…RSITY
…IJKE
UNIVERSITEIT
BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

LINEAR TIME-INVARIANT FILTERS OF INFINITE ORDER FOR NON-STATIONARY PROCESSES

B.B. van der Genugten

FEW 358

LINEAR TIME-INVARIANT FILTERS
OF INFINITE ORDER
FOR NON-STATIONARY PROCESSES

by

B.B. van der Genugten

Abstract

In dynamic linear models linear time-invariant filters of infinite order play an important rôle. E.g. an invertible AR of finite order corresponds to an MA of infinite order but not to one of finite order. To justify the formal operations of composition and inversion of filters regularity conditions concerning convergence have to be checked. This paper discusses the construction of simple classes of non-stationary processes and linear filters of infinite order that are closed under composition and inversion. For such classes the necessary regularity conditions are automatically satisfied.

Concepts as "exponentially decreasing filters" and "non-exponentially increasing processes" come up in a natural way. Some examples illustrate the applications of the concepts.

1. Introduction

Let $\mathcal{L}$ be a class of m-vectorial stochastic processes $X = \{X_t, t \in \mathbf{Z}\}$ on the set of integers $\mathbf{Z}$ and $\mathcal{F}$ a class of filters mapping $\mathcal{L}$ into $\mathcal{L}$. So for each *filter* $A \in \mathcal{F}$ the *input* $X \in \mathcal{L}$ gives the *output* $Y = \{Y_t, t \in \mathbf{Z}\} = A(X) \in \mathcal{L}$, or shortly: $A(\mathcal{L}) \subset \mathcal{L}$, $A \in \mathcal{L}$. Hence $\mathcal{F}(\mathcal{L}) \subset \mathcal{L}$.

Let $A, B \in \mathcal{L}$ and $X \in \mathcal{L}$. Then $Z = B(Y)$ with $Y = A(X)$ can be written as $Z = C(X)$ with $C = B \circ A$, the *composition* of A and B. So the definition of $\mathcal{F}$ implies that $\mathcal{F}$ is *closed* under composition.

Let $\mathcal{G} \subset \mathcal{F}$ be a group with the composition as group-operation. Then by definition the identity I belongs to $\mathcal{G}$ and for each $A \in \mathcal{G}$ there exists a unique $A^{-1} \in \mathcal{G}$ such that $A^{-1} \circ A = I$, the *inverse* A^{-1} of A. The existence of such an $\mathcal{G}$ implies $\mathcal{F}(\mathcal{L}) = \mathcal{L}$ (since $I(\mathcal{L}) = \mathcal{L}$).

In this paper we look for suitable choices of $\mathcal{L}$, $\mathcal{F}$ and $\mathcal{G}$ such that all processes are *one-sided moving averages* of infinite order allowing simple mathematical operations for composition and inversion. We will formulate this more precisely.

In $\mathcal{L}$ we do not distinguish between processes who only differ on a set of probability zero ($X = X'$ a.s. $\Rightarrow X = X'$). For $\mathcal{F}$ we suppose that each $A \in \mathcal{F}$ can be represented uniquely on $\mathcal{L}$ by a sequence $\{A_k, k \in \mathbb{N}_0\}$ of m×m-matrices or shortly: $A = \{A_k, k \in \mathbb{N}_0\}$. Uniqueness refers to

$$(1.2) \qquad A(X) = B(X) \text{ for all } X \in \mathcal{L} \iff A_k = B_k \text{ for all } k \in \mathbb{N}_0.$$

