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OPTIMAL TAXATION ON PROFIT AND
POLLUTION WITH A MACROECONOMIC
FRAMEWORK

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OPTIMAL TAXATION ON PROFIT AND POLLUTION WITHIN A MACROECONOMIC FRAMEWORK

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Abstract

In this paper a macroeconomic model of optimal profit taxation developed by Gradus (1990, Chapter 4) is extended through incorporation of a tax rate on pollution. It is assumed that the representative firm owns two different stocks of capital goods. The first one is productive but also generates pollution, and the second one is non productive but cleans pollution.

The problem is modelled as a Stackelberg differential game such that the government is the leader; the firms and consumers, each represented by one, are the followers playing Nash against each other. Open-loop and feedback equilibria are studied and a numerical example is used to gain some further insights. We show that the open-loop Stackelberg equilibrium leads to a higher level of welfare, but not necessarily to a lower level of pollution.

1. INTRODUCTION

In recent years, during which a clean environment becomes more and more a scarce commodity, economists have shown an increasing interest in the problem of reducing the pollution output of firms. An important question in this respect is what kind of policy instruments the government, in its role as social planner, should choose to reduce the level of pollution. In the economic literature it is argued that from an economic point of view the government should try to diminish the firm's pollution by introducing a pollution tax rate rather than imposing laws and/or production restrictions on the firm.

The influence of pollution tax on the behavior of a profit maximizing firm was studied in a paper by Kort, Van Loon and Luptacik (1990). It turned out that the optimal policy of the firm over time mainly depended on the relationships between the values of the different unit costs, in which also the pollution tax rate occurs. However, a major drawback of this model is that the policy of the government, i.e. fixing the pollution

tax rate, is taken exogenously in the sense that only the behavior of the firm is maximized. By doing this the Lucas critique (cf. Lucas (1976)), which states that the interactions between private and public sector should be evaluated when we want to derive the incidence of different tax rate, is not taken into account. The Lucas critique can be dealt with by modelling the problem as a dynamic game between private and public sector. Starting point of this kind of research is a paper by Fischer (1980), where in a simple two-period model the trade off between capital and labor tax is described. Also the issue of time-inconsistency, which occurs when having distortionary taxation, is discussed in that paper. The main conclusion is that if the government commits its tax policy, it yields a higher level of welfare. Later on the Fischer framework has been extended by many authors to other tax rates (cf. Rogers (1987), Chang (1988)) and political aspects (cf. Persson and Svensson (1989), Alesina and Tabellini (1987)). In this paper the route of optimal taxation will be followed and we show that the pollution tax rate can be time-inconsistent. Moreover, we focus on the policy implications of this conclusion.

Starting point of this paper will be the decentralized market model of Abel and Blanchard (1983) in which the policy of the government was taken exogenously. In this paper a general equilibrium model with utility maximizing consumers and value maximizing firms, which face costs of adjustment, was presented and the incidences of different tax rates were analyzed. We extend this research in two directions. First, we model the government's behavior endogenously, where it maximizes the utility of a representative agent. Here, we assume that the firms and consumers behave atomistically, while the government is the leader within a Stackelberg game. We study the commitment and no-commitment solution of this game. Second, we incorporate a pollution tax rate.

The paper is organized as follows. In Section 2 we model the firms' and consumers' decision problem. Furthermore, the equilibrium in the goods and labor market is described while in Section 3 we present optimal government's behavior under the condition that it takes into account the way that agents make their decisions and that there is an open-loop information structure. The feedback case is studied in Section 4 where also a numerical example is presented. Finally, in Section 5 we conclude this paper.

2. THE FIRM'S AND CONSUMER'S DECISION PROBLEM

For reasons of analytical tractability we assume that there is only one representative type of consumer and firm.

