


A STATISTICAL APPROACH TO THE PROBLEM OF NEGATIVES IN INPUT-OUTPUT ANALYSIS

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A STATISTICAL APPROACH TO THE PROBLEM OF NEGATIVES IN INPUT-OUTPUT ANALYSIS

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Economic Modelling (forthcoming, October 1988)

## Abstract

The purest and theoretically superior method for the construction of in-put-output coefficients is given by the commodity technology model. The commodity technology based input-output coefficients have one shortcoming, however. Some of them turn out negative, which is economically not meaningful. This paper presents a methodology to deal with the problem of negatives in input-output analysis. It allows a statistical assessment of the problem. We are lead to reject the commodity technology model. This conclusion is surprising, at least to us, in view of the theoretical appeal of the model and the empirical smallness of the negatives.

We thank Anton Markink for the excellent programming of the statistical adjustment procedure. The research of the first author has been made possible by a Senior Fellowship of the Royal Netherlands Academy of Arts and Sciences and a grant ( R 46-177) of the Netherlands Organization for the Advancement of Pure Research (Z.W.O.).

## 1. Introduction

The construction of input-output coefficients matrices is complicated by the presence of secondary outputs. Sectors produce not only own or primary output, but also each others or secondary outputs. In textbook input-output analysis coefficients are determined by dividing inputs through by primary output, while secondary output is assumed away. In reality, one must account for secondary products and a number of methods are available for the construction of technical coefficients (ten Raa, Chakraborty and Small, 1984, Fukui and Seneta, 1985, and Viet, 1986).
The purest and theoretically superior method is given by the commodity technology model. This model simply postulates input-output coefficients, calculates the consequent direct requirements for the outputs of each sector and equates the sum to the observed inputs. Thus, for each sector we have a commodity vector equation. These equations can be solved simultaneously for the technical coefficients. The solution is simple, the input-output coefficients matrix is basically the input matrix divided by the output matrix, and has nice properties, such as scaling invariance.
The input-output coefficients based on the commodity technology model have one shortcoming, however. Some of them turn out negative, which is economically not meaningful. This paper presents a methodology to deal with the problem of negatives in input-output analysis. It allows a statistical assessment of the problem. We will be lead to reject the commodity technology model. This conclusion is surprising, at least to us, in view of the theoretical appeal of the model and the empirical smallness of the negatives.
The paper is organized as follows. Section 2 reviews the commodity technology model and shows how it may generate negative input-output coefficients. Section 3 presents a diagnosis of the negatives for U.K. data in order to provide some intuition. Section 4 applies a re-estimation procedure to eliminate the negatives. Results are presented and discussed in Section 5. They confirm the established practice of dealing with negatives by simply setting them zero, but must, nonetheless, reject the model that underlies the construction of coefficients, as the last section concludes.

## 2. The commodity technology model

The System of National Accounts (U.N., 1967) includes an input or "use" table $U$ and an output or "make" table $V$. Entry $u_{i j}$ is the amount of commodity $i$ consumed by industry $j . v_{j k}$ is industry $j$ 's amount of product $k$. The commodity technology model postulates technical coefficients $a_{i k}$, for all sectors (van Rijckeghem, 1967). In particular, industry j requires a $\mathrm{i}_{\mathrm{ik}}$ $v_{j k}$ of input $i$ for output $k$. Its consumption of input $i$ equals the requirements summed over outputs: $u_{i j_{1}}=\Sigma_{k} a_{i k} v_{j k}$. Hence $U=A V^{\top}$ or $A=U V^{-\top}$, where ${ }^{\top}$ denotes transposition and ${ }^{-1}$ inversion. (Since the latter two operations commute, their compositions may be denoted ${ }^{-T}$ without confusion.)
It is instructive to consider the example of a two-sector economy with one sector, say the first one, producing some secondary output:

$$
\mathrm{U}=\left[\begin{array}{ll}
u_{11} & u_{12} \\
u_{21} & u_{22}
\end{array}\right] \text { and } \mathrm{v}=\left[\begin{array}{cc}
v_{11} & v_{12} \\
0 & v_{22}
\end{array}\right]
$$

Then

$$
\begin{aligned}
A= & U V^{-T}=\left[\begin{array}{ll}
u_{11} & u_{12} \\
u_{21} & u_{22}
\end{array}\right]\left[\begin{array}{cc}
v_{11} & 0 \\
v_{12} & v_{22}
\end{array}\right]^{-1}=\left[\begin{array}{ll}
u_{11} & u_{12} \\
u_{21} & u_{22}
\end{array}\right]\left[\begin{array}{cc}
1 / v_{11} & 0 \\
-v_{12} /\left(v_{11} v_{22}\right) & 1 / v_{22}
\end{array}\right] \\
& =\left[\begin{array}{cc}
\left(u_{11}-\frac{u_{12}}{v_{22}} v_{12}\right) / v_{11} & \frac{u_{12}}{v_{22}} \\
\left(u_{21}-\frac{u_{22}}{v_{22}} v_{12}\right) / v_{11} & \frac{u_{22}}{v_{22}}
\end{array}\right] .
\end{aligned}
$$

For sector 2 we have the usual coefficients, $a_{12}=u_{12} / v_{22}$ and $a_{22}=$ $u_{22} / v_{22}$, but for sector 1 we obtain

$$
\begin{equation*}
a_{11}=\left(u_{11}-a_{12} v_{12}\right) / v_{11} \text { and } a_{21}=\left(u_{21}-a_{22} v_{12}\right) / v_{11} . \tag{1}
\end{equation*}
$$

In other words, the technical coefficients are net input over net output where net output is total output net of secondary products and net input is input net of the associated secondary product requirements. In theory
the input requirements of secondary products cannot exceed the total input of the sector, so the coefficients of the input-output table, A, cannot be negative. However, the theory may not be valid in its pure form, or at least the use and make tables are observed with measurement errors. Therefore the input requirements of secondary products may exceed the observed input of the sector. Then the subtraction yields a negative net input and hence one observes a negative technical coefficient, $a_{11}$ or $a_{21}$, in this case. Alternatively, if the use and make data are measured without error and negatives nevertheless arise in the construction of input-output tables, the basic assumption of the commodity technology must be wrong. This incompatibility between theory and empirical outcome is the subject of this study.

## 3. Diagnosis of negative input-output coefficients

The data used in this study are in the System of National Accounts for 1975 of the U.K. (Barker, van der Ploeg and Weale, 1984). The use and make tables are square tables; the size is the number of sectors, 39. (The "Unallocated" sector is omitted.) $U$ and $V$ are reproduced in Tables 1 and 2. The unit of measurement is million pounds. The derived technical coefficients matrix, $A=U V^{-\top}$, is in Table 3. They are multiplied by a factor of 100 , so that the unit is pennies per pound. All tables are in the Appendix.
There are three negatives on digit level two, namely $a_{4,10}=-.007, a_{28,31}$ $=-.015$ and $a_{28,32}=-.005$. Mind that they are multiplied by a factor of 100 in Table 3. The biggest one, $a_{28,31}$, is the only one that persists when indirect requirements are taken into account through the Leontief inverse $(I-A)^{-1}$. No other negatives on digit level two are created in the inverse.

It is well known why the commodity technology model produces negatives. Each commodity is assumed to have its own input structure, irrespective of the sector where it is put together. To identify input structures, sectors are purified by subtraction of secondary activities. Negative net inputs are created if secondary products have input components that, in sum, exceed the actual inputs of the sector at hand, as reported by the use table, U. Recall equation (1).

