[...]SITY
[...]OLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153 5000 LE TILBURG THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

[redacted]

CHARACTERIZATION OF ECONOMIC AGENTS IN ARBITRARY COMMUNICATION STRUCTURES

Robert P. Gilles
Pieter H.M. Ruys

FEW 365

Characterization of Economic Agents in Arbitrary Communication Structures [1]

Robert P. Gilles[2] Pieter H.M. Ruys

Dept. of Economics
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands

December 1988

[1]The authors thank Willy Spanjers, Aart de Zeeuw, Dolf Talman and Ajit Ranade for their useful comments and notes on a previous draft of this paper. Especially we thank Birgit Grodal and Shou Jilin for their contributions in the development of this paper.

[2]This author is financially supported by the Netherlands organization for scientific research (N.W.O.), grant 450-228-009.

Abstract

Relations on the set of economic agents are natural social constraints with respect to individual and social behaviour of those agents. Although the society consists of a complex system of such relations and networks based on these relations, such a relational structure is until now not used in general equilibrium theory. One of the main reasons is that such relations are determining the social behaviour of the economic agents, which is not yet described properly, especially in environments with infinitely many agents.

In this paper we introduce a method for describing an economy with a communication or relational structure, instead of a unstructured set of agents. Such a communication structure consists of a set of agents endowed with relations as the social characteristics of those agents. Furthermore we introduce a topological attribute space. By typifying agents of the population in the attribute space we arrive at a model which explains relations between agents *locally*. This is the explicit formal expression of the fundamental thought in social sciences that social relations between individual agents are interdependent with the individual characteristics of those agents.

1 Introduction

In general equilibrium theory the economy in which a competitive equilibrium can be defined, is usually described by a mapping from the set of agents into some space of individual attributes. The chosen attributes are e.g. preferences on the commodity space, initial endowments, and production possibility sets. Such a mapping is assumed to be a complete description of the economy, and contains all necessary information for defining the equilibrium concept, which is based on specific behaviourial assumptions on the individual agents. It naturally follows that in such a setting of a market system this individual behaviour is oriented by the prices on the markets in the system. The equilibrium price that results, may be considered as information about scarcity of commodities. It is in fact the *only* social parameter in the model. For exhaustive expositions of this model we refer to Debreu (1959), Hildenbrand (1974), and Mas-Colell (1985). The chosen atttibute space is extensively studied in Grodal (1974).

In this paper we will introduce social aspects within the society already *explicitly* in the definition of an economy. In some recent literature this is also done in a more implicit way. Hammond (1987) and Hammond, Kaneko and Wooders (1987) incorporate implicitly the social rule that in coalition formation only finite coalitions are allowable. They however do not base themselves explicitly on the attributes of the agents in the economy. Grodal (1972) also implicitly assumes that there is some interdependency between the (social) determinants in the process of coalition formation and the individual attributes of the agents.

We concern ourselves with a more explicit introduction of social aspects. Traditionally one can introduce those aspects in two explicit forms, namely in *coalitional structures* and in *relational* or *communication structures*. For coalitional approaches we refer to the seminal paper by Aumann and Drèze (1974) and the recent work of Gilles (1987 and 1988a). In this paper we follow the relational approach.

In order to introduce these social aspects properly we depart from two primitive economic concepts: a set of agents with a communication structure, and a space of attributes of these agents. Both concepts can be regarded independently of each other, but in real life they are related, of course. People, for example, who are characterized by speaking the same language usually communicate more easily than people who speak different languages. Similar observations can be made for people living close to each other, or who are working in the same factory. (A nice application of this idea is given by Allen (1982) in the theory of accepting innovations by producers in a communication structure.)

In the traditional approach to economic modelling the distinction between the two primitive concepts is not relevant. There it is assumed that the agents are endowed with a complete communication structure. Eventual characteristics based on incomplete abilities to communicate, and thus pointing to a social communication structure, are just ignored. The result is that a model designer is forced to choose all characteristics such that agents can be treated symmetrically. This is one of the main reasons that there does not exist a general equilibrium model of a production economy with oligopolistic firms in the market.

In our modelling we will explicitly choose for an asymetric treatment of the agents in a communication structure. In the main stream of literature on communication structures one treats agents symmetrically by introducing stochastic concepts into the model. (We refer to the work of Greenberg and Weber (1983), and Kirman et al. (1986) on stochastic communication structures.) Our approach will be more in the line of a seminal paper by Kalai et al. (1978). In this line of reasoning the distinction between a set of agents with a communication structure and a space of characteristics becomes relevant. In those specific, deterministic environments agents cannot be treated symmetricly. If, for example, the characteristic "language" is relevant for the problem to be analysed, it is evident that agents speaking the same language can be treated symmetrically, or can communicate with each other, while agents speaking only very different languages cannot. However, if the languages are not *too* different from each other, there may be agents in both language groups who can understand each other and do communicate. This rather subtle relation between agents and their characteristics will be defined formally in the sequel.

In order to do so, we express the two primitive economic concepts in mathematical objects. Following common reasoning in the main stream literature on the relational approach we represent the communication structure by a ***graph***, while the characteristics are represented by a ***topological attribute space***. Since we want to emphasize the difference between the more vague economic concepts and the exact notions used further, we will denote them by using the terms "population" and "attribute space". The relation mentioned above is defined as a mapping from the population into the attribute space. It is not evident at all, whether such a mapping, called a *characterization*, exists. If it exists, we are able to typify agents in a population by their attributes. An element in the attribute apace is then called a *type*. This paper is concerned with reciprocal relations between agents and types. We derive properties about types from properties about agents, and vice versa.

In the second section of this paper we will formally develop our method of modelling as described above. After the formal introduction of the notions of population, attribute space, and characterization, we analyse as the major feature of the model the concept of *communication* in an incomplete communication structure. We distinguish two kinds of communication, namely *direct communication* and a weaker form of "regular" *communication*. Direct communication is a form of direct contact between agents in the population, while regular communication is a form of contact through finitely many intermediairy agent.

It is commonly assumed that in an economy every agent has to be able to reach every other agent in the economy through this regular form of (indirect) communication. If this is not the case, then we have a situation of several *separate* economies, consisting of groups of agents who can communicate with each other. However there is *no* communication between these separate groups of economic agents, and hence there is no natural communication between these separate economies. Therefore they can be analysed separately. A substantial part of our analysis in this paper is therefore devoted to the question under which conditions on the characterization of a population in an attribute space, there is (regular) communication between all agents in the population. A first step in this analysis is already made in the Section 2 of this paper. There it is stated that this kind of full communication is equivalent to connectedness of types in the attribute space.

In the third section we introduce a special kind of characterizations of populations in suitable attribute spaces. Our principle in this kind of modelling is that in case of an at most countable population we are able to understand and use the communication structure directly in the modelling of certain features, such as cooperative behaviour. (See also Gilles (1988b).) We therefore do not demand that a characterization of such a population in some attribute space has to satisfy that it *reduces* the communication structure. It is still simple enough to use it directly in our analysis. However in the case of an uncountable population, describing situations that occur in the analysis of large economies, it is very problematic to use the communication structure directly, without using certain instruments. In our case we introduce a global condition on the characterization such that the communication structure is indeed reduced to a handable size.

In the setting of these kind of "large" populations the problem of full (regular) communication completely reduces to connectedness of certain clusters of types. These clusters, which we will call *macro-types*, emerge naturally from the characterization of the population, satisfying this global condition. Thus we have introduced a natural form of *aggregation* of types within the setting of our method of modelling. It is this kind of aggregation that is the crux of economic analysis. It makes it possible to understand complex economic problems, since it reduces them to a comprehensable level. In our case that level is the level of macro-types. In the case of at most countable populations this level is identical with the "higher" level of (micro-) types. Our results show that on this low level we are well able to study the problem of full communication in large economies with incomplete communication structures.

Another notion of communication is introduced finally in Section 4. We observe that in daily economic life, there exist certain coalitions of economic agents which are handling certain information streams, services and/or commodities. They form the basic particles of the organization structure of the economy. In this paper we introduce a collection of coalitions in the population, which satisfy some *minimal* set of certain social conditions for being able to perform such tasks as described above. Since the members of this collection only satisfy a minimal set of conditions, they are only *potentially* performers of these tasks. We therefore implicitly describe potential organization structures of economies with incomplete communication structures, called *networks*.

The main problem which is (partially) solved in this paper is that this set of minimal social conditions does not lead to an empty collection of potential organization structures on the economy. We show that for regular cases our set of social conditions does not lead to this problem, i.e., there exist coalitions which satisfy these conditions. In a subsequent paper by Gilles, Ruys and Shou Jilin (1989) we prove that also in pathological economies these conditions do not lead to contradictions.

