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RESEARCH MEMORANDUM

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The theory of wage differentials:
a correction

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The authoritative article by Bhagwati and Srinivasan (1971) tried to prove that when there is a distortionary wage differential between sectors the production possibility curve might have both convex and concave stretches. This was based on the sign of the second derivative. However, their complex equation (15) and their next ones as special cases contain a mistake. This paper presents the correct outcomes. The Bwagwati-Srinivasan conclusions are affected in the following way.

1. The conditions under which the frontier is convex at one specialisation point and concave at the other are somewhat more intricate than those stated by Bhagwati and Srinivasan. A general classification of the conditions leading to different combinations of curvatures at the specialisation points is presented.
2. In the special case of CES production functions, the production possibility frontier will be convex under less stringent condition than those stated by Bhagwati and Srinivasan.

The correct equation for the second derivative is:

$$\frac{d^2Q_1}{dQ_2^2} = \frac{-w(R_2 - R_1)^2}{D^2}\left[\frac{N(\gamma R_1 - R_2)}{(w + R_1)(\gamma w + R_2)} + \frac{(\gamma - 1)R_1R_2f_1^1f_1^2}{D}\right.$$

$$\times\left\{\left\{(R_2 - R)\sigma_1R_1 + (R - R_1)\sigma_2R_2\right\}\left\{\sigma_1(R_2 - R) + \sigma_2(R - R_1)\right\}\right.$$

$$\left.- w(R_2 - R_1)(R_2 - R)(R - R_1)\left\{\sigma_2\frac{d\sigma_1}{dw} - \sigma_1\frac{d\sigma_2}{dw}\right\}\right\}$$

$$\left.\frac{-(\gamma - 1)f_1^1f_1^2}{D}\left\{\sigma_1R_1(R_2 - R) + \sigma_2R_2(R - R_1)\right\}\sigma_1\sigma_2(R_2 - R_1)(R_1R_2 - wR)\right] \quad (15)$$

This result influences the outcomes for the cases of complete specialisation. In the case of complete specialisation in Q_1 the result is:

$$\frac{d^2Q_1}{dP_2^2} = \frac{-w(R_1 - R)^2}{D^2} \left[\frac{N(\gamma R - R_2)}{(w + R)(\gamma w + R_2)} \right.$$

$$\left. + \frac{(\gamma - 1)}{D} f_1^1 f_1^2 \sigma_1^2 (R_2 - R)^2 R^2 \left\{ R_2(1 - \sigma_2) + \sigma_2 w \right\} \right]$$

where $N = -f_1^1 \left\{ (w + R_2)(R_2 - R) \sigma_1 R \right\} \gtrless 0$ as $R_1 \gtrless R \gtrless R_2$

and $D = f_1^2 \left\{ (\gamma w + R_2)(R_2 - R) \sigma_1 R \right\} \lesseqgtr 0$ as $R_1 \gtrless R \gtrless R_2$

For the case of complete specialisation in Q_2 the result is:

$$\frac{d^2Q_1}{dQ_2^2} = \frac{-w(R - R_1)^2}{D^2} \left[\frac{N(\gamma R_1 - R)}{(w + R_1)(\gamma w + R)} \right.$$

$$\left. + \frac{(\gamma - 1)}{D} f_1^1 f_1^2 \sigma_2^2 (R - R_1)^2 R^2 \left\{ R_1(1 - \sigma_1) + \sigma_1 w \right\} \right]$$

where $N = -f_1^1 \left\{ (w + R_1)(R - R_1) \sigma_2 R \right\} \gtrless 0$ as $R_1 \gtrless R \gtrless R_2$

and $D = f_1^2 \left\{ (\gamma w + R_1)(R - R_1) \sigma_2 R \right\} \lessgtr 0$ as $R_1 \gtrless R \gtrless R_2$

These revised outcomes have certain consequences for the conditions under which the second derivative in the neighbourhood of the points of specialisation is negative or positive. These conditions differ from those of Bhagwati and Srinivasan especially with respect to $\sigma_i (i = 1,2)$

In case $R_1 > R > R_2$, so that $N > 0$ and $D < 0$, the second derivative for complete specialisation in Q_1 is negative, i.e. concavity, if both terms in square brackets are positive. This holds if $\gamma R > R_2$, what is certain if $\gamma > 1$ and is possible even if $\gamma < 1$, and either if $\gamma > 1$ and $\sigma_2 > 1$ or if $\gamma < 1$ and $\sigma_2 < 1$. For complete specialisation in Q_2 the second derivative is positive, i.e. convexity, if both terms in square brackets are negative. This holds if $\gamma R_1 < R$, that requires that $\gamma < 1$, and either if $\gamma > 1$ and $\sigma_1 < 1$ or if $\gamma < 1$ and $\sigma_1 > 1$. Thus there is a concavity for complete specialisation in Q_1 and convexity for complete specialisation in Q_2 if $\gamma < 1$, $\gamma R > R_2$, $\sigma_2 < 1$, $\gamma R_1 < R$ and $\sigma_1 > 1$.

