TILBURG UNIVERSITY
KATHOLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

[redacted]

A DIFFERENTIAL GAME BETWEEN GOVERNMENT AND FIRMS: A NON-COOPERATIVE APPROACH

Raymond H.J.M. Gradus

FEW 303

A DIFFERENTIAL GAME BETWEEN GOVERNMENT AND FIRMS: A NON-COOPERATIVE APPROACH

Raymond H.J.M. GRADUS
Tilburg University, Postbox 90153-5000 LE Tilburg, The Netherlands

In this paper we present a differential game where government and firm interact through investment and tax policy. Within an optimal control framework the firm maximizes the future stream of dividend, while the government optimizes a utility function, which depends on government consumption. Attention is paid to different solution concepts (Nash, Stackelberg and Pareto), information structures (open-loop and feedback) and their economic interpretations. Attention is also paid to the problems of time-inconsistency, credibility and reputation, which can arise by using differential games. One of the conclusions is that credibility of governmental policy may have a great influence on the market value of the firm.

1. INTRODUCTION

Governmental policy has a great influence on the way firms behave even in a free market economy. A crucial question in this respect is: how can the government by its policy influence the decisions of the firms so that its own objectives are achieved ? In this paper our interest focusses on the corporate tax policy of the government as an instrument to achieve its goals. A high level of corporate tax may induce a lower level of investment and that may lead to a lower level of corporate tax earnings in the future. So there is a certain trade off between tax earnings now and tax earnings in the future. But if the government asks a low rate to have more tax earnings in the future the question is: will the firm use this money for investment ? So the government's decision about the corporate tax rate is affected by the investment policy of the firm.

In this paper we present a differential game in which the government announcing a certain tax rate and the firms decide whether to invest their money back into the firm or to pay out dividend. We use a differential game to model interactions between government and firms, because the actions carried out by one of the players will influence the outcome for the other. We will pay special attention to different solution concepts like Nash, Stackelberg and Pareto and information structures like feedback and open-loop. The conceptual framework is related to Lancaster (1973) and Hoel (1978). We deal with the problems of time-inconsistency and commitment which can arise by using this technique (e.g. Kydland and Prescott (1977) and Reinganum and Stokey (1985)).

The model for the firms is related to the models designed by, for instance, Leban and Lesourne (1982), Van Loon (1982) and Van Schijndel (1986), which dealt with dividend and investment optimization within an optimal control framework. Following Theil (1964) we take the government as entity with a single objective function.

In section 2 we present the model which is meant to be a heuristic framework for governmental policy and firm behaviour. In section 3 the solutions for the different solution concepts and their economic interpretation are given. Special attention is paid to a comparison of the concepts. In section 4 we discuss two important extensions of the model presented in section 2. For these extensions we only discuss the Nash solution. Finally, in section 5 we make some remarks and suggestions for future research.

2. THE MODEL

2.1. The firm

The model is developed within a deterministic setting. We assume that the firms can be represented by one and this firm behaves as if it maximizes the shareholders' value of the firm, which consists of the sum of the discounted dividends over the planning period. We assume that shareholders have access to perfect capital markets, so that the discount rate equals the market interest rate. At the planning horizon the shareholders will value capital good stock as $bK(T)e^{-rT}$, where it is reasonable to assume that $0 \leq b \leq 1$. So the objective function is:

$$\max_{I} \int_{0}^{T} D(t)e^{-rt}dt + bK(T)e^{-rT}, \qquad (1)$$

in which: T : planning horizon

t : time

D(t) : dividend

I(t) : gross investment

K(t) : capital good stock

r : market interest rate

We assume that there is a linear relation between capital and labour. This is the case if there is a Leontief technology or if a firm operates under constant returns to scale with fixed factor prices. Assuming that there are no problems with attracting labour and, output price and wage rate are constant, we get a linear relation between profit and capital good stock:

$$\begin{aligned} R(t) &= pQ(t) - wL(t) \\ &= (p\kappa - w\frac{\kappa}{\alpha})K(t) \\ &= qK(t), \end{aligned} \qquad (2)$$

where: R(t): profit (before tax payment)

Q(t): output

L(t): labour

p: output price

w: wage rate

κ: capital productivity

α: labour productivity

q: rentability of capital good stock

In subsection 4.1 we will relax the assumption of a linear relation between profit and capital good stock by assuming that the firm operates under decreasing returns to scale. About the financial structure of the firm we assume that investment is completely financed by retained earnings (there is no debt). Furthermore, profit after taxation can be used for investment or to pay out dividend:

$$R(t) - TX(t) = D(t) + I(t), \qquad (3)$$

in which: TX(t): tax payment.

The impact of investment on the production capacity is described by the generally used formulation of net investment:

$$\dot{K}(t) = I(t) - aK(t), \tag{4}$$

where: a: depreciation rate

2.2. The government

For the government too we make some simplifying assumptions. Its utility function depends in general on macro-economic variables like government consumption, employment and so on. In this paper we take a utility function, which depends on government consumption:

$$\max_{\tau}. \int_0^T U(G(t))e^{-rt}dt, \tag{5}$$

where U(.): the utility function for the government

G(t): the government consumption

τ: corporate tax rate

Note that the government also has access to perfect capital markets. In this paper we take into consideration two different utility functions, a linear and a logarithmic one:

$$U(G(t)) = G(t) \quad \text{(section 3 and subsection 4.1.)} \tag{6a}$$

$$U(G(t)) = \ln(G(t)) \quad \text{(subsection 4.2.)} \tag{6b}$$

We assume that the spendings of the government are not productive. For instance we may think of building houses and bridges, hospital care, education and military forces. Furthermore, all tax payments received will immediately be spent and the government is not able to spend more than it receives (i.e. no budgetary deficit):

$$TX(t) = G(t) \tag{7}$$

The tax payments paid by the firm are given by:

$$TX(t) = \tau(t)(R(t) - aK(t)) \tag{8}$$

Finally, corporate tax rate is restricted between τ_1 and τ_2:

$$0 < \tau_1 \leq \tau(t) \leq \tau_2 < 1, \ \tau_1 \neq \tau_2 \tag{9}$$

For the shareholders investing in the firm is more profitable than putting their money into the bank:

$$r < (q-a)(1-\tau_2) \tag{10}$$

Equation (10) ensures that the marginal rentability of investment exceeds the opportunity costs of financial investments.

3. THE SOLUTIONS

3.1. The Nash, Stackelberg and Pareto solution

In the previous section we have described the objectives of both players, the dynamics of the system and the strategy space. As argued by Van der Ploeg (1987) the formal structure of the interaction between government and firm depends on the absence or the presence of binding contracts (feedback or open-loop) and the mood of playing (Nash, Stackelberg and Pareto). In the Stackelberg game we take the government as the leader. We are able to prove that within this model the feedback-Nash and -Stackelberg solution are equal to the open-loop Nash solution (see appendix 1). Therefore, it suffices to only present the open-loop solutions, where we have to remember that open-loop Nash is a candidate for feedback-Nash and -Stackelberg. Because of the linearity of the Hamiltonians in the control variables the solution has a bang-bang structure.

