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DYNAMIC EQUILIBRIUM IN A COMPETITIVE CREDIT MARKET: INTERTEMPORAL CONTRACTING AS INSURANCE AGAINST RATIONING

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DYNAMIC EQUILIBRIUM IN A COMPETITIVE CREDIT MARKET: INTERTEMPORAL CONTRACTING AS INSURANCE AGAINST RATIONING[1)]

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1) The Appendix is published seperately.

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ABSTRACT

Credit rationing has recently been explained as an equilibrium phenomenon under asymmetric information. It is, however, a very costly resolution of informational problems. Thus, it is natural to expect credit mechanisms to arise that lessen the incidence of rationing. Using a dynamic model of credit market equilibrium under asymmetric information, this paper explains how intertemporal credit contracting can eliminate the rationing which arises endogenously with spot contracting. This may explain why billions of dollars in credit commitments are annually issued by commercial banks: these commitments provide borrowers with guaranteed future funds availability and hence insure them against rationing. It is also indicated how a perturbation of the model leads to the persistence of credit rationing despite long-term credit commitments, even though its incidence is reduced by such commitments. The analysis also produces additional predictions.

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I. INTRODUCTION

The purpose of this paper is to examine intertemporal contracting issues in an informationally constrained, competitive credit market in which borrowers have access to collateral. Specifically, our principal goals are twofold. The first is to characterize and prove the existence of a competitive equilibrium in a two-period, universally risk neutral credit market in which the contract choices and repayment behavior of privately informed borrowers convey information to banks through time, and banks can ration credit in any period. The second, more important, goal is to show that the possibility of rationing makes it advantageous for borrowers to purchase commitments from banks that guarantee the future availability of credit at predetermined terms. Thus, even risk neutral borrowers demand "insurance" against future credit rationing.

This research is inspired by two distinct strands of the financial intermediation literature. One is the credit rationing literature, and the other is the literature on credit options, commonly known as "bank loan commitments."

Although the literature on credit rationing is extensive (see, for example, Jaffee and Modigliani (1969), Jaffee and Russell (1976), and particularly the survey by Baltensperger (1978)), it's only recently that there have emerged explanations for rationing as an equilibrium phenomenon resulting from profit-maximizing behavior by competitive banks. The seminal contributions of Stiglitz and Weiss (1981, 1983) have shown that informational constraints can lead banks to ration credit either in a single period model or in a two period model in which poor first period performance is followed by credit denial being used as an ex post efficient incentive mechanism.[1] Besanko and Thakor (1987) have demonstrated that credit rationing can occur in a single period model even when collateral is available, as long as it is constrained.[2] The starting point of our analysis are the Stiglitz and Weiss (S-W) and Besanko and Thakor (B-T) papers. As in S-W (1981) and B-T (1987), we construct a model in which static contracting leads to equilibrium rationing by competitive banks. Unlike S-W (1983), however, dynamic contracting eliminates rationing. This is an important distinction because the focal point of this paper is the argument that the widespread occurrence of credit commitments and other similar long term arrangements between banks and borrowers can be rationalized as "insurance" against credit rationing. Moreover, another distinction between S-W (1983) and our paper is that, unlike S-W (1983), the rationing here is not an ex post "disciplining" device.

Currently, billions of dollars are loaned annually by U.S. banks under bank loan commitments. Although there is now a voluminous literature on loan commitments[3], there does not seem to be a well accepted explanation for why these instruments exist in the first place. Basically, a loan commitment is a contract that guarantees the future availability of credit at an interest rate that is either fixed or a deterministic function of some index rate.[4] The contract has been interpreted as a put option[5] that enables the borrower to acquire credit at a below-market interest rate. However, this view implies that borrowers are purchasing insurance against future random borrowing rates. It is, therefore, incapable of explaining why the bulk of loan commitment demand stems from corporations owned by diversified shareholders, i.e., risk neutral customers. The "practitioner's" view of loan commitments, on the other hand, seems to be that these are arrangements primarily intended to assure borrowers of ready availability of credit in future time periods. To date, however, there is no formal analysis that explains loan commitment demand on these grounds.[6] It is our objective to lend rigor to this intuition. We do this by showing that intertemporal contracting -- of the type inherent in loan commitments -- can aid in minimizing the allocational distortions arising from rationing in (future) spot markets. The intertemporal contracts we derive are rather complex. The contracts specify the current loan terms (first period contract terms) in conjunction with the terms of future credit delivery (second period contract terms). The terms of future delivery are conditioned on first period realizations -- that is, contracts have a "memory" feature in the sense of Rogerson (1985) -- and are binding for the bank.[7] But the borrower always has the option to "walk away" from the contract and take a spot market contract in the second period. (It is for this reason that we do not examine "two-way" binding contracts). While this intertemporal credit contract has a striking resemblance to real world loan commitment contracts, it is difficult to determine the extent to which our theoretically optimal pricing structure corresponds to the pricing structure found in loan commitments (see Melnick and Plaut (1986)). For this reason, we shall refer to our contracts as "intertemporal credit contracts" rather than loan commitments.[8]

The task of showing that intertemporal credit contracting is motivated by borrowers' desire to acquire protection against future rationing is made delicate by the following observation. If equilibrium credit rationing is indeed possible in a given set of circumstances, then it must be true that it is (ex ante) efficient for the (competitive) bank to deny credit in those circumstances. Why, then, is it ever an equilibrium phenomenon for the same bank to agree to an intertemporal contract that prevents it from rationing credit in the same set of circumstances? Any model designed to address this issue must be careful to resolve this apparent paradox.

Our approach is related to papers by Harris and Holmstrom (1982), Cooper and Hayes (1982), and Palfrey and Spatt (1985), all of which examine intertemporal contracting issues. Harris and Holmstrom (1982) analyze dynamic contracting in a labor market with risk averse workers. Their contracting environment is similar to our intertemporal contracts in that long-term commitments made by firms (banks) to workers (borrowers) are assumed to be honored, but commitments by workers to firms are not. However, there are two key differences. First, workers in their model are risk averse; we have universal risk neutrality. Second, they have symmetric information, while asymmetric information is at the heart of our analysis. This also distinguishes our paper from Palfrey and Spatt's (1985) which assumes that the insurer and the risk averse insured are symmetrically informed. Moreover, Palfrey and Spatt (1985) assume that commitments are either honored by both parties or by none. Finally, Cooper and Hayes (1982) allow for asymmetric information but assume the insured are risk averse. Moreover, their contracting regime and equilibrium analysis differ substantially from ours.

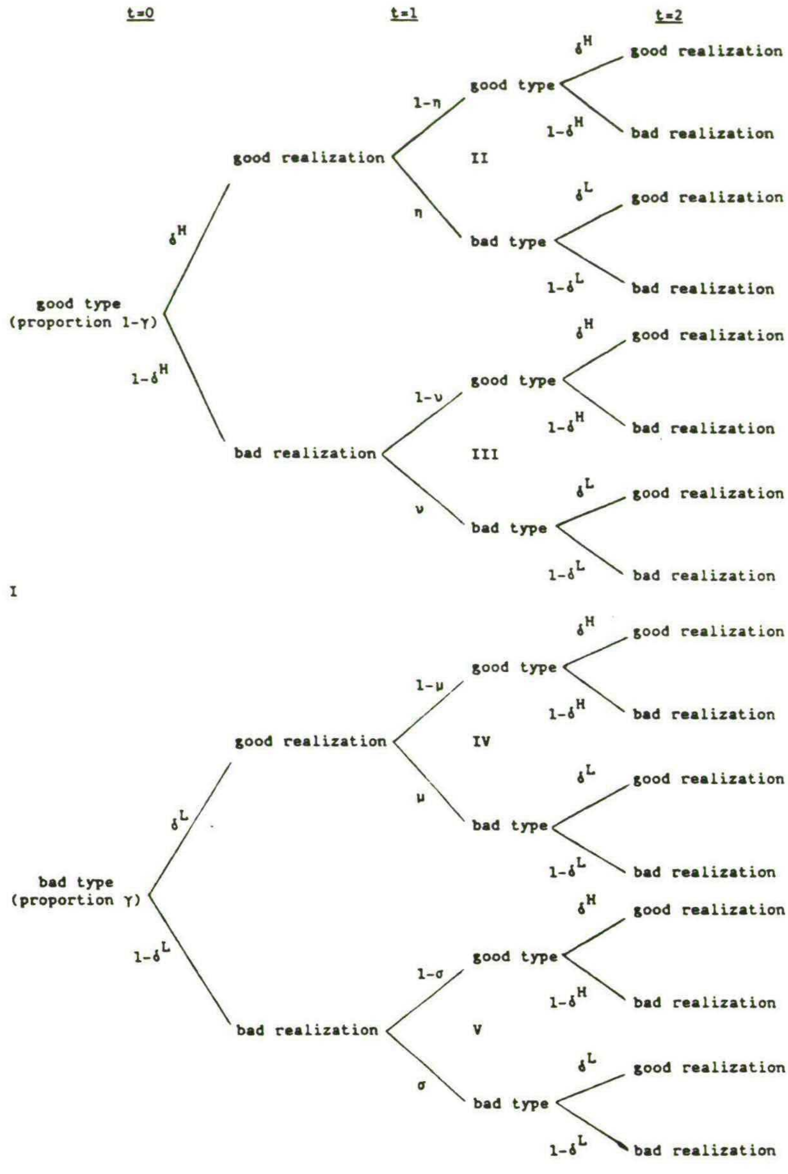
It is striking that we obtain the result that intertemporal contracting is welfare-improving despite the universal risk neutrality assumption. Previous research has been able to establish a benefit for intertemporal contracting only with risk aversion.

Our model -- which is an augmented and dynamic version of the B-T (1987) model -- can briefly be described as follows. There is a large credit market with many banks and borrowers. All agents are risk neutral and the market is perfectly competitive. Each borrower invests in each period in a single period project whose payoff distribution is known only to the borrower. The inputs in both periods are investments which are funded by bank loans. A borrower's project payoffs are positively correlated over time, which implies that a bank learns something about the borrower's type by observing realized (past) returns. However, observing returns never completely resolves the informational asymmetry since a realized return is only a noisy signal of a borrower's type.[9] *A priori* uninformed banks attempt to sort borrowers by offering contracts that specify, for each period, the (i) credit granting probability, (ii) loan interest factor, and (iii) collateral requirement. Two types of contracting structures are studied. Both structures are dynamic in that they incorporate intertemporal linkages -- second period contracts depend on first period contract choices and repayment behavior. The first structure, however, only permits what we call "single period" contracts. With these contracts, the bank is constrained to break even in *each* period. The second structure permits what we call "intertemporal" contracts, which allow for the possibility of intertemporal subsidies to borrowers. Thus, the bank is only constrained to break even *across* the two periods. With single period contracting, the problem reduces to one in which there are two successive spot credit markets. In this setting we obtain an equilibrium in which credit is rationed in the second period following first period default. With intertemporal contracting, however, it is shown that equilibrium credit rationing can be eliminated. The rest of the paper is organized as follows. In Section II, we describe the model and state the main assumptions. Section III contains a definition of the equilibrium, a formal statement of the constrained optimization program that leads to the equilibrium, and the equilibrium solution to the single period contracting problem. In Section IV we present the equilibrium solution to the intertemporal contracting problem. Section V has a comparison of the single period and intertemporal contracting equilibria. Section VI takes up the issue of existence of equilibrium and contains a theorem that establishes the existence of a unique equilibrium. Section VII discusses a perturbation of the model that would lead to the persistence of rationing even with long-term commitments, although such commitments would reduce its incidence. Empirical implications of the analysis are also drawn out. Finally, Section VIII concludes. All formal proofs are contained in the Appendix.

II. THE MODEL

A. Preferences and Market Structure:

All agents are risk neutral. The credit market is perfectly competitive and banks compete for both deposits and loans. Deposits are in perfectly elastic supply at a commonly known market determined interest rate. The economy lives for two periods. The first period begins at t=0 and ends at t=1 and the second begins at t=1 and ends at t=2. Taken together, these assumptions are meant to imply that: (i) the bank's depositors receive in each period an expected return equal to the single period riskless interest rate, (ii) the bank earns zero expected profit, and (iii) the expected utility of each borrower is maximized subject to the relevant informational and breakeven constraints. The single period riskless interest factor (one plus the riskless interest rate) is r and is assumed to be constant over the two periods.



where, γ = proportion bad type borrowers at t=0;
η = probability that a successful good type is of the bad type in the second period;
ν = probability that an unsuccessful good type is of the bad type in the second period;
μ = probability that a successful bad type is of the bad type in the second period;
σ = probability that an unsuccessful bad type is of the bad type in the second period.

Figure 1: The borrowers' types and returns in the two period credit market.

