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A SOCIAL POWER INDEX FOR HIERARCHICALLY STRUCTURED POPULATIONS OF ECONOMIC AGENTS

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A Social Power Index for Hierarchically Structured Populations of Economic Agents* †

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Abstract

This paper presents a model of a finite collection of socially related economic agents. We assume that an agent in an economy is part of some social structure in which he might *dominate* some agents while he himself is dominated by other agents. We consider structures in which these social relations between the agents have some special features. Such a social structure is called a *hierarchically structured population*. We identify two types of social differences between economic agents in a hierarchically structured population. Firstly we show that the agents can be subdivided into groups that can be ordered such that agents in 'higher' groups dominate agents in 'lower' groups. Secondly we show that the communication structure between the agents, in general, will be incomplete.

These social differences lead to different potential influences agents have on economic processes. We introduce an index that measures this potential influence. Such an index will be called a *social power index* because it measures power resulting from the agent's social positions. We also give an characterization of this social power index. Furthermore, we derive the rather striking result that under a general uniformity condition this social power index can be viewed as the representation of the subjective expectations of the economic agents in the hierarchy with respect to their influence on economic processes.

1 Introduction

Economic agents are subjects that participate in some economic organization. Therefore, when analyzing their behaviour, we should not look at each agent isolated from the other agents but should take account of their social relations with one another. In many economic models, such as for example in Debreu (1959) or Ichiishi (1983), economic agents are modelled as subjects that differ from each other only with respect to certain individually determined characteristics such as income, preferences, wealth, and so on. No account is taken of the *social positions* of the economic agents in the economic organization.

In this paper we are primarily interested in the description of social features of economic agents. We present a model in which economic agents have different influences on economic processes within the organization. To illustrate this point, in a model of a perfectly competitive market organization it is assumed that no agent has influence on the marketprices and therefore all agents take these prices as given. In other models, such as for example the monopoly or oligopoly model, not all agents are powerless with respect to the prices. For arbitrary economic processes we now assume that the agents have different *direct* influences on these economic processes. With this direct influence we mean the possibility to set conditions under which the economic processes will take place such as, for example, the power to set the prices under which trade with other agents will take place.

We introduce *social* or *relational power* as the potential influence that economic agents have on economic processes resulting from their social relations with one another within a *hierarchical* economic organization. Much work has been done with respect to the measurement of 'power' of agents in social situations. Next we discuss some of the literature on this problem.

Talking about the 'power' of economic agents in social situations is useless if we do not specify what is meant with 'power'. In different situations the definition of the notion of power can differ considerably. Suppose, for example, that a group of agents has to choose one out of several alternatives. We can talk about the power of an agent as being his influence on the final decision that is taken by the group.

In cooperative game theory a situation in which a group of agents just has to decide whether to accept or reject a certain alternative, can be represented by a *simple game*. A simple game is a function that assigns to each subgroup or coalition of agents the value one if this coalition can guarantee that the alternative will be accepted (such a coalition is called a *winning* coalition) and the value zero if this is not the case. We can talk about the (voting) power of an agent participating in a simple game as being his possibilities to turn losing coalitions into winning ones by cooperating with these coalitions. This power can be measured by a *power index*. The most famous axiomatic power indices for simple games are the *Shapley-Shubik index* and the *Banzhaf index*. Axiomatizations of the Shapley-Shubik index and the Banzhaf index respectively can be found in Dubey (1975) and Dubey and Shapley (1979). Another axiomatic power index for simple games is the one introduced in Deegan and Packel (1978) or its generalization in Packel and Deegan (1980).

Simple games form a subclass of the more general collection of cooperative games with transferable utilities or simply TU-games. A TU-game on a set of agents is a function that assigns a real value to every coalition of agents. For a particular coalition this value represents the pay-off this coalition can attain if the agents in the coalition cooperate. We can talk about the *coalitional power* of an agent in a TU-game as being his possibilities to let coalitions earn more by cooperating with him. Axiomatic power indices for these more general TU-games are the *Shapley value* (Shapley (1953)) of which the Shapley-Shubik index is a restriction for simple games, and the *Banzhaf value* for TU-games which is the generalization of the Banzhaf index. (An axiomatization of the Banzhaf value for TU-games can be found in Lehrer (1988).)

In this paper we introduce a power index that measures the potential influence of economic agents resulting from their social relations. We present a model of a rudimental social organization. Such a rudimental organization is called a *hierarchically structured population*, a concept that has been introduced in Gilles (1990b). We will distinguish two social features of economic agents in a hierarchically structured population. First of all we derive an ordered subdivision of the agents into groups such that agents in 'higher' groups set the conditions under which economic processes with agents in 'lower' groups will take place, i.e., agents in higher groups *dominate* agents in lower groups. Such an ordered hierarchical subdivision of the agents is called an *echelon partition*. It can be seen as a special kind of *coalition structure* as developed and analyzed in e.g. Aumann and Drèze (1974), Owen (1977), and Winter (1989). Each group in an echelon partition is called an *echelon*.

The second social feature that we distinguish in our model deals with the communication possibilities of economic agents. In our model of a social organization the possibilities of communication between the agents, in general, not all pairs of agents are able to communicate directly with one another. This means that there can be pairs of agents that need other agents in order to engage in some binary economic process. Such a limited communication structure can be represented by a graph whose nodes represent the agents and whose edges represent these binary economic relations. Such *communication graphs* are considered in, for example, Myerson (1977), Kalai, Postlewaite and Roberts (1978), Owen (1986), and Borm, Owen and Tijs (1990).

Both social features that we discussed above lead separately to a different type of social power. The first source of social power of an agent is his possibility to set the conditions under which economic processes with lower echelon agents will take place. The second source of social power results from the limited communication structure. Consider two agents who are not able to communicate directly with one another. When it is possible for these two agents to communicate with each other with the help of one or more other agents then these *intermediary* agents will have some influence on the economic process that takes place between these two agents. We will see that within the setting of a hierarchically structured population both sources of social power of an agent are related.

We assume that the trade processes in a hierarchically structured population is consisting of two subsequent stages. In the first stage an agent chooses one of his dominating agents as the one with whom he is engaging into a binary economic (trade) process. Secondly, he actually starts this economic process. This means that an agent only uses a selection of the communication lines with these dominating agents. Which communication lines actually will be used, is described by a special kind of hierarchically structured population indicated as an *echelon tree*. In general there exist more than one echelon tree in a particular hierarchically structured population from which eventually only one emerges. The social power of an agent clearly depends on which situation eventually will occur. Because, given a particular hierarchically structured population, we do not know which echelon tree eventually will occur, the social power in a hierarchically structured population is in fact a *potential* feature of the agents in the population.

We introduce a *social power index* as a function that measures the potential social power that economic agents in a hierarchically structured population have over the economic relations on which they set the conditions.* After the introduction of a social power index we give a specific example, that we indicate as the *BG-index*. This BG-index has seminally been introduced in Gilles (1988). We show that the BG-index can be interpreted as a social power index which measures the social power in a situation in which each echelon tree is given equal probability of occurrence. This can be regarded as an *objective* interpretation or characterization of the BG-index.

Additionally we give a *subjective* characterization of the BG-index. Before giving this subjective analysis we introduce some descriptive concepts, indicating how the economic agents focus at the social or hierarchical power structure in the population. For each agent we derive a probability distribution over the echelon trees representing the agent's expectation about which echelon tree will occur. Given such a probability distribution for an agent we introduce a *subjective social power index* that measures the social power as it is expected by this agent. We derive that under some uniformity condition the average of the subjective social power indices over all agents is equal to the BG-index. This is a generalization of a result as stated in van den Brink (1989). It shows that social power indices, which can be regarded as "objective" distribution rules of social power, can be founded on "subjective" considerations.

This paper is organized as follows. In Section 2 we introduce and analyze the notion of a hierarchically structured population. In particular we identify the two social features of the agents in such a hierarchical organization and discuss the two sources of social power that arise from these social features. Furthermore we describe how the echelon trees in a hierarchically structured population can be constructed.

In Section 3 the concept of a social power index as a measure of social power is introduced and we present the BG-index as a specific example of such a social power index. We also give an objective interpretation of the BG-index.

In Section 4 we introduce the concepts which describe the subjective views of the agents with respect to the power structure in a hierarchically structured population. Furthermore we give a subjective interpretation of the BG-index.

Finally, in Section 5 we give an example that illustrates the objective and subjective interpretations of the BG-index.

We emphasize that in this paper we only consider the social features of eco-

*We remark that this number does not have to be an integer. The main reason for this is that many agents are potentially dominated by more than one agents in the population.

nomic agents. A next step will be to model economic agents that have individual as well as social features. In this respect we refer to, for example, Gilles, Owen and van den Brink (1990) where a hierarchical social structure like the one considered in this paper limits the cooperation possibilities of agents endowed with individual abilities. For a study of the BG-index in a more general setting we refer to van den Brink and Gilles (1990).

2 Hierarchically structured populations

First we introduce some notational conventions. In the sequel $N = \{1, \ldots, n\}$ denotes a finite set of economic agents. For every $i \in N$ and every correspondence $S \colon N \to 2^N$ we define

$$S^0 := \{i\}$$

and, recursively, for every $k \in \mathbf{N}$, where $\mathbf{N} = \{1, 2, \ldots\}$ denotes the set of natural numbers, we define

$$S^k(i) := \bigcup_{j \in S^{k-1}(i)} S(j) = \bigcup_{j \in S(i)} S^{k-1}(j).$$

Note that $S^1 = S$. The main tool in the decription of a hierarchically structured population is a correspondence $S \colon N \to 2^N$, which assigns to every agent $i \in N$ a collection $S(i) \subset N$ of agents, who are dominated *directly* by agent i. The agents in $S^k(i)$, $k \geq 2$ then are dominated *indirectly* by i. Formally this is done as follows.

