker, provided he is matched. Commission fees are assumed to be nonnegative. Besides a commission fee, buyers and sellers incur a relational cost by going to a match maker. This includes costs of effort, search, transportation, etc. The match makers are differentiated maximally with respect to the relational costs. It is assumed that the match makers are symmetric in the sense that if the fractions of buyers and sellers are the same for both match makers, the buyers and sellers get equal trade surpluses at each match maker.

[1] The indivisibility assumption is relaxed by Trejos and Wright (1995).

The focus of our model is on the competition in commission fees between the match makers. Therefore, the mechanism by which trade is performed, is not modeled explicitly. Such a mechanism could be a competitive market (Shapley and Shubik (1972)), or bargaining (Rubinstein and Wolinsky (1985)).

We incorporate a limited willingness to pay into the model, in the form of reservation prices. The reservation price indicates how much a buyer or seller is willing to spend, in terms of the fee and the relational cost, in order to be matched by a match maker. Reservation prices influence the 'potential markets' of the match makers, being the fraction of buyers or sellers at a match maker whose fee and relational cost are covered fully by the reservation price. Following Webers (1995), we can distinguish three different regimes of potential market areas at given prices: *Strong competition*, which is the case if the potential market areas of the two match makers at these prices have a nonempty intersection for both types of agents, *weak competition*, in case the potential market areas of the match makers at these prices have a nonempty intersection for one of the two types of agents and for the other type the intersection is either a point or empty, and *no competition*, in case the potential market areas of the match makers at these prices have an intersection which is either a point or empty for both types of agents. The notion of potential market areas is a generalization of the one formulated by Gabszewicz and Thisse (1986), which holds for 'zero prices'.

The profits of the match makers are determined by their respective market sizes. More precisely, we assume that every buyer and seller goes to the match maker whose sum of fee and relational cost is the lowest. We do not include agents' beliefs about the distribution of buyers and sellers over the middlemen, because we want to focus on the competition in commission fees. If we think of the situation as a two-stage game, in which the middlemen set fees in the first stage, and the buyers and sellers choose a middleman in the second stage simultaneously, the number of equilibria of this game is large. This is caused by the externality associated with bilateral matching: The surplus of every market participant is depending on the distribution of buyers and sellers over the match makers. Since the buyers and sellers choose a match maker simultaneously, no information is available about this distribution[2]. Therefore, many strategies support an equilibrium in the second stage.

The profit of a match maker is determined by the minimum of the sizes of his potential market areas of buyers and sellers, by the assumption that only matched agents pay the commission fee. Therefore, when maximizing profits, a match maker equals the buyer and seller fractions he serves. By this property,

[2]Buyers and sellers run the risk of not being matched. In some sense, this risk could be related to the risk associated with the timely delivery of products (Espinosa (1992)).

the case of unequal densities of sellers and buyers along the circle[3], has to be distinguished from the equal density case. If densities are unequal, no competition and weak competition can only occur in equilibrium. If densities are equal, strong competition may also occur.

Two interesting results follow from the model. First, the restriction on one side of the market implies that for sufficiently high reservation prices, the long side of the market can be 'exploited' completely by the middlemen. Since the short side determines the middlemens' profits entirely, it is not optimal for the firms to compete for the agents on the long side. Hence, the firms' profits tend to infinity if reservation prices become larger and larger. At the same time, the agents on the short side of the market may entirely 'free ride', in the sense that they pay a zero commission fee. In equilibrium, the middlemen even desire to subsidize these agents. The positive effect of the fees on potential market areas is then dominating the negative effect on profits. We restrict ourselves to non-negative fees, although the middlemen have a tendency to subsidize. One may argue that explicit subsidies are not permitted. In the case of equal densities, the asymmetry between the long and short side of the market disappears completely. Second, a large amount of equilibrium indeterminacy is created for equal densities. For unequal densities, this problem does not occur, except for a non-generic set of parameters. The case of equal densities itself is non-generic, however, so that the indeterminacy does not cause too serious problems. The case of equal densities is analyzed in order to provide a benchmark.

The remainder of this paper is organized as follows. In Section 2, the model is formulated. In Section 3, the equilibria of the price-setting game are derived, for the cases of equal densities and unequal densities. Section 4 provides a characterization of the equilibria. Finally, in Section 5, comparative statics is performed between the cases of equal densities and unequal densities.

2 The Model

In the model there are three different parties. First, there are two different types of agents. Agents of type 1 are willing to sell a unit of a homogeneous indivisible good and agents of type 2 are willing to buy a unit of this good. In order to trade they need a third party, say intermediaries, whose service it is to match the sellers and the buyers. These intermediaries are referred to as firms.

[3]It is often assumed in the literature, that either the supply is not binding or the demand functions of the firms are exogenous. In our model, the 'demand functions', i.e., the potential markets, are endogenous. The model can be seen as a 'strategic market coverage' type. Strategic market coverage through advertising was considered by Boyer and Moreaux (1992).

The number of firms is equal to two. Firm j, $j \in \{1,2\}$, charges price or fee ϕ_j^i to agents of type i, $i \in \{1,2\}$, for providing this service. Let ϕ_j denote the tuple of prices $< \phi_j^1, \phi_j^2 >$ for $j \in \{1,2\}$.

Agents of type i, $i \in \{1,2\}$, are located uniformly along a circle with perimeter 1. The density equals α for type 1 agents and β for type 2 agents, where $\alpha, \beta > 0$. For ease of exposition we let $\alpha \leq \beta$, so potential demand is at least as large as potential supply, although all results will hold as well in case $\alpha > \beta$. Firms are located symmetrically along the circle, so they are located at maximum distance from each other. Firm 1's location will be fixed at 0, so firm 2's location is $\frac{1}{2}$.

Both types of agents face identical linear relational costs with unit cost $t > 0$. Furthermore agents of type i, $i \in \{1,2\}$, have reservation price $\overline{p}_i$ for the relational costs and fees charged by any of the two firms, i.e., they want to pay up to an amount $\overline{p}_i$ for the firms' services. The reservation prices are assumed to be given exogenously.

It may happen well that the fees or the relational costs are so high that the reservation price cannot cover these.

Definition 2.1 *The potential market area of firm j, $j \in \{1,2\}$, for agents of type i, $i \in \{1,2\}$, at price ϕ_j^i, denoted by $\mathcal{M}_{ij}(\phi_j^i)$, is the set of agents of type i, for which the sum of the relational cost and the price ϕ_j^i charged by firm j does not exceed the reservation price.*

More formally we get $\mathcal{M}_{i1}(\phi_1^i) = \{x_i \in [0,1] \mid \phi_1^i + tx_i \leq \overline{p}_i \text{ or } \phi_1^i + t(1-x_i) \leq \overline{p}_i\}$ and $\mathcal{M}_{i2}(\phi_1^i) = \{x_i \in [0,1] \mid \phi_2^i + t(\frac{1}{2} - x_i) \leq \overline{p}_i \text{ or } \phi_2^i + t(x_i - \frac{1}{2}) \leq \overline{p}_i\}$ for $i \in \{1,2\}$.

The notion of potential market areas is used to describe the structure of competition among the two firms.

Definition 2.2 *At given prices there is strong competition if the potential market areas of the two firms at these prices have a nonempty intersection for both types of agents, there is weak competition if the potential market areas of the firms at these prices have a nonempty intersection for one of the two types of agents and for the other type the intersection is either a point or empty, and there is no competition at these prices if the potential market areas of the firms at these prices have an intersection which is either a point or empty for both types of agents.*

The size of the potential market area of firm j, $j \in \{1,2\}$, of agents of type i, $i \in \{1,2\}$, at price ϕ_j^i is the total length of the interval of agents of type i for which the sum of the relational cost to firm j and the price of firm j, ϕ_j^i, does not exceed the reservation price $\overline{p}_i$. The minimum of the sizes of the potential market areas of firm j of agents of type 1 and type 2 is called the *market size* of firm j. We denote the market size of firm j, $j \in \{1,2\}$, at prices ϕ_1 and ϕ_2 by $M_j(\phi_1,\phi_2)$. The profits of firm j, $j \in \{1,2\}$, at prices ϕ_1 and ϕ_2 are equal to $(\phi_j^1+\phi_j^2)M_j(\phi_1,\phi_2)$ and are denoted by $\Pi_j(\phi_1,\phi_2)$.

It is easy to verify that the potential market areas of the two firms for agents of type i, $i \in \{1,2\}$, have a nonempty intersection in case $\frac{\phi_1^i+\phi_2^i}{2}+\frac{t}{4} \leq \overline{p}_i$ and have an intersection which is either a point or empty in case $\frac{\phi_1^i+\phi_2^i}{2}+\frac{t}{4} \geq \overline{p}_i$. This means that there are four different regions under concern.