For each $A = \{A_k, k \in \mathbb{N}_0\} \in \mathcal{F}$ and each input $X = \{X_t, t \in \mathbf{Z}\} \in \mathcal{L}$ the corresponding output $Y = \{Y_t, t \in \mathbf{Z}\} \in \mathcal{L}$ is given by

$$(1.1) \qquad Y_t = \sum_0^{\infty} A_k X_{t-k}, \qquad t \in \mathbf{Z}.$$

Here it is assumed that the type of convergence (in probability, r^{th} mean, a.s.) is the same for all $A \in \mathcal{F}$ and $X \in \mathcal{L}$. It follows that the identity $I \in \mathcal{L}$ is represented by $I = \{\delta_{0k} I_m, k \in N_0\}$, where I_m denotes the m×m-matrix. We suppose that $\mathcal{L}$ is a linear space (with the usual componentwise definition of addition and scalar multiplication of stochastic processes). Then $A = \{A_k, k \in \mathbb{N}_0\} \in \mathcal{F}$ is a linear filter. We assume also that $\mathcal{F}$ is a linear space. Then the nulfunction $0 \in \mathcal{F}$ is represented by $\{0, k \in \mathbb{N}_0\}$.

The output $Z = \{Z_t, t \in \mathbb{Z}\}$ of the filter $B = \{B_k, k \in \mathbb{N}_0\} \in \mathcal{F}$ of $y = y_t, t \in \mathbb{Z}\}$, given by

$$Z_t = \sum_0^\infty B_k Y_{t-k} \quad , \quad t \in \mathbb{Z},$$

can be written as $Z = B(Y) = B(A(X)) = C(X)$ with $C = B \circ A \in \mathcal{F}$ the composition of A and B. So we can write

$$Z_t = \sum_0^\infty C_k X_{t-k} \quad , \quad t \in \mathbb{Z},$$

where the composition is represented by $C = \{C_k, k \in \mathbb{N}_0\}$. A formal substitution makes the following *convolution* property desirable:

$$(1.3) \qquad A, B \in \mathcal{F}: C = B \circ A \Leftrightarrow C_k = \sum_0^k B_j A_{k-j} \quad , \quad k \in \mathbb{N}_0.$$

The inversion property (1.3) imposes conditions on the linear spaces $\mathcal{L}$ and $\mathcal{F}$.

Let $A = \{A_k, k \in \mathbb{N}_0\} \in \mathcal{G}$. It follows from (1.3) that $B = \{B_k, k \in \mathbb{N}_0\} \in \mathcal{F}$ is the inverse of A iff

$$(1.4) \qquad \delta_{0k} I_m = \sum_0^k B_j A_{k-j}.$$

The inversion property (1.4) imposes conditions on the choice of a group $\mathcal{G}$ in $\mathcal{F}$.

The convolution property (1.3) for $\mathcal{L}$ and $\mathcal{F}$, and the inversion property (1.4) for $\mathcal{G}$ are exactly the properties that allow careless formal mathematical operations with composition and inversion. Often these properties are only needed for linear subspaces $\mathcal{L}_0 \subset \mathcal{L}$ which are closed under composition themselves (i.e. $\mathcal{F}(\mathcal{L}_0) = \mathcal{L}_0$).

The approach in this paper is different from standard literature (see e.g. Anderson (1971), Hannan (1970) or Brockwell and Davis (1987) in this way that composition and inversion is not considered for a specific process and a specific filter but for whole spaces of processes and filters. This seems to be the appropriate way to justify formal operations.

2. The choice of $\mathcal{L}$, $\mathcal{F}$ and $\mathcal{G}$

It is attractive to make the linear spaces $\mathcal{L}$, $\mathcal{F}$ and the group $\mathcal{G}$ as large as possible. However, the need for convergence in (1.1) prevents this. If $\mathcal{F}$ is large then $\mathcal{L}$ will be small and conversely. Furthermore, the definition (1.1) rules out also classes $\mathcal{L}$ that are not invariant under linear filtering ($A(\mathcal{L}) \neq \mathcal{L}$ for some $A \in \mathcal{F}$), e.g. $\mathcal{L}$ cannot be the class of processes $X = \{X_t, \ t \in \mathbf{Z}\}$ with orthogonal or independent components X_t (except in trivial cases).

The choice $\mathcal{L}_w = \{X\colon X \text{ wide-stationary}\}$ is possible if we take for the corresponding class of filters the linear space

$$(2.1) \qquad \mathcal{F}_a = \{A\colon \sum_0^\infty \|A_k\| < \infty\}$$

The conditions (1.1) - (1.3) are satisfied for $\mathcal{L}_w$ and $\mathcal{F}_a$ with convergence in mean square or absolute convergence a.s as the type of convergence.

