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LINEAR TIME-INVARIANT FILTERS OF  
INFINITE ORDER FOR NON-STATIONARY  
PROCESSES

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LINEAR TIME-INVARIANT FILTERS  
OF INFINITE ORDER  
FOR NON-STATIONARY PROCESSES

by

B.B. van der Genugten

Abstract

In dynamic linear models linear time-invariant filters of infinite order play an important rôle. E.g. an invertible AR of finite order corresponds to an MA of infinite order but not to one of finite order. To justify the formal operations of composition and inversion of filters regularity conditions concerning convergence have to be checked. This paper discusses the construction of simple classes of non-stationary processes and linear filters of infinite order that are closed under composition and inversion. For such classes the necessary regularity conditions are automatically satisfied.

Concepts as "exponentially decreasing filters" and "non-exponentially increasing processes" come up in a natural way. Some examples illustrate the applications of the concepts.

## 1. Introduction

Let  $\mathcal{L}$  be a class of  $m$ -vectorial stochastic processes  $X = \{X_t, t \in \mathbb{Z}\}$  on the set of integers  $\mathbb{Z}$  and  $\mathcal{F}$  a class of filters mapping  $\mathcal{L}$  into  $\mathcal{L}$ . So for each filter  $A \in \mathcal{F}$  the input  $X \in \mathcal{L}$  gives the output  $Y = \{Y_t, t \in \mathbb{Z}\} = A(X) \in \mathcal{L}$ , or shortly:  $A(\mathcal{L}) \subset \mathcal{L}$ ,  $A \in \mathcal{L}$ . Hence  $\mathcal{F}(\mathcal{L}) \subset \mathcal{L}$ .

Let  $A, B \in \mathcal{L}$  and  $X \in \mathcal{L}$ . Then  $Z = B(Y)$  with  $Y = A(X)$  can be written as  $Z = C(X)$  with  $C = B \circ A$ , the *composition* of  $A$  and  $B$ . So the definition of  $\mathcal{F}$  implies that  $\mathcal{F}$  is *closed* under composition.

Let  $\mathcal{G} \subset \mathcal{F}$  be a group with the composition as group-operation. Then by definition the identity  $I$  belongs to  $\mathcal{G}$  and for each  $A \in \mathcal{G}$  there exists a unique  $A^{-1} \in \mathcal{G}$  such that  $A^{-1} \circ A = I$ , the *inverse*  $A^{-1}$  of  $A$ . The existence of such an  $\mathcal{G}$  implies  $\mathcal{F}(\mathcal{L}) = \mathcal{L}$  (since  $I(\mathcal{L}) = \mathcal{L}$ ).

In this paper we look for suitable choices of  $\mathcal{L}$ ,  $\mathcal{F}$  and  $\mathcal{G}$  such that all processes are *one-sided moving averages* of infinite order allowing simple mathematical operations for composition and inversion. We will formulate this more precisely.

In  $\mathcal{L}$  we do not distinguish between processes who only differ on a set of probability zero ( $X = X'$  a.s.  $\Rightarrow X = X'$ ). For  $\mathcal{F}$  we suppose that each  $A \in \mathcal{F}$  can be represented uniquely on  $\mathcal{L}$  by a sequence  $\{A_k, k \in \mathbb{N}_0\}$  of  $m \times m$ -matrices or shortly:  $A = \{A_k, k \in \mathbb{N}_0\}$ . Uniqueness refers to

$$(1.2) \quad A(X) = B(X) \text{ for all } X \in \mathcal{L} \Leftrightarrow A_k = B_k \text{ for all } k \in \mathbb{N}_0.$$

