INCREASING RETURNS AND FIXED MARKET SHARES

by

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1. Introduction.

It is well known, that if a commodity is produced under increasing returs to scale, so that mean costs are decreasing with the level of production, then a supply function does not exist, since the profit function does not attain a finite non zero maximum. So equality of supply and demand cannot be a basis of equilibrium.
If a firm's mean costs are decreasing, then at a given price, either the price is so low that production is not profitable at any level, or there exists some level of sales such that profits are non-negative at this level and profits increase with output. The profit is maximal if output = sales is infinite. Any firm knows that it will not be able to sell that much, and it also knows that its competitors are in the same position.
Given total demand, it is important to each firm, how consumers distribute their demand among firms. We consider a homogeneous commodity, so consumers do not prefer the product of one firm over the product of another firm. Nevertheless each consumer has to decide from which producer he will buy. These decisions as a whole determine each producer's share of the market. Consumers will always buy at the lowest price, so only one price can exist on the market, since any producer asking a higher price, will sell nothing. If some firm lowers the prevailing price, his competitors have to follow him.

The distribution of market shares is a rationing scheme, as it also appears in the recent literature on disequilibrium (see e.g. Drèze [3]). There a rationing scheme is necessary to cope with excess supply (or -demand) at disequilibrium prices. Typically in the case of increasing returns there is always excess supply and therefore a rationing scheme is always needed.
The first decision a producer has to make is if he will produce at all. If he expects to sell so much and at such a price, that this profit is positive, he will produce and be an active producer. If he can only get a negative profit, he will not produce and be a sleeping producer. Secondly he has to decide if he will accept the prevailing market price or if he will fix a lower price.

The market considered in this paper is essentially an n-persons non-zero sum game. The equilibrium concept which is used is a Nash Equilibrium which is a non-coöperative solution: it is assumed that firms do not consider the effect of their behaviour on their competitors (apart from the assumption that they do expect their competitors to follow any price decrease). This may only be plausible if the number od firms is "large" (whatever that means). There certainly exist other equilibrium concepts (coöperative solutions) which might be interesting in the present case. However the Nash Equilibrium approach keeps the analysis nearest to competitive behaviour of firms, in the case of decreasing returns. However the market has also important features of monopolistic competition since, given its share, each firm faces a decreasing demand function.

In most theories, both of partial and of general equilibrium it is assumed that mean costs are decreasing or first decreasing and then increasing ("U-shaped" curves). Bain [1] considers the case of economics to scale up to a certain, but possibly high level. In a recent paper Marshak and Selten [4] introduce a general equilibrium model with non decreasing returns to scale. Their approach is similar to the one in the present paper, however their equilibrium concept is different and they consider a restricted case. Many other papers, e.g. Dierker, Fourgeaud and Neufeind [2] consider general equilibrium solutions in an economy with increasing returns, which are enforced by some planning mechanism; in these papers it is assumed that each commodity is produced by one firm.

The present paper generalizes a result proved in [5].

2. The market.

We consider a market characterized by: (1) a non empty set $N = \{1,2,\dots,n\}$ of potential firms. $\mathcal{N}$ is the set of non empty subsets of N; (2) a cost function $f_i(y)$ for each $i \in N$; $\bar{f}_i(y) = f_i(y)/y$ denotes the mean cost function; (3) a market demand function $x(p)$, defined on the interval $0 < p \leq d$, where d is a (arbitrary high, but finite) maximum price; (4) a market share distribution $\rho : \mathcal{N} \to T \cup \{0\}$, associating to each

set $A \in \mathcal{N}$ of active firms their market shares.
$T = \{\rho_i | \Sigma\rho_i = 1 \text{ and } \rho_i \geqq 0\} \subset R^n$ is the unit simplex in R^n and $\rho(A) = (\rho_1(A), \rho_2(A), \ldots, \rho_n(A))$, where $\rho_i(A) = 0$ if $i \notin A$.

The restriction of the demand function to a bounded interval rules out complications with solutions, where prices become infinite (and production infinitely small), which otherwise would burden the analysis.
The number d should be understood to be arbitrary large.
The market share distribution attributes to each firm its share of the market, given the configuration of active firms that obtains. It describes the aggregate of the choices of consumers among firms. It may result from "weak" preferences of each consumer among firms, depending for instance on distance ("weak" in the sense that a consumer first considers prices and then chooses among the firms with lowest price). The market share distribution might also be generated by random choices of consumers: if all consumer choose at random, the (expected) value of $\rho_i(A)$ would be $1/|A|$, $|A|$ being the number of elements in A.

