Discounts For Qualified Buyers Only

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Abstract

The standard monopoly pricing problem is re-considered when the buyer can disclose his type (e.g. age, income, experience) at some cost. In the optimal sales mechanism with costly disclosure, the seller posts a price list, including a "sticker price" available to any buyer and a schedule of discounts available to those who disclose certain types. Unambiguous welfare implications of such a pricing policy are available in the limiting case when the buyer's type is fully informative: (i) The buyer is better off and the monopolist worse off when disclosure is more costly. (ii) When discounts are sufficiently rare, social welfare is strictly less than if the seller could not offer discounts.

1 Introduction

In classic models of monopoly pricing of an indivisible, perishable good, the seller's information about the buyer is treated as exogenous. For example, a monopolist may know the buyer's willingness to pay ("first-degree price discrimination"), the buyer's payoff-relevant type ("third-degree p.d."), or nothing about the buyer except the overall population from which he is drawn. In some settings, it is natural to view information

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about the buyer as being known to the seller a priori, e.g. employee discounts, as being costlessly observable by the seller, e.g. "ladies' night" discounts at a nightclub, or as being costlessly disclosable by the buyer, e.g. the Kama'aina rate offered only to Hawaiian residents. If so, the monopoly pricing problem reduces to finding an optimal take-it-or-leave-it price to offer buyers in each separate segment.

Other times, such information about buyers is not readily available. Retailers would like to offer discounts to bargain-hunters, but cannot directly verify a customer's price sensitivity. One solution is to offer discounts through channels that target bargain hunters. Newspaper coupons are a classic example, as price-conscious customers are more likely to search for and clip coupons, and many more such channels have now developed on the internet. For instance, Keycode.com offers online coupons on behalf of dozens of nationwide retailers, charging sellers each time a coupon is used. Similarly, Restaurants.com offers coupons worth from $10 - $100 at over 13,000 restaurants (as of May 2010), charging buyers 40 cents for each dollar of discount.

Another solution is for the buyer (or seller) to share (or gather) information directly, again often at a cost. For example, the drug manufacturer Genentech offers lower prices to patients who cannot afford the $50,000 price-tag for its cancer-fighting drug Avastin, through its Avastin Access Solutions program, but only after patients meet with a counselor to review their financial situation and insurance coverage.[1] Similarly, many private colleges and high schools offer generous financial aid, but only to those who prove that they cannot afford to pay full tuition.

This paper takes a first step toward endogenizing the seller's information about the buyer, by allowing the buyer to credibly disclose hard information about himself, at some cost to the buyer and/or to the seller. The optimal sales mechanism in this setting takes the form of what I call a "price-list mechanism". Any buyer who does not disclose faces

[1] Genentech likely has a powerful public-relations motive to provide Avastin to patients who cannot afford to pay full price. This paper's analysis can accommodate such concerns, by modeling Genentech as having a "negative cost of service" for such patients.

a take-it-or-leave-it "sticker price", while those who disclose certain pre-specified types qualify for a customized discount. The seller's information about the buyer is endogenous in this mechanism. In particular, the seller does *not* learn about those buyers who do not purchase its product, nor about those who pay sticker price.

The findings here qualify some well-known welfare comparative statics. Consider the extreme case in which the buyer's type is his true willingness to pay (or "value") for the good. Social welfare is strictly higher under perfect price discrimination, when the buyer's value is known a priori to the seller, than under uniform pricing. Suppose now that the buyer's value is not known a priori, but can be disclosed at some cost. As long as the cost of disclosure is in an intermediate range, so that the buyer's value is disclosed with a small enough but positive probability in the optimal mechanism, I show that expected social welfare is lower than under uniform pricing. In other words, as long as discounts are sufficiently rare in the profit-maximizing mechanism, a regulator could increase social welfare by forcing the seller *not* to offer discounts.

The seller's cost is assumed to be zero in the main model considered here, but all results extend to settings in which the cost of service varies across buyers. Such cost of service can be broadly interpreted to include ancillary benefits of service enjoyed by the seller. For example, MGM Mirage offers free membership in its Players' Club, a casino loyalty program. By identifying themselves (and thereby disclosing their likelihood to lose money at the hotel casino), gamblers in the Players' Club qualify for personalized deals on rooms and other hotel amenities. Similarly, Schneider (2009) finds that auto mechanics charge local customers less on average for a diagnostic exam ($37.70 vs. $59.75), perhaps in part because of the potential for repeat business or positive word of mouth.[2]

The rest of the paper is organized as follows. The introduction continues with a discussion of some related literature. Section 2 provides a self-contained analysis of an

[2] In Schneider's field experiment, a discount was offered when the buyer merely *claimed* to live nearby. Even if such claims are cheap talk, however, not all buyers may be aware that making such a claim will lead to a discount. Thus, such cheap-talk claims can still serve to "disclose" potentially payoff-relevant information about the buyer, namely, that he is aware of this opportunity to get a better price.

illustrative limiting case of the main model in which the buyer's type is fully informative of his value. Section 3 then presents the main model, in which the buyer can disclose an imperfectly informative type. Section 4 contains the bulk of the analysis, including extensions to allow for finitely many types, fixed costs of enabling disclosure, and private costs of service. Section 5 concludes with some comments and directions for future research. Proofs are in the Appendix.

Related literature. Most closely related is Riley and Zeckhauser (1983)'s classic paper on the optimality of posted prices in monopoly pricing. A key feature of posted price mechanisms is that each buyer receives the good with probability zero or one. The optimality of such non-random allocation rules is not obvious once buyer disclosure is possible. For example, might the seller increase its expected profit by sometimes withholding the good from those buyers who do not disclose? This paper shows that, indeed, it is optimal to offer non-disclosing buyers a posted price (the "sticker price"), albeit a higher price than without disclosure. Also related in this vein is the literature on the welfare effects of market segmentation (Schmalensee (1981) and Varian (1985)). Again, the difference here is that the seller's information about the buyer is costly and endogenous.[3]

A complementary literature is that on monopoly menu pricing ("second-degree price discrimination"). The monopolist in that literature extracts more of the total surplus from trade by allowing the buyer to choose among different goods, sorting different types of buyers on volume (e.g. Wilson (1993)), delay (e.g. Chiang and Spatt (1982)) and/or quality (e.g. Deneckere and McAfee (1996)). The main difference here is that the buyer can reveal information about himself *directly* through disclosure rather than indirectly through product choice. Some examples such as movie ticket pricing combine elements of menu pricing (e.g. matinee discounts) with elements of pricing based on disclosed

[3]Since the monopolist's profits are always higher when the buyer's type can be disclosed than under uniform pricing, facilitating disclosure can be viewed as a rent-seeking activity (Posner (1975)). Thus, any welfare gains that might arise from such information revelation could be diminished or reversed by the cost of rent-seeking.

information (e.g. senior citizen and student discounts). In other settings, the buyer's menu of options may itself depend on what the buyer has disclosed. Characterizing the profit-maximizing pricing mechanism in this more general setting is an important area for future research. However, to isolate what is new, this paper focuses on the special case of a single, indivisible, perishable good.

While similar in spirit, this paper is very different from the literatures on disclosure (e.g. Grossman (1981) and Milgrom (1981)) and mechanism design with partially verifiable information (e.g. Green and Laffont (1986) and Bull and Watson (2005)). In these literatures, all messages are costless. Here, sending a "credible message" is costly.[4] Somewhat more related is the literature following Townsend (1979) on costly state verification. For example, Border and Sobel (1987) consider optimal taxation when the taxation authority can verify ("audit") a citizen's wealth at some cost.[5] The analogous question in monopoly pricing, of how to design an optimal sales mechanism when the seller can conduct a costly audit to learn the buyer's type, is interesting and important but remains an open question. In particular, this paper does not address the question of optimal monopoly auditing.

2 Illustrative case: perfect disclosure

Before presenting the formal model, I will develop intuition and some welfare results in a setting in which (i) the buyer is able to disclose his true willingness to pay (or "value") $v \in [0, \infty)$ for the good, at cost $c \geq 0$, and (ii) the seller commits to a *price list*, i.e. a list of take-it-or-leave-it prices depending on whether and what the buyer discloses.

Definition 1 (Price list)**.** A *price list* $p : (\{NO\} \cup D) \to \mathbf{R}$ specifies a take-it-or-leave-

[4] If disclosure is costless, the solution to the seller's mechanism design problem is trivial: withhold the good unless the buyer discloses his type and then set an optimal posted price conditional on his type.

[5] I am unaware of any papers that consider optimal pricing when the seller can verify the buyer's type at some cost. Severinov and Deneckere (2006) consider a monopoly pricing context in which the buyer can *mis*represent his private information at some cost.

it "sticker price" $p(NO)$ to any buyer who does not disclose his value, as well as a "customized price" $p(v)$ to any buyer who discloses value $v \in D \subset [0, \infty)$; D is the set of buyers who qualify for a discount.

Let $F(\cdot)$, $f(\cdot)$ denote the c.d.f. and p.d.f. of the buyer's value.

The model of perfect disclosure considered here is a limiting case of the model to be presented in Section 3, in which the buyer is able to disclose a "type" that is imperfectly informative of his value. This limiting case is of special interest for two reasons. First, the expected profit-maximizing price list has a particularly simple and intuitive structure. Second, unambiguous welfare implications are available that contrast with well-known results from the literature on perfect price discrimination. In particular, whereas perfect price discrimination increases total welfare relative to an optimal posted price when observing the buyer's value is costless, such price discrimination *decreases* total welfare whenever the cost of disclosure is high enough that discounts are sufficiently rare.

I begin by characterizing the optimal price list, as a function of disclosure cost $c \geq 0$.

Proposition 1. *The optimal price list offers customized price* $p^*(v) = v - c$ *for all* $v \in (c, p^*(NO))$ *and sticker price*

$$p^*(NO) \in \arg\max_p \left(\int_p^\infty f(v) \left(v - \frac{1 - F(v)}{f(v)} \right) \mathrm{d}v + \int_{\min\{p,c\}}^p f(v)(v - c)\mathrm{d}v \right). \qquad (1)$$

Corollary 1. *Suppose that "marginal revenue"* $MR(p) = \frac{\mathrm{d}[p(1-F(p))]}{\mathrm{d}[1-F(p)]} = p - \frac{1-F(p)}{f(p)}$ *is strictly increasing in* p*. Then the sticker price* $p^*(NO)$ *is implicitly and uniquely defined by*

$$p^*(NO) - MR(p^*(NO)) = \min\{p^*(NO), c\}. \qquad (2)$$

Intuition. In the standard monopoly pricing problem, the seller is forced to exclude all buyers having values less than the posted price. Given the possibility of disclosure, the seller may now instead offer such buyers the opportunity to qualify for a discount. Let p^* be the optimal posted price. When $c > p^*$, the optimal sticker price $p^*(NO) = p^*$, reflecting the fact that the seller finds it unprofitable to offer any discount that would induce excluded buyers to disclose their values. By contrast, when $c < p^*$, the seller can strictly improve upon the optimal posted price by offering some customized discounts.

Example 1 (Linear demand)**.** Consider the linear-demand case in which $v \sim U[0,1]$, illustrated in Figure 1. Without disclosure, the seller will set price $p^* = \frac{1}{2}$ so that the lowest-value buyer to receive the good has zero marginal revenue. The seller's expected revenue in this case is $\frac{1}{4}$, while the buyer's expected surplus is $\frac{1}{8}$. For any disclosure cost $c \geq \frac{1}{2}$, such uniform pricing remains optimal. When $c < \frac{1}{2}$, by contrast, the seller can increase profits by inducing buyers having values $v \in (c, 1-c)$ to disclose. Each such buyer can be charged a customized price $p^*(v) = v - c$ that extracts all of the surplus net of disclosure costs ("net surplus"), and that gives the seller *more* than that buyer's marginal revenue $MR(v) = 1 - 2v$.

Overall, the seller's expected revenue is illustrated graphically in Figure 1 as the area under the (bolded) *upper envelope* of marginal revenue $MR(v)$, the net surplus $NS(v) = v - c$, and the x-axis. (The seller optimally induces disclosure here iff $NS(v) > \max\{MR(v), 0\}$.) Namely, the seller's expected revenue is $\frac{1}{2} - c + c^2 > \frac{1}{4}$ while the buyer's expected surplus is $c^2/2 < \frac{1}{8}$.