B. Technology:

Each potential borrower can invest $1 each period in a point-input, point-output project. Outputs are only available for periods in which investments are made. The output each period is an end-of-period return which is R (a positive, real valued scalar) if the project is successful and zero otherwise. R is the same for all borrowers as well as for both time periods, and is common knowledge. However, the probability of success varies across borrowers in any given time period and across time periods for any given borrower. In any time period, there are two types of borrowers, "good" (g) and "bad" (b). A borrower's type in period $j \in \{1,2\}$ is denoted by $\kappa_j \in \{g,b\}$. Good borrowers have a success probability of δ^H, and bad borrowers have a success probability of δ^L. We let $0 < \delta^L < \delta^H < 1$. Project returns are positively correlated through time. That is, a borrower starts out at t=0 being of a certain type, and then its type in the second period is probabilistically determined by its first period type and the realized return of its first period project. These conditional transition probabilities are η, ν, μ and σ, which are defined in Figure 1. This figure also pictorially depicts the temporal evolution of borrower types. In order to ensure intertemporally positively correlated project returns, we assume $\nu > \eta$ and $\sigma > \mu$.[10]

C. Endowments:

All potential borrowers have existing endowments of liquidity which are totally invested in other projects. It is inefficient for any borrower to prematurely liquidate its "other" project in order to finance the new ($1 point input, point output) investment project in any period. Hence, all borrowers must finance their new investments with bank loans. Although not optimal to prematurely liquidate, a borrower's existing investment can be offered as collateral to secure the loan for the new investment. For simplicity, each borrower is assumed to start at t=0 with a commonly known end-of-period value of W_0 for its existing investment, i.e., W_0 is constant across borrowers. Hence, the maximum collateral a borrower can offer at t=0 is W_0. Collateral is not augmented through time, but it may be lost. We assume that the first period return from a borrower's existing investment as well as the return from the first period project financed with a bank loan are unavailable for financing the second period project. Thus, a borrower must enter the credit market again at t=1 to acquire a loan for its second period project.[11]

D. Information Structure:

The bank knows the cross-sectional distribution of borrowers' success probabilities at t=0 as well as the conditional transition probabilities that guide the temporal evolution of each borrower's type. Moreover, the bank observes all realized returns. However, the bank does not know any individual borrower's success probability at t=0. That is, at t=0 the bank knows that a fraction γ of the countable infinity of borrowers in the market are bad and a fraction $1-\gamma$ are good,[12] but is unable to distinguish borrowers by type. Except in some special cases, the bank suffers from a similar informational handicap at t=1.

E. Feasible Contract Space:

The bank will attempt to cope with this pre-contract informational asymmetry by designing a menu of credit contracts at t=0 that induce borrowers to truthfully reveal their success probabilities to the bank in a manner consonant with the revelation principle.[13] It is convenient to think of a dynamic strategic credit policy as a vector, $\mathcal{B} = (\mathcal{B}_1, \mathcal{B}_2(y_1, x_1))$ where $\mathcal{B}_1$ is the first period credit policy and $\mathcal{B}_2(y_1, x_1)$ is the second period credit policy available to a borrower whose first period choice of contract from policy $\mathcal{B}_1$ was y_1 and whose realized first period project return was χ_1. Note that χ_1 is a random variable with state space {0,R}. Viewing initial strategic credit policies this way allows us to capture the dynamic structure of the credit market with a simple representation. This should not be taken to imply that borrowers bind themselves to long term contracts. A borrower that chooses $y_1 \in \mathcal{B}_1$ at t=0 might very well decide at t=1 that it does not wish to take any contract from $\mathcal{B}_2(y_1, \chi_1)$ because some other bank in the spot credit market at t=1 offers it more favorable credit terms. It should only be taken to mean that a bank offering $\mathcal{B}_1$ at t=0 is willing to offer the same borrower $\mathcal{B}_2(y_1, \chi_1)$ at t=1.

A <u>credit contract</u> for a given period is defined to consist of: (i) π, the probability with which credit will be granted, (ii) α, the loan interest factor (one plus the loan interest rate) if credit is granted, and (iii) C, the amount of collateral required, where $C \in [0, W_0]$.

The policy $\mathcal{B}_1$ consists of a pair of credit contracts, one for each type at t=0, whereas the policy $\mathcal{B}_2(y_1, \chi_1)$ consists of eight credit contracts, a pair for each initial choice $y_1 \in \mathcal{B}_1$ and for each possible value of χ_1. At t=0, the borrower can be viewed as selecting one contract from $\mathcal{B}_1$. Then, at t=1, it will observe its project return (as will the bank) and select a contract from the <u>pair</u> in $\mathcal{B}_2(y_1, \chi_1)$ that corresponds to its specific (y_1, χ_1) combination.

We permit a disparity in the valuation of collateral by the borrower and the bank by defining the bank's valuation as βC, with $\beta \in [0,1)$.[14] Further, any interaction between a borrower and a bank immediately becomes common knowledge. Thus, any possible information accumulation gain from an enduring bank-borrower relationship is ruled out. Finally, as a matter of convenience, all borrowers' reservation utility constraints are assumed to be slack at the optimum, i.e., equilibrium credit contracts are such that the net surplus accruing to any borrower from any of its projects is nonnegative.

III. EQUILIBRIUM CONCEPT AND THE OPTIMIZATION PROGRAM

A. Equilibrium Concept:

In our model, the (uninformed) bank posts a menu of credit contracts and then each borrower responds by selecting its most favored contract. Thus, we have a (multiperiod) game in which the uninformed agent moves first.[15] To characterize equilibrium we adopt a modified, dynamic version of Riley's (1979) reactive equilibrium.[16]

Let $N = \{1,---,n\}$ denote the set of all possible (competing) banks (the counting measure of N could be infinity) and let $\zeta_i(\mathcal{B}^1,---,\mathcal{B}^n)$ be the (net) expected profit of bank i when the vector of strategic credit policies being offered by all banks at t=0 is $(\mathcal{B}^1,---,\mathcal{B}^n)$. Here we take $\mathcal{B}^i=(\mathcal{B}_1^i,\mathcal{B}_2^i(y_1,x_1))$ as the strategic credit policy of bank $i \in N$. A bank's (net) expected profit is the aggregate revenue from its loans over the two periods minus its payoff to depositors over those two periods. For later use, we define a feasible spot credit contract to be simply a single period credit contract available to the borrower in the spot credit market at either t=0 or t=1 such that the offering bank earns nonnegative expected profit on its single period loan to the borrower type for which the contract was designed. We assume spot credit contracts are available in both time periods.

Definition of Feasible Policies: A dynamic strategic credit policy $\mathcal{B}^i$ of bank i is feasible if

(i) it produces nonnegative expected profits for the offering bank, and

(ii) it contains only those credit contracts that are at least as attractive for the borrowers as the corresponding feasible spot credit contracts at t=0 and t=1.

This definition of feasibility rules out intertemporal contracts that can be "broken" by spot credit contracts at any point in time, i.e., it precludes intertemporal contracts under which 1) the borrower at t=0 prefers an available (feasible) spot credit contract to <u>any</u> $y_1 \in \mathcal{B}_1$, <u>knowing</u> that its choice of the spot contract will preclude choosing any element of $\mathcal{B}_2(y_1,\chi_1)$ at t=1, or 2) the borrower at t=1 prefers an available (feasible) spot credit contract to <u>any</u> contract in $\mathcal{B}_2(y_1,\chi_1)$, given its (y_1,χ_1) realization. This implies that no spot credit contract can exist in any period that lures a borrower away from tne contract choices offered by the bank under its dynamic credit policy. We can now define equilibrium.

<u>Definition of Equilibrium</u>: A dynamic reactive equilibrium (DRE) is a set of feasible strategic credit policies, $\hat{\mathcal{B}} = (\hat{\mathcal{B}}^1,---,\hat{\mathcal{B}}^n)$, for the n banks if:

(a) for any $i \in N$ and any feasible strategic policy $\mathcal{B}^i$ such that

$$\zeta_i(\hat{\mathcal{B}}^1,---,\mathcal{B}^i,---,\hat{\mathcal{B}}^n) > \zeta_i(\hat{\mathcal{B}})$$

$\exists$ another creditor $j \in N$ and another feasible strategic policy $\mathcal{B}^j$ such that

(i) $\zeta_j(\hat{\mathcal{B}}^1,---,\mathcal{B}^i,---\hat{\mathcal{B}}^n) \le \zeta_j(\hat{\mathcal{B}})$

(ii) $\zeta_j(\hat{\mathcal{B}}^1,---,\mathcal{B}^i,---,\mathcal{B}^j,---,\hat{\mathcal{B}}^n) > \zeta_j(\hat{\mathcal{B}}^1,---,\mathcal{B}^i,---,\hat{\mathcal{B}}^j,---,\hat{\mathcal{B}}^n)$

(iii) $\zeta_i(\hat{\mathcal{B}}^1,---,\mathcal{B}^i,---,\mathcal{B}^j,---,\hat{\mathcal{B}}^n) < \zeta_i(\hat{\mathcal{B}})$

(iv) $\forall\, m \in N$, $m \ne i, j$ and all feasible $\mathcal{B}^m$

$$\zeta_j(\hat{\mathcal{B}}^1,---,\mathcal{B}^i,---,\mathcal{B}^j,---,\mathcal{B}^m,---,\hat{\mathcal{B}}^n)$$
$$\ge \zeta_j(\hat{\mathcal{B}}^1,---,\mathcal{B}^i,---,\hat{\mathcal{B}}^j,---,\hat{\mathcal{B}}^m,---,\hat{\mathcal{B}}^n);$$

(b) $\exists$ no $\hat{\mathcal{B}}^i \in \hat{\mathcal{B}}$ such that its feasibility requires that the bank and the borrower be restricted from renegotiating their credit contract when it is mutually beneficial for them to do so.

By imposing condition b), we ensure that the equilibrium concept does not artificially force the bank and the borrower to agree ex ante to long-term strategies that neither may wish to pursue subsequently. It thus rules out from the equilibrium set those intertemporal contracts which give both the borrower and the bank an incentive to renegotiate the contract terms at a future point in time.[17]

It is convenient to think of a borrower reporting its first period type, $\kappa_1 \in \{H, L\}$, to a bank i at t=0. It will receive a first period contract $y_1^i(\kappa_1) \in \mathcal{B}_1^i$ from the bank that is contingent upon its report. Then, at t=1 this borrower will observe its first period payoff realization, $\chi_1 \in \{0, R\}$, and report its second period type, $\kappa_2 \in \{H, L\}$, to bank i. Let $y_2^i(\kappa_1, \chi_1, \kappa_2) \in \mathcal{B}_2(y_1, \chi_1)$ be the second period credit contract awarded to this borrower. Define $\mathcal{B}_0^i(\kappa_1, \chi_1, \kappa_2) \equiv y_1^i(\kappa_1) \cup y_2^i(\kappa_1, \chi_1, \kappa_2) \in \mathcal{B}^i$. For notational ease, let $z \equiv (\kappa_1, \chi_1, \kappa_2) \in \{H, L\} \times \{0, R\} \times \{H, L\}$ denote the "composite type" of the borrower in the second period. From Figure 1 we can see that there are eight possible values of z. Following Riley (1979) we now define the credit policy of bank i as being <u>dynamically strongly informationally consistent</u> (DSINC) if it is feasible and has the following properties:

(i) $y_1^i(\kappa_1) \succsim_{\kappa_1} y_1^i(\tilde{\kappa}_1) \ \forall \ \kappa_1, \tilde{\kappa}_1 \in \{H, L\}$

(ii) $y_2^i(\kappa_1, \chi_1, \kappa_2) \succsim_{\kappa_2} y_2^i(\kappa_1, \chi_1, \tilde{\kappa}_2) \ \forall \ \kappa_2, \tilde{\kappa}_2 \in \{H, L\}$

(iii) $\zeta_i^*(\mathcal{B}_0^i(z), z) = 0 \ \forall \ z \in \{H, L\} \times \{0, R\} \times \{H, L\}$

(iv) it is subgame perfect,

where $\succsim_{\kappa_j}$ denotes the preference ordering of a type - κ_j borrower (j=1,2) and $\zeta_i^*(\mathcal{B}_0^i(z), z)$ is the expected profit of bank i on the two-period contract sequence $\mathcal{B}_0^i(z)$ when such a contract sequence is taken by a borrower of "composite type" z. We now have the following adaptation of one of Riley's (1979) principal results.

THEOREM 1: The DRE credit policy for a given bank is the Pareto dominant member of the family of DSINC credit policies.

B. Some Preliminary Remarks About the Nature of Sorting:

The motivation for the bank to choose credit instruments in addition to the loan interest rate comes from the fact that (first best) contracts which specify only a different loan interest rate for each borrower type are not incentive compatible. The only possible outcome in this case is pooling. However, a pooling allocation that involves all borrowers being charged the same interest rate is inherently susceptible to the introduction of more complex credit contracts that entice only the initially good borrowers away from the pooling allocation. One such complexity is a secured loan, i.e., the loan contract specifies a collateral requirement along with an interest rate. A competitive bank can now offer two contracts. One contract demands some collateral but a low interest rate, whereas the other contract involves no collateral but has a high interest rate. As Bester (1985) and B-T (1987) show, this induces an incentive compatible sorting of borrowers. Good borrowers choose the secured loan contract with a low interest rate, and bad borrowers choose the unsecured loan with a high interest rate. This is not a first best outcome because collateral involves deadweight losses due to the bank's evaluation of it being lower than the borrower's. Now, if the two borrower types are sufficiently disparate, the incentive compatible level of collateral in the secured loan contract will exceed the borrower's available collateral-eligible assets (B-T (1987)). This frustrates sorting. However, incentive compatibility can be restored by setting the credit granting probability on the secured loan contract to be nonzero but less than one. Thus, the equilibrium involves the good borrowers being asked to put up collateral and be sometimes rationed, and the bad borrowers receiving unsecured, high interest rate loans with probability (w.p.) one. This allocation is even more distortionary because rationing is a very costly sorting device in that positive net present value investments are foresaken. All of these preliminaries are proved rigorously in B-T (1987) and are also stated formally for completeness in what follows. The discussion here is intended to motivate our modeling approach.

We will assume throughout that the initial level of collateral-eligible wealth, W_0, is such that the collateral constraint is never binding at t=0. But if the collateral is lost by a borrower who selected a secured contract in the first period (due to failure of the project), the collateral remaining for the second period, W, is less than the level needed for an "optimally" (no rationing) collateralized (separating) contract. This is the sense in which collateral is assumed to be constrained. Since in the second period, we have just a single period game, the B-T (1987) second period results apply. Thus, we have a case in which rationing is encountered in the second period. Figure 1 reveals that the constraint on W_0 leads to rationing of good borrowers in the set of nodes III (to see this, note that only the contracts for good borrowers involve collateral and, therefore, collateral has been lost only in the set of nodes III). So, we should expect rationing of good borrowers in the set of nodes III. (Formal proofs will be presented in subsequent sections.) The major analysis is aimed at proving that intertemporal contracting -- which allows for subsidizing across time periods -- could obviate the need for rationing and improve borrower welfare.