We conclude this paper with a section in which we describe some applications of our method of modelling and some future lines of research.

2 Populations and Typifications

This section deals with the foundations of our model, and the first step in the description of the process of modelling economies with an incomplete communication structure. It is therefore necessary to describe the economic primitive notions of the model, and to begin with a basic analysis of the model. As stated in the introduction it is our purpose to extend the traditional procedure to describe economic agents with the use of certain attributes to the case of incomplete or limited communication between those economic agents.

Our first economic primitive notion is that of a *population*. It consists of a set of economic agents in the economy endowed with a structure of social relationships between the members of the population. Thus a population is a socially structured class of economic agents. We choose to describe a population with a deterministic graph. Hence, we assume that in a static context relations between economic agents have to be given. Moreover every economic agent knows exactly his or her abilities and possibilities to communicate with other agents in the population. In fact we explicitly assume a situation of complete information of the agent with respect to the relational structure of the population. In our opinion it is not proper to describe such a relational structure by means of a stochastic concept. This leads to the following definition of a population:

Definition 2.1 *A pair* $(\mathcal{A}, \mathcal{R})$ *is a* **population** *if*

- $\mathcal{A}$ *is a set of economic agents ;*
- $\mathcal{R} \subset \mathcal{A} \times \mathcal{A}$ *is a symmetric and reflexive relation on the set of agents* $\mathcal{A}$.

It is clear that a population – or *relational structure* – is an undirected graph. In this paper we will assume that this graph can be as well finite as infinite, especially uncountable. In the uncountable case this description forms an alternative for the measure theoretic description of a mass of economic agents. Furthermore we note that in the finite case it does not degenerate to a trivial structure as it is the case with a measure theoretic description. In the graph theoretic description all features of the communication structure remain distinguishable.

We remark that the vertices in the graph describe the economic agents in the population, while the edges of the graph are describing *direct* relationships between those economic agents. In our opinion a direct relation is a very strong form of communication, namely one knows how to reach that agent directly. A weaker form of communication is that one can reach another agent in the population through a chain of other economic agents of which one has a direct relationship with the first of those agents. This leads to the following definition.

Definition 2.2 *Let $(\mathcal{A}, \mathcal{R})$ be a population. Then two agents $\alpha, \beta \in \mathcal{A}$ are* **able to communicate** *if there exists a finite sequence $\gamma_1, \ldots, \gamma_n \in \mathcal{A}$, where n is some integer, such that*

- $\gamma_1 = \alpha$ *;*
- $\gamma_n = \beta$ *;*
- $(\gamma_j, \gamma_{j+1}) \in \mathcal{R}$ *for every* $j = 1, \ldots, n-1$.

The above described principle of (regular) communication is based on the observation that one is only able to send a message to another agent if one is in finite reach of this other agent, i.e., one can reach the other agent within a finite number of steps or "handshakes". In a given population $(\mathcal{A}, \mathcal{R})$ it is obvious that it is known with which agents one is able to communicate directly and in this way to have access to communication with a larger collection of agents in the population. (This is a natural consequence of our full information assumption.)

We note that the notion of direct communication is defined in the context of a population. In the sequel we shall introduce some other concepts describing the same social phenomenon, but defined in the context of derived constructions of a population. These other concepts are basically similar to the above notion of communication in the setting of a population.

In case of *limited* abilities of communication of economic agents, i.e., when the communication structure is incomplete, we explicitly state that those agents have as well individual characteristics as social characteristics. We note that these social characteristics of a certain economic agent are just his or her possibilities of communicating (directly) with other agents in the economy. In the case of a complete communication structure these social characteristics are not *identifiable*. However in the case of an incomplete communication structure they are significant and contribute substantially to the description of agents in the population.

In order to describe these characteristics, as well individual as social, we introduce another primitive concept, namely a collection of *attributes*. By attributes we understand abstract characteristics or abstractions of characteristics. In real life one is endowed with certain quantities of commodities, with certain qualities. If we model such an agent we abstract from a too large a variety of qualities and define an endowment as a vector of quantities of a finite number of commodities. Hence we construct an attribute, which is an abstraction of certain characteristics of the agents in the population. Secondly we only construct those attributes which are relevant and suitable for the problem at hand. So we arrive at a collection of *suitable attributes*.

These suitable attributes therefore consist of individual attributes such as (production) capacities, preferences, and endowments as well as social or socialized attributes such as language, institutional membership, and place of living. Mathematically we represent the collection of all tuples of attributes by some topological space satisfying some (topological) separation property. In traditional general equilibrium theory it is common practice to represent such a collection by a metric or metrizable space. For an exposition of this theory we refer to Grodal (1974) and Hildenbrand (1974).

Definition 2.3 *A topological space (C, τ) is called an* **attribute space** *if it satisfies the T_1-separation property, i.e., for every two points $x, y \in C$, such that $x \neq y$, there exists an open set $V \in \tau$ such that ($x \in V$ and $y \notin V$) or ($y \in V$ and $x \notin V$).*

Note that by this definition we do not exclude Hausdorff or metric spaces to be attribute spaces. Hence, this definition also includes the procedure as practised in traditional general equilibrium theory. There the attribute space (C, τ) is taken to be the metrizable space $\mathcal{P} \times E^{\ell}_{+}$. In this expression ℓ is the number of commodities in the economy, and so the non-negative orthant of the ℓ-dimensional Euclidean space E^{ℓ}_{+} is the *commodity space* of the economy. Furthermore $\mathcal{P}$ is the space of all preference relations on the commodity space E^{ℓ}_{+}. Now the relevant attributes for describing the agents in this economy, and therefore for the analysis of a general equilibrium, are contained in the space $\mathcal{P} \times E^{\ell}_{+}$.

The fundamental problem in empirical *and* theoretical economics is the question whether it is possible to describe the agents in the population $(\mathcal{A}, \mathcal{R})$ with the use of a suitable collection of attributes as described in some attribute space $(\mathcal{C}, \tau)$. In the case of an incomplete communication structure as well the individual as the social characteristics of the agents in the population have to be described by these attributes.

This problem of describing a population with an incomplete communication structure is translated in the following definition of a *characterization*. It expresses the basic principle that the agents in the population can sufficiently be described by the choice of a suitable attribute space. Mostly the description, i.e., the characterization, and the choice of the attribute space depend heavily on each other and on the problem to be analysed.

Definition 2.4 *Let $(\mathcal{A}, \mathcal{R})$ be a population and let $(\mathcal{C}, \tau)$ be an attribute space. A mapping $g : \mathcal{A} \longrightarrow \mathcal{C}$ is a* **characterization** *of $(\mathcal{A}, \mathcal{R})$ in $(\mathcal{C}, \tau)$ if it satisfies the following properties:*

(i) *For every point $a \in g(\mathcal{A})$ there exists an open neighbourhood $U_a \in \tau$ such that*

$$b \in U_a \cap g(\mathcal{A}) \implies (a, b) \in R$$

where $R := \{(x, y) \in \mathcal{C} \times \mathcal{C} \mid \exists (\alpha, \beta) \in \mathcal{R}$ such that $x = g(\alpha)$ and $y = g(\beta)\}$.

(ii) *For every point $a \in g(\mathcal{A})$ and every pair of agents $\alpha, \beta \in g^{-1}(a)$ there exists a finite sequence $\gamma_1, \ldots, \gamma_n \in g^{-1}(a)$, where n is a natural number, such that*

- $\alpha = \gamma_1$;
- $\beta = \gamma_n$;
- $(\gamma_j, \gamma_{j+1}) \in \mathcal{R}$ *for every* $j = 1, \ldots, n-1$.

The pair (A, R) is called a **typification** *of the population $(\mathcal{A}, \mathcal{R})$ in the attribute space $(\mathcal{C}, \tau)$ under the characterization $g : \mathcal{A} \longrightarrow \mathcal{C}$ if $A = g(\mathcal{A}) \subset \mathcal{C}$ and $R = \{(x, y) \in \mathcal{C} \times \mathcal{C} \mid \exists (\alpha, \beta) \in \mathcal{R}$ such that $x = g(\alpha)$ and $y = g(\beta)\}$. If (A, R) is a typification of $(\mathcal{A}, \mathcal{R})$ in $(\mathcal{C}, \tau)$, then the points in $A \subset \mathcal{C}$ are called* **types**.

