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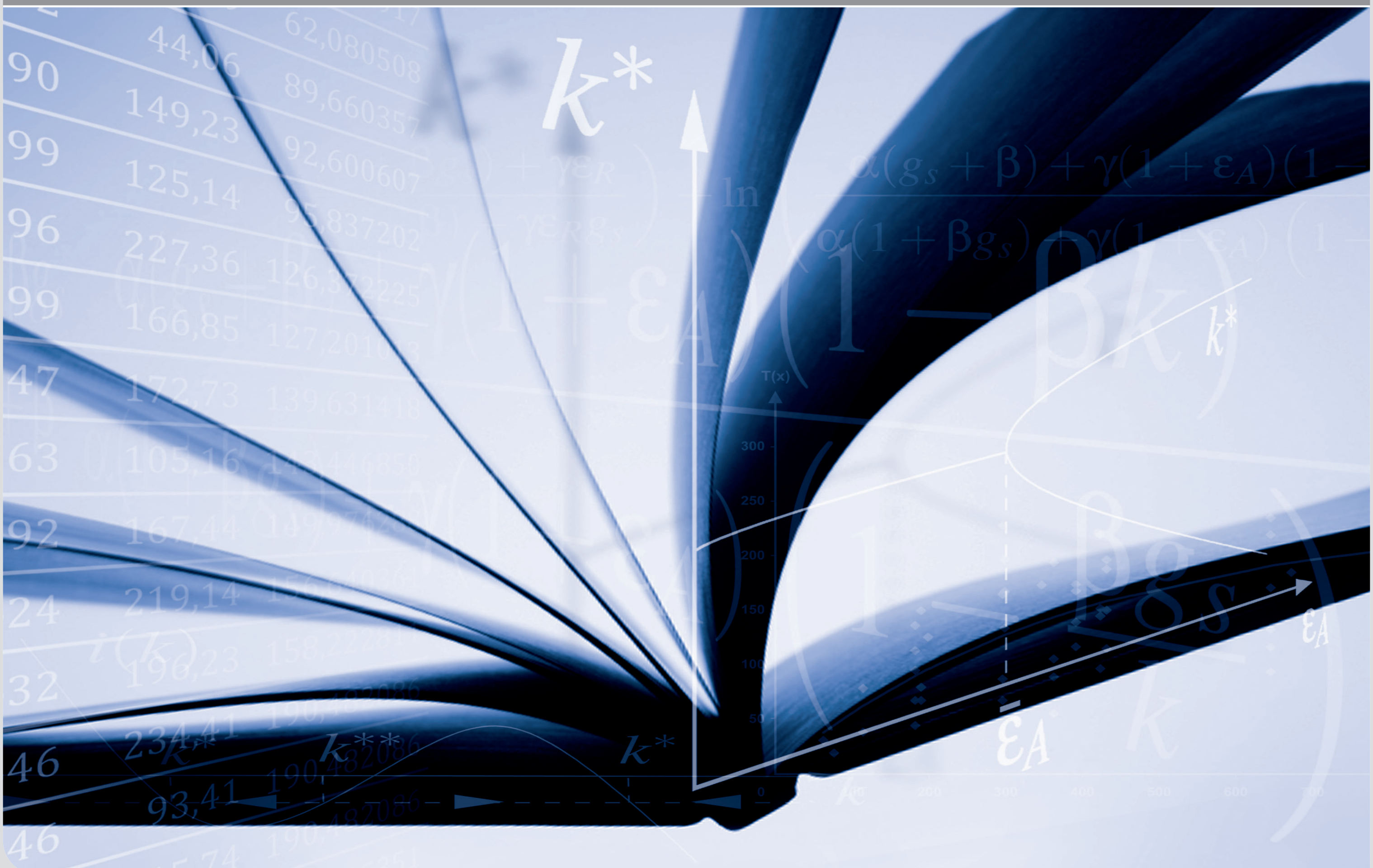

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Estimation of Operational Value-at-Risk in the Presence of Minimum Collection Threshold: An Empirical Study

Anna Chernobai[*], Christian Menn[†], Svetlozar T. Rachev[‡], and Stefan Trück[§]

October 18, 2009

Abstract

The recently finalized Basel II Capital Accord requires banks to adopt a procedure to estimate the operational risk capital charge. Under the Advanced Measurement Approaches, that are currently mandated for all large internationally active US banks, require the use of historic operational loss data. Operational loss databases are typically subject to a minimum recording threshold of roughly \$10,000. We demonstrate that ignoring such thresholds leads to biases in corresponding parameter estimates when the threshold is ignored. Using publicly available operational loss data, we analyze the effects of model misspecification on resulting expected loss, Value-at-Risk, and Conditional Value-at-Risk figures and show that underestimation of the regulatory capital is a consequence of such model error. The choice of an adequate loss distribution is conducted via in-sample goodness-of-fit procedures and backtesting, using both classical and robust methodologies.

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1 Introduction

The three major sources of risks in financial institutions are market risk, credit risk, and operational risk. While the first two have been well understood, the research on operational risk is still a growing area. The scope of risks grouped under operational risks is quite large: the Basel Capital Accord defines operational risk as the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events (BCBS, 2006*a*). Examples of large operational losses due to unauthorized trading, fraud, and human errors include the Orange County (USA, 1994), Barings (Singapore, 1995), Daiwa (Japan, 1995), Société Générale (France, 2008), losses due to natural disasters include those due to hurricanes Andrew and Katrina (USA, 1992 and 2005), terrorist attack of September 11, 2001, to name a few. Arguably, while the recent financial crisis is commonly credited to credit risk, many of its roots can be traced to operational risk. Specifically, the failings of prominent mortgage and financial service companies could be averted had lending practices been founded upon forward-looking market expectations based on fundamentals rather than short-term market movements. As a result, years of improper lending practices led to the mortgage crisis around 2007 and subsequent bailout of a series of U.S. financial institutions by the U.S. Government and have brought to light the grave consequences of inadequate business practices and model errors – yet another type of operational risk.

The significance of operational risk as a major contributor to banks' and insurance companies' risk positions is increasingly recognized by the industry and regulators. Current estimates suggest that the allocation of total financial risk of a bank is roughly 60% to credit, 15% to market and liquidity and 25% to operational risk (Jorion, 2000).[1] Under the Basel II Capital Accord (BCBS, 2001*a*, 2006*a*), each bank is required to adopt a methodology to determine the operational risk capital charge to account for unexpected losses. The U.S. banks are mandated to use the Advanced Measurement Approaches. Under the Loss Distribution Approach (LDA) – one of such approaches – banks compute separately the loss severity and frequency distribution functions for each business line and risk type combination, over a one year period. The total capital charge is then determined as the sum[2] of one year Value-at-Risk (VaR) measures with the confidence level $1-\alpha$ (e.g., α=0.1%), across all combinations, based on the compounded losses.

A number of modeling issues remain in modeling operational risk. One problem is related to internal operational loss data. Data recording is a subject to lower recording thresholds, which for internal databases at set at roughly $10 thousand BCBS (2003).[3] We refer to such data that are not recorded as non-randomly missing data, and to the recorded data as left-truncated and incomplete. If such data truncation is unaccounted for, model errors can carry significant material consequences. *"The choice of loss collection thresholds can significantly affect the calculation of expected loss and,*

[1]Cruz (2002) suggests 50%, 15% and 35%, for credit, market and operational risks, respectively. In the 2008 annual report of JP Morgan Chase, credit risk capital accounted for 63%, market risk capital for 17.5%, and operational risk for 11% of total regulatory capital.

[2]Summing across business line and event types assumes perfect correlation between different cells. In more recent Basel II guidelines (BCBS, 2006*a*), it is recommended that banks use appropriate dependence structures that exist between the cells to produce the aggregate risk capital.

[3]The 2002 Quantitative Impact Study (QIS) revealed that only 5 out of 89 banks that participated in the study had minimum cut-off thresholds below €10 thousand, 59 banks (or 66% of banks) used a threshold of around €10 thousand, and 13 firms (or 15%) had thresholds exceeding that amount (BCBS, 2003). There could be various reasons for truncation. First, data recording is costly. Second, data entry errors that can occur while recording a large number of small losses result in additional operational losses for a firm. Third, smaller losses are easier to hide while larger losses must be reported, which results in smaller losses being under-represented from a complete database if all losses were recorded. And fourth, small frequent losses are perceived as routing and immaterial, so banks often opt to leave them unrecorded.

to some extent, the shape of the estimated loss distribution and estimates of unexpected loss." (BCBS, 2006*b*). Recent guidelines by the Basel Committee make a clear recommendation that data truncation be adequately accounted for.

Chernobai, Menn, Trück and Rachev (2006) showed that, if the truncation is ignored, fitting unconditional distribution to the observed (incomplete) losses would lead to biased estimates of the parameters of both severity and frequency distributions. The magnitude of the effect is dependent on the threshold level and the underlying loss distribution. Under the compound Poisson process model, the severity and frequency distributions of the operational risk are inter-related: while severity of recorded data is biased toward higher losses, for as long as the fraction of missing data (under the truncation point) is estimated to be non-zero, the frequency parameter(s) requires a proportional increase. As a result, the resulting VaR measure would be under-estimated. In this paper, we extend the theoretical framework of Chernobai, Menn, Trück and Rachev (2006) to apply it to real operational loss data and test empirically the implications of the model error associated with misspecified severity and frequency distributions on the estimates of operational risk regulatory capital.[4]

The aim of this paper is two-fold. We analyze the effects of missing data on loss severity and frequency distributions, and then examine the impact of model misspecification on the operational risk capital charge, determined by two alternatives: the VaR and Conditional VaR (CVaR) measures. The paper is organized as follows. Section 2 explains the truncation problem and discusses the methodology for the correct estimation of the severity distribution and the necessary adjustment to the frequency of loss events. Section 3 presents the results of empirical study using 1980-2002 publicly available operational loss data and examines the effects of misspecified and correctly defined distributions on the capital charge, and then carries out goodness-of-fit tests to determine an optimal law for the loss severity. We show that ignoring the missing data leads to misleading (under-estimated) VaR estimates. Section 4 provides a robustness check in which we estimate the operational risk regulatory capital using the principle of robust statistics. Section 5 concludes and states final remarks.

2 Compound Model for Operational Risk

2.1 Compound Poisson Process Model

Following the recommendation by the Basel Committee, we assume that the aggregated operational losses follow a stochastic process $\{S_t\}_{t\geq 0}$ over the time interval $\triangle t$ expressed by the following equation:

$$S_t = \sum_{k=0}^{N_t} X_k, \quad X_k \overset{\text{iid}}{\sim} F_\gamma, \tag{2.1}$$

in which the loss magnitudes are described by the random sequence $\{X_k\}$ assumed to follow the distribution function (cdf) F_γ that belong to a parametric family of continuous probability distributions, and the density f_γ, and the counting process N_t is assumed to take a form of a homogeneous Poisson process (HPP) with intensity $\lambda > 0$ or a non-homogeneous Poisson process (NHPP) with intensity $\lambda(t) > 0$[5]. Depending on the distribution, γ is a parameter vector or a scalar. For sim-

[4] A similar model was applied by Chernobai, Burneçki, Rachev, Trück and Weron (2006) to the natural catastrophe insurance model, where it is shown that the model misspecification of the claims leads to serious under-estimation of the ruin probabilities.

[5] The case of a sinusoidal rate function is considered in Chernobai, Burneçki, Rachev, Trück and Weron (2006).

plicity, we would refer to it as a parameter throughout the paper. We assume that the distribution family is sufficiently well-behaved so that γ can be estimated consistently by Maximum Likelihood (MLE). To avoid the possibility of negative losses we restrict the support of the distribution to the positive half-line $\mathbb{R}_{>0}$. Representation (2.1) assumes independence between frequency and severity distributions. The cdf of the compound Poisson process is given by:

$$P(S_t \le s) = \begin{cases} \sum_{n=1}^{\infty} P(N_t = n)\ F_\gamma^{n*}(s) & s > 0 \\ P(N_t = 0) & s = 0 \end{cases} \tag{2.2}$$

where F_γ^{n*} denotes the n-fold convolution with itself.

In practice, model (2.1) can be used to determine the required capital charge imposed by regulators. It is measured as the $(1-\alpha)^{\text{th}}$ quantile of the cumulative loss distribution (2.2) over a one year period, that defines VaR. $\text{VaR}_{\triangle t, 1-\alpha}$ for the tolerated risk level α and the time interval of length $\triangle t$ (generally $\alpha = 1\% - 5\%$ and $\triangle t$ is one year) is defined as the solution of the following equation:

$$P(S_{t+\triangle t} - S_t > \text{VaR}_{\triangle t, 1-\alpha}) = \alpha \tag{2.3}$$

and the CVaR[6] is defined by:

$$\begin{aligned} \text{CVaR}_{\triangle t, 1-\alpha} : &= \mathbb{E}\left[S_{t+\triangle t} - S_t \mid S_{t+\triangle t} - S_t > \text{VaR}_{\triangle t, 1-\alpha}\right] \\ &= \frac{\mathbb{E}\left[S_{t+\triangle t} - S_t \ ;\ S_{t+\triangle t} - S_t > \text{VaR}_{\triangle t, 1-\alpha}\right]}{\alpha}. \end{aligned} \tag{2.4}$$

Given a sample $\mathbf{x} = (x_1, x_2 \ldots, x_n)$ containing n losses which have occurred during some time interval $\triangle t = T_2 - T_1$, under the imposed assumptions on the structure of F_γ, the task of estimating λ and γ can be performed with the MLE principle (or in case of a NHPP, $\lambda(t)$ is estimated by directly fitting a deterministic function):

$$\hat{\lambda}_{\text{MLE}}(x) = \frac{n}{\triangle t} \quad \text{and} \quad \hat{\gamma}_{\text{MLE}}(x) = \arg\max_{\gamma} \sum_{k=1}^{n} \log f_\gamma(x_k) \tag{2.5}$$

The task of operational loss data analysis is complicated by the presence of missing data that fall to the left of the left truncation point (minimum collection threshold). The estimates in (2.1) would be misleading in the presence of truncation. The question addressed in subsequent analysis is whether ignoring the missing data has a significant impact on the estimation of the frequency parameter ($\lambda(t)$) and the severity parameter γ. From the statistical viewpoint, ignoring of non-randomly missing data would lead to a bias in all estimates. However in practical applications a possible reason to why such thresholds are ignored would be an argument saying that since the major bulk of losses is in excess of the threshold then the small losses can not have a significant impact on the operational VaR that is determined by the upper quantiles of the loss distribution. This paper presents empirical evidence to disprove the argument. In the following section we review the methodology for consistent estimation of loss and frequency distributions, as suggested in Chernobai, Menn, Trück and Rachev (2006).

2.2 Estimation of Complete-Data Severity and Frequency Distributions

In the presence of missing data, we conclude that the observed operational losses follow a *truncated* compound Poisson process. We follow similar notations to those in Chernobai, Menn, Trück and

[6] CVaR is also called Expected Tail Loss (ETL) or Expected Shortfall (ES).