Our collateral constraint can be thought of as follows. Suppose we are in the second period in an information set such that the bank does not know the borrower's second period type but knows that the borrower failed in the first period. Let the borrower have collateral-eligible assets of W and assume W is insufficient to eliminate second period rationing. Given W and the endogenously determined second period rationing probability, suppose an endogenously determined collateral of C is needed with a secured loan in the first period to ensure that the borrower taking the secured loan is not rationed at t=0. Then our assumption is that $C+W < W_0$. Of course, since collateral levels in both periods are endogenously determined, we will need to show that reducing C and augmenting second period collateral availability is not optimal, i.e., it is not optimal to reduce second period rationing in exchange for some first period rationing.

As mentioned earlier, one can substitute, without loss of generality, $\eta=0$ and $\sigma=1$. This implies that a borrower which is good at t=0 and is successful during the first period is good w.p. one in the second period, whereas a bad borrower which is unsuccessful in the first period is bad w.p. one in the second period. This does not sacrifice generality because in the set of nodes II and V, where these substitutions apply, the collateral constraint is never binding. Thus, these nodes can be simplified without affecting the main result.

As mentioned earlier, our assumption that the collateral constraint is not binding at t=0 does not preclude first period rationing since collateral could be "saved" in the first period by accepting some rationing in order to reduce the probability of rationing in the second period. Intuitively, however, it is easy to see that accepting some rationing at t=0 in order to reduce the likelihood of rationing at t=1 is not optimal. This is because rationing at t=0 affects all good types -- including future types -- while rationing at t=1 affects only the initial good types who end up in the good type node of the set of nodes III. We shall later present a formal proof of this claim. For now, we state and solve the representative bank's constrained maximization problem assuming that rationing at t=0 is suboptimal. We initially examine credit market equilibrium where we only allow for single period contracts. That is, intertemporal subsidies are introduced only later. This means that bank i earns zero expected profit on $y_1^i(K_1) \forall K_1$ and on $y_2^i(z) \forall z$. This is a stronger requirement than the zero profit condition stated in the definition of DSINC policies. We retain all the requirements of the DRE. The DRE allocations are always fully separating. By looking at Figure 1, one sees that in each of the sets of nodes I, III and IV, the bank offers two separating contracts.[18] These observations are useful in the formulation of the maximization program of the representative bank.

C. The Maximization Program (Single Period Contracting in the Two Period Game):

Given Theorem 1, we know that the DRE allocations can be obtained by solving for the Pareto dominant DSINC credit policy. Henceforth, we shall deal with a representative bank and drop the superscript denoting a specific bank. When the bank is restricted to earn zero expected profit in each period, the problem is as follows. (The subscript j below is used to number contract nodes in Figure 1).

Maximize $\mathcal{J} = \gamma A_L + [1-\gamma] A_H$ (1)

$\{\alpha_j^i, C_j^i, \pi_j^i\}$

$i=H,L$

$j=I,II,---,V$

$i \neq L$ for $j=II$

$i \neq H$ for $j=V$

subject to

$$U_j(\delta^L|\delta^L) \geq U_j(\delta^H|\delta^L), \; j=I,III \text{ and } IV \quad (2)$$

$$U_j(\delta^H|\delta^H) \geq U_j(\delta^L|\delta^H), \; j=I,III \text{ and } IV \quad (3)$$

$$\delta^i\alpha_j^i - [1-\delta^i]\beta C_j^i = r, \; i\in\{H,L\} \text{ and } j=I,II,---,V \quad (4)$$

$$\left.\begin{array}{l} C_j^i \in [0,W_0], \; i \in\{H,L\}, \; j=I,II \text{ and } IV \\ C_{III}^i \in [0,W_0-C_I^H], \; i \in \{H,L\} \\ C_V^L \in [0,W_0-C_I^L] \end{array}\right\} \quad (5)$$

$$\alpha_j^i \geq 0, \; i \in \{H,L\} \text{ and } j=I,II,---,V \quad (6)$$

$$\pi_j^i \in [0,1], \; i \in \{H,L\} \text{ and } j=I,II,---,V. \quad (7)$$

In this maximization program, $U_j(\delta^i|\delta^k)$ is the total expected utility of a borrower which finds itself in the set of contract nodes j, has a success probability over the next period of δ^k ($k\in\{H,L\}$), and reports its success probability to be δ^i ($i\in\{H,L\}$). A_L and A_H are the expected utilities over the two-period contracting horizon of those borrowers who are initially bad and good respectively. These terms are defined in detail in Table 1. Note that (2) and (3) are incentive compatibility constraints, (4) is the period-by-period zero profit constraint for the bank, and (5), (6) and (7) are feasibility restrictions (including resource constraints). Constraint (5) merely makes precise our earlier stated assumption that the upper bound on C_j^i can be binding in the good type nodes of the set of nodes III and V. We have used the assumption here that rationing at t=0 is inefficient, i.e., $\pi_I^L=\pi_I^H=1$ is assumed for now.

TABLE 1: DEFINITION OF TERMS

$$A_L \equiv \{\delta^L[R-\alpha_I^L] - [1-\delta^L]C_I^L + \delta^L[[1-\mu]\pi_{IV}^H\{\delta^H[R-\alpha_{IV}^H] - [1-\delta^H]C_{IV}^H\}$$

$$+ \mu\, \pi_{IV}^L\{\delta^L[R-\alpha_{IV}^L] - [1-\delta^L]C_{IV}^L\}] + [1-\delta^L]\pi_V^L[\delta^L[R-\alpha_V^L] - [1-\delta^L]C_V^L]\}$$

$$A_H \equiv \{\delta^H[R-\alpha_I^H] - [1-\delta^H]C_I^H + \delta^H\pi_{II}^H[\delta^H[R-\alpha_{II}^H] - [1-\delta^H]C_{II}^H]$$

$$+ [1-\delta^H][[1-\nu]\pi_{III}^H\{\delta^H[R-\alpha_{III}^H] - [1-\delta^H]C_{III}^H\} + \nu\pi_{III}^L\{\delta^L[R-\alpha_{III}^L]$$

$$- [1-\delta^L]C_{III}^L\}]\}.$$

$$U_I(\delta^L|\delta^L) = \delta^L[R-\alpha_I^L] - [1-\delta^L]C_I^L + \delta^L[[1-\mu]\pi_{IV}^H\{\delta^H[R-\alpha_{IV}^H] - [1-\delta^H]C_{IV}^H\}$$

$$+ \mu\, \pi_{IV}^L\{\delta^L[R-\alpha_{IV}^L] - [1-\delta^L]C_{IV}^L\}] + [1-\delta^L]\pi_V^L[\delta^L[R-\alpha_V^L] - [1-\delta^L]C_V^L]$$

$$U_I(\delta^H|\delta^L) = \delta^L[R-\alpha_I^H] - [1-\delta^L]C_I^H + \delta^L\pi_{II}^H\{[[1-\mu]\delta^H + \mu\delta^L][R-\alpha_{II}^H] - [1-[1-\mu]\delta^H - \mu\delta^L]C_{II}^H\}$$

$$+ [1-\delta^L]\pi_{III}^L\{\delta^L[R-\alpha_{III}^L] - [1-\delta^L]C_{III}^L\}$$

$$U_j(\delta^L|\delta^L) = \pi_j^L[\delta^L[R-\alpha_j^L] - [1-\delta^L]C_j^L], \text{ for } j=\text{III, IV and V}$$

$$U_j(\delta^H|\delta^L) = \pi_j^H[\delta^L[R-\alpha_j^H] - [1-\delta^L]C_j^H], \text{ for } j=\text{III and IV}$$

$$U_I(\delta^H|\delta^H) = \delta^H[R-\alpha_I^H] - [1-\delta^H]C_I^H + \delta^H\pi_{II}^H\{\delta^H[R-\alpha_{II}^H] - [1-\delta^H]C_{II}^H\}$$

$$+ [1-\delta^H][[1-\nu]\pi_{III}^H\{\delta^H[R-\alpha_{III}^H] - [1-\delta^H]C_{III}^H\} + \nu\pi_{III}^L\{\delta^L[R-\alpha_{III}^L]$$

$$- [1-\delta^L]C_{III}^L\}]$$

$$U_I(\delta^L|\delta^H) = \delta^H[R-\alpha_I^L] - [1-\delta^H]C_I^L + \delta^H\pi_{IV}^H\{\delta^H[R-\alpha_{IV}^H] - [1-\delta^H]C_{IV}^H\}$$

$$+ [1-\delta^H]\pi_V^L\{[[1-\nu]\delta^H + \nu\delta^L][R-\alpha_V^L] - [1-[1-\nu]\delta^H - \nu\delta^L]C_V^L\}$$

$$U_j(\delta^H|\delta^H) = \pi_j^H[\delta^H[R-\alpha_j^H] - [1-\delta^H]C_j^H], \text{ for } j=\text{II, III and IV}$$

$$U_j(\delta^L|\delta^H) = \pi_j^L[\delta^H[R-\alpha_j^L] - [1-\delta^H]C_j^L], \text{ for } j=\text{III and IV}$$

Individual rationality constraints are superfluous because we assume that these constraints are slack in equilibrium (all borrowers enjoy strictly positive NPV's net of borrowing costs and there are no alternatives to bank loans).[19]

D. Solution Procedure

(i) General Remarks:

Finding the solution to this constrained optimization program directly is rather complicated. However, two observations lead to welcome analytical simplifications. The first simplification is that we can solve directly for the optimal contracts in the sets of nodes II and V. By looking at the bank's zero profit conditions and the objective function (1), one directly concludes that for any $\beta \in [0,1)$, it is in the borrower's interest to choose ("$\hat{}$"'s indicate optimal values in this solution).

$$\hat{C}^{H}_{II} = \hat{C}^{L}_{V} = 0 \tag{8)'$$

and

$$\hat{\pi}^{H}_{II} = \hat{\pi}^{L}_{V} = 1 \tag{8)''$$

which implies that,

$$\hat{\alpha}^{H}_{II} = r/\delta^{H} \text{ and } \hat{\alpha}^{L}_{V} = r/\delta^{L}. \tag{8)'''$$

The intuition for the results (8)' through (8)"' is clear. In the set of nodes II, only good types exist. This is common knowledge, so they should be offered a first best contract. The same is true for the set of nodes V. There only bad types exist, and they should also be awarded a first best contract.

The second simplification is technical in nature. We will take advantage of the fact that one can solve the model in three stages, with the backward induction of dynamic programming. The first stage consists of determining the optimal second period contracts. The optimal second period contracts will be determined under the assumption that the bank was able to establish self-selection by contract choice in the first period. Hence, the bank knows the set of nodes a specific borrower belongs to in the second period. Subsequently, in the second stage, we will solve for the optimal first period contracts, taking the second period contracts as given. Given the earlier assumption that collateral will be unconstrained in the first period (such that, all possible shortages will occur in the second period), this procedure is completely general. Finally, in the third stage, we will show that the intertemporal use of collateral that was assumed indeed represents the optimal policy with respect to the use of collateral.

It is easy to see why this approach is completely general (except for the assumption about the allocation of collateral). Apart from collateral, the first period contracts do not restrict the second period contracts. Hence, these latter contracts should be optimized independently. If the bank does not do this, another bank in the competitive credit market can offer the borrowers utility-maximizing contracts in the second period. Note that this is feasible because any interaction between a borrower and an individual bank immediately becomes common knowledge.

(ii) Stage 1: The Optimal Second Period Contracts:

In this stage we will determine the optimal second period contracts under the assumption that perfect self-selection was established in the first period. Hence, we can successively solve for all the contracts in the sets of nodes II through V.

(a) Contracts II: The results (8)' through (8)"' directly indicate:

$$\hat{\alpha}^{H}_{II} = r/\delta^{H}$$

$$\hat{C}^{H}_{II} = 0$$

$$\hat{\pi}^{H}_{II} = 1$$

(b) Contracts III: In stage 2, it will be apparent that borrowers ending up in these nodes have lost collateral in the first period. Given our assumption regarding the availability of collateral, it follows that collateral is limited to W in the set of nodes III. Therefore, we have to determine

$\{\alpha^i_{III}, C^i_{III}, \pi^i_{III}\}$, for i=L and H, taking into account some binding constraint on collateral. This problem is identical to B-T (1987), Proposition 3. The solution is (see B-T (1987) for a proof)

$$\hat{\alpha}^L_{III} = r/\delta^L;\ \hat{C}^L_{III} = 0;\ \hat{\pi}^L_{III} = 1$$
$$\hat{\alpha}^H_{III} = r[\delta^H]^{-1} - \bar{\delta}^H\beta W[\delta^H]^{-1};\ \hat{C}^H_{III} = W;$$
$$\hat{\pi}^H_{III} = \{\delta^L[R-\hat{\alpha}^L_{III}]\}\{\delta^L[R-\hat{\alpha}^H_{III}] - \bar{\delta}^L W\}^{-1},$$

where $\bar{\delta}^H \equiv 1-\delta^H$, $\bar{\delta}^L \equiv 1-\delta^L$, $W \equiv W_0 - C^H_I$.

