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RESEARCH MEMORANDUM

THE CORRELATION STRUCTURE OF STATIONARY BILINEAR PROCESSES

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Abstract

It is shown that the existence condition for stationary bilinear time series implies the stability condition for the embedded autoregressive model. An explicit expression for the auto correlation function of such bilinear processes is derived in terms of the coefficients of the model and the moments of the error process. It is shown that in general it is impossible to identify the model on the basis of second order properties of the observable process, without additional restrictions on the parameters.

Keywords: bilinear systems, stationary processes, correlation structure Kronecker product of matrices, identification.

1. *Introduction*

In this paper we consider the following class of bilinear models

$$\sum_{i=0}^{p} a_i x_{t-i} = \sum_{k=1}^{p} b_k x_{t-k} \varepsilon_{t-1} + \varepsilon_t, \quad t \in \mathbb{Z} \quad a_0 = 1 \qquad (1.1)$$

where $\{\varepsilon_t, t \in \mathbb{Z}\}$ is a real sequence of i.i.d. random variables defined on some probability space $(\Omega, \mathcal{B}, P)$ with $E(\varepsilon_t) = 0$ and $V(\varepsilon_t) = \sigma^2 < \infty$. From [5] we know that there exists a unique strictly stationary stochastic process $\{x_t, t \in \mathbb{Z}\}$ satisfying (1.1) under a condition involving both the coefficients a_i, b_k and σ^2. This model is a simple nonlinear generalization of the well known auto regressive model of order p and it may be the adequate model in situations where AR or even ARMA models don't give a good fit to the data. Time series analists often choose an appropriate model on the basis of the observed autocorrelation function. Therefore it is important to have an expression for the theoretical autocorrelation function or equivalently the theoretical autocovariance function and to know whether this function uniquely determines the unknown parameters of the model. As in [5], results are more easily obtained when the model is rewritten in vector notation by introducing the random vectors $X_t = (x_t, x_{t-1}, \dots, x_{t-p+1})^T$ $t \in \mathbb{Z}$, (X^T denotes the transpose of X) and the pxp coefficient matrices A and B defined by

$$A = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_{p-1} & -a_p \\ & & \mathbf{I}_{p-1} & & \mathbf{0} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 & \dots & b_p \\ \mathbf{0} & \dots & \mathbf{0} \end{bmatrix}, \qquad (1.2)$$

where $\mathbf{I}_m$ denotes the m×m unit matrix and $\mathbf{0}$ a vector of zeros. Putting $C = (1, 0, \dots, 0)^T \in \mathbb{R}^p$, equation (1.1) can be written as

$$X_t = AX_{t-1} + BX_{t-1}\varepsilon_{t-1} + C\varepsilon_t, \qquad t \in \mathbb{Z}. \qquad (1.3)$$

As in [5], We can, without complicating the discussion, allow A, and B to be arbitrary matrices obeying the existence condition

$$\rho(A\otimes A + \sigma^2 B\otimes B) < 1, \tag{1.4}$$

where $\rho(.)$ denotes the spectral radius of a matrix and $\otimes$ is the Kronecker product. Also the vector C may be any p-vector. It is however noteworthy that when A and B are of the form (1.2), taking **B=0** makes (1.4) equivalent to the well known stability condition that the polynomial $\Sigma a_k z^k$ should have all its zeroes outside the unit circle. We shall see below that (1.4) in fact *implies* that this polynomial satisfies the stability condition.
In 1987 GUEGAN gave some expressions involving moments of bilinear processes (see [6]) , but he did not derive explicit formulae in terms of the coefficients and the moments of the error process (ε_t) as we intend to do. In particular we shall discuss whether the second order properties of the observable process identify the model. The necessary calculations make extensively use of some special matrix theory which we shall briefly outline in the next section.

2. *Preliminaries*

For the basic properties of Kronecker products we refer to [7]. We shall also use the *vec* operator which assigns to the matrix A with columns $a_1, a_2, \ldots, a_p$ the vector $vec(A) = (a_1^T, \ldots, a_p^T)^T$. Generally the *vec* operator is not invertible, but if the dimensions of its argument are prescribed it is. A well known and usefull lemma that relates Kronecker products and the *vec* operator is

▷*LEMMA 1. If the matrices A, B and X are such that the product AXB is well defined, then* $vec(AXB) = (A\otimes B^T)vec(X)$.

PROOF. See [7], th. 2 p. 30. □

From now on we shall suppose that all ordinary matrix products and sums that occur are well defined. As an application of lemma 1 we state the following result on general linear matrix equations.

