Packham, Natalie; Schlögl, Lutz; Schmidt, Wolfgang M.

Working Paper

Credit gap risk in a first passage time model with jumps

CPQF Working Paper Series, No. 22

Provided in cooperation with:
Frankfurt School of Finance and Management

Suggested citation: Packham, Natalie; Schlögl, Lutz; Schmidt, Wolfgang M. (2009) : Credit gap risk in a first passage time model with jumps, CPQF Working Paper Series, No. 22, http://hdl.handle.net/10419/40179

Frankfurt School of Finance & Management
Bankakademie | HfB

Centre for Practical Quantitative Finance

CPQF Working Paper Series
No. 22

Credit gap risk in a first passage time model with jumps

Natalie Packham, Lutz Schlögl and Wolfgang M. Schmidt

November 2009

Authors:

Natalie Packham
Assistant Professor for Quantitative Finance
Frankfurt School of Finance & Management
Frankfurt/Main
n.packham@frankfurt-school.de

Lutz Schlögl
Fixed Income Quantitative Research
Nomura International Plc
London, UK
lutz.schloegl@nomura.com

Wolfgang M. Schmidt
Professor of Quantitative Finance
Frankfurt School of Finance & Management
Frankfurt/Main
w.schmidt@frankfurt-school.de

Publisher:

Frankfurt School of Finance & Management
Phone: +49 (0) 69 154 008-0 ▪ Fax: +49 (0) 69 154 008-728
Sonnemannstr. 9-11 ▪ D-60314 Frankfurt/M. ▪ Germany

Credit gap risk in a first passage time model with jumps

Natalie Packham,* Lutz Schlögl† and Wolfgang M. Schmidt‡

November 2009

Abstract

The payoff of many credit derivatives depends on the level of credit spreads. In particular, credit derivatives with a leverage component are subject to gap risk, a risk associated with the occurrence of jumps in the underlying credit default swaps. In the framework of first passage time models, we consider a model that addresses these issues. The principal idea is to model a credit quality process as an Itô integral with respect to a Brownian motion with a stochastic volatility. Using a representation of the credit quality process as a time-changed Brownian motion, one can derive formulas for conditional default probabilities and credit spreads. An example for a volatility process is the square root of a Lévy-driven Ornstein-Uhlenbeck process. The model can be implemented efficiently using a technique called Panjer recursion. Calibration to a wide range of dynamics is supported. We illustrate the effectiveness of the model by valuing a leveraged credit-linked note.

Keywords: gap risk, credit spreads, credit dynamics, first passage time models, stochastic volatility, general Ornstein-Uhlenbeck processes

JEL classification: G12, G13, G24, C69

1 Introduction

Other than being subject exclusively to default risk, the payoff of some credit derivatives is determined explicitly by the level of CDS spreads, the spreads of credit default swaps. In this case, the dynamics of CDS spreads play a significant role in product valuation. Examples of such products are default swaptions and credit derivatives with a leverage component. The latter are in addition sensitive to *gap risk*, a risk that is linked to the occurrence of jumps in the evolution of credit spreads, even if such jumps do not lead to default. An example of such a product, the *leveraged credit-linked note*, is described in detail in Section 2.4.

There is a significant amount of empirical research that indicates that credit spreads are subject to sudden and unexpected jumps [Johannes, 2000], [Zhou, 2001], [Das, 2002], [Dai and Singleton, 2003], [Tauchen and Zhou, 2006] and [Zhang et al., 2008]. In a recent empirical study on credit spreads, [Schneider et al., 2007] observe that CDS spreads exhibit frequent positive jumps, which typically affect CDS spreads of all maturities. These jumps are attributed to the arrival of bad news. Good news also affect the whole maturity spectrum, but tend to propagate gradually.

The model considered here, a first passage time model with jumps, captures these stylised facts and is suitable for valuing credit derivatives that are subject to gap risk.

*Natalie Packham, Centre for Practical Quantitative Finance, Frankfurt School of Finance & Management, Sonnemannstr. 9-11, 60314 Frankfurt am Main. n.packham@frankfurt-school.de

†Lutz Schlögl, Fixed Income Quantitative Research, Nomura International Plc, lutz.schloegl@nomura.com

‡Wolfgang M. Schmidt, Centre for Practical Quantitative Finance, Frankfurt School of Finance & Management, Sonnemannstr. 9-11, 60314 Frankfurt am Main. w.schmidt@frankfurt-school.de

There are generally two approaches to modelling credit risk: the structural and the reduced-form approach. In reduced-form models, default is not linked to economic variables, but is an unpredictable Poisson-type event, and the main object of the modeller's attention is the hazard rate of the jump process describing default. This approach has been overwhelmingly popular with practitioners, its main advantage is its tractability: It is generally straightforward to fit a given term structure of CDS spreads and the techniques are very similar to those of interest rate modelling. The literature on this type of models is vast, the papers by [Jarrow and Turnbull, 1995], [Lando, 1998] and [Duffie and Singleton, 1999] are only a few classic examples.

From the point of view of spread dynamics, modelling the default time as a totally unpredictable stopping time is not entirely satisfactory. Even with a low initial hazard rate, such a model will assign a non-negligible probability to the possibility of the credit defaulting without a prior movement in the credit spread. Defaults of this type are very uncommon in practice. The default swap market is efficient enough that default events are almost always preceded by a significant widening of credit spreads. It is this spread widening that is the real jump event that a market participant needs to worry about.

The ability of a model to assign probability mass to spread widening scenarios is constrained by the probability assigned to defaults in low spread scenarios, as the model must fit the initial credit spread. From a practical point of view, failure to assign enough probability to spread widening scenarios can lead to a dangerous underpricing of credit spread gap risk. The phenomenology of default that we are trying to capture is the following: a credit with a low default swap spread does not default "out of the blue", but rather some kind of regime change takes place, causing the credit spread to widen, after which the credit may either default or eventually recover.

We implement this idea via a first passage time model where a *credit quality process* exhibits stochastic volatility. In fact, the volatility process is a Levy-driven Ornstein-Uhlenbeck process. A jump in the volatility process is the "regime switch" we alluded to earlier. The current trend is to interpret the class of structural models in a wide sense to include any model where default is modelled as the first hitting time of a certain threshold by an abstract observable credit quality process. In this sense our model is structural, though the term "first passage time model" is technically more accurate. The structural approach, pioneered by [Merton, 1974], has been developed by [Black and Cox, 1976], [Longstaff and Schwartz, 1995], and many others.