For each  $A = \{A_k, k \in \mathbb{N}_0\} \in \mathcal{F}$  and each input  $X = \{X_t, t \in \mathbb{Z}\} \in \mathcal{L}$  the corresponding output  $Y = \{Y_t, t \in \mathbb{Z}\} \in \mathcal{L}$  is given by

$$(1.1) \quad Y_t = \sum_{k=0}^{\infty} A_k X_{t-k}, \quad t \in \mathbb{Z}.$$

Here it is assumed that the type of convergence (in probability,  $r^{\text{th}}$  mean, a.s.) is the same for all  $A \in \mathcal{F}$  and  $X \in \mathcal{L}$ . It follows that the identity  $I \in \mathcal{L}$  is represented by  $I = \{\delta_{0k} I_m, k \in \mathbb{N}_0\}$ , where  $I_m$  denotes the  $m \times m$ -matrix. We suppose that  $\mathcal{L}$  is a linear space (with the usual componentwise

definition of addition and scalar multiplication of stochastic processes). Then  $A = \{A_k, k \in \mathbb{N}_0\} \in \mathcal{F}$  is a linear filter. We assume also that  $\mathcal{F}$  is a linear space. Then the nullfunction  $0 \in \mathcal{F}$  is represented by  $\{0, k \in \mathbb{N}_0\}$ .

The output  $Z = \{Z_t, t \in \mathbb{Z}\}$  of the filter  $B = \{B_k, k \in \mathbb{N}_0\} \in \mathcal{F}$  of  $y = y_t, t \in \mathbb{Z}$ , given by

$$Z_t = \sum_0^{\infty} B_k Y_{t-k}, \quad t \in \mathbb{Z},$$

can be written as  $Z = B(Y) = B(A(X)) = C(X)$  with  $C = B \circ A \in \mathcal{F}$  the composition of  $A$  and  $B$ . So we can write

$$Z_t = \sum_0^{\infty} C_k X_{t-k}, \quad t \in \mathbb{Z},$$

where the composition is represented by  $C = \{C_k, k \in \mathbb{N}_0\}$ . A formal substitution makes the following *convolution* property desirable:

$$(1.3) \quad A, B \in \mathcal{F}: C = B \circ A \Leftrightarrow C_k = \sum_0^k B_j A_{k-j}, \quad k \in \mathbb{N}_0.$$

The inversion property (1.3) imposes conditions on the linear spaces  $\mathcal{L}$  and  $\mathcal{F}$ .

Let  $A = \{A_k, k \in \mathbb{N}_0\} \in \mathcal{G}$ . It follows from (1.3) that  $B = \{B_k, k \in \mathbb{N}_0\} \in \mathcal{F}$  is the inverse of  $A$  iff

$$(1.4) \quad \delta_{0k} I_m = \sum_0^k B_j A_{k-j}.$$

The inversion property (1.4) imposes conditions on the choice of a group  $\mathcal{G}$  in  $\mathcal{F}$ .

The convolution property (1.3) for  $\mathcal{L}$  and  $\mathcal{F}$ , and the inversion property (1.4) for  $\mathcal{G}$  are exactly the properties that allow careless formal mathematical operations with composition and inversion. Often these properties are only needed for linear subspaces  $\mathcal{L}_0 \subset \mathcal{L}$  which are closed under composition themselves (i.e.  $\mathcal{F}(\mathcal{L}_0) = \mathcal{L}_0$ ).

The approach in this paper is different from standard literature (see e.g. Anderson (1971), Hannan (1970) or Brockwell and Davis (1987) in



this way that composition and inversion is not considered for a specific process and a specific filter but for whole spaces of processes and filters. This seems to be the appropriate way to justify formal operations.

## 2. The choice of $\mathcal{L}$ , $\mathcal{F}$ and $\mathcal{G}$

It is attractive to make the linear spaces  $\mathcal{L}$ ,  $\mathcal{F}$  and the group  $\mathcal{G}$  as large as possible. However, the need for convergence in (1.1) prevents this. If  $\mathcal{F}$  is large then  $\mathcal{L}$  will be small and conversely. Furthermore, the definition (1.1) rules out also classes  $\mathcal{L}$  that are not invariant under linear filtering ( $A(\mathcal{L}) \neq \mathcal{L}$  for some  $A \in \mathcal{F}$ ), e.g.  $\mathcal{L}$  cannot be the class of processes  $X = \{X_t, t \in \mathbb{Z}\}$  with orthogonal or independent components  $X_t$  (except in trivial cases).