A *typification* is in fact a described or characterized population in some suitable attribute space, while the definition of the *characterization* itself expresses the principle that we can describe a population by a suitable collection of attributes. The mere existence of such a mapping expresses the requirement that the attribute space

$(\mathcal{C}, \tau)$ is rich enough to describe the communication structure that is considered relevant for the problem in study. The conditions as formulated in Definition 2.4 just are the minimal properties that such a description has to satisfy.

Property 2.4 (i) states the basic principle that social relations can partially be inferred from attributes that describe the individual characteristics of the agents in a population. If two agents are "close" enough in terms of their attributes, then they should be socially related. This basic point is also made by Grodal (1972) in which she defines a core concept which is fully based on this principle applied to coalition formation. The property as given in the definition explicitly is a description of an interdependency *between* social and individual characteristics of agents in the population. The condition that neighbouring types in the typification should also be socially "close" to each other, is therefore an explicit expression of a well established principle in economic modelling.

The second property as defined in 2.4 describes the very nature of a type. It states that agents of the same type can communicate with each other *through* each other. In this case we assume that we choose the collection of attributes such that agents who are described in the same way, i.e., who are indistinguishable in this particular typification, are also a *social group*. This condition seems demanding a lot of the characterization, but it also seems quite natural.

We note that in the standard case of complete communication, these conditions are trivially satisfied. This is mostly also the case for a finite population. (For these finite cases *any* attribute space with enough points will do.) We illustrate the definition above by refering to common practice in general equilibrium theory, where an economy is defined as a mapping $\mathcal{E} : \mathcal{A} \longrightarrow \mathcal{P} \times E^{\ell}_{+}$. All agents are described by their preference, given by a member of $\mathcal{P}$, and their endowments, given by a point in the commodity space E^{ℓ}_{+}. So we are describing the economy with use of a typification, where $\mathcal{E}$ is the characterization of that typification. In such models one is not dealing with the agents themselves, but only with the types, and hence with the typification.

In the sequel we will only consider populations $(\mathcal{A}, \mathcal{R})$ which can be described or characterized in some attribute space $(\mathcal{C}, \tau)$. We remark that a typification of a certain population does not have to be an exhaustive description. Usually the model designer chooses a limited collection of relevant attributes for the problem to be studied. However an appropriate description always satisfies the minimal conditions as stated in Definition 2.4. Next we study some aspects of communication in populations, which can be described in a typification.

Definition 2.5 *Let $(\mathcal{A}, \mathcal{R})$ be a population, which can be characterized in the attribute space $(\mathcal{C}, \tau)$ by the typification (A, R).*
The mapping $F : A \longrightarrow 2^A$, which assigns to every type $a \in A$ the set

$$F(a) \ := \ \{b \in A \mid (a,b) \in R\}$$

is called the **type-relation mapping** *of (A, R).*

The definition above gives a mapping, which for every type $a \in A$, the set $F(a)$ is describing the collection of *related types* of a in the typification (A, R). The next result states some fairly trivial properties of the tools as developed above.

Proposition 2.6 *Let $(\mathcal{A}, \mathcal{R})$ be a population, and let (A, R) be a typification of that population in the attribute space $(\mathcal{C}, \tau)$. Then:*

(a) *$R \subset A \times A$ is a reflexive and symmetric relation on the collection of types A.*

(b) *For every type $a \in A$ it holds that $a \in int(F(a))$, where the interior is taken with respect to the restricted attribute space $(A, \tau|A)$ with $\tau|A := \{E \cap A \mid E \in \tau\}$.*

(c) *For every two types $a, b \in A$ it holds that $a \in F(b)$ if and only if $b \in F(a)$.*

(d) *For every type $a \in A$, the collection $F(a)$ is a closed set of the restricted attribute space $(A, \tau|A)$ if and only if for every two types $a, b \in A$ it holds that if $(a, b) \notin R$, then there exists an open neighbourhood $U_a \in \tau|A$ of a such that for every $c \in U_a$: $(b, c) \notin R$.*

PROOF

(a) **Reflexivity**

Let $a \in A$ and let $\alpha \in g^{-1}(a) \subset \mathcal{A}$. Now it is obvious that $(\alpha, \alpha) \in \mathcal{R}$ and so by definition of R it holds that $(a, a) \in R$.

Symmetry

Let $(a, b) \in R$. Then there exist $\alpha \in g^{-1}(a)$ and $\beta \in g^{-1}(b)$ such that $(\alpha, \beta) \in \mathcal{R}$. Now by symmetry of $\mathcal{R}$ it holds that $(\beta, \alpha) \in \mathcal{R}$ and so by definition $(b, a) \in R$.

(b) This property directly follows from the definition of a typification. For every type $a \in A$ there exists an open neighbourhood in $\tau|A$ which is contained in the set $F(a)$.

(c) This is evident.

(d) **Only if**

Let (A, R) satisfy the property as given in the assertion, so F is a point-closed mapping. Now if for every type $a \in A$ it holds that $F(a) = A$, then it is obvious that the assertion is satisfied. Hence assume that for at least one type $a \in A$ it holds that $F(a) \neq A$. Then we can take $b \in A$ such that $(a, b) \notin R$. (Symmetry)

Then by definition $a \notin F(b)$, and hence by closedness of $F(b)$ there exists an open neighbourhood $U_a \in \tau$ of a, such that $U_a \cap F(b) = \emptyset$.

If

Let (A, R) satisfy the property as stated in the assertion, and let $a \in A$. Now suppose we can take a type $b \notin F(a)$, then by definition $(a, b) \notin R$. So there exists an open neighbourhood $U_b \in \tau$ of b such that $U_b \cap F(a) = \emptyset$. This implies that $A \setminus F(a) \equiv \bigcup\{ U_b \cap F(a) \mid b \notin F(a)\}$ is an open set, and so $F(a)$ has to be a closed set in the restricted attribute space $(A, \tau|A)$.

Q.E.D.

With the tools as developed in the previous proposition and the previous definitions we now can give an exact description of "full" communication in the sense of Definition 2.2. With this notion of full communication we do *not* describe a situation of a complete communication structure in graph theoretic sense, but a situation that all agents in the population are able to communicate regularly with each other in the sense of Definition 2.2.

The next definition gives an equivalent formulation of this notion of complete communication in the setting of a typification. The next theorem confirms that connectedness of the typification is indeed equivalent to complete communication in the corresponding population. (We refer to Definition 5 of the appendix for the technical definition of an irreducible chain in a topological space, and note that we denote by $\mathcal{N}$ the collection of all non-negative integers.)

Definition 2.7 *Let $(\mathcal{A}, \mathcal{R})$ be a population and let (A, R) be a typification of that population in the attribute space $(\mathcal{C}, \tau)$. Then the typification (A, R) is* **type-connected** *if for every two types $a, b \in A$ there exists a finite sequence of types $c_1, \ldots, c_n \in A$, with $n \in \mathcal{N}$, such that the sets $F(c_1), \ldots, F(c_n) \subset A$ form an irreducible chain between $F(a)$ and $F(b)$ in the restricted attribute space $(A, \tau|A)$.*

The notion of type-connectedness describes a situation in which one can reach from one agents related types to another agents related types in a finite number of handshakes. Moreover the properties of an irreducible chain imply that one does not have to make detours in reaching other types.

The first main result of this paper states that it can be proved that the derived notion of type-connectedness is equivalent to the basic notion of regular communication in the setting of a population.

Theorem 2.8 *Let $(\mathcal{A}, \mathcal{R})$ be a population and let (A, R) be a typification of that population in the attribute space $(\mathcal{C}, \tau)$. Then the following assertions are equivalent:*

1. *(A, R) is type-connected ;*
2. *For every two types $a, b \in A$ there exists a finite sequence of types, denoted by $c_1, \ldots, c_n \in A$, such that*
   - *$c_1 = a$ and $c_n = b$;*
   - *for every $j = 1, \ldots, n-1$ it holds that $c_{j+1} \in F(c_j)$;*
3. *Every two agents $\alpha, \beta \in \mathcal{A}$ are able to communicate within the population $(\mathcal{A}, \mathcal{R})$.*

PROOF

1. implies 2.

Take two types $a, b \in A$. Now by the definition of type-connectedness there exists a finite sequence $c_1, \ldots, c_n \in A$ of types such that $c_1 = a$, $c_n = b$, and $F(c_1), \ldots, F(c_n)$ form an irreducible chain between $F(a)$ and $F(b)$ in the restricted attribute space $(A, \tau|A)$.

Hence we can construct a finite sequence $a_1, \ldots a_{2n-1} \in A$ in the following way:

- For $j \in \{1, \ldots, n\}$: $a_{2j-1} = c_j$;

- For $j \in \{1, \ldots, n-1\}$ take any $a_{2j} \in F(c_j) \cap F(c_{j+1}) \neq \emptyset$.