As the name suggests, in a first passage time model, the computation of default probabilities is equivalent to computing the distributions of first passage times. The simplest case is that of a Brownian motion hitting a constant barrier, where a simple closed-form solution exists. This simple set-up however does not allow one to fit a given term structure of credit spreads. Furthermore, in reality, credit spreads exhibit strong jump dynamics. Several extensions where the credit quality process is modelled by a jump-diffusion or a Lévy process were brought forward to overcome these problems, e.g. [Zhou, 2001], [Kiesel and Scherer, 2007], [Baxter, 2007] and [Cariboni and Schoutens, 2007]. However, in all of these cases, computing first passage times is intractable or computationally very demanding.

[Overbeck and Schmidt, 2005] propose a simple solution to the problem of calibrating a first passage time model to a term structure of credit spreads.by considering the first hitting time of a time-changed Brownian motion to a constant barrier. The time change is continuous, strictly increasing and deterministic. Because both the time change and the underlying Brownian motion are continuous, one can easily adapt the analytic formula from the simple Brownian case and obtain an analytic calibration to a term structure of default probabilities.

Our model builds directly on [Overbeck and Schmidt, 2005]. We consider a credit quality process X to be a time-changed Brownian motion with a stochastic, continuous and strictly increasing time change independent of the Brownian motion. The credit quality process X can also be represented as an Itô integral $X = \int_0^{\cdot} \sigma_u dW_u$ with a Brownian motion W and a volatility process σ. Our standard example for the volatility process σ is the square root of an Ornstein-Uhlenbeck process driven by a compound Poisson process. In the time-changed Brownian motion interpretation, we can write X as $X_t = B_{\Lambda_t}$ with B a Brownian motion and $\Lambda_t = \int_0^t \sigma_u^2 du$. Because the time change Λ is continuous and independent of B, we retain all the tractability of the deterministic case, while jumps in the volatility σ induce jumps in the credit spreads, even though the credit quality process X is continuous.

The model and its properties are studied in detail in [Packham et al., 2009]. In the present paper, we focus on the implementation, calibration and on valuation of gap risk. Given the state of the credit quality process and its volatility, one can efficiently compute the whole term structure of default probabilities or credit spreads using a technique called Panjer recursion [Panjer, 1981]. Implementation then boils down to a combination of Monte Carlo simulation for determining the state of the credit quality process and numerical computation of term structures. Calibration to a given term structures is achieved by minimising the root mean square error of model and market term structures. We show that under suitable choices for the dynamics parameters the error of calibration to a term structure of CDS spreads is as small as 10^{-6} basis points, while at the same time the model may be calibrated to a wide range of dynamical behaviours. In particular, even though the CDS spread in the first passage time model vanishes as maturity of the underlying CDS is approached, thereby excluding short-term default events, one may approximate short-term default events by including the possibility of large jumps in the volatility.

The paper is structured as follows: in Section 2 we introduce some notation that is used throughout, and we treat in detail the valuation of CDS spreads and of leveraged credit-linked notes. In Section 3, we briefly introduce the Overbeck-Schmidt model and the extended first passage time model with jumps. We state formulas for conditional default probabilities, from which credit spreads may be determined, and we establish that jumps in the stochastic volatility of the credit quality process translate into jumps in CDS spreads. In Section 4, we derive the algorithm for efficiently computing term structures of default probabilities and credit spreads, given the state of the credit quality process and its volatility. Calibration to implied term structures is treated in Section 5. Since there are currently no liquid market instruments to calibrate the dynamics, we demonstrate the range of viable dynamics that may be obtained in the model; this is done in Section 6. We also discuss the individual parameters that govern the dynamics of the model. Finally, in Section 7, we apply the model to the valuation of leveraged credit-linked notes and default swaptions.

2 Credit derivatives

2.1 Notation

Throughout, let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbf{P})$ be a complete probability space endowed with a filtration $(\mathcal{F}_t)_{t\geq 0}$, representing the information available in the market. In particular, $\mathcal{F}_0$ is $\mathbf{P}$-trivial. We assume that $(\mathcal{F}_t)_{t\geq 0}$ satisfies the usual hypotheses, i.e., $\mathcal{F}_0$ contains all $\mathbf{P}$-null sets of $\mathcal{F}$ and $(\mathcal{F}_t)_{t\geq 0}$ is right-continuous. We also assume that the probability space is rich enough to support any objects that we define. If not otherwise stated, all processes are $(\mathcal{F}_t)_{t\geq 0}$-adapted.

We assume that $\mathbf{P}$ is a risk-neutral measure (presupposing existence of such a measure), that is, $\mathbf{P}$ is a probability measure equivalent to the real-world probability measure and such that discounted prices are $\mathbf{P}$-martingales. It follows that the market is free of arbitrage.

2.2 Credit default swaps

The fundamental product of the credit derivatives market is the *credit default swap (CDS).* Given an underlying entity, such as a company, it is a contract between two counterparties, the protection buyer and the protection seller, that insures the protection buyer against the loss incurred by default of the underlying entity within a fixed time interval. The protection buyer regularly pays a constant premium, the *credit spread* or *CDS spread*, which is fixed at inception, up until maturity of the CDS or the default event, whichever occurs first. This stream of payments is termed the *premium leg* of the CDS. In return, the protection seller agrees to compensate the protection buyer for the loss incurred by default of the underlying entity at the time of default in case this occurs before maturity. This constitutes the *protection leg* of the CDS. The CDS spread that makes the value of the premium leg and the protection leg equal is the *fair CDS spread*.

More precisely, let $r \in \mathbb{R}_+$ denote the default-free interest rate, assumed to be constant for simplicity. Furthermore, assume that the payment at default is a fraction $(1 - R)$ of the notational amount, $R \in [0, 1)$. Denote by τ the random time of the default event. In our setup, τ is an $(\mathcal{F}_t)_{t\geq 0}$-stopping time. The distribution function of τ conditional on the information flow $(\mathcal{F}_t)_{t\geq 0}$ is denoted by $P(t, T) := \mathbf{P}(\tau \leq T|\mathcal{F}_t)$. Denote by $s(t, T)$ the fair credit spread at time t of a CDS with maturity T. Entering into a CDS involves no initial cash-flow, that is, the market value of a CDS at inception is 0; in other words, the discounted fair values of the premium and the default legs are equal. From these considerations, one can show that, on $\{\tau > t\}$, the *fair credit spread* or *fair CDS spread* $s(t, T)$ at time t is given by

$$\frac{s(t,T)}{1-R} = \frac{\int_t^T \mathbf{e}^{-r(u-t)}\,\mathrm{d}P(t,u)}{\int_t^T \mathbf{e}^{-r(u-t)}\,(1-P(t,u))\,\mathrm{d}u}. \tag{1}$$

On $\{\tau \leq t\}$ or for $t \geq T$, we set $s(t, T) = 0$. The mapping $T \mapsto s(t, T)$ is the *term structure of credit spreads* at time t. Given a term structure of CDS spreads $s(t, T)$, $T \geq t$, one can infer the risk-neutral default probabilities $\mathbf{P}(t, T)$, $T \geq t$, from Equation (1), and vice versa.

