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A Maximum Likelihood Estimation Method of
a Three Market Disequilibrium Model

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Abstract

In this paper we propose a maximum likelihood estimation method for three markets. For the consumer we assumed a Johansen-type utility function and for the producer we maximized expected profits under a CES-production technology in two labour inputs: the number of workers and the number of working hours. The representative Walrasian and effective supply and demand functions for the various regimes are presented in Section 1.
The maximum likelihood procedure, displayed in the second section, involves a mutual comparison of probabilities that certain quantity rationing regimes have occurred in the economy described.
Once the most likely quantity rationing regime is defined, the policy maker can apply the most suitable economic measures, e.g. to restore an equilibrium situation. This is the subject of non-Walrasian equilibrium theory, i.e. to adjust the economic policy of the government to the kind of disequilibrium regime.
Section three presents a sectoral analysis of the problem. Since the situation on the commodity market and on the labour markets is not similar for all sectors, we tried to allocate the aggregated manufacturing sector to a number of industrial sectors.We hope that this sectoral approach will give us a better insight in the disequilibria on the different markets.

Table of contents

Introduction

A quantity rationing model has been derived under exogenous prices for a two market model (the commodity market and the labour market) in Kooiman & Kloek (1981), Artus, Laroque and Michel (1982) and for a three market model in Meersman & Plasmans (1982). In the latter paper the labour market has been split into two submarkets: one market for the number of employed people and one market for the (average) number of working hours per employed person. The reason for this splitting is to investigate whether a varying working time has any influence on the umemployment rate and which impact a growing unemployment has in a non-Walrasian economy.

Because we consider a three market model, where in any market the demand can be greater or lower than the supply, eight different disequilibrium regimes are possible. By following the reasoning of Malinvaud (1977) that the regime of underconsumption, where the producer is rationed on both the commodity market as well as on the labour markets, is not likely to occur in reality, we can exclude this regime and also the two related one with changing opposite disequilibria in the labour markets. This means that the number of possible disequilibrium regimes is reduced to five.

The representative producer is supposed to maximize expected profits under a CES-production function in both labour inputs. The representative consumer maximizes a Johansen-type utility function subject to a budget constraint. The supply for the number of workers is derived from the analysis of a Labour Force Participation rate. This paper provides a maximum likelihood procedure to estimate simultaneously the effective demands and supplies for the five remaining regimes.

The paper is organized as follows:

- In the first section we present the necessary formulae for the consumer and the producer as they are derived in a more detailed way in Meersman & Plasmans (1982).
- In the second section we explain the derivation of the maximum likelihood procedure. For each of the five remaining regimes the joint density function is considered as a product of conditional densities and the likelihood function of the complete sample and for all rationings is the sum of the five joint density functions derived.
- In the last section we work out a preliminary version of how tc allocate the supply of commodities, the demand for workers and the demand for labour hours of the manufacturing sector to N different industrial sectors. We formulate the optimization problem of the producer under the restriction of Mukerji aggregation functions for labour inputs and for product output.

1. Derivation of a quantity rationing model with a CES-production function.

1.1. The consumer side of the model with labour supply treated as exogenous*)

We consider a representative consumer (or a body of consumers) who maximizes the following utility function for every period, $t = 0,\ldots,\infty$.

$$U_t = \frac{\beta_1}{\alpha_1}\left(\frac{y_t}{\beta_1}\right)^{\alpha_1} - \frac{\beta_2}{\alpha_2}\left(\frac{x_{2t}}{\beta_2}\right)^{\alpha_2} + \frac{\beta_3}{\alpha_3}\left(\frac{M_t/p_{c,t}}{\beta_3}\right)^{\alpha_3} \tag{1.1}$$

where:

y_t : represents the quantities transacted on the commodity market at period t.

x_{2t} : the average number of hours of work for the individual during period t.

M : nominal money stock

$p_{c,t}$: consumer price index.

and where the parameters have to satisfy the following conditions:

$$0 < \alpha_1, \alpha_2, \alpha_3 < 1$$

$$\beta_1, \beta_2, \beta_3 > 0$$

The budget restriction is given by:

$$p_{c,t}\, y_t + M_t = \{w_t(1-q_t)x_{2t} + N_t\}\ (1-v_t) + M_{t-1} \tag{1.2}$$

with:

q_t : the average ratio denoting the employee's share of the payroll taxes (for Social Security)

w_t : the nominal wage rate per hour of work

N_t : non-labour income

v_t : average personal income tax rate.

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*) See Meersman & Plasmans (1982) for a discussion on the identifiability of the model.

Money is assumed to have an indirect utility. When we summarize over an infinite horizon the utility function for the consumer becomes:

$$U = \sum_{t=0}^{\infty} \ell_t U_t$$

where ℓ_t is a discount factor which attributes less importance to future utilities.

1.1.1. The Walrasian supply and demand functions.

We get the notional or Walrasian quantities, when these functions are only function of the price of the commodity or the wage cost for the labour markets, but where there is no quantity rationing from another market.
In order to define the Walrasian commodity demand and the Walrasian supply of the average number of hours of work, we have to work out the following maximization problem:

$$\max U = \sum_{t=0}^{\infty} \ell_t \left\{ \frac{\beta_1}{\alpha_1} \left(\frac{y_t}{\beta_1} \right)^{\alpha_1} - \frac{\beta_2}{\alpha_2} \left(\frac{x_{2t}}{\beta_2} \right)^{\alpha_2} + \frac{\beta_3}{\alpha_3} \left(\frac{M_t/p_{c,t}}{\beta_3} \right)^{\alpha_3} \right\} \tag{1.3}$$

$$\text{subject to: } p_{c,t} y_t + M_t = \{w_t (1-q_t) x_{2t} + N_t\}(1-v_t) + M_{t-1}$$

This yields the following expressions of Walrasian consumption demand and labour supply:

$$\ell n\, y_t^d = \ell n\, \beta_1 - \frac{1-\alpha_3}{1-\alpha_1} \ell n\, \beta_3 + \frac{1-\alpha_3}{1-\alpha_1} \ell n \frac{M_t}{p_{c,t}} \tag{1.4}$$

$$\ell n\, x_{2t}^s = \ell n\, \beta_2 - \frac{1-\alpha_3}{1-\alpha_2} \ell n\, \beta_3 - \frac{1}{1-\alpha_2} \ell n \left\{ \frac{w_t}{p_{c,t}} (1-q_t)(1-v_t) \right\}$$

$$+ \frac{1-\alpha_3}{1-\alpha_2} \ell n \frac{M_t}{p_{c,t}} \tag{1.5}$$

$\ln x_{1t}^{s}$ = exogenous, where x_{1t}^{s} denotes the average number of workers.*)

1.1.2 The effective demand and supply functions.

The effective quantities are obtained when the functions are not only function of the price or wage component but also of a quantity rationing from another market. The Clower effective demand and supply functions result from the maximization of the trader's preferences taking account of all quantity constraints except those prevailing on that market. The Drèze effective quantities are calculated by taking account of all constraints. Throughout this paper we employ the Clower effective functions.

1.1.2.1 The consumer is rationed in the commodity market.

In order to derive the effective supply of hours of work we make the following assumption: the rationing in the commodity market is reflected in the money stock and assume:

$$\begin{aligned}\ln\left(\frac{M_t}{p_{c,t}}\right) &= \ln\left(\frac{M_t}{p_{c,t}}\right)^{w} + \gamma_1(\ln y_t^d - \ln y_t)\\ &= \ln\left(\frac{M_t}{p_{c,t}}\right)^{w} + \gamma_1 \ln\beta_1 - \gamma_1 \frac{1-\alpha_3}{1-\alpha_1}\ln\beta_3\\ &+ \gamma_1 \frac{1-\alpha_3}{1-\alpha_1}\ln\left(\frac{M_t}{p_{c,t}}\right)^{w} - \gamma_1 \ln y_t \qquad (1.6)\end{aligned}$$

where the superscript w denotes the Walrasian quantities.
This expression will now be substituted into the effective supply of average hours of work:

$$\begin{aligned}\ln x_{2t}^{s} &= \ln\beta_2 + \gamma_1\frac{1-\alpha_3}{1-\alpha_2}\ln\beta_1 - \left\{\frac{1-\alpha_3}{1-\alpha_2} + \gamma_1\frac{(1-\alpha_3)^2}{(1-\alpha_1)(1-\alpha_2)}\right\}\ln\beta_3\\ &- \frac{1}{1-\alpha_2}\ln\left\{\frac{w_t}{p_{c,t}}(1-q_t)(1-v_t)\right\} + \frac{1-\alpha_3}{1-\alpha_2}\left\{1+\gamma_1\frac{1-\alpha_3}{1-\alpha_1}\right\}\ln\left(\frac{M_t}{p_{c,t}}\right)^{w}\\ &- \gamma_1\frac{1-\alpha_3}{1-\alpha_2}\ln y_t \qquad (1.7)\end{aligned}$$

$\ln x_{1t}^{s}$ = exogenous

*) See Meersman & Plasmans (1982, Section 2) and Meersman & Plasmans (1980, pp. 20-32). According to Meersman and Plasmans (1982, appendix) the exogenization of the average number of workers is a sufficient condition for statistical identifiability.