From this construction it easily follows that $a_1 = c_1 = a$ and that $a_{2n-1} = c_n = b$. The second condition of 2, as stated in the theorem, follows from the definition of this sequence and also by noting that

1. for $j = 1, \ldots, n$ it holds that $a_{2j-1} = c_j \in F(a_{2j})$, since $a_{2j} \in F(c_j)$ and F is symmetric in the sense of the proposition above.

2. for $j = 1, \ldots, n-1$ it follows that $a_{2j} \in F(c_{j+1}) = F(a_{2j+1})$.

2. implies 3.
Take two agents $\alpha, \beta \in \mathcal{A}$. Now we can distinguish two cases.

Case I: $g(\alpha) = g(\beta) = a \in A$.
Then by Definition 2.4 α and β are able to commmunicate within $g^{-1}(a) \subset \mathcal{A}$.

Case II: $g(\alpha) = a \neq b = g(\beta)$.
Now by application of assertion 2 it follows that there exists a sequence $c_1, \ldots, c_n \in A$ such that $c_1 = a$, $c_n = b$, and $c_j \in F(c_{j+1})$ for every $j \in \{1, \ldots, n-1\}$.

Hence we can construct the following finite sequence between the two agents α and β, which satisfies all necessary properties of communication in the sense of Definition 2.2.

Choose $\gamma_j^1, \gamma_j^2 \in \mathcal{A}$ $(j = 1, \ldots, n)$ such that:

- $(\gamma_j^2, \gamma_{j+1}^1) \in \mathcal{R}$ for every $j \in \{1, \ldots, n-1\}$;
- $g(\gamma_j^1) = g(\gamma_j^2) = c_j$ for every $j \in \{1, \ldots, n-1\}$;
- $\gamma_1^1 \equiv \alpha$ and $\gamma_n^2 \equiv \beta$.

Such a sequence can indeed be constructed according to the properties of $c_1, \ldots, c_n$.
Since by construction for every $j \in \{1, \ldots, n\}$ it follows that $\gamma_j^1, \gamma_j^2 \in g^{-1}(c_j)$, and hence by application of Definition 2.4 there exists a finite sequence of agents, denoted by $\delta_1^j, \ldots, \delta_{k_j}^j \in g^{-1}(c_j)$, such that the following properties are satisfied:

- $\delta_1^j \equiv \gamma_j^1$;
- $\delta_{k_j}^j \equiv \gamma_j^2$;
- $(\delta_i^j, \delta_{i+1}^j) \in \mathcal{R}$ for every $i \in \{1, \ldots, k_j - 1\}$.

Now the union of all sequences $\delta_1^j, \ldots, \delta_{k_j}^j$, where j ranges from 1 to n, is also a finite sequence in $(\mathcal{A}, \mathcal{R})$ which satisfies all desired properties.

3. implies 1.
Let $a, b \in A$ be two types. Moreover let $\alpha \in g^{-1}(a)$ and $\beta \in g^{-1}(b)$ be two agents of those types.
Now by assertion 3 there exists a finite sequence $\gamma_1, \ldots, \gamma_n \in \mathcal{A}$ such that $\gamma_1 = \alpha$, $\gamma_n = \beta$, and for every $j = 1, \ldots, n-1$ it holds that $(\gamma_j, \gamma_{j+1}) \in \mathcal{R}$.
Next let $\{c_1, \ldots, c_k\} := g(\{\gamma_1, \ldots, \gamma_n\}) \subset A$ be the image of this sequence in the typification (A, R). (Note that $k \leq n$.) We can order this finite sequence in A such that $c_1 = a$, $c_k = b$, and for every $j = 1, \ldots, k-1$ it holds that $(c_j, c_{j+1}) \in R$.
It can easily be established that $F(c_1), \ldots, F(c_k)$ is a chain between $F(a)$ and $F(b)$. (Note that this chain does not necessarily have to be an irreducible chain.) Since this chain is finite, we evidently can take a finite subcollection $\{d_1, \ldots, d_m\} \subset \{c_1, \ldots, c_k\}$, with $m \leq k$, which generates an irreducible chain between $F(a)$ and $F(b)$. This proves the type-connectedness of the typification (A, R).

Q.E.D.

3 Typologies

As one can see, a typification of a population $(\mathcal{A}, \mathcal{R})$ in a certain attribute space $(\mathcal{C}, \tau)$ has the specific property that the topological structure of the attribute space describes the relations between the economic agents *locally*. The assumptions as made in Definition 2.4 refer to this local description of characteristics, as well individual as social, by the characterization in the attribute space $(\mathcal{C}, \tau)$.

In the case of an **at most countable** population these conditions are mostly satisfied, and therefore trivial. It is our assumption that in those cases one can easily analyse the graph theoretic structure of the population, and therefore *all* social characteristics of the economic agents in the population. In those cases it is not necessarily clarifying that certain attributes "explain" social relations between individual agents.

However in the case of an **uncountable** population the properties of a typification are no longer trivially satisfied. Then these conditions on the characterization state that *locally* we can infer the social relations between agents in the population from a suitable set of attributes. The technical reason for this important property is that one cannot understand an uncountable graph anymore, unless one uses additional techniques, such as the technique of description, as developed in the previous section, or a randomization of the graph as is done in Kirman, Oddou and Weber (1986). The uncountable case seems however very important for the theory of large economies, coalition formation in such economies, and perfect competition as is sketched in Gilles and Ruys (1988) and Gilles (1988a).

The next example shows that we are however able to construct typifications of populations in an attribute space with a non-trivial topology, in which these properties of the characterization become meaningless. In those cases the intuitive economic interpretation of those "local" properties is no longer valid, and hence we get an economically trivial characterization of the population. (By triviality we mean in this case that the characterization does not explain anything on the social characteristics of the agents in the population by the suitably chosen collection of attributes. It is obvious that such a characterization does not satisfy the economic foundations of our method of modelling.)

Example 3.1

In this example we construct the following class of populations: Take $\mathcal{A} := I \equiv [0,1]$, and let $\mathcal{R} \subset I \times I$ be any reflexive and symmetric relation on $\mathcal{A} = I$. It is obvious that $(\mathcal{A}, \mathcal{R})$ is a population. Next take the attribute space to be the set $\mathcal{C} := I^I$, endowed with the product topology τ emerging from the Euclidean topology on the unit interval I. So $(\mathcal{C}, \tau)$ is a compact Hausdorff space and hence an attribute space.

Next we construct a characterization of $(\mathcal{A}, \mathcal{R})$ in $(\mathcal{C}, \tau)$ by defining the mapping $g : \mathcal{A} \longrightarrow \mathcal{C}$ which is given by $g(\alpha) := f_\alpha$, for $\alpha \in \mathcal{A} = I$, where $f_\alpha : I \longrightarrow I$ is defined as $f_\alpha(\alpha) = 1$, and $f_\alpha(\beta) = 0$, $\beta \neq \alpha$. Obviously this mapping is a characterization of $(\mathcal{A}, \mathcal{R})$ in $(\mathcal{C}, \tau)$.

However it is also obvious that the typification, which results from this characterization, is trivial in the sense that it does not satisfy the intuitive economic foundation of our modelling that the chosen attributes describe as well the individual as the social characteristics of the agents in the population. In this case the social characteristics are not described or reduced by the description, i.e., the social structure of the typification is evenly complicated as the original population. (We have not gained by describing the agents.) This is illustrated by noting that for every type $a \in I$ there exists a neighbourhood $U_a \in \tau$ such that $(U_a \setminus \{a\}) \cap g(I) = \emptyset$.

From the example above it is clear that we want to restrict ourselves to a subclass of *non-trivial* typifications instead of the large class of typifications itself. Therefore we have to introduce an additional condition on the *global* structure of a typification, which makes those properties of the local structure of the typification economically non-trivial.

Before we state this global property we make some conventions for shortening the statement of the next definitions and results. In the sequel we assume that we have a fixed population $(\mathcal{A}, \mathcal{R})$ and a fixed attribute space $(\mathcal{C}, \tau)$. Furthermore we assume that we can characterize this population in this attribute space by some characterization $g : \mathcal{A} \longrightarrow \mathcal{C}$. Mostly we will denote a typification of $(\mathcal{A}, \mathcal{R})$ in $(\mathcal{C}, \tau)$ by (A, R).

We are now able to formulate the additional condition on the global structure of a typification. A typification, which satisfies this additional property, is called a *typology*.