For $\frac{\phi_1^1+\phi_2^1}{2}+\frac{t}{4} \geq \overline{p}_1$ and $\frac{\phi_1^2+\phi_2^2}{2}+\frac{t}{4} \geq \overline{p}_2$ we have the situation of no competition. The market size of firm j, $j \in \{1,2\}$, is given then by

$$M_j(\phi_1,\phi_2) = min\left\{\frac{2\alpha}{t}(\overline{p}_1-\phi_j^1), \frac{2\beta}{t}(\overline{p}_2-\phi_j^2)\right\}$$

For $\frac{\phi_1^1+\phi_2^1}{2}+\frac{t}{4} \leq \overline{p}_1$ and $\frac{\phi_1^2+\phi_2^2}{2}+\frac{t}{4} \geq \overline{p}_2$ we have the situation of weak competition, where the firms compete for the sellers. The market size of firm j, $j \in \{1,2\}$, is given then by

$$M_j(\phi_1,\phi_2) = min\left\{\frac{\alpha}{t}(\phi_k^1-\phi_j^1+\frac{t}{2}), \frac{2\beta}{t}(\overline{p}_2-\phi_j^2)\right\}$$

with $k \neq j \in \{1,2\}$.

For $\frac{\phi_1^1+\phi_2^1}{2}+\frac{t}{4} \geq \overline{p}_1$ and $\frac{\phi_1^2+\phi_2^2}{2}+\frac{t}{4} \leq \overline{p}_2$ we have the situation of weak competition, where the firms compete for the buyers. The market size of firm j, $j \in \{1,2\}$, is given then by

$$M_j(\phi_1,\phi_2) = min\left\{\frac{2\alpha}{t}(\overline{p}_1-\phi_j^1), \frac{\beta}{t}(\phi_k^2-\phi_j^2+\frac{t}{2})\right\}$$

with $k \neq j \in \{1,2\}$.

Finally, for $\frac{\phi_1^1+\phi_2^1}{2}+\frac{t}{4} \leq \overline{p}_1$ and $\frac{\phi_1^2+\phi_2^2}{2}+\frac{t}{4} \leq \overline{p}_2$ we have the situation of strong competition. The market size of firm j, $j \in \{1,2\}$, is given then by

$$M_j(\phi_1,\phi_2) = min\left\{\frac{\alpha}{t}(\phi_k^1-\phi_j^1+\frac{t}{2}), \frac{\beta}{t}(\phi_k^2-\phi_j^2+\frac{t}{2})\right\}$$

with $k \neq j \in \{1,2\}$.

3 Equilibria

Each firm j, $j \in \{1,2\}$, chooses fees ϕ_j^i, $i \in \{1,2\}$, as to maximize its profits. We define firm j's strategy $\phi_j \in \Phi = [0,\overline{p}_1] \times [0,\overline{p}_2]$ as the tuple of prices charged by firm j. The profits of firm j, $j \in \{1,2\}$, are denoted by $\Pi_j(\phi_1,\phi_2)$. The game in which firms simultaneously choose prices, is referred to as G. For equilibrium analysis we use the Nash equilibrium concept.

Definition 3.1 *A pure Nash equilibrium for the game G is a pair of strategies $(\phi_1^*,\phi_2^*) \in \Phi \times \Phi$ such that $\Pi_1(\phi_1^*,\phi_2^*) \geq \Pi_1(\phi_1,\phi_2^*)\ \forall\ \phi_1 \in \Phi$ and $\Pi_2(\phi_1^*,\phi_2^*) \geq \Pi_2(\phi_1^*,\phi_2)\ \forall\ \phi_2 \in \Phi$.*

Because firms are located symmetrically it makes sense to look for an equilibrium in which both firms choose the same prices. Moreover for both firms demand and supply must be equal in equilibrium. This is stated in the following lemma.

Lemma 1 *At any Nash equilibrium (ϕ_1^*,ϕ_2^*) of the game G, demand and supply are equal for both firms.*

Proof Suppose first that demand is greater than supply. Then increasing the fee for the buyers increases profits because supply will not change. Suppose next that demand is smaller than supply. Then increasing the price for the sellers increases profits. So demand must equal supply in equilibrium.

For the situation $\alpha < \beta$ equilibrium outcomes are given in Proposition 1 and Proposition 2.

Proposition 1 *Let $\alpha < \beta$ be given and let $\overline{p}_1 \leq \frac{(\alpha+3\beta)t}{4\beta}$ for $\overline{p}_2 = \frac{\alpha t}{4\beta}$. Then there exists a unique Nash equilibrium (ϕ_1^*,ϕ_2^*) for the game G given by*

$$\phi_1^* = \phi_2^* = \begin{cases} < \frac{(2\alpha+\beta)\overline{p}_1 - \beta\overline{p}_2}{2(\alpha+\beta)}, \frac{(\alpha+2\beta)\overline{p}_2 - \alpha\overline{p}_1}{2(\alpha+\beta)} > & \text{if } \overline{p}_1 + \overline{p}_2 \leq \frac{(\alpha+\beta)t}{2\beta}, \\ & \quad \frac{\beta}{2\alpha+\beta}\ \overline{p}_2 \leq \overline{p}_1 \leq \frac{\alpha+2\beta}{\alpha}\ \overline{p}_2 \\ < 0, \overline{p}_2 - \frac{\alpha}{\beta}\ \overline{p}_1 > & \text{if } \overline{p}_1 \leq \frac{\beta}{2\alpha+\beta}\ \overline{p}_2,\ \overline{p}_1 \leq \frac{t}{4} \\ < \overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2, 0 > & \text{if } \overline{p}_2 \leq \frac{\alpha}{\alpha+2\beta}\ \overline{p}_1,\ \overline{p}_2 \leq \frac{\alpha t}{4\beta} \\ < \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{\alpha t}{4\beta} > & \text{if } \overline{p}_1 \geq \frac{t}{4},\ \overline{p}_2 \geq \frac{\alpha t}{4\beta}, \\ & \quad \frac{(\alpha+\beta)t}{2\beta} \leq \overline{p}_1 + \overline{p}_2 \leq \frac{(2\alpha+3\beta)t}{4\beta} \\ < \frac{(\alpha+\beta)t}{2\beta} - \overline{p}_2, \overline{p}_2 - \frac{\alpha t}{4\beta} > & \text{if } \overline{p}_1 + \overline{p}_2 \geq \frac{(2\alpha+3\beta)t}{4\beta}, \\ & \quad \frac{\alpha t}{4\beta} \leq \overline{p}_2 \leq \frac{(\alpha+\beta)t}{2\beta} \\ < 0, \overline{p}_2 - \frac{\alpha t}{4\beta} > & \text{if } \overline{p}_2 \geq \frac{(\alpha+\beta)t}{2\beta},\ \overline{p}_1 \geq \frac{t}{4}. \end{cases}$$

Proof See Appendix.

Proposition 2 *Let $\alpha < \beta$ be given and let $\overline{p}_1 > \frac{(\alpha+3\beta)t}{4\beta}$ and $\overline{p}_2 = \frac{\alpha t}{4\beta}$. Then there exists a continuum of Nash equilibria (ϕ_1^*, ϕ_2^*) for the game G characterized by $\phi_1^* = \phi_2^* = <\varphi, 0>$ with $\varphi \in [\frac{(\alpha+2\beta)t}{4\beta}, \overline{p}_1 - \frac{t}{4}]$.*

Proof See Appendix.

To be complete and to provide a benchmark we also give the Nash equilibria in case the agents' densities are the same, i.e., $\alpha = \beta$. This requirement complicates the proofs, because now the situation of strong competition can occur in equilibrium, which gives rise to a lot of indeterminacies. Consequently there are several ranges of reservation prices for which there exist continua of equilibria.

Proposition 3 *Let $\alpha = \beta$ be given and let $\overline{p}_1 + \overline{p}_2 \leq \frac{3t}{2}$ in case $\overline{p}_1 \geq \frac{t}{4}$ and $\overline{p}_2 \geq \frac{t}{4}$. If furthermore $\overline{p}_k < \frac{5t}{4}$ for $\overline{p}_j = \frac{t}{4}$, $j \neq k \in \{1, 2\}$, then there exists a unique Nash equilibrium (ϕ_1^*, ϕ_2^*) for the game G given by*

$$\phi_1^* = \phi_2^* = \begin{cases} <\frac{3\overline{p}_1 - \overline{p}_2}{4}, \frac{3\overline{p}_2 - \overline{p}_1}{4}> & \text{if} \quad \overline{p}_1 + \overline{p}_2 \leq t,\ \frac{\overline{p}_2}{3} \leq \overline{p}_1 \leq 3\overline{p}_2 \\ <0, \overline{p}_2 - \overline{p}_1> & \text{if} \quad \overline{p}_1 \leq \frac{\overline{p}_2}{3},\ \overline{p}_1 \leq \frac{t}{4} \\ <\overline{p}_1 - \overline{p}_2, 0> & \text{if} \quad \overline{p}_1 \geq 3\overline{p}_2,\ \overline{p}_2 \leq \frac{t}{4} \\ <\overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{t}{4}> & \text{if} \quad t \leq \overline{p}_1 + \overline{p}_2 \leq \frac{3t}{2},\ \overline{p}_1 \geq \frac{t}{4},\ \overline{p}_2 \geq \frac{t}{4}. \end{cases}$$

Proof See Appendix.

