OLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS
RESEARCH MEMORANDUM

[redacted]

A SHORT-PERIOD GOODWIN GROWTH CYCLE

Jörg Glombowski
Michael Krüger

FEW 317

A SHORT-PERIOD GOODWIN GROWTH CYCLE

Jörg Glombowski, Tilburg University,
Tilburg, The Netherlands

Michael Krüger, University of Massachusetts,
Amherst MA, U.S.A.

1. The Background[1)]

In one of the better-known passages of "Capital", Marx developed the idea of a cyclical interaction of accumulation, employment and income distribution between workers and capitalists (Marx 1977, pp. 580-582). The essence of this cyclical process can be summarized as follows: A high rate of capital accumulation reduces the "reserve army of labour". This causes an eventual rise in the wage share due to an increase in labour's bargaining power. In due time, this change in income distribution, unfavourable to capitalists, triggers a decline of accumulation and increases the unemployment ratio. The concomitant shift to profits sets the stage for a new start of the same sequence. Marx clearly associated this mechanism with the business cycle of his time when he stated: "Taking them as a whole, the general movements of wages are exclusively regulated by the expansion and contraction of the industrial reserve army, and these again correspond to the periodic changes of the industrial cycle" (Marx 1977, p. 596, also compare pp. 592-593).

In her essay on Marxian economics, Joan Robinson commented in some detail on this part of Marx's theory (Robinson 1969). While she did not reject its general idea, she argued that Marx was mistaken in presenting his model as an explanation of the business cycle: "This cycle Marx identifies with the decennial trade cycle. This identification is an error" (p. 84). Somewhat later she added: "There may be in reality a cycle of the type

----

1) Critical and constructive remarks by R.M. Costrell are gratefully acknowledged. We also benefitted from comments by P. Flaschel en A. de Zeeuw.

which Marx analyses. But if so, it must be of a much longer period than the decennial trade cycle ..." (p. 85). This assessment seems to be the logical consequence of her Keynesian perspective. She quite rightly remarks that product market disequilibria are neglected in Marx's argument, or, to put it another way, that the significance of effective demand is ignored as a relevant determinant of output and employment. As she is convinced that the latter is the clue to any explanation of the business cycle, Marx's cycle has to be something else: a "long-period cycle" (p. 85) or an element of a "long period-theory of employment" (cf. the title of chapter IV).

About a quarter of a century later, Richard Goodwin picked up Marx's idea and put it into a most elegant mathematical form (Goodwin 1967). Being himself one of the outstanding contributors to Keynesian macro-dynamics, he cannot be accused for ignoring the relevance of its specific points of departure. Nevertheless, he accepted - in contradiction to Robinson - Marx's mechanism as a fruitful approach to business cycle theory.

This view was soon to be challenged by A.B. Atkinson in an interesting article on the time dimension of economic models (Atkinson 1969). Among other subjects, he discussed whether the periodicity generated by Goodwin's growth cycle model could approximately lie in the range of empirically observed lengths of post-war business cycles. Although Atkinson chose parameter values and modifications conducive to shorter cycles, he concluded that Goodwin's 1967 model could not serve as a description of business cycles: "In fact the model as it stands may be better suited to explaining the 16-22 year 'Kuznets' cycle than the postwar trade cycle" (Atkinson 1969, p. 151). While Atkinson did not refer to Robinson's critique, his contribution supported her position. He did so, however, not by referring to the alleged superiority of the Keynesian approach. He rather claimed that Goodwin's model failed to pass a specific kind of consistency test: The time dimension of the processes a model generates should roughly correspond to the time dimension of the economic phenomena that it tries to explain. Of course, this is a rather limited test for the value of a model which certainly falls short of a thorough empirical check by econometric methods. (Similar tests could be applied if a model is claimed to be stable or insensitive to small changes in parameters.) But just as such a test is not very demanding, we feel that Atkinson is quite right to urge that models should be able to pass it.

We are not aware of any comments of Goodwin's on the time-period issue raised by Atkinson. Goodwin discussed Robinson's objections to his growth cycle model, however (Goodwin 1983). Apparently, Robinson had criticized the classical perspective of the model or rather its neglect of Keynesian insights in verbal communications: With underutilization of resources, and unemployment in particular, wages and profits would rise together in an upswing and decline together during a recession. By concentrating on income shares, which necessarily add up to one, according to Robinson too much emphasis had been put on the contradictory aspects of wages and profit dynamics. Goodwin accepted this criticism within certain limits. With unemployment governing nominal wage changes and constant mark-up pricing, he agrees that there may be Keynesian phases in the cycle, in which income shares remain more or less constant, while profits and wages move parallel in accordance with net output. He insists, however, that as output growth becomes eventually restricted by declining unemployment, real wages increases will squeeze profits and bring about a downturn. A similar argument is put forward by Goodwin with regard to the lower turning point. He arrives at the following conclusion: "Consequently it is in the region of full employment that the problem of the inverse relation of wages and profits arises since, with labour shortages, the real wage tends to rise strongly. This is the Marxian concept of the fluctuation of the reserve army of labour; it was this concept which I introduced into the model which Joan took exception to, presumably because I had not properly limited the region of its operation" (Goodwin 1983, p. 308).