The <u>profit function</u> of the firm i

$$\pi_i(\rho_i, p) = \rho_i p x(p) - f_i(\rho_i(x(p))$$

is a mapping of market shares $\rho_i \in [0,1]$ and prices $p \in]0,d]$ into the reals.
A <u>solution</u> of the market, is a pair (A,p), where $A \in \mathcal{N}$ is the set of <u>active firms</u> and $p \in]0,d]$ is the <u>market price</u>.
Each active firm $i \in A$ produces and sells $\rho_i(A)x(p)$. Firms $j \in N \setminus A$ are <u>sleeping.</u>

<u>Definition:</u>
A solution (A,p) is <u>feasible</u> if for all $i \in A$: $\pi_i(\rho_i(A),p) \geq 0$;
A solution (A,p) is <u>internally stable</u> if it is feasible and if for all $i \in A$: $p' < p \Rightarrow \pi_i(\rho_i(A),p') < \pi_i(\rho_i(A),p)$;
A solution (A,p) is <u>externally stable</u> if it is feasible and if for all $j \in N \setminus A$: $p' \leq p \Rightarrow \pi_j(\rho_j(A \cup \{j\}),p) \leq 0$;
An <u>equilibrium</u> is a solution that is both internally and externally stable.

So internal stability means that each active firm makes non-negative profit and could not increase its profit by decreasing the price; external stability means that no sleeping firm could make a positive profit by becoming active and fixing a price not higher than the prevailing price p. In an equilibrium no firm could improve by decreasing the price or sleeping in (if he is active) or by becoming active (if he is sleeping). This equilibrium is a Nash equilibrium: each firm's strategy is optimal, given the strategies of the other firms, where each firm's strategy-set consists of (i) being active or sleeping and (ii) the set of all prices lower than p.

These concepts may be illustrated by figure 1: the curve depicts the profit function of some $i \in N$, as a function of p; the market share ρ_i is kept constant.

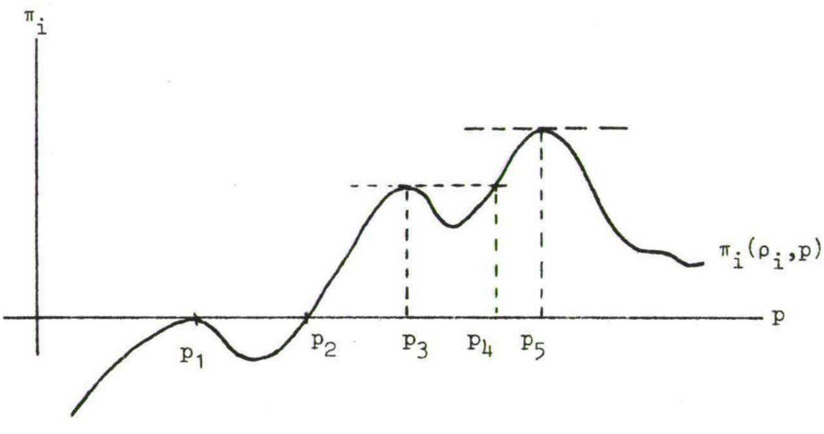

Figure 1.

First suppose that (A,p) is an internally stable solution; $i \in A$ and $\rho_i|A| = \rho_i$: Then $p \in \{p_1, [p_2,p_3], [p_4,p_5]\}$ ($[p_k,p_\ell]$, denoting the interval $p_k \leq p \leq p_\ell$), for if $p > p_5$ ($p_3 < p < p_4$, $p_1 < p < p_2$) then a price decrease to p_5 (p_3,p_1) is profitable for i.