The *mark-to-market value* of an existing CDS position is expressed as the cost of unwinding the transaction by entering into an offsetting CDS position. Assume a CDS contract with maturity T entered at time $v \leq t$ from the point of view of the protection seller. On $\{\tau > t\}$, the value of the position at time t is

$$\begin{aligned} V_t &= s(v,T)\int_t^T \mathbf{e}^{-r(u-t)}\,(1-P(t,u))\,\mathrm{d}u - (1-R)\int_t^T \mathbf{e}^{-r(u-t)}\,P(t,\mathrm{d}u) \\ &= (s(v,T)-s(t,T))\int_t^T \mathbf{e}^{-r(u-t)}\,(1-P(t,u))\,\mathrm{d}u. \end{aligned} \tag{2}$$

If default occurs prior to T, that is, $\tau < T$, we set $V_\tau = -(1 - R)$.

2.3 Shape and dynamics of the term structure

The term structure of credit spreads has been extensively studied.[1] Let us outline some stylised facts about the shape and dynamics of the term structure. A wide variety of term structure shapes has been observed in the market, such as upward sloping, flat, hump-shaped and downward sloping curve, see for example [Helwege and Turner, 1999], [Zhou, 2001], [Fons, 1994] and [Helwege and Turner, 1999].

[1]The term credit spread also refers to the *yield spread*, which is the yield difference of defaultable and default-free zero-coupon bonds of the same maturity. There are some subtle differences between yield spreads and CDS spreads, mainly due to factors such as liquidity of the underlying and restrictions regarding short-selling. However, we assume that stylised facts of the yield spread term structure that can be related to the credit risk component of the underlying entity apply to the CDS term structure as well.

Another common observation is that short-term credit spreads do not tend to zero as maturity tends to zero, but are significantly greater than zero, see e.g. [Duffie and Lando, 2001], [Zhou, 2001], [Duffie and Singleton, 2003, Ch. 3] and [Lando, 2004, Ch. 2]. This indicates that, for any time to maturity, market participants presume a positive probability of unexpected and instantaneous default.

There is also a significant amount of research that indicates that credit spreads are subject to jumps, i.e., in addition to continuous behaviour of credit spreads through time, credit spreads may change by sudden and unexpected jumps, see [Johannes, 2000], [Zhou, 2001], [Das, 2002], [Dai and Singleton, 2003], [Tauchen and Zhou, 2006], [Zhang et al., 2008]. [Schneider et al., 2007] infer the following empirical stylised facts for CDS spreads:

- A jump affects broad ranges of the CDS maturity spectrum. This is economically motivated by the fact that unfavourable events usually affect contracts of both short and long maturities, and similarly, when expectations about the overall credit quality change, the entire term structure of CDS spreads reacts.
- Jumps in CDS spreads are mostly positive. The arrival of bad news such as financial distress causes sudden upward moves in CDS spreads, because protection sellers demand higher compensation for bearing higher risk. Good news, on the other hand, tend to propagate gradually.
- The one-year CDS spread exhibits time-series variation different from CDS spreads of higher maturities.

2.4 Leveraged credit-linked note

Let us now consider a credit derivative whose payoff is sensitive to the occurrence of jumps in credit spreads. A *leveraged credit-linked note* is particularly sensitive to jumps in CDS spreads, even if a jump does not lead to default.[2] The principal idea is that an investor sells protection on an amount of default risk that is a multiple k, the *leverage factor*, of his investment amount. The motivation for taking leveraged exposure is to earn a certain multiple $\tilde{k}$ of the credit spread. Most likely, his investment will not suffice to compensate the loss incurred by default. Therefore, a trigger is agreed to terminate the structure while the cost of closing the position is still likely to be sufficiently covered by the investment amount. The cost of closing the position depends on the level of credit spreads, hence the investor is exposed mainly to spread risk and to default risk only to a lesser extent.

In more detail, the issuer structures the note as follows (see Figure 1): For simplicity, assume an investment amount of 1, which is deposited in a default-free account earning the risk-free rate r. In addition, protection is sold by entering a fair CDS with notional k earning a spread of $k\, s(0,T)$. The investor receives a fixed coupon until either maturity of the note or until a trigger event takes place. The size of the coupon is $r + \tilde{k}\, s(0,T)$ with $\tilde{k}\, s(0,T)$, $\tilde{k} \leq k$, the premium associated with the note, which is generated by the CDS position. The trigger event is defined as follows: denote by V_t^k the mark-to-market value at time t of the underlying CDS position with notional k. The trigger event takes place at time $S = \inf\{t \in (0,T] : V_t^k \leq -K\}$, with $0 < K \leq 1$ a pre-defined trigger level. At S, the note is unwound by withdrawing the investment amount 1 from the deposit account and by closing the CDS position, i.e., by entering the offsetting position, at a cost of $-V_S^k$. Observe that possibly $V_S^k < -1$, in which case the issuer must cover the missing amount required to unwind the CDS position. For this type of risk, called *gap risk*, the issuer is compensated with a premium of $(k - \tilde{k})\, s(0,T)$. In the case where $V_S^k \geq -1$, the investor

[2] A *credit-linked note (CLN)* is a note or bond paying an enhanced coupon to an investor for bearing the credit risk of a reference entity; see [Bielecki and Rutkowski, 2002, Section 1.3.3] for a general description.