The choice  $\mathcal{L}_w = \{X: X \text{ wide-stationary}\}$  is possible if we take for the corresponding class of filters the linear space

$$(2.1) \quad \mathcal{F}_a = \{A: \sum_0^\infty \|A_k\| < \infty\}$$

The conditions (1.1) - (1.3) are satisfied for  $\mathcal{L}_w$  and  $\mathcal{F}_a$  with convergence in mean square or absolute convergence a.s as the type of convergence.

The space  $\mathcal{L}_w$  can be extended to  $\mathcal{L}_a$ , defined by

$$(2.2) \quad \mathcal{L}_a = \{X: E\|X_t\| < \infty \text{ for all } t \in \mathbb{Z}, \limsup_{t \rightarrow -\infty} E\|X_t\| < \infty\}.$$

The conditions (1.1) - (1.3) can be verified again where the type of convergence is absolute convergence a.s. The proof is based on the fact that for absolute convergence a.s of series the convergence of the expectations of the absolute values is sufficient.

The class  $\mathcal{L}_a$  contains not only wide-stationary processes but also non-stationary processes with bounded expectations of the components. In particular  $\mathcal{L}_a$  contains bounded deterministic processes. A suitable large group  $\mathcal{G}_a$  in  $\mathcal{L}_a$  is given by

$$(2.3) \quad \mathcal{G}_a = \{A \in \mathcal{F}_a : \det\{A(z)\} \neq 0 \text{ if } |z| \leq 1,$$

$$\sum_0^\infty \|B_k\| < \infty \text{ where } A^{-1}(z) = \sum_0^\infty B_k z^k, |z| \leq 1\}.$$

Here,  $A(z) = \sum_0^\infty A_k z^k$  denotes the matrix generating function of  $A$ . For each  $A \in \mathcal{F}_a$  we have for the convergence radius  $\rho(A)$  (defined as the minimum of the convergence radii of the components) that  $\rho(A) \leq 1$ . This guarantees the expansion of  $A^{-1}(z) = \sum_0^\infty B_k z^k$  for  $|z| < 1$ , and therefore the definition of  $\{B_k, k \in \mathbb{N}_0\}$  satisfying (1.4). However, the convergence of  $\sum \|B_k\|$  is not guaranteed and has to be added in the definition of  $\mathcal{G}_a$ .

For many applications (e.g. AR, MA, ARMA, ARMAX) the class  $\mathcal{F}_a$  is large,  $\mathcal{L}_a$  is small and  $\mathcal{G}_a$  is complicated. In such applications the filters are often finite or infinite with exponentially decreasing coefficients. On the other hand, the deterministic processes are often unbounded (e.g. ARMAX-models with a polynomial trend). This suggests to restrict the class  $\mathcal{F}_a$  further to *exponentially decreasing filters*  $A = \{A_k, k \in \mathbb{N}_0\}$  defined by

$$\exists \rho > 1: \rho^n A_n \rightarrow 0, \quad n \rightarrow \infty,$$

thereby allowing an extension to *non-exponentially increasing processes*  $X = \{X_t, t \in \mathbb{Z}\}$  defined by

$$\forall \rho > 1: \rho^t X_t \rightarrow 0, \quad t \rightarrow -\infty.$$

Here the type of convergence has to be specified further on. We consider convergence a.s. and convergence in  $r^{\text{th}}$  mean. So we are led to the introduction of the linear space of filters  $\mathcal{F}_e$ , defined by

$$(2.4) \quad \mathcal{F}_e = \{A: \rho^n A_n \rightarrow 0 \text{ if } n \rightarrow \infty \text{ for some } \rho > 1\}$$