Definition 2.1 *A correspondence $S \colon N \to 2^N$ is a* **successor mapping** *on N if it satisfies the following two conditions.*

(i) *S is acyclic, i.e., for every agent $i \in N$ it holds that:*

$$i \notin \widehat{S}(i) := \bigcup_{k=1}^{\infty} S^k(i).$$

(ii) *For every pair of agents $i, j \in N$ there is some $h \in N$ such that*

$$\{i, j\} \subset [\widehat{S}(h) \cup \{h\}].$$

The collection of all successor mappings on N is denoted by $\mathcal{S}^N$.

The first condition stated in Definition 2.1 requires that an agent cannot dominate himself (neither directly nor indirectly). The second condition says that for each pair of agents it holds that either one of the two dominates the other, or there is another agent that dominates both.

In this paper we interpret the "domination" of economic agents as follows. If $i \in N$ and $j \in S(i)$, then agent i sets the conditions under which some binary economic process between agent j and himself has to take place. (For example, i sets the prices under which he and j can exchange commodities.) The agents in $S(i)$ are called the *potential successors* of i according to S. If agent j is a potential successor of i then j is called a *potential predecessor* of i according to S. The collection of all potential predecessors of i according to S is denoted by $S^{-1}(i)$, i.e., $S^{-1}(i) := \{j \in N \mid i \in S(j)\}$. A pair (N, S), where N is a finite set of economic agents and S is a successor mapping on N is called a *hierarchically structured population* on N. The remainder of this section is devoted to the analysis of hierarchically structured populations.

Recursively we can introduce the sets L_k, $k \in \mathbf{N} \cup \{0\}$, as follows

$$L_0 := \emptyset$$

and for every $k \in \mathbf{N}$

$$L_k := \left\{ i \in N \setminus \bigcup_{p=1}^{k-1} L_p \mid S(i) \subset \bigcup_{p=1}^{k-1} L_p \right\}. \tag{1}$$

We now can prove the following theorem.

Theorem 2.2 *Let $S \in \mathcal{S}^N$. There exists a number $M \in \mathbf{N}$ such that $\{L_1, \dots, L_M\}$ is a partition of N consisting of non-empty sets only. Furthermore, L_M is a singleton.*

The proof of this theorem can be found in section 6. The number M is called the *length* of the hierarchically structured population (N, S) and is denoted by $l(S)$. The agent $i_0 \in L_M$ is the unique agent that is not dominated and is called the *leader* in (N, S). The partition $\xi = \{L_1, \dots, L_M\}$ is called the *echelon partition* of (N, S) and can be seen as a hierarchical subdivision of the agents in N induced by S. The elements in the echelon partition are called *echelons*.

Besides this hierarchical subdivision, a successor mapping S also describes the possibilities of the agents to communicate with each other, i.e., their *economic relations*. These communication possibilities are given by the *communication structure* R, which is defined by

$$R := \{\{i,j\} \mid i \in N, j \in S(i)\}.$$

We have distinguished two social features of economic agents in a hierarchically structured population, namely their position in the echelon partition and their communication possibilities. These two social characteristics are related in the following way.

Theorem 2.3 *Let $S \in \mathcal{S}^N$ with $l(S) = M$, and let $\xi = \{L_1, \ldots, L_M\}$ and R respectively be the echelon partition and the communication structure of (N, S).*

1. *For every $1 \leq k, l \leq M$ and every pair of agents $i \in L_k$, $j \in L_l$ it holds that:*

   $i \in S(j)$ *if and only if* $\{i,j\} \in R$ *and* $k < l$;

2. *For every $2 \leq k \leq M$ and every agent $i \in L_k$ there exists an agent $j \in L_{k-1}$ such that $\{i,j\} \in R$;*

3. *For every $1 \leq k \leq M-1$ and every agent $i \in L_k$ there exists an agent $j \in \bigcup_{l=k+1}^{M} L_l$ such that $\{i,j\} \in R$.*

The proof of this theorem can also be found in section 6. Condition 1 in Theorem 2.3 says that if two agents are directly related to each other, then they must be part of different echelons and the agent in the higher echelon dominates the lower echelon agent. In this way the economic relations in R also can be seen as *dominance relations*. Together with this condition, condition 2 says that if an agent is not part of the lowest echelon, then there must be an agent in the echelon right below him that he dominates. Together with condition 1, condition 3 says that if an agent is not part of the highest echelon, then he must be dominated by another agent. The following example illustrates the concepts introduced so far.

Example 2.4 Consider the hierarchically structured population (N, S), where $N = \{1, \ldots, 6\}$ and the successor mapping S is given by:

$$S(1) = \{2,3,4\},\ S(2) = \{4\},\ S(3) = \{5\},\ S(4) = \{6\},\ S(5) = \emptyset,\ S(6) = \emptyset.$$

The echelon partition ξ of (N, S) is given by:

$$\xi = \{\{5,6\}, \{3,4\}, \{2\}, \{1\}\}.$$

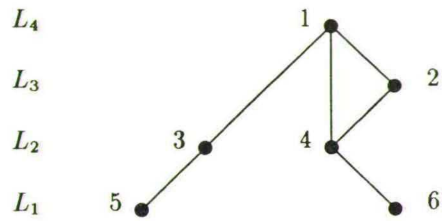


Figure 1: (N, R)

The communication structure R of (N, S) is given by:

$$R = \{\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,5\},\{4,6\}\}.$$

The *communication graph* (N, R) can be drawn in a way such that agents belonging to the same echelon are placed on the same horizontal line (see figure 1).

Thus far we have described a special kind of social organization structure by a hierarchically structured population. How a particular hierarchically structured population arises might depend on individual features, on social features, or on a combination of both. Individual features that might determine the hierarchically structured population are, for example, the initial endowments of the agents. An example of hierarchically structured populations that depend on social features are the ones that are determined by *networks*. (See Gilles (1990a) or Gilles and Ruys (1990).) In this paper we do not address this problem but just take a hierarchically structured population as given.

Different positions in a hierarchically structured population lead to different possibilities to influence economic processes. The influence that an agent has on the economic processes resulting from his social characteristics is refered to as his *social power*. It is clear that the direct influence an agent has on his relations with his potential successors is some source of social power. As mentioned in the introduction there is a second source of social power that arises from the limited communication structure in a hierarchically structured population. Theorem 2.3 directly yields the following result.

Corollary 2.5 *Let $S \in \mathcal{S}^N$. Then the pair (N, R) is a connected graph, i.e., for every pair of agents $i, j \in N$, with $i \neq j$, there exists a finite sequence $c_1, \ldots, c_m \in N$ such that $c_1 = i$, $c_m = j$, and $\{c_k, c_{k+1}\} \in R$ for every $1 \leq k \leq m-1$.*

A sequence $c_1, \ldots, c_m$ as described in Corollary 2.5 is called a *communication path* between i and j. If two agents $i, j \in N$ cannot communicate directly then, according to Corollary 2.5, i and j can communicate indirectly through one or more other agents. These *intermediary* agents in the communication process between i and j have some influence on the economic process that takes place between i and j. This influence is the second source of social power an agent has in a hierarchically structured population.

We argue that both sources of social power of an agent are in some sense identical within the setting of a hierarchically structured population. This follows from the following lemma.

Lemma 2.6 *Let $S \in \mathcal{S}^N$ and let R be its communication structure. For all agents $i, j \in N$, $i \neq j$, there exists a sequence $c_1, \ldots, c_m$ and a positive integer $T \leq m$ such that:*

1. $c_1 = i$
2. $c_k \in S(c_{k+1})$ *for* $k = 1, \ldots, T-1$
3. $c_{k+1} \in S(c_k)$ *for* $k = T, \ldots, m-1$
4. $c_m = j$

Proof of this lemma can be found in section 6. The agent c_T in a communication path as described in Lemma 2.6 is called the *topman* on that communication path. Lemma 2.6 says that there is a communication path between each pair of agents $i, j \in N$ such that each agent on that communication path, except the topman, *directs* himself to one of his potential predecessors. That is, the intermediary agents within such a communication path are dominating each other in an order such that there is a unique agent at the top of this communication chain.

Not all communication paths between two agents in a hierarchically structured population need to be of the form as in Lemma 2.6. However, the following discussion implies that communication paths other than those described in Lemma 2.6 will not be used.

We assume that, in case some economic process takes place between the agents in N, then each agent chooses one of his potential predecessors as the one with whom he is going to engage in a binary economic process. Such an organization structure can be described by a function, the *predecessor function*.

Definition 2.7 *Let $S \in \mathcal{S}^N$ and let the echelon partition of (N, S) be given by $\xi = \{L_1, \ldots, L_M\}$, where $M = l(S)$.*
A function $t: N \backslash L_M \to N \backslash L_1$ is a **predecessor function** *in S if for every $i \in N \backslash L_M$ it holds that $t(i) \in S^{-1}(i)$.*
A pair (N, T) is an **echelon tree** *in (N, S) if $T \in \mathcal{S}^N$ is such that the correspondence $t: N \setminus L_M \to N \setminus L_1$ given by $t(i) = T^{-1}(i)$, for all $i \in N \setminus L_M$ is a predecessor function in S.*
The collection of all correspondences T such that (N, T) is an echelon tree in (N, S) is denoted by $\mathcal{T}_S$.