The space $\mathcal{L}_w$ can be extended to $\mathcal{L}_a$, defined by

$$(2.2) \qquad \mathcal{L}_a = \{X\colon E\|X_t\| < \infty \text{ for all } t \in \mathbf{Z},\ \limsup_{t \to -\infty} E\|X_t\| < \infty\}.$$

The conditions (1.1) - (1.3) can be verified again where the type of convergence is absolute convergence a.s. The proof is based on the fact that for absolute convergence a.s of series the convergence of the expectations of the absolute values is sufficient.

The class $\mathcal{L}_a$ contains not only wide-stationary processes but also non-stationary processes with bounded expectations of the components. In particular $\mathcal{L}_a$ contains bounded deterministic processes. A suitable large group $\mathcal{G}_a$ in $\mathcal{L}_a$ is given by

$$(2.3) \qquad \mathcal{G}_a = \{A \in \mathcal{F}_a : \det\{A(z)\} \neq 0 \text{ if } |z| \le 1,$$

$$\sum_0^\infty \|B_k\| < \infty \text{ where } A^{-1}(z) = \sum_0^\infty B_k\, z^k,\ |z| \le 1\}.$$

Here, $A(z) = \sum_0^\infty A_k\, z^k$ denotes the matrix generating function of A. For each $A \in \mathcal{F}_a$ we have for the convergence radius $\rho(A)$ (defined as the minimum of the convergence radii of the components) that $\rho(A) \le 1$. This guarantees the expansion of $A^{-1}(z) = \sum_0^\infty B_k\, z^k$ for $|z| < 1$, and therefore the definition of $\{B_k, k \in \mathbb{N}_0\}$ satisfying (1.4). However, the convergence of $\Sigma\|B_k\|$ is not guaranteed and has to be added in the definition of $\mathcal{G}_a$.

For many applications (e.g. AR, MA, ARMA, ARMAX) the class $\mathcal{F}_a$ is large, $\mathcal{L}_a$ is small and $\mathcal{G}_a$ is complicated. In such applications the filters are often finite or infinite with exponentially decreasing coefficients. On the other hand, the deterministic processes are often unbounded (e.g. ARMAX-models with a polynomial trend). This suggests to restrict the class $\mathcal{F}_a$ further to *exponentially decreasing filters* $A = \{A_k, k \in \mathbb{N}_0\}$ defined by

$$\exists \rho > 1: \rho^n A_n \to 0 \quad , \quad n \to \infty,$$

thereby allowing an extension to *non-exponentially increasing processes* $X = \{X_t, t \in \mathbb{Z}\}$ defined by

$$\forall \rho > 1: \rho^t X_t \to 0 \quad , \quad t \to -\infty.$$

Here the type of convergence has to be specified further on. We consider convergence a.s. and convergence in r^{th} mean. So we are led to the introduction of the linear space of filters $\mathcal{F}_e$, defined by

$$(2.4) \qquad \mathcal{F}_e = \{A: \rho^n A_n \to 0 \text{ if } n \to \infty \text{ for some } \rho > 1\}$$

and the linear space of processes $\mathcal{L}_r$, defined for $r \ge 0$ by

$$(2.5)\quad \begin{cases} \mathcal{L}_0 = \{X:\ \rho^t X_t \xrightarrow{a.s.} 0 \text{ if } t \to -\infty \text{ for all } \rho > 1\} \\ \mathcal{L}_r = \{X:\ E\|X_t\|^r < \infty \text{ for all } t \in Z, \\ \qquad\qquad \rho^t X_t \xrightarrow{r} 0 \text{ if } t \to -\infty \text{ for all } \rho > 1\},\ r > 0. \end{cases}$$

The notation in (2.5) is motivated by the property (see section 3 for the proof):

$$(2.6)\quad \mathcal{L}_s \subset \mathcal{L}_r \text{ for } 0 \le r \le s.$$

The group $\mathcal{G}_e \subset \mathcal{F}_e$, corresponding to (2.3), can be defined now in a simple way:

$$(2.7)\quad \mathcal{G}_e = \{A \in \mathcal{F}_e:\ \det\{A(z)\} \ne 0 \text{ if } |z| \le 1\},$$

since $A \in \mathcal{F}_e$ is equivalent to $\rho(A) > 1$.

