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RESEARCH MEMORANDUM

CONTROLLED COMMUNICATION NETWORKS

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Controlled Communication Networks

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Abstract

This paper considers a generalisation of communication situations, namely controlled communication networks. Three solution concepts for controlled communication networks are introduced, the Myerson value, the position value and the mixed value, which are inspired by corresponding solution concepts for communication situations, and axiomatic characterisations of these concepts are provided. Further, network games associated with controlled communication networks are considered and it is shown that every TU-game can be obtained as a network game.

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1 Introduction and Preliminaries

In a cooperative TU (Transferable Utility) game (N, v) as modelled by von Neumann & Morgenstern (1944), N is a finite set of players, and the characteristic function v assigns to each subgroup of players a real number which is to be interpreted as the maximal gains (minimal costs) this coalition can achieve by cooperating, regardless of the actions of the other players.

It is generally assumed that each subgroup of players can form and cooperate to obtain its value. However, this approach fails to take into account communication restrictions that may cause deficiencies in cooperation in some coalitions. Myerson (1977) introduced communication graphs to model non-transitive communication restrictions. In such a graph the vertices are the players and an edge between two players represents the fact that these players can communicate directly. The general procedure is the following: given a TU-game (N, v) and a communication graph (N, A), one defines a reward function r on collections of vertices and edges which takes the communication restrictions into account, and then new games are extracted. This model was elaborated further by Owen (1986), Borm, Owen & Tijs (1992), Van den Nouweland, Borm, Owen and Tijs (1991) and Van den Nouweland & Borm (1991). A survey on this subject is given in Borm, Van den Nouweland & Tijs (1991).

In this paper, we generalise communication situations by allowing the vertex set of the graph to differ from the set of players N, and by starting out with a reward function r, instead of deriving it. This means that we can assign different rewards to different connected graphs with the same vertex set. We assume that the players control the vertices and edges of the graph through so-called control games. In section 2 we provide the formal definition of controlled communication networks and we introduce three solution concepts for these networks, the Myerson value, the position value and the mixed value. These solution concepts are characterised axiomatically in section 3. Finally, in section 4 we present an alternative way of constructing a TU-game corresponding to a controlled communication network, and we show that all TU-games can be obtained as such a network game.

Preliminaries

Let $N := \{1, \ldots, n\}$, and $2^N := \{S \mid S \subseteq N\}$. We denote the class of all transferable utility games with player set N by TU^N.

For each $S \in 2^N \backslash \{\emptyset\}$, the unanimity game $(N, u_S) \in TU^N$ is defined by

$$u_S(T) = \begin{cases} 1 & \text{if } S \subseteq T \\ 0 & \text{otherwise} \end{cases}$$

for all $T \subseteq N$.
It is well known that $\{(N, u_S) \mid S \in 2^N \backslash \{\emptyset\}\}$ forms a basis of TU^N.
The Shapley value Φ (cf. Shapley, 1953) is a linear function $TU^N \to \mathbf{R}^N$ defined by

$$\Phi_i(N, u_S) = \begin{cases} \frac{1}{|S|} & \text{if } \ i \in S \\ 0 & \text{if } \ i \in N \backslash S \end{cases}$$

for all $S \in 2^N \backslash \{\emptyset\}$.

2 Controlled Communication Networks

Consider a finite undirected graph (V, E) without loops or parallel edges. We assume that for each vertex $v \in V$ a *simple control game* (N, c_v) with *veto players* is given and, similarly, for each edge $e \in E$ a simple control game (N, c_e) with veto players is given. Here, a TU-game (N, c) is called *simple* if $c(S) \in \{0, 1\}$ for all $S \subseteq N$ and $c(N) = 1$. Moreover, a player $i \in N$ is called a *veto* player of the simple game (N, c), denoted by $i \in \text{veto}(c)$, if i can block a positive outcome, i.e. $c(S) = 0$ for all $S \in 2^N \backslash \{\emptyset\}$ with $i \notin S$. If c_v is the control game for vertex $v \in V$, then a coalition $S \subseteq N$ is allowed to use vertex v if and only if $c_v(S) = 1$. The control games (N, c_e) for edges have a similar interpretation.