Rachev (2006). The available data set collected in the time interval $[T_1, T_2]$ is incomplete due to the non-negative pre-specified thresholds u that defines a partition on $\mathbb{R}_{>0}$ through the events $A_1 = (0, u)$ and $A_2 = [u, \infty)$. Realizations of the loss distribution belonging to A_1 will not enter the data sample – neither the frequency nor the severity of losses below u are recorded (missing data). Realizations in A_2 are fully reported, i.e. both the frequency and the loss amount are specified (observed data). The observed sample is of the form $\mathbf{z} = (n, \mathbf{x})$, where n is the number of observations in A_2 and $\mathbf{x}$ are the values of these concrete observations. Given that the total number of observations in the complete sample is unknown, one possible *joint* density of $\mathbf{z}$ (with respect to the product of counting and Lebesgue measures) consistent with the model specification in Equation (2.1), is given by the following expression:

$$g_{\lambda,\gamma}(\mathbf{z}) = \frac{(\triangle t\ \tilde{\lambda})^n}{n!} e^{-\triangle t\ \tilde{\lambda}} \cdot \prod_{k=1}^{n} \frac{f_\gamma(x_k)}{q_{\gamma,2}}, \tag{2.6}$$

where $\triangle t\lambda$ is replaced with $\Lambda(t)$ for a NHPP, and $q_{\gamma,j}$ denotes the probability for a random realization to fall into set A_j, $j = 1, 2$, observed intensity $\tilde{\lambda} := q_{\gamma,2} \cdot \lambda$ and $\triangle t := T_2 - T_1$ is the length of the sample window. In the representation (2.6), the Poisson process $\tilde{N}_t$ of intensity $\tilde{\lambda}$ (or $\tilde{\lambda}(t)$) that counts only the observed losses exceeding in magnitude u can be thus interpreted as a *thinning* of the original process N_t of intensity λ ($\lambda(t)$) that counts all events in the complete data sample. The maximization of the corresponding log-likelihood function with respect to λ (for the HPP case) and γ can be divided into two separate maximization problems, each depending on only one parameter:

$$\hat{\gamma}_{\text{MLE}} = \arg\max_{\gamma} \log g_\gamma(\mathbf{z}) = \arg\max_{\gamma} \log \left(\prod_{k=1}^{n} \frac{f_\gamma(x_k)}{q_{\gamma,2}} \right), \tag{2.7}$$

$$\hat{\lambda}_{\text{MLE}} = \arg\max_{\lambda} \log g_{\lambda,\hat{\gamma}_{\text{MLE}}}(\mathbf{z}) = \frac{n}{\triangle t \cdot q_{\hat{\gamma}_{\text{MLE}},2}}. \tag{2.8}$$

The MLE estimation of the unknown parameter γ can be done in two ways: performing direct numerical integration or using the two-step Expectation-Maximization algorithm, developed by Dempster et al. (1977). Expectation-Maximization algorithm has been used in a variety of applications such as probability density mixture models, hidden Markov models, cluster analysis, factor analysis, survival analysis. References include McLachlan and Krishnan (1997), Meng and van Dyk (1997), Wulfsohn and Tsiatis (1997), DeCanio and Watkins (1998), among many others, and in the framework of the operational risk modelling in Chernobai, Menn, Trück and Rachev (2006), Bee (2005).

2.3 Implications of Data Misspecification on the Operational Risk Capital Charge

The Basel Capital Accord requires banks to provide operational risk capital charge that would cover the unexpected losses. At the same time they suggest using VaR for computing the capital charge. Some confusion arises from such definition of the capital charge, because providing the capital charge for the unexpected losses would mean that the expected aggregated loss (EL) has to be subtracted from VaR. We therefore analyze the impact of data misspecification on all relevant components - aggregated expected loss, VaR, and also CVaR.[7]

[7] It is also notable that EL does not exist for some very heavy-tailed distributions that possess an infinite mean.

For a compound Poisson process, the aggregated expected loss is computed as a product of the expected frequency and loss distributions:

$$\mathbb{E}S_{\triangle t} = \mathbb{E}N_{\triangle t} \cdot \mathbb{E}X. \tag{2.9}$$

$\text{VaR}_{\triangle t, 1-\alpha}$ was previously defined in Equation (2.3). We fix a tolerated risk level α and a time horizon of length $\triangle t$ - the Basel Committee suggests to use $\triangle t = 1$ year (BCBS (2001*b*)), and α e.g. 0.1%. By definition, VaR equals the capital charge which must be maintained in order to protect against potential operational losses in $\triangle t$ from now that can occur with probability $(1-\alpha)$. Generally no closed-form expression for the cumulative loss distribution is available. The upper quantiles have to be determined numerically through approximations such as the recursive Panjer-Euler scheme, FFT inversion of the characteristic function or simulation (we use the Monte Carlo method in this paper). For the special case of a sub-exponential loss distributions $F \in \mathcal{S}$, such as Lognormal, Pareto and the heavy-tailed Weibull – relevant in the context of operational risk modelling – the tail of the compound process is approximated by Embrechts, Klüppelberg and Mikosch (1997):

$$P(S_{\triangle t} > s) \sim \mathbb{E}N_{\triangle t} \cdot P(X > s), \quad s \to \infty. \tag{2.10}$$

For an example when the losses X follow a Lognormal(μ, σ) distribution, combining Equations (2.8) and (2.7) with (2.3) and (2.10) results in the following expected aggregated loss and approximate VaR estimates:

$$\mathbb{E}S_{\triangle t} = \hat{\lambda}_{\text{MLE}} \triangle t \cdot \exp\left\{\hat{\mu}_{\text{MLE}} + \frac{\hat{\sigma}^2_{\text{MLE}}}{2}\right\}, \tag{2.11}$$

$$\widehat{\text{VaR}}_{\triangle t, 1-\alpha} \sim \exp\left\{\hat{\mu}_{\text{MLE}} + \hat{\sigma}_{\text{MLE}}\ \Phi^{-1}\left(1 - \frac{\alpha}{\hat{\lambda}_{\text{MLE}} \triangle t}\right)\right\} \tag{2.12}$$

for the HPP case, with $\hat{\lambda}_{\text{MLE}} \triangle t$ is replaced by $\hat{\Lambda}(\triangle t)$ for the NHPP case, where $\varphi(\cdot)$ and $\Phi(\cdot)$ denote the density and the distribution function, and $\Phi^{-1}(\cdot)$ denotes the quantile of a standard Normal distribution. A closed-form expression for CVaR exists only for Gaussian loss severity. If the missing data is ignored in the estimations of loss severity and frequency, then for the Lognormal example the bias of the parameters can be expressed analytically as:

$$\begin{aligned} \mathbb{E}\ \hat{\lambda}_{\text{observed}} &= \lambda \cdot \left(1 - \Phi\left(\frac{\log u - \mu}{\sigma}\right)\right), \\ \text{bias}(\hat{\lambda}_{\text{observed}}) &= -\lambda \cdot \Phi\left(\frac{\log u - \mu}{\sigma}\right) \\ &< 0, \end{aligned} \tag{2.13}$$

$$\begin{aligned} \mathbb{E}\ \hat{\mu}_{\text{observed}} &= E\left(\frac{1}{n}\sum \log X_k \mid X_k > u\right) = \mu + \sigma \cdot \frac{\varphi\left(\frac{\log u - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{\log u - \mu}{\sigma}\right)}, \\ \text{bias}(\hat{\mu}_{\text{observed}}) &= \sigma \cdot \frac{\varphi\left(\frac{\log u - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{\log u - \mu}{\sigma}\right)} \\ &> 0, \end{aligned} \tag{2.14}$$

	$\mu_0 = 4$	$\mu_0 = 5$	$\mu_0 = 6.5$
$\sigma_0 = 1.5$	0.48	0.23	0.04
$\sigma_0 = 2$	0.48	0.29	0.10
$\sigma_0 = 2.7$	0.49	0.34	0.17

Table 1: Fraction of missing data, $F_{\gamma_0}(u)$, for the Lognormal(μ_0, σ_0) example with nominal threshold of $u = 50$.

$$
\begin{aligned}
\mathbb{E}\,\hat{\sigma}^2_{\text{observed}} &= E\left(\frac{1}{n}\sum \log^2 X_k - \hat{\mu}^2_{\text{observed}} \,|\, X_k > u\right) \\
&= \sigma^2\left(1 + \frac{\log u - \mu}{\sigma}\cdot\frac{\varphi\left(\frac{\log u - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{\log u - \mu}{\sigma}\right)} - \left(\frac{\varphi\left(\frac{\log u - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{\log u - \mu}{\sigma}\right)}\right)^2\right), \\
\text{bias}(\hat{\sigma}^2_{\text{observed}}) &= \sigma^2\left(\frac{\log u - \mu}{\sigma}\cdot\frac{\varphi\left(\frac{\log u - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{\log u - \mu}{\sigma}\right)} - \left(\frac{\varphi\left(\frac{\log u - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{\log u - \mu}{\sigma}\right)}\right)^2\right) \\
&< 0 \quad \text{since } \log u \text{ small,} \qquad (2.15)
\end{aligned}
$$

where in the first Equation (2.13) λ is replaced by $\lambda(t)$ for the NHPP case. Figure 1 gives an illustration to the biases of the three parameters for a wide range of initial (complete-data) true values of μ and σ. The distances between the ratios of the estimated parameters to the true parameters, represent the relative biases for each case. For the example, a threshold level of $H = 50$ in nominal value was considered, which corresponds to the following cutoff levels: For the same example, Figure 2 demonstrates the biases (represented by the ratios) of the estimated fractions of missing data in the 'naive' and conditional scenarios. The fraction being equal to one indicates the absence of bias. Combining (2.12) and replacing the estimates for μ and σ with their expectations from (2.14) and (2.15), we obtain an approximate estimate of expected aggregated loss and VaR under the data misspecification:

$$
\begin{aligned}
\mathbb{E}S_{\triangle t} &= \left(\lambda + \text{bias}(\hat{\lambda}_{obs})\right)\cdot\exp\left\{\mu + \text{bias}(\hat{\mu}_{obs}) + \frac{\left(\sigma + \text{bias}(\hat{\sigma}_{obs})\right)^2}{2}\right\}, \qquad (2.16) \\
&< \text{true } \mathbb{E}S_{\triangle t},
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\text{VaR}}_{\triangle t, 1-\alpha} &\approx \exp\left\{\mu + \text{bias}(\hat{\mu}_{obs}) + \left(\sigma + \text{bias}(\hat{\sigma}_{obs})\right)\cdot\Phi^{-1}\left(1 - \frac{\alpha}{\left(\lambda + \text{bias}(\hat{\lambda}_{obs})\right)\triangle t}\right)\right\} \qquad (2.17) \\
&< \text{true VaR}_{\triangle t, 1-\alpha},
\end{aligned}
$$

with appropriate adjustments for a NHPP case. The direction of the last inequality (it also holds for CVaR) generally depends on the threshold u and the underlying distribution. For practical purposes in the context of operational risk, the "<" inequality is valid in general cases Chernobai, Menn, Trück and Rachev (2006). Figure 3 illustrates the biases (represented by the ratios) of the expected aggregated loss, VaR and CVaR figures, under the 'naive' and conditional scenarios, for $\lambda = 100$ example. We note that the value of λ has no effect on the ratio of the expected loss, that follows directly from Equation (2.16), and increase of λ to 150, 200 or more has a very negligible impact (bias increases) on the biases of VaR and CVaR (figures are omitted here).

Figure 1: Ratios of estimated parameters to the true (complete-data) parameter values, for the Lognormal example, $u = 50$. (a) μ, 'naive'; (b) μ, conditional; (c) σ, 'naive'; (d) σ, conditional; (e) λ.

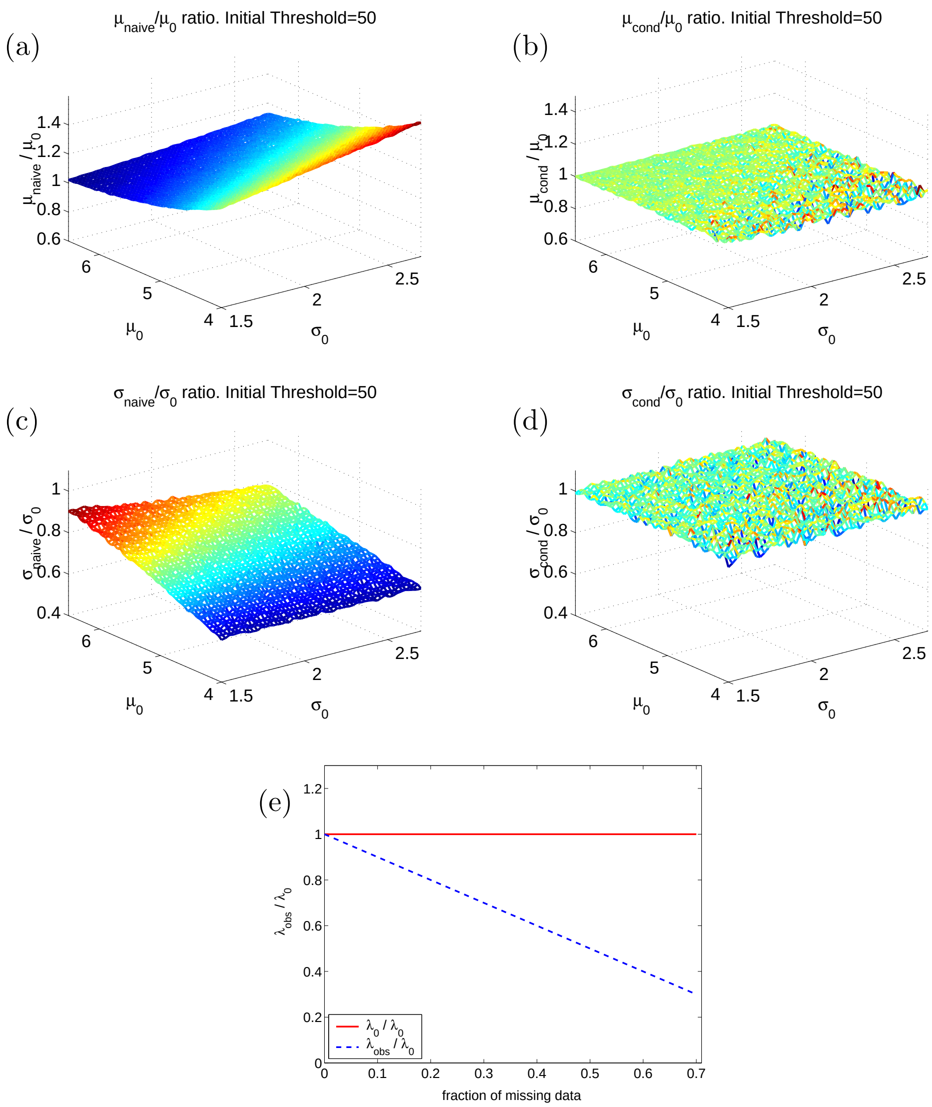
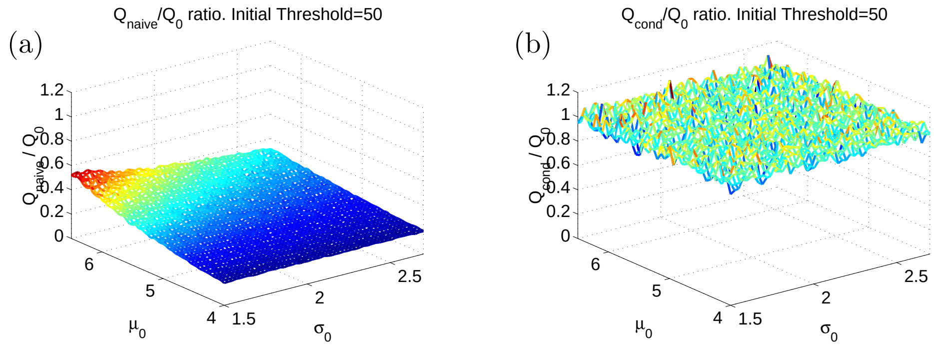


Figure 2: Ratios of estimated fraction of missing data (Q) to the true (complete-data) fraction, for the Lognormal example, $u = 50$. (a) $F(u)$, 'naive'; (b) $F(u)$, conditional.