In each of the cases listed above, a single secondary product accounts for the negative value of the input-output coefficient. Each one shall be taken up in turn. First, $a_{4,10}=-.007$. Sector 10 (chemicals) produces one secondary output with a sizeable petroleum \& natural gas (commodity 4) requirement, namely $v_{10,9}=78.9$ (petroleum products). Nonetheless, sector 10 itself uses no petroleum \& natural gas. The petroleum \& natural gas requirement amounts $\mathrm{a}_{4,9} \mathrm{v}_{10,9}=.62 * 78.9=48.6$ which, after division by primary output $v_{10,10}=6928.0$, accounts precisely for the negative value of $a_{4,10}$. How can the chemical sector produce petroleum products without petroleum? In theory, there are three possible answers: vertical integration, throughput or alternative technology. If the chemical sector were vertically integrated into the petroleum sector, then it could produce petroleum products from petroleum \& natural gas inputs. The latter inputs are not well represented in the chemical sector though, so that vertical integration is not the answer in this case. The second possibility, "throughput", turns out to be the right answer. The chemical sector produces petroleum products out of petroleum products. It has a sizeable petroleum products output, $v_{10,9}=78.9$, as well as input, $u_{9,10}=494.9$. Thus, the first negative, in the chemical sector, is due to the problem associated with products having much own input (ten Raa, Chakraborty and Small, 1984, p. 93). It can be considered as an alternative technology instance, namely one with own input coefficient one and all others zero. (It will not be so extreme in practice, but one petroleum product may be turned another, which essentially manifests an aggregation problem.)
Next take the second negative, $a_{28,31}=-.015$. Sector 31 (water) produces one secondary output with a sizeable construction (commodity 28) requirement, namely $v_{31,28}=73.3$ (construction). The requirement amounts $a_{28,28}$ $v_{31,28}=.8^{*} 73.3=13.3$ which, after division by primary output $v_{31,31}=$ 654.5, accounts precisely for the reduction of $a_{28,31}$ to its negative value. How can the water department produce construction with relatively little construction? This is the mirror image of the first case. Now we have the problem of products with much own input, not in the sector at hand (31), but in the sector of reference of the secondary input structure (28). So the answer is that construction use of construction in its own sector, $u_{28,28}=2836.3$, is big. The third and last negative, $a_{28,32}=-$ .005 , is similar. The construction secondary output, $v_{32,28}$, is again the
source of the problem; its commodity 28 (construction) requirement accounts for the reduction of $a_{28,32}$ to its negative value. Our diagnosis of negative input-output coefficients can now be summarized. The source of the trouble is the presence of much throughput of secondary products, either in the sector under consideration $\left(u_{9,10} \rightarrow v_{10,9}\right.$ which causes negativity of $a_{4,10}$ ), or in the sector of reference of the secondary product ( $u_{28,28}$ which causes negativity of $a_{28,31}$ and $a_{28,32}$ ).
Throughput typically remains within a firm and its statistics are considered worthless relative to interindustry data for reasons of definition of transactions as well as confidentiality. Thus, our diagnosis of the problem of negatives directs attention to the reliability of the data (the use and make tables).

## 4. The reestimation procedure

The negatives generated in the process of constructing an input-output coefficients matrix are clearly a nuisance. Something must be wrong. Either the model underlying the construction is misspecified or the data must be off due to measurement error and so on. We begin to explore the latter case. Our null hypothesis is that the model is correct. Data (U,V) fail to observe nonnegativity of input-output coefficients,

$$
\begin{equation*}
\mathrm{Uv}^{-\top} \geq 0 \tag{2}
\end{equation*}
$$

but this constraint may hold for the true values of the inputs and the outputs. The wedge between data and true values consists of error. The question is if, given our null hypothesis, the errors take probable values. If not, we must reject the commodity technology model.
The situation is reminiscent of accounting theory. This is easily explained by incorporating the value added vector of the System of National Accounts, $y$, in our presentation. For each sector, the value of input and value added must add to the value of output:

$$
\begin{equation*}
U^{\top} e+y=V e \tag{3}
\end{equation*}
$$

where $e$ is the vector with all entries equal to one. Data ( $U, V, y$ ) typically fail to meet this balance constraint. Accountants proceed to adjust the data until constraint (3) is observed. For this purpose a reestimation procedure has been designed by Stone, Champernowne and Meade (1942) and extended by van der Ploeg (1982). We adopt the idea and will reestimate $U$ and $V$ such that constraint (2) instead of (3) is observed.
We need more precise notation. From now on, $u_{i j}$ and $v_{j k}$ refer to true values of inputs and outputs of sector $j$. Attached to them are error terms $\delta_{i j}$ and $\varepsilon_{j k}$. Errors can be positive due to overreporting and negative in case of underreporting. True value plus error makes the datum: Observed data are indexed by a superscript $\circ: u_{i j}^{\circ}$ and $v_{j k}^{\circ}$. It follows that the data equal

$$
u_{i j}^{\circ}=u_{i j}+\delta_{i j}
$$

and

$$
v_{j k}^{\circ}=v_{j k}+\varepsilon_{j k}
$$

These data are sectoral statistics which are obtained by adding establishment figures. Assume that establishments report with errors which are independent and identically distributed. Then, by the central limit theorem, sectoral errors $\delta_{i j}$ and $\varepsilon_{j k}$ are distributed normally. We also assume that these errors are independent, across cells (i,j,k=1, $, \ldots, 39$ ). The first assumption is natural, the second less so. However, the presence of correlations (for example between inputs and outputs within sectors) would modify the reestimation procedure in a straightforward way (van der Ploeg, 1982) without affecting our conclusions.

In mainstream econometrics one needs many observations $u_{i j}^{\circ}$ and $v_{j k}^{\circ}$ for each $i$, $j$ and $k$ to infer the mean and variance of $\delta_{i j}$ and $\varepsilon_{j k}$. In inputoutput analysis, to the contrary, one typically has only one observation. This hampers the application of sound statistical analysis. To proceed our study nonetheless, we have employed subjective information on the accuracy of the data as furnished by the statisticians who gather them. We belief that this direct method of estimating errors in measurement is a good substitute for inference.
As regards the mean of the errors, we assume that in the absence of accounting or economic constraints, statisticians have completed their job
of compiling data as good as they can, that is without systematic bias. Hence the means are zero. With the variances the specification is more delicate. Sir Richard Stone has pushed for revelation of such error information. All that we know is available are the standard deviations reported as percentages of the sectoral statistics underlying Barker, van der Ploeg and Weale (1984). For self-containedness we publish the sectors and the percentages in Table 4.
So the variance of the first datum, $u_{11}^{\circ}$, is $\sigma_{11}^{2}=(5 \% \text { of } 1420.2)^{2}=$ $5,042.4201$. The second one is similar, but the third one is more complicated, since $u_{31}^{\circ}$ is not confined to sectors of the same reliability. Its accuracy will be neither $5 \%$ nor $10 \%$, but some average. The reporting of errors as percentages, suggests that mixed data have geometric mean accuracy. Hence, it is natural to set the variance of $u_{31}^{\circ}$ equal to $\sigma_{31}^{2}=$ $\left(\sqrt{.05^{*} \cdot 10} u_{31}^{\circ}\right)^{2}=.05^{*} \cdot 10 * 3.5^{2}=0.06125$. The variances of all other data are determined in the same way.
We are now in a position to write down the likelihood of real values $(\mathrm{U}, \mathrm{V})$. Its logarithm is

$$
\begin{align*}
L(U, V)= & -\frac{1}{2} \sum_{i, j} \sigma_{i j}^{-2}\left(u_{i j}-u_{i j}^{\circ}\right)^{2}-\frac{1}{2} \sum_{j, k} \tau_{j k}^{-2}\left(v_{j k}-v_{j k}^{\circ}\right)^{2}-\sum_{i, j} \log \left(\sigma_{i j}^{2}\right)- \\
& \sum_{j, k} \log \left(\tau_{j k}^{2}\right)-\frac{1}{2}\left(2 * 39^{2}\right) \log (2 \pi) \tag{4}
\end{align*}
$$

where $\sigma_{i j}^{2}$ is the variance of $u_{i j}$ and $\tau_{j k}^{2}$ is the variance of $v_{j k}$. The basic idea is to find the most likely ( $U, V$ ) that is consistent with non-negativity of input-output coefficients, (2). Since the variances are assumed to be known, maximizing $L$ is equivalent to minimizing $f$ defined by

$$
\begin{equation*}
f(U, V)=\sum_{i, j} \sigma_{i j}^{-2}\left(u_{i j}-u_{i j}^{\circ}\right)^{2}+\sum_{j, k} \tau_{j k}^{-2}\left(v_{j k}-v_{j k}^{\circ}\right)^{2} \tag{5}
\end{equation*}
$$