2.1. The model of the firm

Consider a firm operating in an environment without exogenous uncertainty. The firm produces a homogeneous output by means of its factors capital and labor. The firm's output can be used for different kinds of public and private spendings. With respect to the production function we assume that capital and labor are substitutes and there is a constant returns to scale technology, so that

$$f = f(k, l), f(k, 0) = 0, f(0, l) = 0, f_k > 0, f_l > 0, f_{ll}f_{kk} - f_{kl}^2 = 0, \quad (1)$$

where f , k and l denote the amount of production, the stock of capital goods and the number of employed workers.

As an inevitable by-product production causes pollution. In the literature there is some discussion about the source of pollution. Van der Ploeg and Withagen (1991) take pollution as a linear function of production, Feichtinger and Luptáčík (1987) take a convex function of the labor force. Luptáčík and Schubert (1982) have three sources of pollution: consumption, production and the capital stock. As argued in "Zorgen voor Morgen" (1989) the main source of pollution is captured in the capital force and not in the labor force. Here it is assumed that the amount of pollution is a convex function of capital goods. Furthermore, it is assumed that this amount of pollution can be reduced through investment in a second kind of capital goods which is non productive, but cleans pollution instead. The amount by which pollution output is decreased, is assumed to be a concave function of the stock of those abatement capital goods

$$e = \rho(k) - m(u), \rho'(k) > 0, \rho''(k) \geq 0,$$

$$\rho(0) = 0, m'(u) > 0, m''(u) \leq 0, m(0) = 0, \quad (2)$$

where e , k and u denote the amount of pollution, the polluted and productive capital stock, and the cleaning and non productive capital stock, respectively. The firm is confronted with two taxes, because the government asks a proportional tax on profits and pollution. Furthermore, we assume that both investing in capital goods that are productive, as well as in capital goods that clean pollution, generates internal adjustment costs, which in both cases are a convex function of the investment rate

$$\varphi_1 = \varphi_1(i), \varphi_1'(i) < 0 \text{ if } i > 0, \varphi_1''(i) > 0, \varphi_1(0) = 0, \quad (3)$$

$$\varphi_2 = \varphi_2(a), \varphi_2'(a) < 0 \text{ if } a > 0, \varphi_2''(a) > 0, \varphi_2(0) = 0, \quad (4)$$

where φ_1 , φ_2 , i , a represent the adjustment costs of i , the adjustment costs of a , the investment rate assigned to the productive capital stock and the investment rate assigned to the capital stock that cleans pollution, respectively. According to the information described above the present value of the firm's cash flow can be defined as

$$V_0 = \int_0^{\infty} \{ (1-\tau_1)[f(k, \ell) - w\ell] - i - \varphi_1(i) - a - \varphi_2(a) - \tau_2[p(k) - m(u)] \} \exp\left[-\int_0^t r(v)dv\right] dt, \quad (5)$$

in which τ_1 , τ_2 , w and r denote the proportional tax on profits, the proportional tax on pollution, the wage rate and the interest rate. We assume that the firm takes these taxes and prices as given.

The decision problem of the representative firm is to choose time-paths of investment in both kinds of capital goods and employment that maximize V_0 subject to the accumulation equations:

$$\dot{k} = i - \delta_1 k, \quad (6)$$

$$\dot{u} = a - \delta_2 u, \quad (7)$$

with δ_1 and δ_2 symbolizing the rate of exponential depreciation for both capital goods.

Solving the firm's problem is a straightforward exercise of Pontryagin's maximum principle from which the following necessary conditions for an optimum can be obtained (e.g. Feichtinger and Hartl (1986))

$$-1 - \varphi_1'(i) + q_1 = 0, \quad (8)$$

$$-1 - \varphi_2'(a) + q_2 = 0, \quad (9)$$

$$\dot{q}_1 = (r + \delta_1)q_1 - f_k(1 - \tau_1) + \tau_2 \rho'(k), \quad (10)$$

$$\dot{q}_2 = (r + \delta_2)q_2 - \tau_2 m'(u), \quad (11)$$

$$f_l = w, \quad (12)$$

where the symbols q_1 and q_2 stand for the shadow prices of the polluted and abatement capital. From equations (8) and (9) we obtain:

$$i = i(q_1), \quad i(1) = 0, \quad i'(q_1) > 0, \quad (13)$$

$$a = a(q_2), \quad a(1) = 0, \quad a'(q_2) > 0. \quad (14)$$

From (6), (8) and (10) we get that the steady-state level of productive capital stock satisfies:

$$(1 - \tau_1)f_k = (r + \delta_1)\{1 + \varphi_1'(\delta_1 k^*)\} + \tau_2 \rho'(k^*), \quad k_{\tau_1}^* < 0, \quad k_{\tau_2}^* < 0, \quad k_r^* < 0. \quad (15)$$

On the left-hand side of (15) we have the marginal revenue net from profit taxation, while on the right-hand side we find the marginal costs consisting of the sum of the discount rate and depreciation rate, corrected for the fact that $1 + \varphi_1'(\delta_1 k^*)$ dollars are required for a marginal increase of the polluted capital goods level, and of the extra pollution tax that must be paid when the polluted capital stock increases with one unit.

The equations (7), (9) and (11) lead to the following equation for the steady-state level of the abatement capital goods:¹⁾

$$\tau_2 m'(u^*) = (r + \delta_2) \{1 + \varphi_2'(\delta_2 u^*)\}, \quad u_r^* < 0, \quad u_{\tau_2}^* > 0 \quad (16)$$

Like (15), also (16) is a relation that equates marginal revenue to marginal costs, but now for the cleaning capital goods. Notice that the marginal revenue of these capital goods consist of the decrease in pollution tax due to an extra unit of abatement capital goods.

2.2. The model of the consumer

The welfare of consumers positively depends on private consumption (c), public consumption (g) and negatively on the amount of pollution (e)

$$U_0 = \int_0^{\infty} u(c, g, e) \exp(-\sigma t) dt, \quad u_c > 0, \quad u_g > 0, \quad u_e < 0, \quad (17)$$

where σ is a (constant) rate of time-preference. Similar to Abel and Blanchard (1983) and Van de Klundert and Peters (1986) the consumers maximize U_0 with respect to consumption and subject to the dynamic budget constraint

$$\dot{b} = rb + \pi + wl - c, \quad (18)$$

where b and π are the amount of bonds held by the consumer and the obtained dividends.

Again the standard solution technique can be applied to obtain necessary conditions for an optimum

$$u_c = x, \quad (19)$$

1) Notice that a steady state value of u does not exist if τ_2 comes close to zero. However, we assume that the government's disutility of pollution is that large that it will always fix τ_2 such, that it is sufficiently high to guarantee existence of a steady state value of u .

$$\dot{x} = (\sigma - r)x, \quad (20)$$

where x denotes the costate variable associated to the dynamic budget constraint.

To exclude paths from borrowing forever we assume that there are No-Ponzi-Games

$$\lim_{t \rightarrow \infty} \exp\left(-\int_0^t r(v)dv\right)b(t) = 0. \quad (21)$$

In Subsection 2.1 we did not say anything about the way the firms finance their investment. After paying wages to the worker, the firm has to decide how to distribute profit and finance investment by retained earnings or by issuing new shares or bonds. For example, we can assume that replacement investment is financed out of retained earnings and that net investment is financed by bonds. However, because equity and bonds are treated equally by the tax system and there is no uncertainty, the conditions of the Modigliani-Miller theorem hold, thus all financing schemes are equivalent in the sense that they lead to the same path of total consumption and investment; they differ, however, in terms of institutional arrangements (for a proof of this see Abel and Blanchard (1983, pp. 680-681)).

2.3. The markets

In this economy there are two markets: the goods and the labor market. We assume that the goods market is in equilibrium, so that demand is equal to supply

$$f(k, \ell) = c + g + i + \varphi_1(i) + a + \varphi_2(a). \quad (22)$$

From this equation the interest rate r , which is the relative price between current and future consumption, can be derived (e.g. Abel and Blanchard (1983)).