Note that W is the available collateral in the second period if C^H_I has been lost in the first period. These results are in line with the motivating remarks in Subsection B. A bank which has to sort observationally indistinguishable borrowers is able to reward good borrowers with lower interest rates (without inducing bad borrowers to claim to be good), only if the good borrowers are willing to offer the necessary amount of collateral. The reason for this is that the expected cost of collateral is lower for the good borrowers than for the bad borrowers, simply because the former are less likely to end up in the default state in which they relinquish collateral. Unfortunately, the availability of collateral is constrained in these nodes. This makes the contract for good borrowers still attractive to the (mimicking) bad borrowers. Therefore, the bank needs to make the credit granting probability in the good borrower's contract smaller than one ($\hat{\pi}^H_{III}<1$), in order to discourage the bad borrowers from mimicking.

(c) <u>Contracts IV</u>: Borrowers enter these nodes without having lost any collateral. Therefore, the collateral constraint is not binding. The problem is identical to B-T (1987), Proposition 2. The solution is (see B-T (1987) for a proof)

$$\hat{\alpha}^L_{IV} = r/\delta^L;\ \hat{C}^L_{IV} = 0;\ \hat{\pi}^L_{IV} = 1$$
$$\hat{\alpha}^H_{IV} = r[\delta^H]^{-1} - \bar{\delta}^H\beta\hat{C}^H_{IV}[\delta^H]^{-1};\ \hat{C}^H_{IV} = (\delta r)(\delta^H\bar{\delta}^L-\beta\delta^L\bar{\delta}^H)^{-1};\ \hat{\pi}^H_{IV} = 1.$$

where $\delta \equiv \delta^H - \delta^L$. The intuition underlying these results is similar to that for contracts III. Now collateral availability is not constrained. Hence, separating contracts can be offered without rationing.

(d) Contracts V:. The results (8)' through (8)"' directly indicate:

$$\hat{\alpha}^L_V = r/\delta^L$$

$$\hat{C}^L_V = 0$$

$$\hat{\pi}^L_V = 1$$

This completes Stage 1. The results should be substituted in the maximization program (1) through (7). Then, one can solve for the optimal first period contracts. That is Stage 2.

(iii) Stage 2: The Optimal First Period Contracts:

In the process of finding the optimal first period contracts, we will use the following definitions (in which we will also substitute the optimal second period contracts as determined in Stage 1).

$$Q^L \equiv [1-\mu]\hat{\pi}^H_{IV}\{\delta^H[R-\hat{\alpha}^H_{IV}] - \bar{\delta}^H\hat{C}^H_{IV}\} + \mu\,\hat{\pi}^L_{IV}\{\delta^L[R-\hat{\alpha}^L_{IV}] - \delta^L\hat{C}^L_{IV}\},$$

implying

$$Q^L = \{1-\mu\}\{\delta^H R^H_N - [1-\beta]\bar{\delta}^H\delta r[\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H]^{-1}\} + \mu\,\delta^L R^L_N \tag{9}$$

$$Q^{LL} \equiv \hat{\pi}^L_V\{\delta^L[R-\hat{\alpha}^L_V] - \bar{\delta}^L\hat{C}^L_V\} = \delta^L R^L_N \tag{10}$$

$$T^{HH} \equiv \hat{\pi}^H_{II}\{\delta^H[R-\hat{\alpha}^H_{II}]-\bar{\delta}^H\hat{C}^H_{II}\} = \delta^H R^H_N \tag{11}$$

$$T^H = [1-\nu]\{\delta^L R^L_N[\delta^L R^H_N + \delta^L\bar{\delta}^H\beta W[\delta^H]^{-1} - \bar{\delta}^L W]^{-1}[\delta^H R^H_N - [1-\beta]\bar{\delta}^H W]\}$$
$$+ \nu\delta^L R^L_N \tag{12}$$

where $R^H_N \equiv R-r[\delta^H]^{-1}$ and $R^L_N \equiv R-r[\delta^L]^{-1}$ are the returns net of repayment obligations to the good and bad borrowers respectively in their first best contracts. (Recall that the first best contract is an unsecured loan).
Substitution of the expressions Q^L, Q^{LL}, T^{HH} and T^H in the maximization program specified in (1) through (7) leads to the following simplified program (again

we substitute where possible the optimal second period contracts).

$$\text{Maximize}_{\substack{\{\alpha_I^i, C_I^i\} \\ i\in\{H,L\}}} \quad \mathcal{T} = \gamma[\delta^L[R-\alpha_I^L] - \bar{\delta}^L C_I^L + \delta^L Q^L + \bar{\delta}^L Q^{LL}] + [1-\gamma][\delta^H[R-\alpha_I^H] - \bar{\delta}^H C_I^H + \delta^H T^{HH} + \bar{\delta}^H T^H] \qquad (1)'$$

subject to,

$$U_I(\delta^L|\delta^L) \geq U_I(\delta^H|\delta^L) \qquad (2)'$$

where,

$$U_I(\delta^L|\delta^L) = \delta^L[R-\alpha_I^L] - \bar{\delta}^L C_I^L + \delta^L Q^L + \bar{\delta}^L Q^{LL}$$

$$U_I(\delta^H|\delta^L) = \delta^L[R-\alpha_I^H] - [1-\delta^L]C_I^H + \delta^L\{[1-\mu]\delta^H + \mu\delta^L\}R_N^H + \bar{\delta}^L\delta^L R_N^L$$

$$U_I(\delta^H|\delta^H) \geq U_I(\delta^L|\delta^H) \qquad (3)'$$

where,

$$U_I(\delta^H|\delta^H) = \delta^H[R-\alpha_I^H] - \bar{\delta}^H C_I^H + \delta^H T^{HH} + \bar{\delta}^H T^H$$

$$U_I(\delta^L|\delta^H) = \delta^H[R-\alpha_I^L] - \bar{\delta}^H C_I^L + \delta^H\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H \delta r\{\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H\}^{-1}\} + \bar{\delta}^H\{[1-\nu]\delta^H + \nu\delta^L\}R_N^L$$

$$\delta^i\alpha_I^i + [1-\delta^i]\beta C_I^i = r, \; i \in \{H,L\} \qquad (4)'$$

$$0 \leq C_I^i \leq W_0, \; i \in \{H,L\} \qquad (5)'$$

$$\alpha_I^i \geq 0, \; i \in \{H,L\}. \qquad (6)'$$

We shall formulate the Lagrangian by taking into account only constraint (2)' in addition to the objective function (1)'. The constraint (4)' will be rewritten as $\alpha_I^i = [r/\delta^i]-\{[1-\delta^i]\beta C_I^i/\delta^i\}$, for $i\in\{H,L\}$, and will be substituted directly in the Lagrangian. From the solution, it will be easy to check that (6)' is slack. We will also show that (3)' is slack. Furthermore, (5)' will be recognized explicitly, once we analyze the first order conditions. This leads to the following Lagrangian

$$\begin{aligned}\mathcal{P} = {} & \gamma[\bar{\delta}^L R_N^L - [1-\beta]\bar{\delta}^L C_I^L + \delta^L Q^L + \bar{\delta}^L Q^{LL}] + [1-\gamma][\delta^H R_N^H - [1-\beta]\bar{\delta}^H C_I^H + \delta^H T^{HH} + \bar{\delta}^H T^H] \\ & + \lambda[\delta^L R_N^L - [1-\beta]\bar{\delta}^L C_I^L + \delta^L Q^L + \bar{\delta}^L Q^{LL} \\ & \quad + \{\delta^L R_N^H - [\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H]C_I^H[\delta^H]^{-1} + \delta^L[\{1-\mu\}\delta^H + \mu\delta^L]R_N^H + \bar{\delta}^L\delta^L R_N^L\}]\end{aligned}$$

Differentiating the Lagrangian with respect to C_I^H gives,

$$\partial\mathcal{P}/\partial C_I^H = -[1-\gamma][1-\beta]\bar{\delta}^H - \lambda\delta^L - \bar{\delta}^H\beta[\delta^H]^{-1} + \lambda\bar{\delta}^L = 0.$$

The equality should hold for a solution satisfying $0 < \hat{C}_I^H < W_0$.

Rearranging the first order condition gives,

$$\lambda = [1-\gamma][1-\beta]\bar{\delta}^H\{\bar{\delta}^L - \delta^L\bar{\delta}^H\beta[\delta^H]^{-1}\}^{-1} > 0.$$

This implies that for any interior solution for $\hat{C}_I^H$, the constraint (2)' is binding. Hence, we have the following result

$$U_I(\delta^L|\delta^L) = U_I(\delta^H|\delta^L). \tag{13}$$

Differentiating the Lagrangian with respect to C_I^L gives,

$$\partial\mathcal{P}\ \partial C_I^L = -\gamma[1-\beta]\bar{\delta}^L - \lambda[1-\beta]\bar{\delta}^L < 0.$$

This directly implies that,

$$\hat{C}_I^L = 0. \tag{14}$$

We now solve (13) for $\hat{C}_I^H$ by using (14), the definitions under (2)' and those given in (9) through (12). This yields

$$\hat{C}_I^H = D_1\{\delta^H\bar{\delta}^L - \delta^L\bar{\delta}^H\beta\}^{-1}\delta^H$$

which implies

$$\hat{C}_I^H = D_2\{\delta^H\bar{\delta}_L - \delta^L\bar{\delta}^H\beta\}^{-1}r > 0. \tag{15}$$

where

$$D_1 \equiv r - \delta^L Q^L - \bar{\delta}^L Q^{LL} - \delta^L r[\delta^H]^{-1} - \bar{\delta}^L\delta^L R_N^L + \delta^L[\{1-\mu\}\delta^H + \mu\delta^L]R_N^H$$

$$D_2 \equiv \delta + \mu\delta^L\delta + [1-\mu]\delta^H\delta^L\{[1-\beta]\bar{\delta}^H\delta\}\{\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H\}^{-1}$$

Since $\hat{C}_I^H$ is positive and finite, there exists a W_0 such that $\hat{C}_I^H$ is an interior solution. The optimal values $\hat{\alpha}_I^L$ and $\hat{\alpha}_I^H$ follow directly from (4)', (14) and (15).

Next, we will prove that the solution does not violate (3)'. Note that, from (4)', (14) and (15) we have

$$\alpha_I^H = r[\delta^H]^{-1} - \bar{\delta}^H\beta\hat{C}_I^H\{\delta^H\}^{-1} \text{ and } \alpha_I^L = r[\delta^L]^{-1}.$$

Now, substituting these results in the definitions of $U_I(\delta^H|\delta^H)$ and $U_I(\delta^L|\delta^H)$, and using (11) and (12), we obtain

$$U_I(\delta^H|\delta^H)=\delta^H R_N^H - [1-\beta]\bar{\delta}^H\hat{C}_I^H + \delta^H\delta^H R_N^H$$

$$+\bar{\delta}^H[\{1-\nu\}\delta^L R_N^L\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H W\}D_3^{-1} + \nu\delta^L R_N^L]$$

$$U_I(\delta^L|\delta^H)=\delta^H R_N^L + \delta^H[\delta^H R_N - [1-\beta]\bar{\delta}^H\delta r\{\delta^H\bar{\delta}^L - \delta^L\bar{\delta}^H\beta\}^{-1}]$$

$$+\bar{\delta}^H\{[1-\nu]\delta^H + \nu\delta^L\}R_N^L$$

where $D_3 \equiv \delta^L R_N^H + \delta^L\bar{\delta}^H\beta W(\delta^H)^{-1} - \bar{\delta}^L W$.

This implies that

$$U_I(\delta^H|\delta^H)-U_I(\delta^L|\delta^H)$$

$$=\delta(\delta^L)^{-1}r - [1-\beta]\bar{\delta}^H\hat{C}_I^H + \{\delta^H[1-\beta][1-\delta^H]\delta r\}\{\delta^H\bar{\delta}^L-\beta\delta^L\bar{\delta}^H\}^{-1}$$

$$+[1-\delta^H][1-\nu]D_3^{-1}[\delta^L R_N^L\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H W\} - \delta^H R_N^L].$$

The above expression is strictly positive because

$$\delta[\delta^L]^{-1}r - [1-\beta]\bar{\delta}^H\hat{C}_I^H + \{\delta^H[1-\beta]\bar{\delta}^H\delta r\}\{\delta^H\bar{\delta}^L-\beta\delta^L\bar{\delta}^H\}^{-1}$$

>

$$\delta[\delta^L]^{-1}r - [1-\beta]\bar{\delta}^H[1+\delta^L]\delta r\{\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H\}^{-1} + \{\delta^H[1-\beta]\bar{\delta}^H\delta r\}\{\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H\}$$

>

$$\delta[\delta^L]^{-1}r - [1-\beta]\bar{\delta}^H\delta r\{\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H\}^{-1}$$

$$= \quad \delta^2 r\{\delta^H\bar{\delta}^L - \beta\delta^L\bar{\delta}^H\}^{-1} > 0.$$

Thus, we have proved that constraint (3)' is not binding in the optimal solution.

(iv) The Non-optimality of Rationing at t=0:

Thus far we have assumed that it is optimal to use collateral in the first period as if it is unconstrained and accept all collateral shortages --

with the attendant rationing -- in the second period. We will now establish this as the optimal strategy.

Once rationing is permitted at t=0, we need to be specific about the information the bank has about borrowers which were rationed at t=0 and which try to enter the credit market again at t=1. We know that rationing only occurs in the separating contract for the good borrowers. Hence, the credit market knows that the borrowers rationed at t=0 are good borrowers for sure at t=0. But what is their type at t=1? As we have seen already, the non-rationed borrowers can be either good or bad at t=1. However, for the rationed borrowers the bank does not observe any first period returns (because these borrowers do not invest). Hence, the bank can not allocate the rationed t=0 borrowers over the sets of nodes II and III, and offer them all sorting contracts. But sorting contracts involves dissipative costs. Non-rationed borrowers do not get a sorting contract in the set of nodes II. This effect tends to make the second period contracts for those who were rationed at t=0 and are good types at t=1 worse than the contracts for those who were not rationed at t=0 and are good types at t=1. On the other hand, borrowers rationed at t=0 did not lose any collateral in the first period. So, their collateral is unconstrained in the second period, implying that their second period sorting contracts do not involve rationing. This effect makes the second period contracts for borrowers who were rationed at t=0 and are good types at t=1 slightly better than the contracts for non-rationed good borrowers. Unfortunately, both effects work in opposite directions. Hence, no unambiguous welfare implications of rationing are possible. This does not alter our result about the suboptimality of rationing at t=0, but it substantially complicates the proof. We, therefore, make the following simplifying assumption.