The constraints, A, are given by

$$
\begin{equation*}
A(U, V)=U V^{-T} \geq 0 \tag{6}
\end{equation*}
$$

The use of (6) instead of (3) complicates the application of mathematical statistics, not so much by the inequality sign, but by the nonlinearity of
the constraint in at least one set of variables, namely $V$. The best linear unbiased estimate property of Stone. Champernowne and Meade's (1942) or van der Ploeg's (1982) reestimator is lost if some of the constraints are binding. Furthermore, if the initial estimates are normally distributed, then the adjusted estimates are not necessarily normally distributed. This means that it is difficult to calculate the variances of the reestimated data. However, it is always possible to use the likelihood-ratio test (Silvey, 1975, Sections 7.1 and 7.2) to investigate whether any binding non-negativity constraints are consistent with the prior covariance matrices of the unadjusted data (see Section 5). Since our conclusion will be negative, we do not really need the optimality properties mentioned above. The objective function, $f$, is exceedingly simple. It has linear firstorder and constant second-order derivatives. The function of constraints, $A$, is linear in $U$, but complicated in $V$. We can nevertheless write down the sensitivity of the input-output coefficients with respect to inputs and outputs:

Lemma 1. $\frac{\partial a_{i j}}{\partial u_{r s}}=0$ if $i \neq r, \frac{\partial a_{r j}}{\partial u_{r s}}=w_{s j}$, and $\frac{\partial a_{i j}}{\partial v_{r s}}=-a_{i s} w_{r j}$, where $w_{i j} \quad$ (i, $j=1, \ldots, 39$ ) are the elements of $W=V^{-\top}$.

Proof. See Appendix.

We have also been able to calculate the second order derivatives:

Lemma 2. $\frac{\partial^{2} a_{i, j}}{\partial u_{k \ell} \partial u_{r s}}=0, \frac{\partial^{2} a_{i, j}}{\partial u_{k \ell} \partial v_{r s}}=0(i \neq k), \frac{\partial^{2} a_{k, j}}{\partial u_{k \ell} \partial v_{r s}}=-w_{l s} w_{r j}$,
and $\frac{\partial^{2} a_{i j}}{\partial v_{k} \ell^{\partial v_{r s}}}=a_{i \nmid}{ }^{w_{k s}} w_{r j}+a_{i s} w_{r} \ell^{w_{k j}}$, where $w_{i j}(i, j=1, \ldots, 39)$ are the elements of $W=V^{-T}$.

Proof. See Appendix.

We turn to a routine for non-linear constrained optimization that exploits analytical knowledge of first-order and second-order derivatives: E04WAF of the Numerical Algorithms Group (1984). The computation is complicated
by the prohibitive size of the second-order derivatives matrix, the nonconvexity of the constraint set and the presence of stationary points that are no global solutions. To keep it managable, we aggregate the data. Usually aggregation blurs the analysis, but here it accentuates the problem and the nature of the solution, so no harm is involved at all.
Aggregation is by the rather traditional scheme, specified in Table 5 of the Appendix. The constraint set, (6), remains unchanged. The objective function, (5), must be reinterpreted. The coefficients, that are the variances, are now variances of the aggregated flows. Now, as the data are independently normal distributed, the variances of sums are equal to the sums of variances. In short, the aggregation also applies to the objective function coefficients.

## 5. Results

Table 6 (see Appendix) presents the aggregated inputs, the square roots of their variances as percentages (that is standard deviations) and the reestimates. Table 7 presents the same for the outputs. The percentages are sort of weighted averages of the disaggregated percentage standard deviations. If the flows are zero so that no weights can be determined, then a blank enters. This is no problem, since zero flows remain zero in the maximum likelihood adjustment procedure for finite percentage standard deviations. The standard deviations percentages are sometimes smaller than in the disaggregated case due to the cancelling out of errors.
We wish to draw the reader's attention to two, related, results. First, the maximum likelihood estimation involves the setting of some secondary outputs equal to zero. Second, the adjustment sets some data off the "true" values by more than two standard deviations. We will dwell on both of them.
The solution features zero values of some variables. This is easily explained through the example of the introduction. Non-negativity of the input-output coefficients of sector 1 , (1), requires that its inputs exceed the secondary output requirements. But, if such an input, say $u_{11}$, is zero, then, since standard deviations are given as percentages so that zeros remain zero, the secondary output requirements, $a_{12} v_{12}$, must be zero. Hence $u_{12}$ or $v_{12}$ must be set zero to meet non-negativity of $a_{11}$. In
short, if an input is zero, then the corresponding input requirements of the secondary products of that sector must also be zero. The maximum likelihood readjustment brings this about by setting to zero the secondary outputs with such an input requirement.
In this study. Table 7 shows that secondary outputs $v_{24}, v_{27}, v_{28}, v_{29}$ and $\mathrm{v}_{59}$ are set to zero. Clearly, these constitute significant adjustment steps. They are independent of the standard deviations of the variables and may exceed them by multitudes. For example, if a flow belongs to a sector of which data are accurate up to $5 \%$, then a readjustment towards zero corresponds to 20 standard deviations. This holds for the Mining \& Gas sector, 2. In other words, the data have errors that have much less than even 1\% probability to be observed. This is, of course, very unlikely. Statisticians reject unlikely outcomes. In our context, we shall be forced to reject the model that underlies the reestimation procedure, that is constraint (6) or the commodity technology model for input-output coefficients.
The raw input-output coefficients, $\mathrm{UV}^{-\top}$ based on the aggregated data, as well as the adjusted input-output coefficients, $\mathrm{UV}^{-\top}$ stemming from the constrained optimization problem $(5,6)$, are reported in Table 8 of the Appendix. They are multiplied by a factor of 100 , so that the unit is pennies per pound. It is interesting to note that, basically, our adjustment procedure sets the negatives equal to zero up to digit level 3. That is precisely the common practice of dealing with the problem. Thus routine practice is given a statistical foundation. Also, Table 7 confirms that the coefficient adjustments are minor. However, coefficients are derived constructs. Any change must be conceived as the result of a change in data. Although the change in coefficients is small, the underlying change in data must be large. Big data must be reduced all the way to zero. This involves many standard deviations and, therefore, a long distance in terms of likelihood. So, although the common practice of ignoring the inputoutput coefficients by sweeping them under the carpet seems alright at first sight, statistical analysis renders all this unlikely.
One way of obtaining insight into this question is the use of the likeli-hood-ratio test (Silvey, 1975, Sections 7.1 and 7.2). Since the variances of the unadjusted data are assumed to be known from the Central Statistical Office, twice times the difference in the log-likelihood, (4), equals
(minus) the difference in the "sum of squares", (5), and this is the test statistic of the likelihood ratio test. It is distributed as a $x^{2}(r)$ variate, where $r$ is the number of binding non-negativity constraints. In our case $r=9$ and the test statistic is 1914.2 Since the critical value of $x^{2}(9)$ at the $5 \%$ significance level is 16.92 , the non-negativity constraints are violated at the 5\% level. This leaves no room other than for an empirical rejection of the commodity technology model.

