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Block-tridiagonal linear systems and branched continued fractions

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Block-tridiagonal linear systems and branched continued fractions.

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Abstract.

The convergent of an ordinary continued fraction can be computed by solving a tridiagonal linear system for its first unknown. In this paper this approach is generalized to branched continued fractions and it is shown how the convergent of a branched continued fraction can be considered as the first unknown of a block-tridiagonal linear system. Hence algorithms for the solution of such systems of equations can be used for the computation of convergents of branched continued fractions, which have applications in approximation theory, systems theory, ... In future research special attention will be paid to the use of parallel algorithms.

Block-tridiagonal linear systems and branched continued fractions.

In the case of ordinary continued fractions

$$B_i = b_0^{(i)} + \frac{a_1^{(i)}|}{|b_1^{(i)}} + \frac{a_2^{(i)}|}{|b_2^{(i)}} + \dots \qquad i = 0, 1, 2, \dots \tag{1}$$

forward evaluation of and determinant formulas for

$$C_n^{(i)} = b_0^{(i)} + \sum_{j=1}^{n} \frac{a_j^{(i)}|}{|b_j^{(i)}}$$

are well-known. If we denote $C_n^{(i)} = P_n^{(i)}/Q_n^{(i)}$ then $P_n^{(i)}$ and $Q_n^{(i)}$ can be computed by the following three-term recurrence relation [5]

$$\begin{cases} P_k^{(i)} = b_k^{(i)} P_{k-1}^{(i)} + a_k^{(i)} P_{k-2}^{(i)} \\ Q_k^{(i)} = b_k^{(i)} Q_{k-1}^{(i)} + a_k^{(i)} Q_{k-2}^{(i)} \end{cases} \qquad k = 1, \dots, n \tag{2}$$

with $P_{-1}^{(i)} = 1 = Q_0^{(i)}, P_0^{(i)} = b_0^{(i)}$ and $Q_{-1}^{(i)} = 0$. Using this three-term recurrence relation one can prove that $P_n^{(i)}$ and $Q_n^{(i)}$ are also given by the following determinant formulas [4]

$$P_n^{(i)} = \begin{vmatrix} b_0^{(i)} & -1 & & & \\ a_1^{(i)} & b_1^{(i)} & -1 & & \\ & a_2^{(i)} & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n^{(i)} & b_n^{(i)} \end{vmatrix} \qquad Q_n^{(i)} = \begin{vmatrix} b_1^{(i)} & -1 & & & \\ a_2^{(i)} & b_2^{(i)} & -1 & & \\ & a_3^{(i)} & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n^{(i)} & b_n^{(i)} \end{vmatrix} \tag{3}$$

and hence that, if $Q_n^{(i)} \neq 0$, $C_n^{(i)} = b_0^{(i)} + x_1^{(i)}$ where $x_1^{(i)}$ is the first unknown of the tridiagonal system

$$\begin{pmatrix} b_1^{(i)} & -1 & & & \\ a_2^{(i)} & b_2^{(i)} & -1 & & \\ & a_3^{(i)} & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n^{(i)} & b_n^{(i)} \end{pmatrix} \begin{pmatrix} x_1^{(i)} \\ \vdots \\ x_n^{(i)} \end{pmatrix} = \begin{pmatrix} a_1^{(i)} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4}$$

Let us now generalize (3) and (4) for branched continued fractions [3, 6]

$$B_0 + \frac{a_1|}{|B_1} + \frac{a_2|}{|B_2} + \dots \tag{5}$$

where each of the B_i is an ordinary continued fraction as in (1). A convergent of (3) is denoted by

$$C_{n,m_0,\dots,m_n} = C_{m_0}^{(0)} + \sum_{j=1}^{n} \frac{a_j|}{|C_{m_j}^{(j)}} \tag{6}$$