Secondly suppose (A,p) is externally stable and $i \notin A$ and $\rho_i = \rho_i(A \cup \{i\})$. Then $p \leq p_2$; for if $p > p_2$, i could make a positive profit at some price $p_2 < p' < p$.
For $\rho_i \in [0,1]$, the minimum price of i at share p_i is defined:

$$\check{p}_i(\rho_i) = \begin{cases} \inf\{p \mid \pi_i(\rho_i,p) > 0\} & \text{if for some } p \in \,]0,d] : \pi_i(\rho_i,p) > 0 \\ d & \text{otherwise} \end{cases}$$

Note that it is not excluded that $\pi_i(\rho_i,p) = 0$ for some $p < \check{p}_i(\rho_i)$.
(In de case of figure 1, $\check{p}_i(\rho_i) = p_2$; however $\pi_i(\rho_i,p_1) = 0$).
A feasible solution is externally stable if and only if for all $j \notin A$: $\check{p}_i(\rho_i(A \cup \{j\}) \geq p$.
For $\rho_i \in [0,1]$ and $p \in \,]0,d]$, the restricted maximum price is defined:

$$\hat{p}_i(\rho_i,p) = \begin{cases} \min\{\bar{p} \mid \pi_i(\rho_i,\bar{p}) = \max\limits_{p' < p} \pi_i(\rho_i,p')\} & \text{if } p \geq \check{p}_i(\rho_i) \\ p & \text{otherwise} \end{cases}$$

i.e., i's profit attains a maximum at $\hat{p}$, given the restriction $\hat{p} < p$; if the maximum profit is non-positive, than $\hat{p} = p$.
A feasible solution (A,p) is internally stable if and only if $\hat{p}_i(\rho_i,p) = p$, for all $i \in A$, for then i is not tempted to decrease the price.

3. Assumptions.

A. On the cost function: for all $i \in N$: (1) $f_i(0) = 0$ and for $y > 0$ $f_i(y) > 0$; (2) for $y > 0$, f_i is continuous; (3) for $y > 0$, $f_i(y)$ is increasing; (4) for $y > 0$, $\bar{f}_i(y)$ is decreasing; (5) there exists $c > 0$, such that for all i and y: $\bar{f}_i(y) > c$.

B. On the demand function: For $0 < p < d$: (1) $0 < x(p) < \sim$ and $x(d) \geq 0$; (2) x is continuous; (3) x is decreasing for $0 < p < d$.

C. On the market share distribution: (1) $\rho(A) = 0$ if and only if $A = \emptyset$; (2) if $i \in A \subseteq A'$ and $A \neq A'$, then $\rho_i(A) > \rho_i(A')$.

D. <u>Feasibility:</u> for all $i \in N$, there exists $p \in]0,d]$, such that $px(p) - f_i(x(p)) > 0$.

By assumption (A1) non-zero fixed costs are allowed.
However mean costs are decreasing by (A4), they are always larger than the constant c. We do not make assumptions on the behaviour of marginal costs. By (C1), $\rho(A) \in T$, unless $A = \emptyset$ and by (C2) an active firm's share <u>strictly</u> decreases, if new firms become active. Assumption (D) requires a potential firm to be profitable, at least if he is a monopolist, (which means that firms which do not meet this condition are not included in N). There is at least one such firm, since $N \neq \emptyset$.

<u>Lemma 3:</u> Under assumptions A, B and C:
(a) $\pi_i(0,p) = 0$;
(b) if $p \leq c$ and $\rho_i > 0$: $\pi_i(\rho_i,p) < 0$;
(c) π_i is continuous in $\rho_i > 0$ and $p > 0$;
(d) if $\pi_i(\bar{\rho}_i,p) \geq 0$, then π_i is increasing in ρ_i, for $\rho_i \geq \bar{\rho}_i$;
(e) if $\breve{p}_i(\rho_i) < d$, then $\pi_i(\rho_i,\breve{p}_i(\rho_i)) = 0$;
(f) if $\rho_i \geq \bar{\rho}_i$ and $\breve{p}_i(\bar{\rho}_i) < d$, then $\breve{p}_i(\rho_i) < \breve{p}_i(\bar{\rho}_i)$.

<u>Proof:</u> (a) follows from (A1); (b) from (B1) and (B5); (c) from (A2) and (B2); (d) if $\rho_i' > \rho_i \geq \bar{\rho}_i$, by (A4): $\bar{f}_i(\rho_i'x) < \bar{f}_i(\rho_i,x)$, hence $(p-\bar{f}_i(\rho_i'x))\rho_i'x > (p-\bar{f}_i(\rho_i x))\rho_i x \geq 0$; (e) follows from the continuity of π_i; (f): by (e): $\pi_i(\bar{\rho}_i,\breve{p}_i(\bar{\rho}_i)) = 0$, hence by (d): $\pi_i(\rho_i,\breve{p}_i(\rho_i)) > 0$.