3.1.1. The Nash solution

	$t \in [0,\bar{t})$	$t \in [\bar{t},T]$
tax rate	τ_1	τ_2
investment	max. investment	zero investment

Table 1. The Nash solution (if $\tau_2 \geq \frac{q}{(2-b)(q-a)}$)

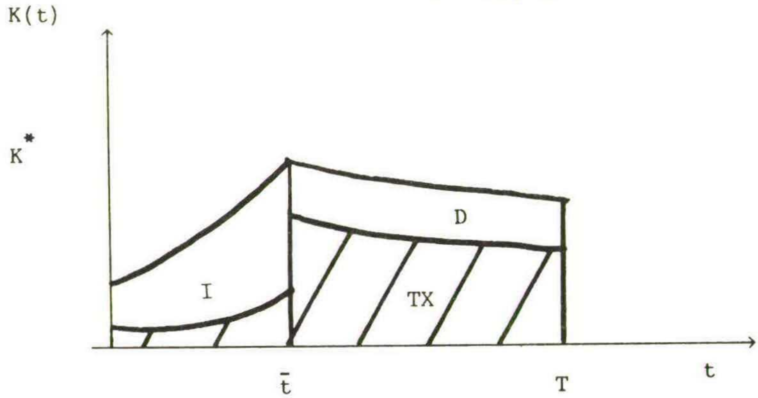

Figure 1. The optimal trajectories of capital good stock, dividend, investment and tax

	$t \in [0,\hat{t})$	$t \in [\hat{t},\bar{t})$	$t \in [\bar{t},T]$
tax rate	τ_1	τ_2	τ_2
investment	max. investment	max. investment	zero investment

Table 2. The Nash and Stackelberg solution (if $\tau_2 < \frac{q}{(2-b)(q-a)}$)

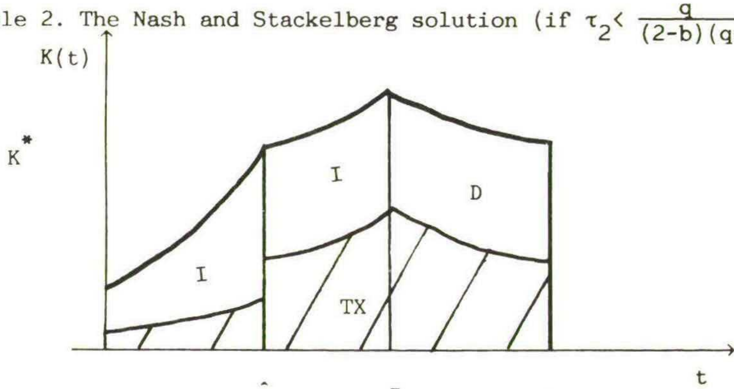

Figure 2. The optimal trajectories

$$\bar{t} = T + \frac{1}{a + r}\ln\{(1 - \frac{a + r}{q-\tau_2(q-a)})/(1 - b\frac{a + r}{q-\tau_2(q-a)})\}$$

$$\frac{d\bar{t}}{dr} < 0,\ \frac{d\bar{t}}{da} < 0,\ \frac{d\bar{t}}{db} > 0,\ \frac{d\bar{t}}{dq} > 0,\ \frac{d\bar{t}}{d\tau_2} < 0 \qquad (11)$$

The Nash solution is seen to consist of two phases at least (see table 1 and 2). The government starts to tax at a low rate and the firm invests at its maximum rate. The reason for the government to ask the low rate is that more money is left to the firm to invest, so that future tax earnings will be greater. In the beginning the firm invests at its maximum rate in order to be able to pay out more dividend in the future. Before the end of the planning period there is always, if $b < 1$, a period during which the government asks the high rate and the firm pays out dividend. Because the end of the planning period comes nearer the shareholders are more interested in collecting dividend and the government in collecting tax earnings. However, the time-points where the government and the firm want to switch their policy will in general not be the same and depend on the parameters of the model. In the Nash solution at the moment that the firm stops investment the government will immediately react by introducing the high rate. In figure 1 we have drawn the optimal trajectories of the capital good stock and other variables under the assumption that (without loss of generality) $q=1$.

Note that there are two possibilities for the Nash solution. In the case that $\tau_2 \geq \frac{q}{(q-a)(2-b)}$ at the time-point $\bar{t}$ the firm stops investment and pays out dividend, while the government raises the tax rate. In spite of the fact that the government wants more investment, it cannot force the firm to invest. In the situation that $\tau_2 < \frac{q}{(q-a)(2-b)}$ (see table 2) the firm keeps on investing even after the government has raised the tax rate. The switch from low to high tax will take place at $\hat{t}$, where $\hat{t}$ can be solved numerically.

3.1.2. The open-loop Stackelberg solution

	$t \in [0,t_1)$	$t \in [t_1,t_2)$	$t \in [t_2,T]$
tax rate	τ_1	τ_1	τ_2
investment	max. investment	zero investment	zero investment

Table 3. The open-loop Stackelberg solution (if $\tau_2 \geq \frac{q}{(2-b)(q-a)}$)

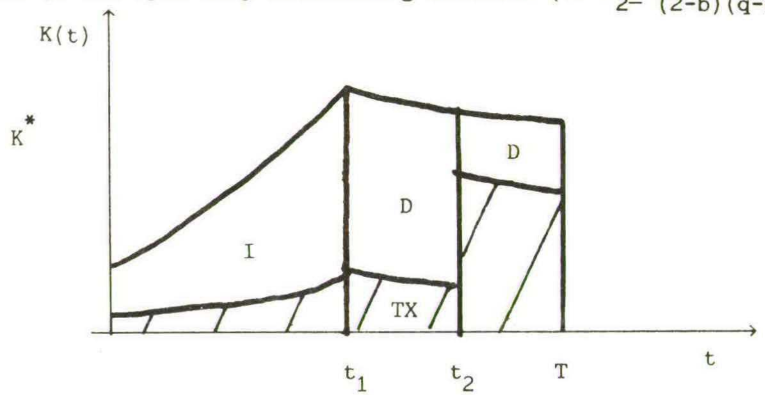

Figure 3. The optimal trajectories

$$t_1 = \min[T+\frac{1}{a+r}\ln\{(1-(2-b)\frac{a+r}{q}\}, T+\frac{1}{a+r}\ln\{(1-\frac{a+r}{q-\tau_1(q-a)})/(1-b\frac{a+r}{q-\tau_1(q-a)})\}]$$

$$\frac{dt_1}{dr} < 0,\ \frac{dt_1}{da} < 0, \frac{dt_1}{dq} > 0,\ \frac{dt_1}{db} > 0,\ \frac{dt_1}{d\tau_2} = 0,\ \frac{dt_1}{d\tau_1} \leq 0 \quad (12)$$

$$t_2 = \min[T, T - \frac{1}{a+i}\ln\{1 + \frac{(1-(2-b)\frac{q-a}{q}\tau_1)(a+i)}{(\tau_2-\tau_1)(q-a)(1-2(a+i/q))}\}]$$

$$\frac{dt_2}{dr} \leq 0,\ \frac{dt_2}{da} \leq 0, \frac{dt_2}{dq} \geq 0,\ \frac{dt_2}{db} \geq 0,\ \frac{dt_2}{d\tau_2} \geq 0,\ \frac{dt_2}{d\tau_1} \geq 0 \quad (13)$$

In the case that $\tau_2 < \frac{q}{(q-a)(2-b)}$ there is no difference between the Nash- and the open-loop Stackelberg solution. The fact that the government may announce its strategy first yields no advantage. However, in the case that $\tau_2 \geq \frac{q}{(q-a)(2-b)}$ there is an interesting difference between the two solutions.

In the Stackelberg game the firm will switch from investment to dividend at the time-point t_1. It can easily be shown that this switch will happen at a later point of time than in the Nash game. The firm's investment period is longer in the Stackelberg solution than in the Nash solution, so there are more capital goods in the economy if Stackelberg is played. The reason for this longer period of investment is that the government will postpone the the high rate $(t_2 > t_1 > \bar{t})$.

In the Nash game the government wants more investment, but it cannot force the firm to do so. In the Stackelberg game, where a phase with zero investment and low tax rate is possible (such a phase could not exist in the Nash game), the government can influence the investment policy in such a way that its own objective function will have a higher value. The government achieves this by announcing at the start of the planning period a relatively long period of low tax. In this way investing at the maximum rate becomes more profitable, so the firm will do this during a longer period. Announcing a period of low tax only makes sense if the government has a greater affinity to investment than the firm (i.e. $\tau_2 \geq \frac{q}{(q-a)(2-b)}$).

3.1.3. The Pareto solution

For our model we can derive the Pareto solution, a well-known concept supposing that the firm and government cooperate. In principle there are infinitely many Pareto solutions, depending on the bargaining power of player 2 against player 1 given by μ (e.g. De Zeeuw (1984)). The problem is now to maximize, with respect to the admissible strategies and dynamics, $J = J_G + \mu J_F$, where J_F and J_G are given by (1) and (5).