Unfortunately, Goodwin does not mention any discussion with Joan Robinson on the problem of the appropriate length of the period generated by his model. It is this question that we are going to consider in the remainder of the present paper.[2)] The modifications of Goodwin's model which will be introduced have a Keynesian background. However, unlike Goodwin himself, we will not take up the issues of price and nominal wage formation,[3)] but rather allow for product market disequilibria, or, what amounts to the same, we will allow for a variable capital coefficient and a variable capacity utilization. By doing so, we try to help Goodwin's model to pass Atkinson's "consistency test". We are afraid that Joan Robinson would not have liked our mix of Marxian and Keynesian ingredients, as the Marxian components remain the dominant elements - as, for that matter, it is the case in Goodwin's defense. We are sure she would have preferred an amalgamation of Marxian and Keynesian ideas along underconsumptionist lines instead (cf. Robinson 1969, pp. 48-51, 71-72).

In section 2 we outline the modified model, which will be reduced to a system of three differential equations in section 3. Its characteristics are discussed in section 4, while the final section contains concluding remarks.

2. The Modified Goodwin-Model

Let us start by briefly summarizing those assumptions of Goodwin's we will not change. They refer first of all to the labour market. Labour supply (A) is assumed to grow with the constant growth rate n, while labour productivity (y) increases with the constant proportional rate m, i.e.

$$\hat{A} = (dA/dt)/A = n \tag{1}$$

$$\hat{y} = (dy/dt)/y = m \tag{2}$$

---

2) The issue under consideration has attracted more attention recently. After having finished this manuscript, we had a chance to see Robert Solow's contribution to the Goodwin-Festschrift, which is going to appear in 1988. Solow, too, takes a sceptical view respect to the neglect of demand side consideration and the period length in Goodwin's model.

3) The aspects neglected here are taken into account in another paper of ours, cf. Glombowski/Krüger 1987.

Ignoring labour hoarding, labour demand (L) is governed by the level of net output (Y),

$$L = Y/y \tag{3}$$

The employment ratio (β) is then obtained by dividing labour demand by labour supply,

$$\beta = L/A \tag{4}$$

Let w be the real wage rate. Its growth rate is assumed to depend positively on the employment ratio. We follow Goodwin in adopting a linear version of this relation, i.e.

$$\hat{w} = -a_1 + a_2\beta \tag{5}$$

where a_1 and a_2 are positive parameters. This relation can be justified by reference to the increase in bargaining power as employment approaches ever higher levels.[4] It should be noted that this assumption is rather Marxian than Keynesian. Keynesians would not deny that low levels of unemployment are likely to cause increments in <u>money</u> wages. Yet they would argue that the very same circumstances that make for high employment would, at the same time, create product market conditions which enabled capitalists to raise prices more or less in line with wage increases.

Next, define the wage share (μ) to be

$$\mu = wL/Y = w/y \tag{6}$$

From (5) and (6) it follows that its growth rate depends on the employment ratio,

---

4) One should notice, however, that the summarizing presentation of bargaining results by a single function like this does not provide much insight into the objectives, the strategies and the learning behaviour associated with the actual process of bargaining decisions.

$$\hat{\mu} = \hat{w} - \hat{y} = -(a_1+m) + a_2\beta \tag{7}$$

We now turn to the product market. Goodwin assumed in the classical fashion that total product could always be sold at once at a constant price. Under these circumstances the level of output would be determined by the capital stock at hand and a constant (technically determined) capital-output ratio. Let us assume instead that the rate of change of output is governed by a trend component and an excess demand part, i.e.

$$\dot{Y} = dY/dt = aY + \delta(I-S) \tag{8}$$

In (8), the parameter a is the trend rate of growth capitalists expect. It should be compatible with factual experiences. As demand (D) is composed of consumption demand (C) and investment (I), while net income (Y) is either consumed (C) or saved (S), the difference between demand and net income (supply), i.e. excess demand, equals the difference between (planned) investment and savings,

$$D-Y = (C+I) - (C+S) = I-S \tag{9}$$

Of course, a (positive) excess demand implies that stocks of finished goods are diminished in order to serve customers, while an excess supply means that present unsold produce is taken on stock for sale later on. The reaction coefficient δ measures how strongly producers will adjust production levels to product market disequilibria. The excess demand component of (8) is a continuous version of the well-known discrete-time dynamic multiplier, the lag being taken as exponentially distributed (cf. Allen 1966, pp. 69-72).