▷*LEMMA 2. Let $A_1,\dots,A_n$ and $B_1,\dots,B_n$ be arbitrary matrices . Then the matrix equation*

$$\sum_{i=1}^{n} A_i X B_i = C \tag{2.1}$$

has a unique solution if and only if the matrix

$$\sum_{i=1}^{n} (A_i \otimes B_i^T)$$

is nonsingular.

PROOF. Since the dimensions of X are prescribed by the number of columns of A_i and the number of rows of B_i, we may apply the *vec* operation on both sides of equation (2.1) . Application of lemma 1 and the rule $vec(P+Q) = vec(P) + vec(Q)$, yields that (2.1) is equivalent with

$$\left(\sum_{i=1}^{n} (A_i \otimes B_i^T) \right) vec(X) = vec(C) \ .$$

The result follows then easily. □

Kronecker products of the form P⊗P and sums of such products as in (1.4) need some special attention. Although they are in general not semi positive definite, the following lemma shows that in some way they behave like nonnegative matrices. It is a special case of a more sophisticated result in [8].

▷*LEMMA 3. Let P and Q be arbitrary square matrices of equal dimensions with real elements. Then $\rho(P\otimes P) \le \rho(P\otimes P+Q\otimes Q)$.*

PROOF. Let $(P)_{ij}$ denote the (i, j)-th element of the matrix P. For sums of Kronecker products we adopt a notation for the elements as introduced in [5], thus $(..)_{ij,ij}$ denotes the (i, j)-th element of the (i, j)-th block when the matrix is partitioned in the obvious way.
Put $\rho = \rho(P\otimes P + Q\otimes Q)$, and let $\varepsilon > 0$ be arbitrary.

For all $n \in \mathbb{N}$ we have

$$(\rho+\varepsilon)^{-n/2}P^n = (\rho+\varepsilon)^{-n/2}\, 2^{-n}[(P+Q) + (P-Q)]^n =$$

$$= (\rho+\varepsilon)^{-n/2}\, 2^{-n} \sum \prod_{k=1}^{n} (P + \delta_k Q)\ , \qquad (2.2)$$

where the summation is over all 2^n sequences $(\delta_k)_{k=1}^{n}$, with $\delta_k = \pm 1$. Hence, using the inequality $|\sum_i a|^2 \le N \sum_i a^2$ where N is the number of terms in the summation, (2.2) implies

$$(\rho+\varepsilon)^{-n}|(P^n)_{ij}|^2 \le (\rho+\varepsilon)^{-n}\, 2^{-n} \sum \left(\prod_{k=1}^{n} (P + \delta_k Q) \right)_{ij}^{2} =$$

$$= (\rho+\varepsilon)^{-n}\, 2^{-n} \sum \left(\prod_{k=1}^{n} [(P + \delta_k Q)\otimes(P + \delta_k Q)] \right)_{ij,ij} =$$

$$= (\rho+\varepsilon)^{-n}\, 2^{-n} \left([(P+Q)\otimes(P+Q) + (P-Q)\otimes(P-Q)]^n \right)_{ij,ij} =$$

$$= (\rho+\varepsilon)^{-n} \left([P\otimes P + Q\otimes Q]^n \right)_{ij,ij} \le (\rho+\varepsilon)^{-n} c_0 n^{p-1} \rho^n$$

for some constant c_0 (see [8]).
Since $\rho(\rho+\varepsilon)^{-1} < 1$ and a matrix power series like $\sum_n M^n$ converges absolutely if and only if $\rho(M) < 1$, it follows that $\sum_n (\rho+\varepsilon)^{-n/2}P^n$ converges absolutely and so that $\rho((\rho+\varepsilon)^{-1/2}P) < 1$, or equivalently $\rho(P) < (\rho+\varepsilon)^{1/2}$. Since ε was arbitrary it follows that $\rho(P) \le \rho^{1/2}$ which is equivalent with $\rho(P\otimes P) \le \rho$. □

COROLLARY 2.1. Condition (1.4) implies $\rho(A\otimes A) < 1$ which in turn implies that $\rho(A) < 1$ and so it follows that the matrix $I_p - A$ is nonsingular. In particular, when A is of the form (1.2), it follows that the polynomial $\Sigma a_k z^k$ has all its zeroes outside the unit circle. More generally, it follows for arbitrary A that the polynomial $det(I_p - Az)$ has all its zeroes outside the unit circle. This is the well known stability condition for the embedded autoregressive model i.e. the model (1.3) with B = 0. Because of symmetry, a similar conclusion can be drawn for the polynomial $det(I_p - \sigma Bz)$.