We distinguish three possible situations:

i) $\mu < 1$ (the government in a strong bargaining position)

ii) $\mu = 1$

iii) $\mu > 1$ (the firm in a strong bargaining position)

	$t \in [0,\tilde{\tilde{t}})$	$t \in [\tilde{\tilde{t}},\tilde{t})$	$t \in [\tilde{t},T]$
tax rate	τ_1	τ_2	τ_2
investment	max. investment	max. investment	zero investment

Table 4. The Pareto-solution (if $0 < \mu < 1$)

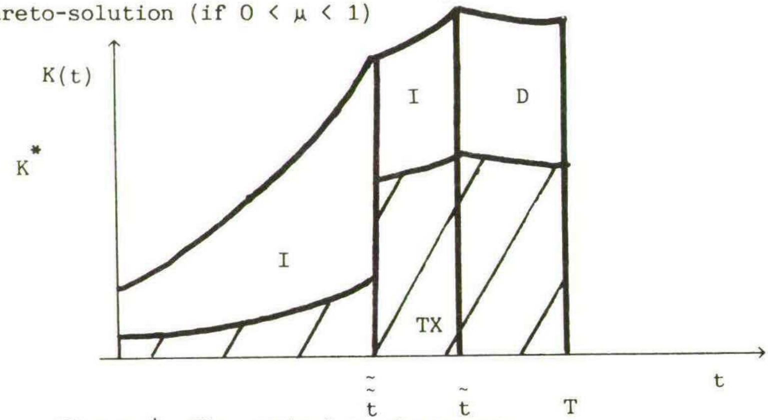

Figure 4. The optimal trajectories

$$\tilde{\tilde{t}} = \tilde{t} - \frac{1}{(q-a)(1-\tau_2)-r}\ln\{\frac{r-(q-a)}{r-(q-a)(1-\tau_2)}(\mu + \frac{\tau_2(q-a)}{(q-a)(1-\tau_2)-r})^{-1}\}$$

$$\frac{d\tilde{\tilde{t}}}{dr} < 0,\ \frac{d\tilde{\tilde{t}}}{da} < 0,\ \frac{d\tilde{\tilde{t}}}{dq} > 0,\ \frac{d\tilde{\tilde{t}}}{db} > 0,\ \frac{d\tilde{\tilde{t}}}{d\tau_2} > 0,\ \frac{d\tilde{\tilde{t}}}{d\tau_1} = 0,\ \frac{d\tilde{\tilde{t}}}{d\mu} < 0 \qquad (14)$$

$$\tilde{t} = \min[T, T + \frac{1}{a+r}\ln\{1 - \frac{(\mu-b)(a+r)}{\mu q+(1-\mu)\tau_2(q-a)}\}]$$

$$\frac{d\tilde{t}}{dr} \leq 0,\ \frac{d\tilde{t}}{da} \leq 0,\ \frac{d\tilde{t}}{dq} \geq 0,\ \frac{d\tilde{t}}{db} \geq 0,\ \frac{d\tilde{t}}{d\tau_2} \geq 0,\ \frac{d\tilde{t}}{d\tau_1} = 0,\ \frac{d\tilde{t}}{d\mu} > 0 \qquad (15)$$

In the first situation, $\mu < 1$, the solution consists of three phases. In the first phase the government asks the low rate and the firm invests all its money. At the time-point $\tilde{\tilde{t}}$ the government switches from the low to the high tax rate. But because of the strong bargaining position of the government the firm continues investment until $\tilde{t}$. The stronger the bargaining position of the government the earlier the tax switch and the later the dividend switch will take place.

	$t \in [0,t^*)$	$t \in [t^*,T]$
tax rate	τ_1	$\tau \in [\tau_1,\tau_2]$
investment	max. investment	zero investment

Table 5. The Pareto-solution (if μ = 1)

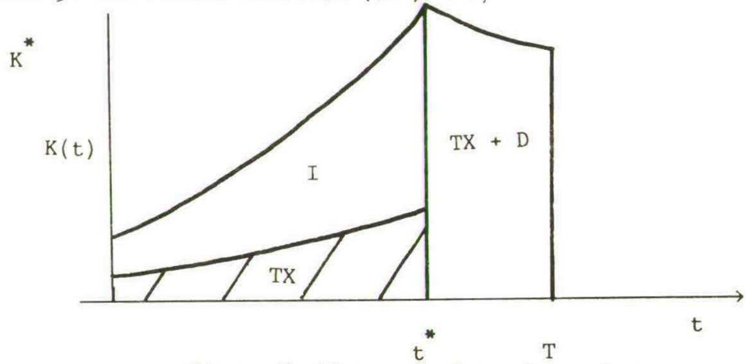

Figure 5. The optimal trajectories

$$t^* = T + \frac{1}{a+r}\ln\{1 - (1-b)\frac{a+r}{q}\}$$

$$\frac{dt^*}{dr} < 0,\ \frac{dt^*}{da} < 0,\ \frac{dt^*}{dq} > 0,\ \frac{dt^*}{db} > 0,\ \frac{dt^*}{d\tau_2} = 0,\ \frac{dt^*}{d\tau_1} = 0 \qquad (16)$$

In the second situation (see table 5), where μ=1, both players have equal bargaining power. Also in this case the planning period begins with low tax and maximal investment. At the time-point t^* the firm starts paying out dividend and the government asks an arbitrary tax rate $\tau \in [\tau_1,\tau_2]$. Because we have maximized the sum of both objectives there is no conclusion possible about the way profit is divided between government and shareholders. The Pareto solution will only give an answer to the question of the value of the sum of both objectives.

	$t \in [0,t')$	$t \in [t',T]$
tax rate	τ_1	τ_1
investment	max. investment	zero investment

Table 6. The Pareto-solution (if μ > 1)

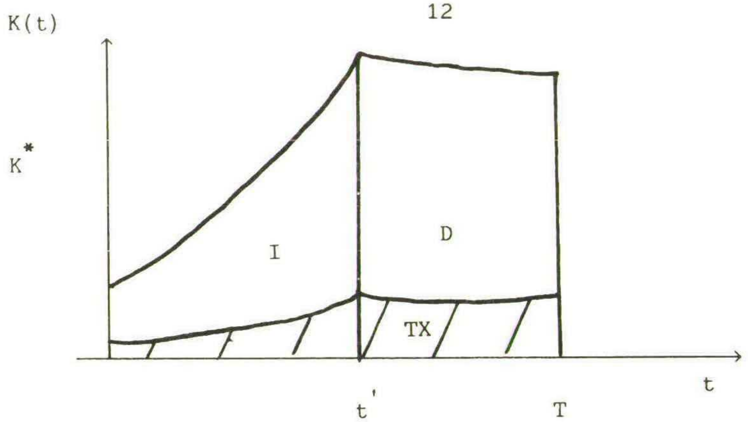

Figure 6. The optimal trajectories

$$t' = T + \frac{1}{a+r}\ln\{1 - \frac{(\mu-b)(a+r)}{\mu q+(1-\mu)\tau_1(q-a)}\}$$

$$\frac{dt'}{dr} < 0,\ \frac{dt'}{da} < 0,\ \frac{dt'}{dq} > 0,\ \frac{dt'}{db} > 0,\ \frac{dt'}{d\tau_2} = 0,\ \frac{dt'}{d\tau_1} < 0,\ \frac{dt'}{d\mu} < 0 \qquad (17)$$

In the situation that the firm is in a strong bargaining position the government sticks to its low tax rate, while the firm switches from investment to dividend at t'. The greater μ, the sooner the dividend switch will take place.