## 6. Conclusion

We find that the magnitude of the adjustments to the use and make data which are required to ensure the non-negativity of the input-output coefficients, based on the commodity technology model, are inconsistent with the distribution of the unadjusted data. This means that we have a statistical basis for the rejection of the commodity technology model. This rejection is particularly surprising given the high level of aggregation we used in our exercise. At such a high level of aggregation there are only a few negative input-output coefficients and their magnitude is tiny, but the adjustments required to satisfy non-negativity are nevertheless sweeping and inconsistent with the data.

It follows that we must accept that different industries have different technologies for producing the same commodity. This is clear when some industries produce more efficiently than others, but even in a perfectly competitive world it may hold. The A-matrix is limited to material inputs, and apparent comparative disadvantages may be offset by lower direct factor costs (fixed capital or labor). Since Kop Jansen and ten Raa (1987) reject the alternatives to the commodity technology model for other reasons, we must abandon the very linear framework of deriving technical unit coefficients (A) from the black-box of input and output flows (U,V). We must account for the output destination of inputs within sectors. In the absence of such information one may continue to compute the pure commodity technology input-output matrix, but limit its application to final demand or value added vectors of which the proportions are close to the ones in the year on which the construction of the technical coefficients is based. One can still suppress the negatives as usual, since their magnitude is small anyway, but within the just described class of admissable scenario's
industrial output or price projections will be positive anyway and the zero setting yields modifications which are redundant. In short, adjustments, even when based on information about reliabilities, make projections along trends worse instead of better. One should either determine the within-sector commodity destination of inputs or limit the applications to scenarios proportioned close to the structure of the economy in the years of construction and leave the negatives as they are.
use tableu


```
toxtiles loat,clo
```

| agricult | 109.0 | 87.4 | 0.9 |
| :---: | :---: | :---: | :---: |
| coalmin | 4.7 | 0.4 | 26.1 |
| mining | 1.4 | 0.0 | 104.6 |
| petr gas | 0.0 | 0.0 | 0.0 |
| food man | 3.6 | 9.8 | 0.8 |
| drink | 0.0 | 0.0 | 0.6 |
| tobacco | 0.0 | 0.0 | 0.0 |
| coal prd | 0.0 | 0.0 | 4.9 |
| petr prd | 74.7 | 19.5 | 173.9 |
| chemical | 341.2 | 38.9 | 83.3 |
| iron stl | 1.1 | 0.6 | 9.6 |
| nofer mt | 0.7 | 0.0 | 4.1 |
| mech eng | 38.0 | 9.9 | 46.4 |
| inst eng | 0.0 | 0.0 | 0.0 |
| elec eng | 0.0 | 0.0 | 0.8 |
| ship bld | 0.0 | 0.0 | 0.0 |
| motor vh | 5.3 | 0.8 | 5.3 |
| aero eqp | 0.0 | 0.0 | 0.0 |
| other vh | 0.0 | 0.0 | 0.0 |
| metl gds | 49.0 | 28.6 | 72.3 |
| textiles | 1554.8 | 608.5 | 24.5 |
| leat, clo | 7.0 | 273.0 | 2.4 |
| bricks | 2.3 | 0.8 | 167.4 |
| timber | 2.2 | 3.1 | 15.6 |
| paper | 78.6 | 31.2 | 62.1 |
| prt publ | 7.3 | 12.2 | 10.1 |
| other min | 28.0 | 76.0 | 46.7 |
| construc | 8.0 | 5.4 | 6.1 |
| gas | 6.4 | 1.7 | 25.9 |
| electr | 78.5 | 12.6 | 90.2 |
| water | 9.3 | 2. 1 | 3.3 |
| rail | 4.2 | 2.5 | 18.5 |
| road | 30.8 | 13.6 | 135.4 |
| other tr | 62.1 | 26.4 | 30.9 |
| communic | 11.0 | 6.3 | 9.5 |
| distribu | 201.9 | 98.9 | 68.2 |
| buss srv | 50.8 | 35.6 | 40.2 |
| profesto | 27.8 | 9.0 | 19.2 |
| misc srv | 169.6 | 45.1 | 119.3 |

timber

| 53.7 |
| :---: |
|  |  |
|  |
| 0.0 |
| 4.3 |
| 0.7 |
| 0.0 |
| 0.2 |
| 65.9 |
| 43.1 |
| 19.4 |
| 7.9 |
| 19.3 |
| 0.0 |
| 6.4 |
| 0.0 |
| 3.8 |
| 0.0 |
| 0.0 |
| 89.9 |
| 102.3 |
| 3.2 |
| 25.4 |
| 683.2 |
| 39.5 |
| 15.3 |
| 78.3 |
| 6.8 |
| 1.8 |
| 21.5 |
| 0.2 |
| 1.2 |
| 36.8 |
| 64.6 |
| 11.2 |
| 51.2 |
| 55.2 |
| 14.2 |
| 66.9 |


| 3.2 | 0.0 | 56.1 | 4.1 |
| :---: | :---: | :---: | :---: |
| 4.5 | 0.1 | 1.1 | 0.4 |
| 5.3 | 0.0 | 3.6 | 294.0 |
| 0.0 | 0.0 | 0.0 | 0.0 |
| 6.7 | 0.0 | 1.3 | 2.9 |
| 0.1 | 0.0 | 0.1 | 0.0 |
| 0.3 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 |
| 57.4 | 28.8 | 44.3 | 107.2 |
| 145.3 | 65.9 | 522.3 | 233.5 |
| 3.7 | 0.9 | 14.0 | 411.8 |
| 11.6 | 11.8 | 16.0 | 171.5 |
| 26.0 | 23.3 | 62.5 | 692.1 |
| 0.0 | 0.0 | 0.0 | 0.0 |
| 0.1 | 0.1 | 2.1 | 202.6 |
| 0.0 | 0.0 | 0.0 | 0.0 |
| 1. 1 | 1. 1 | 1.5 | 35.3 |
| 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 |
| 28.0 | 8.8 | 106.2 | 372.9 |
| 17.1 | 13.5 | 120.7 | 31.1 |
| 0.8 | 0.2 | 6.0 | 4.4 |
| 1.7 | 0.3 | 13.1 | 1318.1 |
| 21.4 | 2.3 | 30.8 | 662.7 |
| 1003.9 | 472.3 | 92.1 | 9.2 |
| 24.6 | 349.1 | 8. 6 | 36.1 |
| 30.2 | 13.4 | 131.3 | 266.4 |
| 4.8 | 10.9 | 4. 3 | 2836.3 |
| 7.4 | 2.4 | 8. 3 | 3.9 |
| 36.1 | 16.1 | 52.0 | 42.6 |
| 3.9 | 0.6 | 2.4 | 0.0 |
| 3.1 | 20.9 | 3. 6 | 6.1 |
| 48.0 | 20.3 | 42.9 | 250.9 |
| 68.6 | 41.2 | 33.5 | 68.3 |
| 8.3 | 47.3 | 12.6 | 53.4 |
| 66.7 | 37.1 | 94.4 | 456.3 |
| 29.8 | 50.2 | 53.0 | 155.9 |
| 13.5 | 27.2 | 22.4 | 40.2 |
| 80.6 | 179.9 | 133.7 | 74.2 |