Furthermore, we assume that there is a reward function r on subsets of vertices and edges $r : 2^V \times 2^E \to \mathbf{R}$, measuring the economic value of subnetworks. Keeping in mind that edges are to model communication channels, it seems reasonable to assume that an edge is useless without both its end points, i.e. for all $W \subseteq V$ and $F \subseteq E$ it holds that $r(W, F) = r(W, F \backslash \{\{v_1, v_2\}\})$ if $\{v_1, v_2\} \in F$ is such that $\{v_1, v_2\}$ is not a subset of W. So, the reward of a network (W, F) does not depend on the edges not in $F(W) := \{\{v, w\} \in F \mid v \in W, w \in W\}$. Moreover, in a network (W, F), W is partitioned into *communication components* in the following way: $C \subseteq W$ is a component within (W, F) if and only if $(C, F(C))$ is a connected subgraph of $(W, F(W))$ and is maximal with respect to this property. The resulting partition of W is denoted W/F.
Correspondingly, we assume that the reward function is additive with respect to these components, i.e.

$$r(W,F) = \sum_{C \in W/F} r(C, F(C))$$

for all $W \subseteq V$ and all $F \subseteq E$.
For simplicity, we assume that r is *zero-normalised*, i.e. $r(\{v\}, \emptyset) = 0$ for all $v \in V$.

A *controlled communication network* is a 6-tuple $\langle N, V, E, \{c_v | v \in V\}, \{c_e | e \in E\}, r\rangle$ as described above. The set of all controlled communication networks with player set N will be denoted CCN^N.

The purpose of this paper is to present and characterize some solutions to the problem of how to distribute the reward $r(V,E)$ among the players in N. Formally, a *solution concept* on CCN^N is a function $\gamma : CCN^N \to \mathbf{R}^N$ assigning $\gamma_i(\mathcal{C})$ to player i in the controlled communication network $\mathcal{C} \in CCN^N$. One way to obtain solution concepts on CCN^N is to construct for each controlled communication network a TU-game corresponding to this network in which the players are the edges and/or vertices of the graph. To this game one can apply a solution concept from cooperative game theory, for example the Shapley value. This yields a value for edges and/or vertices and this value can be distributed among the players according to veto control. Concentrating on vertices this procedure yields the Myerson value (cf. Myerson, 1977) and concentrating on edges it yields the position value (cf. Borm, Owen, and Tijs, 1992). If no such distinction between vertices and edges is made, one obtains the mixed value.

Let $\mathcal{C} = \langle N, V, E, \{c_v | v \in V\}, \{c_e | e \in E\}, r\rangle$ be a controlled communication network. Then the *Myerson value* $\mu(\mathcal{C}) \in \mathbf{R}^N$ is defined by

$$\mu_i(\mathcal{C}) := \sum_{v \in V : i \in \mathrm{veto}(c_v)} \frac{\Phi_v(V, r_E)}{|\,\mathrm{veto}(c_v)|}$$

for all $i \in N$, where the *vertex game* (V, r_E) is a game in which the vertices are the players, defined by $r_E(W) = r(W, E)$ for all $W \subseteq V$.
Further, the *position value* $\pi(\mathcal{C}) \in \mathbf{R}^N$ is defined by

$$\pi_i(\mathcal{C}) := \sum_{e \in E : i \in \mathrm{veto}(c_e)} \frac{\Phi_e(E, r_V)}{|\,\mathrm{veto}(c_e)|}$$

for all $i \in N$, where the *edge game* (E, r_V) is a game in which the edges are the players, defined by $r_V(F) = r(V, F)$ for all $F \subseteq E$.
Finally, the *mixed value* $\rho(\mathcal{C}) \in \mathbf{R}^N$ is defined by

$$\rho_i(\mathcal{C}) := \sum_{j \in V \cup E : i \in \mathrm{veto}(c_j)} \frac{\Phi_j(V \cup E, \tilde{r})}{|\,\mathrm{veto}(c_j)|}$$

for all $i \in N$, where the game $(V \cup E, \tilde{r})$ is defined by $\tilde{r}(J) = r(J \cap V, J \cap E)$ for all $J \subseteq V \cup E$.