3.2. A comparison of the three concepts

The optimal values of control variables (the tax and investment rate) for the different solotion concept (see tables 1 through 6) generate the following values for the objective functions:

i) Nash

$$J_G(I^N,\tau^N) = \frac{(q-a)\tau_1}{(q-a)(1-\tau_1)-r}\{e^{-r\bar{t}}K(\bar{t})-K_0\} + \frac{(q-a)\tau_1}{q-\tau_2(q-a)}e^{-r\bar{t}}K(\bar{t}) \qquad (18)$$

$$J_F(I^N,\tau^N) = e^{-r\bar{t}}K(\bar{t}) \qquad (19)$$

ii) Stackelberg

$$J_G(I^S,\tau^S) = \frac{(q-a)\tau_1}{(q-a)(1-\tau_1)-r}\{e^{-rt_1}K(t_1)-K_0\} + e^{-rt_1}K(t_1) \qquad (20)$$

$$J_F(I^S,\tau^S) = e^{-rt_1}K(t_1) \tag{21}$$

iii) Pareto

$$\mu=1:\ J_G(I^P,\tau^P) = \frac{(q-a)\tau_1}{(q-a)(1-\tau_1)-r}\{e^{-rt^*}K(t^*)-K_0\} + \frac{(q-a)\tau}{q}e^{-rt^*}K(t^*) \tag{22a}$$

$$J_F(I^P,\tau^P) = e^{-rt^*}K(t^*) \tag{23a}$$

$$\mu>1:\ J_G(I^P,\tau^P) = \frac{(q-a)\tau_1}{(q-a)(1-\tau_1)-r}\{e^{-rt'}K(t')-K_0\} + \frac{\mu(q-a)\tau_1}{\mu q+(1-\mu)(q-a)\tau_1}e^{-rt'}K(t')$$

$$J_F(I^P,\tau^P) = e^{-rt'}K(t')\frac{q-(q-a)\tau_2}{\mu q+(1-\mu)(q-a)\tau_1} \qquad \text{(22b) and (23b)}$$

$$\mu<1:\ J_G(I^P,\tau^P) = \frac{(q-a)\tau_1}{(q-a)(1-\tau_1)-r}\{e^{-r\tilde{\tilde{t}}}K(\tilde{\tilde{t}})-K_0\} + \frac{(q-a)\tau_2}{(q-a)(1-\tau_2)-r}\{e^{-r\tilde{t}}K(\tilde{t})-$$

$$e^{-r\tilde{\tilde{t}}}K(\tilde{\tilde{t}})\} + \frac{\mu(q-a)\tau_2}{\mu q+(1-\mu)(q-a)\tau_2}e^{-r\tilde{t}}K(\tilde{t}) \tag{22c}$$

$$J_F(I^P,\tau^P) = e^{-r\tilde{t}}K(\tilde{t})\frac{q-(q-a)\tau_2}{\mu q+(1-\mu)(q-a)\tau_2} \tag{23c}$$

where: J_F and J_G are given by (1) and (5)

: (I^N,τ^N): set of strategies by the Nash solution

: $K(\bar{t})$: capital good stock at time-point $\bar{t}$

A derivation of (18)-(23) is straightforward given the time-points (11)-(17) and the objective functions (A1.3) and (A1.4). So there are many Pareto solutions, depending on the parameter μ, and if $\mu=1$, on the choice of the tax rate in the final phase. A Pareto solution means that, if a set of strategies is chosen a different outcome, at least one of the players has higher costs. The Nash and Stackelberg solution are not Pareto-optimal, so there is a set of Pareto solutions which gives a higher value for both players. In appendix 2 we have calculated the objective functions for an example. In this case the Nash solution is dominated by a great set of

Pareto solutions. Only for small values of μ the objective function for the firm will be lower (see figure 7).

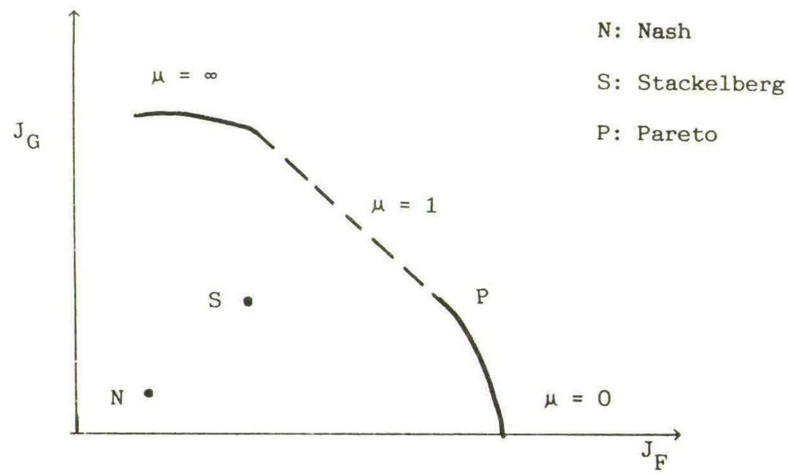

Figure 7. The value of the objective functions by different solution concepts

In comparison with the Nash solution the outcome for the open-loop Stackelberg game is better for both players. Not only is the leader better off, but the follower also reaches a greater value of his objective function. From this point of view there is an incentive to play open-loop Stackelberg. Note that for the example we have calculated (see appendix 2), the outcome for the Stackelberg game is better than for most Pareto solutions. Only for μ close to one the Pareto solution will yield a higher value for J_F and J_G. Also this could be an incentive to play Stackelberg.

However, a well-known disadvantage of an open-loop Stackelberg solution is the possibility of time-inconsistency. In our problem it is easy to understand that if the government has the possibility, at a moment between t_1 and t_2 (see table 3), to make a new initial plan, the high tax rate is the plan, so the solution is time-inconsistent. So if the follower has no reason to believe that the leader will stick to his initial plan open-loop is no longer a useful concept. In this case the feedback-Stackelberg concept, which for this model yields the same solution as the open-loop Nash-concept, can be used. In the recent literature there has been a growing interest in the concept of reputation to solve the problem of time-inconsistency. If the game is played indefinitely many times or if there is a situation of imperfect information, reputational forces can be important to prevent the government from cheating (e.g. Van der Ploeg (1987, sect. 7), Kreps and Wilson (1982)).

We can interpret (1) as the market value of the firm:

$$V(0) = \int_0^T D(t)e^{-rt}dt + bK(T)e^{-rT} \tag{24}$$

As pointed out by for example Hayashi (1982) V(0) will be influenced by tax rules taking into account corporate tax rate and depreciation formulas. Because of the fact that corporate tax policy of the feedback and open-loop Stackelberg solution will be different, the market value of the firm depends on the choice between open-loop or feedback. For both players it is better to play open-loop, but this makes only sense if the firm believes that government will stick to its initial plan. So credibility of governmental policy can play an important role in the market value of the firm and a change in this credibility can cause an enormous fall or raise of the share prices. There can be different reasons why the credibility of governmental policy will change. For example the government has announced a policy of low tax, but because of its high deficit it is no longer credible, that it will stick to this policy. Perhaps this has happened in oktober 1987 to U.S.-government and is one of the possible answers to the fall of the share prices at that moment.

3.3. A further comparison of Nash and Stackelberg solution

In appendix 1 we derived that the firm will switch from investment to dividend exactly at the moment that its valuation of a marginal increase in the capital good stock, i.e. $\lambda_2(t)$(=the firm's shadow price of capital goods), falls below unity. In other words the firm continues to invest if the marginal earnings of an extra dollar investment, i.e. $\lambda_2(t)$, are greater than the marginal costs, i.e. one dollar. In terms of finance theory we can conclude that investment is profitable as long as the net present value of marginal investment is greater than zero. For the government $\lambda_1(t)$ can also be interpreted as the valuation of a marginal increase in capital good stock. We can see this valuation as a measure of government's affinity to the firm's investment.

First, we study the Nash solution. If $\tau_2 < \frac{q}{(q-a)(2-b)}$, the government's valuation of a marginal increase in capital good stock is already below unity before the time-point that the firm switches from investment to dividend, so the government has already switched from low to high tax. If $\tau_2 \geq \frac{q}{(q-a)(2-b)}$, the valuation is still greater than one, at the moment that the firm stops investment. The government wants more investment, because the marginal earnings of an extra dollar investment are greater than the marginal costs. The government compares the stream of future tax earnings due to an extra dollar investment against collecting it now. In spite of this it cannot force the firm to continue investing.

In the Stackelberg case, however, the firm will switch from investment to dividend at that moment that not only its own valuation of a marginal increase in capital good stock equals one, but also the valuation of the government equals one. The government uses its tax policy to manipulate the firm's investment policy in such a direction that at the switching moment λ_1 equals one. This is interesting because we have not only given an economic interpretation of the results, but we have also derived decision rules for governmental policy and firm behaviour. The decision rules can be used to derive the open-loop Stackelberg solutions for more complex models (e.g. Gradus (1987)).