and the linear space of processes  $\mathcal{L}_r$ , defined for  $r \geq 0$  by



$$(2.5) \quad \begin{cases} \mathcal{L}_0 = \{X: \rho^t X_t \xrightarrow{\text{a.s.}} 0 \text{ if } t \rightarrow -\infty \text{ for all } \rho > 1\} \\ \mathcal{L}_r = \{X: E\|X_t\|^r < \infty \text{ for all } t \in Z, \\ \rho^t X_t \xrightarrow{r} 0 \text{ if } t \rightarrow -\infty \text{ for all } \rho > 1\}, r > 0. \end{cases}$$

The notation in (2.5) is motivated by the property (see section 3 for the proof):

$$(2.6) \quad \mathcal{L}_s \subset \mathcal{L}_r \text{ for } 0 \leq r \leq s.$$

The group  $\mathcal{G}_e \subset \mathcal{F}_e$ , corresponding to (2.3), can be defined now in a simple way:

$$(2.7) \quad \mathcal{G}_e = \{A \in \mathcal{F}_e: \det\{A(z)\} \neq 0 \text{ if } |z| \leq 1\},$$

since  $A \in \mathcal{F}_e$  is equivalent to  $\rho(A) > 1$ .

Theorem ( $r = 0$  or  $r \geq 1$ ).

The linear spaces  $\mathcal{F}_e$  and  $\mathcal{L}_r$  satisfy the conditions (1.1) - (1.3), where the type of convergence is specified by  $r$ .

Remark. No conclusion is obtained for  $0 < r < 1$ .

This theorem is the main result of the paper. As long as we restrict inputprocesses to  $\mathcal{L}_r$  are filters to  $\mathcal{F}_e$  (and/or  $\mathcal{L}_e$ ) the usual formal operations connected with composition and inversion are automatically satisfied. Outputprocesses can again be handled as inputprocesses without any restrictions. The proof of the theorem is contained in section 3.

Example 1.

Consider the homogeneous difference equation

$$\sum_0^{\infty} A_k Y_{t-k} = 0, \quad t \in Z$$

and suppose that  $A(z) = \sum A_k z^k$  has a radius of convergence larger than 1 with  $\det\{A(z)\} \neq 0, |z| \leq 1$ . Then  $A \in \mathcal{G}_e$  and so from  $A(Y) = 0$  it follows that  $Y = A^{-1}(0) = 0$  for any  $Y \in \mathcal{F}_e$ . Therefore the only non-exponentially

increasing solution of this homogeneous equation is  $Y_t = 0$  a.s.,  $t \in \mathbb{Z}$ . This agrees with the standard result in the theory of finite linear difference equations.

Example 2.

Consider the ARMAX-model of the form

$$\sum_0^{\infty} A_g Y_{t-g} = B' X_t + \sum_0^{\infty} C_h \epsilon_{t-h}, \quad t \in \mathbb{Z}$$

with  $(\epsilon_t)$  white noise (i.e.  $E\{\epsilon_t \epsilon_s'\} = \delta_{ts} \Sigma$ ) and  $A = \{A_g, g \in \mathbb{N}_0\}$ ,  $C = \{C_h, h \in \mathbb{N}_0\}$  exponentially decreasing. If  $X = (X_t)$  is mean-square non-exponentially increasing, then the right hand side of the equation belongs to  $\mathcal{L}_2$ . Therefore the equation determines uniquely a mean-square non-exponentially increasing process  $Y = (Y_t)$  provided that  $A \in \mathcal{G}_e$  i.e.  $\det\{A(z)\} \neq 0$ ,  $|z| \leq 1$ . The solution is given by

$$Y_t = \sum_0^{\infty} \Phi_h B' X_{t-h} + \sum_0^{\infty} \Psi_h \epsilon_{t-h}, \quad t \in \mathbb{Z},$$

where  $\Phi(z) = A^{-1}(z)$ ,  $\Psi(z) = A^{-1}(z) C(z)$ .