3 Application to Operational Risk Data

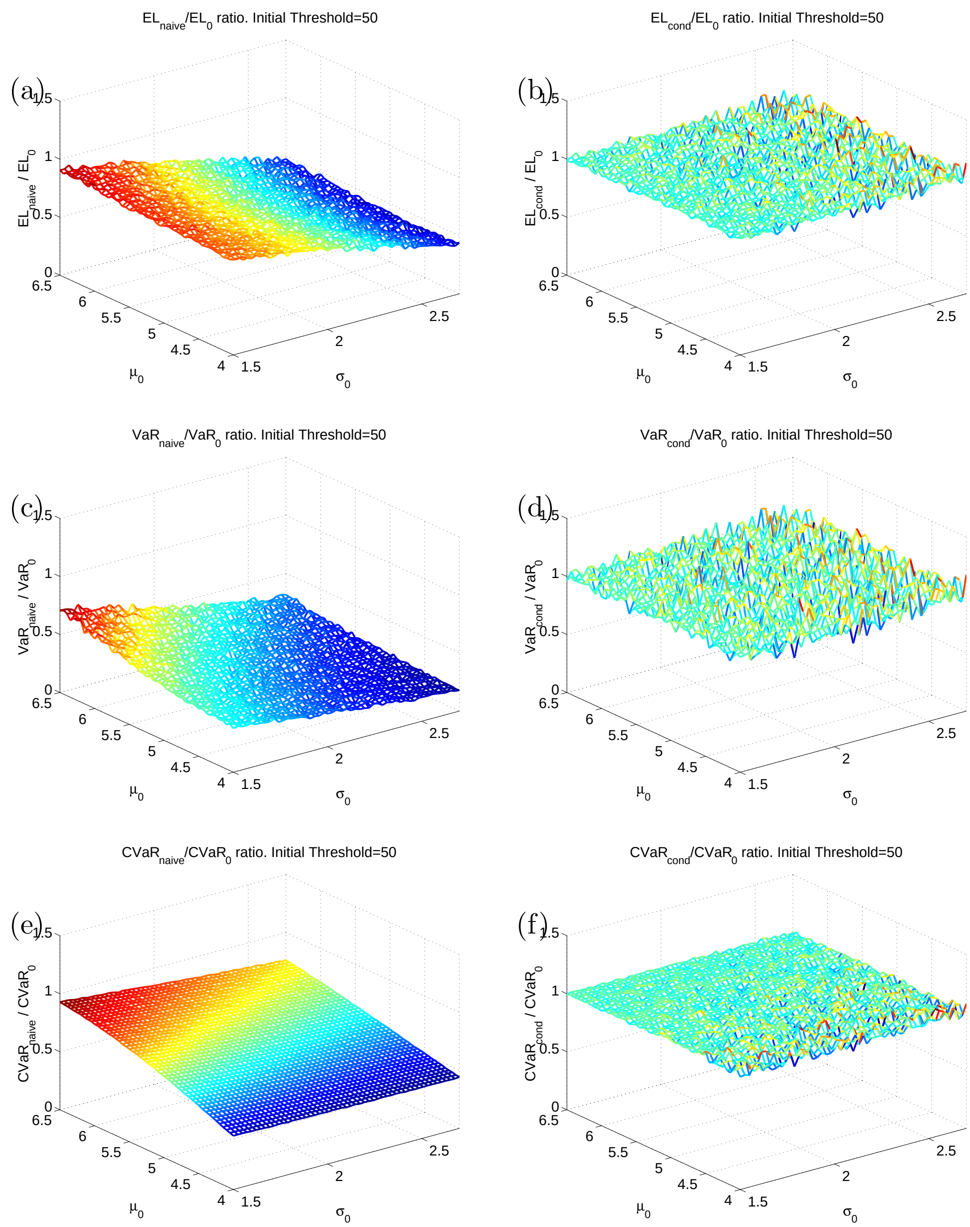
3.1 Purpose of Study and Data Description

In this section we apply the model to real operational risk data, obtained from Zurich IC Squared (IC^2) FIRST Database of Zurich IC Squared (IC^2), an independent consulting subsidiary of Zurich Financial Services Group. The external database is comprised of operational loss events throughout the world. The original loss data cover losses in the period 1950-2002. A few recorded data points were below \$1 million in nominal value, so we excluded them from the analysis, to make it more consistent with the conventional threshold for external databases of \$1 million. Furthermore, we excluded the observations before 1980 because of relatively few data points available (which is most likely due to poor data recording practices). The final dataset for the analysis covered losses in US dollars for the time period between 1980 and 2002. It consists of five types of losses: "Relationship" (such as events related to legal issues, negligence and sales-related fraud), "Human" (such as events related to employee errors, physical injury and internal fraud), "Processes" (such as events related to business errors, supervision, security and transactions), "Technology" (such as events related to technology and computer failure and telecommunications) and "External" (such as events related to natural and man-made disasters and external fraud). The loss amounts have been adjusted for inflation using the Consumer Price Index from the U.S. Department of Labor. The numbers of data points of each type are $n =$849, 813, 325, 67, and 233, respectively.

We would like to point out, that since the data set is external, the estimates of the parameters and VaR and CVaR values are not applicable to any particular bank. The purpose of the empirical study is to apply the model proposed in Section 2 and demonstrate the results, and we recommend to the risk managers to apply the technique to their internal databases.

In the empirical study we focus on two scenarios. The first scenario we refer to as a 'naive' approach in which no adjustments to the missing data are made to the data. The second scenario is the refined approach in which the losses are modelled with truncated (conditional) distributions, given that the losses are larger than or equal \$1 million, and the MLE estimates are obtained according to the Equation (2.8), and the frequency function's parameters of the Poisson counting process are adjusted according to the Equation (2.7).

Figure 3: Ratios of estimated 1-year EL, 95% VaR and 95% CVaR to the true (complete-data) values, for the Lognormal example, $u = 50$, $\lambda = 100$. (a) EL, 'naive'; (b) EL, conditional; (c) $\mathrm{VaR}_{0.95}$, 'naive'; (d) $\mathrm{VaR}_{0.95}$, conditional; (e) $\mathrm{CVaR}_{0.95}$, 'naive', (f) $\mathrm{CVaR}_{0.95}$, conditional.



3.2 Operational Frequency Distributions

We consider two types of a Poisson process: a homogeneous and a non-homogeneous Poisson processes, with cumulative intensity $\lambda \triangle t$ for the HPP, and the cumulative intensity $\Lambda(t)$ for the NHPP. For this particular data set, visual inspection (Figures 4 and A1) of the annually aggregated number of losses suggests that the accumulation is somewhat similar to a cdf-like process. We hence consider two following fitted cubic functions for the NHPP, each with four parameters:

1. *Cubic I*: a Lognormal cdf-like process of form $\Lambda(t) = a + \frac{b \exp\left\{ -\frac{(\log t - d)^2}{2c^2} \right\}}{\sqrt{2\pi} c}$;

2. *Cubic II*: a log-Weibull cdf-like process of form $\Lambda(t) = a - b \exp\left\{ -c \log^d t \right\}$.

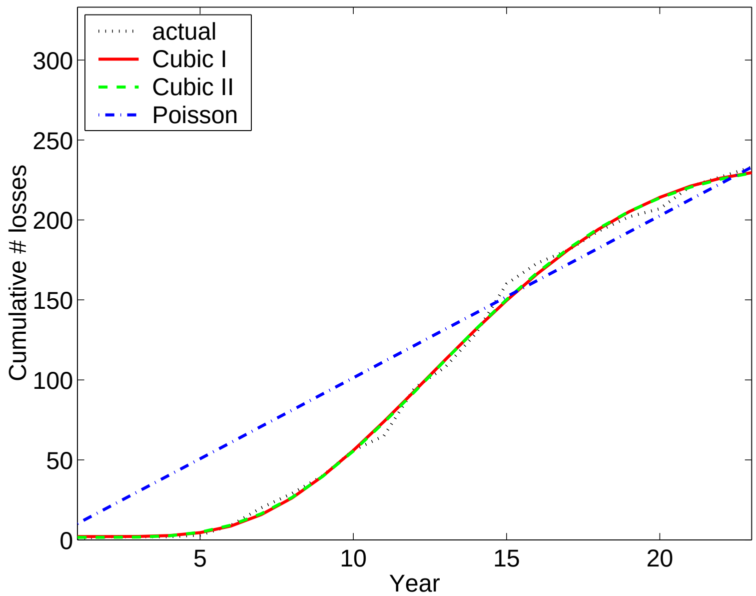
We obtain the four parameters a, b, c, d so that the Mean Square Error is minimized (minimizing Mean Absolute Error instead often led to higher error estimates). For the HPP, the estimate for λ is obtained by simply averaging the annual total number of operational loss events. Other deterministic functions were tried for the cumulative intensity (sinusoidal, tangent, etc.), but did not result in a good fit. The following Table 2 demonstrates the estimated parameters and the Mean Square Error (MSE) and the Mean Absolute Error (MAE) for the cubic cumulative intensities and a simple homogeneous Poisson process with a constant intensity factor. Figure 4 shows the three fits plotted together with the actual aggregated number of events. The cubic fits appear to be superior to the standard Poisson, as illustrated by the Figures 4 and A1, and confirmed by the MSE and MAE error comparison from Tables 2 and A1. In the subsequent analysis, we will

Table 2: Fitted frequency functions to the *"External"* type losses.

Process	Parameter Estimates				MSE	MAE
Cubic I	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	2.02	305.91	0.53	3.21	16.02	2.708
Cubic II	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	237.88	236.30	0.00026	8.27	14.56	2.713
Poisson				λ		
				10.13	947.32	24.67

assume the deterministic cubic (I or II) forms for the operational loss frequency distributions, and will no longer consider the HPP case.

Figure 4: Annual accumulated number of *"External"* operational losses, with fitted cubic and Poisson models.



3.3 Operational Loss Distributions

We restrict our attention to the loss distributions that can be used to model the losses that lie on the positive real half-line. The following distributions for loss severity are considered in the study:

Exponential	$\mathcal{E}xp(\beta)$	$f_X(x) = \beta e^{-\beta x}$ $x \geq 0,\ \beta > 0$
Lognormal	$\mathcal{LN}(\mu, \sigma)$	$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left\{-\frac{(\log x - \mu)^2}{2\sigma^2}\right\}$ $x \geq 0,\ \mu, \sigma > 0$
Gamma	$\mathcal{G}am(\alpha, \beta)$	$f_X(x) = \frac{\beta^\alpha x^{\alpha-1}}{\Gamma(\alpha)} \exp\{-\beta x\}$ $x \geq 0,\ \alpha, \beta > 0$
Weibull	$\mathcal{W}eib(\beta, \tau)$	$f_X(x) = \tau\beta x^{\tau-1} \exp\{-\beta x^\tau\}$ $x \geq 0,\ \beta, \tau > 0$
log-Weibull	$\log \mathcal{W}eib(\beta, \tau)$	$f_X(x) = \frac{1}{x}\tau\beta(\log x)^{\tau-1} \exp\{-\beta(\log x)^\tau\}$ $x \geq 0,\ \beta, \tau > 0$
Generalized Pareto	$\mathcal{GPD}(\xi, \beta)$	$f_X(x) = \beta^{-1}(1 + \xi x \beta^{-1})^{-(1+\frac{1}{\xi})}$ $x \geq 0,\ \beta > 0$
Burr	$\mathcal{B}urr(\alpha, \beta, \tau)$	$f_X(x) = \tau\alpha\beta^\alpha x^{\tau-1}(\beta + x^\tau)^{-(\alpha+1)}$ $x \geq 0,\ \alpha, \beta, \tau > 0$
log-αStable	$\log \mathcal{S}_\alpha(\beta, \sigma, \mu)$	$f_X(x) = \frac{g(\ln x)}{x}, g \in \mathcal{S}_\alpha(\beta, \sigma, \mu)$ no closed-form density $x > 0,\ \alpha \in (0, 2),\ \beta \in [-1, 1],\ \sigma, \mu > 0$
Symmetric αStable	$\mathcal{S}_\alpha\mathcal{S}(\sigma)$	$f_Y(y) = g(y), g \in \mathcal{S}_\alpha(0, \sigma, 0),$ no closed-form density $x = \lvert y \rvert,\ \alpha \in (0, 2),\ \sigma > 0$

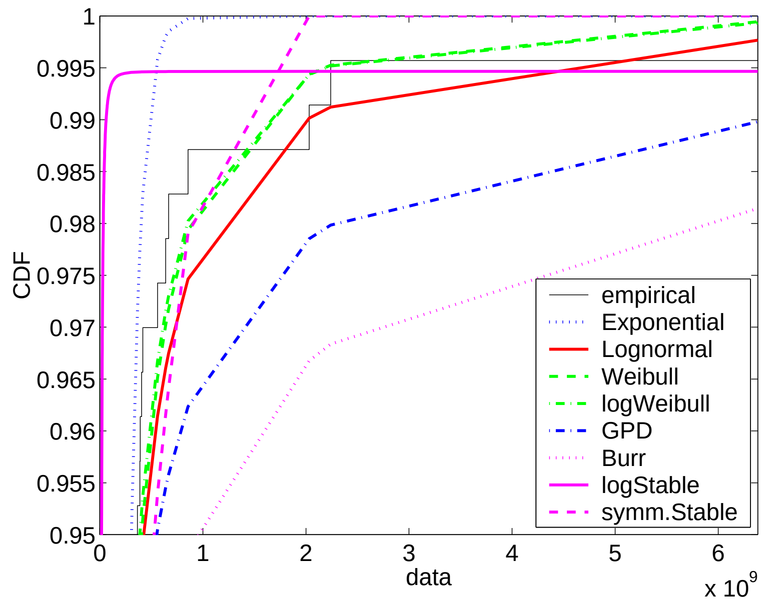
Table 3: Estimated γ and $F_\gamma(u)$ values for the *"External"* type operational loss data.

	γ,$F_\gamma(u)$	'Naive'	Conditional
$\mathcal{E}xp$	β	$9.6756{\cdot}10^{-9}$	$9.7701{\cdot}10^{-9}$
	$F_\gamma(u)$	0.0096	0.0097
$\mathcal{LN}$	μ	16.5789	15.7125
	σ	1.7872	2.3639
	$F_\gamma(u)$	0.0610	0.2111
$\mathcal{G}am$	α	0.3574	$1.5392{\cdot}10^{-6}$
	β	$3.4585{\cdot}10^{-9}$	$1.6571{\cdot}10^{-9}$
	$F_\gamma(u)$	0.1480	≈ 1
$\mathcal{W}eib$	β	$1.1613{\cdot}10^{-4}$	0.0108
	τ	0.5175	0.2933
	$F_\gamma(u)$	0.1375	0.4629
$\log\mathcal{W}eib$	β	$3.1933{\cdot}10^{-12}$	$2.8169{\cdot}10^{-8}$
	τ	9.2660	6.2307
	$F_\gamma(u)$	0.1111	0.3016
$\mathcal{GPD}$	ξ	1.2481	1.5352
	β	$1.2588{\cdot}10^{7}$	$0.7060{\cdot}10^{7}$
	$F_\gamma(u)$	0.0730	0.1203
$\mathcal{B}urr$	α	0.0987	0.1284
	β	$2.5098{\cdot}10^{26}$	$3.2497{\cdot}10^{20}$
	τ	4.2672	3.3263
	$F_\gamma(u)$	0.0145	0.0311
$\log\mathcal{S}_\alpha$	α	1.8545	1.3313
	β	1	-1
	σ	1.1975	2.7031
	μ	16.6536	10.1928
	$F_\gamma(u)$	0.0331	0.9226
$\mathcal{S}_\alpha\mathcal{S}$	α	0.6820	0.5905
	σ	$1.1395{\cdot}10^{7}$	$0.7073{\cdot}10^{7}$
	$F_\gamma(u)$	0.0715	0.1283

Table 3 demonstrates the parameter values γ of the fitted distributions to the *"External"* data set and the estimated fraction of the missing data $F_\gamma(u)$, under the 'naive' and the correct, conditional, approaches. The results for the remaining four loss types are presented in the Appendix. The tables demonstrate that under the truncated fit, more weight is put on the lower magnitude losses, including the missing losses, than what is predicted by the 'naive' model, as indicated by the $F_\gamma(u)$ estimates. The fraction indicates the true 'information loss' due to data misspecification. The location parameters (if relevant) are decreased, the scale parameters increased and the shape parameters (if relevant) decreased under the correct model (for the GPD distribution, the shape parameter corresponds to $1/\xi$), in most cases. Furthermore, the change in the skewness parameter β of the log-αStable law from 1 to -1 for the *"External"*, *"Relationship"* and *"Human"* types indicates that the right tail of the loss distribution under the correct model has a near-exponential decay, comparable to that of the Lognormal.