4. A CONCAVE PROFIT FUNCTION AND OTHER MODIFICATIONS

4.1. A concave profit function

As mentioned in section 2.1., one of the restrictive assumptions of the model is the linear relation between profit and capital good stock (see equation (2)). In this subsection we will relax this assumption by assuming a concave relation between profit (before tax payment) and capital good stock:

$$\frac{dR(K(t))}{dK(t)} > 0, \; \frac{d^2R(K(t))}{dK(t)^2} < 0 \qquad (25)$$

There may be two reasons for this concavity:

-the firm operates on a monopolistic output market (e.g. Van Loon (1982)) with a declining price-sales function

-there is a neo-classical production function, which satisfies the conditions of decreasing returns to scale.

In appendix 3 we give the necessary conditions for the open-loop Nash game. There are two possible situations depending on the value of b. If b equals one, then the firm invests at its maximum rate until the slope of the profit function equals:

$$\frac{dR}{dK} = \frac{r}{1-\tau_2} \qquad (26)$$

After this time-point, where this sales level has been reached, the firm will invest on such a level, that the capital good stock remains constant (I = aK). This result is well-known from dynamic models of the firm (e.g. Van Loon (1982), Van Schijndel (1986)). The time-point at which (26) is reached, depends completely on the initial state and the earnings function. There is also a possibility that during the whole planning period this optimal level of capital good stock, where marginal costs equals marginal earnings, will not be reached. In that case the firm invests at its maximum rate during the whole planning period, because the shareholders prefer capital good stock at T to dividend in [0,T].

In the case that b lies between zero and one, the planning period always ends with dividend pay out and maximum tax. The switch from maximum investment to maximum dividend will take place at $\bar{t}$ given by

$$\bar{t} = T + \frac{1}{a+r}\ln\{(1 - \frac{(1-\tau_2)\frac{dR}{dK}}{a+r})/(b - \frac{(1-\tau_2)\frac{dR}{dK}}{a+r})\} \qquad (27)$$

The firm stops investment at such a time-point that:

$$\frac{dR}{dK}\Big|_{t=\bar{t}} > \frac{i}{1-\tau_2} + a \qquad (28)$$

The greater the value of b the later the dividend switch will take place.

4.2. A logarithmic utility function

The solution of the model presented in section 2 has a bang-bang structure (i.e. the control variable jumps from its lower- to its upperbound (or from its upper- to its lowerbound)). This is because both Hamiltonians are linear in the control variables. In practice an immediate jump from low to high tax rate will not occur. We can easily deal with this problem by replacing the utility function of section 2 through a logarithmic utility function:

$$\int_0^T \ln(G(t))e^{-rt}dt, \qquad (29)$$

In appendix 4 the necessary and sufficient conditions for the Nash game (open-loop and feedback) are given. If the firm invests the following smooth behaviour of tax rate will arise:

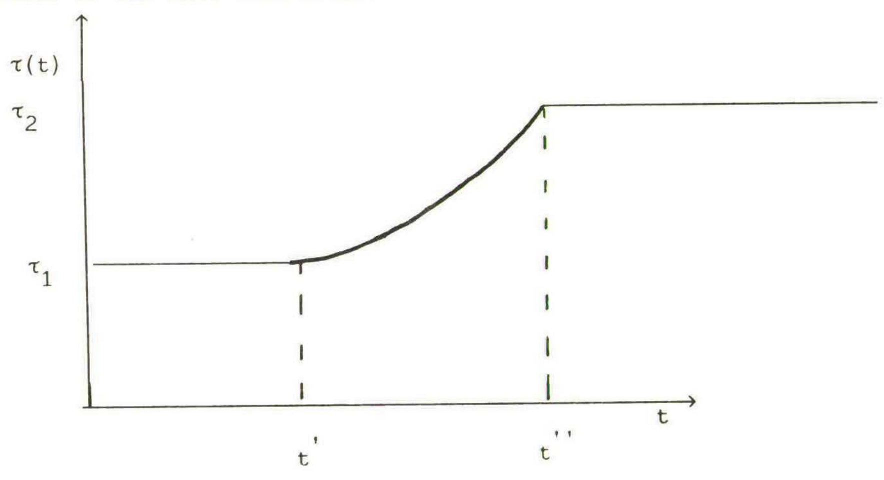

Figure 8. Tax-rate by logarithmic utility function

The time-points t' and t'' are given by:

$$t' = T + \frac{1}{r}\ln(1 - \frac{r}{(q-a)\tau_1}) \tag{30}$$

$$t'' = T + \frac{1}{r}\ln(1 - \frac{r}{(q-a)\tau_2}) \tag{31}$$

5. CONCLUSIONS

For the outcome of the economic process and the way firms behave governmental policy is of great importance. In this paper, therefore, we have modelled the interactions between government and firms as a differential game. By using this technique of differential games we take into account the behavioural relationships within a dynamic environment. Within this game different concepts (Pareto, Nash and Stackelberg) and information structures (open-loop and feedback) are studied. We have compared both non-cooperative solutions (Nash and Stackelberg) with the cooperative solutions(Pareto) and the times that the firm and government will change their policy. Special attention is paid to the difference between the time-inconsistent policy, i.e. open-loop Stackelberg, and the subgame-perfect policy, i.e. feedback Stackelberg. The main conclusion is that the credibility of governmental policy can have a great influence on the market value of the firm.

The model we have presented, has some unrealistic features. In section 4 we discussed two important extensions. We have only dealt with the Nash game. The Stackelberg solutions will be a topic of future research. Especially the case in which we incorporate a logarithmic utility function is of great interest, because then there is a "smooth" behaviour of tax rate, which is of course more realistic. The model still has some unrealistic features even after incorporating these extensions. For example we can incorporate more instruments like wage- and interest control. In this paper we have assumed that government spendings are not productive, but in practice some government spendings will be part of investment. Of course, society is more complex than we described. In spite of this we believe that our model gives a solid description of the interactions between government and firm and gives answers to the importancy of credibility and agreements in economic theory.

ACKNOWLEDGEMENT

The author likes to thank Peter Kort, Piet Verheyen, Aart de Zeeuw (Tilburg University), Paul van Loon, Onno van Hilten (University of Limburg) and Steffen Jørgensen (Copenhagen School of Economics and Business Administration) for their valuable comments. Financial support by the Netherlands Organization for the Advancement of Pure Research is gratefully acknowledged

REFERENCES

Başar, T. and Haurie, A., 1982, Feedback-Stackelberg solution in continouos time multi-level games, in: proceedings of the 21st. IEEE conference on decision and control (Orlando, FL), 664-668

Başar, T., Haurie, A., and Ricci, G., 1985, On the dominance of capitalists leadership in a 'feedback-Stackelberg' solution of a differential game of capitalism, Journal of Economic Dynamics and Control 9, 101-125

Dockner, E, Feichtinger, G. and Jørgensen, S., 1985, Tractable classes of nonzero-sum open-loop Nash differential games: theory and examples, Journal of Optimization Theory and Application 45, 179-197

Gradus, R.H.J.M, 1987, The net present value of governmental policy: a possible way to find the Stackelberg solutions, in: Methods of Operations Research (XII-symposium on Operation research), (Athenäum, Frankurt)

Hayashi, F.H., Tobin's marginal q and average q: A neo-classical interpretation, Econometrica 50, 213-240

Hoel, M., Distribution and growth as a differential game between workers and capitalists, International Economic Review 19, 335-350

Kreps, D.M. and Wilson, R., 1982, Reputation and imperfect information, Journal of Economic Theory 27, 253-279

Kydland, F. and Prescott, E., 1977, Rules rather than discretion: the inconsistency of optimal plans, Journal of Political Economy 85, 473-492

Lancaster, K., 1973, The dynamic inefficiency of capitalism, Journal of Political Economy 81, 1092-1109

Lesourne, J. and Leban, R., La substitution Capital-Travial au cours de la Croissance de l'enterprise, Rev. d'Economic Politique 4, 540-568

Loon, van, P., 1982, Employment in a monopolistic firm, European Economic Review 19, 305-327

Ploeg, van der, F., 1987, Trade unions, investment, and employment: a non-cooperative approach, European Economic Review 39, 1465-1492

Reinganum, J.F. and Stokey, N.L., 1985, Oligopoly extraction of a common property natural resource: the importance of the period of commitment in dynamic games, International Economic Review 26, 161-173

