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NON-COOPERATIVE STRATEGIES FOR DYNAMIC POLICY GAMES AND THE PROBLEM OF TIME INCONSISTENCY: A COMMENT

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Abstract

This is a comment on a recent paper by A.J. Hughes Hallett. We argue that the proposed solution to the non-cooperative bargaining problem assumes information that is not available to the players, suffers from non-uniqueness, often fails to converge and does not satisfy the Nash property. These claims are substantiated with numerical examples. We also argue that the proposed solution to the problem of time inconsistency yields strategies that are not credible.

1. Introduction

The Lucas (1976) critique of econometric policy evaluation has increased the interest in applications of rational expectations and non-cooperative difference/differential game theory to dynamic econometric models, because these techniques allow the optimal determination of government economic policy to influence the behaviour of private sector agents (such as trade unions or firms) and thus the policy structure of econometric models. It is well known that economies with rational expectations or open loop Stackelberg games are noncausal due to the anticipation of future actions of dominant players (such as the Treasury or the Central Bank). In such models the problem of time inconsistency arises due to the incentive of dominant players to renege on announced strategies (Kydland and Prescott, 1977). If there are no binding commitments on the part of each player and such cheating occurs all the time, there is no incentive left for the dominant players to renege and the resulting actions will be optimal and consistent (Hughes Hallett, 1984). However, this solution to the problem of time inconsistency suffers from the fact that the optimal strategies are not credible as the private sector cannot be expected to believe misleading announcements all the time. The feedback Stackelberg (leader-follower) equilibrium concept only considers these strategies that are credible and therefore typically yields sub-optimal strategies for the dominant players, although these strategies are consistent by construction. It follows that there is a trade-off between optimal and credible strategies. A proper discussion of this trade-off involves an analysis of the dynamic evolution of the reputations of the players (cf. Kreps and Wilson, 1982; Barro and Gordon, 1983; Backus and Driffill, 1984), and is beyond the scope of this paper. The above issues are discussed more fully in Section 3, but before this is done we need to discuss Hughes Hallett's (1984) approach to the non-cooperative bargaining problem. We show in Section 2 that the proposed solution is based upon a Stackelberg leader-leader framework and therefore does not satisfy the Nash property, might give rise to disequilibrium situations, and suffers from non-uniqueness. The open loop (e.g. van der Ploeg, 1982) or feedback (e.g. Plasmans and de Zeeuw, 1980) Nash equilibria are the normal approaches to non-cooperative bargaining with players of equal strength (Kydland, 1975) and do not suffer from the problem of time inconsistency. Hughes Hallett (1984) criticises these approaches, as they assume a certain naivety on the part of each player, and therefore suggests his "sophisticated" equilibrium for players of equal strength. The Stackelberg nature of this solution explains why the economy is non-causal and the problem of time inconsistency occurs even though the players are assumed to have equal strength.

Most of the comments on Hughes Hallett (1984) also apply to Brandsma and Hughes Hallett (1982, 1984a, b) and Hughes Hallett and Rees (1983, Chapter 11), but for concreteness we will concentrate our attention on the first paper (denoted by HH).

2. "Conjectural variations" in an open loop Nash equilibrium?

In sections 3 and 5.1 of HH a dynamic difference game is converted to a static game by stacking the policy instruments for all time periods into one vector. Such a solution procedure corresponds to an open loop Nash equilibrium (Kydland, 1975, pp. 327-330; Başar and Olsder, 1982, p. 245), because the information set available to each player at any time of the planning period is the information available prior to the planning exercise. The open loop Nash equilibrium is convenient to calculate, but assumes that players do not have access to the current state of the economy. This is unrealistic and HH would have benefited from calculating feedback or closed loop with memory Nash equilibria (e.g. Başar and Olsder, 1982, Chapter 6; de Zeeuw, 1984, Chapter 4) which have more satisfactory information patterns.