3 Axiomatic characterisations

In this section we provide axiomatic characterisations of the three solution concepts introduced in section 2. It will turn out that all three concepts can be characterised by four axioms, three of them being the same for all three solution concepts.

For concision, if $\mathcal{C}$ is a controlled communication network and $F \subseteq E$ a set of edges we will use $\mathcal{C}^{-F}$ to denote the network where the edges in F have been omitted, and where the reward function has been restricted accordingly. We now introduce some properties.
A solution concept γ on CCN^N is called *efficient* if for each controlled communication network $\mathcal{C}$, γ distributes exactly $r(V, E)$ among the players. In formula:

$$\sum_{i \in N} \gamma_i(\mathcal{C}) = r(V, E).$$

A solution concept γ on CCN^N is called *additive* if it is additive with respect to the reward function (ceteris paribus).
A solution concept γ on CCN^N is said to have the *superfluous edge property* if for all $\mathcal{C} \in CCN^N$ and all edges $e \in E$ that are superfluous for $\mathcal{C}$ it holds that

$$\gamma(\mathcal{C}) = \gamma(\mathcal{C}^{-\{e\}}).$$

Here, an edge $e \in E$ is called *superfluous for* $\mathcal{C}$ if for all $F \subseteq E$,

$$r(V, F) = r(V, F \backslash \{e\}).$$

Note that for an edge e to be superfluous, we only consider the whole set of vertices V; we do not demand $r(W,F) = r(W, F\backslash\{e\})$ for all $W \subseteq V$ and $F \subseteq E$. However, this turns out to be an equivalent demand.

Lemma 1 Let $C \in CCN^N$. Then an edge $e \in E$ is superfluous for C if and only if

$$r(W,F) = r(W, F\backslash\{e\})$$

for all $W \subseteq V$ and all $F \subseteq E$.

Proof. The "if" part is straightforward. For the "only if" part, note that

$$\begin{aligned} r(W,F) &= \sum_{C\in W/F} r(C, F(C)) \\ &= \sum_{C\in W/F(W)} r(C, F(C)) \\ &\overset{(*)}{=} \sum_{C\in V/F(W)} r(C, F(C)) = r(V, F(W)) \end{aligned}$$

for all $F \subseteq E$ and $W \subseteq V$.
Here, equality $(*)$ follows from the fact that r is zero-normalised.
Hence, for a superfluous edge e

$$\begin{aligned} r(W, F\backslash\{e\}) &= r(V, (F\backslash\{e\})(W)) \\ &= r(V, F(W)\backslash\{e\}) \\ &= r(V, F(W)) \\ &= r(W,F) \end{aligned}$$

for all $F \subseteq E$ and $W \subseteq V$. This competes the proof. □

In a graph (V,E), we denote by $D(V,E)$ the set of vertices that have at least one neighbour in the graph, and we will shorten this notation to D whenever this does not lead to confusion.

The fourth property of solution concepts on CCN^N we introduce is anonymity. A controlled communication network is said to be *anonymous* if the reward function only depends on the number of non-isolated vertices and edges, i.e. there exists a function $f : \{0, \dots, |D \cup E|\} \to \mathbf{R}$ such that

$$r(W, F) = f(|(W \cap D) \cup F|)$$

for all $W \subseteq V$ and $F \subseteq E$.
A solution concept γ on CCN^N satisfies *anonymity* if for all anonymous $\mathcal{C} \in CCN^N$, the solution is proportional to the veto power of the players over the non-isolated parts of the graph or, in formula: there exists an $\alpha \in \mathbf{R}$ such that for all $i \in N$

$$\gamma_i(\mathcal{C}) = \alpha \cdot \sum_{j \in D \cup E : i \in \mathrm{veto}(c_j)} \frac{1}{|\,\mathrm{veto}(c_j)|}$$

The mixed value ρ satisfies the four properties mentioned. This is shown in

Lemma 2 The mixed value ρ satisfies efficiency, additivity, anonymity and the superfluous edge property.