water rail
road other tr communic distribu buss srverof srvmiscest

| agricult | 0.0 | 0.0 | 0.0 | 15.3 | 0.0 | 1.5 | 0.0 | 0.0 | 157.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| coal min | 0.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| mining | 0.0 | 4. 6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| petr gas | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| food man | 0.0 | 0.0 | 0.0 | 57.5 | 0.0 | 124.5 | 0.0 | 0.0 | 638.0 |
| drink | 0.0 | 0.0 | 0.0 | 22.2 | 0.0 | 0.0 | 0.0 | 0.0 | 74.3 |
| tobacco | 0.0 | 0.0 | 0.0 | 3.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| coal prd | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| petr prd | 11.0 | 54.0 | 219.1 | 889.3 | 7. 6 | 111.9 | 26.8 | 26.3 | 149.6 |
| chemical | 6.6 | 3.7 | 3.1 | 12.1 | 1.3 | 29.8 | 2.1 | 20.3 | 224.4 |
| iron stl | 14.3 | 22.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| nofer mt | 3.4 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| mech eng | 43.2 | 0.0 | 0.0 | 1.0 | 0.0 | 26.4 | 10.8 | 4.7 | 26.5 |
| inst eng | 0.2 | 0.0 | 0.0 | 2.1 | 0.0 | 0.0 | 0.0 | 26.3 | 18.7 |
| elec eng | 0.6 | 19.4 | 25.4 | 11.5 | 173.2 | 65.4 | 2.4 | 1.4 | 207.0 |
| ship bld | 0.0 | 0.0 | 0.0 | 157.6 | 0.0 | 0.0 | 0.0 | 0.0 | 3.2 |
| motor vh | 0.5 | 0.0 | 117.2 | 22.7 | 3.1 | 8.7 | 0.0 | 0.0 | 448.7 |
| aero eqp | 0.0 | 0.0 | 0.0 | 94.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| other vh | 0.0 | 145.3 | 0.0 | 2.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| metl gds | 3.4 | 12.1 | 8.8 | 5.0 | 13.2 | 17.5 | 4.1 | 0.2 | 63.1 |
| textiles | 0.1 | 0.0 | 0.0 | 1. 1 | 0.0 | 227.9 | 3.7 | 1.7 | 42.8 |
| leat, clo | 0.7 | 4.5 | 4.1 | 0.8 | 4.8 | 22.8 | 0.0 | 0.0 | 11.5 |
| bricks | 3.5 | 18.0 | 0.0 | 0.0 | 3.3 | 34.2 | 0.0 | 1.2 | 35.2 |
| timber | 0.5 | 6.0 | 0.0 | 16.6 | 3.3 | 50.3 | 36.3 | 10.5 | 169.8 |
| paper | 0.6 | 8.7 | 26.3 | 5.8 | 0.0 | 199.1 | 70.5 | 62.2 | 52.4 |
| prt publ | 1.0 | 10.6 | 4.3 | 36.4 | 32.6 | 329.3 | 424.6 | 134.9 | 30.3 |
| other min | 3.7 | 11.4 | 119.5 | 16.5 | 17.5 | 118.6 | 21.6 | 6.5 | 129.4 |
| construc | 3.6 | 3.3 | 2.4 | 14.2 | 12.0 | 78.5 | 326.9 | 43.7 | 42.1 |
| gas | 0.0 | 4. 6 | 1.4 | 5.9 | 6.7 | 55.8 | 10.6 | 14.9 | 42.1 |
| electr | 27.5 | 48.7 | 3.0 | 16.6 | 22.9 | 221.3 | 61.8 | 45.0 | 127.8 |
| water | 10.8 | 0.0 | 0.0 | 2.6 | 3.3 | 30.9 | 17.2 | 5.6 | 11.0 |
| rail | 0.0 | 0.0 | 0.0 | 1.0 | 56.9 | 27.0 | 1.9 | 2.6 | 8.1 |
| road | 0.7 | 2.0 | 27.2 | 18.4 | 1.9 | 1235.3 | 0.0 | 36.3 | 120.1 |
| other tr | 0.3 | 11.1 | 2.3 | 1646.1 | 44.4 | 767.8 | 18.3 | 30.2 | 117.7 |
| communic | 1. 9 | 18.2 | 33.2 | 61.2 | 243.1 | 342.4 | 575.7 | 290.3 | 182.5 |
| distribu | 3.9 | 10.5 | 42.6 | 11.1 | 10.1 | 67.4 | 17.8 | 38.5 | 210.9 |
| buss stv | 4.1 | 2.5 | 167.9 | 210.1 | 60.2 | 504.2 | 2006.5 | 259.1 | 912.4 |
| prof stv | 0.0 | 9.2 | 22.5 | 86.1 | 31.4 | 10.3 | 261.8 | 203.6 | 549.9 |
| miscsty | 4. 3 | 24.2 | 3.0 | 146.0 | 102.6 | 30.0 | 40.0 | 363.4 | 512.8 |

drink tobacco coal prd petr prd chemical

| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.8 |
| 469.4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 297.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 9463.8 | 17.4 | 0.0 | 0.0 | 0.1 | 16.8 |
| 0.0 | 0.0 | 2.0 | 1732.0 | 0.0 | 0.0 | 0.0 | 7.9 |
| 0.0 | 0.0 | 0.0 | 0.0 | 629.1 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 328.9 | 0.0 | 3.1 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 5040.2 | 33.9 |
| 0.0 | 0.0 | 21.8 | 0.0 | 0.0 | 0.2 | 78.9 | 6928.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 248.0 | 0.0 | 5.8 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.0 | 10.6 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 7.3 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.1 |
| 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 6.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 33.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1. 0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 46.5 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |


| 5616.9 | 0.0 |
| ---: | ---: |
| 0.0 | 185.7 |
| 0.0 | 0.3 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
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| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 |  |

iron stl nofer mt mech eng inst eng eleceng ship bld motor vh aero eqpother vhet gds

| agricult | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| coal min | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| mining | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| petr gas | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| food man | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| drink | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| tobacco | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| coal prd | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| petr prd | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| chemical | 0.3 | 7.7 | 17.4 | 5.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3.6 |
| iron stl | 4333.3 | 7.0 | 28.9 | 0.0 | 0.3 | 0.8 | 4.9 | 0.0 | 12.4 | 30.7 |
| nofer mt | 6.6 | 1679.9 | 5.6 | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 | 0.4 | 42.3 |
| mech eng | 40.2 | 4.4 | 7701.6 | 34.3 | 13.0 | 52.3 | 59.1 | 10.5 | 13.8 | 83.2 |
| inst eng | 0.5 | 0.0 | 10.3 | 828.1 | 74.3 | 0.0 | 1.4 | 6.1 | 0.0 | 4.6 |
| eleceng | 2. 4 | 0.0 | 41.0 | 66.7 | 5572.3 | 33.7 | 6.3 | 23.2 | 7.8 | 7.5 |
| ship bld | 0.8 | 0.0 | 48.8 | 0.0 | 0.1 | 1058.8 | 0.0 | 0.0 | 0.0 | 0.4 |
| motor vh | 32.6 | 3.3 | 142.0 | 0.0 | 16.5 | 7.8 | 4729.9 | 2.8 | 128.6 | 30.4 |
| aero eqp | 0.0 | 0.0 | 6.3 | 0.0 | 22.3 | 2.1 | 2.0 | 1653.5 | 0.0 | 0.0 |
| other vh | 0.7 | 0.0 | 36.6 | 0.0 | 0.0 | 0.0 | 1.6 | 0.0 | 721.5 | 0.3 |
| metl gds | 19.3 | 27.6 | 79.4 | 2.8 | 18.0 | 0.5 | 20.3 | 4.5 | 1.1 | 4215.9 |
| textiles | 0.3 | 0.0 | 2.2 | 0.0 | 0.0 | 0.0 | 2.2 | 0.0 | 0.0 | 0.1 |
| leat, clo | 0.0 | 0.0 | 0.0 | 0.4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| bricks | 0.0 | 0.0 | 0.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 |
| timber | 0.1 | 0.0 | 1. 3 | 0.2 | 0.0 | 0.0 | 0.7 | 0.0 | 0.0 | 31.1 |
| paper | 0.0 | 16.2 | 0.3 | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| prt publ | 0.0 | 0.0 | 0.3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| other min | 0.0 | 0.7 | 0.3 | 0.3 | 0.2 | 0.1 | 0.0 | 0.1 | 0.4 | 4.7 |
| construc | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| gas | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| electr | 0.0 | 0.0 | 0.0 | 0.0 | 36.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| water | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| rail | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| road | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| other tr | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| communic | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| distribu | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| buss srv | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| profesty | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| misc stv | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |


|  | panut]uos ${ }^{6} Z$ əTqEL |
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| $\begin{aligned} & \text { n } \\ & \text { on } \end{aligned}$ | $0000000 \mathrm{mOH} 000000000000000000=0000000000$ <br>  |
| $\begin{aligned} & u \\ & J \\ & u \\ & u \\ & n \\ & \text { a } \\ & 0 \\ & 0 \end{aligned}$ |  <br>  |
| $\begin{aligned} & \text { ㅌ } \\ & \text { H } \\ & \text { E } \\ & \text { C } \\ & 0 \end{aligned}$ |  <br>  <br>  |
| $\begin{aligned} & -1 \\ & 0 \\ & a \\ & a \\ & 2 \\ & u \\ & 0 \end{aligned}$ | 0000000000000000000 NOOOHनNNOOOOOOOOOOO <br>  $\xrightarrow[N]{\text { No }}$ |
| $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $000000000 N 000$ roo000r000tana 0000000000000 <br>  |
| L <br> © <br> E <br> E |  <br>  |
| $\begin{aligned} & n \\ & \underset{\sim}{u} \\ & \sim \\ & \sim \\ & \hline \end{aligned}$ |  |
| $\begin{gathered} 0 \\ \vec{v} \\ \dot{\sim} \\ \underset{\sim}{*} \\ \underset{\sim}{2} \end{gathered}$ | $0000 N 00000000$ OOOOOOOHROONRNOOOOOOOOOO <br>  |
| $\begin{aligned} & \oplus \\ & \oplus \\ & \underset{\sim}{-1} \\ & \underset{\sim}{x} \\ & \oplus \end{aligned}$ | $000000000 \mathrm{m0000000000rn00007000000000000}$ <br>  $000000000 \mathrm{molo0000000} \underset{\sim}{N}$ |
|  |  |

make
tablev

## v

|  | water | rail | road | ertr | $m u n i c$ | stribu | ss srv | of srv | isc srv |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| agricult | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.2 |
| coal min | 0.0 | 0.0 | 17.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| mining | 0.0 | 0.0 | 50.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| petr gas | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| food man | 0.0 | 0.0 | 352.8 | 0.0 | 0.0 | 145.7 | 0.0 | 0.0 | 0.0 |
| drink | 0.0 | 0.0 | 111.4 | 0.0 | 0.0 | 129.5 | 0.0 | 0.0 | 0.0 |
| tobacco | 0.0 | 0.0 | 5.5 | 0.0 | 0.0 | -13.9 | 0.0 | 0.0 | 0.0 |
| coal prd | 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.9 | 0.0 | 0.0 | 0.0 |
| petr prd | 0.0 | 0.0 | 9.7 | 0.0 | 0.0 | 1.6 | 0.0 | 0.0 | 0.0 |
| chemical | 0.0 | 0.0 | 73.7 | 0.0 | 0.0 | 175.1 | 0.0 | 0.0 | 0.0 |
| ironstl | 0.0 | 0.0 | 45.3 | 0.0 | 0.0 | 8.9 | 0.0 | 0.0 | 3.8 |
| nofer mt | 0.0 | 0.0 | 12.1 | 0.0 | 0.0 | 9.2 | 0.0 | 0.0 | 0.0 |
| mech eng | 0.0 | 0.0 | 126.1 | 0.0 | 0.0 | 167.6 | 0.0 | 0.0 | 0.0 |
| inst eng | 0.0 | 0.0 | 14.8 | 0.0 | 0.0 | 22.1 | 0.0 | 0.0 | 0.1 |
| elec eng | 0.0 | 0.0 | 63.2 | 0.0 | 0.0 | 49.9 | 0.0 | 0.0 | 15.9 |
| ship bld | 0.0 | 0.0 | 5.3 | 0.0 | 0.0 | 2.6 | 0.0 | 0.0 | 0.1 |
| motor vh | 0.0 | 0.0 | 68.1 | 0.0 | 0.0 | 151.3 | 0.0 | 0.0 | 0.0 |
| aero eqp | 0.0 | 0.0 | 12.0 | 0.0 | 0.0 | -4.0 | 0.0 | 0.0 | 0.0 |
| other vh | 0.0 | 0.0 | 3.0 | 0.0 | 0.0 | 2.4 | 0.0 | 0.0 | 0.0 |
| metl gds | 0.0 | 0.0 | 81.4 | 0.0 | 0.0 | 41.2 | 0.0 | 0.0 | 0.0 |
| textiles | 0.0 | 0.0 | 52.2 | 0.0 | 0.0 | 25.0 | 0.0 | 0.0 | 0.0 |
| leat, clo | 0.0 | 0.0 | 34.7 | 0.0 | 0.0 | 31.3 | 0.0 | 0.0 | 0.0 |
| bricks | 0.0 | 0.0 | 69.8 | 0.0 | 0.0 | 23.6 | 0.0 | 0.0 | 0.0 |
| timber | 0.0 | 0.0 | 96.5 | 0.0 | 0.0 | 62.5 | 0.0 | 0.0 | 0.0 |
| paper | 0.0 | 0.0 | 40.2 | 0.0 | 0.0 | 26.8 | 0.0 | 0.0 | 0.0 |
| prt publ | 0.0 | 0.0 | 56.0 | 0.0 | 0.0 | 30.1 | 0.0 | 0.0 | 0.0 |
| other mn | 0.0 | 0.0 | 44.1 | 0.0 | 0.0 | 64.9 | 0.0 | 0.0 | 0.0 |
| construc | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 21.9 | 0.0 | 0.0 | 0.0 |
| gas | 0.0 | 0.0 | 66.4 | 0.0 | 0.0 | 32.4 | 0.0 | 0.0 | 0.0 |
| electr | 0.0 | 0.0 | 101.5 | 0.0 | 0.0 | 55.5 | 0.0 | 0.0 | 3.3 |
| water | 654.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| rail | 0.0 | 1099.9 | 32.8 | 0.0 | 0.0 | 0.0 | 11.5 | 0.0 | 29.8 |
| road | 0.0 | 0.0 | 2832.7 | 0.0 | 0.0 | 0.0 | 6.9 | 0.0 | 6.5 |
| other tr | 0.0 | 0.0 | 0.0 | 6074.1 | 0.0 | 0.0 | 53.4 | 0.0 | 6.5 |
| communic | 0.0 | 0.0 | 0.0 | 0.0 | 3529.2 | 0.0 | 0.0 | 0.0 | 0.0 |
| distribu | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 15300.0 | 0.0 | 0.0 | 60.0 |
| buss srv | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 7481.2 | 0.0 | 0.0 |
| profesry | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 5867.0 | 0.0 |
| misc srv | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 180.2 | 0.0 | 0.0 | 13207.3 |

