EH

subfaculteit der econometrie

RESEARCH MEMORANDUM

TILBURG UNIVERSITY

DEPARTMENT OF ECONOMICS

Postbus 90153 - 5000 LE Tilburg
Netherlands

The economic interpretation of the
advertising effect of Lydia Pinkham

W.J. Oomens

ABSTRACT

In this paper we want to investigate the economic implications of the advertising effects of Lydia Pinkham. In 1964 Palda [5] computed distributed lag models on annual data. His model implied that advertising effects last about 6 years and his conclusion was that the management of the firm pursued nearly optimally their advertising effort. A test for data interval bias shows now that monthly data should be used in stead of annual data. Several models are estimated with maximum likelihood methods on the monthly data. It turns out that the implied duration of advertising effect is about 8 months and that actual advertising expenditures exceeded the optimal level by 34 percent.

Introduction.

Current marketing expenditure usually influences not only the current sales but also the sales in future periods, if the periods are not taken too long. These influences are called carryover effects: the marketing expenditures have effects on sales that carry over into future periods. According to Kotler [3] two types of carryover effects can be distinguished. One type is the delayed response effect, which arizes from the fact that delays occur between the time marketing dollars are spend and the time induced purchases occur. These delays occur between the period when the marketing expenditure is made and the marketing stimulus appears, between the appearance of the marketing stimulus and the time of its noting by potential buyers, between the time of perception and the time of purchase and finally between the time an order is placed or a purchase is made somewhere in the distribution system and the time the sales are recorded in the company. It should be noticed that these induced sales concern initial purchases by buyers who have bought competiting brands or who never have bought a product of the product class in question in earlier periods.

The other type of carryover effect is the customer holdover effect which arises from the fact that new customers created by the marketing expenditures remain customers for many subsequent periods. This effect can take two different pure forms. The marketing stimulus may increase the number of customers in future periods (new-buyer holdover effect) and the marketing stimulus may increase the average quantity purchased per period by a customer (the increased purchases holdover effect).

Because in practice the effects mentioned above cannot be distinguished they are usually taken together. To analyse the effectiveness of the marketing instruments we don't take the marketing expenditure in one period which has influence on the future sales but the sales in one period which is the consequence of the marketing expenditure in the same period and preceding periods.

A linear model can be written as:

$$y_t = \alpha + \sum_{i=0}^{\infty} \beta_i x_{t-i} + u_t \tag{1}$$

where

$$\sum_{i=0}^{\infty} \beta_i = \beta < \infty \qquad (2)$$

and

$$y_t \quad = \text{sales in period t measured in units or money}$$
$$\alpha, \beta_i = \text{constants}$$
$$x_t \quad = \text{marketing expenditures in period t}$$
$$u_t \quad = \text{a random disturbance term.}$$

The coefficient β_0 measures the short term marginal efficiency or the current effect of the marketing expenditures. The coefficients β_1, β_2,... measure the carryover effects of the marketing expenditures in the preceding periods t-1, t-2,... . The sum of the coefficients β_0, β_1,... represents the long term efficiency.

In general it is assumed that all β-coefficients are non-negative. Model (1) can now be rewritten as a distributed lag model:

$$y_t = \alpha + \sum_{i=0}^{\infty} w_i x_{t-i} + u_t \qquad (3)$$

where

$$w_i \geqslant 0$$

and

$$\sum_{i=0}^{\infty} w_i = 1$$

The w_i-coefficients represent the lag structure and are interpreted as probabilities of a discrete probability distribution.

Using the lag operator L defined as

$$L^k x_t = x_{t-k} \qquad , k = 0,1,2,... \qquad (4)$$

we can rewrite (3):

$$y_t = \alpha + \beta \sum_{i=0}^{\infty} w_i L^i x_t + u_t \quad \text{or} \tag{5}$$

$$y_t = \alpha + \beta W(L)x_t + u_t \tag{6}$$

where

$$W(L) = \sum_{i=0}^{\infty} w_i L^i \tag{7}$$

Very important is the shape and the duration of the lagged effects because of the optimal timing and distribution of marketing expenditure over a planning horizon. Coefficient β determines highly the economic results of the investment in marketing.

We want to investigate the economic implications of the advertising effects of Lydia Pinkham. 54 annual data and 78 montly data are available.