1.1.2.2 The consumer is rationed in the commodity market and on the number of workers.

In following the assumption:

$$\ell n\left(\frac{M_t}{p_{c,t}}\right) = \ell n\left(\frac{M_t}{p_{c,t}}\right)^w + \gamma_4(\ell n\, y_t^d - \ell n\, y_t) + \gamma_5(\ell n\, x_{1t}^s - \ell n\, x_{1t}) \quad (1.8)$$

we get:

$$\ell n_{2t}^s = \ell n\, \beta_2 - \{\frac{1-\alpha_3}{1-\alpha_2} + \gamma_4 \frac{(1-\alpha_3)^2}{(1-\alpha_1)(1-\alpha_2)}\}\, \ell n\, \beta_3 + \gamma_4 \frac{1-\alpha_3}{1-\alpha_2}\, \ell n\, \beta_1$$
$$- \frac{1}{1-\alpha_2}\, \ell n\, \{\frac{w_t}{p_{c,t}}(1-q_t)(1-v_t)\} + \frac{1-\alpha_3}{1-\alpha_2}\{1 + \gamma_4 \frac{1-\alpha_3}{1-\alpha_1}\}\, \ell n\left(\frac{M_t}{p_{c,t}}\right)^w$$
$$+ \gamma_5 \frac{1-\alpha_3}{1-\alpha_2}\, \ell n\, x_{1t}^s - \gamma_4 \frac{1-\alpha_3}{1-\alpha_2}\, \ell n\, y_t - \gamma_5 \frac{1-\alpha_3}{1-\alpha_2}\, \ell n\, x_{1t} \quad (1.9)$$

1.1.2.3 The consumer is rationed in the commodity market and on the average hours of work.

No influence on the number of workers, since it is assumed exogenous.

1.1.2.4 The consumer is rationed on the number of workers and on the average hours of work.

Assuming that:

$$\ell n\left(\frac{M_t}{p_{c,t}}\right) = \ell n\left(\frac{M_t}{p_{c,t}}\right)^w + \gamma_6\, (\ell n\, x_{1t}^s - \ell n\, x_{1t}) + \gamma_7(\ell n\, x_{2t}^s - \ell n\, x_{2t}) \quad (1.10)$$

the effective commodity demand can be written as:

$$\ell n\, y_t^d = \ell n\, \beta_1 + \gamma_7 \frac{1-\alpha_3}{1-\alpha_1}\, \ell n\, \beta_2 - \{\frac{1-\alpha_3}{1-\alpha_1} + \gamma_7 \frac{(1-\alpha_3)^2}{(1-\alpha_1)(1-\alpha_2)}\}\, \ell n\, \beta_3$$
$$- \gamma_7 \frac{(1-\alpha_3)^2}{(1-\alpha_1)(1-\alpha_2)}\, \ell n\, \{\frac{w_t}{p_{c,t}}(1-q_t)(1-v_t)\} +$$

$$\frac{1-\alpha_3}{1-\alpha_1}\{1+\gamma_7\frac{1-\alpha_3}{1-\alpha_1}\}\ \ln\left(\frac{M_t}{P_{c,t}}\right)^w+\gamma_6\frac{1-\alpha_3}{1-\alpha_1}\ln x_{1t}^s$$

$$-\gamma_6\frac{1-\alpha_3}{1-\alpha_1}\ln x_{1t}-\gamma_7\frac{1-\alpha_3}{1-\alpha_1}\ln x_{2t}. \tag{1.11}$$

1.1.2.5. The consumer is rationed in all markets.

If the consumer is rationed in all markets, then the expressions given in 1.1.2.2; 1.1.2.3 and 1.1.2.4 are valid since we consider the Clower effective functions.

1.2. The producer side of the model with a CES-production function.

We consider a representative producer (or a body of producers) who has for each period t ($t = 0,\dots,\infty$) a production function in the number of workers and in the average hours of work per worker. We use a CES-production function in both labour inputs:

$$y_t = A\, e^{\lambda t}\{\delta\, x_{1t}^{-\rho} + (1-\delta)x_{2t}^{-\rho}\}^{-\frac{\mu}{\rho}} \tag{1.12}$$

where:

δ : distribution parameter

ρ : substitution parameter

μ : return to scales parameter

and with $\lambda > 0$, $A > 0$, $0 < \delta < 1$, $\rho > -1$, $\mu > 0$.

The after tax profit function in period t is given by

$$\pi_t = (1-u_t)\ \{p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - c_t\} \tag{1.13}$$

with

u_t : average corporation income tax rate

p_t : wholesale price index

w_t : average wage rate per hour

s_t : average coefficient to calculate the employer's contributions to Social Security

c_t : other costs as capital costs, net depreciation costs, etc.

Let k_t be the discount factor for period t and

$$k_t = 0 \text{ for } t = 0$$

$$k_t = \prod_{\theta=0}^{t} \frac{1}{1+r_\theta} \quad \text{for } t > 0$$

where r_θ = discount rate at the end of period θ.
So, we have:

$$\pi_t = \sum_{\theta=0}^{\infty} k_t(1-u_t) \{ p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - c_t \} \tag{1.14}$$

1.2.1. The Walrasian supply and demand functions.

In order to determine the Walrasian commodity supply and the demands for the number of workers and for the average hours of work we have to maximise (1.14) under the restrictions of (1.13). We get then the following Walrasian quantities:

$$\ell n\, y_t^s = \frac{2}{2-\mu} \ell n\, A + \frac{\mu}{2-\mu} \ell n\, \mu - \frac{\mu}{\rho(2-\mu)} \ell n\, \delta - \frac{\mu}{\rho(2-\mu)} \ell n(1-\delta)$$

$$- \frac{\mu(\rho+2)}{\rho(2-\mu)} \ell n\, 2 + \frac{2\lambda}{2-\mu} t - \frac{\mu}{2-\mu} \ell n\, w_t + \frac{\mu}{2-\mu} \ell n\, p_t - \frac{\mu}{2-\mu} \ell n\,(1+s_t) \tag{1.15}$$

$$\ell n\, x_{1t}^d = \frac{1}{2-\mu} \ell n\, A + \frac{1}{2-\mu} \ell n\, \mu + \frac{1-\mu}{\rho(2-\mu)} \ell n\, \delta - \frac{1}{\rho(2-\mu)} \ell n(1-\delta)$$

$$- \frac{\mu+\rho}{\rho(2-\mu)} \ell n\, 2 + \frac{\lambda}{2-\mu} t - \frac{1}{2-\mu} \ell n\, w_t + \frac{1}{2-\mu} \ell n\, p_t - \frac{1}{2-\mu} \ell n(1+s_t) \tag{1.16}$$

$$\ell n\, x_{2t}^d = \frac{1}{2-\mu} \ell n\, A + \frac{1}{2-\mu} \ell n\, \mu + \frac{1-\mu}{\rho(2-\mu)} \ell n(1-\delta) - \frac{1}{\rho(2-\mu)} \ell n\, \delta$$

$$- \frac{\mu+\rho}{\rho(2-\mu)} \ln 2 + \frac{\lambda}{2-\mu} t - \frac{1}{2-\mu} \ln w_t + \frac{1}{2-\mu} \ln p_t - \frac{1}{2-\mu} \ln(1+s_t) \qquad (1.17)$$

1.2.2. The effective demand and supply functions.

1.2.2.1. The producer is rationed in the commodity market.

$$\ln x_{1t}^d = \frac{1}{\mu} \ln y_t - \frac{1}{\mu} \ln A + \frac{1}{\rho} \ln 2 + \frac{1}{\rho} \ln \delta - \frac{\lambda}{\mu} t \qquad (1.18)$$

$$\ln x_{2t}^d = \frac{1}{\mu} \ln y_t - \frac{1}{\mu} \ln A + \frac{1}{\rho} \ln 2 + \frac{1}{\rho} \ln(1-\delta) - \frac{\lambda}{\mu} t \qquad (1.19)$$

1.2.2.2. Rationing on the number of workers.

$$\ln x_{2t}^d = \frac{1}{a} \ln w_t + \frac{1}{a} \ln(1+s_t) - \frac{1}{a} \ln p_t - \frac{1}{a} \ln \mu - \frac{\lambda}{a} t +$$

$$\frac{1-\delta(\mu+\rho)}{a} \ln x_{1t} \qquad (1.20)$$

$$\ln y_t^s = - \frac{1+\rho\delta}{a} \ln A - \frac{(1-\delta)\mu}{a} \ln \mu + \frac{(1-\delta)}{a} \ln w_t +$$

$$\frac{(1-\delta)\mu}{a} \ln(1+s_t) - \frac{(1-\delta)\mu}{a} \ln p_t - \lambda \frac{1+\rho\delta}{a} t +$$

$$\frac{\mu\{1-\delta(\rho+2)\}}{a} \ln x_{1t} \qquad (1.21)$$

where $a := (\mu+\rho)(1-\delta) - \rho - 1$

1.2.2.3. Rationing on the average hours of work per worker.

$$\ln x_{1t}^{d} = -\frac{1}{b}\ln A - \frac{1}{b}\ln\mu - \frac{1}{b}\ln\delta - \frac{\lambda}{b}t + \frac{1}{b}\ln w_t + $$

$$+ \frac{1}{b}\ln(1+s_t) - \frac{1}{b}\ln p_t + \frac{1-(1-\delta)(\mu+\rho)}{b}\ln x_{2t} \qquad (1.22)$$

$$\ln y_t^s = \frac{\mu(\delta-1)-1}{b}\ln A - \frac{\delta\mu}{b}\ln\mu - \frac{\delta\mu}{b}\ln\delta + \lambda\,\frac{\rho(\delta-1)-1}{b}\,t$$

$$+ \frac{\mu\delta}{b}\ln w_t + \frac{\mu\delta}{b}\ln(1+s_t) - \frac{\mu\delta}{b}\ln p_t + \frac{\mu(\delta\rho+2\delta-\rho-1)}{b}\ln x_{2t}$$

$$(1.23)$$

where $b := \delta(\mu+\rho) - \rho - 1$

1.2.2.4. Rationing on working hours and workers.

$$\ln y_t^s = \ln A + \lambda t + \mu\delta\ln x_{1t} + \mu(1-\delta)\ln x_{2t} \qquad (1.24)$$

2. Formulating an estimation method.

2.1 Description of the regimes.

Because we are working on a model where three markets are allowed, and because there is an excess demand or an excess supply in each market, eight different disequilibrium regimes are possible. We cannot deny the theoretical possibility of the underconsumption regime, where the producer is constrained on all markets. But, according to Malinvaud (1977) this regime, where the producers would like to attract more people than they are currently supplied with, notwithstanding the fact that they will not be able to increase scales (due to insufficient demand) this regime only makes sense in multi-period setting, where stocks of finished, but as yet unsold, produducts can be carried over to the next period. When we use the effective relationships of the first section, where there is no inventory function, the above problem cannot occur. That is the reason why we have excluded this regime and the two related ones, with changing opposite disequilibria on the labour markets. When, however, we start from a model as in Meersman & Plasmans (1982, Section 2) inventories may occur. Table 1 summarizes the basic structure of the model to be considered in this paper.

