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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

DENSITY OF THE F-STATISTIC IN THE LINEAR MODEL WITH ARBITRARILY NORMAL DISTRIBUTED ERRORS

B.B. van der Genugten

FEW 500

DENSITY OF THE F-STATISTIC IN THE LINEAR MODEL WITH ARBITRARILY NORMAL DISTRIBUTED ERRORS

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Abstract

This paper is concerned with the density of the F-statistic in the context of a linear model with normal distributed errors. The covariance matrix of the errors is arbitrary. The result is useful in the study of robustness of the F-test with respect to errors of the first and second kind.

An explicit expression for this density is given in the form of a proper Riemann-integral on a finite interval, suitable for numerical calculation.

Keywords

F-test, F-statistic, ratio of quotient of quadratic forms in normal variables, numerical evaluation of probability densities.

AMS classification: 60E05, 62J05.

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1 Introduction

Let $Y \sim N_n(\mu, \Omega)$, the n-variate normal distribution with expectation μ and covariance matrix Ω. Let L, R be two orthogonal linear subspaces of $\mathbf{R}^n$ of dimensions l and r, respectively ($l \geq 1, r \geq 1, l + r \leq n$). Set

$$X = |Y_L|^2/|Y_R|^2, \quad F = (r/l)X, \tag{1.1}$$

with $Y_L = P_L Y$ and P_L the orthogonal projection matrix belonging to L; Y_R and R are defined similarly.

In this paper we give a relatively simple expression for the density g of X (or equivalently for the density of the F-statistic F). For numeric calculation some eigenvalues and eigenvectors must be computed once and then an one-dimensional proper Riemann-integral on a finite interval must be evaluated for each point $x \in \mathbf{R}$ to get the value $g(x)$.

The result is useful in studying the robustness of the F-test in linear models. Let $Y = Z\beta + \varepsilon$ with $Z \in \mathbf{R}^{n \times k}$ the (non-stochastic) matrix of explanatory variables and $\varepsilon \sim N_n(0, \Omega)$. Then $Y \sim N_n(\mu, \Omega)$ with $\mu = Z\beta \in \mathbf{R}^n$. An (identifiable) hypothesis H_0 in terms of restrictions on β is equivalent to $H_0 : \mu \in L_0$ with L_0 some linear subspace of $\mathcal{R}(Z)$. The usual F-statistic F for testing $H_0 : \mu \in L_0$ against $H_1 : \mu \in \mathcal{R}(Z) - L_0$ is given by F in (1.1), where L and R are determined by $L \perp L_0, L + L_0 = \mathcal{R}(Z)$ and $R \perp \mathcal{R}(Z), R + \mathcal{R}(Z) = \mathbf{R}^n$.

With the usual assumption $\Omega = \sigma^2 I_n$ we have $F \sim F^l_r(\delta)$, the non-central F-distribution with degrees of freedom l, r and non-centrality parameter $\delta = |\mu_L|^2/\sigma^2$. Equivalently, X follows the distribution with density

$$\exp(-\tfrac{1}{2}\delta) \sum_{k=0}^{\infty} \frac{(\delta/2)^k}{k!} p(x; l/2 + k, r/2), \quad x > 0 \tag{1.2}$$

where $p(x; \rho_1, \rho_2)$ stands for the density of the beta-distribution of the second kind given by

$$p(x;\rho_1,\rho_2) = x^{\rho_1-1}(1+x)^{-\rho_1-\rho_2}/B(\rho_1,\rho_2), \quad x>0. \tag{1.3}$$

So with an expression for the density g of X for general μ and Ω we can study the robustness of the F-test for specified probabilities of errors of the first and second kind.

The question of robustness of the F-test is a very old problem. A detailed study for heteroskedasticity and autocorrelation in some special ANOVA-designs can be found in Scheffe (1959), sections 10.2, 10.3 and 10.5. We refer to this book for an overview.

The problem of the distribution of X in (1.1) is a special case of that of the quotient of two quadratic forms in normal variables. The best references in this field within the context of this paper are Lugannani and Rice (1984) and Magnus (1986).

2 Statement of the results

Let (λ_j, h_j), $j=1,\dots,n$ be the eigenvalues and orthogonal eigenvectors of Ω. Set $\alpha_j = |P_R h_j|^2$, $\beta_j = |P_L h_j|^2$. Throughout this paper we assume that $\max(\lambda_j\alpha_j) > \min(\lambda_j\alpha_j)$, $\max(\lambda_j\beta_j) > \min(\lambda_j\beta_j) > 0$. The following theorem 2.1 preceeds the main theorem 2.2 and is interesting in its own.