<u>4. Equilibria in the case of identical firms.</u>

We first assume that all firms have <u>identical</u> cost functions and <u>equal</u> market shares, i.e. $f_i(x) = f(x)$ for all $i \in N$, and $\rho_i(A) = 1/|A|$, where $|A|$ denotes the number of firms in A.
If there exists a feasible solution with m firms, there exist equilibria for any set A, containing at most m firms. The equilibrium prices are lower, the smaller the number of firms in A.

<u>Theorem 4:</u> If all firms have identical cost functions and equal market shares, and if $(\bar{A},\bar{p})$ is feasible and $|\bar{A}| = m$, then there exist prices $p_m > p_{m-1} > \ldots > p_1$, such that if $|A| = k \leq m$, (A,p_k) is an <u>equilibrium</u>.

<u>Proof:</u> Since $(\bar{A},\bar{p})$ is feasible and $|\bar{A}| = m$, $\pi(\frac{1}{m},\bar{p}) \geq 0$ and by lemma 3(d): $\pi(\frac{1}{k},\bar{p}) > 0$, for $k < m$.
Choose:

$$p_m = \begin{cases} \bar{p} & \text{if for all } p < \bar{p} : \pi(\frac{1}{m},p) \leq 0 \\ \ddot{p}(\frac{1}{m}) & \text{if for some } p < \bar{p} : \pi(\frac{1}{m},p) > 0 \end{cases}$$

and for $k < m$:

$$p_k = \ddot{p}(\frac{1}{m})$$

By lemma 3(e), $\pi(\frac{1}{k},p_k) = 0$ and since $\frac{1}{k-1} > \frac{1}{k}$, by lemma 3(f): $p_{k-1} < p_k$ (for $m \geq k > 1$).
Any solution (A,p_k), where $|A| = k \leq m$ is feasible since $\pi(\frac{1}{k},p_k) = 0$, internally stable since $\pi(\frac{1}{k},p) \leq 0$, if $p \leq p_k$ and externally stable, since $\pi(\frac{1}{k+1},p_k) < \pi(\frac{1}{k},p_k) = 0$.

By assumption D, there exists a single-firm feasible solution, i.e. for some $p : \pi(1,p) \geq 0$. Define $\bar{\rho}$ inf $\{0 < \rho \leq 1 | \max_p \pi(\rho,p) > 0\}$, so $\bar{\rho}$ is the lower bound of the market shares at which a firm makes a nonnegative profit at some price p, i.e. if $\frac{1}{|A|} < \bar{\rho}$, then for some p, (A,p) is feasible. If $\bar{\rho} = 0$ feasible solutions, and therefore, by theorem 1, equilibria, exist for any set $A \subset N$. In this case there is a price $\bar{p}$, such that $x(\bar{p}) > 0$ and $\bar{f}(y) < \bar{p}$ for any $y > 0$, which implies: $\bar{p}x > f(x)$ for all x, hence $f(x) \to 0$ if $x \to 0$. If $\bar{\rho} > 0$, the largest feasible solution contains at most k firms, where k is a whole number such that $\frac{1}{k} < \bar{\rho} < \frac{1}{k+1}$. The equilibrium prices $p_k = \ddot{p}(\frac{1}{k})$, considered in the proof, give zero profits to all firms. For any p such that: (1) $p_k \leq p \leq p_{k+1}$: (2) $p \leq \min\{p | \partial\pi(\frac{1}{k},p)/\partial p > 0\}$ and (3) $\pi(\frac{1}{k},p) \geq 0$, (A_k,p) is also an equilibrium, and by the definition of p_k for $k < m$, among these equilibria occur positive-profit equilibria.

Equality of shares and identity of cost functions is a strong condition. The conclusion of the theorem which ensures the existence of an equilibrium for any set of firms smaller than the "maximal feasible set" seems a strong conclusion also. In the next section we consider some generalisations of theorem 4.

5. Equilibria and stable solutions.

By assumption (D) there exist single-firm feasible solutions. This implies that there exist at least one single-firm equilibrium:

Proposition 5.1.: There exists $i \in N$ and $p > 0$, such that $(\{i\},p)$ is an equilibrium.