For every hierarchically structured population (N, S) it holds that $\mathcal{T}_S \subset \mathcal{S}^N$. The agent $t(i)$ is the potential predecessor to which $i \in N \setminus L_M$ directs himself if t is the predecessor function that describes the situation that actually occurs. This agent $t(i)$ is called the *predecessor* of i according to t. It is easy to see that if (N, T) is an echelon tree with communication structure W, then the graph (N, W) is a tree. In such a tree there exists exactly one communication path between each pair of agents and all these paths are of the form as described in Lemma 2.6. In this way the power of an agent resulting from his possibilities to let other agents communicate with one another also depends on which agents are his potential successors.

We remark here that the echelon partition of (N, T) with $T \in \mathcal{T}_S$ need not be the same as the echelon partition of (N, S) itself. This is shown in the following example.

Example 2.8 Consider the hierarchically structured population (N, S) given in Example 2.4.
Agent 4 is the only agent who has more than one potential predecessor, i.e., agent 4 is the only agent who can choose to which agent he is going to direct himself. Therefore there are exactly two predecessor functions in S. These are $t_1: \{2, 3, 4, 5, 6\} \to \{1, 2, 3, 4\}$ which is given by:

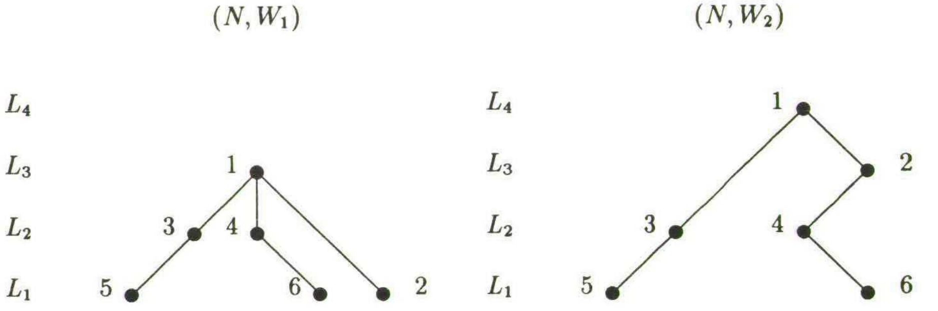


Figure 2: communication graphs of the echelon trees in (N, S)

$$t_1(2) = 1,\ t_1(3) = 1,\ t_1(4) = 1,\ t_1(5) = 3,\ t_1(6) = 4$$

and t_2: $\{2,3,4,5,6\} \to \{1,2,3,4\}$ which is given by:

$$t_2(2) = 1,\ t_2(3) = 1,\ t_2(4) = 2,\ t_2(5) = 3,\ t_2(6) = 4.$$

The echelon partition ξ_1 and communication structure W_1 of the echelon tree belonging to t_1 are given by:

$$\xi_1 = \{\{2,5,6\}, \{3,4\}, \{1\}\}$$

$$W_1 = \{\{1,2\}, \{1,3\}, \{1,4\}, \{3,5\}, \{4,6\}\}$$

and those of the echelon tree belonging to t_2 are given by:

$$\xi_2 = \{\{5,6\}, \{3,4\}, \{2\}, \{1\}\} = \xi$$

$$W_2 = \{\{1,2\}, \{1,3\}, \{2,4\}, \{3,5\}, \{4,6\}\}.$$

The communication graphs of these echelon trees are given in figure 2.
Note that if agent 4 chooses agent 1 as his predecessor then the echelon partition ξ_1 that actually occurs has one echelon less than the echelon partition ξ of (N, S). If agent 4 chooses agent 2 as his final predecessor, then the echelon partition that actually occurs is the same as the echelon partition of (N, S): $\xi_2 = \xi$.

It is easy to see now why the members of $S(i)$ and $S^{-1}(i)$ respectively are called the potential successors and potential predecessors of i in (N, S). If a potential successor of i has more potential predecessors besides i, then it is not known whether this potential successor actually will direct himself to i. The social power an agent has in a hierarchically structured population clearly depends on which echelon tree eventually will occur.

3 Social power indices

In this section we introduce the notion of a *social power index* that measures the (potential) social power of economic agents in a hierarchically structured population. In the previous section we indicated two sources of social power within the setting of a hierarchically structured population. We argued that, for an agent $i \in N$, both sources of social power depend on which agents he dominates directly. Therefore, the social power index that we have in mind should tell us in which way the power over all dominated agents is distributed over the agents in N. The fact that in a hierarchically structured population the leader is the only agent that is not dominated leads us to the following definition of a social power index.

Definition 3.1 *A* **social power index** *on N is a function $\varphi: N \times \mathcal{S}^N \to \mathbf{R}_+$ such that for every $S \in \mathcal{S}^N$ it holds that*

$$\sum_{i \in N} \varphi(i, S) = \#N - 1.$$

The power over the $\#N - 1$ dominated economic agents can be distributed in various ways as long as this power distribution satisfies Definition 3.1. Here we turn to the analysis of one particular social power index, the BG-index.

From the discussion in the previous section it follows that the social power of an agent $i \in N$ in a hierarchically structured population (N, S) depends on which echelon tree eventually will occur. This depends on which one of their potential predecessors the agents in $N \setminus L_M$ choose as their predecessor. In the BG-index we assume that each agent (except the topman) chooses each one of his potential predecessors as his predecessor with equal probability.

Definition 3.2 *The* **BG-index** *is the function $BG: N \times \mathcal{S}^N \to \mathbf{R}_+$, which for every $S \in \mathcal{S}^N$ and for every $i \in N$ is given by*

$$BG(i,S) := \sum_{j \in S(i)} \frac{1}{\sigma(j)},$$

where $\sigma(j) := \#S^{-1}(j)$ *for every* $j \in N \setminus L_M$.

The BG-index of agent i in a hierarchically structured population (N, S) counts $\frac{1}{\#S^{-1}(j)}$ to the social power value of i for each potential successor j of agent i. In other words, in the BG-index the power over a dominated agent is equally distributed over all his potential predecessors. This expresses the fact that nothing is known about the choices of the agents which potential predecessor they choose as their predecessor. This leads us to the following characterization of the BG-index.

Theorem 3.3 *A function* $\varphi: N \times \mathcal{S}^N \to \mathbf{R}_+$ *is equal to the BG-index if and only if it satisfies the following three conditions:*

(i) *For every hierarchically structured population* (N, S) *it holds that*

$$\sum_{i \in N} \varphi(i,S) = \#N - 1.$$

(ii) *For every hierarchically structured population* (N, S) *and every agent* $i \in N$ *it holds that*

$$\varphi(i,S) \geq \#\{j \in S(i) \mid \sigma(j) = 1\}.$$

(iii) *For every hierarchically structured population* (N, S) *and every agent* $i \in N$ *it holds that*

$$\varphi(i,S) = \frac{1}{\#\mathcal{T}_S} \sum_{T \in \mathcal{T}_S} \varphi(i,T).$$

From the first condition it follows that the BG-index indeed is a social power index as introduced in Definition 3.1. The second condition says that an agent at least has full power over all his potential successors that have to direct themselves to him because they have no other potential predecessor. The third condition says that the BG-index for an arbitrary hierarchically structured population (N, S) is equal to the average of these indices over all echelon trees (N, T) in (N, S).

PROOF OF THEOREM 3.3

First we will prove that the BG-index satisfies the three conditions stated in the theorem.
Suppose that $S \in \mathcal{S}^N$ and let $\{L_1, \ldots, L_M\}$ and R respectively be the echelon partition and the communication structure of (N, S). Then

(i) $$\sum_{i \in N} BG(i,S) = \sum_{i \in N} \sum_{j \in S(i)} \frac{1}{\sigma(j)} = \sum_{j \in N \setminus L_M} \sum_{i \in S^{-1}(j)} \frac{1}{\sigma(j)} = \#N - 1.$$

This shows that the BG-index satisfies the first condition.

(ii) For every $i \in N$ it holds that

$$BG(i,S) = \sum_{j \in S(i)} \frac{1}{\sigma(j)} \geq \sum_{\substack{j \in S(i) \\ \sigma(j)=1}} \frac{1}{\sigma(j)} = \#\{j \in S(i) \mid \sigma(j) = 1\}.$$

This shows that the BG-index satisfies the second condition.

(iii) Let $T \in \mathcal{T}_S$. Then

$$\#T^{-1}(i) = \begin{cases} 1 & \text{for every } i \in N \setminus L_M \\ 0 & \text{for the leader } i = i_0 \in L_M \end{cases}$$

Then it is clear that

$$BG(i,T) = \sum_{j \in T(i)} \frac{1}{\#T^{-1}(j)} = \#T(i).$$

Let $\mathcal{T}_S(i,j) := \{T \in \mathcal{T}_S \mid j \in T(i)\}$. It is easy to see that for every $i \in N$ and for every potential successor $j \in S(i)$ it holds that $\frac{\#\mathcal{T}_S(i,j)}{\#\mathcal{T}_S} = \frac{1}{\sigma(j)}$. Then we may deduce that

$$BG(i,S) = \sum_{j \in S(i)} \frac{1}{\sigma(j)} = \frac{1}{t_S} \sum_{j \in S(i)} \#\mathcal{T}_S(i,j)$$

$$= \frac{1}{t_S} \sum_{T \in \mathcal{T}_S} \#T(i) = \frac{1}{t_S} \sum_{T \in \mathcal{T}_S} BG(i,T),$$

where $t_S := \#\mathcal{T}_S$. This shows that the BG-index also satisfies the third condition.