Theorem 2.1.

The density g of X defined by (1.1) is restricted to the interval $I = (\min(\beta_k/\alpha_k),\ \max(\beta_k/\alpha_k))$ and its value at $x \in I$ is given by

$$g(x) = \frac{e^{-\frac{1}{2}\Sigma\delta_k}}{4\pi i}\sum_{j=1}^{n}\alpha_j\lambda_j\int_{-i\infty}^{i\infty}\{1-\delta_j/(1-c_jz)\}e^{\frac{1}{2}\Sigma\delta_k/(1-c_kz)}\prod(1-c_kz)^{-\frac{1}{2}-\delta_{kj}}\,dz \tag{2.1}$$

where min, max, Σ, Π extend over $k=1,\dots,n$ with $\lambda_k>0$ and with

$$\delta_j = (h_j'\mu)^2/\lambda_j, \quad c_j = \lambda_j(\beta_j-\alpha_j x). \tag{2.2}$$

Example 1. ($\Omega = \sigma^2 I_n$, $\mu \in L$)

For $\Omega = \sigma^2 I_n$ we have $\lambda_j = \sigma^2$ for all j. Hence, without loss of generality we may take h_j such that $L = \mathcal{R}(h_1, \ldots, h_l)$, $R = \mathcal{R}(h_{l+1}, \ldots, h_{l+r})$. Then $\delta_j = (h_j'\mu)^2/\sigma^2$ for $j = 1, \ldots, l$ and $\delta_j = 0$ elsewhere. This implies $\delta = \Sigma\delta_k = |\mu_L|^2/\sigma^2$. Furthermore, $\alpha_j = 1$ for $j = l+1, \ldots, l+r$, $\beta_j = 1$ for $j = 1, \ldots, l$; other α- and β-values are equal to 0. This gives $I = (0, \infty)$, $c_j = \sigma^2$ for $j = 1, \ldots, l$, $c_j = -\sigma^2 x$ for $j = l+1, \ldots, l+r$ and $c_j = 0$ for $j = l+r+1, \ldots, n$. Substitution into (2.1) leads for any $x > 0$ to:

$$\begin{aligned} g(x) &= \frac{e^{-\delta/2}}{4\pi i} r\sigma^2 \int_{-i\infty}^{i\infty} e^{\frac{1}{2}\delta/(1-\sigma^2 z)}(1-\sigma^2 z)^{-l/2}(1+\sigma^2 xz)^{-r/2-1} dz = \\ &= e^{-\delta/2} \sum_{k=0}^{\infty} \frac{(\delta/2)^k}{k!} \frac{r}{4\pi i} \int_{-i\infty}^{i\infty} (1-z)^{-(l/2+k)}(1+xz)^{-(r/2+1)} dz. \end{aligned}$$

The integral in the sum is a variation of Pochhammer's contour integral for the beta-function. We have (see also Lugannani and Rice (1984), E1, p. 487):

$$\frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \frac{dz}{(z-a)^\alpha (b-z)^\beta} = \frac{\Gamma(\alpha+\beta-1)}{(b-a)^{\alpha+\beta-1}\Gamma(\alpha)\Gamma(\beta)}$$

where $Re(\alpha+\beta) > 1$ and $a < 0 < b$. This leads to

$$\frac{r}{4\pi i} \int_{-i\infty}^{i\infty} (1-z)^{-(l/2+k)}(1+xz)^{-(r/2+1)} dz = p(x; l/2+k, r/2)$$

where p is defined by (1.3). Hence,

$$g(x) = e^{-\delta/2} \sum_{k=0}^{\infty} \frac{(\delta/2)^2}{k!} p(x; l/2+k, r/2), \quad x > 0,$$

in agreement with (1.2).

The following theorem shows that (2.2) can be written as a proper Riemann-integral on a finite interval.