Assumption: Borrowers who are rationed at t=0 will not abandon their two period projects but will commence investment at t=1. However, due to strategic product market interactions, this delay in investment will cause a decay in the profitability of the project relative to the projects of those borrowers which commenced investment at t=0. The decay in profitability guarantees that the expected second period utility of the investments undertaken by borrowers rationed at t=0 does not exceed the expected second period utility of investments undertaken by those good borrowers which were not rationed at t=0.

The purpose of this restriction is to ensure that rationing does not benefit any borrower. Formally, this leads to the following restriction on the decay parameter ρ, $0 \leq \rho \leq 1$, where ρR is the return on a successful project net of decay. The decay parameter is such that,

$$U_{t=1}(R) \leq U_{t=1}(N)$$

where, $U_{t=1}(R)$ = expected second period utility of a borrower which is at t=1, reported itself to be good at t=0 and was rationed at t=0;

$U_{t=1}(N)$ = expected second period utility of a borrower which is at t=1, reported itself to be good at t=0 and was not rationed at t=0.

Upon substitution of the appropriate expressions (see Figure 1, and the definitions given earlier), this becomes,

$$[1-\Psi]\{\delta^H R_N^H(\rho) - [1-\beta]\bar{\delta}^H \hat{C}^H\} + \Psi\delta^L R_N^L(\rho)$$

$$\leq [\delta^H]^2 R_N^H + \bar{\delta}^H\{[1-\nu]\hat{\pi}_{III}^H\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H \hat{C}_{III}^H\} + \nu\delta^L R_N^L\}$$

where $R_N^i(\rho) = \rho R - r[\delta^i]^{-1}$ for $i \in \{H, L\}$

$\Psi \equiv \bar{\delta}^H \nu$ = probability that a borrower which is rationed at t=0 is a bad borrower in the second period

$\hat{C}^H$ = level of collateral needed for an optimally collateralized (no rationing) single period contract in the second period (note that $\hat{C}^H = \hat{C}_{IV}^H$ because contract IV is also optimally collateralized).

Rewriting the inequality above gives

$$\rho \leq M$$

where

$$M \equiv \{[\delta^H]^2 R_N^H + \bar{\delta}^H\{[1-\nu]\hat{\pi}_{III}^H[\delta^H R_N^H - \{1-\beta\}\bar{\delta}^H \hat{C}_{III}^H] + \nu\delta^L R_N^L\} - r$$
$$-[1-\Psi][1-\beta]\bar{\delta}^H \hat{C}^H\}\{R[\{1-\Psi\}\delta^H - \Psi\delta^L]\}^{-1}.$$

It is possible for M to exceed unity. Thus, we must impose the following parametric restriction on ρ,

$$\rho \leq M \wedge 1,$$

where "$\wedge$" is the "min" operator. For later use, define

$$\Phi \equiv [\delta^H \bar{\delta}_L - \beta\delta^L \bar{\delta}^H][\delta^H]^{-1}, \quad \xi \equiv [\delta^H \bar{\delta}^L - \nu\delta^L \bar{\delta}^H][\delta^H]^{-1}.$$

We now have the following result.

<u>LEMMA 1</u>: Given the assumed restriction on collateral availability that $\hat{C}_I^H \leq W_0 < \hat{C}_I^H + C_{III}^H(\text{opt})$, where $C_{III}^H(\text{opt})$ is the level of collateral necessary for an optimally collateralized contract, and the restriction that $\rho \leq M \wedge 1$, rationing a borrower at t=0 is always inferior to rationing at t=1. Hence,

collateral is most efficiently used at t=0 and should never be "saved" for later use if doing so causes rationing at t=0.

One implication of this lemma is that in a dynamic credit market, if rationing occurs it is more likely to affect a borrower which has borrowed and defaulted rather than one which is borrowing de novo. This is a potentially testable prediction.

(v) Equilibrium With Single Period Contracting:

We now gather all of the results obtained thus far and present the complete equilibrium solution.

THEOREM 2: Assuming that

(1) each bank is constrained to earn zero expected profit in each period.

(2) collateral availability is limited in the sense that
$\hat{C}_I^H \leq W_0 < \hat{C}_I^H - C_{III}^H(\text{opt})$, and

(3) $\rho \leq M \wedge 1$.

the DRE, if it exists, is given by

$$\hat{\alpha}_I^H = r[\delta^H]^{-1} - \bar{\delta}^H \beta \hat{C}_I^H [\delta^H]^{-1},$$

$$\hat{C}_I^H = [\delta + \mu\delta^L\delta + (1-\mu)\delta^L[1-\beta]\bar{\delta}^H\delta\phi^{-1}]r[\phi\delta^H]^{-1},$$

$$\hat{\pi}_I^H = 1,$$

$$\hat{\alpha}_I^L = r/\delta^L,\ \hat{C}_I^L = 0,\ \hat{\pi}_1^L = 1;$$

$$\hat{\alpha}_{II}^H = r/\delta^H,\ \hat{C}_{II}^H = 0,\ \hat{\pi}_{II}^H = 1;$$

$$\hat{\alpha}_{III}^H = r[\delta^H]^{-1} - \bar{\delta}^H \beta \hat{C}_{III}^H [\delta^H]^{-1},\ \hat{C}_{III}^H = W,$$

$$\hat{\pi}_{III}^H = \{\delta^L[R-\hat{\alpha}_{III}^L]\}\{\delta^L[R-\hat{\alpha}_{III}^H]-[1-\delta^L]W\}^{-1}$$

$$\hat{\alpha}^L_{III} = r/\delta^L,\ \hat{C}^L_{III} = 0,\ \hat{\pi}^L_{III} = 1;$$

$$\hat{\alpha}^H_{IV} = r[\delta^H]^{-1} - \bar{\delta}^H \beta \hat{C}^H_{IV}[\delta^H]^{-1},\ \hat{C}^H_{IV} = \delta r[\phi\delta^H]^{-1},\ \hat{\pi}^H_{IV} = 1;$$

$$\hat{\alpha}^L_{IV} = r[\delta^L]^{-1}, \hat{C}^L_{IV} = 0,\ \hat{\pi}^L_{IV} = 1;$$

$$\hat{\alpha}^L_V = r[\delta^L]^{-1},\ \hat{C}^L_V = 0,\ \hat{\pi}^L_V = 1.$$

This theorem points out two sources of welfare losses for good borrowers in the DRE restricted to spot contracting. One source is the dissipative cost associated with the collateral good borrowers put up in the contract nodes I and III, and the other is the possible rationing in contract node III. We show in the next section how intertemporal contracting helps to reduce welfare losses.

IV. DRE WITH INTERTEMPORAL CONTRACTING

A. Introduction and Basic Results

With intertemporal contracting, a bank need not make zero expected profit on a given borrower in <u>each</u> time period. It can tax the borrower in one period and subsidize it in the next. Of course, our competitive equilibrium concept imposes a restriction on allowable tax-subsidy schemes. The restriction is that the bank will not subsidize a borrower's first period contract with the intention of recouping the loss through a positive expected profit contract on that borrower in the second period. The reason is that, after the first period, the borrower has an incentive to switch to another bank in the spot credit market, leaving the original bank with a loss. Thus, the only feasible subsidy is on the second period contract. Note also that the DRE precludes <u>cross-sectional</u> taxes and subsidies, i.e., positive profits for the bank on one borrower and negative profits on another.

We shall see in this section that permitting intertemporal contracting significantly alters the DRE characterized in Theorem 2. One of the contract variables that changes is C_I^H. Since rationing occurs with single period contracting for all $W_0 \in [\hat{C}_I^H, \hat{C}_I^H + C_{III}^H(\text{opt}))$, we need to show that there exists a compact subset [a,b] of $[\hat{C}_I^H, \hat{C}_I^H + C_{III}^H(\text{opt}))$ such that, for all $W_0 \in [a,b]$, rationing occurs in the DRE with single period contracting but not in the DRE with intertemporal contracting. The following technical result helps in establishing the desired result.

<u>LEMMA 2</u>: For all $C_{III}^H \in [0, C_{III}^H(\text{opt})]$, we have

$$\{\delta^H R_N^H - [1-\beta]\bar{\delta}^H C_{III}^H\}\{\delta^L R_N^H - \phi C_{III}^H\}^{-1} \geq \delta^H[\delta^L]^{-1}. \tag{16}$$

With this lemma, we can now establish one of our main results. We will use tildes to denote equilibrium values.

<u>THEOREM 3</u>: Suppose the DRE with intertemporal credit contracting exists and involves $W_0 \in [\tilde{C}_I^H, \tilde{C}_I^H + C_{III}^H(\text{opt}))$, where $\tilde{C}_I^H$ is the optimal first period use of collateral for a given W_0 and a non-binding first period collateral constraint. Then, a sufficient condition for <u>no</u> credit rationing to occur in the DRE at <u>any</u> time is

$$\nu \leq \beta. \tag{17}$$

This theorem can be interpreted as follows. <u>Assuming</u> a nonempty intersection for the feasible sets to which W_0 belongs in Theorems 2 and 3, intertemporal contracting eliminates rationing for those values of W_0 lying in the intersection. We will prove later that this intersection is nonempty.

To see the intuition behind this theorem, one needs to study its proof. It is apparent from the proof that rationing is avoided by improving the borrower's expected utility -- relative to that in the single period contracting DRE -- in each of the pair of contracts in contract node III. The contract for the good borrower is improved by an increase in the credit granting probability. To preserve incentive compatibility, the bad borrower's contract needs to be improved too. However, this adversely affects incentive compatibility at t=0 because borrowers which are bad at t=0 now find it more attractive to mimick the good borrowers at t=0. The reason is that these borrowers are the ones more likely to end up at t=1 as borrowers choosing the contract for bad borrowers in contracts node III. This incentive compatibility problem is resolved by increasing collateral requirements at t=0 for borrowers reporting themselves as good. Note that $1-\beta$ can be interpreted as a "standardized" measure of the costs of resolving the incentive compatibility problem. On the other hand, $1-\nu$ -- the fraction of good borrowers among all borrowers offered contracts node III at t=1 -- may be interpreted as a "standardized" measure of the incremental revenues attributable to the elimination of rationing. A large $1-\nu$ indicates that, when rationing is eliminated, a relatively large proportion of the borrower pool is positively affected. Thus, (17) can be viewed as a "cost-revenue" condition.

Henceforth, the conditions stated in Theorem 3 will be assumed to be satisfied. Consequently, the credit granting probability as a sorting instrument is rendered superfluous. One only needs to deal with interest rates and collateral requirements as sorting instruments. It is fortuitous that these variables enter the model linearly, since we can derive the equilibrium allocations by using linear programming (LP) techniques.

B. Model Specification and the DRE

(i) General Remarks:

The following observations help to significantly simplify the model. Intertemporal contracts enable the banks to offer bad borrowers at t=0 a first best contract over their entire two period time horizon. In the single period contracting DRE described in Theorem 2, bad borrowers at t=0 do not get a first best contract over their entire time horizon. This is because they can end up in the contract IV nodes where separating contracts are offered. The contract for good borrowers in this set involves costly collateral, making the contract worse than first best. However, intertemporal contracting enables the bad borrower at t=0 to pay a higher first period interest rate in exchange for a contract that is first best for those borrowers who are good among all borrowers ending up in the contract IV nodes. Even if an initially bad borrower ends up being bad at t=1, it has paid in the first period for this gain. This construction obviates the need for separating contracts in the contract IV nodes and eliminates collateral costs there, ensuring that borrowers which are bad at t=0 never put up any collateral. Incentive compatibility is not a concern here. This way the bad borrowers at t=0 can be given a contract which yields them an average, per period expected utility equal to first best. Furthermore, the contract cannot be made better than first best because this would imply losses to the bank, forcing it to earn positive profit on the contract for good borrowers at t=0. This is not possible in the DRE. These observations allow us to focus exclusively on the intertemporal contracts for good borrowers at t=0, as long as we take into account the first period incentive compatability constraint that bad borrowers at t=0 prefer their own (first best) contract to the contract for good borrowers at t=0. We can now obtain the DRE by searching for the allocation that maximizes the two period expected utility of the borrower which is good at t=0, subject to the relevant constraints. Given the linearity of the model, this is the same as maximizing the negative of the borrowing costs of the good borrower at t=0 over its two period time horizon. With the help of Figure 1, we see that this implies the following objective function.

$$\underset{\{\alpha_I^H, C_I^H, \alpha_{II}^H, \alpha_{III}^H, C_{III}^H, \alpha_{III}^L\}}{\text{Maximize}} \quad \mathcal{F} = -\delta^H\alpha_I^H - \bar{\delta}^H C_I^H - \delta^H\delta^H\alpha_{II}^H - \bar{\delta}^H[1-\nu][\delta^H\alpha_{III}^H + \bar{\delta}^H C_{III}^H] - \bar{\delta}^H\nu\delta^L\alpha_{III}^L \tag{18}$$

We maximize (18) subject to the following constraints.