Now let $\varphi: N \times \mathcal{S}^N \to \mathbf{R}_+$ be a function that satisfies the three conditions. We next show that it has to be the BG-index.
From the first two conditions it easily follows that for each $S \in \mathcal{S}^N$ and every $T \in \mathcal{T}_S$ it holds that

$$\varphi(i,T) = \#T(i).$$

From the third condition it then follows that for $S \in \mathcal{S}^N$:

$$\varphi(i,S) = \frac{1}{t_S} \sum_{T \in \mathcal{T}_S} \#T(i) = \frac{1}{t_S} \sum_{j \in S(i)} \#\mathcal{T}_S(i,j)$$

$$= \sum_{j \in S(i)} \frac{1}{\sigma(j)} = BG(i,S)$$

This implies that if $\varphi: N \times \mathcal{S}^N \to \mathbf{R}_n$ satisfies the three conditions, then it must be equal to the BG-index.

Q.E.D.

It follows from this proof that the first two conditions uniquely determine the BG-index for echelon trees. According to the third condition, the BG-index measures the potential social power of economic agents in a hierarchically structured population (N,S) if we assume that each echelon tree occurs with the same probability. Therefore we might see the BG-index as an *objective* power index which distributes the social power in some "fair" way. The characterization given in Theorem 3.3 is called the *objective* characterization of the BG-index.

4 A subjective approach to the BG-index

In this section we show that the BG-index also can be seen as a *subjective* social power index, i.e., as an index that measures social power from the viewpoint of the individual agents in the hierarchically structured population.

Consider a particular hierarchically structured (N,S) and an agent $i \in N \setminus L_1$. If a potential successor $j \in S(i)$ has more potential predecessors besides i, then it is not known whether j actually directs himself to i. We suppose that each agent has certain expectations about which ones of his potential successors eventually directs themselves to him. These subjective expectations of the agents are given by a *social expectation function*.

Definition 4.1 *Let $S \in \mathcal{S}^N$. A function $\mu_S: N \times N \to [0,1]$ is a* **social expectation function** *for (N, S) if for every $i \in N$ the following two conditions are satisfied:*

(i) $\mu_S(i,j) = 0$ *if $j \notin S(i)$;*

(ii) $\mu_S(i,j) = 1$ *if $j \in S(i)$ with $\sigma(j) = 1$.*

The probability agent $i \in N$ gives to the occurrence of an echelon tree such that he is the predecessor of agent j is given by $\mu_S(i,j)$. Agent i must expect that he will never be the predecessor of an agent who is no potential successor of him. Furthermore i must expect that he will be the predecessor of a potential successor $j \in S(i)$ with certainty if he is the only potential predecessor of j. It is clear that if $i \in L_1$ then $\mu_S(i,j) = 0$ for all $j \in N$. If $j \in L_M$ then $\mu_S(i,j) = 0$ for all $i \in N$.

Clearly the choices of predecessors by all agents in $N \setminus L_M$ result in a particular echelon tree. Using the social expectation function we can, for each agent in N, derive a probability distribution over $\mathcal{T}_S$. Consider a particular echelon tree (N, T) in (N, S), and an agent $i \in N$. It is easy to see that if $T(i) \neq \emptyset$ then the probability agent i gives to the occurrence of an echelon tree in which he is the predecessor of all agents in $T(i)$ is given by

$$\prod_{j \in T(i)} \mu_S(i,j).$$

For each one of the agents $j \in S(i) \setminus T(i)$, agent i gives probability $1 - \mu_S(i,j)$ to the occurrence of an echelon tree in which he is not the predecessor of j. We assume that agent i expects that all other potential predecessors of j have equal probability to be the predecessor of j. Because in these cases j has $[\#S^{-1}(j) - 1] \geq 1$ other potential predecessors besides i, the probability i gives to the occurrence of an echelon tree in which j directs himself to one particular potential predecessor $h \in S^{-1}(j) \setminus \{i\}$ is given by

$$\frac{1 - \mu_S(i,j)}{\#S^{-1}(j) - 1}.$$

Finally we assume that each agent $i \in N$ expects that for each agent j who is no potential successor of i it holds that all potential predecessors of j have equal probability $\frac{1}{\#S^{-1}(j)}$ to be the predecessor of j. This results in the following functions that yield the expectation of agent i about which echelon tree will occur.

Definition 4.2 *Let $S \in \mathcal{S}^N$, let μ_S be a social expectation function for (N, S) and let $i \in N$. The* **expectation distribution** *of agent i induced by μ_S is the function $p_i : \mathcal{T}_S \to [0,1]$ that is given by*

$$p_i(T) = \prod_{j \in T(i)} \mu_S(i,j) \prod_{j \in S(i) \setminus T(i)} \frac{1 - \mu_S(i,j)}{\sigma(j) - 1} \prod_{j \in N \setminus [S(i) \cup L_M]} \frac{1}{\sigma(j)}, \tag{2}$$

for every $T \in \mathcal{T}_S$.

It can be verified that for each agent $i \in N$ it holds that $p_i(T) \geq 0$ for every $T \in \mathcal{T}_S$ and $\sum_{T \in \mathcal{T}_S} p_i(T) = 1$. Thus the function $p_i : \mathcal{T}_S \to [0,1]$ describes a probability distribution over $\mathcal{T}_S$. This probability distribution reflects agent i's expectation about which echelon tree will occur.

We will illustrate the agent's expectations about the occurrence of echelon trees by discussing three specific types of social expectation functions, namely the cases of pessimistic, neutral, and optimistic expectations. To analyze these cases properly we take a fixed hierarchically structured population (N, S).

We say that agent i has **pessimistic expectations** about his power over his potential successors if the social expectation function μ_S satisfies:

$$\mu_S(i,j) = \begin{cases} 1 & \text{if } j \in S(i) \text{ and } \sigma(j) = 1 \\ 0 & \text{else} \end{cases}$$

From equation (2) it follows that in this case agent i's expectation distribution is such that for every $T \in \mathcal{T}_S$ it holds that

$$p_i(T) = \begin{cases} 0 & \text{if } \{j \in T(i) \mid \sigma(j) \neq 1\} \neq \emptyset \\ \prod_{j \in S(i) \setminus T(i)} \frac{1}{\sigma(j)-1} \prod_{j \in N \setminus [S(i) \cup L_M]} \frac{1}{\sigma(j)} & \text{if } \{j \in T(i) \mid \sigma(j) \neq 1\} = \emptyset \end{cases}$$

This shows that if agent i has pessimistic expectations about his power over his potential successors, then he gives zero probability to the occurrence of all echelon trees in which he is the predecessor of at least one of his potential successors who have more potential predecessors besides himself. If $\mathcal{N}_S(i) := \{T \in \mathcal{T}_S \mid \{j \in T(i) \mid \sigma(j) \neq 1\} = \emptyset\}$, then we can verify that for every $T \in \mathcal{N}_S(i)$ it holds that

$$p_i(T) = \prod_{j \in S(i) \setminus T(i)} \frac{1}{\sigma(j) - 1} \prod_{j \in N \setminus [S(i) \cup L_M]} \frac{1}{\sigma(j)} = \frac{1}{\# \mathcal{N}_S(i)}.$$

This shows that agent i gives equal positive probability to the occurrence of all echelon trees in which he is not the predecessor of any of his potential successors who have more than one potential predecessors.

We say that agent i has **neutral expectations** about his power over his potential successors if the social expectation function μ_S satisfies

$$\mu_S(i,j) = \frac{1}{\sigma(j)} \text{ for all } j \in S(i).$$

From equation (2) it follows that in this case agent i's expectation distribution is such that for every $T \in \mathcal{T}_S$ it holds that

$$p_i(T) = \prod_{j \in N \setminus L_M} \frac{1}{\sigma(j)} = \frac{1}{t_S},$$

where $t_S := \#\mathcal{T}_S$. The equation above asserts that if agent i has neutral expectations, then he gives equal probability of occurrence to each echelon tree in (N, S).

Finally, we say that agent i has **optimistic expectations** about his power over his potential successors if the social expectation function μ_S satisfies

$$\mu_S(i,j) = 1 \text{ for all } j \in S(i).$$

From equation (2) it follows that in this case agent i's expectation distribution is such that for every $T \in \mathcal{T}_S$ it holds that

$$p_i(T) = \begin{cases} 0 & \text{if } T(i) \neq S(i) \\ \prod_{j \in N \setminus [S(i) \cup L_M]} \frac{1}{\sigma(j)} & \text{if } T(i) = S(i). \end{cases}$$

This implies that in case agent i has optimistic expectations about his power over his potential successors, then i gives zero probability to the occurrence of an echelon tree in which he is not the predecessor of all his potential successors. If $\mathcal{D}_S(i) := \{T \in \mathcal{T}_S \mid T(i) = S(i)\}$, then we can verify that for every $T \in \mathcal{D}_S(i)$ it holds that

$$p_i(T) = \prod_{j \in N \setminus [S(i) \cup L_M]} \frac{1}{\sigma(j)} = \frac{1}{\#\mathcal{D}_S(i)}.$$

Thus we may conclude that in case of optimistic expectations agent i gives equal positive probability to the occurrence of all echelon trees in which he is the predecessor of all his potential successors.

Suppose that $\varphi: N \times \mathcal{S}^N \to \mathbf{R}_+$ is a social power index as defined in Definition 3.1. For an echelon tree (N,T) the function $\varphi(\cdot,T): N \to \mathbf{R}_+$ can be seen as a social power index belonging to echelon tree (N,T) which measures social power in a situation that actually might occur. Given the social power index for all echelon trees we can define a subjective social power index which measures social power from the agent's point of view.