Theorem 2.2. (Conditions of theorem 2.1)

$$g(x) = \frac{1}{4\pi}(a^{-1} + b^{-1}) \exp\left(-\tfrac{1}{2}\sum \delta_k\right) \cdot \sum_{j=1}^{n} \alpha_j \lambda_j I_j \left(\prod f_k\right)^{-\frac{1}{2}-\delta_{kj}} \tag{2.3}$$

with

$$\begin{aligned} I_j \; = \; & \int_0^{\pi/2} B_j(t) \cdot \left(\prod A_k(t)\right)^{-\frac{1}{4}-\frac{3}{2}\delta_{kj}} \cdot \exp\left\{\tfrac{1}{2}\sum \delta_k f_k \cos^2 t / A_k(t)\right\} \cdot \\ & \cdot \cos^{\frac{1}{2}n-1} t \cdot \cos\Big[\sum \left\{(\tfrac{1}{2} + \delta_{kj}) \arcsin(\gamma_k \sin t / A_k(t)) - S_k(t)\right\} + \\ & \arcsin(S_j(t)/C_j(t)) \Big] dt \end{aligned} \tag{2.4}$$

where

$$\left.\begin{aligned} & a = \max(\lambda_j \beta_j), && b = x \cdot \max(\lambda_j \alpha_j) \\ & f_j = 1 - \tfrac{1}{2}c_j(a^{-1} - b^{-1}), && \gamma_j = \tfrac{1}{2}c_j(a^{-1} + b^{-1})/f_j \end{aligned}\right\} \tag{2.5}$$

$$\left.\begin{aligned} & A_j(t) = \cos^2 t + \gamma_j^2 \sin^2 t, && C_j(t) = (1 - \delta_j f_j)\cos^2 t + \gamma_j^2 \sin^2 t \\ & S_j(t) = \delta_j f_j \gamma_j \sin t \cos t, && B_j(t) = \{C_j^2(t) + S_j^2(t)\}^{\frac{1}{2}} \end{aligned}\right\} \tag{2.6}$$

Remark. Since $a \geq \max c_j$, $b \geq -\min c_j$ it follows that $f_j > 0$ and $|\gamma_j| \leq 1$.

Corollary. For $\mu = 0$ we have $\delta_j = 0$ for all j. Then $C_j(t) = A_j(t) = B_j(t)$ and $S_j(t) = 0$ and so (2.4) reduces to

$$\begin{aligned} I_j \; = \; & \int_0^{\pi/2} \left(\prod A_k(t)\right)^{-\frac{1}{4}-\frac{1}{2}\delta_{kj}} \cos^{\frac{1}{2}n-1} t \cdot \\ & \cdot \cos\left[\sum (\tfrac{1}{2} + \delta_{kj}) \arcsin(\gamma_k \sin t / A_k(t))\right] dt \end{aligned} \tag{2.7}$$

Example 2. ($\Omega = \sigma^2 I_n$, $\mu = 0$)

Using the results in example 1 we see that $\delta_j = 0$ for all j and $a = \sigma^2, b = \sigma^2 x$. This leads to $f_j = \frac{1}{2}(1 + 1/x)$, $\gamma_j = 1$ for $j = 1, \ldots, l$; $f_j = \frac{1}{2}(1 + x)$, $\gamma_j = -1$ for $j = l + 1, \ldots, l + r$

and $f_j = 1,\ \gamma_j = 0$ for $j = l+r+1,\ldots,n$. Substitution into (2.5)-(2.7) leads for any $x \in I = (0,\infty)$ to

$$g(x) = x^{l/2-1}(1+x)^{-(l+r)/2}2^{(l+r)/2}\frac{r}{2\pi}\int_0^{\pi/2}\cos^{(l+r)/2-1}t.\cos\{(l-r)/2-1\}dt.$$

The integral is a variant for the integral expression for the beta-function. (see Gradshteyn & Ryzhik (1965), 3.632.5, p. 375):

$$\int_0^{\pi/2}\cos^{\alpha+\beta-1}t\cos(\alpha-\beta-1)dt = \pi/\{2^{\alpha+\beta}(\alpha+\beta)B(\alpha,\beta+1)\}$$

where $Re\ \alpha > 0, Re\ \beta > -1$. This leads to $g(x) = p(x;l/2,r/2)$, where p is defined by (1.3).

3 Proof of the theorems

Lemma 3.1. Let (X_1, X_2) have an absolutely continuous distribution with joint characteristic function φ. If $X_2 \geq 0$ a.s. and $E\{X_2\} < \infty$ then $Y := X_1/X_2$ has a density g given by

$$g(y) = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\left(\frac{\partial\varphi(u_1,u_2)}{\partial u_2}\right)\Bigg|_{u_2=-yu_1} du_1. \tag{3.1}$$

Proof. See Cramer (1946), excercise 6, p. 317 or Geary (1944) and for the multivariate generalization Phillips (1985).