Concerning the labor market we assume that unions behave myopically and that they are shortsighted. According to Oswald (1985) this results in a fixed level of wages. It is also possible to model a labor market, where w is determined by supply. As a consequence of this fixed wage assumption,

the capital labor ratio and the marginal productivity of capital are constant, say d and h ,

$$l = hk, \quad (23)$$

$$f'_k = d. \quad (24)$$

In equations (1)-(24) we have an extended version of the decentralized Abel and Blanchard model with pollution. From these equations the optimal values of all variables, except the tax rates τ_1 and τ_2 and government consumption, can be derived. In the next section we derive the optimal values of these variables.

3. OPTIMAL GOVERNMENT POLICIES

Before we formulate the necessary conditions for an optimal solution we make some additional assumptions. First, we assume that the government has the same utility function as the consumers (cf. Turnovsky and Brock (1980)), and that the consumers' preferences are of Cobb-Douglas type:

$$u(c, g, e) = \alpha \ln c + (1-\alpha) \ln g - \varnothing \ln e, \quad 0 < \alpha < 1, \quad \varnothing > 0. \quad (25)$$

Second, there is a balanced budget policy, so that public consumption will be financed from pollution and profit taxation:

$$g = \tau_1 \{f(k, l) - wl\} + \tau_2 \{p(k) - m(u)\}. \quad (26)$$

Third, as already stated before, the government takes into account the way the consumers and firms behave. In this respect it should be noted that the consumers' co-state variable belonging to the budget constraint, which is denoted by x , can be eliminated. Substitution of (22) into (19) gives us a value for x . This elimination of x stems from the fact that the stream of consumption and investment will not be influenced from financial streams. Similar to the Abel and Blanchard model the consumers only play a passive role through clearing the goods market.

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Furthermore, the equilibrium of the goods market gives us the interest rate. Following the approach of e.g. Barro (1979) we treat the interest rate as exogenous to the system.²⁾

By using the information obtained until now, the government's problem can be captured in the following optimal control problem:

$$\max_{\tau_1, \tau_2} \int_0^{\infty} (\alpha \ln c + (1-\alpha) \ln g - \vartheta \ln e) \exp(-\sigma t) dt \quad (28)$$

s.t.

$$\dot{k} = i(q_1) - \delta_1 k, \quad k(0) = k_0 > 0, \quad (29)$$

$$\dot{u} = a(q_2) - \delta_2 u, \quad u(0) = u_0 > 0, \quad (30)$$

$$\dot{q}_1 = (r + \delta_1) q_1 - d(1 - \tau_1) + \tau_2 \rho'(k), \quad (31)$$

$$\dot{q}_2 = (r + \delta_2) q_2 - \tau_2 m'(u), \quad (32)$$

in which:

$$c = (1 - \tau_1) dk + whk - i(q_1) - \varphi_1(i(q_1)) - a(q_2) - \varphi_2(a(q_2)) - \tau_2 \{\rho(k) - m(u)\}, \quad (33)$$

$$g = \tau_1 dk + \tau_2 \{\rho(k) - m(u)\}, \quad (34)$$

$$e = \rho(k) - m(u). \quad (35)$$

The Hamiltonian is defined by:

2) An alternative could be to let the interest rate depend on the state variables and the instruments of the government:

$$r = r(k, u, q_1, q_2, \tau_1, \tau_2) \quad (27)$$

$$\begin{aligned}
H^G = & \alpha \ln[(1-\tau_1)dk + whk - i(q_1) - \varphi_1(i(q_1)) - a(q_2) - \varphi_2(a(q_2)) - \\
& - \tau_2\{\rho(k) - m(u)\}] + (1-\alpha)\ln[\tau_1dk + \tau_2\{\rho(k) - m(u)\}] - \\
& - \emptyset \ln[\rho(k) - m(u)] + \lambda_1\{i(q_1) - \delta_1k\} + \lambda_2\{a(q_2) - \delta_2u\} + \\
& + \nu_1\{(r+\delta_1)q_1 - d(1-\tau_1) + \tau_2\rho'(k)\} + \nu_2\{(r+\delta_2)q_2 - \tau_2m'(u)\}, \quad (36)
\end{aligned}$$

in which:

λ_1 : the government's co-state variable of the stock of productive capital goods

λ_2 : the government's co-state variable of the stock of cleaning capital goods

ν_1 : the government's co-state variable of the firm's co-state variable of the stock of productive capital goods

ν_2 : the government's co-state variable of the firm's co-state variable of the stock of cleaning capital goods

For an interior solution the necessary conditions are as follows:

$$H_{\tau_1}^G = dk\left(-\frac{\alpha}{c} + \frac{1-\alpha}{g}\right) + d\nu_1 = 0, \quad (37)$$

$$H_{\tau_2}^G = (\rho(k) - m(u))\left(-\frac{\alpha}{c} + \frac{1-\alpha}{g}\right) + \nu_1\rho(k) - \nu_2m'(u) = 0, \quad (38)$$

$$\begin{aligned}
\dot{\lambda}_1 = & (\sigma+\delta_1)\lambda_1 - \frac{\alpha}{c} \{(1-\tau_1)b + wh - \tau_2\rho'(k)\} - \frac{(1-\alpha)}{g} \{\tau_1b + \tau_2\rho'(k)\} + \\
& - \nu_1\tau_2\rho''(k) + \frac{\emptyset}{e} \rho'(k), \quad (39)
\end{aligned}$$

$$\dot{\lambda}_2 = (\sigma+\delta_2)\lambda_2 + m'(u) \left\{ \tau_2\left(-\frac{\alpha}{c} + \frac{1-\alpha}{g}\right) - \frac{\emptyset}{e} \right\} + \nu_2\tau_2m''(u), \quad (40)$$

$$\dot{\nu}_1 = (\sigma-r-\delta_1)\nu_1 + \frac{\alpha}{c} \{i'(q_1)(1 + \varphi_1'(i(q_1)))\} - \lambda_1i'(q_1), \quad \nu_1(0) = 0, \quad (41)$$

$$\dot{\nu}_2 = (\sigma-r-\delta_2)\nu_2 + a'(q_2) \left\{ \frac{\alpha}{c} (1 + \varphi_2'(a(q_2))) - \lambda_2 \right\}, \quad \nu_2(0) = 0. \quad (42)$$

In (37) we find the effects of a marginal profit tax change. A marginal momentarily increase of τ_1 results in an extra income for the government of dk , and this also equals the extra loss for the private sector. Therefore, public consumption increases with dk , which results in an extra utility for the government of $dk(1-\alpha)/g$. Also, the private consumption decreases with dk , due to which utility decreases by $dk\alpha/c$.

Another effect of the increase of profit taxation is that owning productive capital goods becomes less attractive to the firm. This "attractiveness" is measured by the firm's co-state variable of productive capital goods q_1 . By solving the differential equation (10) and using the steady state values of q_1 and k as fixed point we obtain the following expression for q_1 :

$$q_1(t) = \int_t^{\infty} \{d(1-\tau_1) - \tau_2 \rho'(k)\} \exp(-(r+\delta_1)(s-t)) ds. \quad (43)$$

From (43) we can conclude that a marginal momentarily increase of τ_1 results in a decrease of q_1 by d units. Due to the fact that ν_1 is the government's co-state variable belonging to q_1 , the government values this decrease by $\nu_1 d$. A decrease of q_1 means that the firm values investments to be less attractive, and therefore future productive capital goods and profits decrease, which results in less profit taxation income for the government. Thus, the government will assign a negative value to a decrease of q_1 , implying that $\nu_1 d$ is negative so that ν_1 must be negative.