Definition 4.3 *Let $S \in \mathcal{S}^N$, let μ_S be a social expectation function for (N,S) and let $\varphi: N \times \mathcal{S}^N \to \mathbf{R}_+$ be a social power index. Furthermore, let p_h be the expectation distribution of agent $h \in N$ induced by μ_S. Agent h's* **subjective expectation** *of φ is the function $E_h(\varphi): N \to \mathbf{R}_+$ which is given by*

$$E_h(\varphi)(i) = \sum_{T \in \mathcal{T}_S} p_h(T)\varphi(i,T) \text{ for every } i \in N.$$

Thus $E_h(\varphi)(i)$ measures the social power of agent i according to agent h. With the assumptions as made in the previous definitions we in fact have constructed a model of subjective expectation patterns with respect to social power in the setting of a hierarchically structured organization of economic processes. With the use of this model we are able to give an approach to the BG-index, which is based on subjective expectations of the agents in the organization. The following result remarkably states that uniform expectations with respect to social power, lead to the same rule for distributing social power, namely the BG-index.

Theorem 4.4 *The function $\varphi: N \times \mathcal{S}^N \to \mathbf{R}_+$ is equal to the BG-index if and only if the following three conditions are satisfied:*

(i) *For every hierarchically structured population (N,S) it holds that*

$$\sum_{i \in N} \varphi(i,S) = \#N - 1.$$

(ii) *For every hierarchically structured population (N,S) and every agent $i \in N$ it holds that*

$$\varphi(i,S) \geq \#\{j \in S(i) \mid \sigma(j) = 1\}.$$

(iii) *For every hierarchically structured population (N,S) and every social expectation function μ_S such that for every $j \in N \setminus L_M$ and every $i \in S^{-1}(j)$*

$\mu_S(i,j) = \mu_S(j)$, *where* $\mu_S(j)$ *is some constant in* $[0,1]$,

it holds that

$$\varphi(\cdot, S) = \frac{\sum_{h\in N} E_h(\varphi)}{\#N},$$

where $E_h(\varphi)$ *is given by Definition 4.3.*

Conditions 1 and 2 are similar to the first two conditions in the *objective* characterization of the BG-index given in Theorem 3.3. So, again these two conditions determine a social power index for each echelon tree in (N, S). The third condition then says that if, for each agent $j \in N \setminus L_M$, it holds that all his potential predecessors in (N, S) have the same subjective expectations regarding the power over him, then the BG-index is equal to the average of the subjective expectations of this rule φ over all agents. Therefore, Theorem 4.4 is a *subjective* characterization of the BG-index.

PROOF OF THEOREM 4.4

First we will prove that the BG-index satisfies the three conditions.

Because the first two conditions are the same as in Theorem 3.3, it follows that the BG-index satisfies these conditions.

Let $S \in \mathcal{S}^N$. Suppose that for all $j \in N \setminus L_M$ it holds that:

$\mu_S(i,j) = \mu_S(j)$, for all $i \in S^{-1}(j)$.

Consider a particular agent $i \in N$.

First we will determine $E_h(BG)(i)$, for all $h \in N$. Let $j \in S(i)$, and let $\mathcal{T}_S(i,j) := \{T \in \mathcal{T}_S \mid j \in T(i)\}$. Then

$$E_h(i,j) := \sum_{T\in\mathcal{T}_S(i,j)} p_h(T)$$

is the probability agent h gives to the occurrence of an echelon tree in which agent i is the predecessor of agent j. Now there have to be considered exactly three cases with respect to agent $h \in N$:

1. Suppose $h = i$.

   Then it immediately follows that:

$$E_i(i,j) = \mu_S(i,j) = \mu_S(j). \tag{3}$$

2. Suppose $h \in S^{-1}(j) \setminus \{i\}$.

   This means that h is another potential predecessor of j. Then h gives probability $1 - \mu_S(j)$ to the occurrence of an echelon tree in which he is not the predecessor of j. We assumed that agent h expects that all other potential predecessors of j have equal probability to be j's predecessor. Therefore it holds that:

$$E_h(i,j) = \frac{1-\mu_S(j)}{\sigma(j)-1} \text{ if } h \in S^{-1}(j) \setminus \{i\}. \tag{4}$$

3. Suppose $h \in N \setminus S^{-1}(j)$.

   Thus h is not a potential predecessor of j. We assumed that, if j is no potential successor of h, then h expects that j will direct himself to each one of his potential predecessors with equal probability. Therefore it holds that:

$$E_h(i,j) = \frac{1}{\sigma(j)} \text{ if } h \in N \setminus S^{-1}(j). \tag{5}$$

This holds for all potential successors j of agent i, so from (3), (4), (5) and the third condition the theorem it follows that for all $h \in N$:

$$E_h(BG)(i) = \sum_{T \in \mathcal{T}_S} p_h(T) BG(i,T) = \sum_{j \in S(i)} \sum_{T \in \mathcal{T}_S(i,j)} p_h(T)$$

$$= \sum_{j \in S(i)} E_h(i,j) = \begin{cases} \sum_{j \in S(i)} \mu_S(j) & \text{if } h = i \\ \sum_{j \in S(h) \cap S(i)} \frac{1-\mu_S(j)}{\sigma(j)-1} + \sum_{j \in S(i) \setminus S(h)} \frac{1}{\sigma(j)} & \text{if } h \neq i \end{cases}$$

Next we establish the following facts:

- If $j \in S(h)$, $h \neq i$, then $\sigma(j) \neq 1$.
- A potential successor j of agent i has $\sigma(j) - 1$ other potential predecessors besides i.

- There are $n - \sigma(j)$ agents in N who are no potential predecessor of j, where $n := \#N$.

With this we can determine the following

$$\sum_{h \in N} E_h(BG)(i) = E_i(BG)(i) + \sum_{h \in N \setminus \{i\}} E_h(BG)(i)$$

$$= \sum_{j \in S(i)} \mu_S(j) + \sum_{h \in N \setminus \{i\}} \left(\sum_{j \in S(h) \cap S(i)} \frac{1 - \mu_S(j)}{\sigma(j) - 1} + \sum_{j \in S(i) \setminus S(h)} \frac{1}{\sigma(j)} \right)$$

$$= \sum_{j \in S(i)} \mu_S(j) + \sum_{\substack{j \in S(i) \\ \sigma(j) \neq 1}} \frac{(\sigma(j) - 1)(1 - \mu_S(j))}{(\sigma(j) - 1)} + \sum_{j \in S(i)} \frac{n - \sigma(j)}{\sigma(j)}$$

$$= \sum_{\substack{j \in S(i) \\ \sigma(j) = 1}} \left(\mu_S(j) + \frac{n - \sigma(j)}{\sigma(j)} \right) + \sum_{\substack{j \in S(i) \\ \sigma(j) \neq 1}} \left(\mu_S(j) + (1 - \mu_S(j)) + \frac{n - \sigma(j)}{\sigma(j)} \right)$$

$$= \sum_{\substack{j \in S(i) \\ \sigma(j) = 1}} \frac{n}{\sigma(j)} + \sum_{\substack{j \in S(i) \\ \sigma(j) \neq 1}} \frac{n}{\sigma(j)} = \sum_{j \in S(i)} \frac{n}{\sigma(j)} \tag{6}$$

Equation (6) immediately implies that

$$\sum_{h \in N} \frac{E_h(BG)(i)}{\#N} = \sum_{j \in S(i)} \frac{1}{\sigma(j)} = BG(i, S).$$

This holds for every agent $i \in N$, and thus we are able to conclude that the BG-index satisfies the third condition.

Next let $\varphi: N \times \mathcal{S}^N \to \mathbf{R}_+$ be a function that satisfies the three conditions.
From the first two conditions it follows that for each $S \in \mathcal{S}^N$ and every $T \in \mathcal{T}_S$ it holds that:

$$\varphi(i, T) = \#T(i) = \sum_{j \in T(i)} \frac{1}{\#T^{-1}(j)} = BG(i, T).$$

From this it follows that

$$\frac{\sum_{h\in N} E_h(\varphi)(i)}{\#N} = \frac{\sum_{h\in N}\sum_{T\in\mathcal{T}_S} p_h(T)\varphi(i,T)}{\#N}$$

$$= \frac{\sum_{h\in N}\sum_{T\in\mathcal{T}_S} p_h(T)BG(i,T)}{\#N} = \frac{\sum_{h\in N} E_h(BG)(i)}{\#N}.$$

With condition 3 it then follows that for every $S \in \mathcal{S}^N$ and for every social expectation function μ_S such that for every agent $j \in N \setminus L_M$ and for every $i \in S^{-1}(j)$ it holds that $\mu_S(i,j) = \mu_S(j)$ ($\mu_S(j) \in [0,1]$), it holds that

$$\varphi(i,S) = \frac{\sum_{h\in N} E_h(BG)(i)}{\#N}.$$

From Equation (6) it is easily established that φ is equal to the BG-index.

Q.E.D.

5 An example

In this section we give an example which illustrates the objective and subjective interpretations of the BG-index.