(A) Non-negativity constraint on the bank's profits: The costs for the bank of lending to a borrower which is good at t=0 is r in each period. Hence, the total expected intertemporal interest and collateral receipts for the bank should not be less than 2r. That is,

$$\delta^H\alpha_I^H + \bar{\delta}^H\beta C_I^H - \delta^H\delta^H\alpha_{II}^H + \bar{\delta}^H[1-\nu][\delta^H\alpha_{III}^H + \bar{\delta}^H\beta C_{III}^H] + \bar{\delta}^H\nu\delta^L\alpha_{III}^L \geq 2r \tag{19}$$

(B) Constraints on second period contracts: Since the bank can tax only first period contracts, in terms of interest rates and collateral, the following restrictions apply,

$$\delta^H\alpha_{III}^H \leq r \tag{20}$$

$$\delta^H\alpha_{III}^H + \bar{\delta}^H\beta C_{III}^H \leq r \tag{21}$$

$$\delta^L\alpha_{III}^L \leq r \tag{22}$$

(C) Incentive compatibility constraint for the contracts III nodes: Here we assume that it is always optimal to use all available collateral in the contract for good borrowers in the contracts node III. In a subsequent lemma, we will formally prove this. The incentive compatibility conditions are, $U_{III}(\delta^H|\delta^H) - U_{III}(\delta^L|\delta^H) \geq 0$,

which implies the following constraints

$$-\delta^H\alpha^H_{III} - \bar{\delta}^H C^H_{III} + \delta^H\alpha^L_{III} > 0 \quad (23)$$

and, $U_{III}(\delta^L|\delta^L) - U_{III}(\delta^H|\delta^L) \geq 0$, which is identical to,

$$-\delta^L\alpha^L_{III} + \delta^L\alpha^H_{III} + \bar{\delta}^L C^H_{III} \geq 0. \quad (24)$$

(D) Incentive compatibility conditions for contract I: As usual, the conditions are, $U_I(\delta^L|\delta^L) - U_I(\delta^H|\delta^L) \geq 0$, which implies (note that 2r is the expected interest cost over two periods for the first best contract for bad borrowers),

$$\delta^L\alpha^H_I - \bar{\delta}^L C^H_I + \delta^L[\{1-\mu\}\delta^H + \mu\delta^L]\alpha^H_{II} + \bar{\delta}^L\delta^L\alpha_{III}{}^H \geq 2r \quad (25)$$

and $U_I(\delta^H|\delta^H) - U_I(\delta^L|\delta^H) \geq 0$. This latter condition is slack as usual. But we shall formally verify this.

We will solve the model subject to the constraints (19), (21), (24) and (25). Subsequently, we will show that the other constraints are slack. As in the single period contract solution, the individual rationality constraints for borrowers are superfluous.

The assumption that $W_0 \in [\tilde{C}^H_I, \tilde{C}^H_I - C^H_{III}(\text{opt}))$ implies $0 \leq C^H_{III} < C^H_{III}(\text{opt})$. From earlier analysis, we know that $C_{III}(\text{opt}) = \delta r[\phi\delta^H]^{-1}$. Thus,

$$0 \leq C^H_{III} < \delta r[\phi\delta^H]^{-1}.$$

Without loss of generality we can rewrite this as

$$C^H_{III} = \theta\delta r[\phi\delta^H]^{-1}, \ \theta \in [0,1). \quad (26)$$

We can now present the main result of this section.

THEOREM 4: Assume that, as long as $0 \leq W_0 - \tilde{C}^H_I < C^H_{III}(\text{opt})$, all available collateral will be demanded of good borrowers in contracts node III. Then, if it exists, the DRE with intertemporal credit contracting is given by

$$\tilde{\alpha}_{I}^{H} = [1+\delta^{H}]r[\delta^{H}]^{-1} - \bar{\delta}^{H}\beta\tilde{C}_{I}^{H}[\delta^{H}]^{-1} + [\bar{\delta}^{H}\delta r\{1-\Theta\}\nu\Phi][\delta^{H}\delta^{H}\Phi]^{-1},$$

$$\tilde{C}_{I}^{H} = \delta r[1+\{1-\Theta\}\xi][\Phi\delta^{H}]^{-1};$$

$$\tilde{\alpha}_{II}^{H} = 0;$$

$$\tilde{\alpha}_{III}^{H} = r[\delta^{H}]^{-1} - \bar{\delta}^{H}\beta\tilde{C}_{III}^{H}[\delta^{H}]^{-1}, \ \tilde{C}_{III}^{H} = \Theta\delta r[\Phi\delta^{H}]^{-1};$$

$$\tilde{\alpha}_{III}^{L} = r[\delta^{L}]^{-1}-[1-\Theta]\delta r[\delta^{L}\delta^{H}]^{-1};$$

$$\tilde{\alpha}_{I}^{L} = r[2 - \{[\delta^{L}]^{2} + \delta^{H}\bar{\delta}^{L}\}\{\delta^{H}\delta^{L}\}^{-1}];$$

$$\tilde{\alpha}_{IV}^{H} = \tilde{\alpha}_{IV}^{L} = r[\delta^{H}]^{-1}, \ \tilde{\alpha}_{V}^{L} = r[\delta^{L}]^{-1}.$$

To complete the characterization of the DRE, we need to establish that the collateral assumption in the above theorem is unnecessary. This is done now.

<u>LEMMA 3</u>: It is optimal to demand all available collateral of good borrowers in the contracts node III as long as $0 \le W_{0} - \tilde{C}_{I}^{H} < C_{III}^{H}(\text{opt})$.

The intuition is as follows. Demanding less collateral of good borrowers in the contracts node III would necessitate a further subsidy to bad borrowers in that node in order to preserve incentive compatibility at t=1. But such a subsidy causes incentive compatibility to break down at t=0. The restoration of incentive compatibility is achieved by demanding higher collateral from every good borrower at t=0. The benefit is a lower collateral requirement for a fraction $[1-\delta^{H}][1-\nu]$ of all borrowers which are good at t=0. However, the dissipation due to an increase in collateral requirements for <u>all</u> good borrowers at t=0 turns out to be greater than the gain due to a reduction in collateral requirements at t=1 for a fraction of these borrowers. This is inefficient unless increasing collateral at t=0 helps reduce rationing at t=1. (A similar strategy of increasing the good borrower's collateral requirement at t=0 is part of the DRE (see Theorem 4) since it eliminates rationing in the contract III nodes. This is optimal because rationing is more dissipative than collateral.) But in this case, our starting point is that there is no rationing at t=0. Given this, it does not pay to shift collateral use from t=1 to t=0.

V. COMPARISON OF SINGLE PERIOD CONTRACTING DRE WITH INTERTEMPORAL CONTRACTING DRE

Before we can conclude that intertemporal contracting indeed eliminates the rationing encountered with single period contracting, we need to verify that the collateral availability assumption made in Theorems 2 and 4 are compatible. This is done below.

LEMMA 4: For any set of parameter values, ∃ some positive and finite W_0 such that the collateral assumptions in Theorems 2 and 4 are simultaneously met. Expressed in terms of exogenous parameters, choices of W_0 that achieve this compatibility satisfy

$$W_0 \in \begin{cases} [W_0^{\ell}, W_0^{u}) & \text{if } \mu\delta^L + [1-\beta]\bar{\delta}^H\delta^L[1-\mu]\phi^{-1} > \xi \\ [\hat{W}_0^{\ell}, W_0^{u}) & \text{if } \mu\delta^L + [1-\beta]\bar{\delta}^H\delta^L[1-\mu]\phi^{-1} \le \xi \end{cases} \tag{27}$$

where $W_0^{\ell} \equiv \delta r[\phi\delta^H]^{-1}[1 + \mu\delta^L + (1-\beta)\bar{\delta}^H\delta^L(1-\mu)\phi^{-1}]$

$\hat{W}_0^{\ell} \equiv \delta r[\phi\delta^H]^{-1}[1+\xi]$

$$W_0^u \equiv 2\delta r[\phi\delta^H]^{-1}$$

With this lemma in hand, we can compare the two DRE's. The major result is that intertemporal contracting eliminates the rationing that occurs in the contract III nodes. Since these nodes follow first period project failure, the implication is that credit rationing, if it occurs, is likely to be encountered after borrower default. This seems consistent with casual empiricism.

Comparing Theorems 2 and 4 shows that $\tilde{\alpha}^L_{III} < \hat{\alpha}^L_{III}$. In fact, with intertemporal contracting, the contract for bad borrowers in the contract III nodes is better than the single period first best contract for such borrowers. The disadvantage of this is that it jeopardizes incentive compatibility at t=0. The advantage is that there is an offsetting positive effect, manifested in a lower collateral requirement now being sufficient to separate borrowers at t=1 following project failure. The combined effect is such that rationing is unnecessary.

A second interesting feature of the DRE with intertemporal contracting is that successful past performance can be rewarded. This reward is striking. Borrowers which report themselves to be good at t=0 <u>and</u> are successful in the first period are rewarded with $\tilde{\alpha}^H_{II}=0$.[20] Notice that asking good borrowers at t=0 to pay a relatively high first period interest rate and then giving them a "free" second period loan is incentive compatible. This is because good borrowers -- more likely to have a successful first period realization -- are more willing than bad borrowers to pay a higher first period interest rate in exchange for a free second period loan.

There is, consequently, an interesting "carrot and stick" distinction between the intertemporal contracting DRE and the single period contracting

DRE. In the former, revelation incentives are positively enforced by offering rewards to good borrowers who indeed succeed. In the latter, revelation incentives are enforced by the threat of ex post punishment (rationing) for good borrowers that fail.

VI. EXISTENCE OF THE DRE

A. Introduction and Basic Results:

We will now examine the question of whether a dynamic equilibrium exists. We will prove the existence of both the single period contracting DRE in Theorem 2 and the intertemporal contracting DRE in Theorem 4. In the former case, defecting banks are limited to only those contracts that break even in each period, although not necessarily those that break even on each type within a period. In the latter case, defecting banks have unrestricted latitude in their choice of contracts.

As in the case of equilibria in static models, defections that can threaten the DRE take the form of nonequilibrium pooling contracts being offered. Since pooling contracts do not sort borrowers, and collateral and rationing are dissipative sorting devices, neither will be used in a pooling contract. Further, an efficient intertemporal pooling contract will utilize all observable information. This implies that an intertemporal pooling contract will specify one interest rate for the first period for all borrowers, and two interest rates -- one for each first period realization -- for the second period.[21] Thus, it is not optimal to have a contract offered at t=0 that pools completely across time and types, offering all borrowers the same interest rate over both periods regardless of first period performance. We now introduce the following additional notation,

α_1 = first period pooling interest rate,

α_2^G = second period pooling interest rate conditional on a good first period realization

α_2^B = second period pooling interest rate conditioned on a bad first period realization.

The following lemma is very useful in the proof of existence.

LEMMA 5: Suppose there exists a (separating) Nash equilibrium in the (single period) spot market at t=1. Then, if the bank is restricted to earn zero expected profit in each period, an optimal pooling contract at t=0 cannot be completely pooling, in the sense that borrowers with bad first period realizations will be offered a pair of (separating) contracts at t=1.

The starting point of this lemma is the assumption that it is possible to separate borrowers at t=1 and the observation that the optimal pooling contract can at best be partially pooling since it must distinguish between borrowers in the second period based on first period outcome. It then goes on to say that, even among the class of such partially pooling contracts, it is inefficient to have contracts that offer the same second period loan interest rate to all the borrowers with bad first period realizations. The lemma asserts that the conditions under which this is true are less restrictive than those needed to sustain a Nash equilibrium -- which we know is fully separating, from the work of Rothschild and Stiglitz (1976) --in the spot market at t=1. It is reasonable to require that a Nash equilibrium (and hence a Riley (1979) equilibrium) exists in the spot market at t=1, so that borrowers have the opportunity to avail of spot credit then as an alternative to utilizing long term commitments.

This lemma is intuitive. The existence of a separating Nash equilibrium in the spot market at t=1 depends on how attractive the pooling allocation is to the good borrowers. The pooling allocation reflects the relative proportion of good and bad borrowers in the population. In the lemma, the pool in question consists of all borrowers with bad first period outcomes. Thus, the proportion of bad borrowers in this pool is high. This implies that the pooling contract has an interest rate closer to the first best interest rate for bad borrowers than that for good borrowers. It is, therefore, unappealing to good borrowers, creating the impetus for competing banks to offer contracts that sort borrowers. This lemma rests on the assumption that the bank is constrained to earn zero expected profit in each period. We will next introduce the possibility of intertemporal subsidies.

B. The Optimal Pooling Contract:

The previous lemma indicates that, with a period-by-period zero profit constraint, the optimal intertemporal pooling contract is likely to involve a pooling interest rate α_1 for all borrowers in the first period, a pooling interest rate α_2^G in the second period for all borrowers with good first period realizations, and a pair of separating contracts in the second period for borrowers with bad first period realizations. We will establish formally that, even with intertemporal subsidies allowed, the optimal pooling contract indeed takes this form as long as collateral is not "too costly."