As we mentioned before, Goodwin assumed the product market to be in permanent equilibrium, which implies that savings always equal planned investment. This is in line with classical reasoning: Workers do not save, while capitalists fully spend on accumulation what they do not spend on consumption. As Goodwin's capitalists do not consume at all, total profits, investment and savings coincide in his model. We assume instead that savings and investment decisions are taken separately. While capitalists are held to accumulate a certain constant percentage (c) of profits (π),

$$I = c\pi, \tag{10}$$

savings are described according to Kaldor's savings function, i.e.

$$S = s_w wL + s_\pi \pi, \tag{11}$$

where s_w and s_π are the constant savings ratios of workers and capitalists, respectively.

Profits are defined as non-wage income, i.e.

$$\pi = Y - wL = (1-\mu)Y \tag{12}$$

Goodwin employs the same definition, but one should notice that its meaning has slightly changed, as profits now include the increment of inventories in case of excess supply and do not include sales of previously produced commodities in case of excess demand. This definition of profits does not cause serious problems as long as it is reasonable to expect that actual surplusses can be sold later. This is exactly what would happen in a non-explosive cyclical development with phases of excess surplusses being followed by times of excess demand and vice versa.

With regard to the parameters of the savings and investment functions we stipulate

$$1 > c > s_\pi > s_w > 0 \tag{13}$$

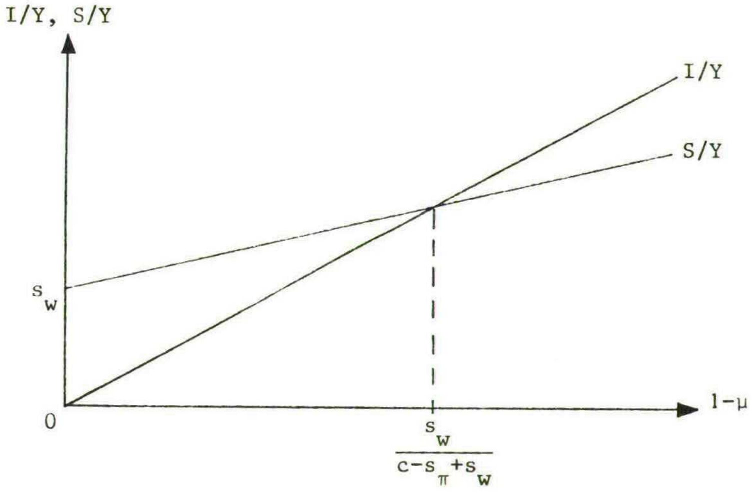
Let us examine the consequences of these assumptions. In figure 1 the savings and investment ratios out of net income are both drawn as functions of the profit share, i.e. $(1-\mu)$.[5] Assumption (13) guarantees that both functions have an intersection for $(1-\mu) \in (0,1)$. Thus there exists a

---

5) Compare the functions (19) and (20) below.

positive profit share smaller than one which is compatible with product market equilibrium.

FIGURE 1: DISTRIBUTION AND PRODUCT-MARKET (DIS)EQUILIBRIUM



There is a second implication of (13) we should mention. Whenever the income distribution should happen to be constant at a level giving rise to a positive excess demand, the concomitant rise in production would not reduce the initial imbalance. Therefore, the dynamic multiplier process is unstable. This is more in line with Harrod's cumulative instability of knife-edge growth than with the short-term stability of Keynes' investment multiplier. Hence, in our present model, it will depend on the dynamics of income distribution whether or not the cumulative unstable process is transformed into a cyclical motion.