The conventional definition of a strategy is a mapping from the information space to the action space, where the action space is the Cartesian product of the sets of possible policy instruments at each point of time and, for the open loop Nash equilibrium, the information space is the Cartesian product of the set of possible initial states of the economy. HH starts off with strategies where the information space consists of the action spaces and the reaction curves, or conjectural variations, of the rival players. The problem with this approach is that the information sets the players are supposed to have access to are in practice never available, so that the game cannot be played. For example, if each player has two policy instruments, then the information set must contain the four reaction coefficients of each rival which are simply not observable. Furthermore, there is some confusion between the iterations of the HH algorithm, designed to suggest new directions in the search for optimal decisions, and the time structure of the game. Presumably, the iterations occur in notional time whereas the economy evolves in real time but this sheds further doubt on how the game might be played in practice.

The conceptual problems discussed above are quite serious, because it can be shown that the HH set of "Nash" solutions do not necessarily give a unique set of actions, the iterations do not necessarily converge and the resulting outcomes are not in Nash equilibrium. Most of these problems arise from introducing information that is not available. Since our allegations are quite severe, it is useful to turn to a more detailed discussion of the HH approach and to provide the reader with some numerical examples to substantiate our claims.

Any dynamic econometric model can be cast into a final form model (e.g. Theil, 1964), that is[1)]

$$y = R^{(1)}x^{(1)} + R^{(2)}x^{(2)} + s \tag{1}$$

where y, $x^{(1)}$, $x^{(2)}$ and s are vectors (stacked for all periods of the finite planning horizon) of endogeneous variables, policy instruments of player 1, policy instruments of player 2 and known uncontrollable exogeneous shocks. The matrices $R^{(1)}$ and $R^{(2)}$ are known matrices for players 1 and 2, which are block-triangular for economies without rational expectations of future events. Our attention is restricted to deterministic games, because the discussion of first-period certainty equivalence and asymmetric information in Section 4 of HH is uncontroversial and a proper account of experimentation, probing and learning of the $R^{(i)}$ would complicate matters unnecessarily. The objective of player i is to minimise a quadratic welfare loss function of the form[2)]

$$w^{(i)} = \frac{1}{2}\, z^{(i)'}Q^{(i)}z^{(i)} + q^{(i)'}z^{(i)} \tag{2}$$

where $z^{(i)} = (y',x^{(i)'})'$. There are no direct externalities in (2), so that the only interactions between the players occur via the dependence of y on $x^{(j)}$, $j \neq i$. Any (open loop, feedback or closed loop with memory) Nash equilibrium, say $\{x^{(1)^*},x^{(2)^*}\}$, must satisfy

$$\begin{aligned} w^{(1)}(x^{(1)^*},x^{(2)^*}) &\leq w^{(1)}(x^{(1)},x^{(2)^*}), \forall x^{(1)} \\ w^{(2)}(x^{(1)^*},x^{(2)^*}) &\leq w^{(2)}(x^{(1)^*},x^{(2)}), \forall x^{(2)} \end{aligned} \tag{3}$$

1) We will stick as close as possible to the notation in HH.

2) The ideal values $y^{(i)^d}$ and $x^{(i)^d}$ can be omitted without loss of generality due to the presence of $q^{(i)}$.

so that there is no individual incentive for any player to deviate from the equilibrium. HH calls strategies that satisfy (3) privately optimal decisions for the non-cooperative Nash game, whilst one of our objectives is to demonstrate that the HH equilibrium concept with conjectural variations does not satisfy (3).

The conventional Nash equilibrium assumes that each player is naive and does not consider the reactions of its rivals to its own actions, that is player i assumes that $\partial x^{(j)}/\partial x^{(i)} = 0$, $j \neq i$, in the calculation of its optimal strategies (e.g. Kydland, 1975; van der Ploeg, 1982). This yields the optimal decisions conditional on the actions of the rivals,

$$x^{(i)*} = - [G_0^{(i)'} Q^{(i)} G_0^{(i)}]^{-1} G_0^{(i)'} \{Q^{(i)} \begin{bmatrix} s+R^{(j)}x^{(j)} \\ \cdots\cdots\cdots \\ 0 \end{bmatrix} + q^{(i)}\} \quad (4)$$

where $G_0^{(i)} = [R^{(i)'} \vdots I]'$ and similarly for player j. The open loop Nash equilibrium satisfies (3) and is calculated as the intersection of the reaction curves (4), that is