Tabel 1 : Regime definitions.

Regime	Commodity market	Labour markets	
		Number of workers	Number of hours
1	$y^d < y^s$	$x_1^d < x_1^s$	$x_2^d < x_2^s$
2	$y^d > y^s$	$x_1^d < x_1^s$	$x_2^d < x_2^s$
3	$y^d > y^s$	$x_1^d > x_1^s$	$x_2^d > x_2^s$
4	$y^d > y^s$	$x_1^d < x_1^s$	$x_2^d > x_2^s$
5	$y^d > y^s$	$x_1^d > x_1^s$	$x_2^d < x_2^s$

The five regimes considered in this paper are displayed in table 1, where y denotes the quantity of the commodity market, x_1 the quantity for the number of workers and x_2 the quantity for the average number of hours per worker. The entries of table 1 are easily obtained as follows. Taking the first regime we have excess supply in the commodity market. Thus the producer is rationed and the consumer is not in this market. At the other markets we find the producer operating on his effective demands, and the consumer on his notional or Walrasian supplies. So we have the notional labour supplies as the actual supplies, and the effective demands as the actual demands. In all markets the level of transactions is assumed to be equal to the minimum of actual demand and supply. As a consequence in each of the three markets either the consumer or the producer is rationed. The concept of a "spill-over" refers to the situation where an economic agent is forced to revise his desired notional level of transactions at one market, once he meets a constraint on the level of transactions in another market. For the producer, the shortcoming of the labour demands respresents the spillover from the commodity market to the labour markets. The shortcoming of the commodity demand displays the spill-over from the labour markets to the commodity market for the consumer.
Applying a similar reasoning to the other rows of the table we obtain which variety of supply and demand is applicable in the labour markets under excess demand in the commodity market. The first regime is recognized as a Keynesian unemployment (general excess supply), the second as a classical unemployment and the third as a repressed inflation regime (general excess demand). Due to the splitting of the labour market, the 4th and 5th regimes are typical.

2.2. Derivation of the likelihood function.

In principle we follow the procedure proposed by Kooiman & Kloek (1981) and Artus, Laroque and Michel (1982). Differences, however, occur owing the introduction of a CES-production function and the consideration of a three market disequilibrium model. For the producer we get the following set of general formulae, where the ε_i's are random error terms:

(i) $\qquad \ell n\ x_1^d = \ell n\ x_1^d(x) + \varepsilon_1$

(ii) $\qquad \ell n\ x_2^d = \ell n\ x_2^d(x) + \varepsilon_2$

$$
\begin{array}{lll}
\text{(iii)} & \ell n\ y^{s} = \ell n\ f(x_1^{d}(x),\ x_2^{d}(x)) + \varepsilon_3 & (2.1) \\
\text{(iv)} & \ell n\ \bar{x}_1^{d} = \ell n\ x_1^{d} - \eta_1(\ell n\ y^{s} - \ell n\ y) - \eta_2(\ell n\ x_2^{d} - \ell n\ x_2) & \\
\text{(v)} & \ell n\ \bar{x}_2^{d} = \ell n\ x_2^{d} - \eta_3(\ell n\ y^{s} - \ell n\ y) - \eta_4(\ell n\ x_1^{d} - \ell n\ x_1) & \\
\text{(vi)} & \ell n\ \bar{y}^{s} = \ell n\ y^{s} - \kappa_1(\ell n\ x_1^{d} - \ell n\ x_1) - \kappa_2(\ell n\ x_2^{d} - \ell n\ x_2) & \\
 & \text{when } 0 \leqq \eta_i \leqq 1 \text{ and } 0 \leqq \kappa_i \leqq 1 \text{ for all } i &
\end{array}
$$

The producer demands labour and supplies commodities. The notional labour demand functions are represented by (i) and (ii) and are explained by their deterministic parts $\ell n\ x_1^d(x)$ and $\ell n\ x_2^d(x)$, where the vector x summarizes all exogenous variables in the model. The functions $\ell n\ x_1^d(x)$ and $\ell n\ x_2^d(x)$ can be either effective or Walrasian according to the kind of rationing regime considered and are given in the first section, y, x_1 and x_2 being market transactions; ε_1 and ε_2 are standing for deviations between stochastic quantities which should be valid if the agent would not be constrained in other markets and the corresponding deterministic quantities derived from economic theory (as e.g. in the previous section). All error terms will be assumed to be independently normally distributed with zero means and constant variances. The notional supply of consumption goods is determined by a CES production function. The constrained or effective demands for labour are displayed in equations (iv) and (v) which can, theoretically, be influenced by a spollover from the commodity market and the other labour market. The effective supply of commodities is represented by equation (vi) where quantity rationings from the labour markets are possible.
Similarly, for the consumer we have:

$$
\begin{array}{lll}
\text{(vii)} & \ell n\ x_1^{s} = \ell n\ x_1^{s}(x) + \varepsilon_4 & \\
\text{(viii)} & \ell n\ x_2^{s} = \ell n\ x_2^{s}(x) + \varepsilon_5 & \\
\text{(ix)} & \ell n\ y^{d} = \ell n\ y^{d}(x) + \varepsilon_6 & (2.2) \\
\text{(x)} & \ell n\ \bar{x}_1^{s} = \ell n\ x_1^{s} - \zeta_1(\ell n\ x_2^{s} - \ell n\ x_2) - \zeta_2(\ell n\ y^{d} - \ell n\ y) & \\
\text{(xi)} & \ell n\ \bar{x}_2^{s} = \ell n\ x_2^{s} - \zeta_3(\ell n\ x_1^{s} - \ell n\ x_1) - \zeta_4(\ell n\ y^{d} - \ell n\ y) & \\
\text{(xii)} & \ell n\ \bar{y}^{d} = \ell n\ y^{d} - \kappa_3(\ell n\ x_1^{s} - \ell n\ x_1) - \kappa_4(\ell n\ x_2^{s} - \ell n\ x_2) & \\
 & \text{where } 0 \leqq \zeta_i \leqq 1 \text{ and } 0 \leqq \kappa_i \leqq 1 \text{ for all } i. &
\end{array}
$$

The consumer demands commodities and delivers labour. Analogously to the producer the notional or Walrasian demand and supply functions are displayed by equations (vii), (viii) and (ix), while the effective expressions are represented by (x), (xi) and (xii).

In a closed economy, demand for goods can simply be defined as the sum of consumption demand, government demand, investment demand and the demand for inventory accumulation. We only consider the demand for consumption goods in this paper. But, by the introduction of foreign trade, the demand for goods changes considerably. First, we have to consider the demand of exports as an additional source of demand for the domestic product. Second, a part of the demand will be directed towards imported goods, and can thus not be considered to be demand for home produced goods. Third, one has to take account of the possibility of spillovers with respect to foreign trade, due to imbalances in the domestic goods market and the labour markets. The importing and exporting equations are:

$$
\begin{array}{lll}
\text{(xiii)} & \ln E^d = \ln E^d(x) + \varepsilon_7 & \\
\text{(xiv)} & \ln I^d = \ln I^d(x) + \varepsilon_8 & (2.3) \\
\text{(xv)} & \ln \bar{E}^d = \ln E^d - \pi_1(\ln y^d - \ln y) - \pi_2(\ln x_1^d - \ln x_1) - \pi_3(\ln x_2^d - \ln x_2) & \\
\text{(xvi)} & \ln \bar{I}^d = \ln I^d + \pi_4(\ln y^d - \ln y) + \pi_5(\ln x_1^d - \ln x_1) + \pi_6(\ln x_2^d - \ln x_2) & \\
 & \text{where } 0 \leqq \pi_i \leqq 1 \text{ for } i = 1,2,\ldots, 6. &
\end{array}
$$

Equation (xvi) can hardly be considered as a constrained demand function because it is more an enhancement of the demand for imported goods by a shortcoming of the domestic supply of goods. Both equations (xv) and (xvi) represent effective quantities while the notional equations are displayed in (xiii) and (xiv).

The likelihood function of one observation on y, x_1 and x_2 can be derived as the sum of five likelihoods, each giving the probability to be in either of the regimes (compare Gourieroux, Laffont and Monfort (1980), Ito (1980)):

$$L = L_1 + L_2 + L_3 + L_4 + L_5$$

where $$L_1 = \int_{\ell n\, y}^{\infty} \int_{\ell n\, x_1}^{\infty} \int_{\ell n\, x_2}^{\infty} g_1(\ell n\, y,\, \ell n\, x_1,\, \ell n\, x_2,\, \ell n\, y^s,\, \ell n\, x_1^s,\, \ell n\, x_2^s,\, \ell n\, E,\, \ell n\, I)\, d\, \ell n\, y^s\, d\, \ell n\, x_1^s\, d\, \ell n\, x_2^s$$

$$L_2 = \int_{\ell n\, y}^{\infty} \int_{\ell n\, x_1}^{\infty} \int_{\ell n\, x_2}^{\infty} g_2(\ell n\, y,\, \ell n\, x_1,\, \ell n\, x_2,\, \ell n\, y^d,\, \ell n\, x_1^s,\, \ell n\, x_2^s,\, \ell n\, E,\, \ell n\, I)\, d\, \ell n\, y^d\, d\, \ell n\, x_1^s\, d\, \ell n\, x_2^s$$