Schijndel, van, G.J.Th.C., 1986, Dynamic behaviour of a value maximizing firm under personal taxation, European Economic Review 33, 1043-1062

Theil, H., 1964, Optimal decision rules for government and industry (North-Holland, Amsterdam)

Wishart, D.M.G. and Olsder, G.J., 1979, Discontinuous Stackelberg solutions, International Journal of Systems Science 10, 1359-1368

Zeeuw, de, A.J., 1984, Difference games and linked econometric models, doctorate thesis, Tilburg University

Appendix 1. The Nash-, Stackelberg- and Pareto solution of the model presented in section 2

First define

$$u_1(t) = \frac{TX(t)}{R(t) - aK(t)} = \text{tax rate} \tag{A1.1}$$

$$u_2(t) = \frac{I(t)}{R(t) - TX(t)} = \text{investment rate} \tag{A1.2}$$

Using the definitions (A1.1) and (A1.2) the model (1)-(10) can be rewritten as follows:

- objective function government:

$$\int_0^T (q-a)u_1Ke^{-rt}dt,\ 0 < \tau_1 \le u_1 \le \tau_2 < 1 \tag{A1.3}$$

- objective function firm:

$$\int_0^T (1-u_2)(q-u_1(q-a))Ke^{-rt} + bK(T)e^{rT},\ 0 \le u_2 \le 1 \tag{A1.4}$$

- state-equation:

$$\mathring{K} = \{(q-u_1(q-a))u_2-a\}K,\ K(0) = K_0 \tag{A1.5}$$

1.1. The open-loop Nash solution

The current-value Hamiltonians are defined by

$$H_1 = (q-a)u_1K + \lambda_1((q-u_1(q-a))u_2-a)K \tag{A1.6}$$

$$H_2 = (1-u_2)(q-u_1(q-a))K + \lambda_2((q-u_1(q-a))u_2-a)K \tag{A1.7}$$

The necessary conditions for an open-loop Nash solution $(u_1^*(t),\ u_2^*(t))$ are:

$$H_1(K^*,u_1^*,u_2^*,\lambda_1) \geqq H_1(K^*,u_1,u_2^*,\lambda_1)\ \forall u_1 \in U_1 \tag{A1.8}$$

$$H_2(K^*,u_1^*,u_2^*,\lambda_2) \geqq H_2(K^*,u_1^*,u_2,\lambda_2) \ \forall u_2 \in U_2 \tag{A1.9}$$

$$\overset{\circ}{\lambda}_1 = -\frac{\partial H_1}{\partial K} + r\lambda_1 = (a+r)\lambda_1-(q-a)u_1^*-\lambda_1((q-u_1^*(q-a))u_2^*) \tag{A1.10}$$

$$\overset{\circ}{\lambda}_2 = -\frac{\partial H_2}{\partial K} + r\lambda_2 = (a+r)\lambda_2-(1-u_2^*)(q-u_1^*(q-a))-\lambda_2((q-u_1^*(q-a))u_2^* \tag{A1.11}$$

$$\lambda_1(T) = 0 \tag{A1.12}$$

$$\lambda_2(T) = b \tag{A1.13}$$

(A1.6) and (A1.8) now imply that

$$u_1^* = \begin{cases} \tau_2 & \text{if} \quad \lambda_1 u_2^* < 1 \\ \tau_1 & \text{if} \quad \lambda_1 u_2^* > 1 \end{cases} \tag{A1.14}$$

Thus the control of the government depends on its shadow price of capital goods and on the control of the firm.
(A1.7) and (A1.9) now imply that

$$u_2 = \begin{cases} 1 & \text{if} \quad \lambda_2 > 1 \\ 0 & \text{if} \quad \lambda_2 < 1 \end{cases} \tag{A1.15}$$

So the firm will invest at its maximum rate if its valuation of a marginal increase in capital stock is greater than one and zero if this valuation is less than one.

It is easy to check that the solution in tables 1 and 2 satisfies the necessary conditions, which are also sufficient because of the fact that the Hamiltonians are linear in the state variable.

1.2. The feedback Nash-solution

Since this game is state-separable (i.e. neither the Hamiltonian maximizing conditions nor the costate equations depend on the state variable) the open-loop Nash solution can also be qualified as the feedback Nash equilibrium (see Dockner et al. (1985)).

1.3. The open-loop Stackelberg solution

In the Stackelberg case we take only the case with the government as the leader into consideration.

The current-value Hamiltonians are defined by:

$$H_1 = (q-a)u_1^*K + \lambda_1((q-u_1^*(q-a))u_2^*-a)K + \pi_1\{(a+r)\lambda_2 - (1-u_2^*)(q-u_1^*(q-a)) - \lambda_2(q-u_1^*(q-a))u_2^*\} \tag{A1.16}$$

$$H_2 = (1-u_2^*)(q-u_1^*(q-a))K + \lambda_2((q-u_1^*(q-a))u_2^*-a)K \tag{A1.17}$$

The necessary conditions are

$$H_1(K^*, u_1^*, u_2^*, \lambda_1,\lambda_2, \pi_1) \geqq H_1(K^*, u_1, u_2^*, \lambda_1,\lambda_2, \pi_1) \ \forall u_1 \in U_1 \tag{A1.18}$$

$$H_2(K^*, u_1^*, u_2^*, \lambda_2) \geqq H_2(K^*, u_1^*, u_2, \lambda_2) \ \forall u_2 \in U_2 \tag{A1.19}$$

$$\overset{\circ}{\lambda}_1 = -\frac{\partial H_1}{\partial K} + r\lambda_1 = (a+r)\lambda_1 - (q-a)u_1^* - \lambda_1((q-u_1^*(q-a))u_2^*) \tag{A1.20}$$

$$\overset{\circ}{\lambda}_2 = -\frac{\partial H_2}{\partial K} + r\lambda_2 = (a+r)\lambda_2 - (1-u_2^*)(q-u_1^*(q-a)) - \lambda_2((q-u_1^*(q-a))u_2^* \tag{A1.21}$$

$$\overset{\circ}{\pi}_1 = -\frac{\partial H_1}{\partial \lambda_2} + r\pi_1 = r\pi_1 - (a+r - ((q-u_1^*(q-a))u_2^*)\pi_1 - (q-u_1^*(q-a))\{\lambda_1K + (1-\lambda_2)\pi_1\}\left[\frac{du_2^*}{d\lambda_2}\right] \tag{A1.22}$$

$$\lambda_1(T) = 0 \tag{A1.23}$$

$$\lambda_2(T) = b \tag{A1.24}$$

$$\pi_1(0) = 0 \tag{A1.25}$$

The costate variables $\lambda_1(t)$, $\lambda_2(t)$ and $\pi(t)$ correspond to $K(t)$ for the government, $K(t)$ for the firm and $\lambda_2(t)$ for the government.
The reaction function for the follower (the firm) is the same as in the Nash solution, while for the government it follows that

$$u_1(t) = \begin{cases} \tau_1 & \text{if} \quad B(t) < 0 \\ \tau_2 & \text{if} \quad B(t) > 0 \end{cases} \tag{A1.26}$$

, where

$$B(t) = K(t) - \lambda_1(t)u_2^*(t)K(t) + \pi_1(t)\{1-u_2^*(t) + u_2^*(t)\lambda_2(t)\} \tag{A1.27}$$

Applying the results by Wishart and Olsder (1979) we can evaluate the costate variable $\pi_1(t)$. The term $\frac{du_2}{d\lambda_2}$ behaves with respect to time as δ-function with a jump at $t = t_1$. The size of this jump is determined by the properties of δ-function (See Wishart and Olsder (1979)). So $\pi_1(t)$ will be zero until the moment that the firm switches from investment to dividend and will have a jump at $t = t_1$

$$\pi_1(t_1) = -\lambda_1(t_1)K(t_1) \tag{A1.28}$$

so that

$$B(t_1) = (1-\lambda_1(t_1))K(t_1) \gtreqless 0 \ \underline{\text{if}} \quad \lambda_1(t_1) \lesseqgtr 1 \tag{A1.29}$$

It depends on the value of $\lambda_1(t_1)$, which policy will be chosen. We have 3 possible situations

1) $\lambda_1(t_1) < 1 \rightarrow u_1^*(t) = \tau_2$, $t \geq t_1$

This will happen if

$$\tau_2 < \frac{1}{2-b} \cdot \frac{q}{q-a} \tag{A1.30}$$

From the transversality condition t_1 can be derived which is the same for the Nash solution.