Proof. Let $\mathcal{C}$ be a controlled communication network. Then

$$\begin{aligned}
\sum_{i \in N} \rho_i(\mathcal{C}) &= \sum_{i \in N} \sum_{j \in V \cup E : i \in \mathrm{veto}(c_j)} \frac{\Phi_j(V \cup E, \tilde{r})}{|\,\mathrm{veto}(c_j)|} \\
&= \sum_{j \in V \cup E} \Phi_j(V \cup E, \tilde{r}) \cdot \sum_{i \in \mathrm{veto}(c_j)} \frac{1}{|\,\mathrm{veto}(c_j)|} \\
&= \sum_{j \in V \cup E} \Phi_j(V \cup E, \tilde{r}) \stackrel{(*)}{=} r(V, E),
\end{aligned}$$

where equality $(*)$ follows from efficiency of the Shapley value Φ. Hence, ρ is efficient. Additivity of ρ follows straightforward by using additivity for Φ.
In order to prove the superfluous edge property, take $\mathcal{C} \in CCN^N$ and $e \in E$ that is superfluous for $\mathcal{C}$. It clearly suffices to prove that $\Phi_e(V \cup E, \tilde{r}) = 0$ and $\Phi_j(V \cup E, \tilde{r}) = \Phi_j(V \cup E \backslash \{e\}, \tilde{r})$ for all $j \in V \cup E \backslash \{e\}$. Using Lemma 1 we easily obtain $\tilde{r}(J) = \tilde{r}(J \backslash \{e\})$ for any $J \subseteq V \cup E$. Hence, e is a zero player in the game $(V \cup E, \tilde{r})$, and consequently $\Phi_e(V \cup E, \tilde{r}) = 0$ and $\Phi_j(V \cup E, \tilde{r}) = \Phi_j(V \cup E \backslash \{e\}, \tilde{r})$ for all $j \in V \cup E \backslash \{e\}$. We

conclude that ρ satisfies the superfluous edge property.
Now let $\mathcal{C} \in CCN^N$ be anonymous, and let $f : \{0, \ldots, |D \cup E|\} \to \mathbf{R}$ be such that $r(W, F) = f(|(D \cap W) \cup F|)$ for all $F \subseteq E$ and $W \subseteq V$. Then all vertices $v \in V \backslash D$ are zero players in the game $(V \cup E, \tilde{r})$, and all $j \in D \cup E$ are symmetric in this game. By symmetry, efficiency and the dummy property of Φ this implies

$$\Phi_j(V \cup E, \tilde{r}) = \begin{cases} \frac{f(|D \cup E|)}{|D \cup E|} & \text{if } j \in D \cup E \\ 0 & \text{if } j \in V \backslash D. \end{cases}$$

Hence, $\rho_j(\mathcal{C}) = \alpha \cdot \sum_{j \in D \cup E : i \in \text{veto}(c_j)} \frac{1}{|\text{veto}(c_j)|}$, where $\alpha := \frac{f(|D \cup E|)}{|D \cup E|}$. □

Before we prove that ρ is characterised by the four properties,we introduce two more definitions.
Let (V, E) be a graph. Then we denote by $\mathcal{R}(V, E)$ the set of (V, E)-admissible reward functions, i.e.

$$\mathcal{R}(V, E) := \{r : 2^V \times 2^E \to \mathbf{R} \mid r \text{ is additive w.r.t. components and zero-normalised}\}.$$

Moreover,

$$\begin{aligned} \mathcal{B}(V, E) \;\; := \;\; & \{u_{W,F} : 2^V \times 2^E \to \mathbf{R} \mid (W, F) \text{ is a connected subgraph of} (V, E), \\ & \text{and } |W| \geq 2\}, \end{aligned}$$

where $u_{W,F}$ is defined by

$$u_{W,F}(W', F') := \begin{cases} 1 & \text{if } W \subseteq W' \text{ and } F \subseteq F' \\ 0 & \text{otherwise.} \end{cases}$$

Then we have the following

Lemma 3 Let (V, E) be a fixed graph. Then $\mathcal{B}(V, E)$ forms a basis of the vector space $\mathcal{R}(V, E)$.