Example 5.1 Consider the hierarchically structured population (N,S), where $N = \{1,\ldots,7\}$ and the successor mapping S is given by:

$$S(1) = \{2,3,4,5\},\ S(2) = \emptyset,\ S(3) = \emptyset,\ S(4) = \{2,6\},\ S(5) = \{3,7\},$$
$$S(6) = \emptyset,\ S(7) = \emptyset.$$

The echelon partition ξ and communication structure R of (N,S) are given by:

$$\xi = \{\{2,3,6,7\},\{4,5\},\{1\}\}$$

and

$$R = \{\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{2,4\},\{3,5\},\{4,6\},\{5,7\}\}.$$

The communication graph (N,R) belonging to (N,S) is given in figure 3 in which the black dots are the agents in the second echelon L_2 and the ringed dot is the agent in the highest echelon L_3. The BG-index of (N,S) is given by:

$$\overline{BG(\cdot,S)} = (3,0,0,1\tfrac{1}{2},1\tfrac{1}{2},0,0),$$

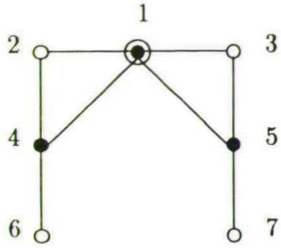


Figure 3: Example 5.1

where $\overline{BG(\cdot,S)} := (BG(1,S),\ldots,BG(7,S))$. The only agents that have a choice possibility with respect to their predecessor are the agents 2 and 3. They both have two potential predecessors and therefore there are four echelon trees in (N,S). These four echelon trees (N,T_k) are given in the first column of table 1.

Consider the first echelon tree. According to condition 2 in Theorem 3.3 it must hold that $\overline{\varphi(\cdot,T_1)} \geq (4,0,0,1,1,0,0)$. Together with condition 1 ($\sum_{i=1}^{7}\varphi(i,T_1) = 6$) this implies that equality holds and $\varphi(i,T_1) = BG(i,T_1)$, with $BG(i,T_1) = \#T_1(i)$. In the second column of table 1 we give the values of $t_k(i)$, $i \in \{1,4,5\}$, $k \in \{1,\ldots,4\}$, where $t_k(i) := \#T_k(i)$. (It is clear that $t_k(i) = 0$ for all $i \in \{2,3,6,7\}$.) Now it is easy to verify that for every agent $i \in N$ it holds that $\sum_{k=1}^{4} \frac{t_k(i)}{4} = BG(i,S)$. This illustrates Theorem 3.3.

In order to illustrate Theorem 4.4 we need to give the subjective expectations of the agents with respect to the power structure. Because the agents 2, 3, 6, and 7 are part of the lowest echelon it follows from Definitions 4.2 and 4.3 that

$$E_h(BG)(i) = BG(i,S) = 0 \quad \text{for all } i \in \{2,3,6,7\} \quad \text{and all } h \in N. \tag{7}$$

Therefore the only agents of interest are the agents 1, 4 and 5. In the last three columns of table 1 we give the probability distributions representing the expectations about the occurrence of echelon trees of the agents 1, 4 and 5 in the case they all have pessimistic, neutral, or optimistic expectations.

Agent 4 has one potential successor for which it is not certain that he will direct himself to 4, namely agent 2. In the case of pessimistic expextations therefore,

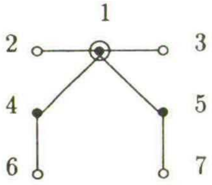
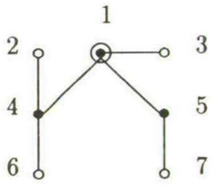
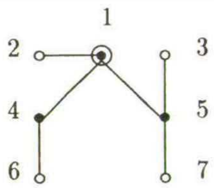
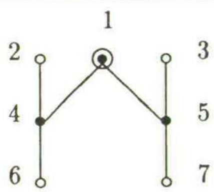
echelon tree	$(t_k(1), t_k(4), t_k(5))$	probability distributions in the case of		
		pessimistic expectations	neutral expectations	optimistic expectations
	$(1,4,1)$	$\frac{1}{2}$ 0 $\frac{1}{2}$	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$	0 1 0
	$(2,3,1)$	0 0 $\frac{1}{2}$	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$	$\frac{1}{2}$ 0 0
	$(1,3,2)$	$\frac{1}{2}$ 0 0	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$	0 0 $\frac{1}{2}$
	$(2,2,2)$	0 1 0	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$	$\frac{1}{2}$ 0 $\frac{1}{2}$

Table 1: Example 5.1

agent 4 gives zero probability to the occurrence of an echelon tree in which he is the predecessor of agent 2. These are the second and fourth echelon trees. The other two echelon trees both are given probability $\frac{1}{2}$ of occurrence from agent 4.

Similarly agent 5 gives zero probability of occurrence to the first and second echelon tree and probability $\frac{1}{2}$ to the occurrence of both the third and fourth echelon tree.

Agent 1 has two potential successors with more than one potential predecessors, namely agents 2 and 3. Therefore agent 1 gives probability one to the occurrence of the fourth echelon tree, which is the only one in which he is not the predecessor of either 2 or 3. The other echelon trees are given probability zero from agent 1.

Next we are able to give the subjective expectations of the BG-index for the agents 1, 4 and 5 in the case of pessimistic expectations.

$$\overline{E_1(BG)} = (2,0,0,2,2,0,0)$$

$$\overline{E_4(BG)} = \frac{1}{2}(4,0,0,1,1,0,0) + \frac{1}{2}(3,0,0,1,2,0,0) = (3\frac{1}{2},0,0,1,1\frac{1}{2},0,0)$$

$$\overline{E_5(BG)} = \frac{1}{2}(4,0,0,1,1,0,0) + \frac{1}{2}(3,0,0,2,1,0,0) = (3\frac{1}{2},0,0,1\frac{1}{2},1,0,0)$$

With (7) it then follows that:

$$\frac{\sum_{c\in N} \overline{E_c(BG)}}{\#N} = (3,0,0,1\frac{1}{2},1\frac{1}{2},0,0) = \overline{BG(\cdot,S)}.$$

Thus condition 3 of Theorem 4.4 is satisfied.

In the case of neutral expectations all agents give equal probability of occurrence $\frac{1}{4}$ to each echelon tree. Then

$$\overline{E_1(BG)} = \overline{E_4(BG)} = \overline{E_5(BG)} = (3,0,0,1\frac{1}{2},1\frac{1}{2},0,0) = \overline{BG(\cdot,S)},$$

and thus the conditions of Theorem 4.4 are satisfied.

In the case of optimistic expectations the agents give zero probability to the occurrence of each echelon tree in which they are not the predecessors of all their potential successors. For agent 4 and 5 this means that they give probability $\frac{1}{2}$ (respectively 0) to the occurrence of each echelon tree to which they give probability zero (respectively $\frac{1}{2}$) in the case of pessimistic expectations.

echelon tree	$(t_i(1), t_i(2))$	probability distributions in the case of					
		pessimistic expectations		neutral expectations		optimistic expectations	
T_1	$(3,0)$	0	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0
T_2	$(2,1)$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	0	1

Table 2: Example 5.2

The only echelon tree in which agent 1 is the predecessor of all his potential successors is the first echelon tree and therefore agent 1 gives probability one to the occurrence of this echelon tree and probability zero to the other echelon trees. The subjective expectations of the BG-index for the agents 1, 4 and 5 in the optimistic case are given by

$$\overline{E_1(BG)} = (4,0,0,1,1,0,0),$$

$$\overline{E_4(BG)} = \frac{1}{2}(3,0,0,2,1,0,0) + \frac{1}{2}(2,0,0,2,2,0,0) = (2\frac{1}{2},0,0,2,1\frac{1}{2},0,0),$$

$$\overline{E_5(BG)} = \frac{1}{2}(3,0,0,1,2,0,0) + \frac{1}{2}(2,0,0,2,2,0,0) = (2\frac{1}{2},0,0,1\frac{1}{2},2,0,0).$$

Again, with (7) it then simply follows that condition 3 of Theorem 4.4 is satisfied.

To complete this paper we give the BG-index for the example used throughout section 2.

Example 5.2 Consider the hierarchically structured population (N, S) given in Example 2.4 whose echelon trees are given in Example 2.8. The BG-index of (N, S) is given by:

$$\overline{BG(\cdot, S)} = (2\frac{1}{2}, \frac{1}{2}, 1, 1, 0, 0).$$

The BG-indices of the echelon trees (N, T_1) and (N, T_2) respectively are given by:

$$\overline{BG(\cdot, T_1)} = (3,0,1,1,0,0) \text{ and } \overline{BG(\cdot, T_2)} = (2,1,1,1,0,0).$$

The only agents that have potential successors with more than one potential predecessor are the agents 1 and 2. In table 2 we give the values of $t_k(i)$, $i \in \{1,2\}$, $k \in \{1,2\}$, and the probability distributions representing the expectations about the occurrence of the echelon trees of agents 1 and 2 in the case they have pessimistic, neutral or optimistic expectations. Both Theorem 3.3 and Theorem 4.4 easily can be verified using this table.

6 Proofs of the theorems of section 2

In order to prove Theorem 2.2 we first prove some lemma's. Let $S \in \mathcal{S}^N$. We introduce an auxiliary mapping $H: \mathbf{N} \cup \{0\} \to 2^N$, which recursively is defined as follows

$$H(0) := \emptyset,$$

and for every $k \in \mathbf{N}$

$$H(k) := \{i \in N \mid S(i) \subset H(k-1)\}.$$

Lemma 6.1 *There exists a finite number $M \in \mathbf{N}$ such that:*

1. $H(k-1) \subset H(k)$, $H(k-1) \neq H(k)$, *for every* $1 \leq k \leq M$*;*
2. $H(k) = N$, *for every* $k \geq M$.

PROOF
The proof of the lemma consists of a number of steps.

(a) For every $k \in \mathbf{N}$: $H(k-1) \subset H(k)$.