The data are unique because the advertising expenditures can be considered as the only explanatory variable of the sales. For this reason we use in future the term advertising expenditures instead of marketing expenditures

The first problem is to decide whether we choose the annual or the monthly data interval for the analysis. For practical reasons in 1964 Palda [5] used the annual data to analyze the duration of the advertising effects. If however the duration of the cumulative effects attributable to advertising is shorter than a year there is a data interval bias in the estimated duration interval. Griliches [2] presents a model-specification with which it is possible to test if the correct datainterval is used. Het discusses a model where the current independent variable influences the current dependant variable and where the disturbance term has a autoregressive structure, which results from a carry-over effect not completely attributed to past values of the independant variable:

$$y_t = \alpha + \beta x_t + u_t \tag{8}$$

$$u_t = \rho u_{t-1} + e_t \qquad |\rho| \leqslant 1 \qquad\qquad (9)$$

Combining the two equations results in:

$$y_t = (1-\rho)\alpha + \rho y_{t-1} + \beta x_t - \rho\beta x_{t-1} + e_t \qquad\qquad (10)$$

If we base ourself on the testequation

$$y_t = b_0 + b_1 y_{t-1} + b_2 x_t + b_2 x_{t-1} + e_t \qquad\qquad (11)$$

the hypothesis that the current effect model is true if

$$b_3 = - b_1 b_2.$$

We have estimated the testequation (11) with the annual and the montly data. Applying the test for data interval bias for the annual data results in

$$y_t = 232 + .765\ y_{t-1} + .609\ x_t - .382\ x_{t-1} \qquad\qquad (12)$$
$$\quad\ \ (.102) \qquad\quad (.136) \qquad\quad (.156)$$

The numbers in paratheses give the standard errors of the regression coefficients. Coefficient $\hat{b}_3 = -.382$ while $-\hat{b}_1 . \hat{b}_2 = - .609 * .765$ $= -.465$. Because the standard error of $\hat{b}_1 \hat{b}_2$ is surely greater than the smaller of the standard errors of $\hat{b}_1$ and $\hat{b}_2$ (.102) the hypothesis that $\hat{b}_3 = -\hat{b}_1\hat{b}_2$ may not be rejected. We have to conclude that the maximum duration interval of advertising effort is shorter than a year and that the cumulative effects if any, should be measured with monthly data. The result of the estimation of the testequation (10) with monthly data is

$$y_t = 398 + .463\ y_{t-1} + .264\ x_t + .209\ x_{t-1} \qquad\qquad (13)$$
$$\quad\ \ (.092) \qquad\quad (.080) \qquad (.801)$$

Coefficient $\hat{b}_3 = .209$ while $-\hat{b}_1 \hat{b}_2 = -.264 * .463 = -.122.$

The hypothesis that advertising expenditures only have effects on current sales should be rejected. This implies that the results from Palda's analysis of the annual data are not correct and in particular the favourable results of his economic analysis of the statistical estimates should not be correct.

The estimation of the cumulative effects of advertising.

The cumulative effects of advertising is estimated with maximum likelihood methods on the montly data. We used geometric lags, the Pascal distribution, rational lags and polynomial lags [4]. The best results we had with the geometric model with the geometric decline after one period. This model is:

$$y_t = \alpha + \beta_0 x_t + \beta_1 \sum_{i=0}^{\infty} (\lambda L)^i x_{t-1} + u_t$$

$$= \alpha + \beta_0 x_t + \frac{\beta_1}{1-\lambda L} x_{t-1} + u_t \tag{14}$$

The Koyck transformation: i.e. multiply (14) throughout by $(1-\lambda L)$ and rearange terms, gives

$$y_t = \alpha' + \lambda y_{t-1} + \beta_0 x_t + \beta_1' x_{t-1} + u_t - \lambda u_{t-1} \tag{15}$$

where

$$\beta_1' = \beta_1 - \lambda \beta_0$$

If u_t are serially independent to start with, the disturbance terms of (15) will be serially correlated. However we can rewrite (15) as

$$y_t = \alpha + (E(y_1)-\alpha)\lambda^{t-1} + \beta_0 z_{1t} + \beta_1' z_{2t} + u_t \tag{16}$$

where

$$z_{1t} = \sum_{i=0}^{t-2} \lambda^i x_{t-i} \qquad \text{and}$$

$$z_{2t} = \sum_{i=0}^{t-2} \lambda^i x_{t-i-1}$$

We know that $0 < \lambda < 1$. For each value of λ we construct the variables λ^{t-1} z_{1t} and z_{2t}, regress y_t on λ^{t-1}, z_{1t} and z_{2t} and choose that value of λ for which the residual sum of squares is a minimum. The variable λ^{t-1} gave technical problems in the estimation procedure because for somewhat larger values of t all values of λ^{t-1} are almost zero. For this reason we drop the variable λ^{t-1} and construct the variables z_{1t} and z_{2t} from the beginning but regress from later periods in the hope that the variable λ^{t-1} from that period takes the value zero.