Based on the estimation fraction of missing data, we agree to exclude the Gamma distribution from further analysis. The proportion of observed data being nearly zero is highly unrealistic.

Figure 5: Upper quantiles of fitted truncated loss distributions to the *"External"* type losses, together with the empirical distribution.



3.4 Goodness-of-Fit Tests for Operational Loss Distributions

3.4.1 Visual Tests

Figure 5 illustrates the upper quantiles of the considered *conditional* distributions (except for Gamma) plotted against the empirical distribution, for the *"External"* type losses. The remaining four cases are in the Appendix, Figure A2. Around the 95th quantile, the Lognormal distribution suggests a good fit for the *"External"* type losses. Overall, Weibull, logWeibull and Lognormal appear to be close to the empirical distribution function. For *"Relationship"* type, Lognormal and log-Stable appear the best, for *"Human"* type Lognormal, log-Stable, Weibull and logWeibull, for *"Process"* type Burr, Weibull and logWeibull, and for *"Technology"* type Weibull and logWeibull, respectively.

3.4.2 EDF Goodness-of-Fit Tests

We test a composite hypothesis that the empirical d.f. (EDF) belongs to an entire family of hypothesized truncated distributions. Since we estimate the parameters via MLE, we do not specify the parameter values in the null expression. The null and alternative hypotheses are summarized as:

$$\begin{aligned} H_0 &: \ F_n(x) \in \widehat{F}(x) \\ H_A &: \ F_n(x) \notin \widehat{F}(x), \end{aligned} \tag{3.1}$$

where $F_n(x)$ is the empirical d.f., and $\widehat{F}(x)$ is the fitted d.f. for this sample. For the 'naive' scenario, $\widehat{F}(x)$ is the naively fitted d.f. with unconditional parameters. After necessary adjustments for the missing data, $\widehat{F}(x)$ for the truncated sample is defined as:

$$\widehat{F}(x) = \begin{cases} \frac{\widehat{F}_{\theta^c}(x) - \widehat{F}_{\theta^c}(H)}{1 - \widehat{F}_{\theta^c}(H)} & x \geq H \\ 0 & x < H, \end{cases} \tag{3.2}$$

We consider seven statistics for the measure of the distance between the empirical and hypothesized d.f.: Kolmogorov-Smirnov (D), Kuiper (V), supremum Anderson-Darling (A), supremum *"upper tail"* Anderson-Darling (A_{up}), quadratic Anderson-Darling (A^2), quadratic *"upper tail"* Anderson-Darling (A^2_{up}) and Cramér-von Mises (W^2), computed as

$$D = \max\left\{D^+, D^-\right\},$$

$$V = D^+ + D^-,$$

$$A = \sqrt{n}\sup_x \left| \frac{F_n(x) - \widehat{F}(x)}{\sqrt{\widehat{F}(x)\big(1 - \widehat{F}(x)\big)}} \right|,$$

$$A_{up} = \sqrt{n}\sup_x \left| \frac{F_n(x) - \widehat{F}(x)}{1 - \widehat{F}(x)} \right|,$$

$$A^2 = n\int_{-\infty}^{\infty} \frac{(F_n(x) - \widehat{F}(x))^2}{\widehat{F}(x)(1 - \widehat{F}(x))} d\widehat{F}(x),$$

$$A^2_{up} = n\int_{-\infty}^{\infty} \frac{(F_n(x) - \widehat{F}(x))^2}{(1 - \widehat{F}(x))^2} d\widehat{F}(x),$$

$$W^2 = n\int_{-\infty}^{\infty} (F_n(x) - \widehat{F}(x))^2 d\widehat{F}(x),$$

where $D^+ = \sqrt{n}\sup_x\{F_n(x) - \widehat{F}(x)\}$ and $D^- = \sqrt{n}\sup_x\{\widehat{F}(x) - F_n(x)\}$, and $\widehat{F}(x)$ is defined in Equation (3.2). Note that the supremum class statistics are multiplied by $\sqrt{n}$ and the quadratic class by n, to make them comparable across samples of different size. The limiting distributions of the test statistics are not parameter-free, so the p-values and the critical values were obtained with Monte Carlo simulations Ross (2001). The A_{up} and A^2_{up} statistics are introduced and studied in Chernobai, Rachev and Fabozzi (2005), and designed to put most of the weight on the upper tail. Results for the *"External"* losses are presented in Table 4. The remaining four cases are presented in the Appendix. For the *"External"* type losses, most of the unconditional distributions show a poor fit under the 'naive' approach, and Weibull and Lognormal show the best fit in terms of both, low statistic values, and high p-values.

3.5 Expected Loss, Value-at-Risk and Conditional Value-at-Risk

In this section, we estimate the expected aggregated loss (EL), VaR and CVaR and examine the impact of ignoring the missing data on the operational risk capital charge. We use a forward-looking approach, and use the functional form of the frequency and the parameters of the severity distribution, obtained from the historical data over 23 year period, to forecast expected total loss, VaR and CVaR one year ahead. We only consider the Cubic I case for the frequency. Table 5 provides the estimates of expected loss (whenever applicable), VaR and CVaR estimates for the

Table 4: Results of in-sample GOF tests for *"External"* type operational losses. p-values (in square brackets) were obtained via 1,000 Monte Carlo simulations.

	KS	V	AD	AD^2	W^2
$\mathcal{E}xp$	6.5941	6.9881	$4.4{\cdot}10^6$	128.35	17.4226
	[<0.005]	[<0.005]	[<0.005]	[<0.005]	[<0.005]
$\mathcal{LN}$	0.6504	1.2144	2.1702	0.5816	0.0745
	[0.326]	[0.266]	[0.469]	[0.120]	[0.210]
$\mathcal{W}eib$	0.4752	0.9498	2.4314	0.3470	0.0337
	[0.852]	[0.726]	[0.384]	[0.519]	[0.781]
$\log\mathcal{W}eib$	0.6893	1.1020	2.2267	0.4711	0.0563
	[0.296]	[0.476]	[0.481]	[0.338]	[0.458]
$\mathcal{GPD}$	0.9708	1.8814	2.7742	1.7091	0.2431
	[0.009]	[<0.005]	[0.284]	[<0.005]	[<0.005]
$\mathcal{B}urr$	1.3266	2.0385	2.8775	2.8954	0.5137
	[0.050]	[0.048]	[0.328]	[0.048]	[0.048]
$\log\mathcal{S}_\alpha$	7.3275	7.4089	37.4863	194.74	24.3662
	[0.396]	[0.458]	[0.218]	[0.284]	[0.366]
$S\alpha S$	0.7222	1.4305	$1.1{\cdot}10^5$	1.7804	0.1348
	[0.586]	[0.339]	[0.990]	[0.980]	[0.265]

Table 5: Estimates of expected aggregated loss, VaR and CVaR (figures must be further scaled $\times 10^{10}$) for *"External"* type losses. Figures are based on 50,000 Monte Carlo samples.

		EL	$\text{VaR}_{0.95}$	$\text{VaR}_{0.99}$	$\text{CVaR}_{0.95}$	$\text{CVaR}_{0.99}$
$\mathcal{E}xp$	'Naive'	0.0207	0.0618	0.0897	0.0790	0.1064
	Conditional	0.0306	0.0798	0.1100	0.0985	0.1283
$\mathcal{LN}$	'Naive'	0.0157	0.0613	0.1697	0.1450	0.3451
	Conditional	0.0327	0.1126	0.4257	0.3962	1.1617
$\mathcal{W}eib$	'Naive'	0.0151	0.0613	0.1190	0.0975	0.1628
	Conditional	0.0208	0.0885	0.2494	0.2025	0.4509
$log\mathcal{W}eib$	'Naive'	-	0.0611	0.1309	0.1059	0.1940
	Conditional	-	0.0839	0.2489	0.2046	0.4909
$\mathcal{GPD}$	'Naive'	-	0.1190	0.8381	2.7082	12.4017
	Conditional	-	0.2562	2.6514	63.4969	314.41
$\mathcal{B}urr$	'Naive'	-	0.4072	8.7417	366.32	1823.79
	Conditional	-	0.7165	15.8905	1502.42	7498.81
$\log\mathcal{S}_\alpha$	'Naive'	-	0.1054	3.7687	-	-
	Conditional	-	0.3879	0.8064	0.6750	1.2641
$\mathcal{S}_\alpha\mathcal{S}$	'Naive'	-	0.1730	1.8319	35.7423	176.64
	Conditional	-	0.4714	7.6647	206.49	1025.16

year 2003, obtained via 50,000 Monte Carlo samples, and compares the figures obtained using the 'naive' approach and the conditional approach. The remaining four cases are in the Appendix.

Table 5 provides evidence to conclude that in most cases, the expected aggregated loss, VaR and CVaR figures appear highly under-estimated if the 'naive' approach is wrongly used, instead of the conditional. Some figures also indicate that the effect is more severe for heavier-tailed distributions. We exclude the Exponential distribution from subsequent consideration due to poor performance of in-sample goodness-of-fit tests.

3.6 Backtesting

In this section, we conduct an out-of-sample backtesting of the models. Before we proceed, we would like to point out that the Exponential distribution showed a poor in-sample fit for all five loss types. We therefore exclude the distribution from any further empirical examination. Additionally, we have demonstrated in the theoretical part of the paper, that fitting unconditional distributions to the operational loss frequency and severity functions would inevitably result in biased estimates. In Section 3.5 we have provided empirical evidence to verify that indeed the capital charge is significantly underestimated (in most cases) if the 'naive' approach is wrongly used.

The goal of this section is to determine which loss distribution fits our five samples best. Examining how well or how badly various considered models predict the true future losses, is, we believe, the key to determining which of the loss distributions is the best to be used for practical purposes. For this purpose, we split our data samples into two parts: (1) the first sample consists of all data points in the 1980-1995 time frame, and will be used for forecasting, and (2) the second sample consists of the remaining data in the 1996-2001 time frame. We use the first sample and the obtained truncated loss distributions' parameter estimates to analyze our models' predicting power regarding the data belonging to the second sample. We conduct the analysis is the following fashion. We assume that our model has a one-step ahead predicting power, with one step equal to one year (due to a scarcity of data, it would be unreasonable to use smaller intervals). The window length of the sample used for calibration is taken to be sixteen years. We start with the data from the first 1980 until 1995, in order to conduct the forecasting about the 1996. First, we estimate the unknown parameters of truncated distributions. Next, to obtain the distribution of the annually aggregated losses we repeat the following a large number (10,000) of times: use the estimated parameters to simulate N losses exceeding the \$25 million threshold, where N is the number of losses in the year that we perform forecasting on as dictated by the fitted frequency function, and aggregate them. At each forecasting step (seven steps total) we shift the window by one year forward and repeat the above procedure. In this way we test the model for both the severity and severity distributions. We have observed that both Cubic I and II models fit the data very well. For simplicity, in this section we only focus on the Cubic I model. Since the observed data is incomplete, we are only able to compare the forecasting power regarding the truncated (rather than complete) data. The analysis is composed with two parts.

In part one, we compare the high quantiles (95, 99 and 99.9) of the forecasted aggregated loss distribution with the corresponding bootstrapped quantiles of the realized losses.[8] Table 6 presents the mean squared error (MSE) and mean absolute error (MAE) estimates for the forecasted high quantiles relative to the corresponding bootstrapped quantiles (left), realized total loss (middle), and the errors of the simulated aggregate losses relative to the actual total loss (right), for the *"External"* type losses. The remaining four cases are in the Appendix. For the *"External"* type losses, clearly the Weibull model provides the lowest estimates for the errors, followed by the logWeibull and log-αStable models. For the remaining four types, Weibull and logWeibull are

[8] The use of bootstrapping and Monte Carlo was suggested by the Basel Committee BCBS (2001*b*),BCBS (2004).

Table 6: Average estimates of forecast errors for *"External"* type aggregated losses. *Left panel:* errors between relative upper quantiles; *middle panel:* errors of forecasted upper quantiles relative to realized loss; *right panel:* overall error between forecasted and realized loss. Figures are based on 10,000 Monte Carlo samples for every year in forecasting period.

		Forecasted upper quantiles vs. upper bootstrapped quantiles		Forecasted upper quantiles vs. actual loss		Overall error: forecasted vs. actual loss	
	quantile	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)
$\mathcal{LN}$	95	0.2284	0.4071	0.2665	0.4508		
	99	2.6679	1.4589	2.8631	1.5220	0.4161	0.1373
	99.9	59.0091	6.9119	60.2081	6.9971		
$\mathcal{W}eib$	95	0.0756	0.2380	0.0981	0.2817		
	99	0.4529	0.6065	0.5358	0.6698	0.0350	0.0835
	99.9	2.7728	1.5693	3.0653	1.6555		
$log\mathcal{W}eib$	95	0.0843	0.2496	0.1078	0.2933		
	99	0.6127	0.6974	0.7058	0.7605	0.0523	0.0873
	99.9	5.6387	2.1352	6.0582	2.2236		
$\mathcal{GPD}$	95	16.7032	3.0645	16.9948	3.1083		
	99	3287.19	41.7376	3292.66	41.8008	$23.2{\cdot}10^{10}$	1877.31
	99.9	$1.1{\cdot}10^{7}$	2454.50	$1.1{\cdot}10^{7}$	2454.59		
$\mathcal{B}urr$	95	1174.90	25.2473	1177.23	25.2909		
	99	$0.3{\cdot}10^{7}$	1026.97	$2.7{\cdot}10^{6}$	1027.03	$3.3{\cdot}10^{12}$	9800.80
	99.9	$5.7{\cdot}10^{10}$	$1.8{\cdot}10^{5}$	$5.7{\cdot}10^{20}$	$1.8{\cdot}10^{5}$		
$\log\mathcal{S}_\alpha$	95	0.0916	0.2684	0.1170	0.3121		
	99	0.7443	0.8117	0.8506	0.8747	0.0792	0.0929
	99.9	6.8469	2.4760	7.2695	2.5635		
$\mathcal{S}_\alpha\mathcal{S}$	95	22.0854	3.8156	22.4520	3.8594		
	99	5606.35	60.8946	5614.50	60.9580	$55.8{\cdot}10^{10}$	4337.63
	99.9	$1.1{\cdot}10^{7}$	2758.87	$1.1{\cdot}10^{7}$	2758.96		

the best, followed by log-αStable and logNormal distributions. GPD, Burr and symmetric Stable over-estimate the true losses, as is suggested by very high error estimates.