The pair of separating contracts for borrowers with bad first period realizations involves a secured loan contract, $\{\alpha_B^H, C_B^H\}$, specifying an interest factor and a collateral requirement, and an unsecured loan contract, $\{\alpha_B^L\}$. The former will be taken by the good borrowers from the pool of those unsuccessful

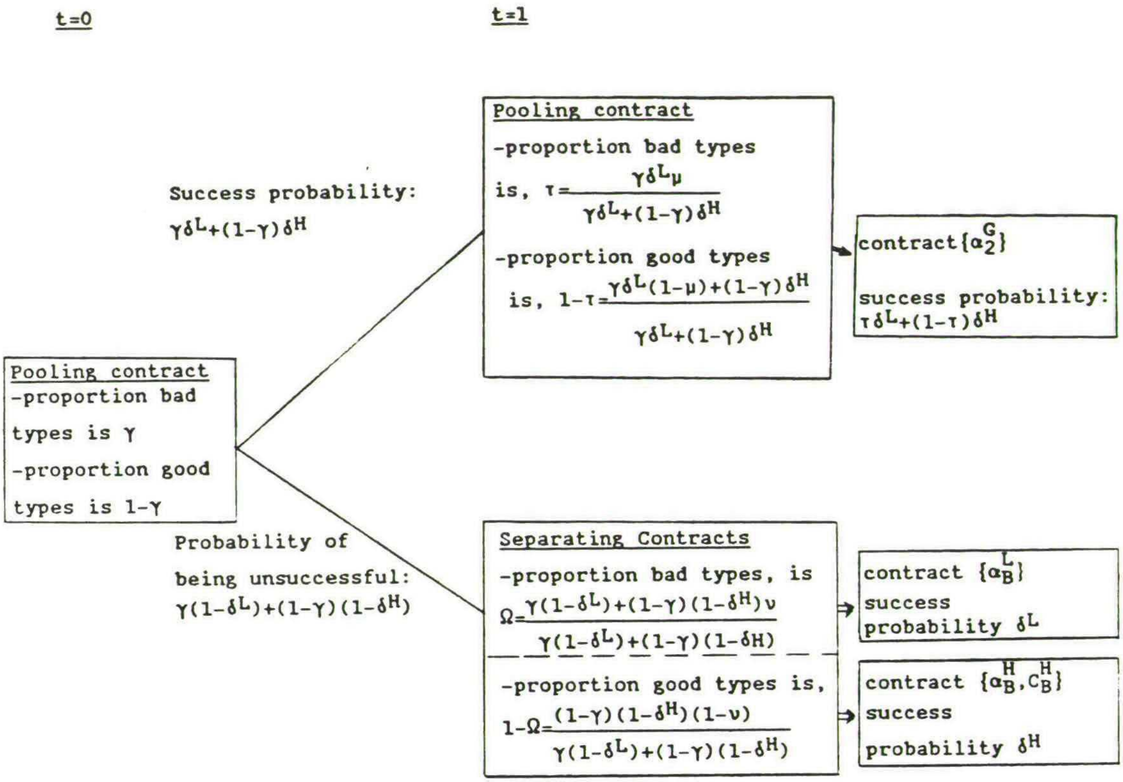


Figure 2: The relevant data in the pooling contract alternative.

in the first period, and the latter will be taken by the bad borrowers from that pool. From Figure 1, we now have <u>Figure 2</u>.

[INSERT FIGURE 2 ABOUT HERE]

In the maximization problem below, we take advantage of the well-known result that maximizing the weighted sum of the expected utilities of the good and bad borrowers is equivalent to maximizing the utility of the good borrower subject to the constraint that the bad borrower get its first best utility (see Spence (1978), for example). Using Figure 2, we get the following objective function.

$$\underset{\{\alpha_1,\alpha_2^G,\alpha_B^H,C_B^H,\alpha_B^L\}}{\text{Maximize}} \quad \mathcal{F} = -\delta^H\alpha_1 - \delta^H\delta^H\alpha_2^G - \bar{\delta}^H[1-\nu][\delta^H\alpha_B^H + \bar{\delta}^H C_B^H] - \bar{\delta}^H\nu\delta^L\alpha_B^L \qquad (28)$$

This objective function should be maximized subject to the following constraints.

(a) <u>The non-negativity condition for the bank's profits</u>: The cost of lending to a borrower is r in both periods. Hence, the total expected interest and collateral receipts for the bank should be at least equal 2r per borrower.

$$[\{1-\gamma\}\delta^H + \gamma\delta^L]\alpha_1 + [\{1-\gamma\}\delta^H\delta^H + \gamma\delta^L\{1-\mu\}\delta^H + \gamma\delta^L\mu\delta^L]\alpha_2^G$$
$$+ [1-\gamma]\bar{\delta}^H[1-\nu][\delta^H\alpha_B^H + \bar{\delta}^H\beta C_B^H] + [\{1-\gamma\}\bar{\delta}^H\nu + \gamma\bar{\delta}^L]\delta^L\alpha_B^L \geq 2r. \qquad (29)$$

(b) <u>The feasibility constraint for the bad borrower's intertemporal utility</u>: In the (separating) intertemporal solution, bad borrowers get a contract over their two period horizon that generates an average expected utility per period that is equal to first best. That is, their expected total interest costs are 2r. Futhermore, in equilibrium good borrowers at t=0 do not covet the contract of bad borrowers. This implies that any feasible pooling contract must offer bad borrowers at t=0 a contract that yields them at least a first level of expected utility over the two periods. That is,

$$\delta^L\alpha_1 + \delta^L[(1-\mu)\delta^H + \mu\delta^L]\alpha_2^G + (1-\delta^L)\delta^L\alpha_B^L \leq 2r \tag{30}$$

(c) Spot market constraints on second period contracts: The second period pooling interest factor for the pool of borrowers successful in the first period is bounded above by the available spot market pooling interest factor. From Figure 2, one can infer that the success probability for this pool is

$\delta^G = \tau\delta^L + [1-\tau]\delta^H$, where $\tau \equiv \gamma\delta^L\mu(\gamma\delta^L + [1-\gamma]\delta^H)^{-1}$. Thus,

$$\delta^G = \{[1-\gamma][\delta^H]^2 + \gamma\delta^L[1-\mu]\delta^H + \gamma[\delta^L]^2\mu\}\{[1-\gamma]\delta^H + \gamma\delta^L\}^{-1}$$

The spot market pooling interest rate in the second period is r/δ^G for this pool. This puts the following constraint on the pooling interest rate α_2^G (substitute in r/δ^G the expression for δ^G)

$$\alpha_2 \leq \{[1-\gamma]\delta^H + \gamma\delta^L\}r\{[1-\gamma][\delta^H]^2 + \gamma\delta^L[1-\mu]\delta^H + \gamma[\delta^L]^2\mu\}^{-1}. \tag{31}$$

With respect to the separating contracts for the pool of borrowers unsuccessful in the first period, the following constraints hold.

$$\delta^H\alpha_B^H + \bar{\delta}^H\beta C_B^H \leq r \tag{32}$$

$$\delta^L\alpha_B^L \leq r \tag{33}$$

(d) Incentive compatibility constraints for the separating contracts for the pool of borrowers unsuccessful in the first period: One incentive compatibility condition is $U_B(\delta^H|\delta^H) - U_B(\delta^L|\delta^H) \geq 0$, which implies

$$-\delta^H\alpha_B^H - \bar{\delta}^H C_B{}^H + \delta^L\alpha_B^L \geq 0 \tag{34}$$

and the other is, $U_B(\delta^L|\delta^L) - U_B(\delta^H|\delta^L) \geq 0$, which implies,

$$-\delta^L\alpha_B^L + \delta^L\alpha_B^H + \bar{\delta}^L C_B{}^H \geq 0. \tag{35}$$

In addition to the constraints above, all variables are also assumed to be non-negative. Since the model is linear, it can be solved by applying the simplex algorithm of linear programming. We will solve the model subject to the constraints (29), (30), (32), (33) and (35). The other conditions will be shown to hold. The solution is presented in the next theorem, for which the following definition is useful. Define

$$F \equiv -[\bar{\delta}^H \nu[1-\gamma] + \gamma\bar{\delta}^L]\delta^L\delta^H\}\{[1-\gamma]\delta^H + \gamma\delta^L\}^{-1}$$
$$+ \{[\bar{\delta}^H]^2[1-\nu][1-\beta]\delta^L\}\phi^{-1} + \bar{\delta}^H\nu\delta^L.$$

<u>THEOREM 5</u>: The optimal intertemporal pooling contract, obtained by maximizing (28) subject to the constraints (29)-(35), is as follows (bars on endogenous contract variables denote optimal values here)

(36) If $F \leq 0$, then

$$\bar{\alpha}_1 = \{2r\}\{[1-\gamma]\delta^H + \gamma\delta^L\}^{-1} - \{[1-\gamma]\bar{\delta}^H + \gamma\bar{\delta}^L\}r\{[1-\gamma]\delta^H + \gamma\delta^L\}^{-1},$$
$$\bar{\alpha}_2^G = 0;$$
$$\bar{\alpha}_B^H = r[\delta^H]^{-1} - \bar{\delta}^H\beta\bar{C}_B^H[\delta^H]^{-1}, \quad \bar{C}_B^H = \delta r[\phi\delta^H]^{-1}, \quad \bar{\alpha}_B^L = r[\delta^L]\}^{-1};$$

(37) If $F > 0$, then

$$\bar{\bar{\alpha}}_1 = 2r\{[1-\gamma]\delta^H + \gamma\delta^L\}^{-1} - \{[1-\bar{\gamma}]\delta^H + \bar{\gamma}\delta^L\}\delta^L r\{\delta^H[\{1-\gamma\}\delta^H + \gamma\delta^L]\}^{-1},$$
$$\bar{\bar{\alpha}}_2^G = 0,$$
$$\bar{\bar{\alpha}}_B^H = r/\delta^H, \quad \bar{\bar{C}}_B^H = 0, \quad \bar{\bar{\alpha}}_B^L = r/\delta^H.$$

This theorem shows that bad borrowers at t=0 are awarded a two period contract that generates a higher average expected utility per period than their first best contract. Given the bank's breakeven condition, this implies an expected utility for good borrowers that is lower than first best. It is a little surprising that the best intertemporal pooling contract is unable to always produce a first best outcome for all borrowers. It is obvious that a pooling contract in a static model cannot generate the first best outcome for the good borrowers. However, it might appear that the possibility of rewarding successful borrowers and punishing unsuccessful borrowers at the end of the first period would lead to first best allocations for both types. But this is not the case. Good borrowers get less than their first best and bad borrowers more than their first best. Moreover, the (partially) pooling solution in (36) indicates that separating contracts will be used for unsuccessful first period borrowers. Separation involves collateral with its attendant deadweight losses. It is easy to show that the solution in (36) is optimal for all $\beta \in [1-u, 1]$ for $u>0$ sufficiently small. That is, if collateral is relatively costless -- β is close to 1 -- then collateral will be used in the optimal pooling contract. The condition $\beta \in [1-u, 1]$ guarantees that $F \leq 0$ -- then it is optimal to not use it. The solution in (37) involves second period contracts that are unsecured for all borrowers; borrowers successful in the first period all receive one second period interest factor and borrowers unsuccessful in the first period all receive another second period interest factor. Thus, there is no separation beyond that possible by observing first period outcomes, and collateral is avoided. This implies that relaxation of the period-by-period zero profit constraint for the bank enables second period pooling to be optimal even following first period failure, as long as collateral is relatively costly.

C. Existence of Equilibrium

In examining existence, we will focus on the pooling contract in (36) as the main threat. This is done for two reasons. First, it makes the exposition parsimonious. Second, except for some differences in parametric restrictions, the pooling contracts in (36) and (37) provide the same insights into the existence of the DRE. We now have the principal result of this section.

<u>THEOREM 6</u>: The following conditions are sufficient for the <u>simultaneous</u> existence (that is, both equilibria exist for the same set of exogenous parameter values) of the single period contracting DRE in Theorem 2 and the intertemporal contracting DRE in Theorem 4:

(a) <u>Single Period Contracting DRE</u>:

$$-\{1-\delta^H[\delta_\gamma^H]^{-1}\}r-[1-\beta]\bar{\delta}^H\hat{C}_1^H - \bar{\delta}^H[1-\nu]J_1 + [1-\beta]\bar{\delta}^H\delta r\Phi^{-1} \geq 0, \tag{38}$$

where $J_1 \equiv \delta^H R_N^H - [1-\beta]\bar{\delta}^H\delta r[\Phi\delta r[\Phi\delta^H]^{-1} - \hat{\pi}_{III}^H[\delta^H R_N^H - \bar{\delta}^H[1-\beta]W\}$

$\bar{\delta} \equiv [1-\gamma]\delta^H + \gamma\delta^L$

$$\delta^H[1-R-r\{[1-\nu]\delta^H + \nu\delta^L\}^{-1} \leq \hat{\pi}_{III}[\delta^H R_N - [1-\beta]\bar{\delta}^H W] \tag{39}$$

$$\bar{\delta}^H[1-\beta][\Phi\delta^H]^{-1} \leq \mu[\{1-\mu\}\delta^H - \mu\delta^L]^{-1} \tag{40}$$

(b) <u>Intertemporal Contracting DRE</u>:

$$[1-\delta^H]r + J_2 - J_3 - [\delta^H]^2[1-\nu][1-\beta][1-\Phi]\delta r[\Phi\delta^H]^{-1} \tag{41}$$

where $J_2 \equiv \delta^H\{-[1-\delta^H] + \gamma\delta\}r\{\delta_\gamma^H\}^{-1}$

$J_3 \equiv \delta^H[1-\beta][1-\{1-\Phi\}\xi]\delta r[\Phi\delta^H]^{-1}$

$\xi \equiv \{\delta^H\delta^L - \nu\delta^L\delta^H\}\{\delta^H\}^{-1}$.

$$\nu \leq \beta \tag{42}$$

$$F \leq 0 \tag{43}$$

(c) The conditions stated in Lemma 4.

Moreover, the set of exogenous parameter values for which all of the above conditions simultaneously hold is nonempty.

This theorem states the joint conditions for the simultaneous existence of the single period contracting DRE in Theorem 2 and the intertemporal contracting DRE in Theorem 4. It guarantees that, under these conditions, there does not exist any (partially or fully) pooling contract that can upset the DRE.