3. *Calculation of the covariance function*

At first we shall introduce some notation that wil be used in the sequel.

$$\sigma^2 = E|\varepsilon_t|^2, \quad \mu_3 = E(\varepsilon_t^3), \quad \mu_4 = E|\varepsilon_t|^4,$$

$$\mu = E(X_t), \quad \tilde{X}_t = X_t - \mu,$$

$$R_k = E(X_t X_{t-k}^T), \quad r_k = E(\tilde{X}_t \tilde{X}_{t-k}^T), \qquad t,k \in \mathbb{Z},$$

$$\Psi_s = E(X_t X_{t-s}^T \varepsilon_t), \quad Y = E(X_t X_t^T \varepsilon_t^2) \qquad s,t \in \mathbb{Z}.$$

Notice that Ψ_0 and Y are symmetric matrices.
As a first step we shall calculate μ. From [5] follows that we can express X_t in $\varepsilon_t, \varepsilon_{t-1}, \ldots$ in the following way

$$X_t = C\varepsilon_t + \sum_{j=1}^{\infty} \prod_{k=1}^{j} (A + B\varepsilon_{t-k}) C\varepsilon_{t-j}, \qquad \text{(a.s.)}. \tag{3.1}$$

Hence, taking expectations on both sides of (1.3) we obtain

$$\mu = A\mu + BC\sigma^2,$$

and since $I_p - A$ is nonsingular by corollary 2.1, we have

$$\mu = (I_p - A)^{-1} BC\sigma^2. \tag{3.2}$$

More complicated is the calculation of the covariance function. Post multiplying both sides of (1.3) with X_{t-s}^T and taking expectations we obtain

$$R_s = AR_{s-1} + B\Psi_{s-1} , \qquad s \geq 1 . \tag{3.3}$$

LEMMA 4. For $s \geq 1$ we have $\Psi_s = \sigma^2 C\mu^T$, and for $s = 0$

$$\Psi_0 = CC^T\mu_3 + \sigma^2(C\mu^T A^T + A\mu C^T) + \sigma^4(CC^T B^T + BCC^T) \tag{3.4}$$

PROOF. For $s \geq 1$ we have

$$\Psi_s = E(X_t X_{t-s}^T \varepsilon_t) = E\{(AX_{t-1} + BX_{t-1}\varepsilon_{t-1} + C\varepsilon_t)X_{t-s}^T \varepsilon_t\} =$$

$$= E(CX_{t-s}^T \varepsilon_t^2) = \sigma^2 C\mu^T ,$$

and for $s = 0$ (using $E(X_t\varepsilon_t) = C\sigma^2$)

$$\Psi_0 = E\{(AX_{t-1} + BX_{t-1}\varepsilon_{t-1} + C\varepsilon_t)(AX_{t-1} + BX_{t-1}\varepsilon_{t-1} + C\varepsilon_t)^T\varepsilon_t\} =$$

$$= A\mu C^T\sigma^2 + E(BX_{t-1}\varepsilon_{t-1}C^T\varepsilon_t^2) + E(CX_{t-1}^T A^T\varepsilon_t^2) + E(CX_{t-1}^T B^T\varepsilon_{t-1}\varepsilon_t^2) +$$

$$+ E(CC^T\varepsilon_t^3) =$$

$$= A\mu C^T\sigma^2 + BCC^T\sigma^4 + C\mu^T A^T\sigma^2 + CC^T B^T\sigma^4 + CC^T\mu_3 \qquad \square$$

From lemma 4 follows immediately

$$r_1 = R_1 - \mu\mu^T = AR_0 + B\Psi_0 - \mu\mu^T = Ar_0 + B\Psi_0 - \sigma^2 BC\mu^T , \tag{3.5}$$

$$r_s = R_s - \mu\mu^T = AR_{s-1} - \mu\mu^T + B\Psi_{s-1} = Ar_{s-1} , \qquad s \geq 2 .$$

Hence

$$r_s = A^{s-1}r_1 , \qquad s \geq 2 . \tag{3.6}$$

Thus the covariance function can be calculated when r_0 or R_0 can be calculated. Two more lemmas are needed.