Note that we have fixed the ARMAX-model (the solution  $Y$ ) by the qualitative restriction  $Y \in \mathcal{L}_e$ . This forms a nice alternative to the usual quantitative specification of starting conditions.

Example 3.

Let  $C(\delta)$  for  $\delta \geq 0$  stand for the centered Cauchy-distribution with characteristic function  $\exp\{-\delta|u|\}$ ,  $u \in \mathbb{R}$ . Consider the (univariate) class  $\mathcal{C}_0$  of strict-stationary centered Cauchy-processes  $X = \{X_t, t \in \mathbb{Z}\}$ , i.e. for some  $\delta \geq 0$  we have  $X_t \sim C(\delta)$ ,  $t \in \mathbb{Z}$  and for any  $m \geq 1$  and  $t_1, \dots, t_m \in \mathbb{Z}$  any linear combination of  $X_{t_1}, \dots, X_{t_m}$  has the distribution  $C(\delta')$  for some  $\delta' \geq 0$ . (see e.g. Ferguson (1962) or Johnson and Kotz (1972)). Then  $\mathcal{C}_0 \subset \mathcal{L}_0$  since for any  $\rho > 1$ :

$$\sum_{-\infty}^0 P\{|\rho^t X_t| \geq \epsilon\} \leq (2\delta/\pi) \sum_0^{\infty} \rho^{-t} < \infty,$$

implying  $\rho^t X_t \rightarrow 0$  a.s.,  $t \rightarrow -\infty$ . So we can use the theorem for  $r = 0$  to conclude that  $\mathcal{F}_e(\mathcal{C}_0) \subset \mathcal{L}_0$ . In fact we have  $\mathcal{F}_e(\mathcal{C}_0) = \mathcal{C}_0$  since for any  $A \in \mathcal{F}_e$  and  $X \in \mathcal{C}_0$  the output  $Y = A(X)$  is also a strict-stationary centered Cauchy-process, i.e.  $Y \in \mathcal{C}_0$ .

This example shows the applicability of the theorem in time series for components with heavy tails. There is no need for the expectation to exist.

### 3. Proofs

#### Proof of (2.6)

Let  $X \in \mathcal{L}_s$  with  $s > 0$ . We have to proof that  $X \in \mathcal{L}_r$  for  $0 \leq r < s$ . For  $r > 0$  this follows from the fact that  $\{E\|\rho^t X_t\|^r\}^{1/r}$  is non-decreasing for increasing  $r$ . For  $r = 0$  we note that  $X_t \in \mathcal{L}_s$  is equivalent to

$$\sum_{-\infty}^0 \rho_1^t E\|X_t\|^s < \infty \text{ for all } \rho_1 > 1.$$

Then, for any  $\rho > 1$  we get for  $\rho_1 = \rho^s$  that

$$\sum_{-\infty}^0 P\{\rho^t \|X_t\| \geq \varepsilon\} \leq \varepsilon^{-s} \sum_{-\infty}^0 \rho^{ts} E\|X_t\|^s < \infty.$$

This implies  $\rho^t X_t \xrightarrow{\text{a.s.}} 0$ ,  $t \rightarrow -\infty$ .

#### Proof of the theorem

At first, we prove that  $Y$  in (1.1) is well-defined according to the type of convergence specified by  $r$ . We use

$$\sum_n^m \|A_k X_{t-k}\| \leq \rho^{-t} \sum_n^m \|A_k \rho^k\| \|\rho^{t-k} X_{t-k}\|.$$

Take  $\rho \in (1, \rho(A))$ . For  $r = 0$  this gives

$$\sum_n^m \|A_k X_{t-k}\| \leq \rho^{-t} \sup_{s \leq t} \rho^s \|X_s\| \cdot \sum_n^{\infty} \|A_k \rho^k\| \rightarrow 0, n \rightarrow \infty.$$