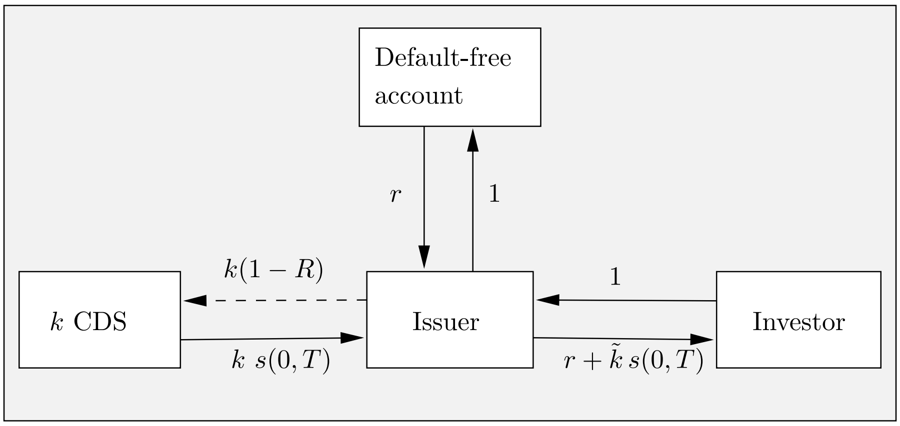

Figure 1: Leveraged credit-linked note with leverage factor k and notional 1. Cash flows at inception and while the note is alive.

receives the remainder of the structure, $1 + V_S^k$. Given K, valuation of the note essentially means determining the fair factor $\tilde{k}$.

Clearly, the trigger time S depends on the evolution of the underlying CDS spread, cf. Equation (2). Furthermore, the amount of the redemption payment $\max(1 + V_S^k, 0)$ is undetermined until S. Assuming a model in which CDS spreads evolve continuously, the mark-to-market value V^k evolves continuously as well. Unless a default takes place, the trigger time is $S = \inf\{t \in (0, T] : V_t^k = -K\}$ and $V_S^k \geq -1$. Hence, a gap event takes place only when default happens without a prior trigger event. On the contrary, assuming a model in which CDS spreads are subject to jumps, upward jumps in CDS spreads translate into downward jumps in the mark-to-market value of the CDS, and possibly $V_S^k < -1$, so the issuer faces gap risk even when no default takes place.

In some models, we can determine the fair factor $\tilde{k}$ by no-arbitrage arguments. Assume first that CDS spreads evolve continuously through time, so V^k evolves continuously as well. Moreover, assume that there is no risk of an unpredictable jump-to-default (i.e., there is no default "totally out of the blue"). In this case, the trigger time is $S = \inf\{t \in (0, T] : V_t^k = -K\}$ and there is no gap risk at all, and the fair factor is $\tilde{k} = k$. Now suppose that CDS spreads are constant, so the note is exposed to default risk only (in which case $V_t^k = 0$, since $s(t, T) = s(0, T)$, for all $t \in [0, T)$). The trigger time then coincides with the default time, in which case the investor loses his invested capital. The payoff of this position is equivalent to the payoff of a short position in $1/(1 - R)$ CDS, so $\tilde{k} = 1/(1 - R)$.

Regardless of any model assumption, we can infer upper and lower bounds for the factor $\tilde{k}$. The upper bound is k as the note's spread pickup is funded by the underlying CDS position. To determine the lower bound, observe that the investor in the leveraged note is exposed to default risk and additionally to spread risk. An investor in a CDS position with notional $1/(1 - R)$ is exposed to the same loss in case of default, but may terminate the investment at the same trigger time S with a smaller loss. Hence, $\tilde{k} > 1/(1 - R)$.

To determine the factor $\tilde{k}$, consider the cash flows to the note issuer discounted to time 0. Observe that the cash flows to the issuer isolate the gap risk component. The risk-neutral value of these cash flows is given by

$$
\begin{aligned}
V_0^{\text{gap}} &= \mathbb{E}\left((k-\tilde{k})s(0,T) \int_0^T \mathbf{e}^{-ru}\mathbf{1}_{\{S>u\}}\,\mathrm{d}u - \mathbf{e}^{-rS}\,\max(-V_S^k - 1, 0) \right) \\
&= (k-\tilde{k})s(0,T) \int_0^T \mathbf{e}^{-ru}\mathbf{P}(S>u)\,\mathrm{d}u \\
&\quad - \int_{(0,T]\times(1,\infty)} \mathbf{e}^{-ru}\,(x-1)\,\mathbf{P}(S\in \mathrm{d}u, -V_S^k \in \mathrm{d}x). \qquad (3)
\end{aligned}
$$

The fair gap risk fee is obtained by setting $V_0^{\text{gap}} = 0$, i.e.,

$$
(k-\tilde{k})s(0,T) = \frac{\int_{(0,T]\times(1,\infty)} \mathbf{e}^{-ru}\,(x-1)\,\mathbf{P}(S\in \mathrm{d}u, -V_S^k \in \mathrm{d}x)}{\int_0^T \mathbf{e}^{-ru}\,\mathbf{P}(S>u)\,\mathrm{d}u}. \qquad (4)
$$

The gap risk component in the valuation formula (3) is an option with payoff $\max(-V_S^k - 1, 0)$ at time S, which the note issuer sells to the investor. The option premium is the stream of payments $\int_0^T (k-\tilde{k})s(0,T)\mathbf{1}_{\{S>u\}}\,\mathrm{d}u$ that is earned while the note is alive, so that Equation (3) may be interpreted as the valuation formula for a gap option.

Clearly, to value a gap option requires a model that includes jumps in the evolution of credit spreads.

3 First passage time model with jumps

In this section we introduce a first passage time model that includes jumps in the evolution of credit spreads. The model is an extension of the Overbeck-Schmidt model, which we introduce first. The proofs of the results stated in this section are all treated in detail in [Packham et al., 2009].

In a first passage time model, the default time τ of a defaultable underlying entity is determined as the first time that a *credit quality process* $X = (X_t)_{t\geq 0}$ hits a barrier b:

$$
\tau = \inf\{\tau \geq 0 : X_t \leq b\}. \qquad (5)
$$

In general, b may itself be stochastic, but in our setting b is constant and $b < X_0$.

3.1 Overbeck-Schmidt model

The principal idea of the Overbeck-Schmidt model (OS-model), [Overbeck and Schmidt, 2005], is to model X as a time-changed Brownian motion. Given a Brownian motion B and a deterministic, strictly increasing and continuous time transformation $\Lambda = (\Lambda_t)_{t\geq 0}$ with $\Lambda_0 = 0$, set

$$
X_t := B_{\Lambda_t}, \quad t \geq 0.
$$

Assume given the distribution of the default time, $F(t) = \mathbf{P}(\tau \leq t)$, $t \geq 0$. If the time-change Λ is given by

$$
\Lambda_t = \left(\frac{b}{\mathrm{N}^{(-1)}\left(\frac{F(t)}{2}\right)} \right)^2, \quad t \geq 0, \qquad (6)
$$

where $\mathrm{N}^{(-1)}$ denotes the inverse of the Normal distribution function, then τ, defined by Equation (5), admits the distribution function $F(t)$, $t \geq 0$. Equation (6) is easily derived via the hitting time distribution of Brownian motion. Furthermore, if $F(t)$, $t \geq 0$, admits a density, then the time change Λ is absolutely continuous, and

$$\Lambda_t = \int_0^t \sigma(s)^2 \, \mathrm{d}s, \tag{7}$$

with $\sigma : [0, \infty) \to [0, \infty)$ a square-integrable function. The quadratic variation $[X, X]$ of X is just $[X, X] = \Lambda$, so that there exists a representation of X as a stochastic integral

$$X_t = \int_0^t \sigma(s) \, \mathrm{d}W_s, \tag{8}$$

for some Brownian motion, see e.g. Section 3.4.A of [Karatzas and Shreve, 1998]. The volatility σ can be interpreted as the *default speed* in the sense that the higher the default speed the higher the likelihood of crossing the default barrier.