Proposition 4 *Let $\alpha = \beta$ be given and let $\overline{p}_1 + \overline{p}_2 \geq \frac{3t}{2}$. Furthermore let $\overline{p}_1 \geq \frac{t}{4}$ and $\overline{p}_2 \geq \frac{t}{4}$. Then there exists a continuum of Nash equilibria (ϕ_1^*, ϕ_2^*) for the game G characterized by $\phi_1^* = \phi_2^* = <\varphi, t - \varphi>$ with $\varphi \in [0, \overline{p}_1 - \frac{t}{4}] \cap [\frac{5t}{4} - \overline{p}_2, t]$.*

Proof See Appendix.

Proposition 5 *Let $\alpha = \beta$ be given. Furthermore let $\overline{p}_1 \geq \frac{5t}{4}$ and $\overline{p}_2 \geq \frac{t}{4}$. Then there exists a continuum of Nash equilibria (ϕ_1^*, ϕ_2^*) for the game G characterized by $\phi_1^* = \phi_2^* = <\varphi, 0>$ with $\varphi \in [t, \overline{p}_1 - \frac{t}{4}]$. Similarly, let $\overline{p}_2 \geq \frac{5t}{4}$ and $\overline{p}_1 \geq \frac{t}{4}$. Then there exists a continuum of Nash equilibria (ϕ_1^*, ϕ_2^*) for the game G characterized by $\phi_1^* = \phi_2^* = <0, \varphi>$ with $\varphi \in [t, \overline{p}_2 - \frac{t}{4}]$.*

Proof See Appendix.

In the appendix it is shown that the set of equilibria characterized in Propositions 3, 4 and 5 is exhaustive in case $\alpha = \beta$.

4 Characterization of equilibria

In order to discuss the different types of equilibria we label the different regions of reservation prices in Propositions 1 and 2 as in Figure 4.1 and summarize the results of the previous section. For the case $\alpha < \beta$ we refer to Table 4.1.

Area	Fees	Profits
I	$< \frac{(2\alpha+\beta)\overline{p}_1-\beta\overline{p}_2}{2(\alpha+\beta)}, \frac{(\alpha+2\beta)\overline{p}_2-\alpha\overline{p}_1}{2(\alpha+\beta)} >$	$\frac{\alpha\beta}{2(\alpha+\beta)t}(\overline{p}_1+\overline{p}_2)^2$
II^a	$< 0, \overline{p}_2 - \frac{\alpha}{\beta}\ \overline{p}_1 >$	$2\frac{\alpha}{t}\ \overline{p}_1(\overline{p}_2 - \frac{\alpha}{\beta}\ \overline{p}_1)$
II^b	$< \overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2, 0 >$	$2\frac{\beta}{t}\ \overline{p}_2(\overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2)$
III	$< \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{\alpha t}{4\beta} >$	$\frac{\alpha}{2}\left(\overline{p}_1 + \overline{p}_2 - \frac{(\alpha+\beta)t}{4\beta}\right)$
IV^a	$< \frac{(\alpha+\beta)t}{2\beta} - \overline{p}_2, \overline{p}_2 - \frac{\alpha t}{4\beta} >$	$\frac{\alpha(\alpha+2\beta)t}{8\beta}$
IV^b	$< 0, \overline{p}_2 - \frac{\alpha t}{4\beta} >$	$\frac{\alpha}{2}\left(\overline{p}_2 - \frac{\alpha t}{4\beta}\right)$

Table 4.1: The different regions in case $\alpha < \beta$.

We can distinguish between three areas of no competition and three areas of weak competition. It is checked easily that the corresponding fees and profits change continuously in and between the areas, except between the areas II^b and IV^a where $\overline{p}_2 = \frac{\alpha t}{4\beta}$ and $\overline{p}_1 > \frac{(\alpha+3\beta)t}{4\beta}$.

Areas I, II^a, II^b: No competition.

In the areas I, II^a and II^b, the reservation price of at least one of the types of agents is so low, that both firms establish 'local monopolies'. In area I, the differences between the reservation prices of the sellers and buyers are sufficiently low to obtain an equilibrium with both fees positive. The fees are such that agents with a higher reservation price also pay a higher fee. This property also holds for the areas II^a and II^b, in which cases the differences between reservation prices are relatively high. In these areas, the firms even actually desire to subsidize the agents with the lowest reservation price. Since we restrict ourselves to non-negative fees, this means that these agents are served for free.

The willingness to subsidize the agents with the lowest reservation price comes from the market externality associated with matching. In order to make a profit, both sellers and buyers are needed. For sufficiently different reservation prices, the demand effect of attracting agents is stronger than the negative price effect on profits. Only the agents with the highest reservation price in that case bring in a positive amount of money.

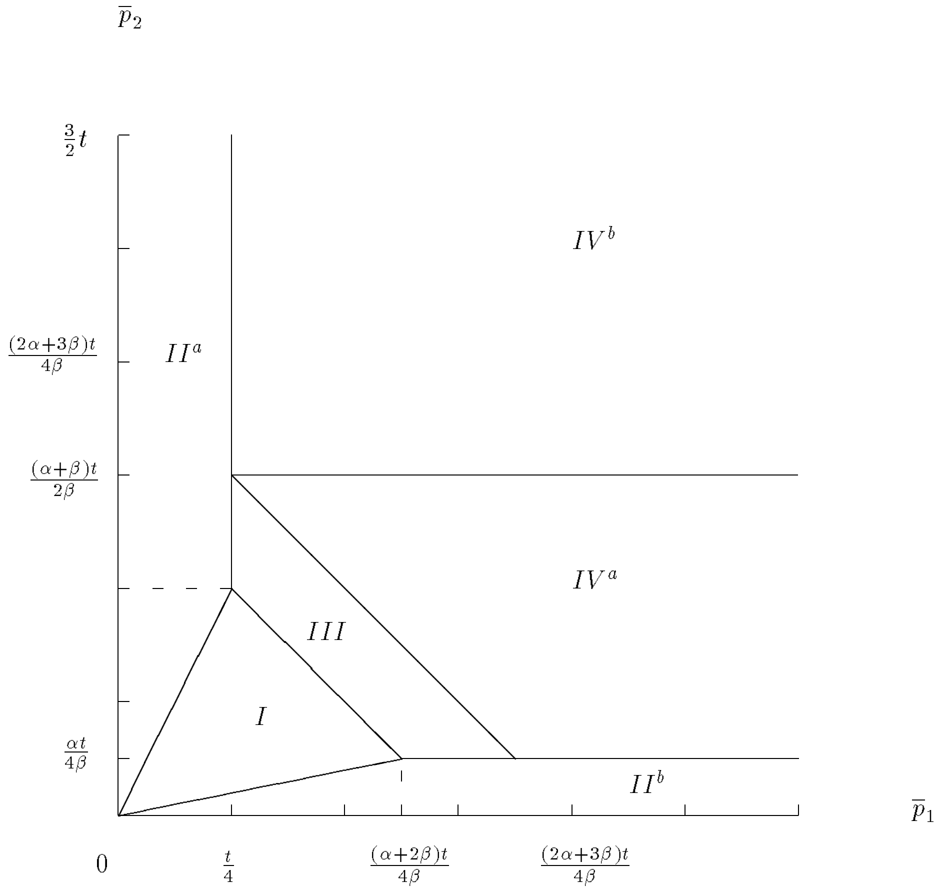

Figure 4.1: The different regions in case $\alpha < \beta$.

Areas III, IV^a, IV^b: **Weak competition**.

In areas III, IV^a and IV^b, the reservation prices are sufficiently high to create a situation of weak competition. In area III, the situations of weak and no competition coincide.

In area III, the reservation prices are still sufficiently low and close to each other to have both type of agents to be treated 'symmetrically'. The sellers located at a distance $\frac{1}{4}$ from the firms have a zero surplus. A fraction $\beta - \alpha$ of the buyers is not served. Firms do not try to capture these buyers, since demand and supply must be equal in equilibrium.

In areas IV^a and IV^b, 'symmetry' between buyers and sellers disappears. Now, the reservation prices are so high, that the sellers located at a distance $\frac{1}{4}$ from both firms claim a positive surplus. The sellers can take advantage of their position in the market, because they form the short side of the market. The negative price effect on profits is more than compensated by the positive effect on the market size by attracting the sellers.

The advantageous market position of a seller in case of high reservation prices is exercised maximally in area IV^b. Similar to the area II^b, the firms desire to subsidize the sellers. This implies that the sellers are served for free. The profits in IV^b are increasing in the reservation price of the buyers, with no upper bound. Since competition on the long side of the market never occurs in equilibrium, the buyers can be charged maximally.

For the case $\alpha = \beta$ the equilibria can be distinguished by the areas I, II^a and II^b as before (with $\alpha = \beta$ substituted) and the areas $\tilde{III}$, $\tilde{IV}^a$, $\tilde{IV}^b$, $\tilde{IV}^c$ as in Figures 4.2a and 4.2b, with corresponding fees and profits as in Table 4.2, where $\varphi \in [0, \overline{p}_1 - \frac{t}{4}] \cap [\frac{5t}{4} - \overline{p}_2, t]$ in area $\tilde{IV}^a$, $\varphi \in [t, \overline{p}_2 - \frac{t}{4}]$ in area $\tilde{IV}^b$, and $\varphi \in [t, \overline{p}_1 - \frac{t}{4}]$ in area $\tilde{IV}^c$.