technical coefficients matrix a

|  |  | agricult coal min |  | mining petr gas food man |  |  | drink | tobacco | 1 prd | chemical |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | agricult | 25.28 | -0.00 | -0.00 | 0.00 | 27.70 | 10.20 | 23.32 | $-0.00$ | -0.00 | 0.42 |  |
|  | coal min | 0.00 | -0.03 | -0.06 | 0.00 | 0.04 | 0.04 | -0.00 | 52.48 | -0.02 | -0.02 |  |
|  | mining | 0.05 | -0.01 | 2.26 | 0.00 | 0.09 | 0.00 | -0.00 | -0.01 | -0.01 | 1.27 |  |
|  | petr gas | 0.00 | 0.00 | 0.00 | 15.82 | 0.00 | 0.00 | -0.00 | -0.18 | 61.60 | -0.70 |  |
|  | food man | 16.24 | -0.00 | -0.00 | 0.00 | 25.93 | 4.71 | 0.00 | -0.02 | 0.06 | 1.77 |  |
|  | drink | 0.28 | -0.00 | -0.00 | 0.00 | 0.13 | 13.67 | 0.00 | -0.00 | 0.01 | 0.04 |  |
|  | tobacco | 0.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 | 0.00 | -0.00 |  |
|  | coal prd | 0.00 | -0.00 | 0.01 | 0.00 | 0.01 | -0.00 | 0.00 | 4.31 | -0.00 | 0.29 |  |
|  | petr prd | 0.77 | 0.83 | 15.54 | 1.75 | 1.66 | 2.90 | 0.45 | 0.44 | 12.47 | 6.84 |  |
|  | chemical | 6.20 | 0.44 | 2.47 | 1.85 | 1.82 | 1.17 | 0.91 | 1.70 | 2.49 | 35.48 |  |
|  | ironstl | 0.01 | 2.49 | 0.01 | 1.89 | 0.20 | 0.01 | -0.00 | 0.01 | 0.04 | 0.12 |  |
|  | nofer mt | 0.02 | -0.01 | -0.01 | 0.00 | 0.26 | 0.02 | 1.00 | $-0.01$ | -0.01 | 0.80 |  |
|  | mecheng | 0.72 | 5.72 | 5.77 | 5.76 | 0.47 | 0.60 | 0.29 | 0.51 | 0.22 | 0.64 |  |
|  | inst eng | -0.00 | -0.00 | 0.00 | 0.00 | -0.00 | -0.00 | 0.00 | $-0.00$ | -0.00 | 0.03 |  |
|  | elec eng | 0.01 | 1.40 | 0.20 | 0.00 | $-0.03$ | -0.08 | -0.01 | 0.03 | -0.00 | -0.02 |  |
|  | ship bld | 0.28 | -0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | $-0.00$ | -0.00 | 0.00 |  |
|  | motor vh | 0.06 | 0.05 | -0.09 | 0.10 | -0.04 | -0.06 | $-0.02$ | -0.00 | -0.00 | -0.00 |  |
| N | aero eqp | 0.00 | -0.00 | -0.00 | 0.00 | -0.00 | -0.00 | 0.00 | $-0.00$ | -0.00 | 0.00 |  |
|  | other vh | 0.36 | -0.00 | -0.00 | 0.00 | $-0.00$ | -0.00 | 0.00 | $-0.00$ | -0.00 | 0.00 |  |
|  | metl gds | 0.27 | 0.52 | 0.80 | 5.19 | 2.05 | 5.29 | 1.18 | 2.27 | 0.54 | 2.77 | 0 |
|  | textiles | 0.50 | 0.08 | -0.06 | 0.00 | 0.05 | -0.07 | 0.01 | 0.05 | -0.00 | 0.36 | 5 |
|  | leat, clo | 0.03 | 0.08 | -0.02 | 0.00 | 0.04 | 0.08 | 0.03 | 0.21 | 0.01 | 0.18 | - |
|  | bricks | 0.22 | 0.17 | 0.54 | 0.00 | 0.37 | 2.78 | -0.01 | -0.01 | $-0.00$ | 0.39 | E |
|  | timber | 0.34 | 1. 10 | 0.18 | 0.00 | 0.15 | 0.23 | 0.10 | 0.05 | 0.03 | 0.18 |  |
|  | paper | 0.25 | 0.02 | 0.62 | 0.00 | 3.06 | 3.15 | 14.39 | -0.03 | 0.00 | 2.21 |  |
|  | prt publ | 0.03 | 0.01 | -0.05 | 0.00 | 0.16 | 0.63 | 3.77 | -0.01 | 0.00 | 0.42 |  |
|  | other mn | 0.28 | 0.26 | 0.75 | 0.00 | 1.08 | 1.24 | 0.38 | $-0.02$ | 0.01 | 1. 14 |  |
|  | construc | 2.21 | 4.60 | -0.02 | 6.19 | 0.07 | 0.31 | 0.07 | $-0.07$ | 0.01 | -0.05 |  |
|  | gas | 0.04 | 0.00 | -0.02 | 0.00 | 0.15 | 0.18 | 0.09 | 4.94 | 0.03 | 1. 00 |  |
|  | - lectr | 0.59 | 4.10 | 4.32 | 0.00 | 0.85 | 0.80 | 0.43 | 1.82 | 0.25 | 2. 52 |  |
|  | water | 0.19 | 0.06 | 0.01 | 0.00 | 0.14 | 0.35 | 0.02 | 0.18 | 0.08 | 0.33 |  |
|  | rail | 0.11 | 0.35 | 7.64 | 0.00 | 0.08 | 0.19 | 0.37 | 1.43 | 0.03 | 0.24 |  |
|  | road | 1.11 | 0.45 | 0.02 | 4.04 | 1.87 | 1.04 | 0.64 | 3. 34 | 0.10 | 1.11 |  |
|  | other tr | 0.38 | 0.39 | 0.71 | 20.88 | 1.80 | 0.68 | 1.73 | 2.80 | 7.36 | 1.69 |  |
|  | communic | 0.30 | 0.14 | 0.28 | 0.00 | 0.11 | 0.06 | 0.20 | 0.04 | 0.02 | 0.22 |  |
|  | distribu | 3.95 | 0.65 | 1.94 | 2. 56 | 6.91 | 5.04 | 1.74 | 0.68 | 2.80 | 3.19 |  |
|  | buss srv | 2.12 | 0.34 | 2.25 | 1. 31 | 0.62 | 0.53 | 0.55 | 0.83 | 0.77 | 0.84 |  |
|  | prof sty | 0.35 | 0.96 | 1.09 | 6.57 | 0.51 | 1.13 | 1.14 | 1.25 | 0.17 | 0.86 |  |
|  | misc srv | 1.36 | 6.57 | 7.17 | 31.25 | 3.80 | 9.64 | 8.37 | 6.16 | 0.30 | 6.19 |  |













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| 빙 | $\sigma$ | ค．$\cap$ | 0 |  |  |  |  | $\Omega$ | － | ＇ror |  |  |  | $\vdash$ | ＋ | 島 | 0 | ¢ | 훙 | 40 | © | $r$ | 0 | 3 | 1 r． |  | 0 | $\bigcirc$ |  |  | m |  |  | 0 | － |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F． 7 | ᄃ | H．O | ＋ |  |  |  |  | 0 | ＋ | 7 |  |  |  | － | （1） | － | ＋ | ． | 0 | J | $\cdots$ | 7 | ＊ | 0 | 7 | 5 | © | 0 | ＋ |  | 0 | ＊ |  | $\bigcirc$ | 9 |
| 40 | 4 | 的 훌 | 5 |  |  |  | © | 5 | 5 | ＋ |  | ＋ | $\sigma$ | 0 | $\times$ | ＋ | 5 | $\square$ | ＋ | $\cdots$ | － | 0 | ก | Th | O | － | ＋ | 0 | 0 |  | $\bigcirc$ | ＋ | B | 0 | $\cdots$ |
| $\bigcirc$ oth | น | ＋ | － |  |  | 2 | $\cdots$ | $n$ | ． |  | － | $\cdots$ | － | ＋ | ＋ | $\cdots$ | － | － | 0 | ＇0 | 0 | ＋ | T | （1） | 5 | 할 | M | $\vdash$ | $\sigma$ | م． | $\Omega$ | m | $\cdots$ | $\cdots$ | $\cdots$ |
|  |  | $\cdots$ | $\square$ | 7 | 7 | 0 | ＊ | ＋ | H | ＇ | D | E | $\cdots$ |  | $\cdots$ |  | 7 |  | H |  |  |  |  | r |  | $\cdots$ |  |  | $\infty$ | n |  |  | 5 |  | ก |
| 0 Un | น | H． 3 |  | $\bigcirc$ | 0 | ＋ | $\bigcirc \square$ | M |  | E | ${ }^{\circ}$ | $\sigma$ | ก | O |  | 9 |  | － |  | $\sigma$ | ¢ | ＊ | 9 |  | 0 | $\Omega$ | ＇ | ＇ | ก | $\cdots$ | \％ | 9 | $\cdots$ | 울 | C |
| $\cdots 7$ | H | $\sigma \mu$ | ＋ | \＄ | $\cdots$ | （1） | ＋ | $E$ | 둥 | $\sigma$ | － | － | K | $\cdots$ | ＊ | 0 | $\leq$ | ค | $\leq$ |  | 5 | 5 | 5 | B | r | 0 | 7 | r | ？ | 5 | \％ | 0 | 5 | $\ldots$ | $\ldots$ |
| $\ll$ | $<$ | F $\cap$ | $n$ | a | $\mapsto$ | n | $\cdots$ un | ก | 3 | $1 \sim$ | － | － | 0 | 0 | on | 0 | 5 | ＇0 | 5 | 2 | $\square$ | $\oplus$ | $\square$ | ＋ | － | $\vdash$ | ค． | a | 0 | r | F | U | $\square$ |  | ＋ |