> Proof: Let $\bar{p} = \min\{\breve{p}_i(1) \mid i \in N\} = \breve{p}_{i_0}(1)$ and $\bar{\bar{p}} = \min\{\breve{p}_j(\{i_0,j\}) \mid j \in N \setminus \{i_0\}\}$. Then any solution $(\{i_0\},p)$ such that $\bar{p} \leqq p < \bar{\bar{p}}$ and $p \leqq \breve{p}_{i_0}(1)$ is an equilibrium.

If (A,p) is a feasible solution, then obviously, if $A' \subset A$, (A',p) is also feasible and by (C2) and lemma 3(f), there exists $p' < p$, such that (A',p') is feasible. It is not true however, as in theorem 4, that feasibility of some solution (A,p) implies the existence of some equilibrium (A,p'). Neither does fasibility imply the existence of an internally stable solution. A feasible solution $(\bar{A},\bar{p})$ is internally stable if no active firm could improve by a price decrease. It is possible, even if for some $i \in A, \pi_i(\rho_i(\bar{A}),p) = 0$ for $p \leq \bar{p}$, that some other active firm $j \in A$ has a restricted maximum price $\hat{p}_i(\rho_i(A),\bar{p}) < \bar{p}$. Clearly in that case j's cost function must be lower and/or his market share larger than i's.

However if a feasible solution occurs at a price such that total consumer expenses $px(p)$ are lower at lower prices, then no active firm can improve by decreasing the price:

Theorem 5.2.: If $(\bar{A},\bar{p})$ is feasible and $p < \bar{p}$ implies $px(p) < \bar{p}x(\bar{p})$, then $(\bar{A},\bar{p})$ is internally stable.

Proof: $(\bar{A},\bar{p})$ is feasible, hence for all $i \in \bar{A}$: $\pi_i(\rho_i(\bar{A}),\bar{p}) \geq 0$.
For $p < \bar{p}$, we have: $px(p) < \bar{p}x(\bar{p})$ and, since by (B3), $x(p) > x(\bar{p})$, for all i: $f_i(\rho_i(\bar{A})x(p)) > f_i(\rho_i(\bar{A})x(\bar{p})$.
Hence $\pi_i(\rho_i(\bar{A}),p) < \pi_i(\rho_i(\bar{A}),\bar{p})$, hence $(\bar{A},\bar{p})$ is internally stable.

Corrollory: If for some p^0, $\frac{d(px(p))}{dp} > 0$ if $c < p \leq p^0$, them any feasible solution (A,p) is internally stable, for $p \leq p^0$.

Particularly, if the total expence function has a single maximum at p^0, any feasible solution (A,p), for $p \leq p^0$, is internally stable. Let the condition of the corrollory hold for some (A,p), where $|A| = m$. Then for any sequence $A \supset A_{m-1} \supset A_{m-2} \supset \ldots \supset A_1$, $(|A_k| = k < m)$, the solutions (A_k,p_k) are feasible and hence internally stable, for $p_k = \max\limits_{A_k} \check{p}_i(\rho_i(A_k))$,

and $p > p_{m-1} > \ldots > p_1$, a result similar to the one of theorem 4.

A feasible solution $(\bar{A},\bar{p})$ is externally stable if no sleeping firm could make a positive profit at a price not above $\bar{p}$. Even if $\bar{p}$ would be the smallest feasible price of $\bar{A}$, i.e. $\bar{p} = \max\limits_i \{\check{p}_i(\rho_i(\bar{A}))\}$, it is not impossible that for some $j \notin \bar{A}$: $\pi_j(\rho_j(\bar{A} \cup \{j\}),p) > 0$. Obviously then j's cost function has to be lower and/or his market share larger than those of the least efficient member of $\bar{A}$. In the case of identical firms this could not occur. This cannot occur either if all firms are similar, i.e. if their cost functions and their market share are not too different. Then feasibility implies existence of an externally stable solution. Similarity of firms is made precise by condition α:

Condition α: There exists a function g, that fullfills assumptions A, B and D and there exist numbers $0 < \varphi < 1$, $0 < \varepsilon < 1$ and $0 < \mu < 1$, such that