In [Overbeck and Schmidt, 2005], the model is used to value products whose payoff does not depend on the level of CDS spreads; for example, by modelling several correlated credit quality processes, one can price multiname products such as first-to-default credit baskets. Although the OS-model is not intended to value products whose payoff depends on the level of the spread, it does exhibit dynamics by specification of the process X. In particular, for $t \leq T$, on $\{\tau > t\}$, the probability of default until T conditional on $\mathcal{F}_t$ is given by

$$P(t, T) = \mathbf{P}(\tau \leq T | \mathcal{F}_t) = 2\mathrm{N}\left(\frac{b - X_t}{\sqrt{\Lambda_T - \Lambda_t}}\right). \tag{9}$$

In turn, by Equation (1) one can determine the time-t term structure of CDS spreads from $\mathbf{P}(\tau \leq T | \mathcal{F}_t)$, $T \geq t$. The dynamics, however, are fully determined by calibration to market-given default probabilities, and it is not possible to assign different dynamics to the same initial term structure of default probabilities. On the other hand, the model is easily analytically calibrated to a given term structure via Equation (6).

We shall extend the OS-model to allow for different dynamics and to allow for jumps in the evolution of conditional default probabilities and credit spreads. At the same time we aim at maintaining tractability in terms of calibration and valuation. For an extensive discussion of the properties of the OS-model we refer to [Packham et al., 2009].

3.2 Credit quality process with stochastic volatility

Let us extend the OS-model by allowing the function σ from Equation (8) to be a stochastic process.

Definition 3.1. The *credit quality process* $X = (X_t)_{t \geq 0}$ of a risky entity is defined to be

$$X_t = \int_0^t \sigma_s \, \mathrm{d}W_s, \quad t \geq 0,$$

where W is an $(\mathcal{F}_t)_{t \geq 0}$-Brownian motion and σ is a strictly positive $(\mathcal{F}_t)_{t \geq 0}$-adapted càdlàg process independent of W with $\mathbf{P}(\int_0^t \sigma_s^2 \, \mathrm{d}s < \infty) = 1$, $t \geq 0$, and $\lim_{t \to \infty} \int_0^t \sigma_s^2 \, \mathrm{d}s = \infty$ $\mathbf{P}$–a.s..[3]

As before, the default time τ of the risky entity associated with the credit quality process X is the first time that X hits a barrier $b < 0$:

$$\tau = \inf\{t \geq 0 : X_t \leq b\}.$$

[3]The requirement $\lim_{t \to \infty} \int_0^t \sigma_s^2 \, \mathrm{d}s = \infty$ $\mathbf{P}$–a.s. ensures that $\tau < \infty$ $\mathbf{P}$–a.s..

Denote the quadratic variation process of X by $\Lambda = (\Lambda_t)_{t\geq 0}$, with $\Lambda_t = \int_0^t \sigma_s^2 \, \mathrm{d}s$. Observe that Λ is continuous, strictly increasing and $(\mathcal{F}_t)_{t\geq 0}$-adapted. By application of the Theorem of Dambis, Dubins-Schwarz (see Section 3.4.B of [Karatzas and Shreve, 1998]) the credit quality process X can be expressed as a time-changed Brownian motion, $X_t = B_{\Lambda_t}$, $t \geq 0$, with Λ the time-change.

Clearly, the credit quality process of Definition 3.1 is a generalisation of the OS-model with an absolutely continuous time-change, Equation (8).

3.3 Conditional default probabilities

In analogy to Equation (9), we have the following formulas for conditional default probabilities.

Proposition 3.2. *Let X be a credit quality process with volatility process σ. Let $\tau = \inf\{t \geq 0 : X_t \leq b\}$ be the associated default time. On $\{\tau > t\}$, the probability of default until time $T > t$, conditional on $\mathcal{F}_t$, is given by*

$$P(t,T) = \mathbf{P}(\tau \leq T | \mathcal{F}_t) = \mathbb{E}\left(2\mathrm{N}\left(\frac{b - X_t}{\sqrt{\Lambda_T - \Lambda_t}}\right) \Big| \mathcal{F}_t\right) \quad \mathbf{P}\text{–}a.s.. \tag{10}$$

Corollary 3.3. *Let X be a credit quality process with volatility process σ, and assume further that (X, σ) has the Markov property. Let τ be the associated default time. Then, for $T > t$, on $\{\tau > t\}$, the conditional default distribution is*

$$P(t,T) = \mathbf{P}(\tau \leq T | X_t, \sigma_t) = \mathbb{E}\left(2\mathrm{N}\left(\frac{b - X_t}{\sqrt{\Lambda_T - \Lambda_t}}\right) \Big| X_t, \sigma_t\right) \quad \mathbf{P}\text{–}a.s.. \tag{11}$$

Corollary 3.4. *Let X be a credit quality process with volatility process σ, and let τ be the associated default time. Then, the default distribution is given by*

$$P(0,T) = \mathbf{P}(\tau \leq T) = 2\mathbb{E}\left(\mathrm{N}\left(\frac{b}{\sqrt{\Lambda_T}}\right)\right), \quad T \geq 0. \tag{12}$$

3.4 Variance as Lévy-driven Ornstein-Uhlenbeck process

We put the model to work by specifying the variance process σ^2 to be a mean-reverting process with positive jumps. Candidates as drivers for the variance are Lévy processes: they incorporate jumps, and we can build Markov processes by specifying the dynamics of the variance with respect to Lévy processes, see [Protter, 2005, Theorem V.32].

For our modelling purpose, it is sufficient to consider variance processes driven by compound Poisson processes, where jumps are rare events – the economic rationale is that jumps in CDS spreads are triggered by the arrival of "bad news" in the market. Nonetheless, the statements in this section apply to infinite activity processes.