3. Reduction of the Model

The model introduced above can be reduced to three differential equations in the employment ratio, the wage share and the capital coefficient. The first of these equations follows immediately from (7):

$$\dot{\mu} = -(a_1+m)\mu + a_2\beta\mu \tag{14}$$

It takes a few steps more to derive the equation for the employment ratio. From (4) we have

$$\hat{\beta} = \hat{L} - \hat{A} \tag{15}$$

while

$$\hat{L} = \hat{Y} - \hat{y} \tag{16}$$

follows from (3). Substituting (16) into (15) and taking (1) and (2) into account, the growth rate of the employment ratio turns out to be equal to the growth rate of net product minus the growth rates of labour productivity and labour supply, i.e.

$$\hat{\beta} = Y - m - n \tag{17}$$

The growth rate of net product can be shown to be a function of the wage share: From (8) we have

$$Y = a + \delta(I/Y-S/Y) \tag{18}$$

The investment ratio is given by

$$I/Y = c(1-\mu) \tag{19}$$

because of (10) and (12), while the savings ratio can be written

$$S/Y = s_w\mu + s_\pi(1-\mu) \tag{20}$$

on behalf of (6), (11) and (12). Making use of the abbreviation

$$g =: c - s_\pi + s_w \tag{21}$$

the relative excess demand becomes

$$I/Y - S/Y = g(1-\mu) - s_w \tag{22}$$

which leads to

$$\hat{Y} = a + \delta[g(1-\mu)-s_w] \tag{23}$$

for the development of net product growth. Substitution of (23) into (17) provides the differential equation for the employment ratio we are looking for:

$$\dot{\beta} = [a-m-n+\delta(g-s_w)]\beta - \delta g\beta\mu \tag{24}$$

Obviously, (14) and (24) form a pair of differential equations in β and μ which are self-sufficient as they do not involve other variables of the system. It seems helpful, however, to add a third equation in the capital coefficient to trace the effects of product market disequilibrium. The capital coefficient (v) is defined as

$$v = K/Y \tag{25}$$

Therefore, its growth rate can be written

$$\hat{v} = \hat{K} - \hat{Y} \tag{26}$$

From (10) and (11) we obtain the growth rate of the capital stock as

$$\hat{K} = c(1-\mu)/v \tag{27}$$

After inserting (18) and (27) into (26) we can derive

$$\dot{v} = c(1-\mu) - (a+\delta g-\delta s_w)v + \delta g\mu v \tag{28}$$

as the differential equation in the capital coefficient.

4. Characteristics of the Solution

We are now prepared to check whether the system (14), (24) and (28) has got a steady state solution, i.e. a triple (β_e, μ_e, v_e) which makes the time derivatives of β, μ and v vanish. A unique non-trivial steady state solution does exist and is given by

$$\beta_e = (m+a_1)/a_2 \tag{29}$$

$$\mu_e = (a-m-n)/(\delta g) + (1-s_w/g) \tag{30}$$

$$v_e = \frac{c(1-\mu_e)}{a + \delta(g-s_w) - \delta g\mu_e} \tag{31}$$

The steady state values of the wage share and the capital coefficient both depend on a, the expected growth rate of demand. Thus it seems that there are a lot of steady states according to different levels of this parameter. From a long-run perspective, however, the assumption a = m+n seems to be natural: A constant employment ratio implies $\hat{Y}$ = m+n because of (17). Then from (18) we obtain

$$I/Y - S/Y = (m+n-a)/\delta \tag{32}$$

Thus whenever a $\neq$ m+n, a steady state with either a constant relative excess demand or a constant relative excess supply emerges. In the excess demand case, sooner or later the initial inventories will be run down, so that demand has to be rationed. In the opposite case, stocks will be piled up in ever larger absolute amounts. One might assume that excess stocks would be destroyed more or less regularly, but such an assumption does not seem to make much sense. In the long run, and especially in a steady state, capitalists should be considered competent enough to correctly anticipate the trend rate of demand growth. Putting

$$a = m+n \tag{33}$$

the equilibrium employment ratio is unaffected, while the others are simplified to become

$$\mu_e = 1 - s_w/g \tag{34}$$

and

$$v_e = cs_w/[g(m+n)] \tag{35}$$

Note that the differential equations for β and v have to be adjusted accordingly.

To check the local stability of the steady state and to get a first idea of the behaviour of our model off the steady state, we consider the solution to the system (14), (24) and (28) linearized around its steady state values. The linearized system reads

$$\begin{bmatrix} \dot{\beta} \\ \dot{\mu} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & -\delta g\beta_e & 0 \\ a_2\mu_e & 0 & 0 \\ 0 & c + \frac{\delta cs_w}{m+n} & -(m+n) \end{bmatrix} \begin{bmatrix} \beta-\beta_e \\ \mu-\mu_e \\ v-v_e \end{bmatrix} \tag{36}$$

where the elements of the (Jacobian) matrix (J) are the partial derivatives of the differential equations with respect to β, μ and v, taken at the equilibrium point. The eigenvalues s can be calculated most easily by developing the last column of the determinant J - sI, putting it equal to zero and solving for s. Proceeding like this, we obtain a pair of purely imaginary eigenvalues,