Table 3，end

| Index | Sector | Accuracy |
| :---: | :---: | :---: |
| 1 | Agriculture etc. | 5\% |
| 2 | Coal Mining | 5\% |
| 3 | Mining nes | 10\% |
| 4 | Petroleum \& Nat. gas | 5\% |
| 5 | Food Manufacturing | 5\% |
| 6 | Drink | 5\% |
| 7 | Tobacco | 5\% |
| 8 | Coal Products | 5\% |
| 9 | Petroleum Products | 5\% |
| 10 | Chemicals | 5\% |
| 11 | Iron \& Steel | 5\% |
| 12 | Non-ferrous Metals | 5\% |
| 13 | Mech. Engineering | 5\% |
| 14 | Instr. Engineering | 5\% |
| 15 | Elect. Engineering | 5\% |
| 16 | Ship Building | 5\% |
| 17 | Motor Vehicles | 5\% |
| 18 | Aerospace Equipment | 5\% |
| 19 | Other Vehicles | 5\% |
| 20 | Metal Goods nes | 5\% |
| 21 | Textiles | 5\% |
| 22 | Leather, Clothing etc. | 5\% |
| 23 | Bricks | 5\% |
| 24 | Timber \& Furniture | 5\% |
| 25 | Paper \& Board | 5\% |
| 26 | Printing \& Publishing | 5\% |
| 27 | Other Manufacturing | 5\% |
| 28 | Construction | 15\% |
| 29 | Gas | 5\% |
| 30 | Electricity | 5\% |
| 31 | Water | 15\% |
| 32 | Rail | 5\% |
| 33 | Road | 40\% |
| 34 | Other Transport | 40\% |
| 35 | Communication | 5\% |
| 36 | Distribution | 50\% |
| 37 | Business Services | 60\% |
| 38 | Professional Service | 60\% |
| 39 | Misc. Services | 60\% |

1. Agriculture etc.
2. Mining \& Gas
3. Food, Drink \& Tobacco
4. Mining \& Gas Products
5. Metals
6. Heavy Manufacturing
7. Light Manufacturing
8. Construction
9. Services
10. Agriculture etc.
11. Coal Mining
12. Mining nes
13. Petroleum \& Nat. gas
14. Food Manufacturing
15. Drink
16. Tobacco
17. Coal Products
18. Petroleum Products
19. Chemicals
20. Iron \& Steel
21. Non-ferrous Metals
22. Mech. Engineering
23. Instr. Engineering
24. Elect. Engineering
25. Ship Building
26. Motor Vehicles
27. Aerospace Equipment
28. Other vehicles
29. Metal Goods nes
30. Textiles
31. Leather, Clothing etc.
32. Bricks
33. Timber \& Furniture
34. Paper \& Board
35. Printing \& Publishing
36. Other Manufacturing
37. Construction
38. Gas
39. Electricity
40. Water
41. Rail
42. Road
43. Other Transport
44. Communication
45. Distribution
46. Business Services
47. Professional Services
48. Misc. Services

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[^0]Table 6
Table 7

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Table
8

Proof of Lemma 1. $A=U V^{-T}=U W$, hence $d A=d U . W+U d W$. To determine $\frac{\partial a_{i j}}{d u_{r s}}$, put $(d U)_{r s}=\partial u_{r s}$ and zeros elsewhere and put $d W=0$ as $W$ depends on $V$ only. Non-r rows of dU being zero, it follows that $\frac{\partial a_{i j}}{\partial u_{r s}}=0$ for $i \neq r$. The $r$-th row of the equation reads $\partial a_{r j}=\partial u_{r s} \cdot w_{s j}, j=1, \ldots, 39$. To determine $\frac{\partial a_{i j}}{\partial v_{r s}}$, put $d U=0$. Now, differentiating $W V^{\top}=I$, we have $(d W) V^{\top}+W d V^{\top}=0$ or $d W=-W\left(d V^{\top}\right) V^{-\top}=-W\left(d V^{\top}\right) W$. Hence $d A=-U W\left(d V^{\top}\right) W=-A\left(d V^{\top}\right) W$. We must put $(d V)_{r s}=\partial v_{r s}$ and zeros elsewhere. Then $\partial a_{i j}=-a_{i s} \partial v_{r s} \cdot w_{r j}$ or $\frac{\partial a_{i j}}{\partial v_{r s}}=$ -a $_{\text {is }}{ }^{W}{ }_{r j}$. Q.E.D.

Proof of Lemma 2. Lemma 1 shows that the first order derivatives with respect to $U$ depend only on $W$ hence $V$. Consequently, the second order derivatives with respect to $U$ vanish. The cross partials vanish for $i \neq k$ by the first part of Lemma 1. If $i=k$ we have $\frac{\partial^{2} a_{k j}}{\partial u_{k} l^{\partial v_{r s}}}=\frac{\partial w_{j}}{\partial v_{r s}}$ by Lemma 1 . To evaluate this, note that $d W=-W\left(d V^{\top}\right) W$ (proof of lemma 1). Put (dV) ${ }_{r s}=$ $\partial v_{r s}$ and zeros elsewhere, then the $(\ell, j)$ th component reads $\partial w_{\ell j}={ }^{-w} \ell_{s}$ $\partial v_{r s} w_{r j}$ or $\frac{\partial w_{l j}}{\partial v_{r s}}=-w_{l s} w_{r j}$. It follows that $\frac{\partial^{2} a_{k j}}{\partial u_{k \ell} \ell_{r s}}=-w_{l s} w_{r j}$. It remains to determine the second order derivatives with respect to V. By Lemma 1 and the product rule, $\frac{\partial^{2} a_{i j}}{\partial v_{k \ell} \partial v_{r s}}=\frac{-\partial}{\partial v_{k \ell}}\left(a_{i s} w_{r j}\right)=-\frac{\partial a_{i s}}{\partial v_{k \ell}} w_{r j}-a_{i s} \frac{\partial w_{r j}}{\partial v_{k \ell}}$. By Lemma 1 and the above expression for the partials of $W$ with respect to $V$, we obtain $\frac{\partial^{2} a_{i j}}{\partial v_{k} \ell^{\partial v_{r s}}}=a_{i} \ell^{w_{k s}} w_{r j}+a_{i s} w_{r} l^{w_{k j}}$.
Q.E.D.

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