We prove this assertion by induction. First note that by definition $H(0) \subset H(1)$.
Let $k \in \mathbf{N}$. Now assume that $H(k-1) \subset H(k)$.
Let $i \in H(k)$, then by definition

$$S(i) \subset H(k-1) \subset H(k).$$

Thus $i \in H(k+1)$, and therefore $H(k) \subset H(k+1)$.

(b) For every $i, j \in N$ it holds that

$$j \in S(i) \implies \#\widehat{S}(j) < \#\widehat{S}(i).$$

Let $i, j \in N$ with $j \in S(i)$. Then by definition for every $k \in \mathbf{N}$ it holds that $S^k(j) \subset S^{k+1}(i)$. Hence,

$$\widehat{S}(j) \subset \bigcup_{k=2}^{\infty} S^k(i) \subset \widehat{S}(i).$$

But $j \in \widehat{S}(i) \setminus \widehat{S}(j)$ and thus $\widehat{S}(j) \neq \widehat{S}(i)$. This implies the assertion.

(c) For every $i \in N$: $i \in H(\#\widehat{S}(i) + 1)$.

Let $i \in N$. We prove the claim by induction on $\#\widehat{S}(i)$.
First we suppose that $\#\widehat{S}(i) = 0$, i.e., $\widehat{S}(i) = \emptyset$. So, $\emptyset = S(i) \subset H(0)$, which implies that $i \in H(1)$.
Let $k \in \mathbf{N}$. Next assume that for all $j \in N$ with $\#\widehat{S}(j) \leq k - 1$ the claim is true, i.e., $j \in H(\#\widehat{S}(j) + 1)$. Furthermore, suppose that $\#\widehat{S}(i) = k$. Then by (b) it holds that $\#\widehat{S}(j) \leq k - 1$ for every $j \in S(i)$. Thus for every $j \in S(i)$ by (a) it holds that $j \in H(\#\widehat{S}(j) + 1) \subset H(k)$ and so $S(i) \subset H(k)$. By definition this implies that $i \in H(k+1) = H(\#\widehat{S}(i) + 1)$.

We introduce the function $r: N \to \mathbf{N}$, which for every $i \in N$ is given by

$$r(i) := \min\{k \mid i \in H(k)\}.$$

By (c) it is clear that r is a well defined function, i.e., for every player $i \in N$ $r(i)$ exists and is a finite number. Furthermore, by definition for every $k \in \mathbf{N}$: $H(k) = \{i \in N \mid r(i) \leq k\}$.
Now we take $M := \max\{r(i) \mid i \in N\}$.
By the statements as proved above it is obvious that for every $1 \leq k \leq M$ it holds that $H(k-1) \subset H(k)$ and that for every $k \geq M$ it holds that $H(k) = N$. Therefore it is left to prove that for every $1 \leq k \leq M$ it holds that $H(k-1) \neq H(k)$. This is done in the following two steps.

(d) For every $i, j \in N$ with $j \in S(i)$ it holds that $r(j) < r(i)$.

By definition $i \in H(r(i))$. Hence, $j \in S(i) \subset H(r(i) - 1)$. This shows that $r(j) \leq r(i) - 1$.

(e) For every $i \in N$ with $r(i) \geq 2$ there is some $j \in N$ such that $j \in S(i)$ and $r(j) = r(i) - 1$.

By (d) it holds that for every $j \in S(i)$: $r(j) \leq k-1$, where $k = r(i)$. Suppose by contradiction that for every $j \in S(i)$ it holds that $r(j) \leq k-2$.
Then $S(i) \subset H(k-2)$ and so $i \in H(k-1)$, which is impossible by the definition of the function r.

We complete the proof of the lemma by remarking that (e) implies that for every $2 \leq k \leq M$ it holds that $H(k-1) \neq H(k)$.
It remains to prove that $H(1) \neq H(0) = \emptyset$. Suppose, by contradiction, that $H(1) = \emptyset$. Hence, there are no players $i \in N$ with $S(i) = \emptyset$, i.e., for every $i \in N$: $S(i) \neq \emptyset$.
Now we construct a sequence $(i_k)_{k \in \mathbf{N}}$ in N as follows. First, choose $i_1 \in N$ arbitrarily. Then, recursively, choose $i_{k+1} \in S(i_k)$, $k \in \mathbf{N}$. We claim that this sequence is infinite. Namely, for every $m \in \mathbf{N}$ it holds that for every $k \geq m+1$: $i_k \in \widehat{S}(i_m)$. By acyclicity of the successor mapping S it is clear that therefore all elements in the sequence have to be distinct. Hence, the sequence has to consist of an infinite number of distinct elements, and so the set of players N has to be infinite. This contradicts the finiteness of N.
This completes the proof of the lemma.

Q.E.D.

With Lemma 6.1 we can derive that the mapping H describes a hierarchy with a finite number of levels. These levels are precisely the echelons L_k, $1 \leq k \leq M$, as introduced in the main text as the following lemma shows.

Lemma 6.2 *Let $S \in \mathcal{S}^N$. Then $\{L_1, \ldots, L_M\}$ is a partition consisting of non-empty sets only.*

PROOF
From (1) and the definition of the mapping H it follows that $\bigcup_{p=1}^{k-1} L_p = H(k-1)$ and thus for all $k \in \mathbf{N}$

$$\begin{aligned} L_k &= \{i \in N \setminus H(k-1) \mid S(i) \subset H(k-1)\} \\ &= \{i \in N \setminus H(k-1) \mid i \in H(k)\} \\ &= H(k) \setminus H(k-1) \end{aligned}$$

Then

1. $\bigcup_{k=1}^{M} L_k = \bigcup_{k=1}^{M} H(k) \setminus H(k-1) = H(M) \setminus H(0) = N$;

2. Let $1 \leq k \neq l \leq M$. Without loss of generality we assume that $k < l$. Then $H(k-1) \subset H(k) \subset H(l-1) \subset H(l)$. This implies that $[H(l) \setminus H(l-1)] \cap H(k) = \emptyset$. With this it follows that $L_k \cap L_l = [H(k) \setminus H(k-1)] \cap [H(l) \setminus H(l-1)] = \emptyset$;

3. Because $H(k) \neq H(k-1)$ it holds that $L_k \neq \emptyset$ for all $1 \leq k \leq M$.

This proves the lemma.

Q.E.D.

Lemma 6.3 *There is a unique agent $i_0 \in N$ for whom $S^{-1}(i_0) = \emptyset$. This agent i_0 is the unique agent in N for whom $\widehat{S}(i_0) = N \setminus \{i_0\}$.*

PROOF

First we remark that there is *at most* one player $i_0 \in N$ for whom it holds that $\widehat{S}(i_0) = N \setminus \{i_0\}$. Furthermore, there exists *at least one* player $j \in N$ for whom it holds that $S^{-1}(j) = \emptyset$.† The lemma is now proved in two subsequent steps.

(a) For $i_0 \in N$ it holds that $\widehat{S}(i_0) = N \setminus \{i_0\}$ if and only if i_0 is the unique player for whom it holds that $S^{-1}(i_0) = \emptyset$.

Only if

Suppose that for $i_0 \in N$ it holds that $\widehat{S}(i_0) = N \setminus \{i_0\}$. Assume, by contradiction, that $S^{-1}(i_0) \neq \emptyset$. Then there is some $j \in N$ such that $i_0 \in S(j)$. But then $\widehat{S}(i_0) \subset \widehat{S}(j)$. Since $j \neq i_0$ we must have $j \in \widehat{S}(i_0) \subset \widehat{S}(j)$, which contradicts acyclicity of the successor mapping S.

Suppose that there is another player $j \in N$ for whom it holds that $S^{-1}(j) = \emptyset$. By assumption it holds that $j \in \widehat{S}(i_0)$, i.e., $j \in S^k(i_0)$ for some $k \in \mathbf{N}$. But this means that $S^{-1}(j) \neq \emptyset$. Contradiction.

If

Suppose that $i_0 \in N$ is the unique player such that $S^{-1}(i_0) = \emptyset$. Assume, however, that $\widehat{S}(i_0) \neq N \setminus \{i_0\}$. Then the set $X := N \setminus \left(\{i_0\} \cup \widehat{S}(i_0)\right)$ is not empty. Thus we may choose $j \in X$ for whom $\#\widehat{S}(j)$ is maximal over X.

By assumption for j it holds that $S^{-1}(j) \neq \emptyset$, and so there must be some

†Note that for every $i \in N$ it holds that $\#\widehat{S}(i) \leq n$. Now choose those $i \in N$ who maximizes the number $\#\widehat{S}(i)$. Clearly these players satisfy the property.

$h \in N$ such that $j \in S(h)$. This implies that $\#\widehat{S}(h) > \#\widehat{S}(j)$. So, $h \notin X$. But this means that either $i_0 = h$ or $h \in \widehat{S}(i_0)$.
The first case is excluded since then $j \in \widehat{S}(h) = \widehat{S}(i_0)$ and, hence, $j \notin X$.
The second case implies that $\widehat{S}(h) \subset \widehat{S}(i_0)$. But this implies also that $j \in \widehat{S}(h) \subset \widehat{S}(i_0)$.

(b) There exists a player $i_0 \in N$ such that $\widehat{S}(i_0) = N \setminus \{i_0\}$.

Suppose, by contradiction, that there is no player who satisfies this condition. Then by (a) there exist at least two players $j, h \in N$, with $j \neq h$, such that $S^{-1}(j) = S^{-1}(h) = \emptyset$. But this property prevents the possibility of the existence of a player $i \in N$ such that $\{j, h\} \subset \widehat{S}(i) \cup \{i\}$. This is in contradiction with the assumptions on the successor mapping S as made in Definition 2.1.