Area	Fees	Profits
I	$< \frac{(2\alpha+\beta)\overline{p}_1 - \beta\overline{p}_2}{2(\alpha+\beta)}, \frac{(\alpha+2\beta)\overline{p}_2 - \alpha\overline{p}_1}{2(\alpha+\beta)} >$	$\frac{\alpha\beta}{2(\alpha+\beta)t}(\overline{p}_1 + \overline{p}_2)^2$
II^a	$< 0, \overline{p}_2 - \frac{\alpha}{\beta}\ \overline{p}_1 >$	$2\frac{\alpha}{t}\ \overline{p}_1(\overline{p}_2 - \frac{\alpha}{\beta}\ \overline{p}_1)$
II^b	$< \overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2, 0 >$	$2\frac{\beta}{t}\ \overline{p}_2(\overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2)$
$\tilde{III}$	$< \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{t}{4} >$	$\frac{\alpha}{2}(\overline{p}_1 + \overline{p}_2 - \frac{t}{2})$
$\tilde{IV}^a$	$< \varphi, t - \varphi >$	$\frac{\alpha}{2}t$
$\tilde{IV}^b$	$< 0, \varphi >$	$\frac{\alpha}{2}\varphi$
$\tilde{IV}^c$	$< \varphi, 0 >$	$\frac{\alpha}{2}\varphi$

Table 4.2: The different regions in case $\alpha = \beta$.

The areas I, II^a and II^b do not change with respect to the situation $\alpha < \beta$, since no competition occurs in equilibrium. The areas associated with competition do change, however. Weak and strong competition coincide in area $\tilde{III}$. For the areas $\tilde{IV}^a$,$\tilde{IV}^b$, $\tilde{IV}^c$ we have strong competition.

Area $\tilde{III}$.

In area $\tilde{III}$, the situations of competition and no competition coincide. Although $\tilde{III}$ is shaped similarly as area III in Figure 4.1, it is larger, however. In order to get competition, the reservation prices have to be larger. The reason is that for the case $\alpha < \beta$, the negative price effect on profits by the lower fees charged under competition is dominated, since only competition for sellers can occur in equilibrium. Firms can 'afford' lower fees for the sellers already for lower reservation prices, since for buyers fees remain monopolistic. In case $\alpha = \beta$, the negative price effect occurs in both market segments.

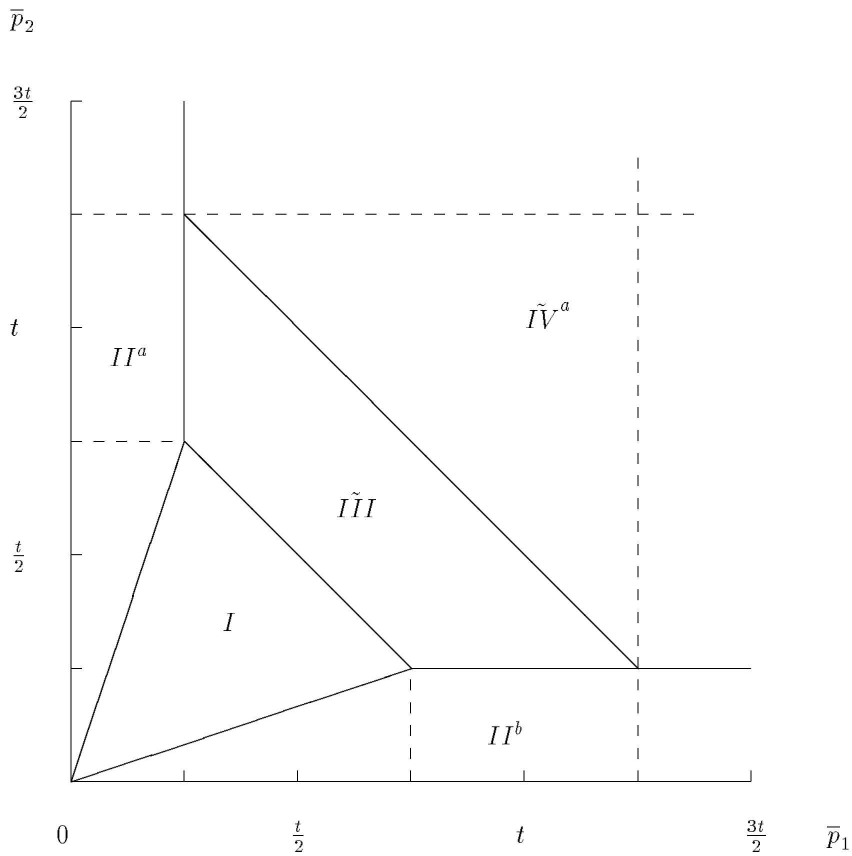

Figure 4.2a: Different regions in case $\alpha = \beta$.

Areas $\tilde{IV}^a, \tilde{IV}^b, \tilde{IV}^c$.

For the case of strong competition, different types of continua of equilibria coexist. For reservation prices in area $\tilde{IV}^a$, for one continuum of equilibria the fees are divided in an arbitrary way, provided their sum is t. Exploitation of one of the market sides does not occur in this equilibrium. Notice that also a 'fair' treatment of agents, that is, $\varphi = \frac{t}{2}$, is allowed as an equilibrium.

Exploitation of one of the market sides comes back in the two other continua of equilibria for the areas $\tilde{IV}^b$ and $\tilde{IV}^c$. In these areas equilibria exist in which one type of agents is served for free and the other type is exploited completely. Equilibria of type $\tilde{IV}^a$, where there is an upper bound on the profits, thus coexist with equilibria of type $\tilde{IV}^b$ or $\tilde{IV}^c$, where there exist equilibria for which the profits tend to infinity if the appropriate reservation price tends to infinity.

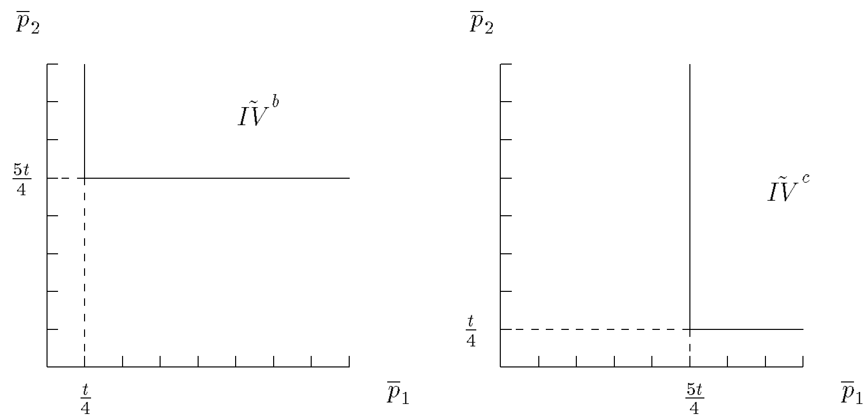

Figure 4.2b: Different regions in case $\alpha = \beta$.

5 Comparative statics

In order to provide some more insight in the differences and similarities between the case $\alpha < \beta$ and the case $\alpha = \beta$ we will discuss equilibrium pricing and equilibrium profits in more detail in this section. In order to use the standard circular model outcome as a benchmark we let $\overline{p}_1 = \overline{p}_2$.

From Section 3 we know that in case $\alpha < \beta$ and $\overline{p}_1 = \overline{p}_2 = \overline{p}$, the equilibrium fees (ϕ_1^*, ϕ_2^*) are given by

$$\phi_1^* = \phi_2^* = \begin{cases} < \frac{\alpha}{\alpha+\beta}\,\overline{p}, \frac{\beta}{\alpha+\beta}\,\overline{p} > & \text{if} \quad \overline{p} \leq \frac{(\alpha+\beta)t}{4\beta} \\ < \overline{p} - \frac{t}{4}, \overline{p} - \frac{\alpha t}{4\beta} > & \text{if} \quad \frac{(\alpha+\beta)t}{4\beta} \leq \overline{p} \leq \frac{(2\alpha+3\beta)t}{8\beta} \\ < \frac{(\alpha+\beta)t}{2\beta} - \overline{p}, \overline{p} - \frac{\alpha t}{4\beta} > & \text{if} \quad \frac{(2\alpha+3\beta)t}{8\beta} \leq \overline{p} \leq \frac{(\alpha+\beta)t}{2\beta} \\ < 0, \overline{p} - \frac{\alpha t}{4\beta} > & \text{if} \quad \overline{p} \geq \frac{(\alpha+\beta)t}{2\beta}. \end{cases}$$

This result is drawn in Figure 5.1.