In case $R_2 > R > R_1$, so that $N < o$ and $D > o$, the second derivative for complete specialisation in Q_1 is negative if both terms in square brackets are positive. This holds if $\gamma R < R_2$, what is certain if $\gamma < 1$ and is possible even if $\gamma > 1$, and either if $\gamma > 1$ and $\sigma_2 < 1$ or if $\gamma < 1$ and $\sigma_2 > 1$. For complete specialisation in Q_2 the second derivative is positive if both terms in square brackets are negative. This holds if $\gamma R_1 > R_1$ what requires that $\gamma > 1$, and either if $\gamma > 1$ and $\sigma_1 > 1$ or if $\gamma < 1$ and $\sigma_1 < 1$. Thus due to the requirement that $\gamma > 1$. There is a possibility of concavity for complete specialisation in Q_1 and convexity for complete specialisation in Q_2 if $\gamma > 1$, $\gamma R < R_2$, $\sigma_2 < 1$, $\gamma R_1 > R$ and $\sigma_1 > 1$.

In order to save space we summarize the different possibilities by presenting next table.

Table 1

	$R_1 > R > R_2$ $(N > o,\ D < o)$			$R_2 > R > R_1$ $(N < o,\ D > o)$		
Concavity in Q_1 and	$\gamma < 1$	$\gamma R > R_2$	$\sigma_2 < 1$	$\gamma > 1$	$\gamma R < R_2$	$\sigma_2 < 1$
Convexity in Q_2		$\gamma R_1 < R$	$\sigma_1 > 1$		$\gamma R_1 > R$	$\sigma_1 > 1$
Convexity in Q_1 and	$\gamma < 1$	$\gamma R < R_2$	$\sigma_2 > 1$	$\gamma > 1$	$\gamma R > R_2$	$\sigma_2 > 1$
Concavity in Q_2		$\gamma R_1 > R$	$\sigma_1 < 1$		$\gamma R_1 < R$	$\sigma_1 < 1$
Concavity in Q_1 and	$\gamma > 1$	$\gamma R > R_2$	$\sigma_2 > 1$	$\gamma < 1$	$\gamma R < R_2$	$\sigma_2 > 1$
Concavity in Q_2		$\gamma R_1 > R$	$\sigma_1 > 1$		$\gamma R_1 < R$	$\sigma_1 > 1$
Convexity in Q_1 and	$\gamma < 1$	$\gamma R < R_2$	$\sigma_2 > 1$	$\gamma > 1$	$\gamma R > R_2$	$\sigma_2 > 1$
Convexity in Q_2		$\gamma R_1 < R$	$\sigma_1 > 1$		$\gamma R_1 > R$	$\sigma_1 > 1$

Finally Bhagwati and Srinivasan consider the case in wich the elasticities of substitution in both sectors are equal and constant. The revised second derivative should read as:

$$\frac{d^2Q_1}{d\,Q_2^2} = \frac{-w(R_2 - R_1)^2}{D^2}\left[\frac{N\ (\gamma R_1 - R_2)}{(w + R_1)(\gamma w + R_2)} + \frac{(\gamma - 1)}{D}\, f_1^1 f_1^2\ (R_2 - R_2)^2\ \sigma R \left\{R_1\ R_2 \sigma(1 - \sigma) + \sigma^2 wR\right\}\right]$$

where $N = -f_1^1\ \sigma \left\{(R_2 - R_1)(R_1R_2 + wR)\right\} \gtrless o$ as $R_1 \gtrless R_2$

and $D = f_1^2\ \sigma \left\{(R_2 - R_1)(R_1R_2 + \gamma\ wR)\right\} \lesseqgtr o$ as $R_1 \gtrless R_2$

In case $R_1 > R > R_2$ throughout convexity is possible if $\gamma R_1 < R_2$, what requires that $\gamma < 1$, and if $\sigma > 1$. In case $R_2 > R > R_1$ throughout convexity is possible if $\gamma R_1 > R_2$, what requires that $\gamma > 1$, and if $\sigma > 1$.

For the CES function $f^i = \left[\alpha_i R_i^{-\varepsilon} + (1 - \alpha_i)\right]^{-\frac{1}{\varepsilon}}$ the revised second derivative becomes:

$$\frac{d^2Q_1}{d\,Q_2^2} = \frac{-w\ (\eta - 1)^3 R_1^{\ 3}\ f_1^1}{D^2}\left[\frac{(\eta - \gamma)\ R_1\,\sigma\ (wR + \eta R_1^2)}{(w + R_1)\ (\gamma w + \eta R_1)} + \frac{(\gamma - 1)\ R\left\{R_1^2\ \eta\ \sigma\ (1 - \sigma) + \sigma^2\ w\ R\right\}}{(\gamma\ w\ R + \eta\ R_1^2)}\right] \tag{16}$$

If $\alpha_1 = \alpha_2$ and $\sigma < 1$ the second derivative is positive because either $1 > \eta > \gamma$ or $\gamma > \eta > 1$. Thus the production possibility curve is indeed convex throughout, although the condition on the elasticity of substitution is less stringent than suggested by Bhagwati and Srinivasan.

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