(2.4) $$L_3 = \int_{\ell n\, y}^{\infty} \int_{\ell n\, x_1}^{\infty} \int_{\ell n\, x_2}^{\infty} g_3(\ell n\, y,\, \ell n\, x_1,\, \ell n\, x_2,\, \ell n\, y^d,\, \ell n\, x_1^d,\, \ell n\, x_2^d,\, \ell n\, E,\, \ell n\, I)\, d\, \ell n\, y^d\, d\, \ell n\, x_1^d\, d\, \ell n\, x_2^d$$

$$L_4 = \int_{\ell n\, y}^{\infty} \int_{\ell n\, x_1}^{\infty} \int_{\ell n\, x_2}^{\infty} g_4\, (\ell n\, y,\, \ell n\, x_1,\, \ell n\, x_2,\, \ell n\, y^d,\, \ell n\, x_1^s,\, \ell n\, x_2^d,\, \ell n\, E,\, \ell n\, I)\, d\, \ell n\, y^d\, d\, \ell n\, x_1^s\, d\, \ell n\, x_2^d$$

$$L_5 = \int_{\ell n\, y}^{\infty} \int_{\ell n\, x_1}^{\infty} \int_{\ell n\, x_2}^{\infty} g_5(\ell n\, y,\, \ell n\, x_1,\, \ell n\, x_2,\, \ell n\, y^d,\, \ell n\, x_1^d,\, \ell n\, x_2^s,\, \ell n\, E,\, \ell n\, I)\, d\, \ell n\, y^d\, d\, \ell n\, x_1^d\, d\, \ell n\, x_2^s$$

The joint density functions g_1 through g_5 of the supply and demand variables relevant to the regime indicated in table 1 can be obtained by describing it as a product of the conditional density functions. To facilitate our notation we introduce the symbol $n(z;\Sigma)$ to denote the joint normal density function of z with zero mean vector and covariance matrix Σ and we use the symbol $N(z;\sigma^2)$ to denote the cumulative normal distribution function of the variate z with mean zero and variance σ^2. According to the given sets of equations for consumers and producers in (2.1), (2.2) and (2.3) we define the following residuals:

$$
\begin{aligned}
\ln u_1 &:= \ln x_1 - \ln x_1^d(x) \\
\ln u_2 &:= \ln x_2 - \ln x_2^d(x) \\
\ln u_3 &:= \ln y - \ln f(x_1^d(x),\ x_2^d(x)) \\
\ln u_4 &:= \ln x_1 - \ln x_1^s(x) \\
\ln u_5 &:= \ln x_2 - \ln x_2^s(x) \\
\ln u_6 &:= \ln y - \ln y^d(x) \\
\ln u_7 &:= \ln E - \ln E^d(x) \\
\ln u_8 &:= \ln I - \ln I^d(x)
\end{aligned}
\tag{2.5}
$$

In this study we deal with the specifications of spillovers on the production side of the economy, i.e. the effective goods supply and the labour demand functions. For the first regime, where we have a general excess supply, we get the following set of equations for the observed quantities by convenient substitution in (2.1), (2.2) and (2.3):

(i) $\ln x_1 = \ln x_1^d(x) + \varepsilon_1 - \eta_1(\ln f(x_1^d(x),\ x_2^d(x)) + \varepsilon_3 - \ln y)$ (i), (iii), (iv) of (2.1)

$$\ln x_1 = \ln x_1^d(x) + \eta_1(\ln y - \ln f(x_1^d(x),\ x_2^d(x))) + \varepsilon_1 - \eta_1\varepsilon_3$$

(ii) $\ln x_2 = \ln x_2^d(x) + \varepsilon_2 - \eta_3(\ln f(x_1^d(x),\ x_2^d(x)) + \varepsilon_3 - \ln y)$ (ii), (iii), (v) of (2.1)

$$\ln x_2 = \ln x_2^d(x) + \eta_3(\ln y - \ln f(x_1^d(x),\ x_2^d(x))) + \varepsilon_2 - \eta_3\varepsilon_3$$

(iii) $$\ln y^s = \ln f(x_1^d(x),\ x_2^d(x)) + \varepsilon_3 \tag{2.6}$$

(iv) $\ln x_1^s = \ln x_1^s(x) + \varepsilon_4 - \zeta_1(\ln x_2^s(x) + \varepsilon_5 - \ln x_2)$ (vii), (viii), (x) of (2.2)

$$\ln x_1^s = \ln x_1^s(x) + \zeta_1(\ln x_2 - \ln x_2^s(x)) + \varepsilon_4 - \zeta_1\varepsilon_5$$

(v) $$\ell n\ x_2^s = \ell n\ x_2^s(x) + \varepsilon_5 - \zeta_3(\ell n\ x_1^s(x) + \varepsilon_4 - \ell n\ x_1)$$

(vii), (viii), (xi) of (2.2)

$$\ell n\ x_2^s = \ell n\ x_2^s(x) + \zeta_3(\ell n\ x_1 - \ell n\ x_1^s(x)) + \varepsilon_5 - \zeta_3\varepsilon_4$$

(vi) $$\ell n\ y = \ell n\ y^d(x) + \varepsilon_6 - \kappa_3(\ell n\ x_1^s(x) + \varepsilon_4 - \ell n\ x_1) - \kappa_4(\ell n\ x_2^s(x) + \varepsilon_5 - \ell n\ x_2)$$

(vii), (viii), (ix), (xii) of (2.2)

$$\ell n\ y = \ell n\ y^d(x) + \kappa_3(\ell n\ x_1 - \ell n\ x_1^s(x)) + \kappa_4(\ell n\ x_2 - \ell n\ x_2^s(x)) +$$

$$+ \varepsilon_6 - \kappa_3\varepsilon_4 - \kappa_4\varepsilon_5$$

(vii) $$\ell n\ E = \ell n\ E^d(x) + \varepsilon_7$$

(viii) $$\ell n\ I = \ell n\ I^d(x) + \varepsilon_8$$

After elaborating the joint density function g_1 as shown in Appendix A, and performing the integration according to the general excess supply regime (2.4), we obtain the following expression for L_1 :

$$\begin{aligned}
L_1 = {} & n(\ell n\ u_7;\ \sigma_7^2)\,n(\ell n\ u_8;\ \sigma_8^2)\ \ n(\ell n\ u_6 - \kappa_3 \ell n\ u_4 - \kappa_4 \ell n\ u_5; \\
& \sigma_6^2 + \kappa_3^2\sigma_4^2 + \kappa_4^2\sigma_5^2)\ \ n(\ell n\ u_4 - \zeta_1 \ell n\ u_5;\ \sigma_4^2 + \zeta_1^2\sigma_5^2) \\
& n(\ell n\ u_5 - \zeta_3 \ell n\ u_4;\ \sigma_5^2 + \zeta_3^2\sigma_4^2) \\
& \{1-N(\ell n\ u_1;\ \sigma_1^2)\} \\
& [1-N(\ell n\ u_2;\ \sigma_2^2)][1-N(\ell n\ u_3 + \frac{\eta_1\sigma_2^2\sigma_3^2}{\sigma_1^2\sigma_2^2 + \eta_3^2\sigma_1^2\sigma_3^2 + \eta_1^2\sigma_2^2\sigma_3^2}(\ell n\ u_1 - \\
& \eta_1\ \ell n\ u_3) + \frac{\eta_3\sigma_1^2\sigma_3^2}{\sigma_1^2\sigma_2^2 + \eta_3^2\sigma_1^2\sigma_3^2 + \eta_1^2\sigma_2^2\sigma_3^2}(\ell n\ u_2 - \eta_3\ \ell n\ u_3); \\
& \sigma_3^2 - \frac{\eta_1^2\sigma_2^2\sigma_3^4 + \eta_3^2\sigma_1^2\sigma_3^4}{\sigma_1^2\sigma_2^2 + \eta_3^2\sigma_1^2\sigma_3^2 + \eta_1^2\sigma_2^2\sigma_3^2})]
\end{aligned}$$

The other integrations can be performed analogously to the first regime. We get then the following expressions:

$$L_2 = n(\ell n\ u_3;\ \sigma_3^2)\ n\ (\ell n\ u_1;\ \sigma_1^2)\ n\ (\ell n\ u_2;\ \sigma_2^2)$$

$$[1-N(\ell n\ u_4 - \zeta_1 \ell n\ u_5 - \zeta_2 \ell n\ u_6;\ \sigma_4^2 + \zeta_1^2\sigma_5^2 + \zeta_2^2\sigma_6^2)]$$

$$[1-N(\ell n\ u_5 - \zeta_3 \ell n\ u_4 - \zeta_4 \ell n\ u_6;\ \sigma_5^2 + \zeta_3^2\sigma_4^2 + \zeta_4^2\sigma_6^2)]$$

$$n(\ell n\ E^d(x) + \pi_1 \ell n\ u_6;\ \sigma_7^2 + \pi_1^2\sigma_6^2)\ n\ (\ell n\ I^d(x) - \pi_4 \ell n\ u_6;$$

$$\sigma_8^2 + \pi_4^2\sigma_6^2)$$

$$[1-N(\ell n\ u_6 - \kappa_3 \ell n\ u_4 - \kappa_4 \ell n\ u_5;\ \sigma_6^2 + \kappa_3^2\sigma_4^2 + \kappa_4^2\sigma_5^2)]$$

$$L_3 = n(\ell n\ E^d(x) + \pi_1 \ell n\ u_6 + \pi_2 \ell n\ u_1 + \pi_3 \ell n\ u_2;\ \sigma_7^2 + \pi_1^2\sigma_6^2 + \pi_2^2\sigma_1^2 + \pi_3^2\sigma_2^2)$$

$$n(\ell n\ I^d(x) - \pi_4 \ell n\ u_4 - \pi_5 \ell n\ u_1 - \pi_6 \ell n\ u_2;\ \sigma_8^2 + \pi_4^2\sigma_4^2 + \pi_5^2\sigma_1^2 + \pi_6^2\sigma_2^2)$$

$$[1-N(\ell n\ u_4 - \zeta_2 \ell n\ u_6;\ \sigma_4^2 + \zeta_2^2\sigma_6^2)][1-N(\ell n\ u_6;\ \sigma_6^2)]$$

$$[1-N(\ell n\ u_5 - \zeta_4 \ell n\ u_6;\ \sigma_5^2 + \zeta_4^2\sigma_6^2)][1-N(\ell n\ u_1 - \eta_2 \ell n\ u_2 -$$