$$s_{1/2} = \pm\, i(\delta g a_2\beta_e\mu_e)^{1/2} \tag{37}$$

and one real eigenvalue,

$$s_3 = -(m+n) \tag{38}$$

The imaginary pair of eigenvalues is associated with the self-sufficient subsystem (14) and (24). This outcome implies that the employment ratio and the wage share will both follow cyclical time paths with constant amplitudes and constant periods in the neighbourhood of the equilibrium point, which means that the equilibrium is locally stable, although not asymptotically stable. The time period (Θ) of the oscillations can be calculated by the formula

$$\Theta = 2\pi(\delta g a_2 \beta_e \mu_e)^{-1/2}, \tag{39}$$

while the amplitudes depend on the initial displacements from the steady state values.[6] We will present some figures below to check whether time periods observable in actual business cycles are likely to be obtained in our model.

The time path of the capital coefficients is made up out of two components: a first component displaying regular oscillations and a second one, associated with the real eigenvalue, which depends on initial conditions and eventually vanishes. Therefore, the capital coefficient will show regular oscillations in the long run with the same period as the other variables, whenever the latter oscillate. Should the wage share and the employment ratio take on their steady state values from the beginning, then any initial difference of the capital coefficient from its steady state value would decrease monotonically with time.

The numerical examples presented here follow Atkinson's examples as far as possible, i.e. we choose the same (size of) parameters as he did. While we allow the parameters c and δ, which are associated with our modifications, to vary as indicated below, we stick to the following values throughout:

$m = 0{,}03 \qquad n = 0{,}01 \qquad a_1 = 0{,}94 \qquad a_2 = 1{,}00$

$s_\pi = 0{,}23 \qquad s_w = 0{,}05$

Note that the "natural rate of growth" ($m+n=0{,}04$) and the equilibrium rate of employment ($\beta_e=0{,}97$) have the same values as in Atkinson's calculations. As far as a_2 is concerned, we take the lowest value that Atkinson chose, i.e. the most <u>unfavourable</u> one for the emergence of short cycles.

The following table shows that there is a broad spectrum of values for c and δ which give rise to periods "acceptable" for a business cycle model.

----

6) Here we only describe the results. More details are given in the appendix.

Table 1: Periods of Cycles

c \ δ	3	6	9	12
0,3	13,925	9,847	8,040	6,963
0,4	8,932	6,316	5,156	4,466
0,5	7,087	5,016	4,092	3,544
0,6	6,055	4,282	3,496	3,028
0,7	5,372	3,800	3,102	2,686

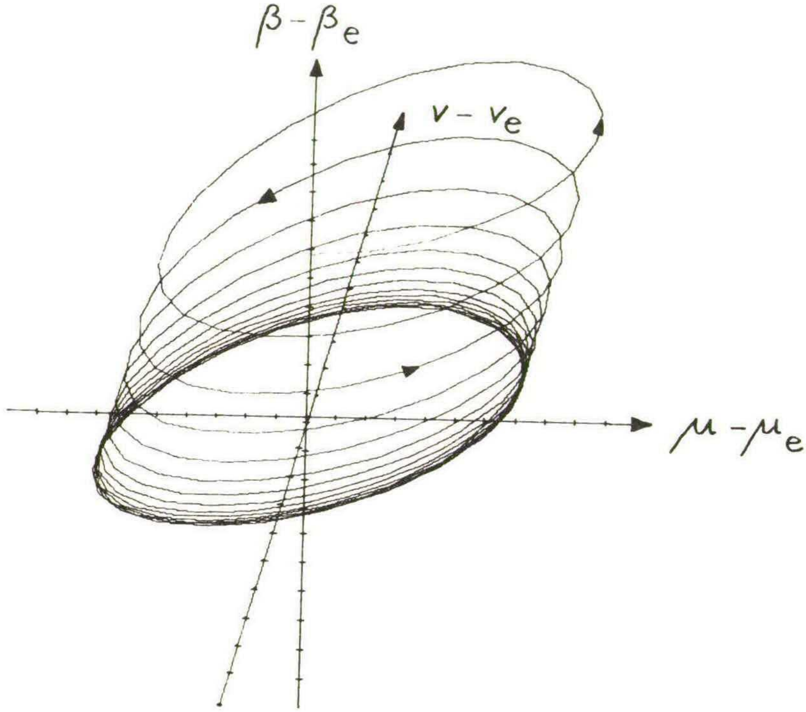
Table 2: Equilibrium Values

c	β_e	μ_e	v_e
0,3	0,970	0,583	3,125
0,4	0,970	0,773	2,273
0,5	0,970	0,844	1,953
0,6	0,970	0,881	1,786
0,7	0,970	0,904	1,683

The equilibrium values do not depend on the reaction coefficient δ, but μ_e and v_e vary with the accumulation quota c as shown in table 2. We only report those values here in order to show that the periods of cycles presented in table 1 do not lead to unreasonable equilibrium values.