Q.E.D.

Theorem 2.2 now directly follows from the lemma's 6.2 and 6.3.

PROOF OF THEOREM 2.3

1. Let $i \in L_k$, $j \in L_l$, $1 \leq k, l \leq M$.

   Only if

   Suppose that $i \in S(j)$. Then, by definition of H it holds that $i \in S(j) \subset H(l-1)$. This implies that $k \leq l - 1$.

   If

   Suppose that $k < l$. $\{i, j\} \in R$ implies that either $i \in S(j)$ or $j \in S(i)$. But if $j \in S(i)$, then $l < k < l$. Thus $i \in S(j)$.

2. Let $i \in L_k$, $2 \leq k \leq M$. Suppose that $S(i) \cap L_{k-1} = \emptyset$. But then $S(i) \subset H(k-2)$. Thus $i \notin L_k$. Statement 2 then follows with statement 1.

3. Let $i \in L_k$, $1 \leq k \leq M - 1$. From Lemma 6.3 it follows that $S^{-1}(i) \neq \emptyset$. Statement 3 then follows with statement 1.

Q.E.D.

PROOF OF LEMMA 2.6

Let $\xi = \{L_1, \ldots, L_M\}$ be the echelon partition of (N, S).
Suppose that $i \in L_k$, $1 \leq k \leq M-1$ and let $i_0 \in L_M$ denote the topman within the hierarchically structured population (N, S). From Theorem 2.3 it directly follows that there exists at least one communication path between i and i_0 satisfying the condition stated in the lemma. Such a communication path can be found using the following algorithm:

STEP 1 Let $c_1 := i$, $p := 1$. GOTO step 2.

STEP 2 IF $c_p \in L_M$ (i.e., $c_p = i_0$), THEN $(c_1, \ldots, c_p)$ is a communication path between i and i_0 satisfying the condition stated in the lemma. STOP.

ELSE there exists a $c_{p+1} \in \bigcup_{a=k+1}^{M-1} L_a$ such that $c_p \in S(c_{p+1})$. (This follows from Theorem 2.3).

GOTO step 3.

STEP 3 Let $p := p + 1$. GOTO step 2.

From the finiteness of $\xi = \{L_1, \ldots, L_M\}$ it follows that this algorithm always leads to a communication path between i and i_0 satisfying the condition stated in the lemma.

Suppose $i \in L_k$, $j \in L_l$, $i \neq j$.
If $\{i, j\} \cap L_M \neq \emptyset$ then it follows from the discussion above that there exists a communication path between i and j satisfying the condition stated in the lemma.

Now suppose that $\{i, j\} \cap L_M = \emptyset$. Then it follows from the discussion above that there exists at least one communication path $\overline{c} = (c_1 = \alpha, \ldots, c_h = i_0)$, $1 \leq h \leq M-k$ between i and i_0, and there exists at least one communication path $\overline{d} = (d_p = j, \ldots, d_1 = i_0)$, $1 \leq p \leq M-l$ between j and i_0 satisfying the condition stated in the lemma. Then it follows from Theorem 2.3 that at least one of the following three situations holds:

1. There exists a communication path $\overline{c} = (c_1 = i, \ldots, c_e = j, \ldots, c_h = i_0)$ between i and i_0 such that j is part of that communication path and $c_k \in S(c_{k+1})$, $1 \leq k \leq h-1$. In this case $(c_1 = i, \ldots, c_e = j)$ is a communication path between i and j satisfying the condition stated in the lemma.

2. There exists a communication path $\overline{d} = (d_p = j, \ldots, d_f = i, \ldots, d_1 = i_0)$ between j and i_0 such that i is part of that communication path and $d_{k+1} \in S(d_k)$, $1 \leq k \leq p-1$. In this case $(d_p = j, \ldots, d_f = i)$ is a communication path between i and j satisfying the condition stated in the lemma.

3. There exists an agent $t \in L_q$, with $\max\{k, l\} < q \leq M$ such that there exist communication paths $(c_1 = i, \ldots, c_e = t, \ldots, c_h = i_0)$ between i and i_0 and $(d_p = j, \ldots, d_f = t, \ldots, d_1 = i_0)$ between j and i_0 such that $c_k \in S(c_{k+1})$, $1 \leq k \leq h-1$ and $d_{k+1} \in S(d_k)$, $1 \leq k \leq p-1$. In this case $(c_1 = i, \ldots, c_e = t = d_f, \ldots, d_p = j)$ is a communication path between i and j satisfying the condition stated in the lemma.

Q.E.D.

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Optimal dynamic investment policies under concave-convex adjustment costs

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Cyclic polling systems: limited service versus Bernoulli schedules

423 M.H.C. Paardekooper
Parallel normreducing transformations for the algebraic eigenvalue problem

424 Hans Gremmen
On the political (ir)relevance of classical customs union theory

425 Ed Nijssen
Marketingstrategie in Machtsperspectief

426 Jack P.C. Kleijnen
Regression Metamodels for Simulation with Common Random Numbers: Comparison of Techniques

427 Harry H. Tigelaar
The correlation structure of stationary bilinear processes

428 Drs. C.H. Veld en Drs. A.H.F. Verboven
De waardering van aandelenwarrants en langlopende call-opties

429 Theo van de Klundert en Anton B. van Schaik
Liquidity Constraints and the Keynesian Corridor

430 Gert Nieuwenhuis
Central limit theorems for sequences with m(n)-dependent main part

431 Hans J. Gremmen
Macro-Economic Implications of Profit Optimizing Investment Behaviour

432 J.M. Schumacher
System-Theoretic Trends in Econometrics

433 Peter M. Kort, Paul M.J.J. van Loon, Mikulás Luptacik
Optimal Dynamic Environmental Policies of a Profit Maximizing Firm

434 Raymond Gradus
Optimal Dynamic Profit Taxation: The Derivation of Feedback Stackelberg Equilibria

435 Jack P.C. Kleijnen
Statistics and Deterministic Simulation Models: Why Not?

436 M.J.G. van Eijs, R.J.M. Heuts, J.P.C. Kleijnen
Analysis and comparison of two strategies for multi-item inventory systems with joint replenishment costs

437 Jan A. Weststrate
Waiting times in a two-queue model with exhaustive and Bernoulli service

438 Alfons Daems
Typologie van non-profit organisaties

439 Drs. C.H. Veld en Drs. J. Grazell
Motieven voor de uitgifte van converteerbare obligatieleningen en warrantobligatieleningen

440 Jack P.C. Kleijnen
Sensitivity analysis of simulation experiments: regression analysis and statistical design

441 C.H. Veld en A.H.F. Verboven
De waardering van conversierechten van Nederlandse converteerbare obligaties

442 Drs. C.H. Veld en Drs. P.J.W. Duffhues
Verslaggevingsaspecten van aandelenwarrants

443 Jack P.C. Kleijnen and Ben Annink
Vector computers, Monte Carlo simulation, and regression analysis: an introduction

444 Alfons Daems
"Non-market failures": Imperfecties in de budgetsector

445 J.P.C. Blanc
The power-series algorithm applied to cyclic polling systems

446 L.W.G. Strijbosch and R.M.J. Heuts
Modelling (s,Q) inventory systems: parametric versus non-parametric approximations for the lead time demand distribution

447 Jack P.C. Kleijnen
Supercomputers for Monte Carlo simulation: cross-validation versus Rao's test in multivariate regression

448 Jack P.C. Kleijnen, Greet van Ham and Jan Rotmans
Techniques for sensitivity analysis of simulation models: a case study of the CO_2 greenhouse effect

449 Harrie A.A. Verbon and Marijn J.M. Verhoeven
Decision-making on pension schemes: expectation-formation under demographic change

450 Drs. W. Reijnders en Drs. P. Verstappen
Logistiek management marketinginstrument van de jaren negentig

451 Alfons J. Daems
Budgeting the non-profit organization
An agency theoretic approach

452 W.H. Haemers, D.G. Higman, S.A. Hobart
Strongly regular graphs induced by polarities of symmetric designs

453 M.J.G. van Eijs
Two notes on the joint replenishment problem under constant demand

454 B.B. van der Genugten
Iterated WLS using residuals for improved efficiency in the linear model with completely unknown heteroskedasticity

455 F.A. van der Duyn Schouten and S.G. Vanneste
Two Simple Control Policies for a Multicomponent Maintenance System

456 Geert J. Almekinders and Sylvester C.W. Eijffinger
Objectives and effectiveness of foreign exchange market intervention
A survey of the empirical literature

457 Saskia Oortwijn, Peter Borm, Hans Keiding and Stef Tijs
Extensions of the τ-value to NTU-games

458 Willem H. Haemers, Christopher Parker, Vera Pless and Vladimir D. Tonchev
A design and a code invariant under the simple group Co_3

459 J.P.C. Blanc
Performance evaluation of polling systems by means of the power-series algorithm

460 Leo W.G. Strijbosch, Arno G.M. van Doorne, Willem J. Selen
A simplified MOLP algorithm: The MOLP-S procedure

461 Arie Kapteyn and Aart de Zeeuw
Changing incentives for economic research in The Netherlands

462 W. Spanjers
Equilibrium with co-ordination and exchange institutions: A comment

463 Sylvester Eijffinger and Adrian van Rixtel
The Japanese financial system and monetary policy: A descriptive review

464 Hans Kremers and Dolf Talman
A new algorithm for the linear complementarity problem allowing for an arbitrary starting point