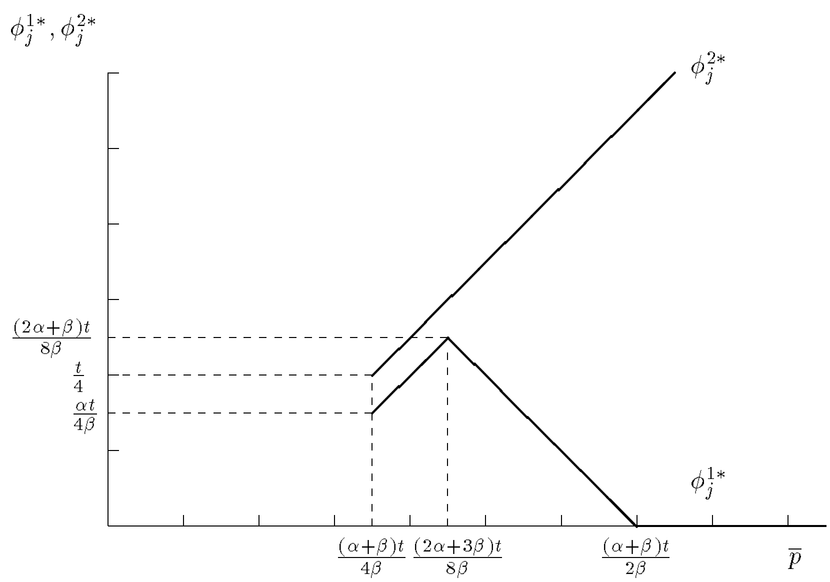

Figure 5.1: Equilibrium fees in case $\alpha < \beta$ for $j \in \{1, 2\}$.

Furthermore we know from Section 3 that in case $\alpha = \beta$ and $\overline{p}_1 = \overline{p}_2 = \overline{p}$ equilibrium prices are given by

$$\phi_1^* = \phi_2^* = \begin{cases} < \frac{\overline{p}}{2}, \frac{\overline{p}}{2} > & \text{if } \ \overline{p} \leq \frac{t}{2} \\ < \overline{p} - \frac{t}{4}, \overline{p} - \frac{t}{4} > & \text{if } \ \frac{t}{2} \leq \overline{p} \leq \frac{3t}{4} \\ < \varphi, t - \varphi > & \text{if } \ \overline{p} \geq \frac{3t}{4},\ \varphi \in [0, \overline{p} - \frac{t}{4}] \cap [\frac{5t}{4} - \overline{p}, t] \\ < \varphi, 0 > & \text{if } \ \overline{p} \geq \frac{5t}{4},\ \varphi \in [t, \overline{p} - \frac{t}{4}] \\ < 0, \varphi > & \text{if } \ \overline{p} \geq \frac{5t}{4},\ \varphi \in [t, \overline{p} - \frac{t}{4}]. \end{cases}$$

This result is drawn in Figure 5.2.

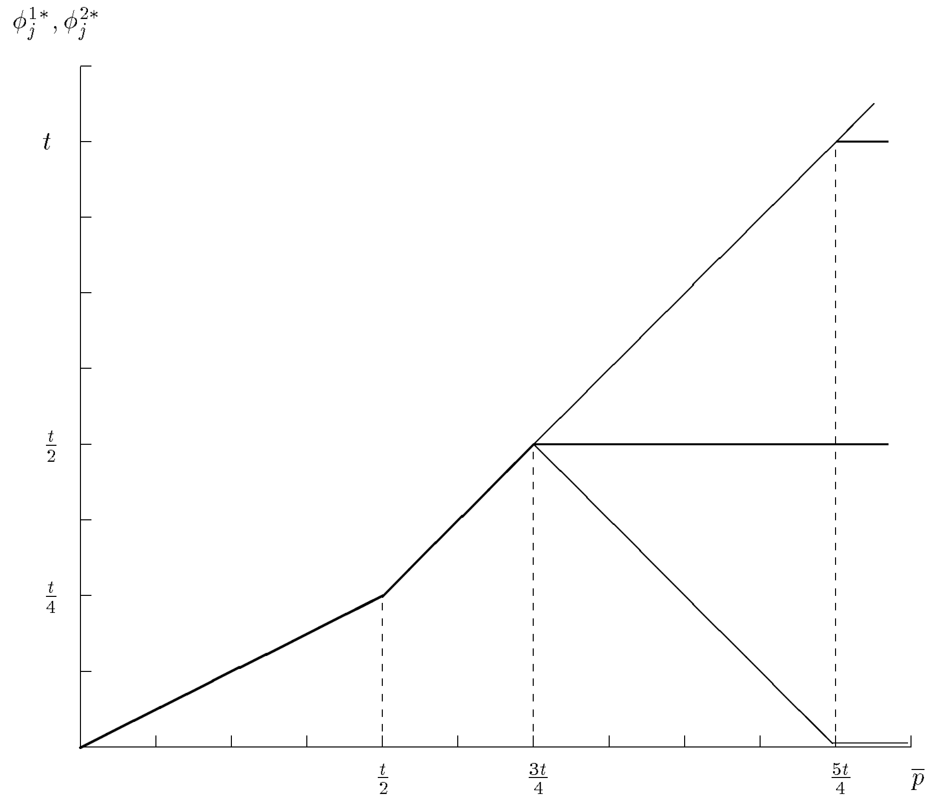

Figure 5.2: Equilibrium fees in case $\alpha = \beta$ for $j \in \{1, 2\}$.

The complication here is that there is a continuum of equilibria for $\overline{p} \geq \frac{3t}{4}$ and that there are even three types of continua for $\overline{p} \geq \frac{5t}{4}$, which gives rise to a coordination problem. Although our purpose is not to solve this coordination problem, we will take the 'fair' solution $\phi_1^* = \phi_2^* =< \frac{t}{2}, \frac{t}{2} >$ for $\overline{p} \geq \frac{3t}{4}$ as a benchmark for the comparison between the case $\alpha < \beta$ and the case $\alpha = \beta$. To our opinion there are several reasons that are in favour of the fair solution. Firstly, the solution for $\overline{p} \leq \frac{3t}{4}$ is also fair. Secondly, the fair solution provides a lower bound on the firms' profits which seems suitable from a social point of view. Thirdly, the fair solution is equal to the solution for the standard circular model (see Webers (1995)).

Recall that the fair solution can be obtained through maximizing profits, which is price times market size. This essentially means that, in case $\alpha = \beta$, there is no matching problem for the social planner. In case $\alpha < \beta$, this is not true if reservation prices are high enough, because the social planner then also is concerned about the agents that are not served.

Firms' profits are drawn in Figure 5.3.

If the reservation price is relatively low, i.e., $\overline{p} \leq \frac{(\alpha+4\beta)t}{8\beta}$, we are in regions I, III, IV^a in case $\alpha < \beta$, and in regions I and $\tilde{III}$ in case $\alpha = \beta$. For $\overline{p} \leq \frac{(\alpha+4\beta)t}{8\beta}$, profits are higher for the situation $\alpha < \beta$ than for the situation $\alpha = \beta$.

If the reservation price is relatively high, i.e., $\overline{p} \geq \frac{(\alpha+4\beta)t}{4\beta}$, we are in region IV^b in case $\alpha < \beta$, and in regions $\tilde{IV}^a$,$\tilde{IV}^b$, $\tilde{IV}^c$ in case $\alpha = \beta$. For $\overline{p} \geq \frac{(\alpha+4\beta)t}{4\beta}$, profits are higher for the situation $\alpha < \beta$ than for the situation $\alpha = \beta$. Competition for the sellers becomes more severe in the latter case, which lowers profits.

If the reservation prices are intermediate, i.e., $\frac{(\alpha+4\beta)t}{8\beta} \leq \overline{p} \leq \frac{(\alpha+4\beta)t}{4\beta}$, profits are higher for the situation $\alpha = \beta$ than for the situation $\alpha < \beta$.