So far we have only considered the linearized version of our equation system. In order to make sure that the same type of system behaviour occurs for the original non-linear equations, we have employed a fourth-order Runge-Kutta formula for numerical integration (cf. Lapidus/Seinfeld 1971, pp. 65-66). Figure 2 shows the trajectories of our variables in three-dimensional state phase for the case c = 0,4 and δ = 3. The equilibrium point forms the origin. Within 120 years we obtain about thirteen cycles, which fits in with the corresponding figure from table 1. Moreover, we can see that the amplitudes of μ and β remain the same, while the cycles converge to a definite one due to the trend component in v. Thus the behaviour of the non-linear model corresponds to the features of the linearized one.

FIGURE 2: THE TIME PATHS OF THE EMPLOYMENT RATIO, THE WAGE SHARE AND THE CAPITAL COEFFICIENT IN THREE-DIMENSIONAL STATE PHASE



Parameters:

$c = 0{,}4$ $\delta = 3$

$\beta_e = 0{,}97$ $\beta(o) = 1$

$\mu_e = \mu(o) = 0{,}77272727$

$v_e = v(o) = 2{,}2727273$

number of iterations = 300

step size = 0,4

number of periods = 120

scale: 1 point = 0,5 % deviation from equilibrium

5. Concluding Remarks

On the basis of our numerical examples, which are not singular, we are able to reject Atkinson's pessimistic conclusion as to the value of Goodwin's model as a basis for business cycle theory. Employing simple modifications, but alternative ones to those suggested by Atkinson, our modified Goodwin model passes his test. The economic reason for this result is that the introduction of excess demand as a factor regulating production dynamics "speeds up" the profit squeeze cycle. Whenever a low wage share induces more investment, more production and a higher employment, the expansion in Goodwin's original model is restricted by capital accumulation, given a constant degree of capacity utilization. In our modified version, however, production growth is more flexible as the capital coefficient may fall due to increases in capacity utilization in the face of excess demand. A stronger reaction of production growth due to excess demand leads to a faster rise in the employment ratio and, thereby, to a quicker reaction of wage rates. The profit squeeze therefore makes itself felt earlier than in the original model. Consequently, the modified model brings about shorter cycles.

We do not claim that by the introduction of our modifications all objections to Goodwin's growth cycle model have been removed. A sufficient number of other serious objections have been raised in the literature (and other modifications have been proposed) to preclude that. Moreover, the purely theoretical question of consistency may be less important than the issue of empirical relevance, which we have not addressed at all. Nevertheless, we think that the specific type of criticism can be rejected which holds that the periods to which this kind of models give rise would necessarily be too long to consider them as candidates for business cycle explanations.

References

Allen, R.G.D. (1966), Mathematical Economics, London (Macmillan), 2nd edition.

Atkinson, A.B. (1969), The Timescale of Economic Models: How Long is the Long Run? Review of Economic Studies 36, pp. 137-152.

Glombowski, J. and M. Krüger (1987), Generalizations of Goodwin's Growth Cycle Model, in: D. Batten/J. Casti/B. Johansson (eds.), Economic Evolution and Structural Adjustment, Berlin (Springer), pp. 260-290.

Goodwin, R.M. (1967), A Growth Cycle, in: C.H. Feinstein (ed.), Socialism, Capitalism and Economic Growth: Essays Presented to Maurice Dobb, Cambridge (Cambridge University Press), pp. 54-61.

Goodwin, R.M. (1983), A Note on Wages, Profits and Fluctuating Growth Rates, Cambridge Journal of Economics 7, pp. 305-309.

Kaplan, W. (1958), Ordinary Differential Equations, Reading (Ma.) (Addison-Wesley).

Lapidus, L. and J.H. Seinfeld (1971), Numerical Solution of Ordinary Differential Equations, New York/London (Academic Press)

Marx, K. (1977), Capital. A Critique of Political Economy, Vol. 1 London (Lawrence & Wishart).

Robinson, J. (1969), An Essay on Marxian Economics, London (Macmillan), reprint of 2nd edition.

Solow, R.M. (1988), Goodwin's Growth Cycle. Reminiscence and Rumination, to appear.