where

$$C_{m_j}^{(j)} = b_0^{(j)} + \sum_{k=1}^{m_j} \frac{a_k^{(j)}|}{|b_k^{(j)}}$$

If we denote $C_{n,m_0,\dots,m_n}$ as $P_{n,m_0,\dots,m_n}/Q_{n,m_0,\dots,m_n}$ then clearly $P_{n,m_0,\dots,m_n}$ and $Q_{n,m_0,\dots,m_n}$ can be computed by applying the three-term reccurence relation (2) to the expression (6) :

$$\begin{cases} P_{k,m_0,\dots,m_k} = C_{m_k}^{(k)} P_{k-1,m_0,\dots,m_{k-1}} + a_k P_{k-2,m_0,\dots,m_{k-2}} \\ Q_{k,m_0,\dots,m_k} = C_{m_k}^{(k)} Q_{k-1,m_0,\dots,m_{k-1}} + a_k Q_{k-2,m_0,\dots,m_{k-2}} \end{cases} \quad k = 1,\dots,n \tag{7}$$

with $P_{-1} = 1 = Q_{0,m_0}$, $P_{0,m_0} = C_{m_0}^{(0)}$ and $Q_{-1} = 0$. As an immediate consequence

$$P_{n,m_0,\dots,m_n} = \begin{vmatrix} C_{m_0}^{(0)} & -1 & & & \\ a_1 & C_{m_1}^{(1)} & -1 & & \\ & a_2 & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n & C_{m_n}^{(n)} \end{vmatrix}$$

$$Q_{n,m_0,\dots,m_n} = \begin{vmatrix} C_{m_1}^{(1)} & -1 & & & \\ a_2 & C_{m_2}^{(2)} & -1 & & \\ & a_3 & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n & C_{m_n}^{(n)} \end{vmatrix}$$

and $C_{n,m_0,\dots,m_n} = C_{m_0}^{(0)} + x_1$ where x_1 is the first unknown of the tridiagonal system

$$
\begin{pmatrix}
C^{(1)}_{m_1} & -1 & & & \\
a_2 & C^{(2)}_{m_2} & -1 & & \\
 & a_3 & \ddots & \ddots & \\
 & & \ddots & & -1 \\
 & & & a_n & C^{(n)}_{m_n}
\end{pmatrix}
\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}
=
\begin{pmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}
$$

Note that in the coefficient matrix of this linear system each $C^{(i)}_{m_i}$ is itself a quotient of determinants. We shall prove in the next theorem that $C_{n,m_0,\ldots,m_n}$ is also the first unknown of a block-tridiagonal linear system where now the partial numerators and denominators $a^{(i)}_j$ and $b^{(i)}_j$ for $j = 0,\ldots,m_i$ and $i = 0,\ldots,n$ of the branched continued fraction (5) appear in the coefficient matrix of the system instead of the $C^{(i)}_{m_i}$. To this end we introduce the notations

$$
\mathcal{B}^{(j)}_{m_j} =
\begin{pmatrix}
b^{(j)}_0 & -1 & & & \\
a^{(j)}_1 & b^{(j)}_1 & -1 & & \\
 & a^{(j)}_2 & \ddots & \ddots & \\
 & & \ddots & & -1 \\
 & & & a^{(j)}_{m_j} & b^{(j)}_{m_j}
\end{pmatrix}
\qquad (m_j + 1) \times (m_j + 1)
$$

$$
\mathcal{A}_j =
\begin{pmatrix}
a_j & 0 & \ldots & 0 \\
0 & & & \\
\vdots & & 0 & \\
0 & & &
\end{pmatrix}
\qquad (m_j + 1) \times (m_{j-1} + 1)
$$

$$
\mathcal{I}_j =
\begin{pmatrix}
1 & 0 & \ldots & 0 \\
0 & & & \\
\vdots & & 0 & \\
0 & & &
\end{pmatrix}
\qquad (m_j + 1) \times (m_{j+1} + 1)
$$

so that $P^{(j)}_{m_j} = \det \mathcal{B}^{(j)}_{m_j}$.