In the second part of the analysis, we test the severity distribution models (without checking for the frequency) via the Likelihood Ratio test, proposed by Berkowitz (2000). We estimate the parameters of the loss distribution from the historic data in the calibration period; then we use this distribution and the estimated parameters to conduct the forecast one year ahead. The parameters are fully specified, so we are able to conduct a simple hypothesis testing. Likelihood Ratio (LR) tests are uniformly most powerful, so applying them would give us an accurate estimate of how likely it is that the realized losses have come from a particular distribution. Under the assumption that a fitted truncated loss distribution F is true, $F \sim \mathcal{U}[0,1]$ under the null. A simple transformation $Y = \Phi^{-1}(F)$ would transform the valued of cdf into a standard Normal random variable Y. The LR test is then applied to Y directly in a usual way:

$$LR = -2(l_0 - l_1),$$

where l_0 and l_1 are, respectively, the log-likelihood under the null parameters $\mu = 0$, $\sigma = 1$, and under the parameters estimated via MLE. The p-values are obtained by referring to the Chi-squared table, with 2 degrees of freedom. Table 7 presents the results for the *"External"* losses, and the

Table 7: LR statistic and p-values (in square brackets) for *"External"* type aggregated losses in the 7-year forecast period.

	LR statistic and p-value							average p-value
Year	1	2	3	4	5	6	7	
$\mathcal{LN}$	6.5946	2.6909	0.2571	0.0677	0.0682	6.6879	3.5182	
	[0.037]	[0.260]	[0.879]	[0.967]	[0.967]	[0.035]	[0.172]	[0.474]
$\mathcal{W}eib$	6.8197	2.7312	0.3206	0.1054	0.1319	6.7159	3.5155	
	[0.033]	[0.255]	[0.852]	[0.949]	[0.936]	[0.035]	[0.172]	[0.462]
$log\mathcal{W}eib$	6.5458	2.3548	0.2483	0.1285	0.1255	6.3278	3.4601	
	[0.038]	[0.308]	[0.883]	[0.938]	[0.939]	[0.042]	[0.177]	[0.475]
$\mathcal{GPD}$	6.8378	3.1089	0.2408	0.0518	0.0462	7.2744	3.5594	
	[0.033]	[0.211]	[0.887]	[0.974]	[0.977]	[0.026]	[0.169]	[0.468]
$\mathcal{B}urr$	9.5129	5.6630	0.6205	≈ 0	≈ 0	9.6215	3.9581	
	[0.009]	[0.059]	[0.733]	[>0.995]	[>0.995]	[0.008]	[0.138]	[0.421]
$\log \mathcal{S}_\alpha$	23.2014	32.0407	15.3027	5.1930	20.0156	6.7474	22.5930	
	[<0.005]	[<0.005]	[<0.005]	[0.075]	[<0.005]	[0.034]	[<0.005]	[0.016]
$\mathcal{S}_\alpha\mathcal{S}$	6.4584	2.5302	0.2017	0.0990	0.0879	7.4456	3.4455	
	[0.040]	[0.282]	[0.904]	[0.952]	[0.957]	[0.024]	[0.179]	[0.477]

remaining four cases are in the Appendix. The symmetric Stable shows the highest average p-values, with logWeibull and GPD the next highest. The highest average p-values were obtained for the Weibull model for the *"Relationship"* and *"Human"* losses, GPD was the best for *"Process"*, and log-αStable was the best for the *"Technology"* losses. The results are slightly surprising compared to the estimated forecast errors, but confirm many conclusions drawn from the in-sample goodness-of-fit tests.

4 Robust Approach

In the previous section, Section 3.6, we tested the forecasting power of the considered loss models. From the first part of the analysis, we concluded that moderately heavy-tailed distributions such as Lognormal and Weibull possess a reasonably good predicting power. The second part of the analysis suggested that the losses considered for the forecast period or more likely to be drawn from heavier-tailed distributions such as GPD or log-αStable. It is very likely that such difference results from the presence of high-magnitude outliers in the data, that leads to accepting heavy-tailed models, whereas inclusion of such outliers in the forecasting models can seriously over-estimate the predicted losses.

In recent years outlier-resistant or so-called robust estimates of parameters are becoming more wide-spread in risk management. Such models – called robust (statistics) models – were introduced by P.J. Huber in 1981 and applied to robust regression analysis, more recent references on robust statistics methods include Huber (2004), Rousseeuw and Leroy (2003), Martin and Simin (2003), Knez and Ready (1997) and Hampel, Ronchetti, Rousseeuw and Stahel (1986). Robust models treat extreme data points as outliers (or some standard procedure is used to detect outliers in the data) which distort the main flow of the loss process. Practitioners are more likely to be searching for a stable model that would capture the mainstream tendency of the operational loss process.

Table 8: Estimated γ and $F_\gamma(u)$ values for the *"External"* type operational loss data, under the *robust* approach.

	$\gamma, F_\gamma(u)$	'Naive'	Conditional
$\mathcal{E}xp$	β	$2.5156 \cdot 10^{-8}$	$2.5805 \cdot 10^{-8}$
	$F_\gamma(u)$	0.0248	0.0255
$\mathcal{LN}$	μ	16.3676	15.8095
	σ	1.5680	1.9705
	$F_\gamma(u)$	0.0518	0.1558
$\mathcal{G}am$	α	0.5532	0.0491
	β	$1.3916 \cdot 10^{-8}$	$6.1244 \cdot 10^{-9}$
	$F_\gamma(u)$	0.1052	≈ 1
$\mathcal{W}eib$	β	1.1900	0.0012
	τ	0.6606	0.4178
	$F_\gamma(u)$	0.1036	0.3185
$\log \mathcal{W}eib$	β	$1.5224 \cdot 10^{-14}$	$2.1389 \cdot 10^{-10}$
	τ	11.2079	7.9597
	$F_\gamma(u)$	0.0879	0.2254
$\mathcal{GPD}$	ξ	0.8995	1.1813
	β	$1.2968 \cdot 10^{7}$	$7.7474 \cdot 10^{6}$
	$F_\gamma(u)$	0.0718	0.1132
$\mathcal{B}urr$	α	0.5477	1.1642
	β	$1.8890 \cdot 10^{9}$	$8.6128 \cdot 10^{5}$
	τ	1.3784	0.8490
	$F_\gamma(u)$	0.0502	0.1451
$\log \mathcal{S}_\alpha$	α	2	2
	β	0.8736	0.4377
	σ	1.1087	1.3992
	μ	16.3674	15.7960
	$F_\gamma(u)$	0.0522	0.1593
$\mathcal{S}_\alpha\mathcal{S}$	α	0.7693	0.6598
	σ	$1.0404 \cdot 10^{7}$	$6.7785 \cdot 10^{6}$
	$F_\gamma(u)$	0.0708	0.1208

Under the robust approach, the focus is on modelling the major bulk of the data that is driving the entire process. Robust models help protect against the outlier bias in parameter estimates and provide with a better fit of the loss distributions to the data than under the classical model. Moreover, outliers in the original data can seriously drive future forecasts in an unwanted (such as worst-case scenario) direction, which is avoided by the robust approach models. Regarding the use of robust methods for the operational risk modelling the Basel Committee stated the following BCBS (2001*a*): *"...data will need to be collected and robust estimation techniques ... will need to be developed."*

Following the idea of robust statistics, for the forecasting purposes we offer a second methodology that involves determining outliers and trimming the top 1-5% of the data. This data adjustment would result in a more robust outlook regarding a general future scenario. Excluding the outliers in the original loss data is likely to noticeably improve the forecasting power of considered loss distributions, and can be used for forecasting of the generic (most likely) scenario of future losses within reasonable boundaries. The resulting operational capital charge estimates would be more

Table 9: Estimates of expected aggregated loss, VaR and CVaR (figures must be further scaled $\times 10^{10}$) for *"External"* type losses, under the *robust* approach. Figures are based on 50,000 Monte Carlo samples.

		EL	$\text{VaR}_{0.95}$	$\text{VaR}_{0.99}$	$\text{CVaR}_{0.95}$	$\text{CVaR}_{0.99}$
$\mathcal{E}xp$	'Naive'	0.0080	0.0238	0.0346	0.0304	0.0409
	Conditional	0.0116	0.0299	0.0414	0.0371	0.0481
$\mathcal{LN}$	'Naive'	0.0088	0.0331	0.0804	0.0661	0.1364
	Conditional	0.0154	0.0580	0.1642	0.1397	0.3334
$\mathcal{W}eib$	'Naive'	0.0076	0.0278	0.0466	0.0393	0.0581
	Conditional	0.0088	0.0354	0.0715	0.0599	0.1066
$log\mathcal{W}eib$	'Naive'	-	0.0289	0.0507	0.0428	0.0675
	Conditional	-	0.0395	0.0865	0.0704	0.1318
$\mathcal{GPD}$	'Naive'	0.0258	0.0463	0.1834	0.2817	1.0771
	Conditional	-	0.0943	0.5604	2.1575	9.9486
$\mathcal{B}urr$	'Naive'	-	0.0751	0.5666	9.8732	48.6049
	Conditional	-	0.0676	0.3246	0.7372	3.1567
$\log \mathcal{S}_\alpha$	'Naive'	-	0.0341	0.0818	0.0684	0.1446
	Conditional	-	0.0570	0.1695	0.1493	0.3841
$\mathcal{S}_\alpha\mathcal{S}$	'Naive'	-	0.0854	0.6791	17.1402	84.8334
	Conditional	-	0.2234	2.4408	129.37	644.14

optimistic than otherwise predicted by the classical model. We emphasize, however, that we are not recommending the use of only one of the two approaches – classical or robust – instead of the other. Rather, in the presence of outliers, we encourage the use of both models for the analysis, and use the robust model as the complement to the classical.

We here consistently exclude the highest 5% of each data set. We reproduce the results for the parameter estimates, capital charge estimates and out-of-sample goodness of fit tests, for the *"External"* type losses.[9] Tables 9, 10 and 11 indicate the following: first, the estimates of expected loss, VaR and CVaR are much more realistic, second, the accuracy of the forecasts has remarkably improved as indicated by much lower error estimates, and third, both tests (forecast error estimates, and the LR test) converge in their indication of the best model: the robust approach confirms that the logWeibull distribution has the best forecast power for the *"External"* type loss, with Weibull, Lognormal, log-Stable, GPD, symmetric Stable and Burr next, in the order from best to poor.

5 Conclusions

In this study we proposed and empirically investigated a methodology for consistent estimation of the loss and frequency distributions for the assumed actuarial model of operational losses in presence of minimum collection thresholds. A compound nonhomogeneous Poisson process was considered for the study. The analysis was conducted using losses of five different loss categories – "Relationship," "Human," "Processes," "Technology," and "External" – obtained from an operational risk loss database.

Our findings demonstrated that ignoring such minimum thresholds leads to severe biases in

[9] We omit the remaining four cases for brevity. Key results and conclusions remain the same.

Table 10: Average estimates of forecast errors for *"External"* type aggregated losses, under the *robust* approach. *Left panel:* errors between relative upper quantiles; *middle panel:* errors of forecasted upper quantiles relative to realized loss; *right panel:* overall error between forecasted and realized loss. Figures are based on 10,000 Monte Carlo samples for every year in forecasting period.

		Forecasted upper quantiles vs. upper bootstrapped quantiles		Forecasted upper quantiles vs. actual loss		Overall error: forecasted vs. actual loss	
	quantile	MSE($\times10^{20}$)	MAE($\times10^{10}$)	MSE($\times10^{20}$)	MAE($\times10^{10}$)	MSE($\times10^{20}$)	MAE($\times10^{10}$)
$\mathcal{LN}$	95	0.0185	0.1153	0.0260	0.1443		
	99	0.1532	0.3498	0.1838	0.3921	0.0143	0.0455
	99.9	1.7033	1.2369	1.8506	1.2937		
$\mathcal{W}eib$	95	0.0037	0.0479	0.0070	0.0749		
	99	0.0136	0.1018	0.0236	0.1442	0.0019	0.0284
	99.9	0.0555	0.2212	0.0838	0.2779		
$log\mathcal{W}eib$	95	0.0035	0.0469	0.0069	0.0753		
	99	0.0140	0.1069	0.0245	0.1489	0.0019	0.0281
	99.9	0.0619	0.2388	0.0918	0.2958		
$\mathcal{GPD}$	95	0.5389	0.5677	0.5729	0.5968		
	99	45.4984	4.9129	45.9027	4.9548	25964.56	1.6577
	99.9	22951.07	108.00	22964.73	108.06		
$\mathcal{B}urr$	95	41.6391	4.3990	41.9815	4.4280		
	99	22565.03	92.0143	22572.25	92.0562	$2.0{\cdot}10^{10}$	630.68
	99.9	$4.1{\cdot}10^{8}$	11255.38	$4.1{\cdot}10^{8}$	11255.44		
$\log\mathcal{S}_\alpha$	95	0.0149	0.0951	0.0210	0.1243		
	99	0.1312	0.2961	0.1568	0.3382	0.0164	0.0413
	99.9	1.6894	1.0598	1.7989	1.1166		
$\mathcal{S}_\alpha\mathcal{S}$	95	3.8490	1.6133	3.9474	1.6423		
	99	604.07	19.7887	605.82	19.8306	$1.5{\cdot}10^{6}$	10.6711
	99.9	$6.2{\cdot}10^{5}$	626.31	$6.2{\cdot}10^{5}$	626.37		

corresponding parameter estimates under the 'naive' approach in which the thresholds are ignored. As a consequence, EL, VaR, and CVaR are underestimated under the 'naive' approach and are generally 1.2 to 5 times higher under the conditional approach, in which truncated loss distributions were fitted to the loss data and frequency was adjusted to account for information loss. A variety of goodness-of-fit measures were used to test the adequacy of different loss distributions. For example, for the *"External"* type losses the Logweibull and Weibull distributions showed the best overall fit, while more heavy-tailed distributions such as Burr and GPD better fit the upper tail for practically all five datasets, supporting the conjecture that the operational loss data is severely heavy-tailed. The findings were supported by out-of-sample forecasting.

An alternative approach, the robust approach, was briefly introduced and applied in the forecasting part of the study. Excluding few highest data points from the dataset allows to investigate the behavior of the major bulk of the data as well as examine the sensitivity of parameters and risk measures to the tail events. In this study, applying the robust methodology resulted in significantly improved forecasts and confirmed the choice of loss distribution obtained under the classical

Table 11: LR statistic and p-values (in square brackets) for *"External"* type aggregated losses in the 7-year forecast period, under the *robust* approach.

	LR statistic and p-value							average p-value
Year	1	2	3	4	5	6	7	
$\mathcal{LN}$	5.7220	3.6199	0.1477	≈0	0.1078	6.5407	2.5591	
	[0.057]	[0.164]	[0.929]	[>0.995]	[0.948]	[0.038]	[0.278]	[0.488]
$\mathcal{W}eib$	5.1081	2.9975	0.0533	≈0	0.0601	5.7516	2.4772	
	[0.078]	[0.223]	[0.974]	[>0.995]	[0.970]	[0.056]	[0.290]	[0.513]
$log\mathcal{W}eib$	4.6784	2.3294	≈0	≈0	0.1273	5.4843	2.4643	
	[0.096]	[0.312]	[>0.995]	[>0.995]	[0.938]	[0.064]	[0.292]	[0.529]
$\mathcal{GPD}$	6.2144	3.7330	0.1203	0.0289	0.2780	7.1288	2.5038	
	[0.045]	[0.155]	[0.942]	[0.986]	[0.870]	[0.028]	[0.286]	[0.473]
$\mathcal{B}urr$	9.2110	6.0794	0.3852	≈0	0.2343	8.9561	2.4562	
	[0.010]	[0.048]	[0.825]	[>0.995]	[0.890]	[0.011]	[0.293]	[0.440]
$\log \mathcal{S}_\alpha$	5.7938	94.1860	0.1588	≈0	125.9073	6.5408	2.5913	
	[0.055]	[<0.005]	[0.924]	[>0.995]	[<0.005]	[0.038]	[0.274]	[0.327]
$\mathcal{S}_\alpha\mathcal{S}$	6.5552	4.4805	0.2889	≈0	0.0702	7.7629	2.4301	
	[0.038]	[0.106]	[0.866]	[>0.995]	[0.966]	[0.021]	[0.297]	[0.470]

approach.