Appendix: Explicit Solution of the Linearized System

The autonomous system of linear differential equations (36) has been obtained by linearizing the original non-linear system (14), (24) and (28) around its unique non-trivial equilibrium point, as given by (29), (30) and (31). The general solution to (36) can be written as follows[7)]

---

7) There are, of course, a lot of treatments on differential equations. For the present case, Kaplan (1958), pp. 224-230 is a very well suited reference.

$$\beta - \beta_e = C_1 c_{11} \exp(s_1 t) + C_2 c_{12} \exp(s_2 t) + C_3 c_{13} \exp(s_3 t)$$

$$\mu - \mu_e = C_1 c_{21} \exp(s_1 t) + C_2 c_{22} \exp(s_2 t) + C_3 c_{23} \exp(s_3 t) \qquad \text{(A1)}$$

$$v - v_e = C_1 c_{31} \exp(s_1 t) + C_2 c_{32} \exp(s_2 t) + C_3 c_{33} \exp(s_3 t)$$

Here, the s_i (i=1,2,3) are the characteristic roots of the Jacobian in (36) as given by (37) and (38); $(c_{1j}, c_{2j}, c_{3j})'$ is the eigenvector corresponding to the j-th root; and the C_j are constants to be determined from initial conditions.

Let us rewrite the non-zero entries of the Jacobian as follows:

$$a_{12} = -\delta g \beta_e$$

$$a_{21} = a_2 \mu_e$$

$$a_{32} = c + \delta c s_w/(m+n)$$

$$a_{33} = -(m+n)$$

Moreover, let

$$(\delta g a_2 \beta_e \mu_e)^{1/2} = b$$

The first eigenvector, i.e. the one associated with root s_1, can be represented by

$$c_{11} = -a_{12} + ib$$

$$c_{21} = a_{21} - ib$$

$$c_{31} = \frac{a_{32}(a_{21}(m+n)-b^2) - iba_{32}(a_{21}+m+n)}{(m+n)^2 + b^2}$$

The second one is the complement of the first. We only have to reverse the signs before the i's to find it. Note that all the eigenvectors are only determined up to multiple constant. Finally, the third eigenvector can be represented by

$$c_{13} = c_{23} = 0, \; c_{33} = 1$$

The eigenvectors can be substituted into (A1). In order to obtain solutions in real numbers, we make use of the following identities

$$\exp(ibt) = \cos bt + i \sin bt$$

$$\exp(-ibt) = \cos bt - i \sin bt$$

Furthermore, we switch to the new pair of constants c_1, c_2 by employing

$$C_1 = (c_1 + ic_2)/2 \text{ and } C_2 = (c_1 - ic_2)/2$$

For the first equation of (A1), we obtain

$$\beta - \beta_e = (C_1c_{11}+C_2c_{12})\cos bt + i(C_1c_{11}-C_2c_{12})\sin bt,$$

which, after substitutions, gives rise to

$$\beta - \beta_e = -(a_{12}c_1+bc_2)\cos bt + (a_{12}c_2-bc_1)\sin bt$$

In the same way, we obtain the solution for μ,

$$\mu - \mu_e = (a_{21}c_1+bc_2)\cos bt + (bc_1-a_{21}c_2)\sin bt$$

The third equation contains an additional exponential term since c_{33} does not vanish. First we get

$$v - v_e = C_1c_{31}(\cos bt + i \sin bt) + C_2c_{32}(\cos bt - i \sin bt) +$$
$$+ C_3c_{33}\exp(-(m+n)t)$$

After all substitutions and rearrangements have been made, we arrive at

$$v - v_e = (c_1g_1 - c_2g_2)\cos bt - (c_1g_2 + c_2g_1)\sin bt + C_3\exp(-(m+n)t),$$

where g_1 and g_2 stand for the following expressions:

$$g_1 = \frac{a_{32}(a_{21}(m+n) - b^2)}{(m+n)^2 + b^2} \qquad g_2 = \frac{a_{32}b(a_{21} + m + n)}{(m+n)^2 + b^2}$$

From the structure of the solution, it is clear that the constants c_1 and c_2 can be derived from the first two equations only, given initial values of β and μ. C_3 will then be determined by adding an initial condition for v.

The solution to the linearized system has the same qualitative features as exhibited by the numerical solution to the non-linear system (cf. figure 2).

In the numerical integration of the original non-linear system we assumed initial conditions of the form $\beta(0) > \beta_e$, $\mu(0) = \mu_e$ and $v(0) = v_e$. If we employ the same type of initial conditions here, the constants can be derived from the equations

$$\beta(0) - \beta_e = -a_{12}c_1 - bc_2$$

$$0 = a_{21}c_1 + bc_2$$

$$0 = c_1g_1 - c_2g_2 + C_3$$

Hence we find

$$c_1 = (\beta(0) - \beta_e)/(a_{21} - a_{12})$$

$$c_2 = -a_{21}(\beta(0) - \beta_e)/(b(a_{21} - a_{12}))$$

$$C_3 = c_2g_2 - c_1g_1$$

Using the same numerical values as in the example given in the text, a graph similar to figure 2 can be shown to emerge from the linearized model.