(1) for all $i \in N$: $g(y) \leq f_i(y) \leq \frac{1}{\varphi}\, g(\varphi y)$;
(2) for all $A \subset N$ and $i \in A$: $\frac{1+\varepsilon}{|A|} \leq \rho_i(A) \leq \frac{1-\varepsilon}{|A|}$
(3) $\varphi > \frac{1+\varepsilon}{1-\varepsilon}\;\frac{1}{1+\mu}$

<u>Theorem 5.3.:</u> Let condition α hold. If $(\bar{A},\bar{p})$ is feasible, $|A| = m$ and $\frac{1}{m} \geq \mu$, then there exist prices $p_m > p_{m-1} > \ldots > p_1$, such that:
(1) $(\bar{A},p_m)$ is <u>externally stable</u>;
(2) if $|A| = k < m$, then (A_k,p_k) is <u>externally stable.</u>

<u>Proof:</u> (α1) implies, for all $y > 0$ and $i \in N$:

$$\bar{g}(y) \leq \bar{f}_i(y) \leq \bar{g}(\varphi y) \qquad \text{(i)}$$

since $\bar{g}$ end $\bar{f}_i$ are decreasing by (A4);

(α1) and α(2) imply, for $A \subset N$, $i \in A$ and $y > 0$:

$$\bar{g}\left(\frac{1+\varepsilon}{|A|}\, y\right) \leq \bar{f}_i(\rho_i(A)y) \leq \bar{g}\left(\frac{\varphi(1-\varepsilon)}{|A|}\, y\right) \qquad \text{(ii)}$$

(α3) implies for $\frac{1}{k} \geq \mu$ and $y > 0$:

$$\bar{g}\left(\frac{\varphi(1-\varepsilon)}{k}\, y\right) < \bar{g}\left(\frac{1+\varepsilon}{k+1}\, y\right) \qquad \text{(iii)}$$

since $\bar{g}$ is decreasing and $\varphi > \frac{1+\varepsilon}{1-\varepsilon} \cdot \frac{1}{1+\mu} \geq \frac{1+\varepsilon}{1-\varepsilon} \cdot \frac{k}{k+1}$.

Since $(\bar{A},\bar{p})$ is feasible, for all $i \in \bar{A}$: $\bar{f}_i(\rho_i(\bar{A})x(\bar{p})) \leq \bar{p}$ and for $i \in A$, $|A| = k < m$, we have by (ii) and (iii):

$$\bar{f}_i(\rho_i(A)x(\bar{p})) < \bar{g}\left(\frac{1+\varepsilon}{k+1}\, x(\bar{p})\right) \leq \bar{f}_i(\rho_i(\bar{A})x(p)) \leq p.$$

Choose

$$p_m = \begin{cases} \bar{p} & \text{if for all } p < \bar{p}\text{: } \bar{g}\left(\frac{\varphi(1-\varepsilon)}{m}\, x(p)\right) \geq \bar{p} \\ \inf\{p \mid \bar{g}\left(\frac{\varphi(1-\varepsilon)}{m}\, x(p)\right) < p\} & \text{if for some } p < \bar{p}\text{: } \bar{g}\left(\frac{\varphi(1-\varepsilon)}{m}\, x(p)\right) < \bar{p} \end{cases}$$

and for $k < m$:

$$p_k = \inf\{p \mid \bar{g}\left(\frac{\varphi(1-\varepsilon)}{k}\, x(p)\right) < p\}$$

By the continuity of $\bar{g}$, $\bar{g}(\frac{\varphi(1-\varepsilon)}{k}\, x(p_k)) = 0$ and since by (A4),

$\bar{g}(\frac{\varphi(1-\varepsilon)}{k-1}\, x(p_k)) < \bar{g}(\frac{\varphi(1-\varepsilon)}{k}\, x(p_k)) = 0$, $p_{k-1} < p_k$.

We have by (ii) and (iii), for $i \in A$: and $j \in N\setminus A$: $\bar{f}_i(\rho_i(A)x(p_k)) \leq$

$\bar{g}(\frac{\varphi(1-\varepsilon)}{k}\, x(p_k)) = p_k < \bar{g}(\frac{1+\varepsilon}{k+1}\, x(p_k)) \leq \bar{f}_j(\rho_j(A \cup \{j\})x(p_k))$.

By the left hand inequalities, (A,p_k) is feasible and by the right hand inequalities (A,p_k) is externally stable.