Lemma 3.2. Let $X \sim N_n(\mu,\Omega), \Omega = TT' > 0$ with $T \in \mathbf{R}^{n\times n}$. Let $X_1 = X'A_1X,\ X_2 = X'A_2X$ with symmetric $A_1, A_2 \in \mathbb{R}^{n\times n}$. Then the joint characteristic function φ of X_1, X_2 is given by

$$\varphi(u_1,u_2) = |I_n - 2iC|^{-\frac{1}{2}}\exp\{-\tfrac{1}{2}\eta'\eta\}\cdot\exp\{\tfrac{1}{2}\eta'(I_n-2iC)^{-1}\eta\} \tag{3.2}$$

where

$$\eta = T^{-1}\mu, \quad C = u_1 T' A_1 T + u_2 T' A_2 T \tag{3.3}$$

Proof. See Magnus (1986), lemma 5, p. 102.

Lemma 3.3. (conditions of lemma 2.2)
If (X_1, X_2) has an absolutely continuous distribution, then the density g of $Y := X_1/X_2$ is given by

$$g(y) = \frac{e^{-\frac{1}{2}\eta'\eta}}{4\pi i} \int_{-i\infty}^{i\infty} e^{\frac{1}{2}\eta' S^{-1}(y,z)\eta} |S(y,z)|^{-\frac{1}{2}} \cdot$$

$$\cdot \left[tr(S^{-1}(y,z) T' A_2 T) + \eta' S^{-1}(y,z) T' A_2 T S^{-1}(y,z)\eta \right] dz \tag{3.4}$$

where

$$S(y,z) := I_n - z(T' A_1 T - y T' A_2 T). \tag{3.5}$$

Proof. We use lemma 3.1 and 3.2 and the formulae

$$\frac{dA^{-1}}{dx} = -A^{-1}\frac{dA}{dx}A^{-1}, \quad \frac{d|A|}{dx} = |A|\, tr\left(A^{-1}\frac{dA}{dx}\right) \quad (|A| \neq 0).$$

Differentiation of (3.2) leads with (3.3) and

$$\frac{\partial}{\partial u_2}|I_n - 2iC|^{-\frac{1}{2}} = i|I_n - 2iC|^{-\frac{1}{2}} tr\{(I_n - 2iC)^{-1} T' A_2 T\}$$
$$\frac{\partial}{\partial u_2}(I_n - 2iC)^{-1} = 2i(I_n - 2iC)^{-1} T' A_2 T (I_n - 2iC)^{-1}$$

to

$$\frac{\partial \varphi(u_1,u_2)}{\partial u_2} = i\varphi(u_1,u_2)\left[tr\{(I_n - 2iC)^{-1} T' A_2 T\} + \right.$$
$$\left. + \eta'(I_n - 2iC)^{-1} T' A_2 T (I_n - 2iC)^{-1}\eta \right].$$

So with (3.5)

$$\varphi(u_1,-yu_1)=|S(y,2iu_1)|^{-\frac{1}{2}}\exp\{-\tfrac{1}{2}\eta'\eta\}\exp\{\tfrac{1}{2}\eta'S^{-1}(y,2iu_1)\eta\}$$

$$\left.\frac{\partial\varphi(u_1,u_2)}{\partial u_2}\right|_{u_2=-yu_1} = i\varphi(u_1,-yu_1)\left[tr\{S^{-1}(y,2iu_1)T'A_2T\}+\right.$$
$$\left.+\eta'S^{-1}(y,2iu_1)T'A_2TS^{-1}(y,2iu_1)\eta\right].$$

Substitution of these expressions into (3.1) together with $z=2iu_1$ leads to (3.4).

Proof of theorem 2.1.