To summarize: an increase of profit taxation implies an increase of public consumption, which is of course a positive effect for the government. But, the negative effects are the decrease of private consumption and the decrease of attractiveness of investing for the firm. The latter results in less future productive capital goods so that future profit taxation income of the government decreases. Now we are able to see that (37) implies that for an interior solution all marginal effects of an increase of τ_1 sum up to zero. This kind of trade off is well known from Ramsey types of models (e.g. Ramsey (1927)).

The implications of a marginal pollution tax change can be found in (38). A marginal increase of the pollution tax rate results in extra taxation

income for the government of $\rho(k) - m(u)$. This leads to a shift from private consumption to public consumption and the change in government utility of this consumption effect equals $(\rho(k) - m(u))(-\alpha/c + (1-\alpha)/g)$. Another effect of a higher pollution tax rate is that pollution output becomes more costly to the firm, and therefore the attractiveness of owning polluted capital goods decreases. This attractiveness is measured by q_1 and from (43) we obtain that increasing τ_2 momentarily with one unit leads to a decrease of q_1 with $\rho'(k)$. The government values this decrease with $\nu_1 \rho'(k)$.

But, pollution output becoming more costly also implies that owning cleaning capital goods, which diminish pollution, becomes more attractive to the firm. This is measured by the firm's co-state variable of cleaning capital goods q_2 . According to an analogous derivation as for (43), q_2 can be expressed by:

$$q_2(t) = \int_t^{\infty} \tau_2 m'(u) \exp(-(r+\delta_2)(s-t)) dt. \quad (44)$$

Hence, increasing τ_2 by one unit implies an increase of q_2 by $m'(u)$ units, where $m'(u)$ equals the extra decrease of pollution output due to a marginal increase of u . The government values this increase of q_2 by $-\nu_2 m'(u)$, because ν_2 is the government's co-state variable of q_2 . The fact that the cleaning capital goods become more attractive to the firm is positively valued by the government, because it implies that the firm's pollution output will decrease. Hence, ν_2 will be negative.

To summarize: an increase of the pollution tax rate has a consumption and a pollution effect. Due to a higher tax income there will be a shift from private to public consumption. The fact that polluting the environment becomes more expensive for the firm implies that the attractiveness of the polluted capital goods decreases and the attractiveness of the cleaning capital goods increases. Equation (38) says that within the interior solution the consumption effect and the pollution effect of a marginal increase of the pollution tax rate sum up to zero.

From the negativity of ν_1 and (37) we can conclude that marginal utility from public consumption $((1-\alpha)/g)$ is less than marginal utility from private consumption. The government gives up a piece of its government consumption to stimulate capital accumulation. This is contrary to Fischer (1980) where marginal utility from private consumption equals marginal utility from public consumption.

So far, we described an optimal taxation plan for the government. However, this optimal plan is time-inconsistent, because there is an incentive for the government to reoptimize and reconsider its tax strategy at some later date. From the point of view of stimulating productive capital accumulation the government should announce a policy of low profit tax rate and low pollution tax rate. But, on the other hand more productive capital implies more pollution output and, therefore, it would be recommendable to accompany stimulation of accumulating productive capital goods with stimulating accumulation of cleaning capital goods. This can be done by announcing a high pollution tax rate. Hence there are two contrary mechanisms working on fixing the pollution tax rate. On the one hand, it should be low to stimulate productive capital accumulation and on the other hand it should be high to diminish pollution output.