Definition 3.2 *A typification (A, R) of the population $(\mathcal{A}, \mathcal{R})$ in the attribute space $(\mathcal{C}, \tau)$ is a* **typology** *of $(\mathcal{A}, \mathcal{R})$ in $(\mathcal{C}, \tau)$ if there exists an at most countable sequence $(C_n)_{n \in \mathcal{N}}$ of topologically connected subsets of the restricted attribute space $(A, \tau|A)$ such that $A = \bigcup_{n=1}^{\infty} C_n$.*

Note that this additional condition excludes typifications such as constructed in the example. From mathematics we learn that as a consequence of this additional property there exists a *unique*, at most countable sequence of components of the restricted attribute space $(A, \tau|A)$. (A component of a topological space is defined as a maximal connected subset of that topological space. See also the appendix.) We remark that these components of A are pairwise disjoint, and we will denote them by $(A_n)_{n \in \mathcal{N}}$. This unique sequence of components will be called the *subdivision* of (A, R).

We now know that if we assume that there exists a typology of a population in some suitable attribute space, then there is always a non-trivial description of "closeness" of agents by refering to their types. So neighbouring types in the typology are now always close in the social structure of the population itself. This also means that the description or characterization in a typology always is economically meaningful. We remark that the choice of the attribute space, or the collection of relevant attributes, determines the typology.

But evenly important is that it is a natural consequence that we have described the population by an at most countable number of collections of topologically neighbouring types. In economic terms we will call these collections *macro-types*. This unique subdivision of the collection of (micro-) types into macro-types is a natural form of *aggregation*. By describing agents in an incomplete communication structure with the use of a suitable collection of attributes we "naturally" get a subdivision into macro-types, describing some kind of discrete characteristic.

The assumption that *this* is possible, is one of the fundamental axioms of social sciences, especially economics. Although this axiom is never stated explicitly, it is used very frequently. One of the most trivial examples is the definition of total demand and supply on a market as the sum – or, more general, the integral – of individual demand and supply. If we have a theory, which is giving a description of a (micro-) type, then with the use of a typology we get a very informative construction. This can be compared with the parametrization of a function: only in case of a finite parametrization it is interesting to use it as a description. In this case in fact we get a class-structure in the population. (Formally: Let $(A_n)_{n\in\mathcal{N}}$ be the unique subdivision of the typology (A, R) of $(\mathcal{A}, \mathcal{R})$ in $(\mathcal{C}, \tau)$, then for every $n \in \mathcal{N}$, the subset $g^{-1}(A_n) \subset \mathcal{A}$ is a *class* of agents of the same macro-type.)

For the reasons mentioned above, we call the requirement that a population can be characterized in a typology, i.e., representing agents into types, which can be exhaustively be aggregated to an at most countable number of macro-types, an axiom of descriptive modelling.

Axiom of Descriptive Modelling

For every population $(\mathcal{A}, \mathcal{R})$ there exists a typology (A, R) in a *suitable* attribute space $(\mathcal{C}, \tau)$.

The first implication of this axiom is that one of the main tasks of descriptive economics is to find this "suitable" attribute space and therefore a suitable characterization for some empirical problem. It is the art of the designer of economic models to choose the attributes suitably. In this paper we will not address this problem.

The second implication is that our form of regular communication, as defined in Definition 2.2, can be guaranteed *within* the components of the typology. This is illustrated by the following results on equivalent statements of communication in typologies. To state these results properly we assume that we have a fixed population $(\mathcal{A}, \mathcal{R})$ which, by the axiom of descriptive modelling, can be characterized in some suitable attribute space $(\mathcal{C}, \tau)$. Hence, if we call a pair (A, R) "a typology", we mean to say that (A, R) is a typology of $(\mathcal{A}, \mathcal{R})$ in this attribute space $(\mathcal{C}, \tau)$.

The next definition describes a way of reducing a typology to the bare global structure by describing relations *between* macro-types only.

Definition 3.3 *Let (A, R) be a typology, and let $(A_n)_{n \in \mathcal{N}}$ be its unique subdivision into components. A pair $(\bar{A}, \bar{R})$, with $\bar{A} \subset \mathcal{C}$ and $\bar{R} \subset \bar{A} \times \bar{A}$, is a* **condensation** *of (A, R) if there exists a surjective mapping $C : A \longrightarrow \bar{A}$ which satisfies the following properties:*

1. *for any $n \in \mathcal{N}$ and any pair of types $a, b \in A_n$ it holds that $C(a) = C(b)$;*
2. *for any two integers $n, m \in \mathcal{N}$, $n \neq m$, and any two types $a \in A_n$ and $b \in A_m$ it holds that $C(a) \neq C(b)$;*
3. *$(\bar{a}, \bar{b}) \in \bar{R}$ if and only if there exist integers $n, m \in \mathcal{N}$, $n \neq m$, and two types $a \in A_n$ and $b \in A_m$ such that*
   - *$C(a) = \bar{a}$;*
   - *$C(b) = \bar{b}$;*
   - *$(a, b) \in R$.*

The previous definition gives a construction of an at most countable graph, describing the bare macro-type structure of a typology. The condensation of a typology just describes all *external* relations between the components or macro-types in the typology. We now define another form of connectedness, in terms of relations between macro-types only, and thus specifically in the setting of a typology instead of a typification, as is the case in the previous section. Again we note that these constructions become trivial in case of an at most countable population.

Definition 3.4 *A typology (A, R) is* **component-connected** *if there exists a condensation $(\bar{A}, \bar{R})$ of (A, R) which is a finitely connected graph, i.e., for every two points $\bar{a}, \bar{b} \in \bar{A}$ there exists a finite sequence $\bar{c}_1, \ldots, \bar{c}_n \in \bar{A}$ $(n \in \mathcal{N})$ such that $\bar{c}_1 = \bar{a}$, $\bar{c}_n = \bar{b}$, and for every $j \in \{1, \ldots, n-1\}$ it holds that $(\bar{c}_j, \bar{c}_{j+1}) \in \bar{R}$.*

We remark that component-connectedness is quite a natural condition on typologies. It just prescribes that there exist communication lines between all macrotypes. Hence in the class-structure of the population there exists communication between classes, and so no class is socially isolated.

The next result links the notion of type-connectedness of a typification with the above concept of component-connectedness. Since any typology is a typification, as defined in Definition 3.2, a typology can also be type-connected. In combination with the main result of the previous section we therefore have also made a link with the notion of regular communication in populations, as defined in Definition 2.2.

Theorem 3.5 *Let (A, R) be a typology. Then (A, R) is type-connected if and only if (A, R) is component-connected.*

PROOF

Let $(A_n)_{n \in \mathcal{N}}$ denote the unique subdivision of (A, R). Moreover let the pair $(\bar{A}, \bar{R})$ be a condensation of (A, R), and let C be the mapping as defined in Definition 3.3. Now decribe the set $\bar{A}$ as follows: $\bar{A} = \{\bar{a}_n \in A \mid \bar{a}_n = C(A_n) \text{ for } n \in \mathcal{N}\}$.

If

Suppose that (A, R) is component-connected. For the proof that all types are connected in the sense of type-connectedness we need the following claim:

CLAIM

For any integer $n \in \mathcal{N}$, every pair of types $a, b \in A_n$ is connnected, i.e., there exists a finite sequence $(c_1, \ldots, c_k)$ in A_n such that $c_1 = a$, $c_k = b$, and $(c_j, c_{j+1}) \in R$ for every $j = 1, \ldots, k-1$, and moreover for every $|j - h| > 1$ it holds that $(c_j, c_h) \notin R$.

Proof of the claim

Apply lemma (10.3.8) of Császàr (1978) – see also the appendix – on the sets $F(a)$, $F(b)$ as subsets of the connected subspace A_n of the restricted

attribute space $(A, \tau|A)$ by taking as open covering of A_n, the collection $\mathcal{B} := \{\text{int}(F(a)) \mid a \in A_n\}$. This lemma asserts the existence of an irreducible chain of elements in $\mathcal{B}$ between $F(a)$ and $F(b)$.
Now let $\{F(d_j) \mid j = 1, \ldots, m\}$ be the collection which can derived from this irreducible chain. Now by reordering and extending this collection we can easily construct a sequence $(c_1, \ldots, c_k)$ which is just the sequence as asserted in the claim. The completes the proof of the claim.

To complete the proof of the if-part of the theorem we take two types $a, b \in A$ and consider two cases:

Case I: There exists an integer $n \in \mathcal{N}$ such that as well $a \in A_n$ as $b \in A_n$. Then direct application of the claim asserts that a and b are connected in the sense of type-connectedness.