$$\begin{bmatrix} x^{(1)*} \\ \\ x^{(2)*} \end{bmatrix} = - \begin{bmatrix} G_0^{(1)'} \; Q^{(1)} \; G_0^{(1)} & \vdots & G_0^{(1)'} \; Q^{(1)} \begin{matrix} R^{(2)} \\ \cdots \\ 0 \end{matrix} \\ \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\ G_0^{(2)'} \; Q^{(2)} \begin{matrix} R^{(1)} \\ \cdots \\ 0 \end{matrix} & \vdots & G_0^{(2)'} \; Q^{(2)} \; G_0^{(2)} \end{bmatrix}^{-1} \quad (5)$$

$$\begin{bmatrix} G_0^{(1)'} \{Q^{(1)} \begin{matrix} s \\ \cdot\cdot \\ 0 \end{matrix} + q^{(1)} \} \\ \cdots\cdots\cdots \\ G_0^{(2)'} \{Q^{(2)} \begin{matrix} s \\ \cdot\cdot \\ 0 \end{matrix} + q^{(2)} \} \end{bmatrix} .$$

The nonlinear extension of this open loop Nash equilibrium has been developed by Rustem and Zarrop (1979) and van der Ploeg (1982). An application to the analysis of conflict between government, trade unions and firms within the context of a multisectoral dynamic model of the U.K. economy (see Barker et al., 1980) is also reported in van der Ploeg (1982), so that interesting applications to large-scale, nonlinear econometric models are quite feasible.

HH argues that the open loop Nash equilibrium is inconsistent, because each player computes an optimal reaction to his opponents' decisions whilst ignoring the counterreactions of his opponents who are doing the same. But zero conjectural variations are the essence of the Nash concept! Let us nevertheless consider the "Nash" approach adopted in HH. Instead of adopting zero conjectural variations, player i has formed an estimate of the reactions of its rivals, say $\partial x^{(j)}/\partial x^{(i)} = D^{(j)} \neq 0$, even though these reaction coefficients are not observable to player i. It follows that, instead of (5), the "sophisticated" equilibrium must satisfy

$$\begin{bmatrix} x^{(1)^*} \\ \\ x^{(2)^*} \end{bmatrix} = - \left[\begin{array}{c:c} G^{(1)'} \; Q^{(1)} \; G_0^{(1)} & G^{(1)'} \; Q^{(1)} \begin{bmatrix} R^{(2)} \\ \cdots \\ 0 \end{bmatrix} \\ \hdashline G^{(2)'} \; Q^{(2)} \begin{bmatrix} R^{(1)} \\ \cdots \\ 0 \end{bmatrix} & G^{(2)'} \; Q^{(2)} \; G_0^{(2)} \end{array} \right]^{-1} \left[\begin{array}{c} G^{(1)'} \; \left\{ Q^{(1)} \begin{bmatrix} s \\ \cdot\cdot \\ 0 \end{bmatrix} + q^{(1)} \right\} \\ \hdashline G^{(2)'} \; \left\{ Q^{(2)} \begin{bmatrix} s \\ \cdot\cdot \\ 0 \end{bmatrix} + q^{(2)} \right\} \end{array} \right] . \qquad (6)$$

where $G^{(i)} = \{(R^{(i)} + R^{(j)} D^{(j)})' \,\vdots\, I\}'$. It is (due to the ambiguous information structure) not clear where the conjectural variations, $D^{(j)}$, come from, but HH suggests an iterative scheme in notional time for finding a fixed point between $D^{(1)}$, $D^{(2)}$, $x^{(1)^*}$ and $x^{(2)^*}$. The proposed scheme is based upon the fact that any conjectural variation at notional time s implies a new conjectural variation at notional time s+1, that is

$$D_{s+1}^{(i)} = -[G_s^{(i)'} \; Q^{(i)} \; G_0^{(i)}]^{-1} \; G_s^{(i)'} \; Q^{(i)} \begin{bmatrix} R^{(j)} \\ \cdots \\ 0 \end{bmatrix}$$