Welfare implications. If disclosure is sufficiently costly, a profit-maximizing seller simply offers a uniform price with no opportunities to qualify for a discount. On the other hand, when disclosure is costless, the optimal price list amounts to costless, perfect price discrimination. Thus, if $c = 0$, total welfare under the optimal price list is greater than that under the optimal uniform price. More generally, how does welfare under the optimal price list vary with the cost of disclosure, and compare with that under optimal uniform pricing? Proposition 2 shows that the buyer and seller have conflicting interests to raise or lower the cost of disclosure, and that total welfare under the optimal price list is less than that under the optimal uniform price whenever the cost of disclosure is in an intermediate range so that the seller *sometimes but sufficiently rarely* sells the good at a discount.

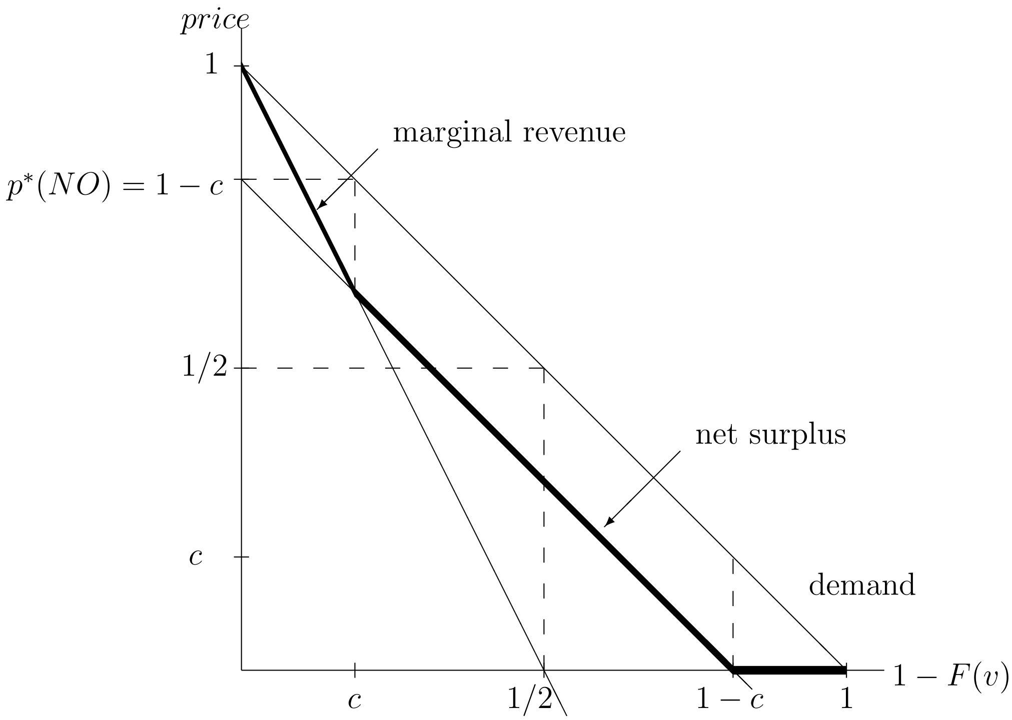

Figure 1: Optimal sticker price given linear demand.

Proposition 2. *The seller's expected profit is non-increasing in c while the buyer's ex post surplus is non-decreasing in the disclosure cost c. Further, as long as the buyer receives a discount sufficiently rarely, expected total welfare is strictly lower under the optimal price list than under the optimal uniform price.*

Intuition: Consider again the case of linear demand. As the cost of disclosure c decreases, the optimal sticker price $p^*(NO) = 1 - c$ increases along with the set of buyer-types who receive customized discounts. However, since such buyers earn zero surplus, every buyer-type is at least weakly worse off as c decreases. Overall, as the cost of disclosure decreases from $\frac{1}{2}$ to any level $c < \frac{1}{2}$, expected buyer welfare decreases by an amount equal to the highlighted trapezoid in Figure 3.

By contrast, for all buyers having values $v \in (c, 1-c)$, the seller is able to extract the full net surplus $v - c$, whereas under uniform pricing such buyers would have generated either marginal revenue $1 - 2v$ or nothing. ($v - c > \max\{1 - 2v, 0\}$ for all $v \in (c, 1-c)$.)

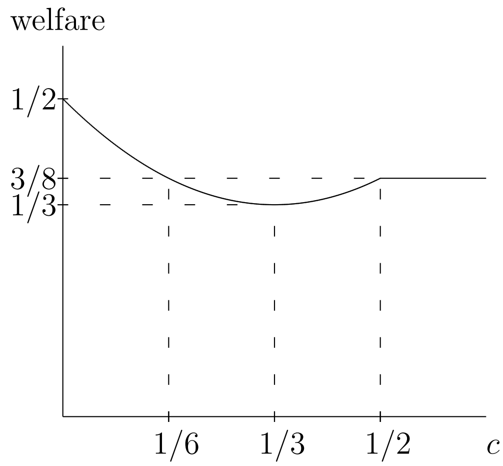

Figure 2: Total welfare depending on the cost of disclosure c.

Overall, as the cost of disclosure decreases from $\frac{1}{2}$ to any level $c < \frac{1}{2}$, expected seller profit increases by an amount equal to the highlighted triangle in Figure 3.

As $c \to \frac{1}{2}$, the seller's expected profit increase is only second-order, while the buyer's expected welfare loss is first-order. Consequently, there is a net expected welfare loss whenever $c \approx \frac{1}{2}$ so that discounted transactions are sufficiently rare.

Example 1 continued. As shown earlier, total welfare under the optimal price list is $1/2 - c + 3c^2/2$, which is (i) minimized at $c = 1/3$, (ii) maximized at $c = 0$, and (iii) equals the total welfare under optimal uniform pricing at $c = 1/2$ and $c = 1/6$. Overall, total welfare under the optimal price list is less than under the optimal uniform price iff $c \in (1/6, 1/2)$. Since the buyer's value is uniformly distributed on $[0, 1]$ and he receives a discount iff $v \in (c, 1 - c)$ (and never if $c > 1/2$), this welfare finding can be re-stated in terms of the probability that the buyer receives a discount. Namely, an optimal price list generates less total welfare than an optimal uniform price iff the buyer receives a discount less than 2/3 of the time.

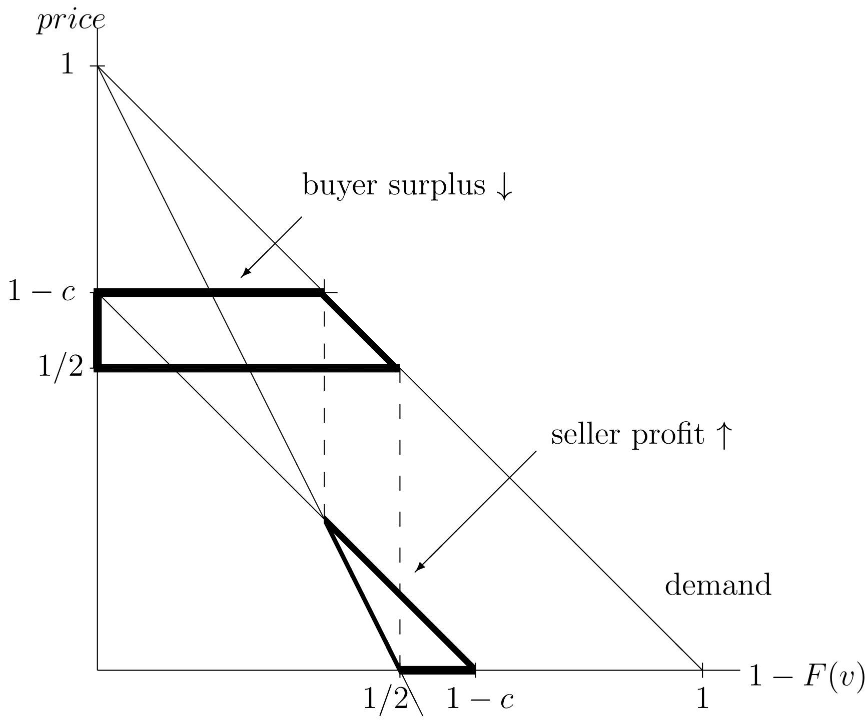

Figure 3: Welfare effects of costly disclosure of buyer values.

3 Model

A risk-neutral monopoly seller has an indivisible, perishable good and faces a single, risk-neutral buyer. This buyer has willingness to pay (or "value") $v \in V \subset \mathbf{R}_+$ and "type" $t \in T$, each his private information. As shorthand, I will refer to a buyer with value v and type t simply as "buyer (v, t)". The buyer's type has density $g(\cdot)$ over measurable type-space T while, conditional on type t, the buyer's value is distributed according to c.d.f. $F(\cdot|t)$ with p.d.f. $f(\cdot|t)$. If the seller were to offer the object at price p to type-t buyers, expected revenue would be $R(p,t) = p(1 - F(p|t))$. I shall assume that $R(p,t)$ is strictly concave for all t.

The seller commits to a *sales mechanism with costly disclosure*. Buyer messages in such a mechanism consist of both cheap-talk from a set X as well as, possibly, "disclosure" of the buyer's type. (See discussion point (a) below.) Let $M = X \times (T \cup \{\emptyset\})$ denote the space of potential messages, $M_\emptyset = X \times \{\emptyset\}$ those available to all buyers and $M_t = X \times \{t\}$

those available only to type-t buyers that serve to disclose type t. The sales mechanism itself specifies an allocation function $q(m)$ specifying the probability that the buyer gets the object and a corresponding payment function $z(m)$.

Disclosure is potentially costly, for both the buyer and the seller; in particular, the buyer pays $c_B(t) \geq 0$ and the seller pays $c_S(t) \geq 0$ whenever the buyer discloses type t, where $c(t) = c_B(t) + c_S(t) \geq 0$ for all types t. (See point (b).)

Most of the analysis in the paper will focus on a sub-class of sales mechanisms with costly disclosure, called "price-list mechanisms".

Definition 2 (Price-list mechanism)**.** A *price-list mechanism* is a sales mechanism with costly disclosure in which the buyer is offered a take-it-or-leave-it "sticker price" $p(NO)$ should he not disclose, or a take-it-or-leave-it "customized price" $p(t)$ should he disclose type $t \in D$, for some $D \subset T$.[6]

Without loss of generality, we may further restrict attention to price lists in which $p(t) \leq p(NO) - c_B(t)$ for all $t \in D$; otherwise, buyers of type t strictly prefer to buy at the sticker price without disclosing.

Discussion of the model. (a) In a standard setting in which buyer disclosure is not possible, any extensive-form interaction between the buyer and seller is equivalent to a direct-revelation mechanism by the Revelation Principle. With costly disclosure, however, the extensive form matters. I have chosen here to focus on a scenario in which the buyer and seller *cannot* engage in a "conversation" with potentially several rounds of communication, in which the cost of disclosure is incurred only if and when the buyer discloses his type. Instead, the buyer here simply sends a single message. Such a conversation-less model may be a better fit with some applications, e.g. if the buyer must decide whether

[6]More formally, a price-list mechanism is one with cheap-talk message space $X = V$, allocation probability $q(v, \emptyset) = 1$ if $v > p(NO)$, $q(v,t) = 1$ if $t \in D$ and $v > p(t) + c_B(t)$, and $q(m) = 0$ otherwise; (iii) payment $z(v, \emptyset)) = p(NO)$ if $v > p(NO)$, $z(v,t) = p(t)$ if $t \in D$ and $v > p(t) + c_B(t)$, and $z(m) = 0$ otherwise.

to incur the cost of being able to disclose *before* communication begins. For instance, patients in Genentech's Avastin Access Solution program (mentioned in the introduction) must gather their financial and insurance information and attend their interview regardless of how that interview is conducted.

(b) The costs to the seller triggered by disclosure are by assumption unavoidable. For example, retailers typically pay a fee whenever a buyer finds and uses an online coupon at a coupon-aggregator website such as Keycode.com; using the coupon can be viewed as "disclosing" that he is the sort of buyer who seeks out online coupons at that site.