The best result is

$$\hat{y}_t = 571 + .264 \, z_{1t} + .224 \, z_{2t} \qquad t = 12, 13, \ldots, \lambda = .57$$
$$\quad\;\; (83) \quad (.089) \qquad (.088)$$

or

$$\hat{y}_t = 571 + .264 \sum_{i=0}^{t-2} .57^i \, x_{t-1} + .224 \sum_{i=0}^{t-2} .57^i \, x_{t-i-1}.$$

The mean lag is $\dfrac{\beta_1' + \lambda \beta_0}{(\beta_0 + \beta_1')(1-\lambda)} = 1.8$ months.

Now we want to compute the 90% duration interval i.e. the number of months in which 90% of the total cumulative advertising effect has occured. If one defines r_n as the reponse of the dependant variable y at time t_{t+n} to a unit pulse of the independant variable x at time t_1 then $\beta_n = \sum_{i=0}^{n} r_i$ is the cumulative response over n periods.

If $\beta = \lim_{n \to \infty} \beta_n$ then the 90% duration interval is the least integer n^* such that $\dfrac{\beta_{n^*}}{\beta} \geqslant .9$.

First of all r_i should be defined. Therefore we take the equation:

$$y_t = \alpha' + \lambda y_{t-1} + \beta_0 x_t + \beta_1' x_{t-1}$$

Consider the expected response in y from a unit transitory change in x.

That is we assume

$$y_{t-1} = x_{t-1} = 0$$
$$x_t = 1 \quad \text{and}$$
$$x_{t+1} = 0 \quad i > 0.$$

Then

$$r_0 = y_t = \beta_0$$
$$r_1 = y_{t+1} = \beta_1' + \lambda\beta_0$$
$$r_i = y_{t+i} = \lambda^{i-1}(\beta_1' + \lambda\beta_0) \quad i = 2,3,\ldots$$

The total cumulative effect

$$\beta = \beta_0 \sum_{i=0}^{\infty} \lambda^i(\beta_1' + \lambda\beta_0) = \frac{\beta_0 + \beta_1'}{1-\lambda} \, .$$

The cumulative response over n periods

$$\beta_n = \beta - (r_{n+1} + r_{n+2} + \ldots)$$

$$= \beta - (\lambda r_n + \lambda^2 r_n + \ldots)$$

$$= \beta - \frac{\lambda r^n}{1-\lambda} \, .$$

We find

$$\frac{\beta_n}{\beta} = \frac{\beta_0 + \beta_1' - \lambda^n(\beta_1' + \beta_0)}{\beta_0 + \beta_1'}$$

The smallest non negative integer n^* such that

$$\frac{\beta_{n^*}}{\beta} = 1 - \lambda^{n^*} \left[\frac{\beta_1' + \lambda\beta_0}{\beta_0 + \beta_1'}\right] \geqslant .9 \quad \text{is found as follows}$$

$$\log .1 \geqslant n^* \log \lambda + \log \left[\frac{\beta_1' + \lambda\beta_0}{\beta_0 + \beta_1'}\right]$$

or   *)

$$n^* \leqslant \frac{\log .1 - \log \frac{\beta_1' + \lambda \beta_0}{\beta_0 + \beta_1}}{\log \lambda}$$

We find $n^* \leqslant 3.6$.

Thus after 3.6 months already 90% of the total expected cummulative effect of advertising has occured. After about 8 months no cumulative advertising effect may be expected.

<u>The economic interpretation of the statistical estimates.</u>

The estimated short-term effect of advertising is

$$\mu(ST) = \frac{dy_t}{dx_t} = \beta_0 = .264.$$

The short term advertising elasticity is

$$n_x(ST) = \frac{dy_t}{dx_t} \frac{\bar{x}}{\bar{y}} = .264 \cdot \frac{601}{1276} = .124$$

where $\bar{x}$ and $\bar{y}$ are the mean value of advertising expenditures and sales respectively over a period of 78 months.

To derive the long run effect of advertising we take the equilibrium equation

$$y_e = \alpha + \frac{\beta_0 + \beta_1'}{1 - \lambda} x_e$$

where y_e and x_e are the equilibrium values of sales and advertising expenditures.