$$= \Omega^{(i)}[D_s^{(j)}] \neq D_s^{(i)}$$

where $G_s^{(i)} = \{(R^{(i)}+R^{(j)}D_s^{(j)})' \; \vdots \; I\}'$, and using (6) implies new actions for the players at notional time s+1. Upon substitution of the iterative scheme (7) into (6) (or, to be more precise, equation (14) of HH), one obtains after considerable algebraic manipulation

$$x_{s+1}^{(i)} = -\{G_s^{(i)'} \; Q^{(i)}[G_s^{(i)} + \begin{bmatrix} R^{(j)} \\ \cdots \\ 0 \end{bmatrix} (D_{s+1}^{(j)} - D_s^{(j)})]\}^{-1}$$

$$G_s^{(i)'}\{Q^{(i)} \begin{bmatrix} I \\ \cdot\cdot \\ 0 \end{bmatrix} [s + R^{(j)}(F_{s+1}^{(j)}s + k_{s+1}^{(j)})] + q^{(i)}\} \qquad (8)$$

for i,j = 1,2 and i ≠ j, where

$$(F_{s+1}^{(i)} \; \vdots \; k_{s+1}^{(i)}) = -(G_s^{(i)'} \; Q^{(i)} \; G_0^{(i)})^{-1} G_s^{(i)'} \{Q^{(i)} \begin{bmatrix} I \\ \cdot\cdot \\ 0 \end{bmatrix} \vdots \; q^{(i)}\}.$$

Equation (8) gives the iterative scheme directly in terms of the decisions of the players.[1)] A fixed point for the conjectural variations of player 1, say $D_*^{(2)}$, satisfies

$$D_*^{(2)} = \Omega^{(2)}[\Omega^{(1)}(D_*^{(2)})]$$

and similarly for player 2. HH gives an algorithm for calculating the fixed-point equilibrium, $x^{(1)^*} = x^{(1)^*}(D_*^{(1)}, D_*^{(2)})$, $x^{(2)^*}$ =

1) It corresponds to equation (16) of HH and corrects some non-trivial mistakes in that equation. We hope that (16) was not used to calculate the empirical results in HH.

$x^{(2)^*}(D_*^{(1)},D_*^{(2)})$, $D_*^{(1)}$ and $D_*^{(2)}$, but does not discuss issues of uniqueness or convergence of the iterations defined by (7) and (8).[1] This is a pity, since we show in the examples below that there are typically many different fixed-point equilibria and some of them need not be stable. This seems, quite apart from the curious information requirements, a serious criticism of the proposed solution concept. Furthermore, we will show that the set of fixed points defined by (9) does not only contain multiple "Nash" equilibria but that none of them need satisfy the Nash property (3) and none of the players need necessarily obtain improvements in their welfare.

Example 1 (Hughes Hallett, 1984, pp. 389-390):

Consider the scalar game $y = x^{(1)} + x^{(2)} - 1$ and $w^{(i)} = y^2 + x^{(i)^2}$, so that $R^{(i)} = 1$, $s = -1$, $Q^{(i)} = 2I$ and $q^{(i)} = 0$. The Nash equilibrium (3) follows from the intersection of the reaction curves (4) $x^{(i)^*} = (1-x^{(j)})/2$, so that $x^{(i)^*} = \frac{1}{3}$, $y^* = -\frac{1}{3}$ and $w^{(i)^*} = 2/9$. This also follows directly from (6) or (8) (or from (16) in HH), because $G_0^{(i)} = (1\ 1)'$, $F_1^{(i)} = -\frac{1}{2}$, $D_1^{(i)} = -\frac{1}{2}$ and therefore the outcome of iteration 1, $x_1^{(i)} = \frac{1}{3}$, is the Nash solution. HH argues that $x^{(1)^+} = x^{(2)^+} = 0.4$ is a better solution, because $y^+ = -0.2$, $w^{(i)^+} = 0.2$ and therefore gives a 10% improvement for both players. This is a peculiar argument, because

$$w^{(2)^+} = 0.2 \nleq (0.4 + x^{(2)} - 1)^2 + x^{(2)^2} = w^{(2)}(0.4,x^{(2)})$$

for $0.2 < x^{(2)} < 0.4$. If $x^{(1)} = 0.4$, then 0.2 is not the minimum loss for player 2, but 0.18 is for $x^{(2)} = 0.3$! $x^{(i)^+}$ is therefore not a Nash equilibrium solution. It is well known that, given the presence of externalities, it is possible to find Pareto improvements over the Nash