This lemma is an easy corollary of the fact that the set of unanimity games $\{(N, u_S) \mid S \in 2^N \backslash \{\emptyset\}\}$ forms a basis of TU^N, and that the reward functions are zero-normalized. However, the proof is rather technical, and therefore we omit it.

Theorem 4 The mixed value ρ is the unique solution concept on CCN^N satisfying efficiency, additivity, anonymity and the superfluous edge property.

Proof. According to Lemma 2 ρ satisfies the four properties. Hence, we only have to show that there is at most one solution concept satisfying these properties. Suppose γ is a solution concept on CCN^N that satisfies the four properties. Using lemma 3 and additivity of γ and ρ, we see that it suffices to prove $\gamma = \rho$ for situations in which $r = \beta u_{W,F}$ for some $\beta \in \mathbf{R}$ and some connected subgraph (W, F) with $|W| \geq 2$. Hence, let $\mathcal{C} \subseteq CCN^N$ be a controlled communication network with $r = \beta u_{W,F}$ for some $\beta \in \mathbf{R}$ and some connected subgraph (W, F) with $|W| \geq 2$. Since every edge e in $E \backslash F$ is superfluous for $\mathcal{C}$, the superfluous edge property implies that $\gamma(\mathcal{C}) = \gamma(\mathcal{C}^{-E\backslash F})$.
Furthermore,

$$r(W', F') = \beta u_{W,F}(W', F') = \begin{cases} \beta & \text{if } W \subseteq W' \text{ and } F' = F \\ 0 & \text{otherwise} \end{cases}$$

for all $W' \subseteq V$ and $F' \subseteq F$.
Since (W, F) is a connected graph, it holds that $D(V, F) = W$. So, defining $f : \{0, \ldots, |D \cup E|\} \to \mathbf{R}$ by

$$f(k) = \begin{cases} \beta & \text{if } k = |W \cup F| \\ 0 & \text{otherwise} \end{cases}$$

we see that the controlled communication network $\mathcal{C}^{-E\backslash F}$ is anonymous. Now, by anonymity of γ we know that there exists an $\alpha \in \mathbf{R}$ such that

$$\gamma_i(\mathcal{C}^{-E\backslash F}) = \alpha \cdot \sum_{j \in W \cup F : i \in \text{veto}(c_j)} \frac{1}{|\text{veto}(c_j)|}$$

for all $i \in N$.
Using efficiency, we obtain

$$\beta = r(V, F) = \sum_{i \in N} \gamma_i(\mathcal{C}^{-E\backslash F}) = \sum_{i \in N} \alpha \cdot \sum_{j \in W \cup F : i \in \text{veto}(c_j)} \frac{1}{|\text{veto}(c_j)|} = \alpha \cdot |W \cup F|.$$

Hence, $\alpha = \beta \cdot |W \cup F|^{-1}$ and recalling $\gamma(\mathcal{C}) = \gamma(\mathcal{C}^{-E\backslash F})$, we see

$$\gamma_i(\mathcal{C}) = \frac{\beta}{|W \cup F|} \cdot \sum_{j \in W \cup F : i \in \text{veto}(c_j)} \frac{1}{|\text{veto}(c_j)|} = \rho_i(\mathcal{C}).$$

□

We proceed by providing axiomatic characterisations of both the Myerson value and the position value. Both values can be characterised by efficiency, additivity, the superfluous edge property and an anonymity axiom. The anonymity axioms we need are *vertex anonymity* and *edge anonymity*.