Theorem.
If $Q_{n,m_0,\ldots,m_n} \neq 0$ then $C_{n,m_0,\ldots,m_n} = C_{m_0}^{(0)} + x_0^{(1)}$ where $x_0^{(1)}$ is the first unknown of the block-tridiagonal linear system

$$\begin{pmatrix} \mathcal{B}_{m_1}^{(1)} & -\mathcal{I}_1 & & & \\ \mathcal{A}_2 & \mathcal{B}_{m_2}^{(2)} & -\mathcal{I}_2 & & \\ & \mathcal{A}_3 & \ddots & \ddots & \\ & & \ddots & & -\mathcal{I}_{n-1} \\ & & & \mathcal{A}_n & \mathcal{B}_{m_n}^{(n)} \end{pmatrix} \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} = \begin{pmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{8}$$

with $X_j = (x_0^{(j)}, \ldots, x_{m_j}^{(j)})^t$.

For the proof we need the following two lemmas.

Lemma 1.

$$\begin{vmatrix} \mathcal{B}_{m_1}^{(1)} & -\mathcal{I}_1 & & & \\ \mathcal{A}_2 & \mathcal{B}_{m_2}^{(2)} & -\mathcal{I}_2 & & \\ & \mathcal{A}_3 & \ddots & \ddots & \\ & & \ddots & & -\mathcal{I}_{n-1} \\ & & & \mathcal{A}_n & \mathcal{B}_{m_n}^{(n)} \end{vmatrix} = Q_{n,m_0,\ldots,m_n} Q_{m_1}^{(1)} \cdots Q_{m_n}^{(n)}$$

Proof. For $n = 1$ the left hand side reduces to

$$\det \mathcal{B}_{m_1}^{(1)} = P_{m_1}^{(1)}$$

We also know from (7) that for $n = 1$

$$Q_{1,m_0,m_1} = C_{m_1}^{(1)} = \frac{P_{m_1}^{(1)}}{Q_{m_1}^{(1)}}$$

and hence that

$$Q_{1,m_0,m_1} Q_{m_1}^{(1)} = P_{m_1}^{(1)} = \det \mathcal{B}_{m_1}^{(1)}$$

Suppose the lemma is valid for $Q_{k,m_0,\ldots,m_k} (k = 1, \ldots, n)$. We shall prove it then for $Q_{n+1,m_0,\ldots,m_{n+1}}$. A Laplacian expansion [1] of

$$\begin{vmatrix} \mathcal{B}_{m_1}^{(1)} & -\mathcal{I}_1 & & & \\ \mathcal{A}_2 & \mathcal{B}_{m_2}^{(2)} & -\mathcal{I}_2 & & \\ & \mathcal{A}_3 & \ddots & \ddots & \\ & & \ddots & & -\mathcal{I}_n \\ & & & \mathcal{A}_{n+1} & \mathcal{B}_{m_{n+1}}^{(n+1)} \end{vmatrix}$$

along the last $(m_{n+1}+1)$ rows reveals that the above determinant equals

$$\det \mathcal{B}^{(n+1)}_{m_{n+1}} \cdot \begin{vmatrix} \mathcal{B}^{(1)}_{m_1} & -\mathcal{I}_1 & & & \\ \mathcal{A}_2 & \mathcal{B}^{(2)}_{m_2} & -\mathcal{I}_2 & & \\ & \mathcal{A}_3 & \ddots & \ddots & \\ & & \ddots & & -\mathcal{I}_{n-1} \\ & & & \mathcal{A}_n & \mathcal{B}^{(n)}_{m_n} \end{vmatrix} +$$