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Appendix

Table A1: Fitted frequency functions to the operational losses.

Process	Parameter Estimates				MSE	MAE
"Relationship"						
Cubic I	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	34.13	1364.82	0.63	3.32	76.57	7.05
Cubic II	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	930.29	896.17	0.0010	6.82	69.08	6.57
Poisson				λ		
				36.91	5907.45	65.68
"Human"						
Cubic I	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	33.49	1436.56	0.65	3.43	68.05	6.89
Cubic II	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	950.20	917.11	0.0008	6.80	61.59	6.60
Poisson				λ		
				35.35	6600.38	65.33
"Processes"						
Cubic I	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	9.44	2098.96	1.04	4.58	22.50	3.64
Cubic II	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	2034.25	2024.77	0.0007	4.79	23.06	3.65
Poisson				λ		
				14.13	1664.82	36.57
"Technology"						
Cubic I	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	0.79	120.20	0.58	3.47	3.71	1.28
Cubic II	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$		
	137.68	138.39	0.0006	6.32	4.89	1.67
Poisson				λ		
				3.35	217.04	13.42

Figure A1: Fitted frequency functions to the operational losses. Top left: *"Relationship"*, top right: *"Human,"* bottom left: *"Processes,"* bottom right: *"Technology,"*

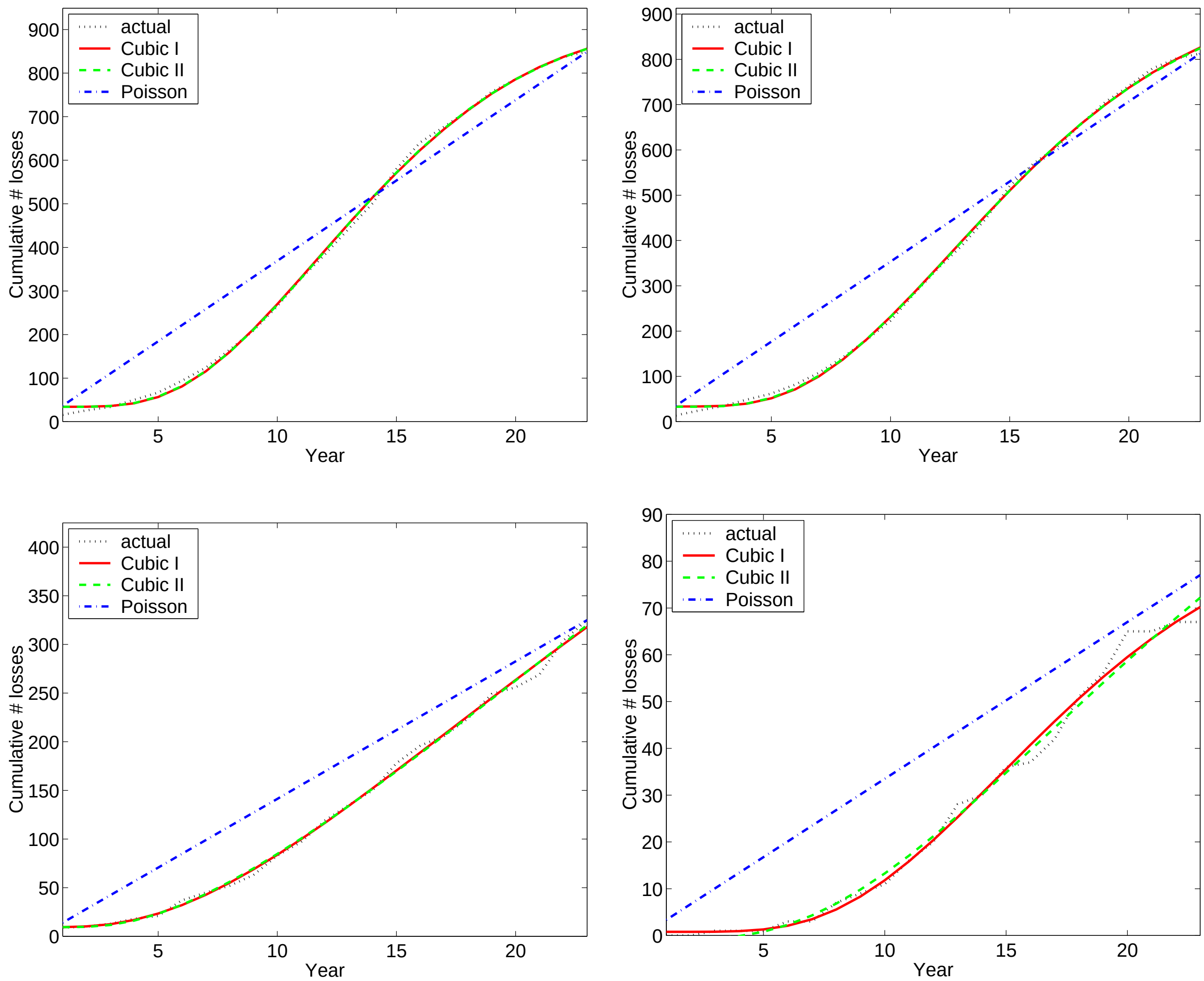

Table A2: Estimated γ and $F_\gamma(u)$ values for the *"Relationship," "Human," "Processes,"* and *"Technology"* type operational loss data.

		"Relationship"		*"Human"*		*"Processes"*		*"Technology"*	
	γ,$F_\gamma(u)$	'Naive'	Conditional	'Naive'	Conditional	'Naive'	Conditional	'Naive'	Conditional
$\mathcal{E}xp$	β	$1.1128{\cdot}10^{-8}$	$1.1253{\cdot}10^{-8}$	$7.2216{\cdot}10^{-9}$	$7.2741{\cdot}10^{-9}$	$3.5020{\cdot}10^{-9}$	$3.5143{\cdot}10^{-9}$	$1.2914{\cdot}10^{-8}$	$1.3083{\cdot}10^{-8}$
	$F_\gamma(u)$	0.0111	0.0112	0.0072	0.0073	0.0035	0.0035	0.0128	0.0130
$\mathcal{LN}$	μ	16.6771	16.1911	16.5878	15.4627	17.5163	17.1600	16.6176	15.1880
	σ	1.6956	2.0654	1.8590	2.5642	2.0215	2.3249	1.9390	2.7867
	$F_\gamma(u)$	0.0457	0.1250	0.0679	0.2603	0.0336	0.0751	0.0742	0.3112
$\mathcal{G}am$	α	0.4018	$7.9092{\cdot}10^{-7}$	0.3167	$6.9763{\cdot}10^{-8}$	0.3450	0.0247	0.4217	$7.5176{\cdot}10^{-6}$
	β	$4.4708{\cdot}10^{-9}$	$1.9614{\cdot}10^{-9}$	$2.2869{\cdot}10^{-9}$	$1.1679{\cdot}10^{-9}$	$1.2082{\cdot}10^{-9}$	$0.5480{\cdot}10^{-9}$	$5.4458{\cdot}10^{-9}$	$2.3538{\cdot}10^{-9}$
	$F_\gamma(u)$	0.1281	≈ 1	0.1628	≈ 1	0.1104	≈ 1	0.1250	≈ 1
$\mathcal{W}eib$	β	$6.1038{\cdot}10^{-5}$	0.0032	0.0002	0.0240	0.0001	0.0021	$6.3668{\cdot}10^{-5}$	0.0103
	τ	0.5528	0.3538	0.4841	0.2526	0.4938	0.3515	0.5490	0.2938
	$F_\gamma(u)$	0.1189	0.3479	0.1501	0.5441	0.0923	0.2338	0.1177	0.4485
$\log\mathcal{W}eib$	β	$0.5128{\cdot}10^{-12}$	$0.2694{\cdot}10^{-8}$	$14.3254{\cdot}10^{-12}$	$30.7344{\cdot}10^{-8}$	$2.4894{\cdot}10^{-12}$	$0.1091{\cdot}10^{-8}$	$1.9309{\cdot}10^{-12}$	$11.0647{\cdot}10^{-8}$
	τ	9.8946	7.0197	9.8946	7.0197	9.1693	7.1614	9.4244	5.7555
	$F_\gamma(u)$	0.0938	0.2386	0.1221	0.3718	0.0687	0.1479	0.1023	0.3329
$\mathcal{GPD}$	ξ	-1.0882	-1.2852	-1.3761	-1.6562	-1.4754	-1.6147	-1.5823	-2.0925
	β	$1.5516{\cdot}10^{7}$	$1.0558{\cdot}10^{7}$	$1.1441{\cdot}10^{7}$	$0.6135{\cdot}10^{7}$	$2.9230{\cdot}10^{7}$	$2.2886{\cdot}10^{7}$	$1.0470{\cdot}10^{7}$	$0.3446{\cdot}10^{7}$
	$F_\gamma(u)$	0.0604	0.0855	0.0792	0.1344	0.0328	0.0413	0.0851	0.2029
$\mathcal{B}urr$	α	0.4817	5.1242	0.0938	0.0922	0.8661	14.3369	0.0645	0.0684
	β	$3.4832{\cdot}10^{9}$	$1.0221{\cdot}10^{4}$	$5.1819{\cdot}10^{27}$	$2.8463{\cdot}10^{27}$	$4.3835{\cdot}10^{6}$	$1.1987{\cdot}10^{4}$	$1.7210{\cdot}10^{35}$	$8.7406{\cdot}10^{20}$
	τ	1.4077	0.4644	4.4823	4.4717	0.8884	0.3829	5.8111	5.2150
	$F_\gamma(u)$	0.0365	0.2575	0.0131	0.0195	0.0405	0.2097	0.0227	0.8042
log-αStable	α	1.9097	1.9340	1.6294	1.4042	2.0000	2.0000	2.0000	2.0000
	β	1	-1	1	-1	0.9697	0.8195	0.7422	0.8040
	σ	1.1584	1.5198	1.1395	2.8957	1.4294	1.6476	1.3715	1.9894
	μ	16.7182	15.9616	16.8464	10.5108	17.5163	17.1535	16.6181	15.1351
	$F_\gamma(u)$	0.0303	0.1742	0.0083	0.8793	0.0336	0.0760	0.0747	0.3195
$\mathcal{S}_\alpha\mathcal{S}$	α	0.7377	0.6592	0.6724	0.6061	0.5902	0.5478	0.1827	0.1827
	σ	$1.3695{\cdot}10^{7}$	$0.9968{\cdot}10^{7}$	$1.1126{\cdot}10^{7}$	$0.7143{\cdot}10^{7}$	$2.7196{\cdot}10^{7}$	$1.9925{\cdot}10^{7}$	$0.1676{\cdot}10^{7}$	$0.1676{\cdot}10^{7}$
	$F_\gamma(u)$	0.0558	0.0841	0.0742	0.1241	0.0358	0.0536	0.3723	0.3723

Figure A2: Upper quantiles of fitted truncated loss distributions to operational losses, together with the empirical distribution. Top left: *"Relationship,"* top right: *"Human,"* bottom left: *"Process,"* bottom right: *"Technology."*

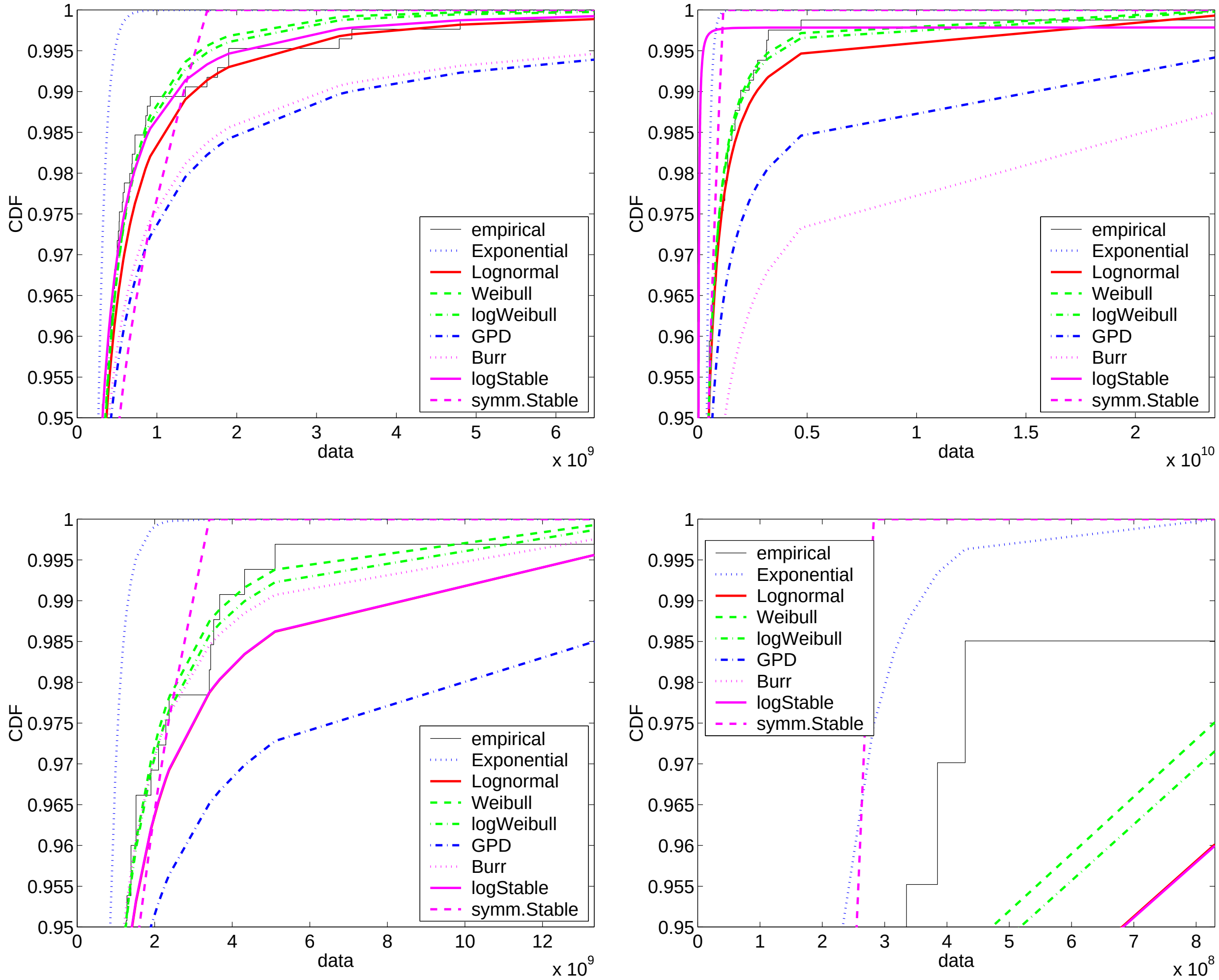

Table A3: Results of in-sample GOF tests for *"Relationship"* type operational losses. p-values (in square brackets) were obtained via 1,000 Monte Carlo simulations.