<u>Theorem</u> ($r = 0$ or $r \ge 1$).
The linear spaces $\mathcal{F}_e$ and $\mathcal{L}_r$ satisfy the conditions (1.1) - (1.3), where the type of convergence is specified by r.
<u>Remark</u>. No conclusion is obtained for $0 < r < 1$.

This theorem is the main result of the paper. As long as we restrict inputprocesses to $\mathcal{L}_r$ are filters to $\mathcal{F}_e$ (and/or $\mathcal{L}_e$) the usual formal operations connected with composition and inversion are automatically satisfied. Outputprocesses can again be handled as inputprocesses without any restrictions. The proof of the theorem is contained in section 3.

<u>Example 1</u>.
Consider the homogeneous difference equation

$$\sum_0^\infty A_k\, Y_{t-k} = 0 \quad , \quad t \in \mathbf{Z}$$

and suppose that $A(z) = \Sigma A_k z^k$ has a radius of convergence larger than 1 with $\det\{A(z)\} \ne 0$, $|z| \le 1$. Then $A \in \mathcal{G}_e$ and so from $A(Y) = 0$ it follows that $Y = A^{-1}(0) = 0$ for any $Y \in \mathcal{F}_e$. Therefore the only non-exponentially increasing solution of this homogeneous equation is $Y_t = 0$ a.s., $t \in \mathbb{Z}$. This agrees with the standard result in the theory of finite linear difference equations.

Example 2.

Consider the ARMAX-model of the form

$$\sum_0^\infty A_g Y_{t-g} = B' X_t + \sum_0^\infty C_h \varepsilon_{t-h} \quad , \quad t \in \mathbb{Z}$$

with (ε_t) white noise (i.e. $E\{\varepsilon_t \varepsilon_s'\} = \delta_{ts} \Sigma$) and $A = \{A_g, g \in \mathbb{N}_0\}$, $C = \{C_h, h \in \mathbb{N}_0\}$ exponentially decreasing. If $X = (X_t)$ is mean-square non-exponentially increasing, then the right hand side of the equation belongs to $\mathcal{L}_2$. Therefore the equation determines uniquely a mean-square non-exponentially increasing proces $Y = (Y_t)$ provided that $A \in \mathcal{G}_e$ i.e. $\det\{A(z)\} \neq 0$, $|z| \leq 1$. The solution is given by

$$Y_t = \sum_0^\infty \Phi_h B' X_{t-h} + \sum_0^\infty \Psi_h \varepsilon_{t-h} \quad , \quad t \in \mathbb{Z},$$

where $\Phi(z) = A^{-1}(z)$, $\Psi(z) = A^{-1}(z)\, C(z)$.

Note that we have fixed the ARMAX-model (the solution Y) by the qualitative restriction $Y \in \mathcal{L}_e$. This forms a nice alternative to the usual quantitative specification of starting conditions.

Example 3.

Let $C(\delta)$ for $\delta \geq 0$ stand for the centered Cauchy-distribution with characteristic function $\exp\{-\delta|u|\}$, $u \in \mathbb{R}$. Consider the (univariate) class $\mathcal{C}_0$ of strict-stationary centered Cauchy-processes $X = \{X_t, t \in Z\}$, i.e. for some $\delta \geq 0$ we have $X_t \sim C(\delta)$, $t \in \mathbb{Z}$ and for any $m \geq 1$ and $t_1, \ldots, t_m \in \mathbb{Z}$ any linear combination of $X_{t_1}, \ldots, X_{t_m}$ has the distribution $C(\delta')$ for some $\delta' \geq 0$. (see e.g. Ferguson (1962) or Johnson and Kotz (1972)). Then $\mathcal{C}_0 \subset \mathcal{L}_0$ since for any $\rho > 1$:

$$\sum_{-\infty}^{0} P\{|\rho^t X_t| \geq \varepsilon\} \leq (2\delta/\pi) \sum_0^\infty \rho^{-t} < \infty,$$

implying $\rho^t X_t \to 0$ a.s., $t \to -\infty$. So we can use the theorem for $r = 0$ to conclude that $\mathcal{F}_e(\mathcal{C}_0) \subset \mathcal{L}_0$. In fact we have $\mathcal{F}_e(\mathcal{C}_0) = \mathcal{C}_0$ since for any $A \in \mathcal{F}_e$ and $X \in \mathcal{C}_0$ the output $Y = A(X)$ is also a strict-stationary centered Cauchy-process, i.e. $Y \in \mathcal{C}_0$.

This example shows the applicability of the theorem in time series for components with heavy tails. There is no need for the expectation to exist.

3. Proofs

Proof of (2.6)

Let $X \in \mathcal{L}_s$ with $s > 0$. We have to proof that $X \in \mathcal{L}_r$ for $0 \le r < s$. For $r > 0$ this follows from the fact that $\{E\|\rho^t X_t\|^r\}^{1/r}$ is non-decreasing for increasing r. For $r = 0$ we note that $X_t \in \mathcal{L}_s$ is equivalent to

$$\sum_{-\infty}^{0} \rho_1^t \, E\|X_t\|^s < \infty \text{ for all } \rho_1 > 1.$$

Then, for any $\rho > 1$ we get for $\rho_1 = \rho^s$ that

$$\sum_{-\infty}^{0} P\{\rho^t \|X_t\| \ge \varepsilon\} \le \varepsilon^{-s} \sum_{-\infty}^{0} \rho^{ts} \, E\|X_t\|^s < \infty.$$

This implies $\rho^t X_t \xrightarrow{\text{a.s.}} 0$, $t \to -\infty$.

Proof of the theorem

At first, we prove that Y in (1.1) is well-defined according to the type of convergence specified by r. We use

$$\sum_{n}^{m} \|A_k X_{t-k}\| \le \rho^{-t} \sum_{n}^{m} \|A_k \rho^k\| \, \|\rho^{t-k} X_{t-k}\|.$$

Take $\rho \in (1, \rho(A))$. For $r = 0$ this gives

$$\sum_{n}^{m} \|A_k X_{t-k}\| \le \rho^{-t} \sup_{s \le t} \rho^s \|X_s\| . \sum_{n}^{\infty} \|A_k \rho^k\| \to 0, \; n \to \infty.$$

For r = 1 we have

$$\sum_n^m \|A_k\, X_{t-k}\| \le \rho^{-t} \sup_{k\ge n}\|A_k\, \rho^k\| . \sum_n^m \rho^{t-k}\|X_{t-k}\|,$$

implying

$$E\{\sum_n^m \|A_k\, X_{t-k}\|\} \le \rho^{-t} \sup_{k\ge n}\|A_k\, \rho^k\| . \sum_n^\infty \rho^{t-k}\, E\|X_{t-k}\| \to 0.$$

Finally, for r > 1 with $\rho > 1$ determined by $1/\rho + 1/r = 1$ we have

$$\sum_n^m \|A_k\, X_{t-k}\| \le \rho^{-t}(\sum_n^m \|A_k\, \rho^k\|)^{1/r}\, (\sum_n^m \|\rho^{t-k}\, X_{t-k}\|^r)^{1/r},$$

implying

$$E\|\sum_n^m A_k\, X_{t-k}\|^r \le \rho^{-rt}\, (\sum_n \|A_k\, \rho^k\|)^{r/p} . \sum_n^\infty E\|\rho^{t-k}\, X_{t-k}\|^r \to 0,\ n \to \infty.$$

Now we prove that $Y \in Z$ i.e. that Y is non-exponentially increasing. Take $\rho > 1$. For r = 0 we get for $1 < \rho_0 < \min(\rho, \rho(A))$:

$$\|\rho^t\, Y_t\| = \|\sum_0^\infty (A_k\, \rho_o{}^k)(\rho_o^{t-k}\, X_{t-k})(\rho/\rho_0)^t\|$$

$$\le \sum_0^\infty \|A_k\|\rho_0{}^k . \sup_{j\le 0} \rho_0{}^j\|X_j\| . (\rho/\rho_0)^t \to 0,\ t \to -\infty.$$