VII. EXTENSIONS AND EMPIRICAL IMPLICATIONS

A. Model Extensions

Our analysis indicates that borrowers may purchase credit commitments to insure themselves against future rationing. Are there any borrowers, then, that would actually get rationed? Although strictly within the context of our model the answer is no, one can think of a (slight) perturbation of the model that would produce borrowers which do not purchase commitments in equilibrium, and hence risk being rationed. Suppose there was a third group of borrowers which enter the credit market at t=0 but are uncertain of needing any credit at t=1. That is, these borrowers have some probability, $\omega \in (0,1)$, of having a project available to invest in at t=1. Such borrowers do not invest in the second period if no project is available. The realization of project availability is independent of the realization of the borrower's first period project payoff. Call such a borrower "possibly transitory" (p.s.). Suppose p.s. borrowers are observationally identical to all other borrowers at t=0, but each has a first period success rpobability of δ^{H}. Also, if a p.s. borrower invests in the second period, it is no different from a borrower that was good at t=0 and had the same first period project payoff as that p.s. borrower. Now, in an intertemporal contracting DRE, since <u>all</u> borrowers receive contracts that tax them in the first period and then subsidize them in the second period for some first period realizations, p.s. borrowers will not purchase intertemporal contracts if ω is sufficiently small. Rather, they will take single period contracts similar to that described in Theorem 2. (Of course, the equilibrium will look different with p.s. borrowers because the incentive compatibility conditions will change.) For some parameter values, such borrowers will get rationed with a nonzero probability at t=1, following first period project failure.

The model can further be extended by having p.s. borrowers being of one of two types at t=0 and selecting separating single period contracts then. In this case, at t=0 we will have each borrower first deciding whether to purchase a single period contract or an intertemporal contract, and then deciding whether to report itself as good or bad.

B. Empirical Implications

The following predictions have emerged from our analysis.

(1) Borrowers that purchase credit commitments are less likely to be rationed than borrowers that borrow in the spot credit market.

(2) Among the borrowers not purchasing intertemporal contracts, those borrowers that paid lower first period interest rates (reported their type as "good") and then defaulted are more likely to be rationed than those borrowers that paid higher first period interest rates (reported their types as "bad") and then defaulted. Alternatively, those who defaulted on secured loans are more likely to be rationed than those who defaulted on unsecured loans.

(3) *Ceteris paribus*, credit contracts received by those who have defaulted in the past are worse than contracts received by those who have repaid in the past.

(4) If rationing exists, it is more likely to be encountered *after* observing a poor performance by the firm for some time rather than at the outset.

It should be emphasized that tests of these predictions must be conducted with care. As in other asymmetric information models, these predictions apply to observationally identical cohorts of borrowers. If the chosen sample aggregates observationally distinct groups, it is possible that differences in contract terms based on observable differences in borrowers will dominate the differences induced by incentive compatibility considerations. Thus, the sample should be chosen to include only borrowers that are identical -- except in their contract choices -- based on all publicly available information.

VIII. CONCLUSION

We have analyzed a dynamic model of credit market equilibrium in which borrowers can choose between spot contracts and long term credit commitments from banks. Moreover, each credit contract can be a secured or an unsecured loan. Banks may ration credit at any time. In this context, we have shown that intertemporal credit contracts have a "memory" feature; future contracts depend on past repayment behavior. Moreover, a borrower that does not purchase a credit commitment and borrows exclusively in the spot market may be rationed in the second period if it defaults in the first period. A borrower can protect itself against rationing by purchasing a commitment from a bank that guarantees future liquidity. It is quite striking that we obtain this result despite universal risk neutrality.

We have also established the existence of a (unique) dynamic equilibrium. In doing so, we have shown that the most efficient pooling contract does not pool completely across time and types, i.e., it is only partially pooling under reasonable conditions. However, the dynamic equilibrium involves fully separating contracts. Our analysis differs from earlier dynamic models of credit market equilibrium such as Diamond (1986) and Spatt (1986) in that those are models of borrower reputation and do not involve collateral. Consequently, they address very different issues. Our analysis also differs from Rogerson's (1985) repeated moral hazard model which does not consider pre-contract asymmetric information.

The intuition formalized in this paper is that it is the possibly widespread occurrence of credit rationing that encourages the development of forward credit markets. We do not claim that rationing can be completely eliminated through contractual mechanisms. Rather, we wish to emphasize that credit contracting innovations, stimulated by rationing, can lessen the incidence of rationing.

LIST OF KEY SYMBOLS

δ^L = the success probability for a bad borrower;

δ^H = the success probability for a good borrower;

π^i_j = the credit granting probability in the set of contracts j for borrower type i ($i \in \{L, H\}$, $j \in \{I, II, III, IV, V\}$);

α^i_j = the interest factor (= one plus the interest rate) in the set of contracts j for borrower type i($i \in \{L, H\}$, $j \in \{I, II, III, IV, V\}$);

β = measure of the bank's evaluation of a borrower's collateral. That is, $1 collateral has a value of β to the bank;

γ $(1-\gamma)$ = the proportion of bad (good) borrowers at t=0;

η $(1-\eta)$ = " " " in the set of nodes II;

ν $(1-\nu)$ = " " " " " III;

μ $(1-\mu)$ = " " " " " IV;

σ $(1-\sigma)$ = " " " " " V;

Ψ $(1-\Psi)$ = the proportion of bad (good) borrowers at t=1 within the pool of borrowers rationed at t=0;

Ω $(1-\Omega)$ = the proportion of bad (good) borrowers within the pool of borrowers with bad first period realizations;

τ $(1-\tau)$ = the proportion of bad (good) borrowers within the pool of borrowers with good first period realizations;

λ = Lagrange multiplier;

ρ = decay parameter for delayed investment projects;

Θ = the shortage of collateral parameter, $\Theta \in [0, 1)$; if $\Theta = 1$, no shortage applies;

$\delta^G \equiv [1-\tau]\delta^H + \delta^L$

$\bar{\delta} \equiv [1-\gamma]\delta^H + \gamma\delta^L$

$\Phi \equiv \{\delta^H[1-\delta^L] - \beta\delta^L[1-\delta^H]\}(\delta^H)^{-1}$

$\xi \equiv \{\delta^H[1-\delta^L] - \nu\delta^L[1-\delta^H]\}(\delta^H)^{-1}$

$C^H_{III}(opt) \equiv [\delta^H - \delta^L]r[\Phi\delta^H]^{-1}$ = the level of collateral in an optimally collateralized single period contract;

R = the return on the investment project if successful;

r = the risk free interest factor (= one plus the risk free interest rate);

W_0 = the initial (t=0) level of available collateral for each individual borrower;

$W \equiv W_0 - C_I$ = the available collateral in the second period if C_I has been lost in the first period;

C_j^i = the collateral asked in the set of contracts j from a type i borrower ($i \in \{L, H\}$; $\gamma \in \{I, II, III, IV, V\}$);

$U_j(\delta^k | \delta^i)$ = the expected ability for a type i borrower who chooses the type k contracts, starting from the set of nodes j;

α_1 = the first period pooling interest rate;

α_2^G = the second period pooling interest rate conditioned on a good first period realization;

$\delta \equiv \delta^H - \delta^L$

$\bar{\delta}^i \equiv 1 - \delta^i$, $i \in \{H, L\}$

$R_N^j \equiv R - r[\delta^j]^{-1}$, $j \in \{H, L\}$

$\mathfrak{B}^i$ ≡ dynamic strategic credit policy of bank i

$\mathfrak{B}_1$ ≡ first period credit policy

$\mathfrak{B}_2(y_1, \chi_1)$ ≡ second period credit policy applicable to borrower with first period contract choice y_1 and first period realization χ_1.

N ≡ set of all possible competing banks (there are n banks)

ζ_i ≡ net expected profit of bank i

κ_1 ≡ borrower's first period type

κ_2 ≡ borrower's second period type

$z \equiv (\kappa_1, \chi_1, \kappa_2)$ is borrower's composite type

y_2 ≡ borrower's second period contract choice

$\mathcal{F}$ ≡ bank's objective function

$\mathcal{P}$ ≡ Lagrangian

α_2^B = the second period pooling interest rate conditioned on a bad first period realization:

In Lemma 5 it is established that the bad return pool within an intertemporal pooling contract gets a separating contact. The following variables are defined for that case.

α_B^L = second period interest factor for bad types in the pool of unsuccessful first period borrowers;

α_B^H = second period interest factor for good types in the pool of unsuccessful first period borrowers;

C_B = second period collateral asked from a good types in the pool of unsuccessful first period borrowers:

Some additional symbols:

"^" on top of variables indicates the (separating) single-period-contract solution:

"~" on top of variables indicates the (separating) intertemporal contract solution:

"—" or "=" on top of variables indicates the (intertemporal) pooling contract solution.

FOOTNOTES

1) Recently, Stiglitz and Weiss (1987) have explored the macroeconomic implications of their rationing models.

2) Thus, Besanko and Thakor's (1987) work may be viewed as a generalization of Bester's (1985). Bester shows that unconstrained collateral availability eliminates the rationing one encounters in Stiglitz and Weiss (1981).

3) See, for example, Campbell (1978), Thakor, Hong, and Greenbaum (1981) and Thakor (1982).

4) These are floating rate commitments which involve the bank lending in the future at a rate that is equal to the prevailing prime rate plus a fixed add-on or the prevailing prime times a fixed multiple.

5) See Thakor, Hong and Greenbaum (1981), for example.

6) There are some recent papers that have provided explanations for loan commitment demand by risk neutral borrowers (see, for example, Boot, Thakor and Udell (1987a), Thakor (1987), and Kanatas (1987)). However, they do not explain the role of loan commitments as a guarantee against future credit rationing.

7) We thus abstract from ex post breach of contract issues. These are explicitly analyzed in Boot, Thakor and Udell (1987b).

8) Our view in this paper is that an important function of intertemporal credit contracts is the provision of "insurance" against rationing. The equilibrium contracts we characterize are a special form of a general intertemporal credit contract, and the loan commitments commonly found in practice represent another special form. Both guarantee funds availability.

9) Taken in conjunction with borrower limited liability, this implies that costless resolutions through payoff-contingent contracts of the Bhattacharya (1980) type are infeasible.

10) In fact, in our formal analysis we will assume $\eta = 0$, $\sigma = 1$, as an extreme simplification consistent with the assumptions $\nu > \eta$ and $\sigma > \mu$.

11) This assumption appears to be strong, but it is not. We could simply assume that the second period investment is so large that even a successful first period borrower could not completely self-finance. The second period project return in the successful state could then be taken as aR, where a > 1 is a sufficiently large number to ensure that the project has a positive net present value. This will also lead to a model structure in which there is first period failure and it is impossible to fund the second period project without a bank loan.

12) Thus, faced with a randomly chosen borrower at t=0, the bank simply takes γ and $1-\gamma$ as the probabilities that the borrower is bad and good, respectively.

13) See, for example, Myerson (1981).

14) This value dissipation can be thought of as arising from the (transactions) costs incurred by the bank in taking possession of and liquidating the collateral put up by a borrower that has defaulted.

15) As Stiglitz and Weiss (1984) point out, the "problem" in such games is usually nonexistence of equilibrium in pure strategies, although existence with mixed strategies is generally attainable (Dasgupta and Maskin (1986a, 1986b)).

16) Apart from the fact that this provides us with a way of conceptualizing an equilibrium that exists, the recent work of Engers and Fernandez (1987) has shown that the intuitive stability-type criterion of Cho and Kreps (1987) justifies the Riley reactive equilibrium as "reasonable". Admittedly, the argument does remain somewhat heuristic since the game-theoretic refinements of Nash equilibria have formally been applied to games in which the informed move first, whereas the uninformed move first in the Riley framework. Notice, however, that we actually prove that a Nash equilibrium exists, i.e., we establish conditions stronger than those needed for a DRE to exist. This is done for two reasons. First, despite our choice of the DRE as the equilibrium concept to use, we wanted our equilibrium allocations to be relatively robust with respect to the equilibrium concept adopted. Second, proving the existence of a Nash equilibrium is, in this case, more illuminating in terms of the insights it produces. Our adoption of the DRE in the first place is to ensure the existence of equilibrium when the model is extended to a continuum of types.

17) This may be viewed as a form of subgame perfection requirement. Subgame perfection requires that, starting from any decision node in an extensive form representation of the game (whose event sequence is in Figure 1) such that the game when restricted to that and the succeeding nodes is a "proper" subgame (in the Kreps and Wilson (1982) sense, the contracts and corresponding strategies restricted to that subgame constitute a Nash equilibrium for the subgame. Note that there is no "problem" in applying this criterion in our context, even though the bank's information set at the start of the second period may not be a singleton, because the bank always has sufficient information at any point in time to offer a pair of separating contracts that are incentive compatible then, and the bank (the uninformed agent) moves first.

Engers and Fernandez (1987) show that the reactive equilibrium is subgame perfect in a static setting. We impose condition (b) in the definition of equilibrium to ensure that subgame perfection holds in our dynamic extension. Note, however, that this condition is only meant to imply that the bank and the borrower will not have an incentive to renegotiate the second period contract after the first period. We do not permit the bank and the borrower to renegotiate the first period contract after the first period.

18) Because, $\eta = 0$ and $\sigma = 1$, only one contract is offered in each of the sets of nodes II and V.

19) Note that the assumption that borrowers have access to no other credit sources besides bank loans is not restrictive. With an alternative credit source that is not as advantageous to the borrower as a bank loan, one simply rescales things so that the borrower's utility of borrowing from the alternative source is netted out.

20) If interest rates are constrained to be nonnegative (i.e., $\alpha \geq 1$), then the second period interest rate will be set at zero. Although the

details of the DRE will change, its qualitative characteristics will be unaltered.

21) This makes it possible to reward/punish first period performance.

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