1) We note that the application to analysing conflict between government and labour within the context of the Klein I model, reported in Section 8 of HH, stops after the first iteration of the algorithm.

outcomes but these will not be individually rational in the sense of (3)! If the players were "sophisticated" in the HH sense, the reaction curves at notional time s are given by $x_s^{(i)^*} = (1-x^{(j)})(1+D_s^{(j)})/(2+D_s^{(j)})$ and iteration (7) becomes $D_{s+1}^{(i)} = F_{s+1}^{(i)} R^{(j)} = -(1+D_s^{(j)})/(2+D_s^{(j)})$. Starting with $D_1^{(i)} = -\frac{1}{2}$ one finds that after another iteration $D_2^{(j)} = -\frac{1}{3}$, so that $x_2^{(i)} = \frac{1}{4}$, $y_2 = -\frac{1}{2}$ and $w_2^{(i)} = \frac{5}{16}$. Both players end up with worse pay-offs than in the conventional Nash equilibrium and it is therefore not clear what is meant with the "down-hill search directions" of the HH algorithm. There are two fixed points corresponding to (9), $D_*^{(i)} = -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$, which give rise to the multiple equilibria $x^{(i)^*} = \frac{1}{2} \mp \frac{1}{10}\sqrt{5}$, $y^* = -\frac{1}{5}\sqrt{5}$ and $w^{(i)^*} = \frac{1}{2} \mp \frac{1}{10}\sqrt{5} = 0.28$ or 0.72. Each of the fully iterated HH solutions produce worse results for both players than the conventional Nash equilibrium (s=1). Furthermore they do not satisfy the Nash property (3), because it can be shown that

$$w^{(1)^*} = \frac{1}{2} \mp \frac{1}{10}\sqrt{5} \nleq w^{(1)}(x^{(1)}, \frac{1}{2} \mp \frac{1}{10}\sqrt{5})$$

for, say, $x^{(1)} = \frac{1}{4} + \frac{1}{20}\sqrt{5}$. The HH solution has little to do with the Nash equilibrium solution, but is much closer in spirit to the Stackelberg leader-leader situation and it is therefore no surprise that the players end up worse than with zero conjectural variations. Finally it follows from $\Omega'[D_*^{(j)}] = -1/(2+D_*^{(j)})^2 = -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$ that one of the equilibria ($D_*^{(i)} = -\frac{3}{2} + \frac{1}{2}\sqrt{5}$) is stable whilst the other is unstable. It also follows that the iterative scheme (7) proceeds in a cyclical manner and that, if the initial conjectures are large enough ($D_0^{(i)} < -2$), the iterations diverge resulting in a disequilibrium game where expectations are never fulfilled ("price war").

<u>Example 2 (de Zeeuw, 1982)</u>:

Consider a symmetric game with two policy instruments for each player and two target variables defined by $R^{(i)} = I$, $s = (1,1)'$, $Q^{(i)} = I$ and $q^{(i)} = 0$. The reaction curves (4) become $x^{(i)^*} =$

$-\frac{1}{2}(x^{(j)}+s)$, so that the Nash solution (5) gives $x^{(i)^*} = (-\frac{1}{3}, -\frac{1}{3})'$, $y^* = (\frac{1}{3}, \frac{1}{3})$ and $w^{(i)^*} = \frac{2}{9}$. The HH approach with conjectural variations yields the fixed-point solution characterised by $x^{(i)^*} = D_*^{(i)}(x^{(j)^*}+s)$, $D_*^{(1)} = D_*^{(2)}$ and

$$(D_*^{(i)})^2 + 3D_*^{(i)} + I = 0. \tag{9}$$

Despite the simple and symmetric structure of the game, it has many solutions and it is not immediately obvious which are dominant and which are stable. This non-uniqueness is especially in applications a very unpleasant feature. It can be verified (after considerable algebraic manipulation) that the HH set of solutions to (9) is given by:

(1) $$\left\{D_*^{(i)} = \begin{bmatrix} p & q \\ \frac{-p^2-3p-1}{q} & -3-p \end{bmatrix}, \quad p,q \in \mathbb{R}, \; q \neq 0,\right.$$

$$x^{(i)^*} = \frac{1}{5}\begin{bmatrix} p+q-1 \\ -4-p-(\frac{p^2+3p+1}{q}) \end{bmatrix},$$

$$y^* = \begin{bmatrix} \frac{3}{5} + \frac{2}{5}(p+q) \\ -\frac{3}{5} - \frac{2}{5}[p+(\frac{p^2+3p+1}{q})] \end{bmatrix},$$

$$w^{(i)*} = \frac{1}{10}(\frac{p^2+3p+1}{q})^2 + (\frac{1}{5}p + \frac{2}{5})$$

$$\left.(\frac{p^2+3p+1}{q}) + \frac{1}{10}(2p^2+q^2+2pq+6p+2q+7)\right\};$$

(2) $\{D_*^{(i)} = (-\frac{3}{2} \pm \frac{1}{2}\sqrt{5})I,\ x^{(i)^*} = (-\frac{1}{2} \pm \frac{1}{10}\sqrt{5})s,\ y^* = (\pm \frac{1}{5}\sqrt{5})s,$

$w^{(i)^*} = \frac{1}{2} \mp \frac{1}{10}\sqrt{5}\};$

(3) $$\{D_*^{(i)} = \begin{bmatrix} -\frac{3}{2} \pm \frac{1}{2}\sqrt{5} & 0 \\ r & -\frac{3}{2} \mp \frac{1}{2}\sqrt{5} \end{bmatrix},\ r \in \mathbb{R},$$

$$x^{(i)^*} = \begin{bmatrix} -\frac{1}{2} \pm \frac{1}{10}\sqrt{5} \\ -\frac{1}{2} \mp \frac{1}{10}\sqrt{5} + \frac{1}{5}r \end{bmatrix},\ y^* = \begin{bmatrix} \pm \frac{1}{5}\sqrt{5} \\ \mp \frac{1}{5}\sqrt{5} + \frac{2}{5}r \end{bmatrix},$$

$$w^{(i)^*} = \frac{1}{2} + \frac{r}{10}(r \mp \sqrt{5} - 1)\}.$$

There is clearly an infinite number of solutions! The stability of these fixed points follows from the iterations

$$D_{s+1}^{(i)} = -(2I+D_s^{(j)})^{-1}(I+D_s^{(j)}) = (2I+D_s^{(j)})^{-1} - I, \tag{10}$$

that is if all the eigenvalues of the Jacobian, evaluated at $D_*^{(j)}$,

$$\frac{\partial \text{ vec } D_{s+1}^{(j)}}{\partial \text{ vec } D_s^{(j)}} = -\{(2I+D_s^{(j)})'^{-1} \otimes (2I+D_s^{(j)})^{-1}\} \tag{11}$$

are inside the unit circle the local stability of the iterative scheme in (10) in the neighbourhood of $D_*^{(j)}$ is guaranteed. The eigenvalues for the three sets of fixed points are given by $\{-\frac{3}{2} + \frac{1}{2}\sqrt{5},\ -\frac{3}{2} - \frac{1}{2}\sqrt{5},\ 1,1\}$, $\{-\frac{3}{2} \pm \frac{1}{2}\sqrt{5},\ 4 \text{ times}\}$ and $\{-\frac{3}{2} + \frac{1}{2}\sqrt{5},\ -\frac{3}{2} - \frac{1}{2}\sqrt{5},\ 1,1\}$, respectively, so that $D_*^{(i)} = (-\frac{3}{2} + \frac{1}{2}\sqrt{5})I$ is the only stable fixed point. The stable solution dominates all members of the third class of solutions, because the lowest possible welfare loss, $w^{(i)^*} = \frac{7}{20} \mp \frac{1}{20}\sqrt{5}$ for r =