For $r \ge 1$ the proof follows the same way.
The space $\mathcal{L}_r$ is large enough to guarantee (1.2). Hence, it remains to prove (1.3). A formal substitution gives

$$Z_t = \sum_{j=0}^\infty B_j(\sum_{k=0}^\infty A_k\, X_{t-j-k}) = \sum_{j=0}^\infty (\sum_{k=j}^\infty B_j\, A_{k-j})X_{t-k} =$$

$$= \sum_{k=0}^\infty (\sum_{j=0}^k B_j\, A_{k-j})X_{t-k} = \sum_{k=0}^\infty C_k\, X_{t-k}.$$

For $r = 0$ we have proved (1.3) if we can justify the reversal of summation. However, this follows with $\rho \in (1, \min(\rho(A), \rho(B))$ from:

$$\sum_{j=0}^{\infty} (\sum_{k=j}^{\infty} \|B_j\| \|A_{k-j}\| \|X_{t-k}) \le \rho^{-t} \sum_{j=0}^{\infty} (\sum_{k=j}^{\infty} \|B_j\ A_{k-j}\ \rho^{k}\|). \sup_{k\le 0} \|\rho^{t-k}\ X_{t-k}\|$$

$$= \rho^{-t} (\sum_{j=0}^{\infty} \|B_j\| \rho^{j}) (\sum_{k=0}^{\infty} A_k\ \rho^{k}). \sup_{k\le 0} \|\rho^{t-k}\ X_{t-k}\| < \infty.$$

For $r \ge 1$ we get from (2.6) that Z is well-defined. Then (1.1) gives that $Z \in \mathcal{L}_r$. This completes the proof.

References

Anderson, T.W. (1971) - The Statistical Analysis of Time Series - Wiley, New York.

Brockwell, P.J. and Davis, R.A. (1987) - Time Series: Theory and Methods - Springer Verlag, Heidelberg.

Ferguson, T.S. (1962) - A representation of the symmetric bivariate Cauchy distribution - Ann. Math. Statist. 33, 1256-1266.

Johnson, N.L. and Kotz, S. (1972) - Distributions in Statistics: Continuous Multivariate Distributions - Wiley, New York.

Hannan, E.J. (1970) - Multiple Time Series - Wiley, New York.

IN 1987 REEDS VERSCHENEN

242 Gerard van den Berg
Nonstationarity in job search theory

243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions

244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk

245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation
of subjective poverty definitions

246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance Criteria

247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en strukturele afhankelijkheden

248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group

249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic Investment Policy

250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical approach

251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically distributed errors

252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten

253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen

254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint products: A comment

255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?

256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium concepts

257 P.H.M. Ruys, G. van der Laan
Computation of an industrial equilibrium

258 W.H. Haemers, A.E. Brouwer
Association schemes

259 G.J.M. van den Boom
Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining

260 A.W.A. Boot, A.V. Thakor, G.F. Udell
Competition, Risk Neutrality and Loan Commitments

261 A.W.A. Boot, A.V. Thakor, G.F. Udell
Collateral and Borrower Risk

262 A. Kapteyn, I. Woittiez
Preference Interdependence and Habit Formation in Family Labor Supply

263 B. Bettonvil
A formal description of discrete event dynamic systems including perturbation analysis

264 Sylvester C.W. Eijffinger
A monthly model for the monetary policy in the Netherlands

265 F. van der Ploeg, A.J. de Zeeuw
Conflict over arms accumulation in market and command economies

266 F. van der Ploeg, A.J. de Zeeuw
Perfect equilibrium in a model of competitive arms accumulation

267 Aart de Zeeuw
Inflation and reputation: comment

268 A.J. de Zeeuw, F. van der Ploeg
Difference games and policy evaluation: a conceptual framework

269 Frederick van der Ploeg
Rationing in open economy and dynamic macroeconomics: a survey

270 G. van der Laan and A.J.J. Talman
Computing economic equilibria by variable dimension algorithms: state of the art

271 C.A.J.M. Dirven and A.J.J. Talman
A simplicial algorithm for finding equilibria in economies with linear production technologies