A solution concept γ on CCN^N is said to be *vertex anonymous* if for every controlled communication network $\mathcal{C} \in CCN^N$ such that there exists a function $f : \{0, \ldots, |D(V, E)|\} \to \mathbf{R}$ with $r(W, E) = f(|D \cap W|)$ for all $W \subseteq V$, there is an $\alpha \in \mathbf{R}$ such that for all $i \in N$

$$\gamma_i(\mathcal{C}) = \alpha \cdot \sum_{v \in D : i \in \text{veto}(c_v)} |\text{veto}(c_v)|^{-1}.$$

A solution concept γ on CCN^N is called *edge anonymous* if for every $\mathcal{C} \in CCN^N$ such that there exists a function $f : \{0, \ldots, |E|\} \to \mathbf{R}$ with $r(V, F) = f(|F|)$ for all $F \subseteq E$, there is an $\alpha \in \mathbf{R}$ such that for all i in N

$$\gamma_i(\mathcal{C}) = \alpha \cdot \sum_{e \in E : i \in \text{veto}(c_e)} |\text{veto}(c_e)|^{-1}.$$

Theorem 5

i) The Myerson value μ is the unique solution concept on CCN^N that satisfies efficiency, additivity, the superfluous edge property and vertex anonymity.

ii) The position value π is the unique solution concept on CCN^N that satisfies efficiency, additivity, the superfluous edge property and edge anonymity.

The proof of theorem 5 runs along the same lines as the proof of lemma 2 and theorem 4 and therefore it is left to the reader.

Remark. The axiomatic characterisations of the Myerson value and the position value provided in theorem 5 are similar to axiomatic characterisations provided in Borm, Owen and Tijs (1992) for TU-communication situations. However, the reader should note that our characterisations hold for all controlled communication networks whereas Borm, Owen and Tijs (1992) had to restrict to cycle-free communication graphs.

4 Network games

In the previous sections we approached the problem how to divide the reward $r(V,E)$ of a controlled communication network amongst the players in an indirect way, by first determining the value of vertices and edges and then distributing these values among the veto players in the corresponding control games. In this section we will describe a direct way of dealing with the problem.

Let $\mathcal{C}$ be a controlled communication network. Now we define an associated game with player set N in the following way:
Let $S \subseteq N$. Then $V(S) := \{v \in V \mid c_v(S) = 1\}$ is the set of all vertices that coalition S can control, and $E(S) := \{e \in E \mid c_e(S) = 1\}$ is the set of edges that coalition S can control. Correspondingly, coalition S can obtain

$$v_{\mathcal{C}}(S) := r(V(S), E(S)).$$

Hence, we associate with $\mathcal{C} \in CCN^N$ the network game $(N, v_{\mathcal{C}})$ as defined above.
Consequently, some solution concept for TU-games could be applied to the game $(N, v_{\mathcal{C}})$.

This approach seems interesting, because a number of games associated with economic situations can be seen to be network games in a more or less natural way. Some examples are sequencing games (Curiel, Pederzoli & Tijs, 1989), permutation games (Tijs *et al.*, 1984) and assignment games (Shapley & Shubik, 1972).
However, some scepticism is in place here, because every TU-game is a network game in a trivial way:
Let (N, v) be an arbitrary TU-game. We proceed by defining a controlled communication network corresponding to (N, v). Let $V := N$ and $E := \{\{i,j\} \mid i,j \in N, i \neq j\}$. Hence, (V, E) is the complete graph with vertex set N. The control game for each vertex $i \in N$ is $(N, u_{\{i\}})$ and the control game for each edge $\{i,j\}$ is $(N, u_{\{i,j\}})$. Hence, every player is a dictator for his own vertex and an edge between two vertices is controlled by

the two players it connects. Finally, the reward function r assigns $v(S)$ to the subgraph $(S, E(S))$ for all $S \subseteq N$, and is extended in some feasible way to all subsets of vertices and edges. It is easily seen that the network game associated with the controlled communication network described above is the game (N, v).

Note that in the above discussion we did not restrict to zero-normalised games. However, the restrictions to zero-normalised reward functions was only made for simplicity and is not essential. Hence, we do not have to worry about it here.

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