$$(-1)^{1+m_n} \begin{vmatrix} a_{n+1} & -1 & & & \\ 0 & b^{(n+1)}_1 & \ddots & & \\ \vdots & a^{(n+1)}_2 & \ddots & & \\ & & \ddots & & -1 \\ 0 & & & a^{(n+1)}_{m_{n+1}} & b^{(n+1)}_{m_{n+1}} \end{vmatrix} \cdot \begin{vmatrix} \mathcal{B}^{(1)}_{m_1} & -\mathcal{I}_1 & & & \\ \mathcal{A}_2 & \mathcal{B}^{(2)}_{m_2} & \ddots & & \\ & \ddots & \ddots & & \\ & & & \mathcal{B}^{(n-1)}_{m_{n-1}} & 0 \\ & & & \mathcal{A}_n & Z \end{vmatrix}$$

where

$$Z = \begin{pmatrix} -1 & & & & -1 \\ b^{(n)}_1 & \ddots & & & 0 \\ a^{(n)}_2 & \ddots & & & \vdots \\ & \ddots & & -1 & \\ & & a^{(n)}_{m_n} & b^{(n)}_{m_n} & 0 \end{pmatrix}$$

This expression can immediately be simplified as

$$P^{(n+1)}_{m_{n+1}} Q_{n,m,\ldots,m_n} Q^{(1)}_{m_1} \cdots Q^{(n)}_{m_n} + (-1)^{1+m_n} a_{n+1} Q^{(n+1)}_{m_{n+1}} \begin{vmatrix} \mathcal{B}^{(1)}_{m_1} & -\mathcal{I}_1 & & & \\ \mathcal{A}_2 & \mathcal{B}^{(2)}_{m_2} & \ddots & & \\ & \ddots & \ddots & & \\ & & & \mathcal{B}^{(n-1)}_{m_{n-1}} & 0 \\ & & & \mathcal{A}_n & Z \end{vmatrix}$$

By making a Laplacian expansion along the columns of Z and using the fact that $\det Z = (-1)^{1+m_n} Q^{(n)}_{m_n}$ it can further be simplified as

$$P^{(n+1)}_{m_{n+1}} Q_{n,m_0,\ldots,m_n} Q^{(1)}_{m_1} \cdots Q^{(n)}_{m_n} + a_{n+1} Q^{(n+1)}_{m_{n+1}} Q^{(n)}_{m_n} Q_{n-1,m_0,\ldots,m_{n-1}} Q^{(1)}_{m_1} \cdots Q^{(n-1)}_{m_{n-1}}$$

On the other hand we can write from (7)

$$Q_{n+1,m_0,\ldots,m_{n+1}} = \frac{P^{(n+1)}_{m_{n+1}}}{Q^{(n+1)}_{m_{n+1}}} Q_{n,n_0,\ldots,m_n} + a_{n+1} Q_{n-1,m_0,\ldots,m_{n-1}}$$

from which we obtain

$$Q_{n+1,m_0,\dots,m_{n+1}} Q_{m_1}^{(1)} \dots Q_{m_{n+1}}^{(n+1)} = \\ P_{m_{n+1}}^{(n+1)} Q_{n,m_0,\dots,m_n} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)} + a_{n+1} Q_{m_{n+1}}^{(n+1)} Q_{n-1,m_0,\dots,m_{n-1}} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)}$$

Since this right hand side coincides with a Laplacian expansion for

$$\begin{vmatrix} \mathcal{B}_{m_1}^{(1)} & -\mathcal{I}_1 & & & \\ \mathcal{A}_2 & \mathcal{B}_{m_2}^{(2)} & -\mathcal{I}_2 & & \\ & \mathcal{A}_3 & \ddots & \ddots & \\ & & \ddots & & -\mathcal{I}_n \\ & & & \mathcal{A}_{n+1} & \mathcal{B}_{m_{n+1}}^{(n+1)} \end{vmatrix}$$

our lemma is proved. ∎

Lemma 2.

$$\begin{vmatrix} \mathcal{B}_{m_0}^{(0)} & -\mathcal{I}_0 & & & \\ \mathcal{A}_1 & \mathcal{B}_{m_1}^{(1)} & -\mathcal{I}_1 & & \\ & \mathcal{A}_2 & \ddots & \ddots & \\ & & \ddots & & -\mathcal{I}_{n-1} \\ & & & \mathcal{A}_n & \mathcal{B}_{m_n}^{(n)} \end{vmatrix} = P_{n,m_0,\dots,m_n} Q_{m_0}^{(0)} \dots Q_{m_n}^{(n)}$$