272 Th.E. Nijman and F.C. Palm
Consistent estimation of regression models with incompletely observed exogenous variables

273 Th.E. Nijman and F.C. Palm
Predictive accuracy gain from disaggregate sampling in arima - models

274 Raymond H.J.M. Gradus
The net present value of governmental policy: a possible way to find the Stackelberg solutions

275 Jack P.C. Kleijnen
A DSS for production planning: a case study including simulation and optimization

276 A.M.H. Gerards
A short proof of Tutte's characterization of totally unimodular matrices

277 Th. van de Klundert and F. van der Ploeg
Wage rigidity and capital mobility in an optimizing model of a small open economy

278 Peter M. Kort
The net present value in dynamic models of the firm

279 Th. van de Klundert
A Macroeconomic Two-Country Model with Price-Discriminating Monopolists

280 Arnoud Boot and Anjan V. Thakor
Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

281 Arnoud Boot and Anjan V. Thakor
Appendix: "Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

282 Arnoud Boot, Anjan V. Thakor and Gregory F. Udell
Credible commitments, contract enforcement problems and banks: intermediation as credibility assurance

283 Eduard Ponds
Wage bargaining and business cycles a Goodwin-Nash model

284 Prof.Dr. hab. Stefan Mynarski
The mechanism of restoring equilibrium and stability in polish market

285 P. Meulendijks
An exercise in welfare economics (II)

286 S. Jørgensen, P.M. Kort, G.J.C.Th. van Schijndel
Optimal investment, financing and dividends: a Stackelberg differential game

287 E. Nijssen, W. Reijnders
Privatisering en commercialisering; een oriëntatie ten aanzien van verzelfstandiging

288 C.B. Mulder
Inefficiency of automatically linking unemployment benefits to private sector wage rates

289 M.H.C. Paardekooper
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues

290 Pieter H.M. Ruys
Industries with private and public enterprises

291 J.J.A. Moors & J.C. van Houwelingen
Estimation of linear models with inequality restrictions

292 Arthur van Soest, Peter Kooreman
Vakantiebestemming en -bestedingen

293 Rob Alessie, Raymond Gradus, Bertrand Melenberg
The problem of not observing small expenditures in a consumer expenditure survey

294 F. Boekema, L. Oerlemans, A.J. Hendriks
Kansrijkheid en economische potentie: Top-down en bottom-up analyses

295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note

296 Arthur van Soest, Peter Kooreman
Estimation of the indirect translog demand system with binding non-negativity constraints

IN 1988 REEDS VERSCHENEN

297 Bert Bettonvil
Factor screening by sequential bifurcation

298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure

299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry

300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method

301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note

302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in lineair models using cross sections, panels or both

303 Raymond H.J.M. Gradus
A differential game between government and firms: a non-cooperative approach

304 Leo W.G. Strijbosch, Ronald J.M.M. Does
Comparison of bias-reducing methods for estimating the parameter in dilution series

305 Drs. W.J. Reijnders, Drs. W.F. Verstappen
Strategische bespiegelingen betreffende het Nederlandse kwaliteitsconcept

306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg
Regression sampling in statistical auditing

307 Isolde Woittiez, Arie Kapteyn
A Model of Job Choice, Labour Supply and Wages

308 Jack P.C. Kleijnen
Simulation and optimization in production planning: A case study

309 Robert P. Gilles and Pieter H.M. Ruys
Relational constraints in coalition formation

310 Drs. H. Leo Theuns
Determinanten van de vraag naar vakantiereizen: een verkenning van materiële en immateriële factoren

311 Peter M. Kort
Dynamic Firm Behaviour within an Uncertain Environment

312 J.P.C. Blanc
A numerical approach to cyclic-service queueing models