Once the productive capital is installed, the government has an incentive to renege on its announcement and to introduce a higher tax rate on both profit and pollution. Notice that the government's co-states belonging to the firm's co-states of both capital goods must equal zero at the start of the planning period, because the firm's co-states are free to jump at that point of time and, therefore, they become effectively additional policy instruments for the government. If the government has the possibility at some later point of time to make a new initial plan, those co-states belonging to the firm's co-states again become zero. At a moment that almost all capital is installed, there is an incentive for the government to increase the tax rates, so much that marginal utility from private consumption equals marginal utility from public consumption, i.e. $g/c = (1-\alpha)/\alpha$. The more the ν_1 deviates from zero the more the government can gain by manipulating q_1 through letting the tax rates τ_1 and τ_2 deviate from the original plan, thus through cheating the firm. The same holds for ν_2 in connection with manipulating q_2 through changing the tax rate

τ_2 . In this way $|\nu_1|$ and $|\nu_2|$ can be interpreted as the government's costs for sticking to its announced plan.

If the firm has no reason to believe that the government will stick to its initial plan, the concept used in this section, which corresponds to an open-loop equilibrium of a Stackelberg game, is no longer a useful concept. In the literature three main streams can be qualified for solving the problem of time-inconsistency. The first attempt is what is called the loss of leadership (cf. Buiter (1983)). In this view the government gives up its role as leader and the interactions between private sector and government is viewed as a Nash rather than a Stackelberg dynamic game. The acceptance of this view would, however, mean the denial of existence of policies which have announcement effects, such as tax policies. Secondly, memory strategies, threats and incentives can be used to sustain the time-inconsistent solution (cf. Backus and Driffill (1985), Barro and Gordon (1983)). Thirdly, we can use recursive or so-called feedback methods. The present government's leadership is preserved with respect to the private sector, but it is lost with respect to future governments, which are free to optimize.

The aim of the next section is to use the third approach to solve the time-inconsistency problem. Hence, for the model under consideration we derive the feedback Stackelberg solution in the next section.

4. THE FEEDBACK STACKELBERG EQUILIBRIUM

In this section we derive the feedback Stackelberg equilibrium for the model described in the previous sections. By constructing this equilibrium we assume that the firms ignore their influence on the level of taxation. So, this equilibrium can be interpreted as the nocommitment equilibrium with "atomistic" behavior of firms and consumers. Due to these additional assumptions it is possible to obtain the feedback Stackelberg equilibrium by putting the co-state variables ν_1 and ν_2 equal to zero³⁾ (cf. conditions (37)-(42)).

3) For a proof that it is possible to construct the feedback Stackelberg equilibrium in such a way in these kinds of models, see Gradus (1990, pp. 152-155).

From equations (37) and (38) it follows that along the equilibrium the following equation holds: $g/c = 1-\alpha/\alpha$. In the feedback Stackelberg equilibrium the marginal utilities from private and public consumption are equal. In the open-loop equilibrium the government gives up a piece of its government consumption to stimulate accumulation. In the feedback equilibrium there is no reason to do so because the firms do not believe such an announcement due to the fact that, in absence of any commitment, the government can deviate at any time.

It should be noticed that it is not possible to obtain the pollution and profit tax rate from equations (37) and (38) in a direct way. However, from point of view of the government there seems to be no reason why the level of pollution and profit tax should be different, because both taxes cause the same distortions. In general it seems reasonable that the profit tax rate will be higher in the feedback Stackelberg equilibrium. Therefore, there will be less polluted capital in this economy and the shadow price of capital, i.e. q_1 , is lower in the feedback Stackelberg equilibrium. General statements about a comparison of the pollution tax rate in the feedback and open-loop Stackelberg equilibrium cannot be made. As already stated in Section 3 the government could impose a high pollution tax rate to stimulate accumulation of the cleaning capital stock. So, it is possible that the level of pollution in the feedback Stackelberg equilibrium is lower than in the open-loop Stackelberg equilibrium.