Proof. For $n = 0$ we know from (7) that

$$P_{0,m_0} = C_{m_0}^{(0)} = \frac{P_{m_0}^{(0)}}{Q_{m_0}^{(0)}}$$

and hence

$$P_{0,m_0} Q_{m_0}^{(0)} = P_{m_0}^{(0)} = \det \mathcal{B}_{m_0}^{(0)}$$

The rest of the inductive proof is completely analogous to that of lemma 1 and is left to the reader. ∎

Let us now try to prove our main result.

Proof of the theorem. Remark that for $n = 1$ (6) reduces to

$$C_{1,m_0,m_1} = C^{(0)}_{m_0} + \cfrac{a_1}{b^{(1)}_0 + \sum\limits_{k=1}^{m_1} \left. \frac{a^{(1)}_k}{b^{(1)}_k} \right|}$$

where $C_{1,m_0,m_1} - C^{(0)}_{m_0}$ is the first unknown $x^{(1)}_0$ of the tridiagonal linear system

$$\begin{pmatrix} b^{(1)}_0 & -1 & & & \\ a^{(1)}_1 & b^{(1)}_1 & -1 & & \\ & a^{(1)}_2 & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a^{(1)}_n & b^{(1)}_n \end{pmatrix} \begin{pmatrix} x^{(1)}_0 \\ \vdots \\ x^{(1)}_{m_1} \end{pmatrix} = \mathcal{B}^{(1)}_{m_1} \begin{pmatrix} x^{(1)}_0 \\ \vdots \\ x^{(1)}_{m_1} \end{pmatrix} = \begin{pmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

More generally, a Laplacian expansion of

$$P_{n,m_0,\ldots,m_n} Q^{(0)}_{m_0} \cdots Q^{(n)}_{m_n} = \begin{vmatrix} \mathcal{B}^{(0)}_{m_0} & -\mathcal{I}_0 & & & \\ \mathcal{A}_1 & \mathcal{B}^{(1)}_{m_1} & -\mathcal{I}_1 & & \\ & \mathcal{A}_2 & \ddots & \ddots & \\ & & \ddots & & -\mathcal{I}_{n-1} \\ & & & \mathcal{A}_n & \mathcal{B}^{(n)}_{m_n} \end{vmatrix}$$

along the first $(m_0 + 1)$ rows learns us that this determinant also equals
$(\det \mathcal{B}^{(0)}_{m_0}) \cdot Q_{n,m_0,\ldots,m_n} Q^{(1)}_{m_1} \cdots Q^{(n)}_{m_n} +$

$$(-1)^{1+m_0} \begin{vmatrix} -1 & & & & -1 \\ b^{(0)}_1 & \ddots & & & 0 \\ a^{(0)}_2 & \ddots & & & \vdots \\ & \ddots & & -1 & \\ & & a^{(0)}_{m_0} & b^{(0)}_{m_0} & 0 \end{vmatrix} \cdot \begin{vmatrix} Y & -\mathcal{I}_1 & & & \\ 0 & \mathcal{B}^{(2)}_{m_2} & \ddots & & \\ \vdots & \mathcal{A}_3 & \ddots & & \\ & & \ddots & & -\mathcal{I}_{n-1} \\ 0 & & & \mathcal{A}_n & \mathcal{B}^{(n)}_{m_n} \end{vmatrix}$$

where

$$Y = \begin{pmatrix} a_1 & -1 & & & \\ 0 & b^{(1)}_1 & \ddots & & \\ \vdots & a^{(1)}_2 & \ddots & & \\ & & \ddots & & -1 \\ 0 & & & a^{(1)}_{m_1} & b^{(1)}_{m_1} \end{pmatrix}$$