$$\frac{1}{B}\{(\ell n\ u_2 - \eta_4 \ell n\ u_1)(\eta_4\sigma_1^2\sigma_3^2 - \eta_2\sigma_2^2\sigma_3^2 + \eta_4\kappa_2^2\sigma_1^2\sigma_2^2 - \eta_2\kappa_1^2\sigma_1^2\sigma_2^2$$

$$- \kappa_1\kappa_2\sigma_1^2\sigma_2^2 + 2\eta_4\kappa_1^2\sigma_1^4 - \eta_2\eta_4\kappa_1\kappa_2\sigma_1^2\sigma_2^2) - (\ell n\ u_3 - \kappa_1 \ell n\ u_1 - \kappa_2 \ell n\ u_2)$$

$$(\eta_4\kappa_2\sigma_1^2\sigma_2^2 + \eta_2\eta_4\kappa_1\sigma_1^2\sigma_2^2 - \kappa_1\sigma_1^2\sigma_2^2 + \eta_4^2\eta_2\kappa_2\sigma_1^2\sigma_2^2 - 2\eta_4^2\kappa_1\sigma_1^4)\}$$

$$\sigma_1^2 + \eta_2^2\sigma_2^2 + \frac{1}{B}\{(\eta_4\sigma_1^2 + \eta_2\sigma_2^2)(\eta_4\sigma_1^2\sigma_3^2 - \eta_2\sigma_2^2\sigma_3^2 + \eta_4\kappa_2^2\sigma_1^2\sigma_2^2 -$$

$$\eta_2\kappa_1^2\sigma_1^2\sigma_2^2 - \kappa_1\kappa_2\sigma_1^2\sigma_2^2 + 2\eta_4\kappa_1^2\sigma_1^4 - \eta_2\eta_4\kappa_1\kappa_2\sigma_1^2\sigma_2^2) +$$

$$(\kappa_1\sigma_1^2 - \eta_2\kappa_2\sigma_2^2)(\eta_4\kappa_2\sigma_1^2\sigma_2^2 - \eta_2\eta_4\kappa_1\sigma_1^2\sigma_2^2 - \kappa_1\sigma_1^2\sigma_2^2 -$$

$$2\eta_4^2\kappa_1\sigma_1^4 + \eta_4^2\eta_2\kappa_2\sigma_1^2\sigma_2^2)\}]$$

$(\text{met } B := \sigma_2^2\sigma_3^2 + \kappa_1^2\sigma_1^2\sigma_2^2 + \eta_4^2\sigma_1^2\sigma_3^2 + \eta_4^2\kappa_2^2\sigma_1^2\sigma_2^2 + 2\eta_4\kappa_1\kappa_2\sigma_1^2\sigma_2^2)$

$[1-N(\ln u_2 - \eta_4 \ln u_1 + \frac{1}{C}\{(\eta_4\sigma_1^2\sigma_3^2 + \eta_4\kappa_2^2\sigma_1^2\sigma_2^2 + \eta_2\sigma_2^2\sigma_3^2$

$+ \eta_2\kappa_1^2\sigma_1^2\sigma_2^2 + \kappa_2\sigma_1^2\sigma_2^2 - \eta_2\eta_4\kappa_1\kappa_2\sigma_1^2\sigma_2^2)(\ln u_1 - \eta_2 \ln u_2) -$

$(\eta_2\eta_4\kappa_2\sigma_1^2\sigma_2^2 - \eta_2\kappa_1\sigma_1^2\sigma_2^2 - \kappa_2\sigma_1^2\sigma_2^2 + \eta_2^2\eta_4\kappa_1\sigma_1^2\sigma_2^2)$

$(\ln u_3 - \kappa_1 \ln u_1 - \kappa_2 \ln u_2)\}; \; \sigma_2^2 + \eta_4^2\sigma_1^2 - \frac{1}{C}\{(\eta_4\sigma_1^2\sigma_3^2 +$

$\eta_4\kappa_2^2\sigma_1^2\sigma_2^2 + \eta_2\sigma_2^2\sigma_3^2 + \eta_2\kappa_1^2\sigma_1^2\sigma_2^2 + \kappa_2\sigma_1^2\sigma_2^2 - \eta_2\eta_4\kappa_1\kappa_2\sigma_1^2\sigma_2^2)$

$(\eta_4\sigma_1^2 + \eta_2\sigma_2^2) + (\eta_2\eta_4\kappa_2\sigma_1^2\sigma_2^2 - \eta_2\kappa_1\sigma_1^2\sigma_2^2 - \kappa_2\sigma_1^2\sigma_2^2 +$

$\eta_2^2\eta_4\kappa_1\sigma_1^2\sigma_2^2)(\kappa_2\sigma_2^2 - \eta_4\kappa_1\sigma_1^2)\}$

$(\text{met } C := \sigma_1^2\sigma_3^2 + \kappa_2^2\sigma_1^2\sigma_2^2 + \eta_2^2\sigma_2^2\sigma_3^2 + \eta_2^2\kappa_1^2\sigma_1^2\sigma_2^2 + 2\eta_2\kappa_1\kappa_2\sigma_1^2\sigma_2^2)$

$[1-N(\ln u_3 - \kappa_1 \ln u_1 - \kappa_2 \ln u_2 - \frac{1}{D}\{(-\kappa_1\sigma_1^2\sigma_2^2 + \eta_2\eta_4^2\kappa_2\sigma_2^4$

$-\eta_4\kappa_2\sigma_1^2\sigma_2^2 + \eta_2\eta_4\kappa_1\sigma_1^2\sigma_2^2)(\ln u_1 - \eta_2 \ln u_2) - (-\eta_2\kappa_1\sigma_1^2\sigma_2^2 +$

$\eta_2\eta_4\kappa_2\sigma_1^2\sigma_2^2 - \kappa_2\sigma_1^2\sigma_2^2 + \eta_2^2\eta_4\kappa_1\sigma_1^2\sigma_2^2)(\ln u_2 - \eta_4 \ln u_1)\};$

$\sigma_3^2 + \kappa_1^2\sigma_1^2 + \kappa_2^2\sigma_2^2 - \frac{1}{D}\{(-\kappa_1\sigma_1^2\sigma_2^2 + \eta_2\eta_4^2\kappa_2\sigma_2^4 - \eta_4\kappa_2\sigma_1^2\sigma_2^2 +$

$\eta_2\eta_4\kappa_1\sigma_1^2\sigma_2^2)(-\kappa_1\sigma_1^2 + \eta_2\kappa_2\sigma_2^2) - (-\eta_2\kappa_1\sigma_1^2\sigma_2^2 + \eta_2\eta_4\kappa_2\sigma_1^2\sigma_2^2$

$-\kappa_2\sigma_1^2\sigma_2^2 + \eta_2^2\eta_4\kappa_1\sigma_1^2\sigma_2^2)(-\kappa_2\sigma_2^2 + \eta_4\kappa_1\sigma_1^2)\}$

$(\text{met } D := \sigma_1^2\sigma_2^2 + \eta_2^2\eta_4^2\sigma_1^2\sigma_2^2 - 2\eta_2\eta_4\sigma_1^2\sigma_2^2)$

$L_4 = n(\ln E^d(x) + \pi_1 \ln u_6 + \pi_3 \ln u_2; \; \sigma_7^2 + \pi_1^2\sigma_6^2 + \pi_3^2\sigma_2^2)$

$n(\ln I^d(x) - \pi_4 \ln u_6 - \pi_5 \ln u_2; \; \sigma_8^2 + \pi_4^2\sigma_6^2 + \pi_5^2\sigma_2^2)$

$$[1-N(\ell n\ u_3 - \kappa_2 \ell n\ u_2;\ \sigma_3^2 + \kappa_2^2\sigma_2^2)][1-N(\ell n\ u_3;\ \sigma_3^2)]$$

$$[1-N(\ell n\ u_5 - \zeta_3 \ell n\ u_4 - \zeta_4 \ell n\ u_6;\ \sigma_5^2 + \zeta_3^2\sigma_4^2 + \zeta_4^2\sigma_6^2)][1-N(\ell n\ u_1;\sigma_1^2)]$$

$$[1-N(\ell n\ u_6 - \kappa_3 \ell n\ u_4;\ \sigma_6^2 + \kappa_3^2\sigma_4^2)][1-N(\ell n\ u_2 + \frac{1}{E}\{\eta_2\sigma_2^2\sigma_3^2(\ell n\ u_1$$

$$-\eta_2 \ell n\ u_2) + \kappa_2\sigma_1^2\sigma_2^2\ (\ell n\ u_3 - \kappa_2 \ell n\ u_2)\};\ \sigma_2^2 - \frac{1}{E}\{\eta_2^2\sigma_2^4\sigma_3^2 + \kappa_2^2\sigma_1^2\sigma_2^4\})]$$

(met $E := \sigma_1^2\sigma_3^2 + \kappa_2^2\sigma_1^2\sigma_2^2 + \eta_2^2\sigma_2^2\sigma_3^2$)

$$L_5 = n(\ell n\ E^d(x) + \pi_1 \ell n\ u_6 + \pi_2 \ell n\ u_1;\ \sigma_7^2 + \pi_1^2\sigma_6^2 + \pi_2^2\sigma_1^2)$$

$$n(\ell n\ I^d(x) - \pi_4 \ell n\ u_4 - \pi_5 \ell n\ u_1;\ \sigma_8^2 + \pi_4^2\sigma_6^2 + \pi_5^2\sigma_1^2)$$

$$n(\ell n\ u_4 - \zeta_1 \ell n\ u_5 - \zeta_2 \ell n\ u_6;\ \sigma_4^2 + \zeta_1^2\sigma_5^2 + \zeta_2^2\sigma_6^2)$$

$$[1-N(\ell n\ u_5 - \zeta_4 \ell n\ u_6;\ \sigma_5^2 + \zeta_4^2\sigma_6^2)][1-N(\ell n\ u_2;\ \sigma_2^2)]$$

$$[1-N(\ell n\ u_6 - \kappa_4 \ell n\ u_5;\ \sigma_6^2 + \kappa_4^2\sigma_5^2)][1-N(\ell n\ u_3;\ \sigma_3^2)]$$

$$[1-N(\ell n\ u_1 + \frac{1}{F}\{\eta_3\sigma_1^2\sigma_3^2(\ell n\ u_2 - \eta_3 \ell n\ u_1) + \kappa_1\sigma_1^2\sigma_2^2(\ell n\ u_3 - \kappa_1 \ell n\ u_1)\};$$

$$\sigma_1^2 - \frac{1}{F}\{\eta_3^2\sigma_1^4\sigma_3^2 + \kappa_1^2\sigma_1^4\sigma_2^2\})]$$

(met $F := \sigma_2^2\sigma_3^2 + \kappa_1^2\sigma_1^2\sigma_2^2 + \eta_3^2\sigma_1^2\sigma_3^2$)

3. Sectoralization of the model

Because the situation on the labour markets and also on the commodity market is not the same for all sectors, it is our objective to allocate the model of the aggregated manufacturing sector, as described in the previous section, to a number of industrial sectors (e.g. construction, food, chemistry, textile, metal industry and woodworks) and a residual sector. This sectoral analysis will give us a more detailed insight in the disequilibria in the different markets considered.