At first suppose $\Omega>0$ or, equivalently, $\lambda_j>0$ for all j. We use (3.4) and (3.5) with $A_1=P_L$ and $A_2=P_R$. Since $\Omega=\Sigma\lambda_jh_jh_j'$ we can take $T=\Sigma\lambda_j^{\frac{1}{2}}h_jh_j'$. This gives successively

$$T'P_LT=\Sigma\beta_j\lambda_jh_jh_j',\ T'P_RT=\Sigma\alpha_j\lambda_jh_jh_j'$$
$$S=S(y,z)=\Sigma(1-c_jz)h_jh_j',\ S^{-1}=\Sigma(1-c_jz)^{-1}h_jh_j'$$
$$|S|^{-\frac{1}{2}}=\Pi(1-c_jz)^{-\frac{1}{2}},\ tr(S^{-1}T'P_RT)=\Sigma(1-c_jz)^{-1}\alpha_j\lambda_j$$
$$\eta=\Sigma\delta_j^{\frac{1}{2}}h_j,\ \eta'\eta=\Sigma\delta_j,\ \eta'S^{-1}\eta=\Sigma\delta_j(1-c_jz)^{-1}$$
$$\eta'S^{-1}T'P_RTS^{-1}\eta=\Sigma\alpha_j\lambda_j\delta_j(1-c_jz)^{-2}.$$

Substitution into (3.4) with the Kronecker symbol $\delta_{kj}=1$ if $k=j$, $\delta_{kj}=0$ if $k\neq j$ leads to (2.1).

The integrand in (2.1) has singular points in the half plane $Re\ z>0$ iff $x<\beta_j/\alpha_j$ for some j and singular points in $Re\ z<0$ iff $x>\beta_j/\alpha_j$ for some j. So $g(x)=0$ if $x>\max(\beta_j/\alpha_j)$ or $x<\min(\beta_j/\alpha_j)$. This concludes the proof of the theorem for $\Omega>0$. The general case follows by continuity arguments with respect to the eigenvalues λ_j of Ω.

Proof of theorem 2.2.

We make into (2.1) the substitution $s=(b-a-2abz)/(b+a)$ and $c=(b-a)/(b+a)$. Then $1-c_kz=(1+\gamma_ks)/f_k$ and so

$$g(x) = \frac{e^{-\frac{1}{2}\Sigma\delta_k}}{8\pi i}(a^{-1}+b^{-1})\sum_{j=1}^{n}\alpha_j\lambda_j\left(\prod f_k\right)^{\frac{1}{2}+\delta_{kj}} I_j(c)$$

with

$$I_j(c) = \int_{-i\infty+c}^{i\infty+c}\left\{\prod(1+\gamma_k s)^{-\frac{1}{2}-\delta_{kj}}\right\}\{1-\delta_j f_j/(1+\gamma_j s)\}e^{\frac{1}{2}\Sigma\delta_k f_k/(1+\gamma_k s)}ds.$$

The integrand has singular points at $s=-1/\gamma_k$. Since $a \geq \max c_k$, $b \geq -\min c_k$ we have $|\gamma_k| \leq 1$ and so all singular points are outside $\{s: \ |Re\ s| < 1\}$. Therefore $I_j(c)$ does not depend on c provided that $|c| < 1$. Since $|b-a|/(b+a) < 1$ we may replace the particular value $c=(b-a)/(b+a)$ by $c=0$. This gives the intermediate result

$$g(x) = \frac{e^{-\frac{1}{2}\Sigma\delta_k}}{8\pi}(a^{-1}+b^{-1})\sum_{j=1}^{n}\alpha_j\lambda_j\left(\prod f_k\right)^{\frac{1}{2}+\delta_{kj}} I_j$$

with

$$I_j = I_j(0) = \int_{-\infty}^{\infty}\left\{\prod(1+i\gamma_k u)^{-\frac{1}{2}-\delta_{kj}}\right\}\{1-\delta_j f_j/(1+i\gamma_j u)\}e^{\frac{1}{2}\Sigma\delta_k f_k/(1+i\gamma_k u)}du.$$

We rewrite this expression in the form of a Riemann integral on a finite interval. Substitution of $u = \mathrm{tg} t$, $du = \cos^2 t dt$ together with

$$\begin{aligned}
1+i\gamma_k u &= A_k(t)\cos t\cdot\exp\{i\arcsin(\gamma_k\sin t/A_k(t))\}\\
1-\delta_j f_j/(1+i\gamma_j u) &= A_j^{-1}(t)B_j(t)\exp\{i\arcsin(S_j(t)/A_j(t))\}\\
\exp\{\tfrac{1}{2}\delta_k f_k/(1+i\gamma_k u)\} &= \exp\{\tfrac{1}{2}\delta_k f_k\cos^2 t/A_k(t)\}\exp\{-iS_k(t)\}
\end{aligned}$$

leads to (2.3), (2.4).

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