4 Sales mechanisms with costly disclosure

In this section, I will characterize the expected profit-maximizing sales mechanism with costly disclosure. Section 4.1 establishes that this optimal mechanism takes the form of a price-list mechanism. Section 4.2 then characterizes the optimal price-list mechanism.

4.1 Optimality of price-list mechanisms

Before proceeding, it is helpful to review briefly why a single posted price is the optimal sales mechanism without disclosure, subject to the usual interim individual-rationality (IR) and interim incentive-compatibility (IC) constraints.

Review: why a posted price is optimal without disclosure. *Revelation Principle.* Without disclosure, it is without loss to restrict attention to direct-revelation mechanisms. Let $S(v) = vq(v) - z(v)$ denote the interim expected surplus of a buyer having value v in such a mechanism. (Recall that $q(m)$ and $z(m)$ are the buyer's probability of receiving the good and payment, respectively, after sending message m.)

Envelope Theorem. IC requires $S(v) = \max_{v'} vq(v') - z(v')$ so that $S(v) = S(0) + \int_0^v q(\tilde{v})d\tilde{v}$ by the Envelope Theorem. Buyer payment is then $z(v) = vq(v) - S(v) = vq(v) - \int_0^v q(\tilde{v})d\tilde{v} - S(0)$. Thus, the seller's expected revenue depends only on (i) the allocation probabilities $q(v)$ for all $v \in V$ and (ii) the surplus $S(0)$ of a buyer having zero value. In particular,

$$\begin{aligned} R(q(\cdot), S(0)) &= \int_0^\infty \left(vq(v) - \int_0^v q(\tilde{v}) d\tilde{v} \right) f(v) \mathrm{d}v - S(0) \\ &= \int_0^\infty q(v) MR(v) f(v) \mathrm{d}v - S(0) \end{aligned}$$

where $MR(v) = \frac{\mathrm{d}[v(1-F(v))]}{\mathrm{d}[1-F(v)]}$ is the "marginal revenue" of the buyer having value v.

Zero surplus to zero-value buyers. Since expected revenue is decreasing in the zero-value surplus $S(0)$, it is without loss to focus on mechanisms in which $S(0) = 0$.

Marginal revenue. The optimal mechanism design problem now reduces to the problem of choosing allocation probabilities. Selling to buyers having value v raises more revenue from such buyers directly, for marginal ex ante gain $f(v)v$, but forces the seller also to increase the interim surplus of all buyers having higher values, for marginal ex ante loss $(1 - F(v))$; the net effect on expected revenue is $MR(v)f(v)$. The solution to this problem is well-known: sell the good with probability one (zero) to all buyers having value greater than (less than) v^*, where $v^* \in \arg\max_v \int_v^\infty MR(v)f(v)dv$.

All together, the optimal sales mechanism is to offer a take-it-or-leave-it price equal to v^*.

Discussion: what changes with disclosure? Not surprisingly, some standard tools of optimal mechanism design remain powerful in this paper's setting. For example, let $S(v,t)$ denote the interim expected surplus of buyer (v,t). All buyers having the *same type* t can only distinguish themselves through cheap talk. Thus, the Envelope Theorem implies that $S(v,t) = S(0,t) + \int_0^v q(\tilde{v},t)d\tilde{v}$ for all types t. In particular, the seller's expected revenue can be expressed as a function of (i) the allocation probabilities $\{q(v,t) : v \in V, t \in T\}$, (ii) the zero-value surpluses $\{S(0,t) : t \in T\}$, and (iii) the event YES in which the buyer discloses his type.

On the other hand, the Revelation Principle does not apply here. Indeed, even assuming that the buyer sends a *non-random* message may entail loss of generality; in principle, the buyer might randomize over whether to send a cheap talk message or a costly disclosing message. (In fact, the buyer sends a non-random message in the optimal mechanism; see Lemma 1.)

Most importantly, the possibility of disclosure adds novel constraints to the mechanism design problem. In particular, incentive-compatibility requires that buyer (v,t) disclose its type whenever $S(v,t) > \min_{t'} S(v,t')$; otherwise some other buyer (v,t') would be able to profitably deviate by "mimicking" buyer (v,t). Because of this "disclosure IC constraint", it is not even clear that the seller will always offer zero surplus to zero-value buyers. In particular, even though raising $S(0,t)$ lowers the seller's expected profit by (i) forcing the seller to offer type-t buyers more surplus and (ii) forcing the seller to induce more type-t buyers to disclose, it also could help the seller by (iii) allowing the seller not to induce disclosure by other types $t' \neq t$. Put more briefly, raising the surplus offered to type-t buyers could in principle increase profits by slackening the disclosure IC constraint for other types of buyers. (In fact, zero-value buyers all receive zero surplus in the optimal mechanism; see Lemma 4.)

Theorem 1. *The optimal sales mechanism with costly disclosure satisfying interim incentive-compatibility (IC) and interim individual-rationality (IR) (or, simply, "the optimal mechanism") is a price-list mechanism.*

Theorem 4.1 is the most technically challenging result of the paper; the rest of this section outlines its proof.

Part I: Non-random disclosure. Let $M(v,t)$ be the *set* of messages sent with positive probability by buyer (v,t). Lemma 1 establishes that, in the optimal sales mechanism, buyer (v,t) is either certain to disclose t or certain not to disclose t. Thus, it is in fact without loss to assume that the buyer sends a single message.

Definition 3 (Non-random disclosure)**.** A sales mechanism with costly disclosure has *non-random disclosure* if $\Pr(M(v,t) \subset M_\emptyset \text{ or } M(v,t) \subset M_t) = 1$.

Lemma 1. *The optimal sales mechanism has non-random disclosure.*

Sketch of Lemma 1's proof: Buyers who disclose randomly must be indifferent between disclosure and non-disclosure. If so, the seller can increase expected profit by breaking this indifference in favor of non-disclosure, for a combination of reasons. First, because of buyer (v,t)'s indifference, inducing non-disclosure does not force the seller to increase the expected surplus offered to type-t buyers. Second, since non-disclosure was already an option for buyer (v,t) in the original mechanism, no type-t' buyer has any new incentive to change its behavior, for all other types $t' \neq t$. All together, all buyer surpluses remain the same after this change. On the other hand, inducing non-disclosure increases total surplus by dissipating less through disclosure costs, increasing seller profit.

Definition 4 (Mechanisms with non-random disclosure)**.** Any sales mechanism with non-random disclosure can be characterized by a disclosure set $YES = \{(v,t) : M(v,t) \subset M_t\}$, non-disclosure set $NO = \{(v,t) : M(v,t) \subset M_\emptyset\}$, allocation probabilities $\{q(v,t) : v \in V, t \in T\}$, and buyer surpluses $\{S(v,t) : v \in V, t \in T\}$.[7]

Since NO is the complement of YES (up to a zero-measure set), any mechanism with non-random disclosure can be described simply as a triplet $(YES, q(\cdot,\cdot), S(\cdot,\cdot))$.

Part II: The seller's "relaxed problem".

Lemma 2. *A mechanism with non-random disclosure* $(YES, q(\cdot,\cdot), S(\cdot,\cdot))$ *satisfies IR and IC only if, for all* (v,t)*,*

$$S(0,t) \geq 0 \text{ and } S(v,t) = \int_0^v q(\tilde{v},t)\mathrm{d}\tilde{v} + S(0,t) \tag{3}$$

$$v' > v \Rightarrow q(v',t) \geq q(v,t) \tag{4}$$

$$S(v,t) > \min_{t'} S(v,t') \Rightarrow (v,t) \in YES. \tag{5}$$

[7] Buyer (v,t)'s payment $z(v,t) = vq(v,t) - S(v,t) - c_B * 1_{(v,t)\in YES}$.

The seller's problem. The seller's expected profit in mechanism $(YES, q(\cdot,\cdot), S(\cdot,\cdot))$ equals the total surplus generated from allocating the object, minus total disclosure costs, minus buyer surplus. Expressed in terms of "marginal revenue",[8]

$$E[PROFIT] = \int\int \big(vq(v,t) - c(t) * 1_{\{(v,t)\in YES\}} - S(v,t)\big) f(v|t)g(t)\mathrm{d}v\mathrm{d}t \tag{6}$$
$$= \left(\int \left(vq(v,t) - \int_0^v q(\tilde{v},t)\mathrm{d}\tilde{v} - c(t) * 1_{\{(v,t)\in YES\}}\right) f(v|t)\mathrm{d}v\right) g(t)\mathrm{d}t - E[S(0,t)]$$
$$= \int \left(\int q(v,t)MR(v,t)f(v|t)\mathrm{d}v - c(t)\Pr\left((v,t)\in YES|t\right)\right) g(t)\mathrm{d}t - E[S(0,t)] \tag{7}$$

Definition 5 (Marginal revenue)**.** The "marginal revenue" associated with buyer (v,t) is $MR(v,t) = \frac{\mathrm{d}[v(1-F(v|t))]}{\mathrm{d}(1-F(v|t))} = v - \frac{1-F(v|t)}{f(v|t)}$.[9]

Thus, the seller's objective is to select $(YES, q(\cdot,\cdot), S(0,\cdot))$ to maximize (7) subject to buyer surpluses $S(v,t) = S(0,t) + \int_0^v q(\tilde{v},t)\mathrm{d}\tilde{v}$ satisfying the appropriate IR and IC constraints.

IR and IC constraints. Let $S(v,t;v',t') = S(v',t') - (v'-v)q(v',t')$ denote buyer (v,t)'s surplus upon "mimicking" buyer (v',t'). IR requires that $S(v,t)\geq 0$ for all (v,t). And since buyer (v,t) can only feasibly mimic (v',t') if either $t'=t$ or $(v',t')\in NO$, IC requires that $S(v,t)\geq S(v,t;v',t)$ for all (v,t) and all $v'\neq v$ and that $S(v,t)\geq S(v,t;v',t')$ for all (v,t) and all $(v',t')\in NO$.

The seller's relaxed problem. Rather than solve the seller's problem directly, I shall proceed to solve first the "relaxed problem" of finding $(YES^*, q^*(\cdot,\cdot), S^*(0,\cdot)$ that maximize (7) subject to the *necessary* conditions (3-5) of IR and IC. Then, I will show that the solution to the relaxed problem satisfies IR and IC and hence is itself optimal.

Part III: Disclosure and allocation. Suppose that $(YES^*, q^*(\cdot,\cdot)), S^*(0,\cdot))$ solves the relaxed problem, and let $S^*(v,t) = S^*(0,\cdot) + \int_0^v q^*(\tilde{v},t)\mathrm{d}\tilde{v}$ be the induced buyer

[8]The first equality above follows from (3) while the second follows from integration by parts.

[9]$MR(v,t) = \frac{dR(1-F(v|t),t)}{dq}$, where $R(q,t) = qF^{-1}(1-q|t)$ denotes the seller's expected revenue when the good is sold with probability q.

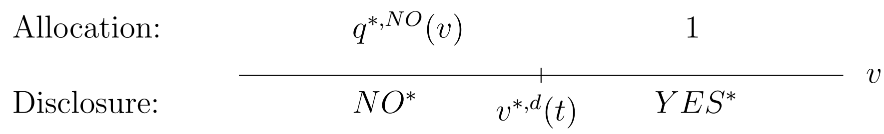

Figure 4: Type-t buyers' allocation probability and disclosure (Lemmas 3-4).

surpluses. Lemma 3 provides several properties of any such solution, listed in the order in which they are proven in the Appendix.