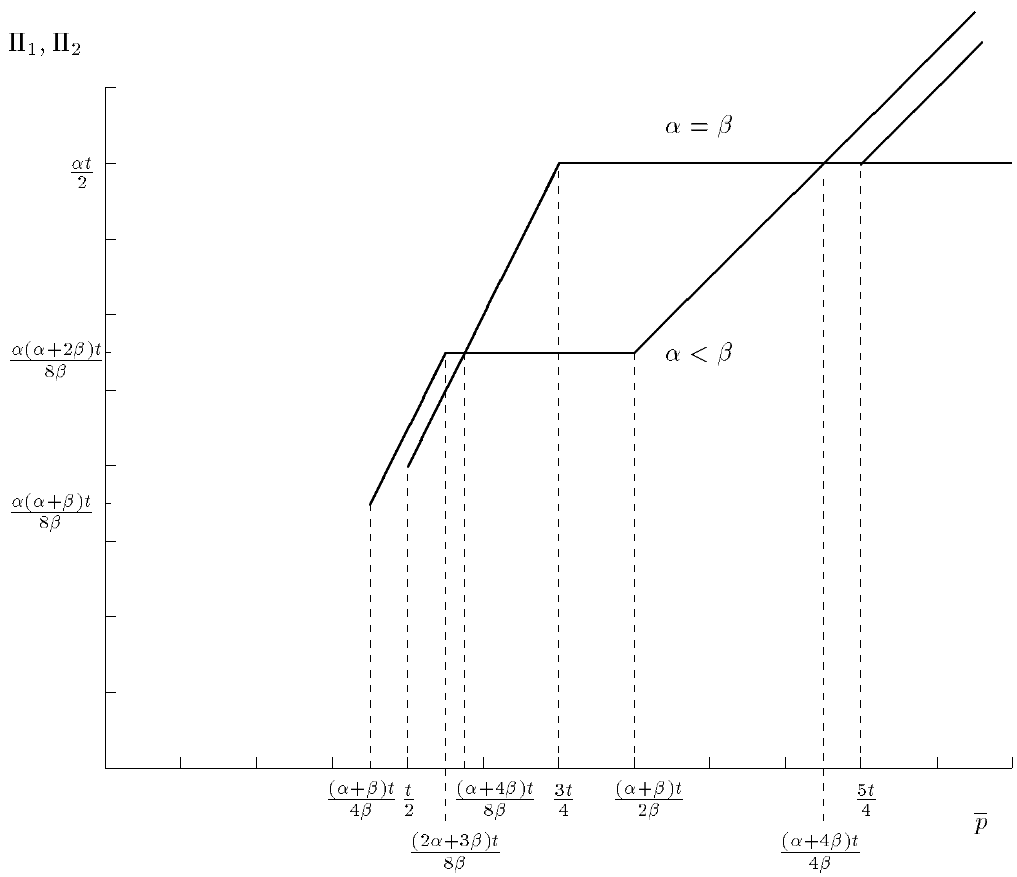

Figure 5.3: Equilibrium profits.

6 Appendix

In order to prove the propositions we first specify the four relevant maximization problems. In the region of prices where there is no competition firms choose prices ϕ_j^1 and ϕ_j^2 that maximize

$$(\phi_j^1 + \phi_j^2)\ min\left\{\frac{2\alpha}{t}(\overline{p}_1 - \phi_j^1), \frac{2\beta}{t}(\overline{p}_2 - \phi_j^2)\right\} \tag{6.1}$$

subject to the price constraints

$$\begin{array}{ll} \overline{p}_1 \leq \frac{\phi_1^1+\phi_2^1}{2} + \frac{t}{4}, & 0 \leq \phi_j^1 \leq \overline{p}_1 \\ \overline{p}_2 \leq \frac{\phi_1^2+\phi_2^2}{2} + \frac{t}{4}, & 0 \leq \phi_j^2 \leq \overline{p}_2. \end{array}$$

In the region of prices where there is weak competition and the firms compete for sellers firms choose prices ϕ_j^1 and ϕ_j^2 that maximize

$$(\phi_j^1 + \phi_j^2)\ min\left\{\frac{\alpha}{t}(\phi_k^1 - \phi_j^1 + \frac{t}{2}), \frac{2\beta}{t}(\overline{p}_2 - \phi_j^2)\right\} \tag{6.2}$$

subject to the price constraints

$$\begin{array}{ll} \overline{p}_1 \geq \frac{\phi_1^1+\phi_2^1}{2} + \frac{t}{4}, & 0 \leq \phi_j^1 \leq \overline{p}_1 \\ \overline{p}_2 \leq \frac{\phi_1^2+\phi_2^2}{2} + \frac{t}{4}, & 0 \leq \phi_j^2 \leq \overline{p}_2. \end{array}$$

In the region of prices where there is weak competition and the firms compete for buyers firms choose prices ϕ_j^1 and ϕ_j^2 that maximize

$$(\phi_j^1 + \phi_j^2)\ min\left\{\frac{2\alpha}{t}(\overline{p}_1 - \phi_j^1), \frac{\beta}{t}(\phi_k^2 - \phi_j^2 + \frac{t}{2})\right\} \tag{6.3}$$

subject to the price constraints

$$\begin{array}{ll} \overline{p}_1 \leq \frac{\phi_1^1+\phi_2^1}{2} + \frac{t}{4}, & 0 \leq \phi_j^1 \leq \overline{p}_1 \\ \overline{p}_2 \geq \frac{\phi_1^2+\phi_2^2}{2} + \frac{t}{4}, & 0 \leq \phi_j^2 \leq \overline{p}_2. \end{array}$$

In the regions of prices where there is strong competition firms choose prices ϕ_j^1 and ϕ_j^2 that maximize

$$(\phi_j^1 + \phi_j^2)\ min\left\{\frac{\alpha}{t}(\phi_k^1 - \phi_j^1 + \frac{t}{2}), \frac{\beta}{t}(\phi_k^2 - \phi_j^2 + \frac{t}{2})\right\} \tag{6.4}$$

subject to the price constraints

$$\begin{array}{ll} \overline{p}_1 \geq \frac{\phi_1^1+\phi_2^1}{2} + \frac{t}{4}, & 0 \leq \phi_j^1 \leq \overline{p}_1 \\ \overline{p}_2 \geq \frac{\phi_1^2+\phi_2^2}{2} + \frac{t}{4}, & 0 \leq \phi_j^2 \leq \overline{p}_2. \end{array}$$

Proof of Proposition 1

First consider the situation of no competition. Because demand and supply have to be equal in equilibrium, we can substitute $\phi_j^2 = \overline{p}_2 - \frac{\alpha}{\beta}\ \overline{p}_1 + \frac{\alpha}{\beta}\ \phi_j^1$ into maximization problem (6.1) for $j \in \{1,2\}$. Note that one of the constraints becomes redundant. If we denote the vector of Lagrange multipliers by $\lambda_j \in \mathbb{R}_+^5$, the corresponding Lagrangian for firm j, $j \in \{1,2\}$, reads $\mathcal{L}_j(\phi_j^1, \lambda_j) = \left(\frac{\alpha+\beta}{\beta}\ \phi_j^1 + \overline{p}_2 - \frac{\alpha}{\beta}\ \overline{p}_1\right)(2(\overline{p}_1 - \phi_j^1)) - \lambda_{j1}(2\overline{p}_1 - \phi_1^1 - \phi_2^1 - \frac{t}{2}) - \lambda_{j2}(\frac{\alpha}{\beta}\ \overline{p}_1 + \overline{p}_2 - \frac{\alpha}{\beta}\ \phi_j^1 - \phi_k^2 - \frac{t}{2}) - \lambda_{j3}(-\phi_j^1) - \lambda_{j4}(\phi_j^1 - \overline{p}_1) - \lambda_{j5}(\overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2 - \phi_j^1)$ with $k \neq j \in \{1,2\}$. Firm j, $j \in \{1,2\}$, thus wants to maximize $\mathcal{L}_j(\phi_j^1, \lambda_j)$ with respect to ϕ_j^1 and $\lambda_j \in \mathbb{R}_+^5$. The first order conditions for profit maximization for firm j, $j \in \{1,2\}$, can be written then as

$$\left\{ \begin{array}{l} 2\left(\frac{2\alpha+\beta}{\beta}\right)\overline{p}_1 - 2\overline{p}_2 - 4\left(\frac{\alpha+\beta}{\beta}\right)\phi_j^1 + \lambda_{j1} + \frac{\alpha}{\beta}\lambda_{j2} + \lambda_{j3} - \lambda_{j4} + \lambda_{j5} = 0 \\ \lambda_{j1}(2\overline{p}_1 - \phi_1^1 - \phi_2^1 - \frac{t}{2}) = 0 \\ \lambda_{j2}(\frac{\alpha}{\beta}\ \overline{p}_1 + \overline{p}_2 - \frac{\alpha}{\beta}\ \phi_j^1 - \phi_k^2 - \frac{t}{2}) = 0 \\ \lambda_{j3}(-\phi_j^1) = 0 \\ \lambda_{j4}(\phi_j^1 - \overline{p}_1) = 0 \\ \lambda_{j5}(\overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2 - \phi_j^1) = 0 \\ (2\overline{p}_1 - \phi_1^1 - \phi_2^1 - \frac{t}{2}) \leq 0 \\ (\frac{\alpha}{\beta}\ \overline{p}_1 + \overline{p}_2 - \frac{\alpha}{\beta}\ \phi_j^1 - \phi_k^2 - \frac{t}{2}) \leq 0 \\ (-\phi_j^1) \leq 0 \\ (\phi_j^1 - \overline{p}_1) \leq 0 \\ (\overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2 - \phi_j^1) \leq 0 \\ \lambda_{jl} \geq 0, l \in \{1,2,3,4,5\}. \end{array} \right.$$