$\frac{1}{2} \pm \frac{1}{2}\sqrt{5}$, is greater than the welfare loss the players incur under the stable solution, $w^{(i)^*} = \frac{1}{2} - \frac{1}{10}\sqrt{5}$. However, the stable solution of the HH set of solutions produces worse pay-offs than the Nash pay-offs. Finally, the only point that does satisfy (3), i.e. the Nash equilibrium $x^{(i)^*} = -\frac{1}{3}s$, is not an element of the HH set of "Nash" solutions.[1)]

For the two examples discussed above, it turned out that there was only one stable solution out of an infinite number of possible HH solutions. It seems that what is required to establish the HH sophisticated outcomes for non-cooperative decision making as a sensible concept is a general theorem that proves the uniqueness of a stable solution. Until this has been achieved, and it is not clear whether it can be achieved, some doubt remains with respect to the algorithm. In any case, it suggests that the user of the HH approach must perturb a proposed solution in order to establish the stability of conjectures.

The criticisms of the HH solution levied in this section are quite independent of the debate on time inconsistency and open loop vs. feedback Nash equilibria and apply to static as well as dynamic games. The next section of this comment discusses our dissatisfaction with the way HH deals with the problem of time inconsistency in dynamic games.

1) This follows from the equations $x^{(i)^*} = \frac{-1}{3}s = D_*^{(i)}(x^{(i)^*} + s)$ and (9) being inconsistent.

3. Time inconsistency and the problem of credibility in noncausal models

Before we discuss HH's solution to the problem of time inconsistency, we discuss the properties of some more conventional dynamic games. The most straightforward solution concept is the open loop Nash equilibrium (e.g. Kydland, 1975, Section 3; Başar and Olsder, 1982, Section 6.2.1; Rustem and Zarrop, 1979; van der Ploeg, 1982), which assumes zero conjectural variations for each player ($D^{(1)} = D^{(2)} = 0$) and static information patterns. There is no gain from re-optimising at a later date and therefore no problem with time inconsistency, because none of the players takes account of the forward-looking behaviour of its rivals. However, the feedback Nash equilibrium (e.g. Kydland, 1975, Section 2; Başar and Olsder, 1982, Section 6.2.2; Plasmans and de Zeeuw, 1982; de Zeeuw, 1984) can yield radically different actions as the players are now assumed to have access to the current state of the economy. Consider as an example the problem of an oligopoly with restricted entry and exit harvesting a renewable resource with zero extraction costs, iso-elastic demand and serially uncorrelated shocks to the natural replenishment rate. It can then be shown that the open loop extraction rates obey Hotelling-type arbitrage rules and are therefore efficient whilst the feedback (or credible) equilibrium leads to excessive extraction rates or even extinction of the resource (van der Ploeg, 1984). Furthermore, it can be shown that, in contrast to centralised decision making, players will benefit from adopting closed loop policy rules with memory as such rules dominate both open loop and feedback rules (Başar and Olsder, 1982, Section 6.3; de Zeeuw, 1984, Section 4.3). None of these more realistic information patterns are discussed in HH, since there attention is restricted to open loop equilibria and dynamic programming is avoided.