Clearly a solution (A,p_k) is also externally stable if $p_k \leq p \leq p_{k+1}$, for p_k as defined in the proof.

Secondly we consider the case, where firms can be ordered by their efficiency. Then a less efficient firm cannot block a solution of more efficient firms, if the price is not too high.

Condition β: if $i,j \in A \subset N$ and $i < j$, then for all $y > 0$:

$\bar{f}_i(\rho_i(A)y) \leqq \bar{f}_j(\rho_j(A)y)$.

The firms in N are numbered according to their "market-efficiency" i.e. mean costs of i at his share of total sales are smaller than those of j. Condition β holds particulary if: (i) for all $y > 0$: $f_i(y) \leqq f_j(y)$ and (ii) if $i,j \in A$: $\rho_i(A) \geqq \rho_j(A)$, i.e. low cost firms have high shares, but condition β also covers cases where a systematically high market share compensates higher costs at the same levels of production.

Theorem 5.4.: Let condition β hold. Define $A_k = \{1,2,\dots,k\}$, for $k \leq n$. If $(A_m,\bar{p})$ is feasible, then there exist prices $p_m < p_{m-1} < \dots < p_1$, such that (A_k,p_k) is externally stable.

Proof: By condition β and the feasibility of $(A_m,\bar{p})$:

$\bar{f}_k(\rho_k(A_m)x(p)) \leq \bar{f}_m(\rho_m(A_m)x(\bar{p})) \leq \bar{p}$, for $k \leq m$

and by (C2) and lemma 3(f), $\bar{f}_\ell(\rho_k(A_k)x(\bar{p})) < \bar{p}$, for $\ell \leq k < m$.

Choose

$$p_m = \begin{cases} \bar{p} & \text{if for all } p < \bar{p}\text{: } \bar{f}_m(\rho_m(A_m)x(p)) \geq p \\ \ddot{p}(\rho_m(A_m)) & \text{if for some } p < \bar{p}\text{: } \bar{f}_m(\rho_m(A_m)x(p) < p. \end{cases}$$

and for $k < m$:

$$p_k = \ddot{p}(\rho_k(A_k))$$

By lemma 1(e), $\bar{f}_k(\rho_k(A_m)x(p_k)) = p_k$.
Hence any solution (A_k,p_k) is feasible. By (C2) and lemma 3(f):
$p_{k-1} < p_k$.
For $j > k$: $p_k = \bar{f}_k(\rho_k(A_k x(p_k)) < \bar{f}_k(\rho_k(A_k \cup \{j\})x(p_k))$
$\leq \bar{f}_j(\rho_k(A_k \cup \{j\})x(p_k))$, hence (A_k,p_k) is externally stable.

Whether externally stable solutions are equilibria, depends on the behaviour of the restricted maximum price.
Theorem 5.2 on the one hand and theorems 5.3 and 5.4 on the other hand can be combined, to give sufficient conditions for the existence of an equilibrium, or of a sequence of equilibria. If condition α or β hold and if a feasible solution occurs at a price such that total expenses are lower at lower prices, then the externally stable solutions considered in theorems 5.3 and 5.4 are equilibria.

6. Conclusion.

The most illuminating result seems to be the one of theorem 4, showing that in the case of identical firms a sequence of equilibria exist where equilibrium prices decrease with the number of active firms. Simple examples show, that the difference between single firm equilibrium prices and many firm equilibrium prices may be large. Theorems 5.2, 5.3 and 5.4 show that the conclusion of theorem 4 remains true in a possibly wide set of cases: existence of many firm feasible solutions at prices where the price elasticity is smaller than 1 does not seem to be an exceptional case; the main parts of the technology to produce most products are public, hence cost functions will not be very different, so that conditions α and/or β may be fullfilled; where large differences in market shares exist, usually the high share firm will not be less efficient thatn the low share firm, and than β holds. Further generalisations of theorem 4 seem possible.

It was assumed, that market-shares were fixed, depending only on the composition of the set of active firms. A more realistic approach would be to make these shares also dependent on selling expenses (advertising etc.) of firms. The model becomes far more complex in this case. It seems however that the structures of the model remains the same; in [5] a result similar to the one of theorem 4 was found for identical firms.

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PREVIOUS NUMBERS.