Case II: There exist two distinct integers $k, m \in \mathcal{N}$ such that $a \in A_k$ and $b \in A_m$. By the definition of component-connectedness of (A, R) there exists a finite sequence in $\bar{A}$, say $(b_1, \ldots, b_n)$, such that $b_1 = \bar{a}_k$, $b_n = \bar{a}_m$, $(b_j, b_{j+1}) \in \bar{R}$ for every $j \in \{1, \ldots, n-1\}$, and $(b_j, b_h) \notin \bar{R}$ if $|j - h| > 1$. Now we construct the following sequence in A:

- $c_{11} = a$;
- $c_{j2} \in C^{-1}(b_j)$ for $j = 1, \ldots, n-1$, and
  $c_{j1} \in C^{-1}(b_j)$ for $j = 2, \ldots, n$ such that
  $(c_{j2}, c_{j+1,1}) \in R$ for $j = 1, \ldots, n-1$;
  (The existence of such pairs is guaranteed by Definition 3.3 and component-connectedness.)
- $c_{n2} = b$.

Now for every $j \in \{1, \ldots, n\}$ the claim asserts the existence of a finite sequence of intermediate agents between $c_{j1}, c_{j2} \in C^{-1}(b_j) \equiv A_j$, and thus by the fact that $(c_{j2}, c_{j+1,1}) \in R$ for $j = 1, \ldots, n-1$, the union of all these finite sequences is again a finite sequence, which is irreducible. Hence a and b are connected in the sense of type-connectedness.

Only if

Suppose that (A, R) is type-connected. Take two points $\bar{a}, \bar{b} \in \bar{A}$. We will show that $\bar{a}$ and $\bar{b}$ are finitely connected within $(\bar{A}, \bar{R})$.

Take types $a \in C^{-1}(\bar{a})$ and $b \in C^{-1}(\bar{b})$, then by assumption a and b are connected in the sense of type-connectedness. Hence there exists a finite sequence $c_1, \ldots, c_n \in A$ such that $c_1 = a$, $c_n = b$, and for every $j \in \{1, \ldots, n-1\}$ it holds that $(c_j, c_{j+1}) \in R$.

Now define $\{\bar{c}_1, \ldots, \bar{c}_k\} := C(\{c_1, \ldots, c_n\}) \subset \bar{A}$, with obviously $k \leq n$. Then we can order this set such that $\bar{c}_1 = \bar{a}$, $\bar{c}_k = \bar{b}$, and for every $j \in \{1, \ldots, k-1\}$ it holds that $(\bar{c}_j, \bar{c}_{j+1}) \in \bar{R}$.
Hence we have proved that $(\bar{A}, \bar{R})$ is finitely connected, and so (A, R) is component-connected.

Q.E.D.

The next corollary links the theorem above with the main result of the previous section. The proof is evident.

Corollary 3.6 *Let (A, R) be a typology. Then the following statements are equivalent:*

(i) *Every two agents $\alpha, \beta \in \mathcal{A}$ are able to communicate within the population $(\mathcal{A}, \mathcal{R})$.*

(ii) *(A, R) is type-connected.*

(iii) *(A, R) is component-connected.*

The corollary above not only summarizes the results of our analysis of communication within a population satisfying the axiom of descriptive modelling, but also illustrates the strength of this axiom with respect to reduction of properties to the level of the condensation of a typology. Here communication is reduced to finitely connectedness of the condensation as a graph.

4 Networks

In the previous sections we have developed a theory in which communication in a typified population is analyzed. This analysis learns that in case of the axiom of descriptive modelling we can reduce the notion of regular communication, as introduced in Definition 2.2, to the notion of links between macro-types in the setting of the typology, called component-connectedness. In this section we introduce another notion concerning communication, namely an organization structure of the population, which can handle *all* communication in the population.

Our point of departure is the observation that in normal cases certain coalitions in a population take care of certain information streams, services, or commodities. For instance the class of physicians takes care of "health" of agents in the population, while labour unions take care of conditions in working situations and wages. All these kind of coalitions, with specific properties with respect to communication and reachability, together form an organization structure of the total population, and therefore of the whole economy. In fact the organization structure as developed by these specific coalitions concerns *all* agents in the population. Hence by describing these kind of coalitions, we are in fact also describing an organization structure of the *total* economy.

In this section we introduce a collection of coalitions in the population, which are *potential* candidates to be such special coalitions taking care of certain services, or commodities. In fact we state a number of minimal conditions which such a coalition has to satisfy, and so we construct a class of coalitions satisfying these minimal conditions. A coalition satisfying these minimal conditions is called a *network* within the population. We note that these minimal conditions only state something on the communicative aspects of these networks. Hence, networks are only *potential candidates* for an actual organization structure.

To complete the description of those special coalitions we have to describe a subclass of the collection of networks in the population, which satisfy additional properties. These additional properties should concern the abilities or capacities of the members in these networks, and therefore be based on the typology of the population in a suitable attribute space. (This problem will not be addressed to in this paper.)

Our starting point is the axiom of descriptive modelling. So in the statements of the formal definition of networks we depart from a given population $(\mathcal{A}, \mathcal{R})$ and a given suitable attribute space $(\mathcal{C}, \tau)$. Now we denote by (A, R) a typology of $(\mathcal{A}, \mathcal{R})$ in $(\mathcal{C}, \tau)$. We are now able to give a formal description of the notion of network as described above.

Definition 4.1 *Let (A, R) be a typology.*
A set of types $N \subset A$ is a **network** *in (A, R) if it satisfies the following properties:*

Reachability

$\{F(a) \mid a \in N\}$ is a covering of A ;

Connectivity

$(N, R|N)$ is type-connected as a subgraph of (A, R) in the sense of Definition 2.7, where $R|N := R \cap (N \times N)$;

Minimality

N is a minimal collection in the sense that for every member $a \in N$, the subcollection $N \setminus \{a\}$ does not satisfy as well the reachability property as the connectivity property.

The collection of all networks in the typology (A, R) will be denoted by $\Psi(A, R)$, or simply by Ψ.

We note that in the definition above we describe a collection of types which satisfies some properties with respect to communication only. Firstly, every (other) type can reach a type in the network. Secondly, the network is internally completely communicative, i.e., every member can reach any other member within a finite number of steps via network members only. Finally, a network satisfies a rudimentary efficiency condition with respect that there is disposed of superfluous "members" who are inessential for maintaining the two important properties of reachability and connectivity. Hence all members are essential for maintaining these properties of the network as a whole.

Again we remark that the notion of network expresses a potential or latent collection of types, which have the minimal *social* properties for handling exclusively a certain information stream, service, or commodity. Hence it expresses a coalition with the social position which fits for the performance of such a task.

Secondly, the definition above describes a concept which can be used to describe the organization structure of an economy. A certain suitable collection of networks can form an organization structure, and may have great influence on the behaviour of agents in this economy. In Gilles (1988b) it is described how to construct such potential organization structures from the networks in the population. Moreover it is explained how one can derive from these structures an index expressing the *potential* power of the agents in the population.

In this paper we will not address the question how to construct other properties to arrive at a subcollection of networks, which will actually handle certain services or commodities. Before we can solve that problem we have to answer the question whether there exist networks. Here we will only solve this question for "normal" cases. The next theorem states under which conditions $\Psi \neq \emptyset$. For extensions of this existence result we refer to Gilles, Ruys and Shou Jilin (1989), where also some other network-like concepts are introduced.

Theorem 4.2 *Let (A, R) be a typology.*

(a) *If there exists a network in (A, R), then (A, R) is component-connected.*

(b) *If the restricted attribute space $(A, \tau|A)$ is compact, then there exists a finite network in (A, R) if and only if (A, R) is component-connected.*

PROOF

Proof of (a)

Let $(\bar{A}, \bar{R})$ be the condensation of the typology (A, R) and let C be the mapping belonging to it. (See also Definition 3.3.) Now suppose that $\Psi \neq \emptyset$. We will prove that $(\bar{A}, \bar{R})$ is a finitely connected graph.

Define $\bar{N} := C(N) \subset \bar{A}$. Obviously $\bar{N}$ is at most countable and finitely connected within $(\bar{A}, \bar{R})$, i.e., for every two points $a, b \in \bar{N}$ there exists a finite sequence in $\bar{N}$, say $(c_1, \ldots, c_n)$, such that $c_1 = a$, $c_n = b$, and for every $j = 1, \ldots, n-1$ it holds that $(c_j, c_{j+1}) \in \bar{R}$. (Connectivity of the network.)