Due to symmetry the first order conditions are solved by $\phi_j^* =< \phi^{1*}, \phi^{2*} >$ for $j \in \{1,2\}$. Solving these equations we get

$$\phi_1^* = \phi_2^* = \left\{ \begin{array}{ll} < \frac{(2\alpha+\beta)\overline{p}_1 - \beta\overline{p}_2}{2(\alpha+\beta)}, \frac{(\alpha+2\beta)\overline{p}_2 - \alpha\overline{p}_1}{2(\alpha+\beta)} > & \text{if } \ \overline{p}_1 + \overline{p}_2 \leq \frac{(\alpha+\beta)t}{2\beta}, \\ & \quad \frac{\beta}{2\alpha+\beta}\ \overline{p}_2 \leq \overline{p}_1 \leq \frac{\alpha+2\beta}{\alpha}\ \overline{p}_2 \\ < 0, \overline{p}_2 - \frac{\alpha}{\beta}\ \overline{p}_1 > & \text{if } \ \overline{p}_1 \leq \frac{\beta}{2\alpha+\beta}\ \overline{p}_2,\ \overline{p}_1 \leq \frac{t}{4} \\ < \overline{p}_1 - \frac{\beta}{\alpha}\ \overline{p}_2, 0 > & \text{if } \ \overline{p}_2 \leq \frac{\alpha}{\alpha+2\beta}\ \overline{p}_1,\ \overline{p}_2 \leq \frac{\alpha t}{4\beta} \\ < \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{\alpha t}{4\beta} > & \text{if } \ \overline{p}_1 + \overline{p}_2 \geq \frac{(\alpha+\beta)t}{2\beta},\ \overline{p}_1 \geq \frac{t}{4}, \\ & \quad \overline{p}_2 \geq \frac{\alpha t}{4\beta}. \end{array} \right.$$

The last thing we have to do is to check whether or not (any of) these solutions can be improved upon. For all the solutions it holds that deviating by setting a higher price for the sellers (and consequently also for the buyers) decreases profits. The more interesting situation is deviating by setting a lower price for the sellers, which of course cannot occur in case the other firm charges prices $< 0, \overline{p}_2 - \frac{\alpha}{\beta}\,\overline{p}_1 >$. If the other firm charges $< \overline{p}_1 - \frac{\beta}{\alpha}\,\overline{p}_2, 0 >$, deviating by setting a lower price for the sellers decreases profits, because demand cannot increase. If the other firm charges $< \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{\alpha t}{4\beta} >$, deviating by setting a lower price for the sellers decreases profits as long as $\overline{p}_1 + \overline{p}_2 \leq \frac{(2\alpha+3\beta)t}{4\beta}$. Finally, if the other firm charges $< \frac{(2\alpha+\beta)\overline{p}_1 - \beta\overline{p}_2}{2(\alpha+\beta)}, \frac{(\alpha+2\beta)\overline{p}_2 - \alpha\overline{p}_1}{2(\alpha+\beta)} >$, deviating by setting a lower price for the sellers decreases profits. For the solution $\phi_1^* = \phi_2^* = < \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{\alpha t}{4\beta} >$ we thus have to impose the additional requirement that $\overline{p}_1 + \overline{p}_2 \leq \frac{(2\alpha+3\beta)t}{4\beta}$.

Next, consider the situation of weak competition. Because demand and supply have to be equal in equilibrium, we can substitute $\phi_j^2 = \overline{p}_2 - \frac{\alpha}{2\beta}\left(\phi_k^1 - \phi_j^1 + \frac{\gamma_1}{2}\right)$ into maximization problem (6.2) for $j \neq k \in \{1,2\}$. We need not consider maximization problem (6.3) because $\alpha < \beta$. If we denote the vector of Lagrange multipliers by $\lambda_j \in \mathbb{R}_+^6$, the corresponding Lagrangian for firm j, $j \in \{1,2\}$, reads $\mathcal{L}_j(\phi_j^1, \lambda_j) = \left(\frac{\alpha+2\beta}{2\beta}\phi_j^1 - \frac{\alpha}{2\beta}\phi_k^1 + \overline{p}_2 - \frac{\alpha t}{4\beta}\right)(\phi_k^1 - \phi_j^1 + \frac{t}{2}) - \lambda_{j1}(\phi_1^1 + \phi_2^1 + \frac{t}{2} - 2\overline{p}_1) - \lambda_{j2}(2\overline{p}_2 - t + \frac{\alpha t}{2\beta} - 2\phi_k^2 + \frac{\alpha}{\beta}(\phi_k^1 - \phi_j^1)) - \lambda_{j3}(-\phi_j^1) - \lambda_{j4}(\phi_j^1 - \overline{p}_1) - \lambda_{j5}(\phi_k^1 + \frac{t}{2} - \frac{2\beta}{\alpha}\,\overline{p}_2 - \phi_j^1) - \lambda_{j6}(\phi_j^1 - \phi_k^1 - \frac{t}{2})$. Firm j, $j \in \{1,2\}$ thus wants to maximize $\mathcal{L}_j(\phi_j^1, \lambda_j)$ with respect to ϕ_j^1 and $\lambda_j \in \mathbb{R}_+^6$. The first order conditions for profit maximization for firm j, $j \in \{1,2\}$, can be written then as

$$
\begin{cases}
-\overline{p}_2 - \frac{\alpha+2\beta}{\beta}\phi_j^1 + \frac{\alpha+\beta}{\beta}\phi_k^1 + \frac{(\alpha+\beta)t}{2\beta} - \lambda_{j1} + \frac{\alpha}{\beta}\lambda_{j2} + \lambda_{j3} - \lambda_{j4} + \lambda_{j5} - \lambda_{j6} = 0 \\
\lambda_{j1}(\phi_1^1 + \phi_2^1 + \frac{t}{2} - 2\overline{p}_1) = 0 \\
\lambda_{j2}(2\overline{p}_2 - t + \frac{\alpha t}{2\beta} - 2\phi_k^2 + \frac{\alpha}{\beta}(\phi_k^1 - \phi_j^1)) = 0 \\
\lambda_{j3}(-\phi_j^1) = 0 \\
\lambda_{j4}(\phi_j^1 - \overline{p}_1) = 0 \\
\lambda_{j5}(\phi_k^1 + \frac{t}{2} - \frac{2\beta}{\alpha}\,\overline{p}_2 - \phi_j^1) = 0 \\
\lambda_{j6}(\phi_j^1 - \phi_k^1 - \frac{t}{2}) = 0 \\
(\phi_1^1 + \phi_2^1 + \frac{t}{2} - 2\overline{p}_1) \leq 0 \\
(2\overline{p}_2 - t + \frac{\alpha t}{2\beta} - 2\phi_k^2 + \frac{\alpha}{\beta}(\phi_k^1 - \phi_j^1)) \leq 0 \\
(-\phi_j^1) \leq 0 \\
(\phi_j^1 - \overline{p}_1) \leq 0 \\
(\phi_k^1 + \frac{t}{2} - \frac{2\beta}{\alpha}\,\overline{p}_2 - \phi_j^1) \leq 0 \\
(\phi_j^1 - \phi_k^1 - \frac{t}{2}) \leq 0 \\
\lambda_{jl} \geq 0, l \in \{1,2,3,4,5,6\}.
\end{cases}
$$

Due to symmetry the first order conditions are solved by $\phi_j^* = < \phi^{1*}, \phi^{2*} >$ for

$j \in \{1, 2\}$. Solving these equations we get

$$\phi_1^* = \phi_2^* = \begin{cases} < 0, \overline{p}_2 - \frac{\alpha t}{4\beta} > & \text{if } \ \overline{p}_2 \geq \frac{(\alpha+\beta)t}{2\beta}, \ \overline{p}_1 \geq \frac{t}{4} \\ < \frac{(\alpha+\beta)t}{2\beta} - \overline{p}_2, \overline{p}_2 - \frac{\alpha t}{4\beta} > & \text{if } \ \overline{p}_1 + \overline{p}_2 \geq \frac{(2\alpha+3\beta)t}{4\beta}, \ \frac{\alpha t}{4\beta} \leq \overline{p}_2 \leq \frac{(\alpha+\beta)t}{2\beta} \\ < \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{\alpha t}{4\beta} > & \text{if } \ \overline{p}_1 + \overline{p}_2 \leq \frac{(2\alpha+3\beta)t}{4\beta}, \ \overline{p}_1 \geq \frac{t}{4}, \ \overline{p}_2 \geq \frac{\alpha t}{4\beta}. \end{cases}$$

Finally we have to check whether or not (any of) these solutions can be improved upon. As we have seen before we have to impose the additional requirement that $\overline{p}_1 + \overline{p}_2 \geq \frac{(\alpha+\beta)t}{2\beta}$ for the solution $< \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{\alpha t}{4\beta} >$.

Because $\alpha < \beta$, the situation of strong competition cannot occur. Combining these results yields Proposition 1.

Q.E.D.