	KS	V	AD	AD^2	W^2
$\mathcal{E}xp$	11.0868	11.9973	$1.3{\cdot}10^7$	344.37	50.5365
	[<0.005]	[<0.005]	[<0.005]	[<0.005]	[<0.005]
$\mathcal{LN}$	0.8056	1.3341	2.6094	0.7554	0.1012
	[0.082]	[0.138]	[0.347]	[0.043]	[0.086]
$\mathcal{W}eib$	0.5553	1.0821	3.8703	0.7073	0.0716
	[0.625]	[0.514]	[0.138]	[0.072]	[0.249]
$\log \mathcal{W}eib$	0.5284	1.0061	3.0718	0.4682	0.0479
	[0.699]	[0.628]	[0.255]	[0.289]	[0.514]
$\mathcal{GPD}$	1.4797	2.6084	3.5954	3.7165	0.5209
	[<0.005]	[<0.005]	[0.154]	[<0.005]	[<0.005]
$\mathcal{B}urr$	1.3673	2.4165	3.3069	3.1371	0.4310
	[0.032]	[<0.005]	[0.309]	[<0.005]	[0.011]
$\log \mathcal{S}_\alpha$	1.5929	1.6930	3.8184	3.8067	0.7076
	[0.295]	[0.295]	[0.275]	[0.290]	[0.292]
$S\alpha S$	1.1634	2.0695	$1.4{\cdot}10^5$	4.4723	0.3630
	[0.034]	[<0.005]	[>0.995]	[0.992]	[<0.005]

Table A4: Results of in-sample GOF tests for *"Human"* type operational losses. p-values (in square brackets) were obtained via 1,000 Monte Carlo simulations.

	KS	V	AD	AD^2	W^2
$\mathcal{E}xp$	14.0246	14.9145	$2.4{\cdot}10^6$	609.15	80.3703
	[<0.005]	[<0.005]	[<0.005]	[<0.005]	[<0.005]
$\mathcal{LN}$	0.8758	1.5265	3.9829	0.7505	0.0804
	[0.032]	[0.039]	[0.126]	[0.044]	[0.166]
$\mathcal{W}eib$	0.8065	1.5439	4.3544	0.7908	0.0823
	[0.103]	[0.051]	[0.095]	[0.068]	[0.188]
$\log \mathcal{W}eib$	0.9030	1.5771	4.1343	0.7560	0.0915
	[0.074]	[0.050]	[0.115]	[0.115]	[0.217]
$\mathcal{GPD}$	1.4022	2.3920	3.6431	2.7839	0.3669
	[<0.005]	[<0.005]	[0.167]	[<0.005]	[<0.005]
$\mathcal{B}urr$	2.2333	3.1970	4.7780	7.0968	1.2830
	[0.115]	[0.115]	[0.174]	[0.115]	[0.115]
$\log \mathcal{S}_\alpha$	9.5186	9.5619	36.2617	304.61	44.5156
	[0.319]	[0.324]	[0.250]	[0.312]	[0.315]
$S\alpha S$	1.1628	2.1537	$5.8{\cdot}10^5$	11.9320	0.2535
	[0.352]	[0.026]	[0.651]	[0.971]	[0.027]

Table A5: Results of in-sample GOF tests for *"Process"* type operational losses. p-values (in square brackets) were obtained via 1,000 Monte Carlo simulations.

	KS	V	AD	AD^2	W^2
$\mathcal{E}xp$	7.6043	8.4160	$3.7{\cdot}10^6$	167.60	22.5762
	[<0.005]	[<0.005]	[<0.005]	[<0.005]	[<0.005]
$\mathcal{LN}$	0.6584	1.1262	2.0668	0.4624	0.0603
	[0.297]	[0.345]	[0.508]	[0.223]	[0.294]
$\mathcal{W}eib$	0.6110	1.0620	1.7210	0.2069	0.0338
	[0.455]	[0.532]	[0.766]	[0.875]	[0.755]
$\log\mathcal{W}eib$	0.5398	0.9966	1.6238	0.1721	0.0241
	[0.656]	[0.637]	[0.832]	[0.945]	[0.918]
$\mathcal{GPD}$	1.0042	1.9189	4.0380	2.6022	0.3329
	[0.005]	[<0.005]	[0.128]	[<0.005]	[<0.005]
$\mathcal{B}urr$	0.5634	0.9314	1.6075	0.2639	0.0323
	[0.598]	[0.800]	[0.841]	[0.794]	[0.840]
$\log\mathcal{S}_\alpha$	0.6931	1.1490	2.0109	0.4759	0.0660
	[0.244]	[0.342]	[0.534]	[0.202]	[0.258]
$S\alpha S$	1.3949	1.9537	$3.3{\cdot}10^5$	6.5235	0.3748
	[0.085]	[0.067]	[0.931]	[0.964]	[0.102]

Table A6: Results of in-sample GOF tests for *"Technology"* type operational losses. p-values (in square brackets) were obtained via 1,000 Monte Carlo simulations.

	KS	V	AD	AD^2	W^2
$\mathcal{E}xp$	3.2160	3.7431	27.6434	27.8369	2.9487
	[<0.005]	[<0.005]	[<0.005]	[<0.005]	[<0.005]
$\mathcal{LN}$	1.1453	1.7896	2.8456	1.3778	0.2087
	[<0.005]	[0.005]	[0.209]	[<0.005]	[<0.005]
$\mathcal{W}eib$	1.0922	1.9004	2.6821	1.4536	0.2281
	[<0.005]	[<0.005]	[0.216]	[<0.005]	[<0.005]
$\log\mathcal{W}eib$	1.1099	1.9244	2.7553	1.5355	0.2379
	[<0.005]	[<0.005]	[0.250]	[<0.005]	[<0.005]
$\mathcal{GPD}$	1.2202	1.8390	3.0843	1.6182	0.2408
	[<0.005]	[<0.005]	[0.177]	[<0.005]	[<0.005]
$\mathcal{B}urr$	1.1188	0.9374	2.6949	2.0320	0.3424
	[0.389]	[0.380]	[0.521]	[0.380]	[0.380]
$\log\mathcal{S}_\alpha$	1.1540	1.7793	2.8728	1.3646	0.2071
	[<0.005]	[0.007]	[0.208]	[<0.005]	[<0.005]
$S\alpha S$	2.0672	2.8003	$2.7{\cdot}10^5$	19.6225	1.4411
	[>0.995]	[>0.995]	[>0.995]	[>0.995]	[0.964]

Table A7: Estimates of expected aggregated loss, VaR, and CVaR (figures must be further scaled $\times 10^{10}$) for *"Relationship"* type losses. Figures are based on 50,000 Monte Carlo samples.

		EL	$\text{VaR}_{0.95}$	$\text{VaR}_{0.99}$	$\text{CVaR}_{0.95}$	$\text{CVaR}_{0.99}$
$\mathcal{E}xp$	'Naive'	0.1348	0.2231	0.2704	0.2515	0.2959
	Conditional	0.1422	0.2322	0.2763	0.2598	0.3016
$\mathcal{LN}$	'Naive'	0.1105	0.2832	0.5386	0.4662	0.8685
	Conditional	0.1634	0.4662	1.0644	0.9016	1.9091
$\mathcal{W}eib$	'Naive'	0.1065	0.2203	0.2996	0.2700	0.3505
	Conditional	0.1284	0.3187	0.5121	0.4430	0.6689
$log\mathcal{W}eib$	'Naive'	-	0.2235	0.3193	0.2845	0.3873
	Conditional	-	0.3332	0.5902	0.5049	0.8386
$\mathcal{GPD}$	'Naive'	-	0.8240	4.1537	9.6367	41.5129
	Conditional	-	1.5756	11.3028	52.8928	249.17
$\mathcal{B}urr$	'Naive'	-	2.8595	31.5637	1234.95	6139.69
	Conditional	-	1.5713	11.5519	25.9142	114.20
$\log \mathcal{S}_\alpha$	'Naive'	-	1.9124	7488.08	inf	inf
	Conditional	-	0.4359	0.9557	0.8277	1.7443
$\mathcal{S}_\alpha\mathcal{S}$	'Naive'	-	2.1873	17.3578	329.99	1627.38
	Conditional	-	4.5476	56.2927	376.09	1822.93

Table A8: Estimates of expected aggregated loss, VaR, and CVaR (figures must be further scaled $\times 10^{10}$) for *"Human"* type losses. Figures are based on 50,000 Monte Carlo samples.

		EL	$\text{VaR}_{0.95}$	$\text{VaR}_{0.99}$	$\text{CVaR}_{0.95}$	$\text{CVaR}_{0.99}$
$\mathcal{E}xp$	'Naive'	0.0305	0.4657	0.5452	0.5145	0.5848
	Conditional	0.0316	0.4818	0.5618	0.5312	0.6041
$\mathcal{LN}$	'Naive'	0.1981	0.4970	0.9843	0.8534	1.6652
	Conditional	0.4171	1.2161	3.4190	3.3869	9.4520
$\mathcal{W}eib$	'Naive'	0.1993	0.4017	0.5507	0.4945	0.6456
	Conditional	0.2881	0.7997	1.5772	1.3232	2.3746
$log\mathcal{W}eib$	'Naive'	-	0.4174	0.6184	0.5460	0.7732
	Conditional	-	0.8672	1.8603	1.5569	3.0576
$\mathcal{GPD}$	'Naive'	-	3.9831	33.5741	3945.75	19685.73
	Conditional	-	12.1150	168.64	67596.68	$3.4 \cdot 10^5$
$\mathcal{B}urr$	'Naive'	-	85.5620	2690.44	$2.1 \cdot 10^6$	$1.1 \cdot 10^7$
	Conditional	-	94.8281	3042.32	$7.7 \cdot 10^6$	$3.8 \cdot 10^7$
$\log \mathcal{S}_\alpha$	'Naive'	-	$1.9 \cdot 10^7$	$7.2 \cdot 10^{24}$	inf	inf
	Conditional	-	2.2737	4.2319	3.6742	6.7179
$\mathcal{S}_\alpha\mathcal{S}$	'Naive'	-	6.2811	77.4762	554.19	2691.79
	Conditional	-	14.5771	203.24	3922.83	19403.45

Table A9: Estimates of expected aggregated loss, VaR, and CVaR (figures must be further scaled $\times 10^{10}$) for *"Process"* type losses. Figures are based on 50,000 Monte Carlo samples.

		EL	$\text{VaR}_{0.95}$	$\text{VaR}_{0.99}$	$\text{CVaR}_{0.95}$	$\text{CVaR}_{0.99}$
$\mathcal{E}xp$	'Naive'	0.5140	0.8175	0.9664	0.9109	1.0504
	Conditional	0.5407	0.8522	1.0058	0.9483	1.0904
$\mathcal{LN}$	'Naive'	0.5622	1.5508	3.5665	3.1201	6.9823
	Conditional	0.8457	2.5610	6.5625	5.7823	13.9079
$\mathcal{W}eib$	'Naive'	0.4170	0.8800	1.2102	1.0891	1.4311
	Conditional	0.5131	1.2761	2.1308	1.8257	2.8578
$log\mathcal{W}eib$	'Naive'	-	0.9611	1.4498	1.2794	1.8514
	Conditional	-	1.4780	2.6511	2.2575	2.8255
$\mathcal{GPD}$	'Naive'	-	12.5930	131.25	1121.25	5467.11
	Conditional	-	20.8700	262.52	4384.77	21648.21
$\mathcal{B}urr$	'Naive'	-	6.8569	52.0391	206.06	962.88
	Conditional	-	1.7987	4.1859	3.9723	9.7191
$\log \mathcal{S}_\alpha$	'Naive'	-	1.5613	3.5159	2.9589	6.0887
	Conditional	-	2.5394	6.7070	5.9289	14.3725
$\mathcal{S}_\alpha\mathcal{S}$	'Naive'	-	38.7627	529.99	$1.3{\cdot}10^5$	$6.4{\cdot}10^5$
	Conditional	-	74.9073	1280.02	$1.8{\cdot}10^6$	$8.9{\cdot}10^6$

Table A10: Estimates of expected aggregated loss, VaR, and CVaR (figures must be further scaled $\times 10^{10}$) for *"Technology"* type losses. Figures are based on 50,000 Monte Carlo samples.

		EL	$\text{VaR}_{0.95}$	$\text{VaR}_{0.99}$	$\text{CVaR}_{0.95}$	$\text{CVaR}_{0.99}$
$\mathcal{E}xp$	'Naive'	0.0232	0.0598	0.0828	0.0741	0.0964
	Conditional	0.0306	0.0712	0.0963	0.0867	0.1102
$\mathcal{LN}$	'Naive'	0.0324	0.1202	0.3593	0.2970	0.7303
	Conditional	0.0958	0.2898	1.2741	1.5439	5.4865
$\mathcal{W}eib$	'Naive'	0.0226	0.0798	0.1368	0.1159	0.1795
	Conditional	0.0358	0.1454	0.3625	0.2958	0.6180
$log\mathcal{W}eib$	'Naive'	-	0.0861	0.1683	0.1399	0.2408
	Conditional	-	0.1670	0.4747	0.3885	0.8817
$\mathcal{GPD}$	'Naive'	-	0.4415	5.6954	56.3367	276.03
	Conditional	-	1.6249	54.4650	92471.16	$4.6{\cdot}10^5$
$\mathcal{B}urr$	'Naive'	-	2.8840	158.94	$5.3{\cdot}10^6$	$2.6{\cdot}10^7$
	Conditional	-	9.0358	855.78	$8.3{\cdot}10^7$	$4.2{\cdot}10^8$
$\log \mathcal{S}_\alpha$	'Naive'	-	0.1222	0.3560	0.3024	0.7435
	Conditional	-	0.2990	1.2312	1.6447	5.9933
$\mathcal{S}_\alpha\mathcal{S}$	'Naive'	-	$4.9{\cdot}10^5$	$3.2{\cdot}10^9$	$9.4{\cdot}10^{23}$	$4.7{\cdot}10^{24}$
	Conditional	-	$7.1{\cdot}10^6$	$6.9{\cdot}10^{10}$	$1.5{\cdot}10^{26}$	$7.6{\cdot}10^{26}$

Table A11: Average estimates of forecast errors for *"Relationship"* type aggregated losses. *Left panel:* errors between relative upper quantiles; *middle panel:* errors of forecasted upper quantiles relative to realized loss; *right panel:* overall error between forecasted and realized loss. Figures are based on 10,000 Monte Carlo samples for every year in forecasting period.

		Forecasted upper quantiles vs. upper bootstrapped quantiles		Forecasted upper quantiles vs. actual loss		Overall error: forecasted vs. actual loss	
	quantile	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)
$\mathcal{LN}$	95	0.1260	0.3356	0.1974	0.4259		
	99	0.9316	0.8823	1.4409	1.1805	0.1225	0.1780
	99.9	10.5910	3.0870	13.1306	3.5322		
$\mathcal{W}eib$	95	0.0837	0.2262	0.0636	0.2260		
	99	0.1731	0.3680	0.2335	0.4681	0.0335	0.1348
	99.9	0.5521	0.7183	0.8549	0.9056		
$log\mathcal{W}eib$	95	0.0846	0.2368	0.0760	0.2508		
	99	0.2031	0.4193	0.3072	0.5387	0.0382	0.1401
	99.9	0.7933	0.8496	1.3684	1.1500		
$\mathcal{GPD}$	95	8.6863	2.8066	9.9388	3.0156		
	99	516.75	22.0579	531.19	22.3719	$2.6{\cdot}10^4$	3.4405
	99.9	$1.1{\cdot}10^5$	320.35	$1.1{\cdot}10^5$	320.79		
$\mathcal{B}urr$	95	31.6051	5.1915	34.0472	5.4006		
	99	2892.27	48.9572	2926.30	49.2709	$16.1{\cdot}10^5$	16.7084
	99.9	$31.1{\cdot}10^5$	1483.69	$31.2{\cdot}10^5$	1484.14		
$\log \mathcal{S}_\alpha$	95	0.1246	0.3250	0.1886	0.4088		
	99	0.7544	0.7781	1.2114	1.0588	0.1378	0.1744
	99.9	9.8656	2.9407	12.3724	3.3795		
$\mathcal{S}_\alpha\mathcal{S}$	95	117.28	10.1415	121.96	10.3504		
	99	$1.35{\cdot}10^4$	109.31	$1.36{\cdot}10^4$	109.62	$6.0{\cdot}10^6$	25.4668
	99.9	$5.0{\cdot}10^6$	2125.12	$5.0{\cdot}10^6$	2125.56		

Table A12: Average estimates of forecast errors for *"Human"* type aggregated losses. *Left panel:* errors between relative upper quantiles; *middle panel:* errors of forecasted upper quantiles relative to realized loss; *right panel:* overall error between forecasted and realized loss. Figures are based on 10,000 Monte Carlo samples for every year in forecasting period.