The nature of the solutions examined may be further classified by a numerical example. Assume quadratic adjustment costs for both functions

$$\varphi_1(i) = \eta_1 \cdot i^2, \quad (45)$$

$$\varphi_2(a) = \eta_2 \cdot a^2, \quad (46)$$

and a Cobb-Douglas production function

$$f(k, \ell) = k^\beta \ell^{1-\beta}. \quad (47)$$

Here, it is assumed that the amount of pollution is a linear function of k and u (cf. (2)), i.e.:

$$p(k) = \rho k, \quad (48)$$

$$m(u) = \mu u. \quad (49)$$

Furthermore, choose the following parameter values: $w = 0.5$, $\eta_1 = \eta_2 = 4$, $\beta = 0.375$, $\delta_1 = \delta_2 = 0.05$, $\alpha = 0.9$, $\sigma = 0.03$, $\phi = 0.2$, $\rho = 1$, $m = 1, 2$. The steady-state values for the open-loop and feedback Stackelberg equilibrium can be found by a numerical procedure and they are given in Table 1.

[insert Table 1]

This example clearly reveals the difference between the open-loop and feedback equilibria. The feedback equilibrium yields a higher value of the steady-state profit tax rate and a lower level of the polluted but productive capital stock than the open-loop equilibrium (see Table 1). Furthermore, also the pollution tax rate is higher in the feedback equilibrium. Therefore, the cleaning but nonproductive capital stock will be higher. The result of these two effects is that for this example the amount of pollution in the feedback equilibrium is lower. Moreover, it should be noticed that in the open-loop case the share of public consumption goods in total output is not so high as in the feedback case but private consumption and total utility will be higher because there is more capital. Finally, as we should expect the utility level in the feedback equilibrium is lower than in the open-loop equilibrium. The decrease in utility caused by the lower productive capital stock is greater than the increase in utility caused by the lower level of pollution.

5. CONCLUSIONS

In this paper the impact of profit and pollution tax on dynamic firm behavior is studied within a general equilibrium model. The tax rates are determined endogenously by assuming that the government maximizes utility. The problem is formulated as a Stackelberg differential game with the government as leader, which is studied for the open-loop as well as the feedback case.

The optimal level of the firms' capital stock is determined by an equality between net marginal revenue and marginal costs, where the latter consist of the cost of capital, corrected for adjustment costs, plus marginal pollution tax expenses. In the feedback equilibrium the government fixes the profit tax rate such, that marginal utility from private consumption equals marginal utility from public consumption. This is not optimal in the open-loop case, where the government announces a policy of low profit tax in order to stimulate economic growth. This announcement is credible, because open-loop implies that the government is committed.

In the open-loop Stackelberg equilibrium the pollution tax rate is such that marginal consumption and pollution effects sum up to zero. The pollution effect arises from the fact that in the open-loop case it is possible for the government to deviate from short term optimality. This effect states that announcing a policy of high pollution tax implies that investing becomes less attractive to the firms, because more production generates more pollution as an inevitable byproduct and the latter is punished more heavily. On the other hand the firms are stimulated more to spend more money in ways to diminish their own pollution. The summation of these two effects gives us the pollution effect. In the feedback equilibrium this pollution effect does not play a role in determining the pollution tax rate due to the lack of credibility of policy announcements. In the paper the above conclusions are confirmed in the results of a numerical example. Another result of this example was that in the feedback case the pollution tax rate was higher, implying more abatement investments of the firms resulting in less pollution. We guess that the reason for the pollution tax rate being higher is that in this way the government tries to compensate the fact that in the feedback solution income from profit tax rate is not so high as in the open-loop solution, because in the latter case the higher capital stock resulted in far more profits.

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Table 1. A numerical example.

	Open-loop Stackelberg	Feedback Stackelberg
τ_1	0	0.18
τ_2	0.05	0.18
q_1	6.14	3.41
k	12.85	6.01
ν_1	-1.37	0
q_2	1.32	2.64
u	0.79	4.09
ν_2	-0.09	0
c	15.69	6.63
g	0.61	0.77
e	11.90	1.10
i	0.64	0.30
$\varphi_1(i)$	1.65	0.36
a	0.04	0.20
$\varphi_2(a)$	0.006	0.17
U	6.911	5.423 ($= c^\alpha g^{1-\alpha} e^{-\theta}$)

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