The estimated long-term marginal sales effect of advertising is

$$\mu(LT) = \frac{dy_e}{dx_e} = \frac{\beta_0 + \beta_1'}{1 - \lambda} = \beta = \frac{0,264 + 0,224}{1 - 0,57} = 1.135.$$

*) Compare [1].

The corresponding long-term elasticity is

$$\eta_x'(LT) = \mu(LT) \frac{\bar{x}}{\bar{y}} = .535.$$

According to the Dorfman-Steiner theorem in the optimal situation should hold

$$\mu(LT) = \frac{1}{w_m}$$

where

w_m = marginal percentage of gross margin $\frac{p-MC}{p}$ and

MC = non advertising cost.

Pinkham's total expenses, exclusive of advertising and capital cost averaged 25 percent of sales during the entire period of 52 years. Assuming the cost structure did not change essentially during the last 78 months of this period and imputing capital cost at 10 percent we have

$$\frac{p-MC}{p} = .65.$$

We conclude there is a great difference between

$$\frac{1}{w_m} = 1.54 \qquad \text{and}$$

$$\mu(LT) = 1.14.$$

From the Dorfman Steiner theorem we can compute the value of the advertising-sales ratio at optimum:

$$\mu(LT) = \frac{y}{x} \eta_x(LT) = \frac{1}{w_m}.$$

$$\frac{x}{y} = \eta_x(LT) \cdot w_m$$

$$= \eta_x(LT) \frac{p-Mc}{p}$$

$$= .535 \times .65 = .35.$$

The estimated optimal advertising sales ratio is .35 while the mean

value of the observed advertising-sales ratio is .47. We conclude that actual advertising expenditures exceed the optimal level by 34 percent. An additional confirmation of the conclusion that the firm is substantially overspending on advertising is obtained when the firm's marginal return on advertising is computed:

$$w \; \mu(LT) \; = \; \frac{p-c}{p} \; \mu(LT),$$

where c = variable cost

The percentage of gross margin is .75 for Lydia Pinkham. We get

$$w.\mu(LT) \; = \; .75 \times 1.135 \; = \; .85$$

Thus a dollar spent in advertising produces approximately a net return of only 85 cents.

Conclusion

The economic analysis of the statistical results of models which are estimated on the annual data are not correct. The long implied duration intervals of advertising effects and the corresponding optimistic econo-mic interpretation are the result of data interval bias and should be rejected. Monthly data should be used to overcome this bias. Several models are estimated of which the geometric model with a geometric delay after one month gave the best statistical results. To get consistent estimates we used maximum likelihood methods. This method is troublesome because iterative procedures should be used to find optimal values for the decayrate. The parameter estimates of this model indicates that the advertising elasticities are too low and that the firm has been over-spending substantially.

References

[1]   Clarke, D.G., 'Econometric measurement of the duration of adverti-
       sing effect on sales', Journal of Marketing Research, 1976, 13,
       347-357.

[2]   Griliches, Z., 'Distributed lags: A survey', Econometrica, 1967,
       35, 16-49.

[3]   Kotler, Ph., Marketing Decision Making: a model building approach,
       Holt, Rinehart and Winston, Inc., New York, 1971.

[4]   Oomens, W.J., 'Vertragingsverdelingen in de marketing', Jaarboek
       van de Nederlandse Vereniging van Marktonderzoekers 1980, 155-165.

[5]   Palda, K.S., The measurement of cumulative advertising effects,
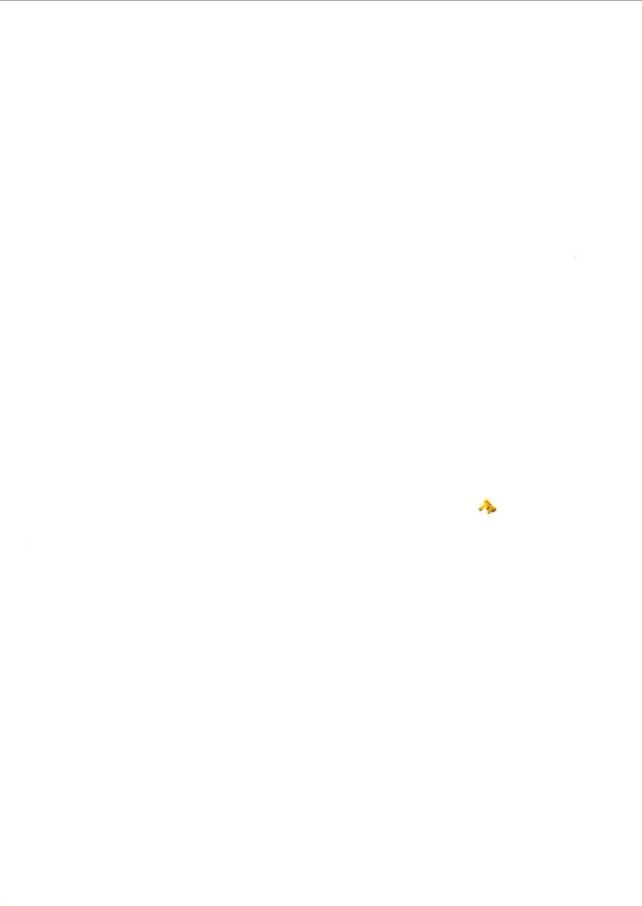   Prentice Hall, Englewood Cliffs, N.Y., 1964.