Lemma 3 (Properties of the solution)**.** *The following properties must hold in any solution to the relaxed problem, for a full-measure set of buyers:*

(a) $(v,t) \in YES^*$ *iff* $S^*(v,t) > \min_{t'} S^*(v,t')$*;*

(b) $\exists$ *non-decreasing* $q^{*,NO}(\cdot)$ *such that* $(v,t) \in NO^*$ *implies* $q^*(v,t) = q^{*,NO}(v)$*;*

(c) $v'' > v' > v$*,* $(v,t) \in NO^*$*, and* $(v',t) \in YES^*$ *together imply* $(v'',t) \in YES^*$*;*

(d) $v' > v$*,* $v \in NO^*$*, and* $v' \in YES^*$ *together imply* $q^*(v',t) = 1$*; and*

(e) $\exists$ *"disclosure thresholds"* $v^{*,d}(t)$ *for all* t*, such that* $(v,t) \in YES^*$ *if* $v > v^{*,d}(t)$ *and* $(v,t) \in NO^*$ *if* $v < v^{*,d}(t)$*.*

Lemma 4 (Zero surplus to zero-value buyers)**.** *In any solution to the relaxed problem,* $S^*(0,t) = 0$ *for a full-measure set of types* $t \in T$*.*

Lemmas 3-4 have several notable implications, summarized in Figure 4. First, any buyer who does not disclose his type receives the good with a probability $q^{*,NO}(v)$ that depends on his value but not his type. Second, the buyer discloses his type iff his value lies above a threshold $v^{*,d}(t) \leq \infty$. Third, any buyer who discloses his type receives the good with probability one.

Given these results, the seller's relaxed problem reduces to choosing an allocation probability $q^{*,NO}(v) \in [0,1]$ for every value v and a disclosure threshold $v^{*,d}(t) \in V \cup \{\infty\}$

for every type t, so as to maximize

$$\int_{(v,t):v<v^{*,d}(t)} q^{*,NO}(v)MR(v,t)f(v|t)g(t)\mathrm{d}v\mathrm{d}t + \int_{(v,t):v>v^{*,d}(t)} \left(MR(v,t) - c(t)\right) f(v|t)g(t)\mathrm{d}v\mathrm{d}t, \quad (8)$$

subject *only* to the monotonicity constraint that $q^{*,NO}(\cdot)$ is non-decreasing. (Since $q^*(v,t) = q^{*,NO}(v)$ for all $v < v^{*,d}(t)$ and $q^*(v,t) = 1$ for all $v > v^{*,d}(t)$, $q^*(v,t)$ is non-decreasing in v for all t iff $q^{*,NO}(v)$ is non-decreasing in v.)

For the moment, treat the disclosure thresholds $v^{*,d}(\cdot)$ as fixed and consider the problem of how to optimally select non-decreasing allocation probabilities $q^{*,NO}(\cdot)$ for non-disclosers. This problem is equivalent to the standard monopoly pricing problem, when faced with an *isolated* market segment in which values have c.d.f. $\tilde{F}(v) = \Pr(v' < v|v' < v^{*,d}(t))$. As is well known, the seller maximizes profits by selling the object with probability one (zero) when $v > v^{*,NO}$ ($v < v^{*,NO}$), where the threshold $v^{*,NO}$ can be interpreted as a take-it-or-leave-it price.

Lemma 5 (Allocation threshold for non-disclosers)**.** *In any solution to the relaxed problem, there exists an "allocation threshold"* $v^{*,NO}$ *such that* $q^{*,NO}(v) = 0$ *if* $v < v^{*,NO}$ *and* $q^{*,NO}(v) = 1$ *if* $v > v^{*,NO}$.

Part IV: Optimality of a price-list mechanism.

Lemma 6 (Price-list mechanism)**.** *The solution to the seller's relaxed problem is a price-list mechanism in which (i) the set of types that qualify for a discount* $D^* = \{t \in T : v^{*,d}(t) < v^{*,NO}\}$, *(ii) each type* $t \in D^*$ *is offered customized price* $p^*(t) = v^{*,d}(t) - c_B(t)$, *and (iii) the sticker price* $p^*(NO) = v^{*,NO}$ *for all* $t \in D^*$.

Lemma 6 completes the proof of Theorem 1. By Lemma 6, the solution to the seller's relaxed problem is a price-list mechanism $(D^*, p^*(NO), p^*(t) : t \in D^*)$ such that $p^*(t) \leq p^*(NO) - c_B(t)$ for all $t \in D^*$. Any such mechanism clearly satisfies IR and IC, rather than just IR and the necessary conditions of IC required in the relaxed problem. Thus, this mechanism is indeed optimal among all sales mechanisms with non-random disclosure.

4.2 Optimal price-list mechanism

In Section 4.1, I showed that the optimal sales mechanism with costly disclosure is a price-list mechanism. Here, I will characterize the optimal such mechanism.

The seller's objective is to select the set of "discount types" $D \subset T$ who will be eligible for a discount and the schedule of prices $\{p(NO), p(t) : t \in D\}$ so as to maximize expected profit, subject to the constraint that all discount types must at least weakly prefer to disclose and buy at price $p(t)$ than not disclose and buy at price $p(NO)$. That is, the seller selects $\{D, p(NO), p(t) : t \in D\}$ to maximize expected profit

$$\Pi(D; p(\cdot)) = \int_{t\in D} (p(t) - c_S(t))\,(1 - F(p(t) + c_B(t)|t))\,g(t)\mathrm{d}t + \int_{t\notin D} p(NO)\,(1 - F(p(NO)|t))\,g(t)\mathrm{d}t$$

subject to $p(t) \leq p(NO) - c_B(t)$ for all $t \in D$.

Theorem 2. *The set of types D^* who qualify for a discount and the schedule of prices $p^*(\cdot)$ in an optimal price-list mechanism satisfy:*

$$p^*(NO) \in \arg\max_p \int_{t\notin D^*} p\,(1 - F(p|t))\,g(t)\mathrm{d}t \tag{9}$$

$$p^*(t) = \arg\max_p\,(p - c_S(t))\,(1 - F(p + c_B(t)|t)) \;\; \textit{for all } t \in D^* \tag{10}$$

$$t \notin D^* \Leftrightarrow p^*(NO)\,(1 - F(p^*(NO)|t)) > \max_{p\leq p^*(NO)-c_B(t)} (p - c_S(t))\,(1 - F(p + c_B(t)|t)) \tag{11}$$

Discussion: The sticker price (9) is the optimal monopoly price against the endogenous set of buyer types who do not disclose. Similarly, the discount (10) offered to each type $t \in D^*$ is the optimal monopoly price against a type-t buyer, in an environment in which the buyer has value distributed as $(v - c_B(t))|t$ and the seller has constant marginal cost equal to $c_S(t)$. (*Intuition*: Disclosure costs are not sunk when the seller sets the type-t price nor when type-t buyers decide whether to buy the good.)

While natural, these properties of the sticker price $p^*(NO)$ and customized prices $\{p^*(t) : t \in D\}$ are not obvious. Since the incentive to disclose depends on the sticker price, one might expect the optimal sticker price to be constrained by the customized prices being offered. To see why this does not arise, suppose for the sake of contradiction that $p^*(t) = p^*(NO) - c_B(t)$ for some $t \in D^*$. Type-t buyers are then indifferent between disclosing and paying $p^*(NO)$, or disclosing and paying $p^*(t)$. However, the seller is not indifferent, since she sells to the same set of buyers (those of type t with values greater than $p^*(NO)$) but profits $c(t)$ more when such buyers purchases at sticker price. Thus, the seller would have been better off forcing type-t buyers to pay sticker price.

Least obvious is how to construct the optimal set of discount types D^*. (11) establishes an important property of D^* that characterizes it in terms of the optimal sticker price $p^*(NO)$. In particular, type-t buyers will be induced to disclose iff, *against type-t buyers only*, the customized price (10) with disclosure is strictly more profitable than the sticker price (9) without disclosure. In this sense, the problem of whether to offer customized discounts is separable across types.

Computing the optimal price-list mechanism. Theorem 2 suggests a numerical approach to compute the optimal price-list mechanism. For any candidate sticker price $p(NO)$, (11) uniquely determines the optimal set of discount types $D(p(NO)$ *if* $p(NO)$ is the optimal sticker price. Conversely, given a disclosing set D, first-order condition (9) uniquely determines the optimal sticker price $p(D)$ *if* D is the optimal disclosing set. All together, the optimal sticker price and optimal disclosing set must satisfy the fixed-point condition that $p(D(p^*(NO))) = p^*(NO)$ and $D(p(D^*)) = D^*$. While this is a necessary condition of the optimal price-list mechanism, it is not sufficient. In principle, one must identify all such fixed points. The optimal mechanism corresponds to whichever fixed point generates the greatest expected profit for the seller.

Example 2. Suppose that the buyer's type $t \sim U[0,1]$, the buyer's value $v|t \sim U[0,t]$ conditional on type t, and the buyer pays disclosure cost $c_B(t) = c(t)$ while the seller pays nothing.

For any given sticker price $p(NO)$, what is the optimal set of types $D(p(NO))$ to be offered a customized price $p(t) \leq p(NO) - c(t)$? (If $p(t) > p(NO) - c(t)$, then every type-t

buyer will prefer to not disclose and buy the good at sticker price.) If pooled at the sticker price, type-t buyers generate expected profit $p(NO)(1 - F(p(NO)|t)) = \frac{p(NO)(t-p(NO))}{t}$. On the other hand, if offered customized price $p(t)$, they generate expected profit of at most $\max_{p(t)\leq p-c} p(t)(1 - F(p(t) + c)|t)) = \max_{p(t)\leq p-c} \frac{p(t)(t-p(t)-c)}{t}$. As can be easily checked, the optimal customized price $p^*(t) = \max_{p(t)\leq p-c} \frac{p(t)(t-p(t)-c)}{t} = \frac{t-c}{2}$ when $\frac{t-c}{2} < p - c(t)$; otherwise, customized price revenue is at most $(p(NO) - c)(1 - F(p(NO)|t)) < p(NO)(1-F(p(NO)|t))$, in which case the seller strictly prefers to sell at the sticker price only. So, consider the case in which $\frac{t-c}{2} < p(NO)-c(t)$ or, equivalently, $t < 2p(NO)-c(t)$. By (11), the seller offers type-t buyers a discounted price $p^*(t) = \frac{t-c}{2}$ iff

$$\frac{(t-c)^2}{4t} > \frac{p(NO)(t-p(NO))}{t} \Rightarrow t < 2p(NO) + c - 2\sqrt{p(NO)c}. \tag{12}$$

In particular, the set of buyer-types *not* offered a discount in the optimal price-list mechanism is an increasing interval of the form $[t^*, 1]$, where $t^* = 2p^*(NO) + c - 2\sqrt{p^*(NO)c}$.

Finally, by (9), $p^*(NO)$ is determined by the first-order condition:

$$\begin{aligned}
&\int_{t^*}^{1} \frac{\mathrm{d}\left[\frac{p^*(NO)(t-p^*(NO))}{t}|t))\right]}{\mathrm{d}p} g(t)\mathrm{d}t = 0\\
\Leftrightarrow &\int_{t^*}^{1} \frac{t-2p^*(NO)}{t}\mathrm{d}t = (1-t^*) + 2p^*(NO)\ln t^* = 0\\
\Leftrightarrow &\, p^*(NO) = \frac{1-t^*}{-2\ln t^*}.
\end{aligned}$$

Proposition 3 summarizes these findings.

Proposition 3. *In the optimal price-list mechanism in Example 2, the seller offers sticker price $p^*(NO)$ as well as customized prices $p^*(t) = \frac{t-c}{2}$ to buyers who disclose types $t < t^*$, where $(p^*(NO), t^*)$ solve the following system of equations:*

$$p^*(NO) = \frac{1-t^*}{-2\ln t^*} \tag{13}$$

$$t^* = 2p^*(NO) + c - 2\sqrt{p^*(NO)c} \tag{14}$$

4.3 Extensions

Finitely many types. Online shoppers frequently qualify for discounts by providing a "promotional code", proving to the seller that they are aware of the code. Such codes may be distributed to a targeted buyer segment through an email or marketing campaign, in which case awareness proves that the buyer belongs to this market segment. Or, they may be available on websites that may only be searched by a subset of potential buyers, in which case awareness proves that the buyer is the sort who searches the website. In either case, there are just two "types" of buyer, whereas the baseline model assumes a continuum of buyers.[10]

Fortunately, all results extend *directly* to settings with finitely many buyer types, under an appropriate re-interpretation. To see why, imagine for the moment a hypothetical situation in which the buyer can disclose an uninformative "label" drawn uniformly from $[0, 1]$ as well as one of finitely many payoff-relevant types $t \in T$. Since a density now exists over the enlarged type-space $T \times [0, 1]$, this paper characterizes the optimal sales mechanism, which takes the form of a price-list mechanism. By Theorem 2, the seller is not indifferent between offering a sticker price or a customized price to any type of buyer in this optimal mechanism. Consequently, buyers having the same payoff-relevant type but different labels must either all disclose or all not disclose their types and labels in the optimal mechanism. In particular, this mechanism remains optimal in the model of interest, with finitely many types but no labels.