Since a sectoralization deals with specification of spillovers at the production side of the economy, we stick to the supply of goods in each sector (y_j^s for $j = 1,\ldots,k$), the demand for the number of workers (x_{1j}^d for $j = 1,\ldots,k$) and the demand for labour hours (x_{2j}^d for $j = 1,\ldots,k$). In view that the producers are going to maximize their collective surplus under the restrictions of a Mukerji aggregation function, we can formulate the model as:

$$\text{Max} \sum_{j=1}^{k} p_{y_j} y_j^s - \sum_{j=1}^{k} w_{x_{ij}} x_{ij}^d \quad \text{for } i = 1,2 \tag{3.1}$$

$$\text{subject to: } x_i = \left(\sum_{j=1}^{k} \delta_{ij} (x_{ij}^d)^{\rho_{ij}}\right)^{\frac{1}{\rho_i}} \quad \text{for } i = 1,2 \tag{3.2}$$

$$y = \left(\sum_{j=1}^{k} \delta_j (y_j^d)^{\rho_i}\right)^{\frac{1}{\rho_0}} \tag{3.3}$$

with: $\rho_{ij} \neq 1$, $\rho_{ij} \neq 0$, $\rho_i \neq 0$, $\rho_0 \neq 0$

and with: p_{y_j} : (given) prices of output y_j ($j = 1,\ldots k$)

$w_{x_{ij}}$: (given) wages per hour and per man for each sector j.

We can solve this maximization problem by means of the method of Lagrange as indicated and performed in Appendix B. We become then the next set of equations:

$$\boxed{\ln x_{1j}^{d} = \alpha_{01j} + \alpha_{11j} \ln \frac{w_{x_{1j}}}{\sum_{j=1}^{k} w_{x_{1j}}} + \alpha_{21j} \ln \frac{x_1}{\sum_{\ell \neq j=1}^{k} x_{1\ell}^{d}}} \tag{3.4}$$

$$\boxed{\ln x_{2j}^{d} = \alpha_{02j} + \alpha_{12j} \ln \frac{w_{x_{2j}}}{\sum_{j=1}^{k} w_{x_{2j}}} + \alpha_{22j} \ln \frac{x_2}{\sum_{\ell \neq j=1}^{k} x_{2\ell}^{d}}} \tag{3.5}$$

$$\boxed{\ln y_{j}^{s} = \alpha_{0j} + \alpha_{1j} \ln \frac{p_{y_j}}{\sum_{j=1}^{k} p_{y_j}} + \alpha_{2j} \ln \frac{y}{\sum_{\ell \neq j=1}^{k} y_{j\ell}^{s}}} \tag{3.6}$$

where α_{21j}, α_{22j} and α_{2j} are allocation elasticities and α_{11j}, α_{12j} and α_{1j} are wage or price substitution elasticities.

Theoretically, one would expect the coefficients α_{11j}, α_{12j} and α_{1j} to be negative. But in view of fluctuations in relative wages and prices, we could expect unstable estimates for this coefficients. Strongly negative parameters would indicate a high degree of competition between the wages and prices of the different sectors on the markets. From the estimated results of the COMET-model (Barten, d'Alcantara, Carrin (1975)), we may conclude that these parameters must have a value somewhere between -1 and 0. The allocation elasticities are in general nonnegative and, leaving on the same COMET-experience, they will mostly be elastic (larger than 1).

Appendix A

We start here with the equations derived in (2.6)

$$\ell n\ x_1 = \ell n\ x_1^d(x) + \eta_1(\ell n\ y - \ell n\ f(x_1^d(x),\ x_2^d(x))) + \varepsilon_1 - \eta_1\varepsilon_3$$

$$\ell n\ x_2 = \ell n\ x_2^d(x) + \eta_3(\ell n\ y - \ell n\ f(x_1^d(x),\ x_2^d(x))) + \varepsilon_2 - \eta_3\varepsilon_3$$

$$\ell n\ y^s = \ell n\ f(x_1^d(x),\ x_2^d(x)) + \varepsilon_3$$

$$\ell n\ x_1^s = \ell n\ x_1^s(x) + \zeta_1(\ell n\ x_2 - \ell n\ x_2^s(x)) + \varepsilon_4 - \zeta_1\varepsilon_5$$

$$\ell n\ x_2^s = \ell n\ x_2^s(x) + \zeta_3(\ell n\ x_1 - \ell n\ x_1^s(x)) + \varepsilon_5 - \zeta_3\varepsilon_4$$

$$\ell n\ y = \ell n\ y^d(x) + \kappa_3(\ell n\ x_1 - \ell n\ x_1^s(x)) + \kappa_4(\ell n\ x_2 - \ell n\ x_2^s(x))$$

$$+ \varepsilon_6 - \kappa_3\varepsilon_4 - \kappa_4\varepsilon_5$$

$$\ell n\ E = \ell n\ E^d(x) + \varepsilon_7$$

$$\ell n\ I = \ell n\ I^d(x) + \varepsilon_8$$

$g_1(\ell n\ y,\ \ell n\ x_1,\ \ell n\ x_2,\ \ell n\ y^s,\ \ell n\ x_1^s,\ \ell n\ x_2^s,\ \ell n\ E,\ \ell n\ I)$ can be factorized as:

$$g^1(\ell n\ y|\ell n\ x_1,\ \ell n\ x_2)g^2(\ell n\ x_1|\ell n\ y)g^3(\ell n\ x_2|\ell n\ y)$$

$$g^4(\ell n\ y^s|\ell n\ y)g^5(\ell n\ x_1^s|\ell n\ x_1)g^6(\ell n\ x_2^s|\ell n\ x_2)g^7(\ell n\ E)g^8(\ell n\ I)$$

The last two factors can directly be obtained from (vii) and (viii) in (2.5) as $n(\ell n\ u_7;\ \sigma_7^2)$ and $n(\ell n\ u_8;\ \sigma_8^2)$ respectively.
Since all error terms are independent g^1, g^5 and g^6 can be written respectively as:

$$n(\ln u_6 - \kappa_3 \ln u_4 - \kappa_4 \ln u_5;\ \sigma_6^2 + \kappa_3^2\sigma_4^2 + \kappa_4^2\sigma_5^2)$$

$$n(\ln u_4 - \zeta_1 \ln u_5;\ \sigma_4^2 + \zeta_1^2\sigma_5^2)$$

$$n(\ln u_5 - \zeta_3 \ln u_4;\ \sigma_5^2 + \zeta_1^2\sigma_4^2)$$

The remaining factors taken together constitute the joint density function of $\ln x_1$, $\ln x_2$ and $\ln y^s$. It is obtained from (i), (ii) and (iii) with mean vector:

$$\begin{pmatrix} \ln x_1^d(x) + \eta_1(\ln y - \ln f(x_1^d(x),\ x_2^d(x))) \\ \ln x_2^d(x) + \eta_3(\ln y - \ln f(x_1^d(x),\ x_2^d(x))) \\ \ln f(x_1^d(x),\ x_2^d(x)) \end{pmatrix}$$

and with covariance matrix:

$$\begin{bmatrix} \sigma_1^2 + \eta_1^2\sigma_3^2 & \eta_1\eta_3\sigma_3^2 & -\eta_1\sigma_3^2 \\ \eta_1\eta_3\sigma_3^2 & \sigma_2^2 + \eta_3^2\sigma_3^2 & -\eta_3\sigma_3^2 \\ -\eta_1\sigma_3^2 & -\eta_3\sigma_3^2 & \sigma_3^2 \end{bmatrix}$$

From the formulae for conditional means and variances for a multi-normal distribution*) the conditional normal density functions can be computed as:

---

*) If a k-vector x is assumed to be normally distributed with mean vector μ and variance-covariance matrix Ω, then the conditional probability density of $(x_1|x_2)$, where x_1 is an ℓ-subvector of x and x_2 is the resulting $(k-\ell)$-subvector of x, is also normal with mean vector $\mu_1 + \Omega_{12}\Omega_{22}^{-1}(x_2-\mu_2)$ and variance-covariance matrix $\Omega_{11} - \Omega_{12}\ \Omega_{22}^{-1}\Omega_{21}$, where μ_i and Ω_{ij} $(i,j = 1,2)$ are the correspondingly partitioned vectors and matrices of μ and Ω (see, e.g. Mood & Graybill (1963), Chapter 9).