IN 1982 REEDS VERSCHENEN

107 A.J. de Zeeuw
     Hierarchical decentralized optimal control in econometric policy
     models.

108 A. Kapteyn en T. Wansbeek
     Identification in Factor Analysis.

109 G. van der Laan en A.J.J. Talman
     Simplical Algorithms for finding Stationary Points, a unifying
     description.

110 P. Boot
     Ekonomiese betrekkingen tussen Oost en West Europa.

111 P.B. van der Genugten
     The asymptotic behaviour of the estimated generalized least squares
     method in the linear regression model.

112 J. P.C. Kleijnen en A.J. van Reeken
     Principle of computer charging in a university-like organization.

113 H. Tigelaar
     The informative sample size for dynamic multiple equation systems
     with moving average errors.

114 Drs. H.G. van Gemert, Dr. R.J. de Groof en Ir. A.J. Markink
     Sektorstruktuur en Economische Ontwikkeling.

115 P.H.M. Ruys
     The tripolar model: a unifying approach to change.

116 A.J. de Zeeuw
     Policy solutions for Mini-Interplay, a Linked Model for two Common
     Market Countries.

117 F.J.M. van Doorne en P.H.M. Ruys
     Die Struktur einer Sprechhandlung in Habermas' Forschungsprogramm.
     Formale Analyse mit den Mitteln des tripolaren Modells.

118 A.J.J. Talman en G. van der Laan
     Simplicial Approximation of Solutions to the Non-Linear Complemen-
     tarity Problem.

119 P. Boot
     The East German Planning System Reconsidered.

120 J.P.C. Kleijnen
     Regression Metamodel Summarization of Model Behaviour.

121 P.G.H. Mulder en A.L. Hempenius
     Evaluating life time in a competing risks model for a chronic
     desease.

IN 1982 REEDS VERSCHENEN (vervolg)

122 A.J.J. Talman en G. van der Laan
     From Fixed Point to Equilibrium.

123 J.P.C. Kleijnen
     Design of simulation experiments.

124 H.L. Theuns en A.M.L. Passier-Grootjans
     Internationaal toerisme; een gids in de algemene basisliteratuur en
     het bronnenmateriaal.

125 J.H.F. Schilderinck
     Interregional Structure of the EUROPEAN community.
     Part I: Imports and Exports, Sub-divided by Countries Aggregated
     According the Branches of the European Community Interregional
     Input-Outputtables 1959, 1965, 1970 and 1975.

IN 1983 REEDS VERSCHENEN:

126 H.H. Tigelaar
Identification of noisy linear systems with multiple arma inputs.

127 J.P.C. Kleijnen
Statistical Analysis of Steady-State Simulations: Survey of Recent
Progress.

128 A.J. de Zeeuw
Two notes on Nash and Information.

129 H.L. Theuns en A.M.L. Passier-Grootjans
Toeristische ontwikkeling – voorwaarden en systematiek; een selec-
tief literatuuroverzicht.

130 J. Plasmans en V. Somers
A Maximum Likelihood Estimation Method of a Three Market Disequili-
brium Model.

131 R. van Montfort, R. Schippers, R. Heuts
Johnson S_U-transformations for parameter estimation in arma-models
when data are non-gaussian.

132 J. Glombowski en M. Krüger
On the Rôle of Distribution in Different Theories of Cyclical
Growth.

133 J.W.A. Vingerhoets en H.J.A. Coppens
Internationale Grondstoffenovereenkomsten.
Effecten, kosten en oligopolisten.