As an explicit example we model the variance process as a *Lévy-driven Ornstein-Uhlenbeck process* (LOU process). If it is driven by a pure-jump process with positive jumps, an LOU process moves up by jumps and decays exponentially in-between the jumps. Models where an asset price's variance is driven by an LOU process were first considered by [Barndorff-Nielsen and Shephard, 2001]. For details on LOU processes, see also [Norberg, 2004], [Schoutens, 2003, Chapter 5] and [Cont and Tankov, 2004, Chapter 15.3.3]. Let us specify the credit-quality process model with the variance driven by an LOU process.

Proposition 3.5. *Let W be an $(\mathcal{F}_t)_{t\geq 0}$-Brownian motion, and let Z be an $(\mathcal{F}_t)_{t\geq 0}$-subordinator (i.e., a Lévy process with nondecreasing paths) independent of W. Furthermore, let $a \in \mathbb{R}_+$*

and let θ be a bounded and càdlàg function, such that σ^2 (defined below) is strictly positive. The stochastic process $X = (X_t)_{t\geq 0}$,

$$X_t = \int_0^t \sigma_s \, \mathrm{d}W_s, \quad t \geq 0, \tag{13}$$

with σ^2 the solution of

$$\mathrm{d}\sigma_t^2 = a(\theta(t) - \sigma_{t-}^2) \, \mathrm{d}t + \mathrm{d}Z_t \tag{14}$$

is a credit quality process in the sense of Definition 3.1. Moreover, (X, σ) is a Markov process with respect to $(\mathcal{F}_t)_{t\geq 0}$.

If Z is a compound Poisson process, the variance and the time-change increment are given by

$$\sigma_t^2 = \mathbf{e}^{-at} \, \sigma_0^2 + \int_0^t \mathbf{e}^{-a(t-u)} \, a \, \theta(u) \, \mathrm{d}u + \sum_{0<u\leq t} \mathbf{e}^{-a(t-u)} \, \Delta Z_u \tag{15}$$

$$\begin{aligned} \Lambda_T - \Lambda_t = \int_t^T \sigma_u^2 \, \mathrm{d}u &= \left(1 - \mathbf{e}^{-a(T-t)}\right) \frac{\sigma_t^2}{a} \\ &+ \int_t^T \theta(u) \left(1 - \mathbf{e}^{-a(T-u)}\right) \mathrm{d}u + \frac{1}{a} \sum_{t<u\leq T} \left(1 - \mathbf{e}^{-a(T-u)}\right) \Delta Z_u. \end{aligned} \tag{16}$$

Equation (15) is verified by applying the Itô formula to $\mathbf{e}^{at} \, \sigma_t^2$ (which establishes *a fortiori* that a solution to Equation (14) exists). The time-change increments is obtained by integrating each term of Equation (15). A sample path of σ^2 and of $\Lambda = \int_0^t \sigma_s^2 \, \mathrm{d}s$ is given in Figure 2.

3.5 Jumps in default probabilities and credit spreads

The continuity of the credit quality process X and the associated time-change Λ are crucial to derive the formula for conditional default probabilities, Equation (10), from which credit spreads can be computed. But recall that we wished to build a model that incorporates jumps in credit spreads. It turns out that for a credit quality process with the variance driven by an LOU process, jumps in the variance process propagate to default probabilities and credit spreads.

Proposition 3.6. *Let X be a credit quality process with LOU variance process σ^2 as in Proposition 3.5. Let $\tau = \inf\{t > 0 : X_t \leq b\}$ be the associated default time. Fix $T > 0$ and let $(P(t,T))_{t\leq T}$ be the associated conditional default probability process. Then, for $\mathbf{P}$-almost all $\omega \in \{\tau > t\}$, $(P(t,T))_{t\leq T}$ is a process whose jumps are positive and*

$$\Delta\sigma_t^2(\omega) = 0 \iff \Delta P(t,T)(\omega) = 0, \quad t < T.$$

A straightforward consequence of this Proposition is that a jump at σ_t^2 triggers a jump in conditional default probabilities $P(t,T)$, for all maturities $T > t$.

Proposition 3.7. *Let $(s(t,T))_{0\leq t\leq T}$ be the CDS spread process for maturity T. Then $(s(t,T))_{0\leq t\leq T}$ is càdlàg, and for $t \leq T$ and for $\mathbf{P}$-almost all $\omega \in \{\tau > t\}$,*

$$\Delta s(t,T)(\omega) > 0 \iff (\Delta P(t,u)(\omega) > 0, \ \textit{for some } u \in (t,T]).$$