For  $r = 1$  we have

$$\sum_n^m \|A_k X_{t-k}\| \leq \rho^{-t} \sup_{k \geq n} \|A_k \rho^k\| \cdot \sum_n^m \rho^{t-k} \|X_{t-k}\|,$$

implying

$$E\left\{\sum_n^m \|A_k X_{t-k}\|\right\} \leq \rho^{-t} \sup_{k \geq n} \|A_k \rho^k\| \cdot \sum_n^m \rho^{t-k} E\|X_{t-k}\| \rightarrow 0.$$

Finally, for  $r > 1$  with  $\rho > 1$  determined by  $1/\rho + 1/r = 1$  we have

$$\sum_n^m \|A_k X_{t-k}\| \leq \rho^{-t} \left(\sum_n^m \|A_k \rho^k\|\right)^{1/r} \left(\sum_n^m \rho^{t-k} \|X_{t-k}\|^r\right)^{1/r},$$

implying

$$E\left\|\sum_n^m A_k X_{t-k}\right\|^r \leq \rho^{-rt} \left(\sum_n^m \|A_k \rho^k\|\right)^{r/p} \cdot \sum_n^m E\|\rho^{t-k} X_{t-k}\|^r \rightarrow 0, \quad n \rightarrow \infty.$$

Now we prove that  $Y \in Z$  i.e. that  $Y$  is non-exponentially increasing. Take  $\rho > 1$ . For  $r = 0$  we get for  $1 < \rho_0 < \min(\rho, \rho(A))$ :

$$\begin{aligned} \|\rho^t Y_t\| &= \left\| \sum_0^\infty (A_k \rho_0^k) (\rho_0^{t-k} X_{t-k}) (\rho/\rho_0)^t \right\| \\ &\leq \sum_0^\infty \|A_k\| \rho_0^k \cdot \sup_{j \leq 0} \rho_0^j \|X_j\| \cdot (\rho/\rho_0)^t \rightarrow 0, \quad t \rightarrow -\infty. \end{aligned}$$

For  $r \geq 1$  the proof follows the same way.

The space  $\mathcal{L}_r$  is large enough to guarantee (1.2). Hence, it remains to prove (1.3). A formal substitution gives

$$\begin{aligned} Z_t &= \sum_{j=0}^\infty B_j \left( \sum_{k=0}^\infty A_k X_{t-j-k} \right) = \sum_{j=0}^\infty \left( \sum_{k=j}^\infty B_j A_{k-j} \right) X_{t-k} = \\ &= \sum_{k=0}^\infty \left( \sum_{j=0}^k B_j A_{k-j} \right) X_{t-k} = \sum_{k=0}^\infty C_k X_{t-k}. \end{aligned}$$

For  $r = 0$  we have proved (1.3) if we can justify the reversal of summation. However, this follows with  $\rho \in (1, \min(\rho(A), \rho(B)))$  from:

$$\begin{aligned} \sum_{j=0}^{\infty} \left( \sum_{k=j}^{\infty} \|B_j\| \|A_{k-j}\| \|X_{t-k}\| \right) &\leq \rho^{-t} \sum_{j=0}^{\infty} \left( \sum_{k=j}^{\infty} \|B_j\| \|A_{k-j}\| \rho^k \right) \cdot \sup_{k \leq 0} \|e^{t-k} X_{t-k}\| \\ &= \rho^{-t} \left( \sum_{j=0}^{\infty} \|B_j\| \rho^j \right) \left( \sum_{k=0}^{\infty} \|A_k\| \rho^k \right) \cdot \sup_{k \leq 0} \|e^{t-k} X_{t-k}\| < \infty. \end{aligned}$$

For  $r \geq 1$  we get from (2.6) that  $Z$  is well-defined. Then (1.1) gives that  $Z \in \mathcal{L}_r$ . This completes the proof.

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