Fixed costs of enabling disclosure. Newspaper coupons are similar to promotional codes, in that using a coupon "discloses" that the buyer uses coupons, but with the extra feature that the seller must pay a *fixed cost* to place the coupon in the paper and thereby enable buyer disclosure. By contrast, the baseline model assumes that all disclosure

[10]Another notable feature of this example is that buyers cannot feasibly disclose that they do *not* know the promotional code. This can be accommodated within the baseline model, by assuming an infinite cost to disclose unawareness.

costs are marginal costs, paid only upon disclosure. Fortunately, it is simple to extend the analysis to endogenize the set of types that can be disclosed. Suppose that, for all $T' \subset T$, the seller must incur fixed cost $C(T')$ to enable the buyer to disclose that his type is t for any $t \in T'$. This paper characterizes the seller's subsequent variable profit $\pi(T')$ in the optimal price-list mechanism when types T' can be disclosed. To maximize profits, then, the seller will enable disclosure of all types in $\arg\max_{T' \subset T} (\pi(T') - C(T'))$.

Private cost of service. The cost of service may vary across buyers in a way that is unknown to the seller. For example, closing a sale often brings ancillary *benefits* of service, the value of which may depend on the buyer's type. To accommodate this, consider an extension in which the buyer's private information consists of a value v, a (potentially negative) cost of service s, and a disclosable type t. Let $S(v, s, t)$ denote the expected surplus that is offered to buyer (v, s, t) in a given mechanism. As in the baseline analysis, incentive-compatibility (IC) requires that $S(v, s, t) = S(0, s, t) + \int_0^v q(\tilde{v}, s, t)\mathrm{d}\tilde{v}$ for all (s, t), where $q(v, s, t)$ is the probability that buyer (v, s, t) receives the good. At the same time, IC requires that $S(v, s, t) = S(v, s', t) \equiv S(v, t)$ for all (v, t) and all costs s, s', since buyer (v, s, t) can earn $S(v, s', t)$ by "mimicking" buyer (v, s', t), and vice versa. In particular, these conditions together imply that $q(v, s, t) = q(v, s', t) \equiv q(v, t)$ for all costs s', s.

The seller's objective in this richer setting is therefore very similar to that in the baseline case with known cost of service. Namely, the seller seeks to maximize an objective equal to that in (7) *minus* an extra term $\int \int q(v, t)E[s|v, t]f(v|t)\mathrm{d}v g(t)\mathrm{d}t$ related to the expected cost of service. The rest of the analysis of Section 4 carries through with only minor modifications. In particular, the optimal sales mechanism with costly disclosure is still a price-list mechanism.

5 Concluding Remarks

Standard monopoly pricing models of a single, indivisible, perishable good take as given what the monopolist knows about the distribution of buyer values: either values are perfectly known (perfect price discrimination), some payoff-relevant characteristic is known (market segmentation), or nothing is known (uniform pricing). This paper endogenizes what the monopolist knows about buyers when setting prices, in a setting with costly disclosure of a payoff-relevant characteristic. The optimal sales mechanism takes a familiar form: the seller offers a "sticker price" to any buyer, as well as a pre-specified list of discounts to qualifying buyers (Theorem 1).

This optimal sales mechanism bears a close resemblance to standard, optimal monopoly market segmentation. In particular, the optimal sticker price is equal to the optimal monopoly price against the endogenous segment of buyers who choose not to disclose their type (Theorem 2). However, there are important differences. For one thing, since disclosure is costly, the practice of perfect price discrimination need not increase total welfare. Indeed, as long as the fraction of buyers receiving fully-extractive customized prices is small enough, one may infer that total welfare is lower than if price discrimination were not possible (Proposition 2).

I conclude by discussing some significant issues not addressed by this paper's analysis, which might be interesting topics for future work.

Listing costs. The optimal pricing mechanism derived here can be viewed as a list of prices: a "sticker price" available to any buyer, as well as a schedule of discounts available to certain buyer types. An implicit assumption here is that the seller incurs no extra cost when adding another price to this list. Consequently, the optimal mechanism exhibits a potentially unrealistic proliferation of discounts. A worthwhile topic for future work would be to examine the impact of listing costs on what discounts are offered, as well as on seller profits and buyer welfare.

Fairness concerns. Buyers may view some price-discrimination practices as unfair, and such fairness concerns may be important in shaping the set of discounts that a firm offers. For instance, in the context of restaurant pricing in Singapore, Sweden, and the United States, Kimes and Wirtz (2003) find that customers view coupons, time-of-day pricing, and lunch/dinner pricing as fair, but view weekday/weekend pricing and table location pricing as unfair. More broadly, fairness concerns may be an important factor limiting the practice of price discrimination. Amazon famously faced a customer backlash when it was found in 2000 to offer different prices to online customers having different purchasing histories (Ward (2000)), while Best Buy faced bad press and an investigation of its pricing practices in 2007 when it was discovered that prices offered in its brick-and-mortar stores differed from those offered on the internet (Marco (2007)). Both firms subsequently discontinued these pricing practices.

Future benefits. The model here assumes non-negative disclosure costs, but this is not realistic in some important settings. For example, sellers of experience goods and services often offer first-time buyer discounts, e.g. the nationwide tanning salon L.A. Tan offers a "Free $50 tanning value" coupon to new customers only. To restrict such a discount to first-time buyers, the seller needs to check and update a database listing all users of its product who have ever claimed the first-timer discount. Updating such a database may provide future benefits to the seller and hence correspond to a negative disclosure cost, if it enables the seller to extract more revenue from its relationship with the buyer. Of course, if buyers are rational, they will demand a sufficiently attractive discount today to undo any such future revenue-extraction benefit enjoyed by the seller. In that case, *total* disclosure cost would be positive. On the other hand, if the database allows the seller to provide more valuable products and services and thereby increase total surplus in the relationship, total disclosure costs would be negative.

As this example suggests, negative disclosure costs arise naturally when the seller and/or buyer get some future benefit from disclosure today. Indeed, search engines, social networks and other information intermediaries often provide their services for free, in exchange for their users' willingness to share information about themselves that can then be used to customize advertisements or other product offerings. While this paper's analysis can be easily generalized to accommodate negative total disclosure costs – the optimal mechanism will always induce disclosure – research is needed to understand more deeply the role of future benefits in relationships with disclosure. For one thing, whereas the buyer here must either reveal his type fully or else not at all, future work could attempt to endogenize what information is shared, and when.

A Appendix

A.1 Proof of Proposition 1 and its corollary

Proof. Suppose that the seller offers sticker price $p(NO)$. To induce type-v buyers to disclose, the seller must offer customized price $p(v) \le \min\{v-c, p(NO)-c\}$. In particular, the seller will not find it profitable to induce disclosure from any buyer having value $v \ge p(NO)$ or $v \le c$. On the other hand, all buyers having values $v \in (c, p(NO))$ refuse to pay the sticker price but can be profitably induced to disclose. Further, the optimal customized price for any such type is clearly that which extracts all of the surplus, i.e. $p^*(v) = v - c$. All together, the seller's expected profit given sticker price $p(NO) = p$ and optimally-induced disclosure of buyer-types $v \in (c, p)$ equals

$$\Pi(p) = p(1 - F(p)) + \int_{\min\{p,c\}}^{p} f(v)(v-c)\mathrm{d}v \tag{15}$$

$$= \int_{p}^{\infty} f(v)\left(v - \frac{1-F(v)}{f(v)}\right)\mathrm{d}v + \int_{\min\{p,c\}}^{p} f(v)(v-c)\mathrm{d}v. \tag{16}$$

This completes the proof of Proposition 1, since $p^*(NO)$ maximizes (16). The corollary follows immediately from $\frac{\mathrm{d}\Pi(p)}{\mathrm{d}p} = f(p)\left(\frac{1-F(p)}{f(p)} - c\right)$. □

A.2 Proof of Proposition 2

Proof. Part I: Buyer surplus and seller profit. The set of values (or "types") $v \in [0, 1-c]$ that receive zero ex post surplus is non-increasing in c, while (16) implies that the sticker-price paid by all other buyer-types is also non-increasing in c. Thus, the buyer's ex post surplus is non-decreasing in c. Let $\Pi(c)$ be the seller's expected profit, viewed now as a function of disclosure cost c, and let $p^*(NO; c)$ be the optimal sticker price given disclosure cost c. By the Envelope Theorem applied to (16),

$$\frac{\mathrm{d}\Pi(c)}{\mathrm{d}c} = -\int_{\min\{p^*(NO;c),c\}}^{p^*(NO;c)} f(v)\mathrm{d}v \leq 0 \tag{17}$$

so that the seller's expected profit is non-increasing in c.

Part II: Total welfare. Let $p^* = \arg\max_p p(1-F(p))$ be the optimal uniform price. For all $c > p^*$, $p^*(NO; c) = p^*$ and ex post welfare is the same with or without the possibility of disclosure. Suppose that the cost of disclosure decreases from c_h to c_l, for any $c_l < c_h \leq p^*$. There are three effects on total welfare. First, buyers having value $v \in (c_l, c_h)$ now receive the good (after disclosure at cost c_l), for an expected welfare gain of *at most* $(c_h - c_l)(F(c_h) - F(c_l))$. Second, buyers having value $v \in (c_h, p^*(NO; c_h))$ disclose at lower cost, for expected welfare gain $(c_h - c_l)(F(p^*(NO; c_h)) - F(c_h))$. Finally, buyers having value $v \in (p^*(NO; c_h), p^*(NO; c_l))$ now disclose, for an expected welfare loss $c_l(F(p^*(NO; c_l)) - F(p^*(NO; c_h)))$.

Consider now $c_h = p^*$ and $c_l = p^* - \Delta$, where $\Delta > 0$. Since $p^*(NO; p^*) = p^*$, the second effect disappears and the expected welfare gain associated with lowering the disclosure cost from p^* to $p^* - \Delta$ is at most

$$\Delta(F(p^*) - F(p^* - \Delta)) - (p^* - \Delta)(F(p^*(NO; p^* - \Delta)) - F(p^*)). \tag{18}$$

To prove that total welfare falls as disclosure costs fall from p^* to $p^* - \Delta$ for small enough Δ, it suffices to show that $\lim_{\Delta \to 0} \frac{F(p^*(NO;p^*-\Delta)) - F(p^*)}{\Delta} > 0$. Since $F(\cdot)$ has well-defined density, this condition holds iff $\lim_{\Delta \to 0} \frac{p^*(NO;p^*-\Delta) - p^*}{\Delta} > 0$.

By (16), $p^*(NO; c)$ satisfies necessary condition $f(p^*(NO; c))c = 1 - F(p^*(NO; c))$ for all $c \leq p^*$. In particular, the total derivative $\frac{\mathrm{d}[f(p^*(NO;c))c+F(p^*(NO;c))]}{\mathrm{d}c} = 0$. Since $F(\cdot), f(\cdot)$ are assumed to have well-defined derivatives, $\frac{\mathrm{d}p^*(NO;c)}{\mathrm{d}c} = \frac{-f(p^*(NO;c))}{f(p^*(NO;c))+cf'(p^*(NO;c))} < 0$ exists.

We conclude that total welfare is strictly increasing in disclosure cost c, over the range $c \in (p^* - \Delta, p^*)$ for some $\Delta > 0$. Let $\gamma(c) = F(p^*(NO;c)) - F(c)$ be the probability that the buyer receives a discount. Equivalently, we have shown that total welfare is strictly increasing in c whenever $\gamma(c) < \gamma(p^* - \Delta)$. □

A.3 Proof of Lemma 1

Proof. By the Envelope Theorem, $\frac{\partial(\max_m S(m;v,t))}{\partial v} = q(m(v,t))$. In particular, buyer (v,t)'s probability of receiving the good $q(m_1) = q(m_2) \equiv q(v,t)$ for all $m_1, m_2 \in M(v,t)$. The buyer's expected surplus $S(v,t) = \int_0^v q(\tilde{v},t)\mathrm{d}\tilde{v} + S(0,t)$. In particular, the buyer's payment *net of buyer disclosure cost* $z(m_1) + c_B(t) * 1_{m_1 \in M_t} = z(m_2) + c_B(t) * 1_{m_2 \in M_t} = vq(v,t) - \int_0^v q(\tilde{v},t)\mathrm{d}\tilde{v} - S(0,t)$ for all $m_1, m_2 \in M(v,t)$.