2) $\lambda_1(t_1) > 1 \rightarrow u_1^*(t) = \tau_1, \ t \geq t_1$

This will happen if

$$\tau_1 > \frac{1}{2-b} \cdot \frac{q}{q-a} \tag{A1.31}$$

From the transversality condition (A1.24) t_1, which is given by equation (12) can be derived.

3) $$\lambda_1(t_1) = 1 \rightarrow u_1^*(t) = \tau_1, \ t_1 \leq t < t_2$$
$$= \tau_2, \ t_2 \leq t \leq T \tag{A1.32}$$

This situation will occur if

$$\tau_1 \leq \frac{1}{2-b} \cdot \frac{q}{q-a} \tag{A1.33}$$

and

$$\tau_2 \geq \frac{1}{2-b} \cdot \frac{q}{q-a} \tag{A1.34}$$

The time-points t_1 and t_2 can be derived from $\lambda_1(t_1) = 1$ and $\lambda_2(t_1) = 1$.

1.4. The feedback Stackelberg solution

In general it is not easy to derive the feedback Stackelberg solution for continuous time games (see Basar and Haurie (1982)).
However, for this game we can derive the following necessary conditions:

$$-\frac{\partial V_1(t,K)}{\partial t} = \max_{u_1 \in [\tau_1, \tau_2]} \{(q-a)u_1 K + \frac{\partial V_1(t,K)}{\partial K} \{(q-u_1(q-a))$$

$\gamma_2(t,K)-a)K\}\}$

(A1.35)

$$-\frac{\partial V_2(t,K)}{\partial t} = \max_{u_2 \in [0,1]} \{(1-u_2)\{q-\gamma_1(t,K,u_2)(q-a)\}K$$

$$+\frac{\partial V_2(t,K)}{\partial K}.\{(q-\gamma_1(t,K,u_2)(q-a))u_2-a\}K\} \tag{A1.36}$$

$$V_1(T,K(T)) = 0 \tag{A1.37}$$

$$V_2(T,K(T)) = b \tag{A1.38}$$

$$(q-a)\gamma_1(t,K,u_2)K + \frac{\partial V_1(t,K)}{\partial K}\{(q-\gamma_1(t,K,u_2)(q-a))u_2-a\}K$$

$$\geqq (q-a)u_1K + \frac{\partial V_1(t,K)}{\partial K}\{(q-u_1(q-a))u_2-a)K\} \ \forall u_1 \in U_1,\ u_2 \in U_2 \tag{A1.39}$$

, where γ_1, and γ_2 are mappings such that

$$\gamma_1(t,K,u_2) \to u_1(t) \in [\tau_1,\tau_2]$$

$$\gamma_2(t,K) \to u_2(t) \in [0,1]$$

and V_1 and V_2 are the value function for the government and the firm. It would be straightforward to check that the following linear value function is a solution of (A1.35) - (A1.39):

$$V_i(t,k) = \lambda_i(t).K \tag{A1.40}$$

, where $\lambda_1(t)$ and $\lambda_2(t)$ are given by (A1.10) - (A1.13). So the open-loop Nash is a candidate for the feedback Stackelberg solution.

1.5. The Pareto-solution

To find the Pareto-solution we have to maximize $J = J_1 + \mu J_2$, $0 < \mu < \infty$ subject to (A.1.5), where μ measures the relative importance of player 1 against player 2 and is assumed to be given. This is a standard optimal control problem, which is easy to solve. We give no details about the derivation:

$$H = \{(q-a)u_1 + \mu(1-u_2)(q-u_1(q-a))\}K + \Psi((q-u_1(q-a))u_2-a)K \quad \text{(A1.41)}$$

$$\mathring{\Psi} = (r+a)\Psi - (q-a)u_1 - \mu(1-u_2)(q-u_1(q-a)) - (q-u_1(q-a))u_2\Psi \quad \text{(A1.42)}$$

$$\Psi(T) = b \quad \text{(A1.43)}$$

3 situations

$$- \mu = 1: \begin{cases} u_1 = \tau_1,\ u_2 = 1 & \text{if } \Psi > 1 \\ u_1 \text{ is singular, } u_2 = 0 & \text{if } \Psi \leqq 1 \end{cases} \quad \text{(A1.44)}$$

$$- \mu > 1: \begin{cases} u_1 = \tau_1,\ u_2 = 1 & \text{if } \Psi > \mu \\ u_1 = \tau_1,\ u_2 = 0 & \text{if } \Psi \leqq \mu \end{cases} \quad \text{(A1.55)}$$

$$- \mu < 1: \begin{cases} u_1 = \tau_1,\ u_2 = 1 & \text{if } \Psi > 1 \\ u_1 = \tau_2,\ u_2 = 1 & \text{if } \mu < \Psi \leqq 1 \\ u_1 = \tau_2,\ u_2 = 0 & \text{if } \Psi \leqq \mu \end{cases} \quad \text{(A1.56)}$$

Appendix 2. An example

Assume without loss of generality that $q = 1$ and furthermore assume that: $a = 0.1$, $r = 0.05$, $b = 0$, $K_0 = 1$, $T = G$ and

$$\begin{matrix} \tau_1 = 1/4 < \\ \tau_2 = 3/4 > \end{matrix} \frac{1}{2}\cdot\frac{10}{9} = \frac{5}{9}.$$

So we are in the situation of tables 1 and 3.

1) Nash:

$\bar{t} = 1,873$

$J_1^N(u_1,u_2) = 6,154$

$J_2^N(u_1,u_2) = 2,673$

2) Stackelberg:

$t_1 = 3,622$

$t_2 = 4,449$

$J_1^S(u_1,u_2) = 6,696$

$J_2^S(u_1,u_2) = 9.799$

3) Pareto:

($\tilde{t}$: dividend switch, $\tilde{\tilde{t}}$: tax switch)

$\lim_{u\to\infty} : \tilde{t} = 4,566$ $\quad J_1(u_1,u_2) = 10,925 \quad J_2(u_1,u_2) = 17,353$

$\lim_{u\downarrow 1} : \tilde{t} = 4,917$ $\quad J_1(u_1,u_2) = 12,281 \quad J_2(u_1,u_2) = 16,750$

$\lim_{u\uparrow 1} : \tilde{t} = \tilde{\tilde{t}} = 4,917$ $\quad J_1(u_1,u_2) = 22,005 \quad J_2(u_1,u_2) = 7,023$

$\lim_{u\downarrow 0} : \tilde{t} = 6,\ \tilde{\tilde{t}} = 4,683$ $\quad J_1(u_1,u_2) = 25,026 \quad J_2(u_1,u_2) = 0$

Appendix 3. The open-loop Nash solution for the case with a concave profit function

The model becomes:

$$\int_0^T u_1(S(K)-aK)e^{-rt}dt \tag{A3.1}$$

$$\int_0^T (1-u_2)(S(K)-u_1(S(K)-aK))e^{-rt} + bK(T)e^{-rt} \tag{A3.2}$$

$$\mathring{K} = u_2(S(K)-u_1(S(K)-aK)) - aK \tag{A3.3}$$

The Hamiltonians:

$$H_1 = u_1(S(K)-aK) + \lambda_1\{u_2(S(K)) - u_1(S(K)-aK)) - aK\} \tag{A3.4}$$

$$H_2 = (1-u_2)(S(K) - u_1(S(K)-aK)) + \lambda_2\{u_2(S(K)) - u_1(S(K)-aK)) - aK\} \tag{A3.5}$$

with

$$\mathring{\lambda}_1 = r\lambda_1 - u_1\left[\frac{dS}{dK} - a\right] - \lambda_1\left[u_2\left\{\frac{dS}{dK} - u_1\left[\frac{dS}{dK} - a\right]\right\}-a\right] \tag{A3.6}$$

$$\mathring{\lambda}_2 = r\lambda_2\ (1-u_2)\left[\frac{dS}{dK} - u_1\left[\frac{dS}{dK} - a\right]\right] - \lambda_2\left[u_2\left\{\frac{dS}{dK} - u_1\left[\frac{dS}{dK} - a\right]\right\}-a\right] \tag{A3.7}$$

$$\lambda_1(T) = 0 \tag{A3.8}$$

$$\lambda_2(T) = b \tag{A3.9}$$

If we assume that $S(K) - aK > 0$ then (A.1.14) and (A.1.15) still holds:
Note that there are two possible situations:

i) $-0 \leqq b < 1 \rightarrow \lambda_2(T) = b < 1$

If $u_2 = 0$ then $\mathring{\lambda}_2 = (r+a)\lambda_2 - \left[\frac{dS}{dK} - \tau_2\left[\frac{dS}{dK} - a\right]\right]$ (A3.10)