$$n(\ell n\ u_1 - \eta_1\ \ell n\ u_3 - [\eta_1\eta_3\sigma_3^2, -\eta_1\sigma_3^2] \begin{bmatrix} \sigma_2^2 + \eta_3^2\sigma_3^2 & -\eta_3\sigma_3^2 \\ -\eta_3\sigma_3^2 & \sigma_3^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \ell n\ u_2 - \eta_3\ \ell n\ u_3 \\ \ell n\ y^s - \ell n\ f(x_1^d(x),\ x_2^d(x)) \end{bmatrix} ;\ \sigma_1^2 + \eta_1^2\sigma_3^2 - [\eta_1\eta_3\sigma_3^2, -\eta_1\sigma_3^2]$$

$$\begin{bmatrix} \sigma_2^2 + \eta_3^2\sigma_3^2 & -\eta_3\sigma_3^2 \\ -\eta_3\sigma_3^2 & \sigma_3^2 \end{bmatrix}^{-1} \begin{bmatrix} \eta_1\eta_3\sigma_3^2 \\ -\eta_1\sigma_3^2 \end{bmatrix})$$

$$n(\ell n\ u_2 - \eta_3\ \ell n\ u_3 - [\eta_1\eta_3\sigma_3^2, -\eta_3\sigma_3^2] \begin{bmatrix} \sigma_1^2 + \eta_1^2\sigma_3^2 & -\eta_1\sigma_3^2 \\ -\eta_1\sigma_3^2 & \sigma_3^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \ell n\ u_1 - \eta_1\ \ell n\ u_3 \\ \ell n\ y^s - \ell n\ f(x_1^d(x),\ x_2^d(x)) \end{bmatrix} ;\ \sigma_2^2 + \eta_3^2\sigma_3^2 - [\eta_1\eta_3\sigma_3^2, -\eta_3\sigma_3^2]$$

$$\begin{bmatrix} \sigma_1^2 + \eta_1^2\sigma_3^2 & -\eta_1\sigma_3^2 \\ -\eta_1\sigma_3^2 & \sigma_3^2 \end{bmatrix}^{-1} \begin{bmatrix} \eta_1\eta_3\sigma_3^2 \\ -\eta_3\sigma_3^2 \end{bmatrix})$$

and $$n(\ln y^s - \ln f(x_1^d(x),\ x_2^d(x)) - [-\eta_1\sigma_3^2, -\eta_3\sigma_3^2] \begin{bmatrix} \sigma_1^2 + \eta_1^2\sigma_3^2 & \eta_1\eta_3\sigma_3^2 \\ \eta_1\eta_3\sigma_3^2 & \sigma_2^2 + \eta_3^2\sigma_3^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \ln u_1 - \eta_1 \ln u_3 \\ \ln u_2 - \eta_3 \ln u_3 \end{bmatrix} ;\ \sigma_3^2 - [-\eta_1\sigma_3^2, -\eta_3\sigma_3^2] \begin{bmatrix} \sigma_1^2 + \eta_1^2\sigma_3^2 & \eta_1\eta_3\sigma_3^2 \\ \eta_1\eta_3\sigma_3^2 & \sigma_2^2 + \eta_3^2\sigma_3^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} -\eta_1\sigma_3^2 \\ -\eta_3\sigma_3^2 \end{bmatrix})$$

The joint density function g_1 can then be written as:

$$n(\ln u_7;\ \sigma_7^2)\ n\ (\ln u_8;\ \sigma_8^2)\ n\ (\ln u_6 - \kappa_3 \ln u_4 - \kappa_4 \ln u_5;$$

$$\sigma_6^2 + \kappa_3^2\sigma_4^2 + \kappa_4^2\sigma_5^2)\ n\ (\ln u_4 - \zeta_1 \ln u_5;\ \sigma_4^2 + \zeta_1^2\sigma_5^2)$$

$$n(\ln u_5 - \zeta_3 \ln u_4;\ \sigma_5^2 + \zeta_1^2\sigma_4^2)$$

$$n(\ln u_1 - \eta_1 \ln u_3 - [\eta_1\eta_3\sigma_3^2, -\eta_1\sigma_3^2] \begin{bmatrix} \sigma_2^2 + \eta_3^2\sigma_3^2 & -\eta_3\sigma_3^2 \\ -\eta_3\sigma_3^2 & \sigma_3^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \ln u_2 - \eta_3 \ln u_3 \\ \ln y^s - \ln f(x_1^d(x),\ x_2^d(x)) \end{bmatrix} ;\ \sigma_1^2 + \eta_1^2\sigma_3^2 - [\eta_1\eta_3\sigma_3^3, -\eta_1\sigma_3^2]$$

$$\begin{bmatrix} \sigma_2^2 + \eta_3^2\sigma_3^2 & -\eta_3\sigma_3^2 \\ -\eta_3\sigma_3^2 & \sigma_3^2 \end{bmatrix}^{-1} \begin{bmatrix} \eta_1\eta_3\sigma_3^2 \\ -\eta_1\sigma_3^2 \end{bmatrix}$$

$$n(\ell n\ u_2 - \eta_3\ \ell n\ u_3 - [\eta_1\eta_3\sigma_3^2, -\eta_3\sigma_3^2] \begin{bmatrix} \sigma_1^2 + \eta_1^2\sigma_3^2 & -\eta_1\sigma_3^2 \\ -\eta_1\sigma_3^2 & \sigma_3^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \ell n\ u_1 - \eta_1\ \ell n\ u_3 \\ \ell n\ y^s - \ell n\ f(x_1^d(x),\ x_2^d(x)) \end{bmatrix} ;\ \sigma_2^2 + \eta_3^2\sigma_3^2 - [\eta_1\eta_3\sigma_3^2, -\eta_3\sigma_3^2]$$

$$\begin{bmatrix} \sigma_1^2 + \eta_1^2\sigma_3^2 & -\eta_1\sigma_3^2 \\ -\eta_1\sigma_3^2 & \sigma_3^2 \end{bmatrix}^{-1} \begin{bmatrix} \eta_1\eta_3\sigma_3^2 \\ -\eta_3\sigma_3^2 \end{bmatrix})$$

$$n(\ell n\ y^s - \ell n\ f(x_1^d(x),\ x_2^d(x)) - [-\eta_1\sigma_3^2, -\eta_3\sigma_3^2] \begin{bmatrix} \sigma_1^2 + \eta_1^2\sigma_3^2 & \eta_1\eta_3\sigma_3^2 \\ \eta_1\eta_3\sigma_3^2 & \sigma_2^2 + \eta_3^2\sigma_3^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \ell n\ u_1 - \eta_1\ \ell n\ u_3 \\ \\ \ell n\ u_2 - \eta_3\ \ell n\ u_3 \end{bmatrix} \quad ;\ \sigma_3^2 - [\ -\eta_1\sigma_3^2,\ -\eta_3\sigma_3^2] \begin{bmatrix} \sigma_1^2 + \eta_1\sigma_3^2 & \eta_1\eta_3\sigma_3^2 \\ \\ \eta_1\eta_3\sigma_3^2 & \sigma_2^2 + \eta_3^2\sigma_3^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} -\eta_1\sigma_3^2 \\ \\ -\eta_3\sigma_3^2 \end{bmatrix} \Bigg)$$

Appendix B

Here we repeat the formulation of the problem as indicated in the third section of this paper:

$$\text{Max} \sum_{j=1}^{k} p_{y_j} \, y_j^s - \sum_{j=1}^{k} w_{x_{ij}} \, x_{ij}^d \qquad (i = 1,2) \tag{B.1}$$

$$\text{subject to: } x_i = \left(\sum_{j=1}^{k} \delta_{ij} (x_{ij}^d)^{\rho_{ij}}\right)^{\frac{1}{\rho_i}} \tag{B.2}$$

$$y = \left(\sum_{j=1}^{k} \delta_j (y_j^s)^{\rho_j}\right)^{\frac{1}{\rho_0}} \tag{B.3}$$

where x_{1j}^d, x_{2j}^d and y_j^s are the decision variables $(j = 1,2,\ldots,k)$

We can solve this maximization problem by using the Lagrangean function:

$$\begin{aligned} \text{Max } L = & \sum_{j=1}^{k} p_{y_j} \, y_j^s - \sum_{j=1}^{k} w_{x_{ij}} \, x_{ij}^d \\ & - \lambda\left\{x_i - \left(\sum_{j=1}^{k} \delta_{ij} (x_{ij}^d)^{\rho_{ij}}\right)^{\frac{1}{\rho_i}}\right\} \\ & + \mu\left\{y - \left(\sum_{j=1}^{k} \delta_j (y_j^s)^{\rho_j}\right)^{\frac{1}{\rho_0}}\right\} \end{aligned}$$

where λ and μ are the Lagrangean parameters.

We take now the first order conditions by setting the first order partial derivatives equal to zero:

$$\frac{\partial L}{\partial x_{ij}^{d}} = - w_{x_{ij}} - \lambda \left[- \frac{1}{\rho_i} \left(\sum_{j=1}^{k} \delta_{ij} (x_{ij}^{d})^{\rho_{ij}} \right)^{\frac{1}{\rho_i} - 1} \right] \rho_{ij} \delta_{ij} (x_{ij}^{d})^{\rho_{ij} - 1} = 0 \tag{B.4}$$

$$\frac{\partial L}{\partial y_{j}^{s}} = p_{y_j} + \mu \left\{ - \frac{1}{\rho_0} \left(\sum_{j=1}^{k} \delta_{j} (y_{j}^{s})^{\rho_{j}} \right)^{\frac{1}{\rho_0} - 1} \right\} \rho_{j} \delta_{j} (y_{j}^{s})^{\rho_{j} - 1} = 0 \tag{B.5}$$

Rewriting equations (B.4) and (B.5):

$$- w_{x_{ij}} + \lambda \left(\frac{1}{\rho_i}\right) \underbrace{\left(\sum_{j=1}^{k} \delta_{ij} (x_{ij}^{d})^{\rho_{ij}} \right)}_{\substack{\Big\downarrow (B.2) \\ (x_i)^{\rho_i}}}^{\frac{1-\rho_i}{\rho_i}} \delta_{ij} \, \rho_{ij} (x_{ij}^{d})^{\rho_{ij} - 1} = 0$$

$$w_{x_{ij}} = \frac{\lambda}{\rho_i} (x_i)^{1-\rho_i} \delta_{ij} \, \rho_{ij} (x_{ij}^{d})^{\rho_{ij} - 1} \tag{B.6}$$

and taking account of (B.3):

$$p_{y_j} = \frac{\mu}{\rho_0} y^{1-\rho_0} \delta_j \, \rho_j (y_j^{s})^{\rho_j - 1} \tag{B.7}$$