Suppose f.s.o.c. that there is a positive measure of buyers that disclose with probability between zero and one in the optimal mechanism, i.e. for each such buyer there exists $m_1(v,t), m_2(v,t) \in M(v,t)$ such that $m_1(v,t) \in M_\emptyset$ and $m_2(v,t) \in M_t$. The seller can strictly increase expected profit from these buyers by inducing each to send only the non-disclosing message $m_1(v,t)$: payment from the buyer increases by $c_B(t)$ while the seller avoids disclosure cost $c_S(t)$. At the same, no other buyer has any new incentive to deviate since the non-disclosing message $m_1(v,t)$ was already available to all buyers. Thus, all other buyers remain equally profitable and the seller can strictly increase expected profit, contradicting the assumption that the original mechanism was optimal. □

A.4 Proof of Lemma 2

Proof. IR implies $S(0,t) \geq 0$, while IC implies $S(v,t) = \int_0^v q(\tilde{v},t)\mathrm{d}\tilde{v} + S(0,t)$; see the proof of Lemma 1. As usual, IC also implies the monotonicity constraint (4). Let $S(v',t;v,t)$ denote buyer (v',t)'s surplus when mimicking buyer (v,t). $S(v',t;v,t) = S(v,t) + (v'-v)q(v,t)$, while vice versa $S(v,t;v',t) = S(v',t) - (v'-v)q(v',t)$. IC requires $S(v',t) - S(v',t;v,t)) + S(v,t) - S(v,t;v',t) = (v'-v)(q(v',t) - q(v,t)) \geq 0$. Namely, $v' > v$ implies $q(v',t) \geq q(v,t)$. Finally, suppose f.s.o.c. that $S(v,t) > S(v,t')$ for some $t' \neq t$, but $(v,t) \in NO$. $S(v,t') \geq S(v,t';v,t)$ by IC while $S(v,t';v,t) = S(v,t)$ by $(v,t) \in NO$, a contradiction. $\square$

A.5 Proof of Lemma 3

Proof of Lemma 3(a): $S^*(v,t) > \min_{t'} S^*(v,t')$ implies $(v,t) \in YES^*$ by (5). And, since disclosure is costly, the seller maximizes (7) by only inducing buyer (v,t) to disclose when (5) requires disclosure. Thus, the set of buyers such that $(v,t) \in YES^*$ and $S^*(v,t) = \min_{t'} S^*(v,t')$ must have zero measure. $\square$

Proof of Lemma 3(b): For all v, let $T(v) = \arg\min_t S^*(v,t)$. By Lemma 3(a), $(v,t) \in NO^*$ iff $t \in T(v)$. Let $t(v)$ denote any selection from $\{T(v) : v \in V\}$. For any $v' > v$, buyers $(v,t(v))$, $(v',t(v'))$ can mimic each other. Buyer $(v,t(v))$'s surplus from mimicking $(v',t(v'))$ equals $S^*(v',t(v')) - (v'-v)q^*(v',t(v'))$, while buyer $(v',t(v'))$'s surplus from mimicking $(v,t(v))$ equals $S^*(v,t(v)) + (v'-v)q^*(v,t(v))$. Thus, IC requires

$$S^*(v,t(v)) \geq S^*(v',t(v')) - (v'-v)q^*(v',t(v')) \tag{19}$$

$$S^*(v',t(v')) \geq S^*(v,t(v)) + (v'-v)q^*(v,t(v)). \tag{20}$$

By (19-20), $q^*(v',t(v')) \geq \frac{S^*(v',t(v')) - S^*(v,t(v))}{v'-v} \geq q^*(v,t(v))$. Since this is true for *any* selection $t(v)$, $\inf_{t \in T(v')} q^*(v',t) \geq \sup_{t \in T(v)} q^*(v,t)$. Namely, $q^*(v,t) = q^*(v,t')$ for a full-measure set of types $t,t' \in T(v)$, for a full-measure set of values. Further, by (19-20), $\lim_{\varepsilon \to 0} \frac{S^*(v+\varepsilon,t(v+\varepsilon)) - S^*(v,t(v))}{\varepsilon} = \lim_{\varepsilon \to 0} q^*(v+\varepsilon,t(v))$, and $\lim_{\varepsilon \to 0} \frac{S^*(v,t(v)) - S^*(v-\varepsilon,t(v-\varepsilon))}{\varepsilon} = \lim_{\varepsilon \to 0} q^*(v-\varepsilon,t(v))$. Namely, again for a full-measure set of buyers $(v,t) \in NO^*$, $q^*(v,t) = q^{*,NO}(v)$, where we define $q^{*,NO}(v) \equiv \frac{\mathrm{d}[\min_{t'} S^*(v,t')]}{\mathrm{d}v}$.

Proof of Lemma 3(c): Suppose for the sake of contradiction that there exists $v'' > v' > v$ such that $(v,t) \in NO^*$, $(v',t) \in YES^*$, and $(v'',t) \in NO^*$. By Lemma 3(a), $S^*(v',t) > \min_{t'} S^*(v',t')$ while $S^*(\tilde{v},t) = \min_{t'} S^*(\tilde{v},t')$ for $\tilde{v} \in \{v,v''\}$. By continuity, there exists

$\underline{v} = \inf\{\tilde{v} < v' : (v,t) \in YES^* \text{ for all } v \in (\tilde{v}, v')\} < v'$ and $\overline{v} = \sup\{\tilde{v} > v' : (v,t) \in YES^*$ for all $v \in (v', \tilde{v})\} > v'$. Further, $S^*(\tilde{v}, t) = \min_{t'} S^*(\underline{v}, t')$ for $\tilde{v} \in \{\underline{v}, \overline{v}\}$.

Consider changing type-t buyers' allocation probabilities, from $q^*(\cdot, t)$ to $\hat{q}(\cdot, t)$:

$$\begin{aligned}\hat{q}(v,t) &= q^*(v,t) \text{ for all } v \notin (\underline{v}, \overline{v}) \\ &= q^{*,NO}(v,t) \text{ for all } v \in (\underline{v}, \overline{v})\end{aligned}$$

Let $\hat{S}(v,t) = S^*(0,t) + \int_0^v \hat{q}(\tilde{v}, t)\mathrm{d}\tilde{v}$ denote type-t buyer surpluses when the mechanism is changed in this way. Note that $\hat{S}(v,t) = S^*(v,t)$ for all $v \notin (\underline{v}, \overline{v})$ while $\hat{S}(v,t) = \min_{t'} S(v,t')$ for all $v \in (\underline{v}, \overline{v})$. Thus, after this change, the seller can increase profit from type-t buyers by no longer inducing disclosure of buyers (v,t) for $v \in (\underline{v}, \overline{v})$. (Further, no *other* buyers must now be induced to disclose.)

To establish the contradiction and complete the proof, it suffices to show that this change also has a positive effect on expected revenue from type-t buyers, namely that

$$\int \hat{q}(v,t) MR(v,t) f(v|t)\mathrm{d}v - \int q^*(v,t) MR(v,t) f(v|t)\mathrm{d}v \tag{21}$$

$$= \int_{\underline{v}}^{\overline{v}} \left(\hat{q}(v,t) - q^*(v,t)\right) MR(v,t) f(v|t)\mathrm{d}v > 0 \tag{22}$$

Here, the equality follows from the definition of $\hat{q}(\cdot, t)$, while the inequality follows from three facts:[11] (i) $\int_{\underline{v}}^{\overline{v}} \left(\hat{q}(v,t) - q^*(v,t)\right)\mathrm{d}v = 0$, since $(\underline{v}, t), (\overline{v}, t) \in NO^*$ implies that $\int_{\underline{v}}^{\overline{v}} \hat{q}(v,t)\mathrm{d}v = \int_{\underline{v}}^{\overline{v}} q^*(v,t)\mathrm{d}v = \min_{t'} S^*(\overline{v}, t') - \min_{t'} S^*(\underline{v}, t')$; (ii) $\int_{\underline{v}}^{v} \left(\hat{q}(\tilde{v},t) - q^*(\tilde{v},t)\right)$ $\mathrm{d}\tilde{v} < 0$ for all $v \in (\underline{v}, \overline{v})$, since $(v,t) \in YES^*$ for all such values; and (iii) $MR(v,t)f(v|t)$ $= vf(v|t) + F(v|t) - 1$ is strictly increasing in v since, by assumption, revenue $R(p,t) = p(1 - F(p|t)$ is strictly concave in p for all t.[12] $\square$

Proof of Lemma 3(d): Define $v^l(t) = \inf\{v : (v,t) \in NO^*\}$ and $v^h(t) = \sup\{v : (v,t) \in NO^*\}$, with $v^l(t) = v^h(t) = \infty$ should $(v,t) \in YES^*$ for all v. By continuity and

[11] Let $h(\cdot), k(\cdot)$ be any functions such that $k(v)$ is strictly increasing in v, $\int_{\underline{v}}^{v} h(\tilde{v})\mathrm{d}\tilde{v} < 0$ for all $v \in (\underline{v}, \overline{v})$, and $\int_{\underline{v}}^{\overline{v}} h(\tilde{v})\mathrm{d}\tilde{v} = 0$. Then $\int_{\underline{v}}^{\overline{v}} h(v)k(v)\mathrm{d}v > 0$. (Details straightforward and omitted.) Here, let $h(v) = \hat{q}(v,t) - q^*(v,t)$ and $k(v) = MR(v,t)f(v|t)$.

[12] $\frac{\mathrm{d}[p(1-F(p|t)]}{\mathrm{d}p} = -MR(p,t)f(p|t)$. This concavity assumption is used nowhere else in the proof.

definition of $v^h(t)$, $S^*(v^h(t),t) = \min_{t'} S^*(v^h(t),t')$ and $S^*(v,t) > \min_{t'} S^*(v,t')$ for all $v > v^h(t)$. So, $\lim_{v \searrow v^h(t)} \frac{S^*(v,t)-S^*(v^h(t),t)}{v-v^h(t)} \geq \lim_{v \searrow v^h(t)} \frac{\min_{t'} S^*(v,t')-\min_{t'} S^*(v^h(t),t')}{v-v^h(t)}$, implying $\lim_{v \searrow v^h(t)} q^*(v,t) \geq q^{*,NO}(v^h(t))$ by (3). Since $q^*(\cdot,t)$ is non-decreasing, we conclude that $q^*(v,t) \geq q^{*,NO}(v^h(t))$ for all $v > v^h(t)$.

Consider any type-t allocation probabilities $\hat{q}(\cdot,t)$ such that $\hat{q}(v,t) = q^*(v,t)$ for all $v \leq v^h(t)$ and $\hat{q}(v,t)$ is non-decreasing in v for all $v \geq v^h(t)$. This monotonicity guarantees incentive-compatibility (IC) within type-t buyers, while IC across buyer types is automatically satisfied by the fact that buyer $(v,t) \in YES^*$ for all $v > v^h(t)$. Thus, the optimal allocation probabilities $q^*(\cdot,t)$ must maximize the seller's expected profit *from disclosing type-t buyers in isolation*,

$$\{q^*(v,t) : v > v^h(t)\} \in \arg \max_{\{q(v,t):v>v^h(t)\}} \int_{v^h(t)}^{\infty} (q(v,t)MR(v,t) - c(t))\, f(v|t)\mathrm{d}v \qquad (23)$$

subject *only* to the condition that $v' > v > v^h(t)$ implies $q^*(v',t) \geq q^*(v,t) \geq q^*(v^h(t),t)$. (Disclosure costs in (23) are paid regardless of whether the buyer receives the object.)