Let $\bar{t}$ be the time-point of investment switch, $\lambda_2(\bar{t}) = 1$,

If $\quad \left.\frac{dS}{dK}\right|_{t = \bar{t}} > \frac{r}{1-\tau_2} + a \longrightarrow \mathring{\lambda}_2 < 0,\ \bar{t} \leqq t \leqq T$ (A3.11)

If $\quad \left.\frac{dS}{dK}\right|_{t = \bar{t}} = \frac{r}{1-\tau_2} + a \longrightarrow \mathring{\lambda}_2 = 0,\ \bar{t} \leqq t \leqq T$

($\mathring{\lambda}_2 = 0$ is not possible because of transversality conditions)

ii) $\quad -b = 1 \rightarrow \lambda_2(T) = 1$

Let $\bar{t}$ be the time-point where $\lambda_2(\bar{t}) = 1$

if $\quad u_2 = \frac{aK}{S(K)-aK}$ then

$$\mathring{\lambda}_2 = (r+a)\lambda_2 - \frac{S(K)}{S(K)-aK} \times \left[\frac{dS}{dK} - u_1\left[\frac{dS}{dK} - a\right]\right] - \frac{aK}{S(K)-aK} \times \lambda_2\left[\frac{dS}{dK} - u_1\left\{\frac{dS}{dK} - a\right\}\right] \quad \text{(A3.12)}$$

$\mathring{\lambda}_2 = 0$ (because of transversality conditions) if and only if

$$\frac{dS}{dK} = \frac{r}{1-\tau_2} + a. \quad \text{(A3.13)}$$

Appendix 4. <u>The Nash solution for the model with a logarithmic utility function</u>

The model can be rewritten in the same form as in appendix 1 and we get the following form:

$$\int_0^T \{\ell n(u_1(q-a)K)\}e^{-rt}dt \quad \text{(A.5.1)}$$

$$\int_0^T \{(1-u_2)(q-u_1(q-a))K\}e^{-rt}dt + bK(T)e^{-iT} \quad \text{(A.5.2)}$$

$$\mathring{K} = \{u_2(q-u_1(q-a)) - a\}.K \quad \text{(A.5.3)}$$

The (current-value) Hamiltonian for the government has the following form:

$$H_1 = \ell n\ K + \ell n\ u_1 + \ell n(q-a) + \lambda_1\{u_2(q-u_1(q-a)) - a\}K$$

Using the transformation $x = \ell n\ K$ and writing the (current-value) of the transformed problem:

$$H_1 = x + \ell n\ u_1 + \ell n(q-a) + \Psi_1\{u_2(q-u_1(q-a)) - a\}$$

The necessary (and sufficient) conditions for the government are

$$\mathring{\Psi}_1 = r\Psi_1 - \frac{\partial H_1}{\partial x} = r\Psi_1 - 1 \qquad \text{(A.5.6)}$$

$$\frac{\partial H_1}{\partial u_1} = \frac{1}{u_1} - \Psi_1 u_1(q-a) \gtreqless 0 \leftrightarrow u_1 = \begin{cases} \tau_2 \\ \cdot\ \dot{\cdot} \\ \tau_1 \end{cases} \qquad \text{(A.5.7)}$$

or

if $\quad \Psi_1 \geq \frac{1}{(q-a)\tau_1}$ then $u_1 = \tau_1$

if $\quad \frac{1}{(q-a)\tau_1} < \Psi_1 < \frac{1}{(q-a)\tau_2}$ then $u_1 = \frac{1}{(q-a)\Psi_1}$

if $\quad \Psi_1 \leq \frac{1}{(q-a)\tau_1}$ then $u_1 = \tau_2$

From (A.5.5) and $\Psi(T) = 0$ we can derive

$$\Psi_1(t) = \frac{1}{r}\ (1-e^{r(t-T)}) \qquad \text{(A.5.8)}$$

So the feasible strategy for the government is

$$u_1(t) = \tau_1,\ t \in [0,t')$$

$$u_1(t) = \frac{r}{(q-a)(1-e^{r(t-T)})},\ t \in [t',t'') \qquad \text{(A.5.9)}$$

$$u_1(t) = \tau_2,\ t \in [t'',T]$$

, where

$$t' = T + \frac{1}{r} \ln(1 - \frac{r}{(q-a)\tau_1}) \qquad (A.5.10)$$

$$t'' = T + \frac{1}{r} \ln(1 - \frac{r}{(q-a)\tau_2}) \qquad (A.5.11)$$

IN 1987 REEDS VERSCHENEN

242 Gerard van den Berg
Nonstationarity in job search theory

243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions

244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk

245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation
of subjective poverty definitions

246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance Criteria

247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en strukturele afhankelijkheden

248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group

249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic Investment Policy

250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical approach

251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically distributed errors

252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten

253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen

254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint products: A comment

255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?

256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium concepts

257 P.H.M. Ruys, G. van der Laan
Computation of an industrial equilibrium

258 W.H. Haemers, A.E. Brouwer
Association schemes

259 G.J.M. van den Boom
Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining

260 A.W.A. Boot, A.V. Thakor, G.F. Udell
Competition, Risk Neutrality and Loan Commitments

261 A.W.A. Boot, A.V. Thakor, G.F. Udell
Collateral and Borrower Risk

262 A. Kapteyn, I. Woittiez
Preference Interdependence and Habit Formation in Family Labor Supply

263 B. Bettonvil
A formal description of discrete event dynamic systems including perturbation analysis

264 Sylvester C.W. Eijffinger
A monthly model for the monetary policy in the Netherlands

265 F. van der Ploeg, A.J. de Zeeuw
Conflict over arms accumulation in market and command economies

266 F. van der Ploeg, A.J. de Zeeuw
Perfect equilibrium in a model of competitive arms accumulation

267 Aart de Zeeuw
Inflation and reputation: comment

268 A.J. de Zeeuw, F. van der Ploeg
Difference games and policy evaluation: a conceptual framework

269 Frederick van der Ploeg
Rationing in open economy and dynamic macroeconomics: a survey

270 G. van der Laan and A.J.J. Talman
Computing economic equilibria by variable dimension algorithms: state of the art

271 C.A.J.M. Dirven and A.J.J. Talman
A simplicial algorithm for finding equilibria in economies with linear production technologies

272 Th.E. Nijman and F.C. Palm
Consistent estimation of regression models with incompletely observed exogenous variables

273 Th.E. Nijman and F.C. Palm
Predictive accuracy gain from disaggregate sampling in arima - models

289 M.H.C. Paardekooper
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues

290 Pieter H.M. Ruys
Industries with private and public enterprises

291 J.J.A. Moors & J.C. van Houwelingen
Estimation of linear models with inequality restrictions

292 Arthur van Soest, Peter Kooreman
Vakantiebestemming en -bestedingen

293 Rob Alessie, Raymond Gradus, Bertrand Melenberg
The problem of not observing small expenditures in a consumer expenditure survey

294 F. Boekema, L. Oerlemans, A.J. Hendriks
Kansrijkheid en economische potentie: Top-down en bottom-up analyses

295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note

296 Arthur van Soest, Peter Kooreman
Estimation of the indirect translog demand system with binding nonnegativity constraints

IN 1988 REEDS VERSCHENEN

297 Bert Bettonvil
Factor screening by sequential bifurcation

298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure

299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry

300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method

301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note

302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in lineair models
using cross sections, panels or both