Proof of Proposition 2
For $\overline{p}_2 = \frac{\alpha t}{4\beta}$ and $\overline{p}_1 > \frac{(\alpha+3\beta)t}{4\beta}$ let the other firm's strategy be given by $< \varphi, 0 >$ with $\varphi \in [\frac{(\alpha+2\beta)t}{4\beta}, \overline{p}_1 - \frac{t}{4}]$. Deviating by setting a lower price for the sellers cannot increase profits, because the price for the buyers is zero. Deviating by setting a (little) higher price for the sellers, say $\varphi + \Delta$ with $\Delta \geq 0$, and consequently setting the highest possible price for the buyers, i.e., ϕ^2 such that $\phi^2 + \frac{\alpha}{\beta}(\frac{1}{4} - \frac{\Delta}{2t}) = \overline{p}_2$, results in profits equal to $(\varphi + \overline{p}_2 + \frac{(\alpha+2\beta)t}{2\beta t}\Delta - \frac{\alpha}{4\beta})(\frac{\gamma_1}{2} - \frac{\Delta}{t})$ which are maximal for $\Delta = 0$ because $\varphi \geq \frac{(\alpha+2\beta)t}{4\beta}$. Deviating by setting a much higher price, i.e., $\varphi + \Delta$ and $\overline{p}_2 + \frac{\alpha}{\beta}(\varphi + \Delta - \overline{p}_1)$ where $\Delta \geq \Delta^* = 2\overline{p}_1 - \frac{t}{2} - 2\varphi$ results in profits $\frac{2\alpha}{t}(\varphi + \Delta + \overline{p}_2 + \frac{\alpha}{\beta}(\varphi + \Delta - \overline{p}_1))(\overline{p}_1 - \varphi - \Delta)$, which is never optimal. The reason is that the derivative of these profits with respect to Δ is equal to $\overline{p}_1 - \overline{p}_2 - 2\varphi - 2\Delta + \frac{2\alpha}{\beta}(\overline{p}_1 - \varphi - \Delta)$ which is negative at Δ^*.

Q.E.D.

Proof of Proposition 3
For $\alpha = \beta$ the solution to the situation of no competition is the same as for $\alpha < \beta$. The only difference with the first part of the proof of Proposition 1 is that the solution $\phi_1^* = \phi_2^* = < \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{\alpha t}{4\beta} > = < \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{t}{4} >$ cannot be improved upon for a larger range of reservation prices, i.e., for all reservation prices satisfying $\overline{p}_1 + \overline{p}_2 \leq \frac{3t}{2}$. If the other firm charges prices $< \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{t}{4} >$, deviating by setting lower prices, say $< \overline{p}_1 - \frac{t}{4} - \Delta, \overline{p}_2 - \frac{t}{4} - \Delta >$, yields profits $(\overline{p}_1 + \overline{p}_2 - \frac{t}{2} - 2\Delta)(\frac{t}{2} + \Delta)$. The derivative of these profits with respect to Δ is equal to $\overline{p}_1 + \overline{p}_2 - \frac{3t}{2} - 3\Delta$. So deviating is not optimal as long as $\overline{p}_1 + \overline{p}_2 \leq \frac{3t}{2}$.

If the other firm charges prices $< \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{t}{4} >$, deviating by setting higher prices, say $< \overline{p}_1 - \frac{t}{4} + \Delta, \overline{p}_2 - \frac{t}{4} + \Delta >$, yields profits $(\overline{p}_1 + \overline{p}_2 - \frac{t}{2} + 2\Delta)(\frac{t}{2} - 2\Delta)$. The derivative of these profits w.r.t. Δ is equal to $2t - 2\overline{p}_1 - 2\overline{p}_2 - 4\Delta$. This means that deviating by setting higher prices is not optimal as long as $\overline{p}_1 + \overline{p}_2 \geq t$.

Q.E.D.

Proof of Proposition 4 and Proposition 5

Consider the situation of strong competition. Because demand and supply have to be equal in equilibrium, we can substitute $\phi_j^2 = \phi_k^2 + \phi_j^1 - \phi_k^1$ into maximization problem (6.4) for $j \neq k \in \{1,2\}$. If we denote the vector of Lagrange multipliers by $\lambda_j \in \mathbb{R}_+^6$, the corresponding Lagrangian for firm j, $j \in \{1,2\}$, reads $\mathcal{L}_j(\phi_j^1, \lambda_j) = (2\phi_j^1 + \phi_k^2 - \phi_k^1)(\phi_k^1 - \phi_j^1 + \frac{t}{2}) - \lambda_{j1}(-\phi_j^1) - \lambda_{j2}(\phi_j^1 - \overline{p}_1) - \lambda_{j3}(\phi_k^1 - \phi_k^2 - \phi_j^1) - \lambda_{j4}(\phi_j^1 + \phi_k^2 - \phi_k^1 - \overline{p}_2) - \lambda_{j5}(\phi_1^1 + \phi_2^1 + \frac{t}{2} - 2\overline{p}_1) - \lambda_{j6}(2\phi_k^2 + \phi_j^1 - \phi_k^1 + \frac{t}{2} - 2\overline{p}_2)$. The first order conditions for profit maximization for firm j, $j \in \{1,2\}$, can be written then as

$$\begin{cases} 3\phi_k^1 - \phi_k^2 - 4\phi_j^1 + t + \lambda_{j1} - \lambda_{j2} + \lambda_{j3} - \lambda_{j4} - \lambda_{j5} - \lambda_{j6} = 0 \\ \lambda_{j1}(-\phi_j^1) = 0 \\ \lambda_{j2}(\phi_j^1 - \overline{p}_1) = 0 \\ \lambda_{j3}(\phi_k^1 - \phi_k^2 - \phi_j^1) = 0 \\ \lambda_{j4}(\phi_j^1 + \phi_k^2 - \phi_k^1 - \overline{p}_2) = 0 \\ \lambda_{j5}(\phi_1^1 + \phi_2^1 + \frac{t}{2} - 2\overline{p}_1) = 0 \\ \lambda_{j6}(2\phi_k^2 + \phi_j^1 - \phi_k^1 + \frac{t}{2} - 2\overline{p}_2) = 0 \\ (-\phi_j^1) \leq 0 \\ (\phi_j^1 - \overline{p}_1) \leq 0 \\ (\phi_k^1 - \phi_k^2 - \phi_j^1) \leq 0 \\ (\phi_j^1 + \phi_k^2 - \phi_k^1 - \overline{p}_2) \leq 0 \\ (\phi_1^1 + \phi_2^1 + \frac{t}{2} - 2\overline{p}_1) \leq 0 \\ (2\phi_k^2 + \phi_j^1 - \phi_k^1 + \frac{t}{2} - 2\overline{p}_2) \leq 0 \\ \lambda_{jl} \geq 0, l \in \{1,2,3,4,5,6\}. \end{cases}$$

Due to symmetry the first order conditions are solved by $\phi_j^* = < \phi^{1*}, \phi^{2*} >$ for $j \in \{1,2\}$. Solving these equations we get

$$\phi_1^* = \phi_2^* = \begin{cases} < \varphi, t - \varphi > & \text{if} \quad 0 \leq \varphi \leq \overline{p}_1 - \frac{t}{4},\ \frac{5t}{4} - \overline{p}_2 \leq \varphi \leq t \\ < 0, \varphi > & \text{if} \quad t \leq \varphi \leq \overline{p}_2 - \frac{t}{4} \\ < \varphi, 0 > & \text{if} \quad t \leq \varphi \leq \overline{p}_1 - \frac{t}{4} \\ < \overline{p}_1 - \frac{t}{4}, \overline{p}_2 - \frac{t}{4} > & \text{if} \quad \overline{p}_1 + \overline{p}_2 \leq \frac{3t}{2} \end{cases}$$

where $\overline{p}_1 \geq \frac{t}{4}$ and $\overline{p}_2 \geq \frac{t}{4}$.

The last thing we have to do is to check whether or not (any of) these solutions can be improved upon.

Recall that any solution $\phi_1^* = \phi_2^* =< \mu, \nu >$ to (6.4) satisfies $0 \leq \mu \leq \overline{p}_1 - \frac{t}{4}$ and $0 \leq \nu \leq \overline{p}_2 - \frac{t}{4}$. First consider the situation where $0 < \mu < \overline{p}_1 - \frac{t}{4}$ and $0 < \nu < \overline{p}_2 - \frac{t}{4}$. If a firm deviates by setting slightly lower prices, say $\mu - \Delta$ and $\nu - \Delta$ for some $\Delta > 0$, profits are $(\mu + \nu - 2\Delta)(\frac{t}{2} + \Delta)$. The derivative of these profits with respect to prices is equal to $\mu + \nu - \gamma_1 - 4\Delta$, so deviating by setting lower prices is not optimal as long as $\mu + \nu \leq t$. Similarly we find that deviating by setting higher prices is not optimal as long as $\mu + \nu \geq t$. Combining these results gives that $\mu + \nu = t$. If prices increase more, the situation of no competition occurs. This requires that $\Delta \geq \Delta^* = 2\overline{p}_1 - \frac{t}{2} - 2\mu$. Profits are equal then to $2(2\mu + \overline{p}_2 - \overline{p}_1 - 2\Delta)(\overline{p}_1 - \mu - \Delta)$. One can check that the derivative of these profits is negative at Δ^*, so deviating to the situation of no competition cannot be optimal. Next consider the situation where one of the two prices is zero. Then we need only consider deviations by setting higher prices. As shown before this means that $\mu + \nu \geq t$. Note that the situation where both prices are zero cannot occur. Finally consider the situation where $\mu = \overline{p}_1 - \frac{t}{4}$ and (consequently) $\nu = \overline{p}_2 - \frac{t}{4}$. As shown in Proposition 3, this can only be Nash as long as $t \leq \overline{p}_1 + \overline{p}_2 \leq \frac{3t}{2}$.

Q.E.D.

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