This problem is very similar to the standard monopoly pricing problem, where the seller is faced with a type-t buyer whose value is truncated to be at least $v^h(t)$. The only difference is that the seller is constrained to sell the good to all buyers with probability at least $q^*(v^h(t),t) \geq 0$. Nonetheless, it is easy to see that solution to this maximization is "bang-bang", as in the standard case. Define

$$v_0(t) \in \arg \max_{v \geq v^h(t)} \int_{v}^{\infty} MR(\tilde{v},t)\mathrm{d}\tilde{v}.$$

The solution has $q^*(v,t) = 1$ for all $v > v_0(t)$ and $q^*(v,t) = q^*(v^h(t),t)$ for all $v \in (v^h(t), v_0(t))$.

To complete the proof, I need to show that $v^h(t) \geq v^0(t)$. Suppose f.s.o.c. that $v^0(t) > v^h(t)$. As shown earlier, $q^*(v,t) = q^{*,NO}(v^h(t)) \leq q^{*,NO}(v)$ for all $v \in (v^h(t), v^0(t))$. Thus, $S^*(v,t) = S^*(v^h(t),t) + \int_{v^h(t)}^{v} q^*(\tilde{v},t)\mathrm{d}\tilde{v} = \min_{t'} S^*(v^h(t),t') + \int_{v^h(t)}^{v} q^*(\tilde{v},t)\mathrm{d}\tilde{v} \leq \min_{t'} S^*(v,t')$, contradicting $(v,t) \in YES^*$.

Proof of Lemma 3(e): $(0,t) \in NO^*$ by Lemma 4,[13] while $(v,t) \in YES^*$ implies $(v',t) \in$

[13] While presented later in the text, the proof of Lemma 4 does not use Lemma 3(e).

YES^* for all $v' > v$ by Lemma 3(c). Thus, $v^{*,d}(t) = \sup\{v : (v,t) \in NO^*\}$. $\square$

A.6 Proof of Lemma 4

Proof. $\min_t S^*(0,t) = 0$ since otherwise the seller can increase profit by equally reducing $S^*(0,t)$ for all t. Suppose f.s.o.c. that $\max_t S^*(0,t) > 0$ and let $\hat{t} \in \arg\max_t S^*(0,t)$. Since $S^*(0,\hat{t}) > \min_t S^*(0,t)$, buyer $(0,\hat{t}) \in YES^*$. By the proof of Lemma 3, $S^*(v,t) > \min_{t'} S^*(v,t')$ for all $v < v^l(t)$ and, if $v^l(t) < \infty$, $S^*(v^l(t),t) = \min_{t'} S^*(v^l(t),t')$, where $v^l(t) \equiv \inf\{v : (v,t) \in NO^*\}$ and $v^l(t) = \infty$ if $(v,t) \in YES^*$ for all v. To complete the proof, I need to show that $v^l(t) = 0$.

Suppose f.s.o.c. that $v^l(t) > 0$. I shall consider two cases separately.

First case: $q^*(v,t) = q^*(0,t)$ for all $v < v^l(t)$. Consider a change in which, for some $\varepsilon > 0$, the seller (i) raises type-t allocation probabilities from $q^*(v,t)$ to $\hat{q}^\varepsilon(v,t) = q^*(v,t) + \varepsilon$ for all $v < v^l(t)$, leaving other allocation probabilities unchanged, (ii) lowers zero-value surplus from $S^*(0,t)$ to $\hat{S}^\varepsilon(0,t) = S^*(0,t) - \varepsilon v^l(t)$, and (iii) does not change buyer disclosure. Let $\hat{S}^\varepsilon(v,t) = S^*(v,t) - \varepsilon(v^l(t) - v)$ denote the induced buyer surplus for all $v < v^l(t)$. (Buyer (v,t)'s surplus does not change for all $v \geq v^l(t)$.) For small enough $\varepsilon > 0$, such a change does not violate any of the constraints of the seller's relaxed problem.

IR constraint (3): $\hat{S}^\varepsilon(0,t) > 0$ as long as $\varepsilon < \frac{S^*(0,t)}{v^l(t)}$, where $\frac{S^*(0,t)}{v^l(t)} > 0$.

Monotonicity IC constraint (4): It suffices to show that $q^*(\cdot,t)$ is discontinuous at $v^l(t)$, since then $\hat{q}^\varepsilon(v^l(t)-,t) = q^*(v^l(t)-,t) + \varepsilon < q^*(v^l(t)+,t)$ for all small enough ε.[14] In fact, I will establish a stronger fact that $q^*(v^l(t)-,t) < q^{*,NO}(v^l(t)-)$. (This desired discontinuity of $q^*(\cdot,t)$ at $v^l(t)$ follows since $q^*(v^l(t)+,t) = q^{*,NO}(v^l(t)+)$ and $q^{*,NO}(\cdot)$ is non-decreasing by Lemma 3(b) and since $(v^l(t),t) \in NO^*$.)

Suppose f.s.o.c. that $q^*(v^l(t)-,t) \geq q^{*,NO}(v^l(t)-,t)$, so that $q^*(\hat{v},t) \geq q^{*,NO}(v^l(t)-,t)$

[14] $h(x-) = \lim_{\varepsilon\to 0} h(x-\varepsilon)$ and $h(x+) = \lim_{\varepsilon\to 0} h(x+\varepsilon)$ denote left- and right-limits, respectively. Notation to account for possible discontinuities is suppressed in most proofs, but here it is important to account carefully for discontinuities.

for any $\hat{v} < v^l(t)$. By definition of $v^l(t)$, $S^*(v^l(t), t) = \min_{t'} S^*(v^l(t), t')$. By Lemma 2 and Lemma 3(b), $\frac{\mathrm{d}S^*(\hat{v},t)}{\mathrm{d}v} = q^*(\hat{v}, t)$ while $\frac{\mathrm{d}[\min_{t'} S^*(\hat{v},t')]}{\mathrm{d}v} = q^{*,NO}(\hat{v})$, where $q^{*,NO}(\hat{v}) \leq q^{*,NO}(v^l(t)-)$ by Lemma 3(b). Thus, $\min_{t'} S^*(\hat{v}, t') \geq S^*(v^l(t), t) - \int_{\hat{v}}^{v^l(t)} q^{*,NO}(v^l(t)-)\mathrm{d}v \geq S^*(v^l(t), t) - \int_{\hat{v}}^{v^l(t)} q^*(v, t)\mathrm{d}v = S^*(\hat{v}, t)$. This contradicts the definition of $v^l(t)$, since $\hat{v} < v^l(t)$ implies $(\hat{v}, t) \in YES^*$ and hence $S^*(\hat{v}, t) > \min_{t'} S^*(\hat{v}, t')$ by Lemma 3(a).

Disclosure IC constraint (5): It suffices to show that, for small enough ε, $\hat{S}^\varepsilon(v, t) > \min_{t'} S^*(v, t')$ for all $v < v^l(t)$. (Otherwise, some type-t buyers would now have an incentive to mimic other types who currently do not disclose, forcing the seller to incur the cost of inducing these other buyers to disclose.)

For any $\varepsilon < q^{*,NO}(v^l(t)-, t) - q^*(v^l(t)-, t)$, $\frac{\mathrm{d}\hat{S}^\varepsilon(v^l(t),t)}{\mathrm{d}v} < \frac{\mathrm{d}\min_{t'} S^*(v^l(t),t)}{\mathrm{d}v}$, so that $\hat{S}^\varepsilon(v, t) > \min_{t'} S^*(v, t')$ for all $v \in \left(v^l(t) - \delta, v^l(t)\right)$ and all small enough δ. Let $\Delta(\delta) = \min_{v \in [0, v^l(t) - \partial]} \left(\hat{S}^\varepsilon(v, t) - \min_{t'} S^*(v, t')\right)$. Since buyer surplus changes by at most $\varepsilon v^l(t)$, $\hat{S}^\varepsilon(v, t) > \min_{t'} S^*(v, t')$ for all $v \in [0, v^l(t) - \partial]$ as long as $\varepsilon < \frac{\Delta(\delta)}{v^l(t)}$.

Seller profit from type-t buyers increases: Total expected surplus is increasing in allocation probabilities but decreasing in the set of buyers who must be induced to disclose. Since the change considered here increases allocation probabilities and leaves disclosure unchanged, total expected surplus is higher than before. On the other hand, type-t buyer surpluses have decreased: $\hat{S}(v, t) < S^*(v, t)$ for all $v < v^l(t)$ and surpluses are unchanged for other buyers. Thus, the seller's expected profit from type-t buyers has strictly increased, contradicting the presumed optimality of the original mechanism.

Second case: there exists $\hat{v} > 0$ such that $q^*(0+, t) < q^*(v, t) < q^*(v^l(t)-, t)$. The proof for this case is very similar, though it is somewhat more complicated to define the new probabilities $\hat{q}^\varepsilon(v, t)$ so as to respect the monotonicity constraint. Define $\hat{\Delta} = \max_{v \leq \hat{v}} \left(S^*(v, t) - \min_{t'} S^*(v, t')\right)$; $\hat{\Delta} > 0$ by continuity of buyer surplus, since $\hat{v} < v^l(t)$ implies $S^*(v, t) - \min_{t'} S^*(v, t') > 0$ for all $v \in [0, \hat{v}]$. Finally, define $\hat{\varepsilon} > 0$ implicitly by

$$\int_0^{\hat{v}} \left(q^{\hat{\varepsilon}}(v, t) - q^*(v, t)\right) \mathrm{d}v = \hat{\Delta}/2$$

where, for any $\varepsilon > 0$,

$$q^{\varepsilon}(v,t) = \min\left\{q^*(v,t) + \varepsilon, q^*(\hat{v},t)\right\} \text{ for all } v \in [0,\hat{v}].$$

Clearly, $q^{\hat{\varepsilon}}(v,t) \geq q^*(v,t)$ for all $v \leq \hat{v}$, while $q^{\hat{\varepsilon}}(v,t) > q^*(v,t)$ for all small enough v.

Consider a change to the mechanism in which the seller (i) raises type-t allocation probabilities from $q^*(v,t)$ to $q^{\hat{\varepsilon}}(v,t)$ for all $v \leq \hat{v}$, leaving all other allocation probabilities unchanged, (ii) lowers zero-value surplus from $S^*(0,t)$ to $\hat{S}(0,t) = S^*(0,t) - \hat{\Delta}/2$, and (iii) does not change buyer disclosure. Let $\hat{S}(v,t) = \hat{S}(0,t) + \int_0^v q^{\hat{\varepsilon}}(\tilde{v},t)\mathrm{d}\tilde{v}$ denote the resulting surplus for all type-t buyers.

Such a change does not violate any of the constraints of the seller's relaxed problem. *IR constraint (3):* $\hat{S}(0,t) \geq \Delta/2 > 0$. *Monotonicity IC constraint (4):* By construction, $q^{\hat{\varepsilon}}(v,t)$ is non-decreasing in v for $v \in [0,\hat{v}]$ and $q^{\hat{\varepsilon}}(\hat{v},t) = q^*(\hat{v},t)$. *Disclosure IC constraint (5):* $\hat{S}(v,t) = S^*(v,t)$ for all $v \geq \hat{v}$ since $\hat{S}(\hat{v},t) = S^*(\hat{v},t)$ and allocation probabilities are unchanged to values $v > \hat{v}$. Thus, it suffices to show that $\hat{S}(v,t) > \min_{t'} S^*(v,t')$ for all $v < \hat{v}$. By definition of $\hat{\Delta}$ and construction of $q^{\hat{\varepsilon}}(\cdot,t)$, $\hat{S}(v,t) \geq S^*(v,t) - \Delta/2 \geq \min_{t'} S^*(v,t') + \Delta/2$ for all $v \leq \hat{v}$.