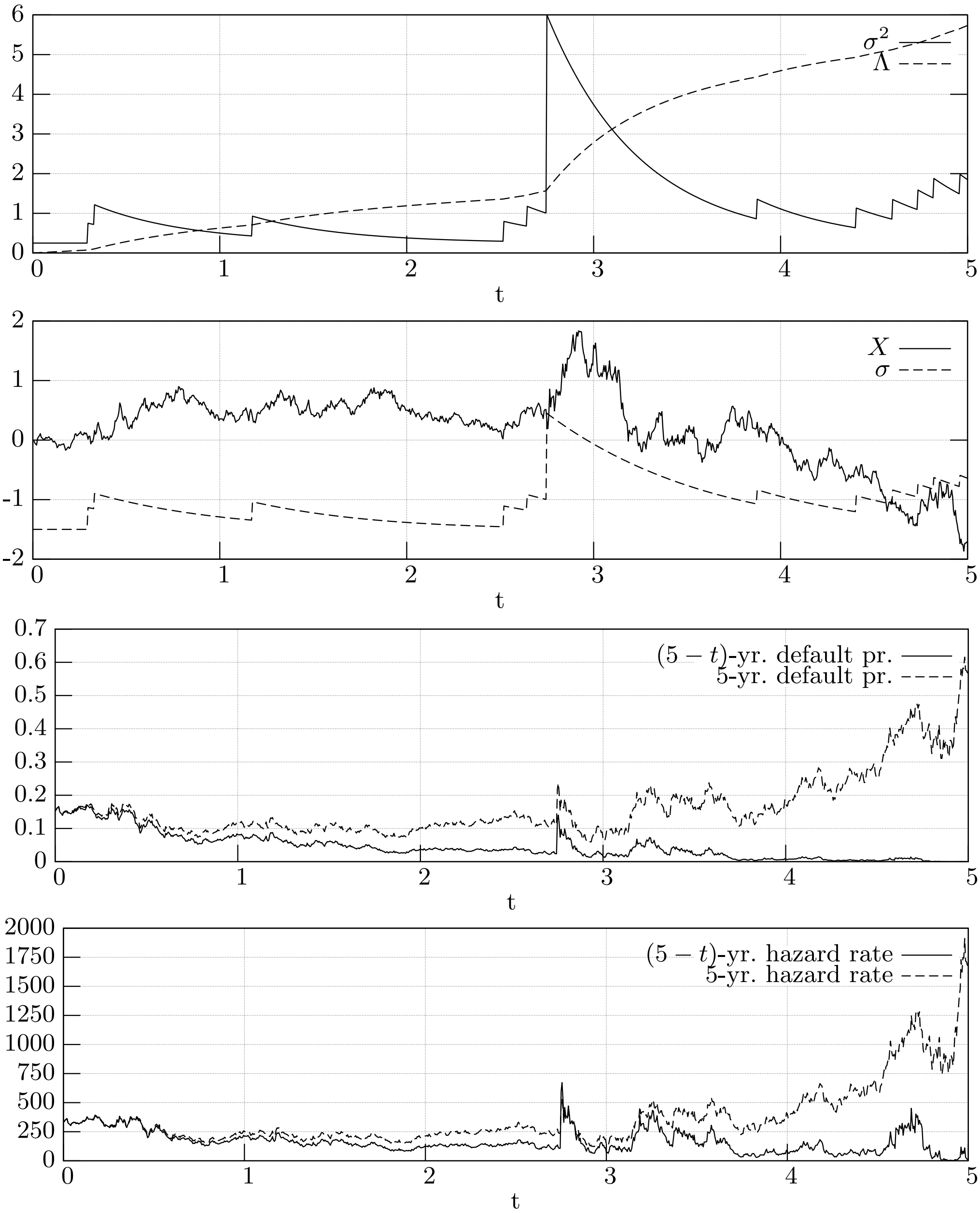

Figure 2: Realization of variance process and credit quality process. *Top:* variance process σ^2, Equation (15) (continuous line, left axis); time-change Λ, Equation (16) (dashed line, right axis). *Second from top:* credit quality process X, Equation (13) (continuous line, left axis); volatility σ (dashed line, right axis). *Second from bottom:* 5-year default probability process $P(t,T)$, Equation (11), with decaying time-to-maturity (continuous line) and with fixed time-to-maturity (dashed line). *Bottom:* Approximation of credit spread $s(t,T)$ (term hazard rate, see Appendix A), with decaying time-to-maturity (continuous line) and with fixed time-to-maturity (dashed line).
Parameters: $a = 2$, $\theta \equiv 0.25$, $\sigma_0^2 = 0.25$; σ^2 is driven by a compound Poisson process with jump intensity $\lambda = 2$ and discrete jump size distribution with jump sizes $0.05, 5$ with probabilities $0.95, 0.05$, respectively. The barrier is $b = -3$.

Consider the variance process in Figure 2: it has a large jump, which is also clearly visible in the default probability process and in the credit spread.

Note that the credit quality process model does not include events where credit spreads jump for selected maturities only. However, this is compatible with the observation that credit spreads tend to jump together, see Section 2.3.

Finally, it is easily seen that a jump in the variance process cannot lead to default $\mathbf{P}$–a.s.. It suffices to observe that $\tau = \inf\{t > 0 : X_t \le b\}$ is a predictable stopping time, whereas the jumps of the driving compound Poisson process are totally inaccessible. However, we shall see later that jump-to-default events can be approximated by large jumps in the variance.

4 Computation of default probabilities and credit spreads

The implementation of the LOU variance model is a combination of Monte Carlo simulation and analytical computation. On $\{\tau > t\}$, and conditional on X_t, σ_t^2, default probabilities $P(t,T) = \mathbf{P}(\tau \le T | X_t, \sigma_t^2)$, $T > t$, can be computed numerically, so that Monte Carlo simulation reduces to simulating X_t and σ_t^2. The advantage of such an algorithm is that valuation of a product involving $P(t,T)$ or $s(t,T)$ requires simulation only until t instead of T. For example, to value a default swaption one needs to simulate merely until option expiry instead of maturity of the underlying CDS (this will be treated in Section 7).

4.1 Jump size distribution of time-change Λ

Assume the credit quality process model (X, σ^2) of Proposition 3.5, with σ^2 a LOU process driven by a compound Poisson process Z. We wish to compute conditional default probabilities $\mathbf{P}(\tau \le T | X_t, \sigma_t)$, $0 \le t \le T$, according to Equation (11). Inspection of the formula for the time-change increments $\Lambda_T - \Lambda_t$, Equation (16), reveals that computation of the conditional expectation (11) essentially entails computing the distribution of

$$L_{t,T} := \sum_{t<u\le T} \left(1 - \mathbf{e}^{-a(t-u)}\right) \Delta Z_u.$$

Let Z have jump intensity λ and jump size $Y > 0$. For every $t \le T$, the random variable $L_{t,T}$ follows a compound Poisson distribution (see [Sato, 1999, Chapter 22] or [Norberg, 2004]),

$$L_{t,T} \sim \mathrm{CPO}(\lambda(T-t), (1 - \mathbf{e}^{-a(T-S)})Y), \tag{17}$$

with S uniformly distributed on $(t,T]$, i.e., $S \sim U(t,T)$, and independent of Y. Moreover, $L_{t,T} \overset{\mathcal{L}}{=} L_{0,T-t}$, hence it suffices to compute the distributions of $L_T := L_{0,T}$. The following result states the distribution of the compounding variate $(1 - \mathbf{e}^{a(T-S)})\, Y$ of L_T.

Lemma 4.1. *For $T > 0$, let $S \sim U(0,T)$ and let Y be a $\mathbf{P}$–a.s. strictly positive random variable independent of S. The distribution of $\left(1 - \mathbf{e}^{a(T-S)}\right) Y$ is given by*

$$F(x) = \mathbb{E}\left(-\frac{\ln(1 - x/Y)}{aT} \mathbf{1}_{\{[0,1-\mathbf{e}^{-aT})\}}(x/Y)\right) + \mathbf{P}\left(Y \le \frac{x}{1 - \mathbf{e}^{-aT}}\right), \quad x \in \mathbb{R}. \tag{18}$$

Proof. Conditioning under Y yields

$$F(x) = \mathbf{P}\left(\left(1 - \mathbf{e}^{-a(T-S)}\right) Y \le x\right) = \mathbb{E}\left(\mathbf{P}\left(\left(1 - \mathbf{e}^{-a(T-S)}\right) Y \le x \middle| Y\right)\right). \tag{19}$$

number of simulations	CPU time (seconds)	MSE (at $t = 5$)
1000	164.28	0.9369
2000	333.70	0.2725
5000	1966.54	0.0589
10000	8554.59	0.0403

Table 1: Monte Carlo simulation vs. Panjer recursion. CPU time (Panjer recursion): 331.33 CPU secs.