Take $a, b \in \bar{A}$, then by definition of a network there exist $c, d \in \bar{N}$ such that $(a, c) \in \bar{R}$ and $(b, d) \in \bar{R}$. (Reachability) Moreover by the statement above, there exists a finite sequence between c and d within $\bar{N}$, say $(s_2, \ldots, s_{n-1})$ with $s_2 = c$ and $s_{n-1} = d$. Now take $s_1 := a$ and $s_n := b$, then by the property of connectedness of the sequence between c and d, it follows that for every $j \in \{1, \ldots, n-1\}$: $(s_j, s_{j+1}) \in \bar{R}$. So we have proved that $(\bar{A}, \bar{R})$ is a finitely connected graph.

Proof of (b)[1]

Suppose that the restricted attribute space $(A, \tau|A)$ is compact.

Since the "only if"–part of the proof is evident by application of assertion (a) to this specific situation, we have only to concern ourselves with the proof of the "if"–part.

Suppose that (A, R) is component-connected, or, equivalently, type-connected. Since $(A, \tau|A)$ is compact, we know that the open covering, $\{U_a \in \tau|A \mid a \in A\}$, where U_a is chosen as an open neighbourhood of $a \in A$ such that $(a, b) \in R$ for all $b \in U_a$, has a finite subcovering. (For the existence of such neighbourhoods we refer to Definition 2.4 (ii).) Let denote the finite subset of types generating this subcovering by W.

Next we define

$$Q := \{N \subset A \mid N \text{ is finite and satisfies reachability and connectivity }\}$$

First we show that $Q \neq \emptyset$.
It is clear that $W \subset A$ is finite and satisfies reachability. Next define the mapping $\Delta : W \times W \longrightarrow 2^A$, where $\Delta(a, b)$ is the finite set $\{c_1, \ldots, c_n\} \subset A$, which can be ordered such that $c_1 = a$, $c_n = b$, and for every $j \in \{1, \ldots, n-1\}$ it holds that $(c_j, c_{j+1}) \in R$. (The existence of such subsets is guaranteed by type-connectedness of (A, R).)
Let $\Phi := \bigcup_{\pi \in W \times W} \Delta(\pi)$. Now by the fact that $W \subset \Phi \subset A$, and hence $F(\Phi) \supset F(W) = A$, it is clear that Φ has to satisfy the reachability condition. Furthermore by construction of Φ, it also has to satisfy the connectivity assumption. Moreover we know that Φ is finite, and thus $\Phi \in Q \neq \emptyset$.

Take any totally ordered subcollection $\mathcal{B}$ of Q. (The ordering of $\mathcal{B}$ is with respect to inclusion.) It is clear that $\cap\mathcal{B}$ is a lower bound of the collection $\mathcal{B}$ with respect to inclusion, and that $\cap\mathcal{B} \neq \emptyset$. From the finiteness assumption on Q it is also clear that $\cap\mathcal{B} \in Q$.
Now by application of Zorn's lemma there exists a minimal element in the collection Q, say N^*. Obviously this minimal set N^* is finite. Moreover it satisfies reachability and connectivity, and so it has to be a network in (A, R).

Q.E.D.

[1]We thank Shou Jilin for his helpful comments for the construction of this part of the proof.

5 Concluding Remarks

In this paper we have developed a uniform and general method concerning the description of economic agents in an arbitrary communication structure. Our basic assumptions were threefold. Firstly, we assumed that in a full information and static context a deterministic graph is the best mathematical tool to describe such (incomplete) communication structures. Secondly, we explicitly stated that we followed the traditional method in descriptive economics by characterizing agents with respect to a suitable collection of attributes. Thirdly, we assumed that social relationships, and therefore communication in the relational structure of a population, are interdependent with the individual characteristics of the agent. (See also Grodal (1972) defending the same principle.)

Basing our modelling on these three principles or axioms, we developed the concept of a *typification*. Here the conditions on this construction as described in Definition 2.4 are explicitly describing this third axiom on interdependency of characteristics and relations between agents in the population. We remark that all conditions as formulated in the definition of a typification are "local".

With an example we however showed that a typification does not have to express these economic principles, as described above, properly. We therefore introduced an additional "global" property to secure this defect. Hence, we arrive at the subcollection of *typologies*. In this context we stated the second principle as mentioned above, namely that any population can be described by a typology, provided one takes the suitable collection of attributes. This principle and the belief that one can find such a suitable collection of attributes, is the crux of economics, as can be shown by refering to the existing literature.

In this paper we only gave the basic principles of modelling agents in an arbitrary communication structure, and analyzed the basic properties of communication. We note that this analysis is only non-trivial in case of a large population, e.g., the uncountable case. This case is however of great interest in the theory of perfect competition, as is shown by Kirman et al. (1986).

Secondly, we introduced the concept of *network* in the setting of our model, describing a coalition with potential power in the organization of information streams, the organization of agents in the economy, and the handling of certain services and/or commodities. We showed that in normal cases, i.e., in cases that the population is at most a continuum, the collection of networks is always non-empty. With the notion of network we introduced an efficient tool in the (future) analysis of behaviour in the setting of incomplete communication structures. The explicit potential power of a network can be used in the description of coalition formation, social or relational constraints in non-cooperative behaviour, and naturally in cooperative behaviour. Moreover it is possible to think of mixtures of both kinds of behaviour, i.e., non-cooperative behaviour with threats of cooperative blocking.

Some answers to the main question after more general results on the existence of networks is given in Gilles, Ruys and Shou Jilin (1989). There it is shown that in nearly all situations there exist networks, while in *every* situation, even the pathological ones, there exist network-like coalitions. A fundamental consequence of these results is that we always can use networks freely in describing behaviour of agents in situations with limited communication.

An application of the method as introduced in this paper on coalition formation is given in Gilles and Ruys (1988). There a way is sketched how to arrive at a model of a large economy with a coalitional structure from a situation with an incomplete communication structure. In such a model with a coalitional structure the limitations on communication are expressed in constraints in coalition formation. This can be made explicit by using a model of coalition formation based on a typology of a population as developed in this paper. An example is given where networks are explicitly used as generators of these allowable, or truely feasible coalitions.

Networks and systems of networks are used in Gilles (1988b) to describe possible organization structures in a *finite* economy with limitations regarding communication between agents. Based on these potential structures a power index is derived, describing the power of the position of the agent in the communication structure of the economy.

Finally we remark that all results and models as mentioned above indicate the possibilities of developing equilibrium concepts, based on mixtures of cooperative and non-cooperative elements in the behaviour of agents as is needed e.g. in the theory of industrial organization. It is quite natural that incomplete communication has consequences for as well non-cooperative as cooperative behaviour. Future research will certainly focus on the consequences of cooperative elements in non-cooperative equilibrium concepts. We think that methods such as described in this paper will contribute to the foundations of these kind of models.

References

Allen, B. (1982), "Some Stochastic Processes of Independent Demand and Technological Diffusion of an Innovation Exhibiting Externalities among Adopters", *International Economic Review*, 23, 595-608.

Aumann, R.J. (1964), "Markets with a Continuum of Traders", *Econometrica*, 32, 39-50.

Aumann, R.J. and J.H. Drèze (1974), "Cooperative Games with Coalition Structures", *International Journal of Game Theory*, 3, 217-237.

Császàr, A. (1978), *General Topology*, Adam Hilger Ltd, Bristol.

Debreu, G. (1959), *Theory of Value*, Wiley, New York.

Gilles, R.P. (1987), *Economies with Coalitional Structures and Core-like Equilibrium Concepts*, Research Memorandum 256, Tilburg University, Tilburg.

Gilles, R.P. (1988a), *On Perfect Competition in an Economy with a Coalitional Structure*, Research Memorandum 298, Tilburg University, Tilburg.

Gilles, R.P. (1988b), *A Discussion Note on Power Indices Based on Hierarchical Network Systems in Finite Economies*, Reeks Ter Discussie 88.07, Tilburg University, Tilburg.

Gilles, R.P., and P.H.M. Ruys (1988), *Relational Constraints in Coalition Formation*, Research Memorandum 309, Tilburg University, Tilburg.

Gilles, R.P., P.H.M. Ruys and Shou Jilin (1989), *On the Existence of Networks in Relational Models*, Forthcoming Research Memorandum, Tilburg University, Tilburg.

Greenberg, J., and S. Weber (1983), "A Core Equivalence Theorem with an Arbitrary Communication Structure", *Journal of Mathematical Economics*, 11, 43-55.

Grodal, B. (1972), "A Second Remark on the Core of an Atomless Economy", *Econometrica*, 40, 581-583.

Grodal, B. (1974), "A Note on the Space of Preference Relations", *Journal of Mathematical Economics*, 1, 279-294.