As in the first case above, the seller's expected profit from type-t buyers increases because total expected surplus increases while buyers' surpluses weakly decrease. This is again a contradiction, completing the proof. □

A.7 Proof of Lemma 5

Let $T^{NO}(v) = \{t \in T : v^{*,d}(t) > v\}$ be those types that the buyer does *not* disclose given value v. ($v^{*,d}(t)$ is defined in Lemma 3.) Define $\widetilde{f}(v) = \int_{t \in T^{NO}(v)} f(v|t)g(t)\mathrm{d}t$ and $\widetilde{MR}(v) = \frac{\int_{t \in T^{NO}(v)} MR(v,t) f(v|t)g(t)\mathrm{d}t}{\int_{t \in T^{NO}(v)} f(v|t)g(t)\mathrm{d}t}$. The first term of the objective (8) now becomes $\int q^{*,NO}(v)\widetilde{MR}(v)\widetilde{f}(v)dv$, so that maximizing this term is equivalent to maximizing revenue in a (hypothetical) market segment consisting of only the non-disclosing buyers, having marginal revenue $\widetilde{MR}(\cdot)$ and density $\widetilde{f}(\cdot)$.[15] The solution to this problem is

[15] $\widetilde{f}(\cdot)$ can be viewed as the density of a distribution with an atom of mass $\Pr((v,t) \in YES^*)$ at zero.

well-known (see e.g. Section 6 of Bulow and Roberts (1989)) and amounts to a take-it-or-leave-it price. In particular, there exists a threshold $v^{*,NO}$ (corresponding to a price to non-disclosers) such that $q^{*,NO} = 0$ for all $v < v^{*,NO}$ and $q^{*,NO} = 1$ for all $v > v^{*,NO}$. $\square$

A.8 Proof of Lemma 6

Proof. By definition, a price-list mechanism $(D, p(NO), p(t) : t \in D)$ is a sales mechanism with non-random disclosure in which (i) for all $t \in D$, $q(v,t) = S(v,t) = 0$ for all $v < p(t)+c_B(t)$ while $q(v,t) = 1$ and $S(v,t) = v-p(t)-c_B(t)$ for all $v > p(t)+c_B(t)$, and (ii) for all $t \notin D$, $q(v,t) = S(v,t) = 0$ for all $v < p(NO)$ while $q(v,t) = 1$ and $S(v,t) = v-p(NO)$ for all $v > p(NO)$. Let $D^* = \{t : v^{*,d}(t) < \infty\}$. By Lemmas 3-5, there exist thresholds $\{v^{*,NO}, v^{*,d}(t) : t \in D^*\}$ such that (i) for all $t \in D^*$, $q^*(v,t) = S^*(v,t) = 0$ for all $v < \min\{v^{*,NO}, v^{*,d}(t)\}$ while $q(v,t) = 1$ and $S(v,t) = v - \min\{v^{*,NO}, v^{*,d}(t)\}$ for all $v > \min\{v^{*,NO}, v^{*,d}(t)\}$, (ii) for all $t \notin D^*$, $q(v,t) = S(v,t) = 0$ for all $v < v^{*,NO}$ while $q(v,t) = 1$ and $S(v,t) = v - v^{*,NO}$ for all $v > v^{*,NO}$. Thus, this is a price-list mechanism with disclosing types D^*, sticker price $p^*(NO) = v^{*,NO}$, and customized prices $p^*(t) = \min\{v^{*,NO}, v^{*,d}(t) - c_B(t)\}$ for all $t \in D^*$.

To complete the proof, it remains to show that $v^{*,NO} > v^{*,d}(t)$ for all $t \in D^*$, since then $p^*(t) = v^{*,d}(t) - c_B(t)$. Suppose f.s.o.c. that $v^{*,d}(t) \geq v^{*,NO}$ for some $t \in D^*$ and fix $\hat{v} > v^{*,d}(t)$ so that $(\hat{v}, t) \in YES^*$. Type-t buyers receive the good iff $v > v^{*,NO}$, so $S^*(\hat{v}, t) = \hat{v} - v^{*,NO}$. However, $(\hat{v}, t') \in NO^*$ and $S^*(\hat{v}, t') = \hat{v} - v^{*,NO}$ for every $t' \notin D$. Thus, $S^*(\hat{v}, t) = \min_{t'} S^*(\hat{v}, t')$ and hence $(\hat{v}, t) \in NO^*$ by Lemma 3(a), a contradiction. $\square$

A.9 Proof of Theorem 2

Proof. Part I: discount condition (10). Consider any price-list mechanism with sticker price $p(NO)$ and disclosing set $D \ni t$. Since disclosure costs $c_B(t), c_S(t)$ are incurred by the buyer and seller when a sale occurs to a buyer of type t, but not otherwise, the seller's problem when setting the customized price $p(t)$ is equivalent to that of a monopolist facing a buyer whose value is distributed as $(v - c_B(t))|t$ given marginal cost $c_S(t)$ and a price ceiling of $p(NO) - c_B(t)$. (If the price ceiling is violated, no type-t buyer will ever choose to disclose, contradicting the presumption that $t \in D$.) Thus, <u>*if*</u> $p(NO) - c_B(t) > p^*(t)$, then $p^*(t)$ is the optimal customized price for type-t buyers.

Since $p^*(t) \leq p(NO) - c_B(t)$ by disclosure incentive-compatibility, to complete the proof it suffices to show that $p^*(t) > p^*(NO) - c_B(t)$ for the *optimal* sticker price $p^*(NO)$. Suppose f.s.o.c. that $p^*(t) = p^*(NO) - c_B(t)$, i.e. that $\max_{p \leq p^*(NO) - c_B(t)}(p - c_S(t))(1 - F(p + c_B(t)|t)) = (p^*(NO) - c(t))(1 - F(p^*(NO)|t))$. Yet revoking type-$t$ buyers' eligibility for a discount and forcing them to pay sticker price would yield strictly greater expected profit from them, $p^*(NO)(1 - F(p^*(NO)|t))$, a contradiction.

Part II: sticker-price condition (9). Consider any price-list mechanism with disclosing set D, sticker price $p(NO)$, and discounts $p(t) \leq p(NO) - c_B(t)$ for all $t \in D$. Increasing the sticker price from $p(NO)$ to $p(NO) + \varepsilon$ is feasible and profitable for small enough $\varepsilon > 0$ *unless*

$$\int_{t \notin D} \frac{\mathrm{d}[p(NO)(1 - F(p(NO)|t))]}{\mathrm{d}p} g(t)\mathrm{d}t \leq 0. \tag{24}$$

Decreasing the sticker price from $p(NO)$ to $p(NO) - \varepsilon$ is feasible as well, as long as each discount $p(t)$ is also decreased to $\min\{p(t), p(NO) - \varepsilon - c_B(t)\}$ so as to maintain disclosure incentive-compatibility for all types $t \in D$. Such a pricing change is profitable for all small enough $\varepsilon > 0$ *unless*

$$\int_{t \notin D \text{ and } t \in D : p(t) = p(NO) - c_B(t)} \frac{\mathrm{d}[p(NO)(1 - F(p(NO)|t))]}{\mathrm{d}p} g(t)\mathrm{d}t \geq 0. \tag{25}$$

However, as shown in the proof of (10), $p^*(t) < p^*(NO) - c_B(t)$ for all $t \in D^*$. Thus, the sticker price $p^*(NO)$ in any optimal price-list mechanism must satisfy

$$\int_{t \notin D^*} \frac{\mathrm{d}[p^*(NO)(1 - F(p^*(NO)|t))]}{\mathrm{d}p} g(t)\mathrm{d}t = 0. \tag{26}$$

Note that (26) is the standard first-order condition of a monopoly seller who faces a buyer having value randomly distributed as $v|t \notin D^*$, and uniquely identifies $p^*(NO)$ as the solution to (9). (Uniqueness follows from the assumption that $v - \frac{1-F(v)}{f(v)}$ is strictly increasing in v.)

Disclosing-set condition (11). "$\Leftarrow$". Suppose that type-t buyers are induced to disclose in the optimal price-list mechanism, i.e. $t \in D^*$. As shown in the proof of (10), $p^*(t) < p^*(NO) - c_B(t)$, so that $\max_{p \leq p^*(NO)-c_B(t)} (p - c_S(t))(1 - F(p + c_B(t)|t))$ is realized at customized price $p^*(t)$. Suppose for the sake of contradiction (f.s.o.c.) that $p^*(NO)(1 - F(p^*(NO)|t)) > (p^*(t) - c_S(t))(1 - F(p^*(t) + c_B(t)|t))$. If so, the seller can increase expected profit by revoking type-t buyers' eligibility for a discount, contradicting the presumption of optimality.

"$\Rightarrow$". Suppose that $t \notin D^*$ and let $\hat{p}(t) = \arg\max_{p \leq p^*(NO)-c_B(t)} (p - c_S(t))(1 - F(p + c_B(t)|t))$. Suppose f.s.o.c. that $p^*(NO)(1 - F(p^*(NO)|t)) < (\hat{p}(t) - c_S(t))(1 - F(\hat{p}(t) + c_B(t)|t))$. If so, the seller can increase expected profit by inducing type-t buyers to disclose their type with a customized price $p(t) = \hat{p}(t)$, contradicting the presumption of optimality. Finally, define

$$Z = \{t \in T : p^*(NO)(1 - F(p^*(NO)|t)) = (\hat{p}(t) - c_S(t))(1 - F(\hat{p}(t) + c_B(t)|t))\}. \quad (27)$$

To complete the proof, it suffices to show $Z \cap \overline{D}^* = \emptyset$ since then the seller *strictly* prefers not to induce any types $t \notin D^*$ to disclose in the optimal price-list mechanism.

As shorthand, let $p^*(NO; \overline{D})$ denote the optimal sticker price given non-disclosing set $\overline{D} \subset T$; in particular, the optimal sticker price $p^*(NO) = p^*(NO; \overline{D}^*)$ and $p^*(NO; t) = \arg\max_p p(1 - F(p|t))$ denotes the optimal monopoly price without disclosure when faced with a buyer known to be type t.

Suppose f.s.o.c. that $Z \cap \overline{D}^* \neq \emptyset$. If so, the seller can increase expected profit by (i) inducing every type $t \in Z \cap \overline{D}^*$ to disclose with a customized price of $\hat{p}(t)$ and (ii) changing the sticker price from $p^*(NO)$ to $p^*(NO; \overline{D}^* \setminus Z)$. By (27), inducing types $t \in Z \cap \overline{D}^*$ to disclose has no effect on the seller's expected profit from such buyers. Further, "re-optimizing" the sticker price for all other types $\overline{D}^* \setminus Z$ if anything allows the seller to increase expected profits from those non-disclosing types that remain. Thus, such a two-fold modification to the original price-list mechanism must weakly increase seller expected profit *if* it does not violate any disclosure IC constraints, i.e. as long as $p^*(NO; \overline{D}^* \setminus Z) \geq \hat{p}(t) + c_B(t)$ for all $t \in D^* \cup Z$. To complete the proof, it therefore suffices to show that $p^*(NO; \overline{D}^* \setminus Z) \geq p^*(NO)$ since then all disclosure IC constraints become more slack.

$t \in Z$ implies $p^*(NO; t) < p^*(NO)$. Suppose f.s.o.c. that $p^*(NO; t) \geq p^*(NO)$. If so,

$$\begin{aligned}(\hat{p}(t) - c_S(t))(1 - F(\hat{p}(t) + c_B(t)|t)) &< \hat{p}(t)(1 - F(\hat{p}(t)|t)) \\ &\leq p^*(NO)(1 - F(p^*(NO)|t)),\end{aligned}$$

violating the definition of Z. The first inequality follows directly from $c_B(t), c_S(t) \geq 0$ and $c_B(t) + c_S(t) > 0$, while the second follows indirectly from the presumption that $p^*(NO; t) \geq p^*(NO)$. Namely, (i) $\frac{\mathrm{d}[p(1-F(p|t))]}{\mathrm{d}p} > 0$ for all $p < p^*(NO; t)$ since $v - \frac{1-F(v)}{f(v)}$ is strictly increasing in v, (ii) $\hat{p}(t) \leq p^*(NO)$ by the definition of $\hat{p}(t)$, and (iii) $p^*(NO) \leq p^*(NO; t)$ by presumption. By this contradiction, we conclude $p^*(NO; t) < p^*(NO)$ and in particular that $\frac{\mathrm{d}[p^*(NO)(1-F(p^*(NO)|t))]}{\mathrm{d}p} < 0$ for all $t \in Z$. Consequently,

$$\int_{t \notin D^* \setminus Z} \frac{\mathrm{d}\left[p^*(NO)\left(1 - F(p^*(NO)|t)\right)\right]}{\mathrm{d}p} g(t)\mathrm{d}t > 0.$$

so that $p^*(NO; \overline{D}^* \setminus Z)$, the optimal sticker price against buyer-types $\overline{D}^* \setminus Z$, is strictly greater than $p^*(NO)$. This completes the proof. □

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