This expression can immediately be simplified as

$$P_{n,m_0,\ldots,m_n} Q^{(0)}_{m_0} \cdots Q^{(n)}_{m_n} =$$

$$P^{(0)}_{m_0} Q_{n,m_0,\ldots,m_n} Q^{(1)}_{m_1} \cdots Q^{(n)}_{m_n} + Q^{(0)}_{m_0} \begin{vmatrix} Y & -I_1 & & & \\ 0 & B^{(2)}_{m_2} & \ddots & & \\ \vdots & A_3 & \ddots & & \\ & & \ddots & & -I_{n-1} \\ 0 & & & A_n & B^{(n)}_{m_n} \end{vmatrix}$$

The value $C_{n,m_0,\ldots,m_n}$ we are interested in is thus given by

$$\begin{aligned} C_{n,m_0,\ldots,m_n} &= \frac{P_{n,m_0,\ldots,m_n}}{Q_{n,m_0,\ldots,m_n}} \\ &= \frac{P_{n,m_0,\ldots,m_n} Q^{(0)}_{m_0} \cdots Q^{(n)}_{m_n}}{Q_{n,m_0,\ldots,m_n} Q^{(0)}_{m_0} \cdots Q^{(0)}_{m_n}} \end{aligned}$$

From lemma 2 and the last Laplacian expansion we know that this quotient equals

$$\frac{P^{(0)}_{m_0}}{Q^{(0)}_{m_0}} + \frac{\begin{vmatrix} Y & -I_1 & & & \\ 0 & B^{(2)}_{m_2} & \ddots & & \\ \vdots & A_3 & \ddots & & \\ & & \ddots & & -I_{n-1} \\ 0 & & & A_n & B^{(n)}_{m_n} \end{vmatrix}}{Q_{n,m_0,\ldots,m_n} Q^{(1)}_{m_1} \cdots Q^{(n)}_{m_n}}$$

Using lemma 1 the second term in this expression is apparently the first unknown $x^{(1)}_0$ of our block-tridiagonal linear system. ■

If we try to describe the result of the theorem we can look upon it as follows. Formula (4) for ordinary continued fractions (1) generalizes to formula (8) for branched continued fractions (5) by replacing

$$\begin{aligned} b^{(i)}_j &\rightarrow B^{(j)}_{m_j} \\ a^{(i)}_j &\rightarrow A_j \\ -1 &\rightarrow -I_j \end{aligned}$$

Continuing this idea it is easy to see that for two-branched continued fractions

$$B^{(0)}_0 + \sum_{j=1}^{\infty} \frac{a^{(0)}_j|}{|B^{(0)}_j} + \sum_{i=1}^{\infty} \frac{a_i|}{\left|B^{(i)}_0 + \sum_{j=1}^{\infty} \frac{a^{(i)}_j|}{|B^{(i)}_j}\right.}$$

with

$$B_j^{(i)} = b_{j0}^{(i)} + \sum_{k=1}^{\infty} \frac{a_{jk}^{(i)} \rfloor}{\lfloor b_{jk}^{(i)}}$$

which result by inserting an ordinary continued fraction for each denominator $b_j^{(i)}$ in (5), a formula similar to (8) can be proved where now within $\mathcal{B}_{m_i}^{(i)}$ each $b_j^{(i)}$ is in its turn replaced by a block of the form

$$\begin{pmatrix} b_{j0}^{(i)} & -1 & & \\ a_{j1}^{(i)} & b_{j1}^{(i)} & \ddots & \\ & \ddots & \ddots & \\ & & & \end{pmatrix}$$

This procedure can be repeated k times and so a general determinant representation can be given for the convergent of a k-branched continued fraction. It is our purpose to discuss parallel algorithms for the computation of (6) by introducing parallel algorithms for the solution of block-tridiagonal linear systems like (8). The computation of this type of convergents arises in approximation theory [2], systems theory, and other applications which are under investigation [3].

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