LEMMA 5. The matrices Y and R_0 satisfy the following matrix equation

$$Y - BYB^T\sigma^2 = AR_0A^T\sigma^2 + \Gamma , \tag{3.7}$$

where Γ is defined by

$$\Gamma = CC^T\mu_4 + \mu_3(C\mu^TA^T + A\mu C^T) + \mu_3\sigma^2(CC^TB^T + BCC^T) + \sigma^2(A\Psi_0B^T + B\Psi_0A^T) \qquad (3.8)$$

PROOF. In a similar way as in the proof of lemma 4 we have

$$\begin{aligned} Y = {} & CC^T\mu_4 + \mu_3(C\mu^TA^T + A\mu C^T) + \mu_3\sigma^2(CC^TB^T + BCC^T) + \\ & + E(AX_{t-1}X_{t-1}^TA^T\varepsilon_t^2) + E(AX_{t-1}X_{t-1}^TB^T\varepsilon_{t-1}\varepsilon_t^2) + \\ & + E(BX_{t-1}X_{t-1}^TA^T\varepsilon_{t-1}\varepsilon_t^2) + E(BX_{t-1}X_{t-1}^TB^T\varepsilon_{t-1}^2\varepsilon_t^2) = \\ = {} & \Gamma + \sigma^2AR_0A^T + \sigma^2BYB^T \end{aligned}$$

which proves the lemma. □

LEMMA 6. The matrices Y and R_0 satisfy the following matrix equation

$$R_0 - AR_0A^T = BYB^T + M \qquad (3.9)$$

where M is defined by

$$M = CC^T\sigma^2 + (A\Psi_0B^T + B\Psi_0A^T) \qquad (3.10)$$

PROOF. As in the preceding lemmas we substitute the right hand side of (1.3) for X_t in $E(X_tX_t^T)$ and calculate the expectations. This gives

$$R_0 = AR_0A^T + A\Psi_0B^T + B\Psi_0A^T + BYB^T + \sigma^2CC^T$$

and the result follows easily. □

The next step is to solve (3.7) and (3.9) for R_0 and Y. Solving (3.9) for BYB^T and substituting the result in (3.7) yields

$$Y = \sigma^2R_0 + \Gamma - \sigma^2M \qquad (3.11)$$

Substitution of this expression in again (3.7) gives the following matrix equation for R_0

$$R_0 - AR_0A^T - \sigma^2 BR_0B^T = M - \sigma^2 BMB^T + B\Gamma B^T \qquad (3.12)$$

By lemma 1 and (1.4) this equation has a unique solution which is given by

$$vec(R_0) = (I - A\otimes A - \sigma^2 B\otimes B)^{-1}\Big((A\otimes A)vec(M) + (B\otimes B)vec(\Gamma)\Big) + vec(M) \qquad (3.13)$$

as can be seen after some minor calculations. (The subscript to indicate the dimensions of the unit matrix is dropped for notational convenience.) Clearly, substitution of (3.4) in (3.8) and (3.10), and subsequently in (3.13) gives a very large formula for R_0 expressed in A, B, C and the moments of the error process. Therefore we shall pay some attention to a special case.

4. *The univariate model with symmetric error distribution*

Since the error distribution is symmetric we have $\mu_3 = 0$. In this section we consider the univariate model (1.1), or equivalently the p-variate model where the matrices A and B are given by (1.2) and where $C = (1,0,\ldots,0)^T$. For practical purposes this is the most important case. We shall show, however, that without any additional restrictions on the parameters it is fundamentally impossible to identify this model on the basis of the second order properties of the observed process. In order to do so, put $b_1 = 0$. Then we have $BC = 0$, and so by (3.2) it follows $\mu = 0$. Furthermore we obtain $\psi_0 = \psi_s = 0$, $s = 1,2,\ldots$ which implies

$$R_s = AR_{s-1}, \qquad s = 1,2,\ldots . \qquad (3.14)$$

It also implies $\Gamma = \mu_4 CC^T$, and $M = \sigma^2 CC^T$, so (3.13) reduces to

$$vec(R_0) = (I - A\otimes A - \sigma^2 B\otimes B)^{-1}(C\otimes C) \ . \qquad (3.15)$$

From (3.14) it follows that the observable process has essentially the same correlation structure as an autoregressive process and so when $p \geq 2$, it is impossible to decide whether $B = 0$ or $B \neq 0$ by investigation of the estimated autocorrelation function of (x_t). In fact we have that the process $(X_t - AX_{t-1})$ is white noise, so (X_t) really is an autoregressive process with $(\eta_t) = (BX_{t-1}\varepsilon_{t-1} + C\varepsilon_t)$ as error process. Using (3.9) and (3.15), straightforward calculation shows that the variables η_t have covariance matrix Σ_η given by

$$vec(\Sigma_\eta) = (I - A\otimes A)(I - A\otimes A - \sigma^2 B\otimes B)^{-1}(C\otimes C) \ . \qquad (3.16)$$

It is easily seen that Σ_η is singular when B is of the form (1.2). From [4] we know that in that case it is not sure that A and Σ_η are identifiable. But even if they were, it is clear that additional assumptions concerning B are needed in order to identify B and σ^2.

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