Hammond, P.J. (1987), "Markets as Constraints: Multilateral Incentive Compatibility in Continuum Economies", *Review of Economic Studies*, 54, 399-412.

Hammond, P.J., M. Kaneko and M. Wooders (1987), *Continuum Economies with Finite Coalitions: Core, Equilibrium, and Widespread Externalities*, Mimeo, University of Toronto.

Hildenbrand, W. (1974), *Core and Equilibria of a Large Economy*, Princeton UP, Princeton.

Kalai, E., A. Postlewaite and J. Roberts (1978), "Barriers to Trade and Disadvantageous Middlemen: Nonmonotonicity of the Core", *Journal of Economic Theory*, 19, 200-209.

Kirman, A., C. Oddou and S. Weber (1986), "Stochastic Communication and Coalition Formation", *Econometrica*, 54, 129-138.

Mas-Colell, A. (1985), *The Theory of General Economic Equilibrium. A Differentiable Approach*, Cambridge UP, Cambridge.

Muto, S., M. Nakayama, J. Potters, and S.H. Tijs (1987), *On Big Boss Games*, Report 8719, Dept. of Mathematics, Catholic University of Nijmegen.

Myerson, R.B. (1977), "Graphs and Cooperation in Games", *Mathematics of Operations Research*, 2, 225-229.

Quigley, F.D. (1970), *Manual of Axiomatic Set Theory*, Meredith Corporation, New York.

Shitovitz, B. (1973), "Oligopoly in Markets with a Continuum of Traders", *Econometrica*, 41, 467-586.

Appendix : Topological Spaces

In this appendix we will discuss some definitions and theorems as formulated in Császàr (1978). For further elaborations of the topics as discussed here, we refer to that textbook on general topology.

The pair (E, τ) is called a *topological space* if E is a set of points and τ is a collection of subsets of E such that the empty set $\emptyset \in \tau$ and τ is closed for taking arbitrary unions and finite intersections. The elements in the collection τ are called *open sets* in (E, τ).

Let (E, τ) be a topological space, and $F \subset E$. The *interior* of F, which we denote by $\text{int}(F)$, is the largest open set $H \in \tau$ such that $H \subset F$.

Definition 1 *Let (E, τ) be a topological space, and let $x \in E$ be a point in that space. A class of sets $\mathcal{V}_x \subset 2^E$ is* **the neighbourhood system** *of x if it satisfies the following properties:*

1. *For every set $F \in \mathcal{V}_x$, $x \in F$.*
2. *If $H \in \mathcal{V}_x$ and $F \supset H$, then also $F \in \mathcal{V}_x$.*
3. *If H is an open set and $x \in H$, then $H \in \mathcal{V}_x$.*

The elements in the neighbourhood system $\mathcal{V}_x$ of x are called **neighbourhoods** *of x in the topology τ.*

Definition 2 *Let (E, τ) be a topological space, and let $\mathcal{V}_x$ be the neighbourhood system of $x \in E$. A class $\mathcal{B} \subset \mathcal{V}_x$ of neighbourhoods is* **a neighbourhood base** *of x in τ if it satisfies the following properties:*

1. *For every set $F \in \mathcal{B}$, $x \in F$.*
2. *For every neighbourhood of x, $H \in \mathcal{V}_x$, there is a $F \in \mathcal{B}$ such that $F \subset H$.*
3. *For every two $H, F \in \mathcal{B}$ with $H \cap F \neq \emptyset$, there exists $G \in \mathcal{B}$ such that $G \subset H \cap F$.*

In the sequel we take a fixed topological space (E, τ). For this fixed space we shall derive some properties. We now come to one of the basic results of topology.

Theorem 3 *Let $x \in E$, and let $\mathcal{V}_x$ be its neighbourhood system in τ. Then the class of open neighbourhoods of x, $\mathcal{V}_x \cap \tau$ is a neighbourhood base for $\mathcal{V}_x$.*

We are now able to define the main topological tools which are needed in the proofs of the theorems as developed in this paper.

Definition 4

(a) *A set F in the topological space (E, τ) is* **connected** *if there do not exist two open subsets $A, B \in \tau$ such that $A \cap B \cap F = \emptyset$ and $(A \cup B) \cap F = F$.*

(b) *A set F in the topological space (E, τ) is a* **component** *of (E, τ) if F is connected and for every connected set $F \subset C \subset E$ it holds that $F = C$.*

We remark that components of a topological space are just maximal connected subsets in that space. The next definition is of major importance in the statements on type-connectedness of typifications of populations in attribute spaces.

Definition 5 *Let $B, C \subset E$ be two subsets of the topological space (E, τ), and let $n \in \mathcal{N}$ be a non-negative integer. A finite sequence of subsets in E, denoted by $D_1, \ldots, D_n \subset E$, is an* **irreducible chain** *between B and C if it satisfies the following properties:*

- *$D_1 = B$ and $D_n = C$;*
- *for every number $j = 1, \ldots, n-1$ it holds that $D_j \cap D_{j+1} \neq \emptyset$;*
- *for all numbers h and j with $|h - j| > 1$ it holds that $D_h \cap D_j = \emptyset$.*

For the next theorem we remind that a collection of open sets $\mathcal{B} \subset \tau$ is an *open cover* of the topological space (E, τ) if $\cup \mathcal{B} = E$.

Theorem 6 ((10.3.8) of Császàr) *Let $C \subset E$ be a connected set, $\mathcal{B}$ be an open cover of the set C, and A, B be two subsets of C such that $A \neq \emptyset \neq B$. Then there exists an irreducible chain between A and B consisting of elements of $\mathcal{B}$.*

We conclude this appendix by stating some definitions and results on connectedness and locally connectedness.

Definition 7 *The topological space (E, τ) is* **locally connected** *if every point $x \in E$ has a neighbourhood base consisting of connected sets only.*

Theorem 8 *A topological space is locally connected if and only if it has a base consisting of connected sets only.*

Theorem 9 ((10.2.3) of Császàr) *Any open subspace of a locally connected space is locally connected as well.*

We finally remark that a topological space (E, τ) is *compact* if every open cover of (E, τ) has a finite subcover. Furthermore a topological space (E, τ) is a *continuum* if it is a compact and connected topological space. In the theory of large economies one usually takes the space of agents to be a continuum.

IN 1987 REEDS VERSCHENEN

242 Gerard van den Berg
Nonstationarity in job search theory

243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions

244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk

245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation of subjective poverty definitions

246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance Criteria

247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en strukturele afhankelijkheden

248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group

249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic Investment Policy

250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical approach

251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically distributed errors

252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten

253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen

254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint products: A comment

255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?

256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium concepts

289 M.H.C. Paardekooper
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues

290 Pieter H.M. Ruys
Industries with private and public enterprises

291 J.J.A. Moors & J.C. van Houwelingen
Estimation of linear models with inequality restrictions

292 Arthur van Soest, Peter Kooreman
Vakantiebestemming en -bestedingen

293 Rob Alessie, Raymond Gradus, Bertrand Melenberg
The problem of not observing small expenditures in a consumer expenditure survey

294 F. Boekema, L. Oerlemans, A.J. Hendriks
Kansrijkheid en economische potentie: Top-down en bottom-up analyses

295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note

296 Arthur van Soest, Peter Kooreman
Estimation of the indirect translog demand system with binding non-negativity constraints

IN 1988 REEDS VERSCHENEN

297 Bert Bettonvil
Factor screening by sequential bifurcation

298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure

299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry

300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method

301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note

302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in lineair models
using cross sections, panels or both

303 Raymond H.J.M. Gradus
A differential game between government and firms: a non-cooperative approach

304 Leo W.G. Strijbosch, Ronald J.M.M. Does
Comparison of bias-reducing methods for estimating the parameter in dilution series

305 Drs. W.J. Reijnders, Drs. W.F. Verstappen
Strategische bespiegelingen betreffende het Nederlandse kwaliteitsconcept

306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg
Regression sampling in statistical auditing

307 Isolde Woittiez, Arie Kapteyn
A Model of Job Choice, Labour Supply and Wages

308 Jack P.C. Kleijnen
Simulation and optimization in production planning: A case study

309 Robert P. Gilles and Pieter H.M. Ruys
Relational constraints in coalition formation

310 Drs. H. Leo Theuns
Determinanten van de vraag naar vakantiereizen: een verkenning van materiële en immateriële factoren

311 Peter M. Kort
Dynamic Firm Behaviour within an Uncertain Environment

312 J.P.C. Blanc
A numerical approach to cyclic-service queueing models