		Forecasted upper quantiles vs. upper bootstrapped quantiles		Forecasted upper quantiles vs. actual loss		Overall error: forecasted vs. actual loss	
	quantile	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)
$\mathcal{LN}$	95	0.0061	0.8881	1.9712	1.3778		
	99	16.0585	3.9059	22.1948	4.6600	13.7183	0.5331
	99.9	478.23	21.2076	522.73	22.2719		
$\mathcal{W}eib$	95	0.2386	0.4025	0.5234	0.6901		
	99	1.1650	0.9607	2.8835	1.6854	0.2607	0.3286
	99.9	11.6062	3.2353	18.8769	4.2933		
$log\mathcal{W}eib$	95	0.3137	0.4948	0.7277	0.8196		
	99	2.3749	1.4061	4.7648	2.1604	0.4318	0.3577
	99.9	29.3964	5.1937	40.6156	6.2559		
$\mathcal{GPD}$	95	694.06	25.8942	721.22	26.3880		
	99	$1.5{\cdot}10^5$	373.03	$1.5{\cdot}10^5$	373.78	$14.3{\cdot}10^{10}$	1796.50
	99.9	$7.5{\cdot}10^8$	24126.68	$7.5{\cdot}10^8$	24127.74		
$\mathcal{B}urr$	95	97987.89	285.09	98258.80	285.58		
	99	$8.5{\cdot}10^7$	8688.99	$8.5{\cdot}10^7$	8689.75	$0.6{\cdot}10^{15}$	$1.6{\cdot}10^5$
	99.9	$4.9{\cdot}10^{12}$	$1.8{\cdot}10^6$	$4.9{\cdot}10^{12}$	$1.8{\cdot}10^6$		
$\log \mathcal{S}_\alpha$	95	0.8942	0.7879	1.7607	1.2803		
	99	11.9707	3.2636	17.2067	4.0170	2.6761	0.4797
	99.9	289.35	15.3948	322.95	16.4575		
$\mathcal{S}_\alpha\mathcal{S}$	95	671.54	25.3614	697.49	25.8550		
	99	$1.4{\cdot}10^5$	365.68	$1.4{\cdot}10^5$	366.43	$6.3{\cdot}10^9$	473.22
	99.9	$7.7{\cdot}10^8$	$2.2{\cdot}10^4$	$7.7{\cdot}10^8$	$2.2{\cdot}10^4$		

Table A13: Average estimates of forecast errors for *"Process"* type aggregated losses. *Left panel:* errors between relative upper quantiles; *middle panel:* errors of forecasted upper quantiles relative to realized loss; *right panel:* overall error between forecasted and realized loss. Figures are based on 10,000 Monte Carlo samples for every year in forecasting period.

		Forecasted upper quantiles vs. upper bootstrapped quantiles		Forecasted upper quantiles vs. actual loss		Overall error: forecasted vs. actual loss	
	quantile	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)
$\mathcal{LN}$	95	2.5846	1.4030	2.1980	1.3718		
	99	19.7641	4.2121	26.0670	4.7600	3.6324	0.8224
	99.9	290.95	16.0800	339.94	17.7763		
$\mathcal{W}eib$	95	2.2836	0.9438	0.5316	0.6703		
	99	3.2796	1.2998	1.6572	1.1983	0.6991	0.6419
	99.9	9.0910	2.6978	11.2902	3.0559		
$log\mathcal{W}eib$	95	2.2026	0.9875	0.6936	0.7703		
	99	4.0327	1.6497	3.4160	1.7046	0.7913	0.6661
	99.9	15.7046	3.7568	21.4673	4.3466		
$\mathcal{GPD}$	95	384.25	17.4088	407.12	18.2343		
	99	97300.89	275.10	97809.74	276.29	$1.7{\cdot}10^{13}$	15951.40
	99.9	$6.2{\cdot}10^{8}$	17619.28	$6.2{\cdot}10^{8}$	17620.00		
$\mathcal{B}urr$	95	5.3751	2.0950	6.7041	2.2717		
	99	248.36	13.3752	277.35	14.6014	$6.7{\cdot}10^{5}$	5.1213
	99.9	33069.50	160.90	33685.29	162.63		
$\log\mathcal{S}_\alpha$	95	2.3570	1.2818	1.6089	1.1205		
	99	15.7130	3.3234	20.0033	3.8551	4.6436	0.7726
	99.9	280.03	13.5974	319.64	15.2891		
$\mathcal{S}_\alpha\mathcal{S}$	95	2176.66	44.5025	2238.36	45.3302		
	99	$7.8{\cdot}10^{5}$	839.45	$7.9{\cdot}10^{5}$	840.66	$9.9{\cdot}10^{10}$	3033.42
	99.9	$0.6{\cdot}10^{10}$	62936.26	$0.6{\cdot}10^{10}$	62937.95		

Table A14: Average estimates of forecast errors for *"Technology"* type aggregated losses. *Left panel:* errors between relative upper quantiles; *middle panel:* errors of forecasted upper quantiles relative to realized loss; *right panel:* overall error between forecasted and realized loss. Figures are based on 10,000 Monte Carlo samples for every year in forecasting period.

		Forecasted upper quantiles vs. upper bootstrapped quantiles		Forecasted upper quantiles vs. actual loss		Overall error: forecasted vs. actual loss	
	quantile	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)	MSE($\times 10^{20}$)	MAE($\times 10^{10}$)
$\mathcal{LN}$	95	0.1879	0.4135	0.2029	0.4316		
	99	3.6536	1.8314	3.7435	1.8571	1.0549	0.1402
	99.9	136.90	11.1882	137.62	11.2242		
$\mathcal{W}eib$	95	0.0294	0.1624	0.0345	0.1806		
	99	0.1819	0.4168	0.2022	0.4426	0.0140	0.0535
	99.9	1.1294	1.0467	1.1985	1.0824		
$log\mathcal{W}eib$	95	0.0439	0.1979	0.0501	0.2162		
	99	0.3556	0.5804	0.3838	0.6063	0.0272	0.0620
	99.9	3.1991	1.7705	3.3291	1.8065		
$\mathcal{GPD}$	95	45.7139	5.4619	45.9212	5.4803		
	99	77821.03	203.46	77829.78	203.49	$1.8{\cdot}10^{13}$	$2.0{\cdot}10^{4}$
	99.9	5.1	40154.50	$5.1{\cdot}10^{9}$	40154.53		
$\mathcal{B}urr$	95	1853.93	34.1046	1854.92	34.1228		
	99	$4.2{\cdot}10^{7}$	4135.69	$4.2{\cdot}10^{7}$	4135.71	$1.5{\cdot}10^{20}$	$5.4{\cdot}10^{7}$
	99.9	$1.6{\cdot}10^{13}$	$2.3{\cdot}10^{6}$	$1.6{\cdot}10^{13}$	$2.3{\cdot}10^{6}$		
$\log \mathcal{S}_\alpha$	95	0.1901	0.4161	0.2053	0.4344		
	99	3.3955	1.7774	3.4864	1.8031	11.9464	0.1610
	99.9	147.59	11.8642	148.36	11.8998		
$\mathcal{S}_\alpha\mathcal{S}$	95	$8.2{\cdot}10^{20}$	$1.1{\cdot}10^{10}$	$8.2{\cdot}10^{20}$	$1.1{\cdot}10^{10}$		
	99	$9.4{\cdot}10^{29}$	$3.7{\cdot}10^{14}$	$9.4{\cdot}10^{29}$	$3.7{\cdot}10^{14}$	$6.0{\cdot}10^{61}$	$2.9{\cdot}10^{28}$
	99.9	$1.1{\cdot}10^{43}$	$1.2{\cdot}10^{21}$	$1.1{\cdot}10^{43}$	$1.2{\cdot}10^{21}$		

Table A15: LR statistic and p-values (in square brackets) for *"Relationship"* type aggregated losses in the 7-year forecast period.

	LR statistic and p-value							average p-value
Year	1	2	3	4	5	6	7	
$\mathcal{LN}$	0.3579	2.3588	1.9911	1.4466	2.0630	0.3518	2.5297	
	[0.836]	[0.308]	[0.370]	[0.485]	[0.357]	[0.839]	[0.282]	[0.497]
$\mathcal{W}eib$	0.0619	1.5978	0.8392	1.2538	2.0261	0.1808	2.7085	
	[0.970]	[0.450]	[0.657]	[0.534]	[0.363]	[0.914]	[0.258]	[0.592]
$log\mathcal{W}eib$	0.1383	1.8481	1.2071	1.3434	2.1209	0.2174	2.5593	
	[0.933]	[0.397]	[0.547]	[0.511]	[0.346]	[0.897]	[0.278]	[0.559]
$\mathcal{GPD}$	0.6618	3.0483	3.2423	1.6954	1.8640	0.7013	2.4631	
	[0.718]	[0.218]	[0.198]	[0.428]	[0.394]	[0.704]	[0.292]	[0.422]
$\mathcal{B}urr$	0.6511	2.8692	3.3531	1.6386	1.7583	0.7717	2.5161	
	[0.722]	[0.238]	[0.187]	[0.441]	[0.415]	[0.680]	[0.284]	[0.424]
$\log \mathcal{S}_\alpha$	0.3433	1.6031	2.0380	5.6319	2.1734	0.3646	9.4886	
	[0.842]	[0.449]	[0.361]	[0.060]	[0.337]	[0.833]	[0.009]	[0.413]
$\mathcal{S}_\alpha\mathcal{S}$	0.2621	24.6599	2.5327	1.1871	1.3401	0.3538	2.2774	
	[0.877]	[$<$0.005]	[0.282]	[0.552]	[0.512]	[0.838]	[0.320]	[0.483]

Table A16: LR statistic and p-values (in square brackets) for *"Human"* type aggregated losses in the 7-year forecast period.

	LR statistic and p-value							average p-value
Year	1	2	3	4	5	6	7	
$\mathcal{LN}$	0.5022	4.6756	0.1023	2.5790	0.9439	4.7796	0.7730	
	[0.778]	[0.097]	[0.950]	[0.275]	[0.624]	[0.092]	[0.679]	[0.499]
$\mathcal{W}eib$	0.2541	3.7590	0.1958	2.5951	0.6551	4.7877	0.7553	
	[0.881]	[0.153]	[0.907]	[0.273]	[0.721]	[0.091]	[0.686]	[0.530]
$log\mathcal{W}eib$	0.3783	4.4179	0.0884	2.4636	0.8178	4.4334	0.7605	
	[0.828]	[0.110]	[0.957]	[0.292]	[0.664]	[0.109]	[0.684]	[0.520]
$\mathcal{GPD}$	0.8031	5.4367	0.1657	2.5592	1.3462	5.3795	0.8338	
	[0.669]	[0.066]	[0.921]	[0.278]	[0.510]	[0.068]	[0.659]	[0.453]
$\mathcal{B}urr$	0.6539	5.4566	0.2840	2.5620	0.8786	7.4635	0.8278	
	[0.721]	[0.065]	[0.868]	[0.278]	[0.645]	[0.024]	[0.661]	[0.466]
$\log \mathcal{S}_\alpha$	0.2067	4.5401	0.1326	2.6464	11.1638	50.2250	0.8028	
	[0.902]	[0.103]	[0.936]	[0.266]	[0.004]	[$<$0.005]	[0.670]	[0.412]
$\mathcal{S}_\alpha\mathcal{S}$	19.5913	57.8253	16.5836	22.7072	20.0307	∞	0.8759	
	[$<$0.005]	[$<$0.005]	[$<$0.005]	[$<$0.005]	[$<$0.005]	[$<$0.005]	[0.645]	[0.092]

Table A17: LR statistic and p-values (in square brackets) for *"Process"* type aggregated losses in the 7-year forecast period.

	LR statistic and p-value							average p-value
Year	1	2	3	4	5	6	7	
$\mathcal{LN}$	4.7303	7.9488	20.5106	$\approx$0	5.9690	1.2397	7.6575	
	[0.094]	[0.019]	[<0.005]	[>0.995]	[0.051]	[0.538]	[0.022]	[0.246]
$\mathcal{W}eib$	5.8358	9.0067	19.8166	$\approx$0	5.3046	1.3434	7.1111	
	[0.054]	[0.012]	[<0.005]	[>0.995]	[0.071]	[0.511]	[0.029]	[0.239]
$log\mathcal{W}eib$	5.4614	8.6240	20.0130	$\approx$0	5.6225	1.2509	7.1481	
	[0.065]	[0.013]	[<0.005]	[>0.995]	[0.060]	[0.535]	[0.028]	[0.243]
$\mathcal{GPD}$	3.3761	7.3430	21.4076	0.0575	6.7186	1.2567	8.0513	
	[0.185]	[0.025]	[<0.005]	[0.972]	[0.035]	[0.534]	[0.018]	[0.253]
$\mathcal{B}urr$	4.4869	8.0195	21.2706	0.0192	6.4037	1.3078	7.1694	
	[0.106]	[0.018]	[<0.005]	[0.990]	[0.041]	[0.520]	[0.028]	[0.243]
$\log \mathcal{S}_\alpha$	4.7196	8.0265	36.6496	2.5550	14.3856	1.2909	7.7456	
	[0.094]	[0.018]	[<0.005]	[0.279]	[<0.005]	[0.524]	[0.021]	[0.134]
$\mathcal{S}_\alpha\mathcal{S}$	4.1486	inf	inf	0.1850	40.7501	1.6664	23.4444	
	[0.125]	[<0.005]	[<0.005]	[0.912]	[<0.005]	[0.435]	[<0.005]	[0.210]

Table A18: LR statistic and p-values (in square brackets) for *"Technology"* type aggregated losses in the 7-year forecast period.

	LR statistic and p-value							average p-value
Year	1	2	3	4	5	6	7	
$\mathcal{LN}$	1.7031	0.7748	7.9165	1.8076	-	3.9816	-	
	[0.427]	[0.679]	[0.019]	[0.405]	[-]	[0.137]	[-]	[0.333]
$\mathcal{W}eib$	1.4969	0.9152	8.5419	1.9915	-	4.1399	-	
	[0.473]	[0.633]	[0.014]	[0.370]	[-]	[0.126]	[-]	[0.323]
$log\mathcal{W}eib$	1.4175	0.9414	8.3460	2.3232	-	4.0876	-	
	[0.492]	[0.625]	[0.015]	[0.313]	[-]	[0.130]	[-]	[0.315]
$\mathcal{GPD}$	2.1113	0.7520	6.8921	1.8079	-	3.6926	-	
	[0.348]	[0.687]	[0.032]	[0.405]	[-]	[0.158]	[-]	[0.326]
$\mathcal{B}urr$	3.1543	0.4183	8.2779	1.3857	-	4.3545	-	
	[0.207]	[0.811]	[0.016]	[0.500]	[-]	[0.113]	[-]	[0.330]
$\log \mathcal{S}_\alpha$	1.7229	0.7572	7.8918	1.7649	-	3.9782	-	
	[0.423]	[0.685]	[0.019]	[0.414]	[-]	[0.137]	[-]	[0.335]
$\mathcal{S}_\alpha\mathcal{S}$	4.3899	5.5676	14.5400	5.6148	-	6.4994	-	
	[0.111]	[0.062]	[<0.005]	[0.061]	[-]	[0.039]	[-]	[0.055]

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