Now we multiply equation (B.6) with x_{ij}^{d} and sum it over all sectors in order to get aggregate nominal wages:

$$\sum_{j=1}^{k} x_{ij}^{d} \; w_{x_{ij}} = \frac{\lambda}{\rho_i} (x_i)^{1-\rho_i} \sum_{j=1}^{k} \delta_{ij} \, \rho_{ij} (x_{ij}^{d})^{\rho_{ij}}$$

$$= \frac{\lambda}{\rho_i} x_i \frac{\sum_{j=1}^{k} \delta_{ij} \, \rho_{ij} (x_{ij}^{d})^{\rho_{ij}}}{x_i^{\rho i}}$$

$$= \frac{\lambda}{\rho_i} x_i \frac{\sum_{j=1}^{k} \delta_{ij} \, \rho_{ij} (x_{ij}^{d})^{\rho_{ij}}}{\sum_{\ell=1}^{k} \delta_{i\ell} (x_{i\ell}^{d})^{\rho_{i\ell}}}$$

Define now:

$$\alpha_{ij} := \frac{\delta_{ij} (x_{ij}^{d})^{\rho_{ij}}}{\sum_{\ell=1}^{k} \delta_{i\ell} (x_{i\ell}^{d})^{\rho_{i\ell}}}$$

so that

$$\sum_{j=1}^{k} \alpha_{ij} = 1$$

then:

$$\sum_{j=1}^{k} x_{ij}^{d} \; w_{x_{ij}} = \frac{\lambda}{\rho_i} x_i \underbrace{\sum_{j=1}^{k} \alpha_{ij} \, \rho_{ij}}_{\downarrow \; \bar{\rho}_i} \qquad \text{(for } i = 1,2\text{)}$$

$$\frac{\lambda}{\rho_i} = \frac{\sum_{j=1}^{k} x_{ij}^{d} \; w_{x_{ij}}}{x_i \; \bar{\rho}_i}$$

$$\lambda = \frac{\rho_i \sum_{j=1}^{k} x_{ij}^d \, {}^w x_{ij}}{x_i \, \bar{\rho}_i} \tag{B.8}$$

We can now substitute the expression (B.8) for λ into (B.6) and this results in:

$${}^w x_{ij} = \frac{\sum_{j=1}^{k} x_{ij}^d \, {}^w x_{ij}}{x_i \, \bar{\rho}_i} (x_i)^{1-\rho_i} \, \delta_{ij} \, \rho_{ij} (x_{ij}^d)^{\rho_{ij}-1}$$

$${}^w x_{ij} \, x_i \, \bar{\rho}_i = \sum_{\ell \neq j}^{k} x_{i\ell}^d \, {}^w x_{i\ell} \, . \, {}^w x_{ij} \, x_{ij}^d (x_i)^{1-\rho_i} \, \delta_{ij} \, \rho_{ij} (x_{ij}^d)^{\rho_{ij}-1}$$

$$(x_{ij}^d)^{\rho_{ij}} = \frac{{}^w x_{ij} \, x_i \, \bar{\rho}_i}{\sum_{\ell \neq j}^{k} {}^w x_{i\ell} \, x_{i\ell}^d \, {}^w x_{ij} \, (x_i)^{1-\rho_i} \, \delta_{ij} \, \rho_{ij}}$$

$$x_{ij}^d = \left(\frac{{}^w x_{ij}}{\sum_{j=1}^{k} {}^w x_{ij}} \; \frac{x_i^{\rho_i}}{\sum_{\ell \neq j}^{k} x_{i\ell}^d} \; \frac{\sum_{j=1}^{k} \alpha_{ij} \, \rho_{ij}}{\delta_{ij} \, \rho_{ij}} \right)^{\frac{1}{\rho_{ij}}} \tag{B.9}$$

By taking the logarithms of this function (B.9) we get fot x_{1j}^d

$$\ell n \, x_{1j}^d = \frac{1}{\rho_{1j}} \ell n \frac{{}^w x_{1j}}{\sum_{j=1}^{k} {}^w x_{1j}} + \frac{1}{\rho_{1j}} \ell n \frac{x_1^{\rho_1}}{\sum_{\ell \neq j}^{k} x_{1\ell}^d} + \frac{1}{\rho_{1j}} \ell n \frac{\sum_{j=1}^{k} \alpha_{1j} \, \rho_{1j}}{\delta_{1j} \, \rho_{1j}}$$

$$\ell n\ x_{1j}^{d} = \frac{1}{\rho_{1j}}\ \ell n\ \frac{{}^{w}x_{1j}}{\sum_{j=1}^{k} {}^{w}x_{1j}} + \frac{\rho_1 - 1}{\rho_{1j}}\ \ell n\ \frac{x_1}{\sum_{\ell \neq j} x_{1\ell}^{d}} + \frac{1}{\rho_{1j}}\ \ell n\ \frac{\sum_{j=1}^{k} \alpha_{1j}\ \rho_{1j}}{\delta_{1j}\ \rho_{1j}}$$

Defining:

$$\alpha_{01j} := \frac{1}{\rho_{1j}}\ \ell n\ \frac{\sum_{j=1}^{k} \alpha_{1j}\ \rho_{1j}}{\delta_{1j}\ \rho_{1j}}$$

$$\alpha_{11j} := \frac{1}{\rho_{1j}}$$

$$\alpha_{21j} := \frac{\rho_1 - 1}{\rho_{1j}}$$

we get

$$\ell n\ x_{1j}^{d} = \alpha_{01j} + \alpha_{11j}\ \ell n\ \frac{{}^{w}x_{1j}}{\sum_{j=1}^{k} {}^{w}x_{1j}} + \alpha_{21j}\ \ell n\ \frac{x_1}{\sum_{\ell \neq j}^{k} x_{1\ell}^{d}} \qquad (B.10)$$

Similarly we get for $\ell n\ x_{2j}^{d}$:

$$\ell n\ x_{2j}^{d} = \alpha_{02j} + \alpha_{12j}\ \ell n\ \frac{{}^{w}x_{2j}}{\sum_{j=1}^{k} {}^{w}x_{2j}} + \alpha_{22j}\ \ell n\ \frac{x_2}{\sum_{\ell \neq j}^{k} x_{2\ell}^{d}} \qquad (B.11)$$

with the same interpretation for the parameters α.

In an analogous way we can perform the same computation for $\ell n\ y_j^s$ which yields the following expression:

$$\ell n\ y_j^s = \alpha_{0j} + \alpha_{1j}\ \ell n \frac{p_{y_j}}{\sum\limits_{j=1}^{k} p_{y_j}} + \alpha_{2j}\ \ell n \frac{y}{\sum\limits_{\ell \neq j}^{k} y_\ell^s} \tag{B.12}$$

with:

$$\alpha_{0j} := \frac{1}{\rho_j}\ \ell n \frac{\sum\limits_{j=1}^{k} \alpha_j\ \rho_j}{\delta_j\ \rho_j}$$

$$\alpha_{1j} := \frac{1}{\rho_j}$$

$$\alpha_{2j} := \frac{\rho_0 - 1}{\rho_j}$$

BIBLIOGRAPHY

- ARTUS, P., LAROQUE, G and MICHEL, G. (1982) Estimation of a quarterly macroeconomic model with quantity rationing. Paper presented at the "Colloque International sur les développements récents de la modélisation macroéconomique", Paris, 13-15 sept. 1982.

- BARTEN, A.P., D'ALCANTARA, G. and CARRIN, G.J. (1975) COMET model: A medium-term macroeconomic model for the European Economic community. European Economic Review, pp. 63-115.

- DRAZEN, A. (1980) Recents developments in macroeconomic disequilibrium theory, Econometrica 48, pp. 283-305.

- GINSBURGH, V., TISHLER, A. and ZANG, I. (1980) Alternative estimation methods for two-regime models, European Economic Review, pp. 207-227.

- GOURIEROUX, C., LAFFONT, J.J. and MONFORT, A. (1980) Disequilibrium econometrics of simultaneous equation systems, Econometrica 48, pp. 75-96.

- ITO, T. (1980) Methods of estimation for multimarket disequilibrium models, Econometrica 48, pp. 97-126.

- KOOIMAN, P. and KLOEK, T. (1980) An aggregate two market disequilibrium model with foreign trade, Working Paper, Econometric Institute, Rotterdam.

- KOOIMAN, P. and KLOEK, T. (1980) The specification of spillovers in empirical disequilibrium models, Report 8031/E, Econometric Institute, Rotterdam.

- KOOIMAN, P. and KLOEK, T. (1981) An empirical two market disequilibrium model for dutch manufacturing, Working Paper, Econometric Institute, Rotterdam.

- KOOIMAN, P. (1982) Using business survey data in empirical disequilibrium models, London School of Economics and Political Science.

- MADDALA, G.S. and NELSON, F.D. (1974) Maximum likelihood methods for models of markets in disequilibrium, Econometrica 42, pp. 1013-1030.

- MEERSMAN, H. and PLASMANS, J. (1980) Goederen- en arbeidsmarkt in onevenwicht : een micro- en macro-economische analyse, SESO-report 80/87, UFSIA, Antwerp.

- MEERSMAN, H. and PLASMANS, J. (1980) A disequilibrium analysis of the commodity market, employment and hours of work, SESO-report 80/105, UFSIA, Antwerp.

- MEERSMAN, H. and PLASMANS J. (1982) An Econometric Quantity Rationing Model for the Labour Market. Reeks "Ter Discussie" no. 82.21 University of Tilburg.

- MOOD, A.M. and GRAYBILL, F.A. (1963), Introduction to the Theory of Statistics, Mc. Graw Hill Book C.J., New York.

- SNEESSENS, H.R. (1981) Theory and Estimation of macroeconomic rationing models, Springer-Verlag, Berlin.

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