IN 1987 REEDS VERSCHENEN

242 Gerard van den Berg
Nonstationarity in job search theory

243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions

244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk

245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation
of subjective poverty definitions

246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance Criteria

247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en strukturele afhankelijkheden

248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group

249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic Investment Policy

250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical approach

251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically distributed errors

252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten

253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen

254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint products: A comment

255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?

256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium concepts

274 Raymond H.J.M. Gradus
The net present value of governmental policy: a possible way to find the Stackelberg solutions

275 Jack P.C. Kleijnen
A DSS for production planning: a case study including simulation and optimization

276 A.M.H. Gerards
A short proof of Tutte's characterization of totally unimodular matrices

277 Th. van de Klundert and F. van der Ploeg
Wage rigidity and capital mobility in an optimizing model of a small open economy

278 Peter M. Kort
The net present value in dynamic models of the firm

279 Th. van de Klundert
A Macroeconomic Two-Country Model with Price-Discriminating Monopolists

280 Arnoud Boot and Anjan V. Thakor
Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

281 Arnoud Boot and Anjan V. Thakor
Appendix: "Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

282 Arnoud Boot, Anjan V. Thakor and Gregory F. Udell
Credible commitments, contract enforcement problems and banks: intermediation as credibility assurance

283 Eduard Ponds
Wage bargaining and business cycles a Goodwin-Nash model

284 Prof.Dr. hab. Stefan Mynarski
The mechanism of restoring equilibrium and stability in polish market

285 P. Meulendijks
An exercise in welfare economics (II)

286 S. Jørgensen, P.M. Kort, G.J.C.Th. van Schijndel
Optimal investment, financing and dividends: a Stackelberg differential game

287 E. Nijssen, W. Reijnders
Privatisering en commercialisering; een oriëntatie ten aanzien van verzelfstandiging

288 C.B. Mulder
Inefficiency of automatically linking unemployment benefits to private sector wage rates

289 M.H.C. Paardekooper
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues

290 Pieter H.M. Ruys
Industries with private and public enterprises

291 J.J.A. Moors & J.C. van Houwelingen
Estimation of linear models with inequality restrictions

292 Arthur van Soest, Peter Kooreman
Vakantiebestemming en -bestedingen

293 Rob Alessie, Raymond Gradus, Bertrand Melenberg
The problem of not observing small expenditures in a consumer expenditure survey

294 F. Boekema, L. Oerlemans, A.J. Hendriks
Kansrijkheid en economische potentie: Top-down en bottom-up analyses

295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note

296 Arthur van Soest, Peter Kooreman
Estimation of the indirect translog demand system with binding non-negativity constraints

IN 1988 REEDS VERSCHENEN

297 Bert Bettonvil
Factor screening by sequential bifurcation

298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure

299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry

300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method

301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note

302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in lineair models using cross sections, panels or both

303 Raymond H.J.M. Gradus
A differential game between government and firms: a non-cooperative approach

304 Leo W.G. Strijbosch, Ronald J.M.M. Does
Comparison of bias-reducing methods for estimating the parameter in dilution series

305 Drs. W.J. Reijnders, Drs. W.F. Verstappen
Strategische bespiegelingen betreffende het Nederlandse kwaliteitsconcept

306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg
Regression sampling in statistical auditing

307 Isolde Woittiez, Arie Kapteyn
A Model of Job Choice, Labour Supply and Wages

308 Jack P.C. Kleijnen
Simulation and optimization in production planning: A case study

309 Robert P. Gilles and Pieter H.M. Ruys
Relational constraints in coalition formation

310 Drs. H. Leo Theuns
Determinanten van de vraag naar vakantiereizen: een verkenning van materiële en immateriële factoren

311 Peter M. Kort
Dynamic Firm Behaviour within an Uncertain Environment

312 J.P.C. Blanc
A numerical approach to cyclic-service queueing models

313 Drs. N.J. de Beer, Drs. A.M. van Nunen, Drs. M.O. Nijkamp
Does Morkmon Matter?

314 Th. van de Klundert
Wage differentials and employment in a two-sector model with a dual labour market

315 Aart de Zeeuw, Fons Groot, Cees Withagen
On Credible Optimal Tax Rate Policies

316 Christian B. Mulder
Wage moderating effects of corporatism
Decentralized versus centralized wage setting in a union, firm, government context