Another important solution concept is the open loop Stackelberg equilibrium (Kydland, 1975; Başar and Olsder, 1982, Section 7.2), which assumes zero conjectural variations for the follower ($D^{(1)} = 0$) and rational anticipation of the reactions of the follower by the leader ($D^{(2)} = -[G_0^{(2)'} Q^{(2)} G_0^{(2)}]^{-1} G_0^{(2)'} Q_1^{(2)} R^{(1)}$ where $Q^{(2)} = (Q_1^{(2)} \vdots Q_2^{(2)})$). It is well known that there is an incentive for the leader to renege on announced strategies, because the economy it faces today depends on announcements about future policy due to the anticipatory and non-predetermined behaviour of the shadow prices of the optimising follower. Such cheating could well be beneficial to both parties. For example, a government may announce taxation of the supply of labour tomorrow, in order to induce agents to accumulate capital today, and then improve economic welfare by the taxation of capital, instead of labour, tomorrow (Fischer, 1980). Although this dissembling behaviour of the government could improve efficiency, there is clearly a credibility problem as the announced strategies will, after repeated cheating, no longer be believed by the private sector. It is important to realise that the announcement of contingent tax rules, rather than a discretionary sequence of tax rates, can be beneficial even in deterministic environments and might, via the use of threat strategies, ensure the attainment of a command optimum. Note, however, that the announcement of such rules still suffers from the problems of time inconsistency (see the discussion of global Stackelberg equilibrium in Başar and Olsder, 1982, Section 7.4)[1] and therefore incredibility. The feedback Stackelberg equilibrium (Başar and Olsder, 1982, Section 7.3) does not rely on prior commitments and is by construction time consistent, because at each stage of the dynamic programming process the corresponding sub-game gives a Stackelberg equilibrium conditional on the state of the economy at that stage.[2] Such a credible equilibrium typically makes the leader worse off. Furthermore, with the feedback Stackelberg equilibrium the leader can, in contrast with the open loop or global Stackelberg equilibrium, be worse off than in the corresponding Nash equilibrium. Again none of these intricated informational issues play a role in Sections 2 and 6 of HH, since there the focus is on the calculation of an open loop Stackelberg equilibrium

1) Furthermore, the determination of such rules is typically indeterminate, due to the non-classical nature of the control problem, and therefore requires the presence of a self-disciplined government (Papavassilopoulos and Cruz, 1979).

2) Note that, unlike the case of Nash equilibrium (with linear strategies), the feedback and global (or closed loop with no memory) Stackelberg equilibrium give different results due to the forward-looking character of the follower's reactions.

and issues of credibility are not discussed.[1)] If the models used in HH were cast in state space rather than final form, the dynamic programming calculations of the feedback Stackelberg equilibrium might have been performed and issues of credibility might have been touched upon. Unfortunately, the discussion in HH, e.g. "The time or dynamic inconsistency and suboptimality of a sequence of decisions derived by recursive optimisation (e.g. dynamic programming) ... is well known" (HH, p. 390), does not seem to realise that dynamic programming yields consistent strategies by construction!

However, let us turn to the main effort of HH which seems to provide a more sophisticated equilibrium ($D^{(1)} = D_*^{(1)}$, $D^{(2)} = D_*^{(2)}$) than the Nash or Stackelberg equilibrium (see Section 2). The implicit leader-leader game in HH implies non-causality for both players, because $R^{(i)} + R^{(j)}D_*^{(j)}$ is no longer block-triangular even when $R^{(i)}$ is, and therefore suffers from time inconsistency. In Section 5.3 and the Appendix of HH a solution to the problem of time inconsistency is given. It is based on re-optimisation after each period and a policy-revision algorithm is provided to ensure feedback for the realisation of exogeneous shocks (Athans, et al., 1975) and to take advantage of the incentive to renege. The resulting policies are claimed to be optimal and time consistent.[2)] However, we argue that these policies rely on perpetual breaking of constracts and that they will therefore eventually loose credibility and not be implemented in practice.

In summary, HH confuses issues of time inconsistency and proposes a dissembling solution to the time inconsistency problem which lacks credibility.

1) In this context the statement "No special optimisation technique is needed because noncausality is avoided" (HH, p. 393) is particularly puzzling.

2) It is shown that there exists a problem such that their equilibrium results in a time consistent solution to that problem. The values of the Lagrange multipliers belonging to the original game are used for the modification. The revised problem has no interpretation and these arguments have to be considered a technical gimmick.

4. Conclusions

The HH set of sophisticated Nash equilibria relies on ambiguous information sets, contains an infinite number of solutions but not the Nash equilibrium, and most of these solutions are never reached as they are unstable. Furthermore, the pay-offs to the players are typically worse than in the conventional Nash equilibrium and the proposed equilibria are more closely related to the Stackelberg leader-leader than the Nash follower-follower game. One of the main problems is that a general theorem, which establishes the uniqueness of a stable solution, is not provided by HH and might not even exist. The problem of time inconsistency is closely related to the open loop and feedback equilibrium concepts and was not properly discussed, so an attempt has been made to clarify some of the issues involved. In particular, we argued that the optimal and consistent strategies proposed in HH are not credible.

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