Define $g_x(y) := \mathbf{P}\left(\left(1 - \mathbf{e}^{-a(T-S)}\right) y \leq x\right)$, $y > 0$. By the independence of S and Y, the conditional probability of Equation (19) is given by $g_x(Y)$. Since $S \in [0, T]$,

$$g_x(y) = \begin{cases} 0, & x \leq 0, \\ 1, & x \geq (1 - \mathbf{e}^{-aT})y, \\ -\frac{\ln(1-x/y)}{aT}, & x \in (0, (1 - \mathbf{e}^{-aT})y), \end{cases}$$

and the claim follows by inserting into Equation (19). □

4.2 Panjer recursion

The distribution of L_T can be computed efficiently using *Panjer recursion*, see [Panjer, 1981] or [McNeil et al., 2005, Chapter 10]. This method is based on a recursive evaluation formula for a family of compound distributions. In our implementation it has proven to be numerically more stable to assume a discrete distribution of the compounding variate, although the distribution function of the compounding variate in Equation (18) is continuous. For the compound Poisson case, the method works as follows: Suppose N is a Poisson distributed random variable with intensity λ and let the compounding variate Y take values in the nonnegative integers. Set $f(i) = \mathbf{P}(Y = i)$, $i = 1, 2, \ldots$. For a random variable $L \sim \mathrm{CPO}(\lambda, Y)$, its distribution $g(i) = \mathbf{P}(L = i)$, $i = 1, 2, \ldots$, is given by

$$g(i) = \sum_{n=0}^{i} f^{n*}(i)\mathbf{P}(N = n), \quad i = 1, 2, \ldots,$$

where $f^{n*}(i)$ denotes the n-fold convolution product of f at i. The number of computations required for determining $g(i)$ is of the order i^2. The result by Panjer states that

$$g(i) = \frac{\lambda}{i} \sum_{j=1}^{i} j\, f(j)\, g(i-j), \quad i = 1, 2, \ldots,$$

in which case the number of computations required for determining $g(i)$ is of the order i. By proper scaling on an equidistant grid, the method can also be used for discrete nonnegative compounding variates not restricted to integers.

To illustrate the pickup in computational speed using Panjer recursion, we compare the computation of the distributions of Λ_{t_i}, with $t_i = i/10$, $i = 0, \ldots, 200$, for points $x = (x_i)_{i=0,\ldots,8000}$, using Monte Carlo simulation and Panjer recursion. For the Monte Carlo simulation, Λ_t was simulated at 200 time points with 8000 grid points each, with 1000, 2000, 5000 and 10000 simulations. The CPU times of the simulations are given in Table 1. In addition, the simulation results are compared with the computation using Panjer recursion by considering the simulation mean square error (MSE) relative to the value obtained by Panjer recursion.

Require: $t_1 = 0 < \ldots < t_N$ // time grid
Require: $x_1 = 0 < \ldots < x_M$ // space grid
Require: $u_1, \ldots, u_J$ // desired maturities
Require: K // number of simulations
Require: b // default barrier
Require: a, θ, λ, σ_0^2, F // volatility process parameters, F jump size distribution
1: // Panjer recursion
2: **for** $i = 1$ to N **do**
3: **for** $j = 1$ to J **do**
4: compute $\mathbf{P}(L_{t_i} \in [x_{j-1}, x_j))$
5: **end for**
6: **end for**
7: // simulation step
8: **for** $k = 1$ to K **do**
9: $\tau^k \leftarrow \infty$ // default time of k-th simulation
10: **for** $j = 1$ to J **do**
11: simulate $\sigma_{u_j}^k$ and $X_{u_j}^k$
12: sample $d \leftarrow \mathbf{1}_{\{\min_{u_{j-1} < s \leq u_j} X_s \leq b\}}$ cond. on $X_{u_{j-1}}$ and X_{u_j} // (see text)
13: **if** $d = 1$ or $X_{u_j}^k \leq b$ **then**
14: $\tau^k \leftarrow u_j$
15: $P^k(u_j, u_j + t_i) \leftarrow 1$, $s^k(u_j, t_i) \leftarrow 0$ // for all $i = 1, \ldots, N$
16: **next** k // exit k-th simulation
17: **end if**
18: **for** $i = 1$ to N **do**
19: $h \leftarrow (1 - \mathbf{e}^{-at_i})\, \sigma_{u_j}^2 / a + \int_{u_j}^{u_j + t_i} \theta(r) \left(1 - \mathbf{e}^{-a(u_j + t_i - r)}\right) \mathrm{d}r$
20: $P^k(u_j, u_j + t_i) \leftarrow 2 \sum_{m=1}^{M} \mathrm{N}\left(\frac{b - X_{u_j}}{\sqrt{h + x_{m-1}/a}}\right) \mathbf{P}\left(L_{t_i} \in [x_{m-1}, x_m)\right)$
21: $s^k(u_j, u_j + t_i) \leftarrow -(1 - R) \ln(1 - P(u_j, u_j + t_i))/t_i$ // credit triangle
22: **end for**
23: **end for**
24: **end for**

Algorithm 1: Computation of conditional default probabilities

4.3 Algorithm

Suppose we wish to compute default probabilities $P(u_j, u_j + t_i)$, $j = 1, \ldots J$, $i = 1, \ldots, N$.[4] The full simulation algorithm, outlined below, is given in pseudo-code in Algorithm 1. For each t_i, we compute the distribution of L_{t_i} on an equidistant space grid $x_1, \ldots, x_M$. Next, we simulate K paths of (σ, X), yielding $(\sigma_{u_j})_{j=1,\ldots,J}^k$ and $(X_{u_j})_{j=1,\ldots,J}^k$, $k = 1, \ldots, K$. For each u_j, we check if default has occurred. However, simulating on a discrete time grid underestimates the occurrence of the default event. Hence, in addition we sample an indicator variable that determines whether default has occurred between two time points. This is realised by applying a well-known result to determine the barrier hitting event of a Brownian bridge. Taking into account that X is a Brownian motion with a continuous time-change, the indicator takes value 1 with probability $\mathbf{e}^{-2(b - X_{u_{j-1}})(b - X_{u_j})/(\Lambda_{u_j} - \Lambda_{u_{j-1}})}$, $j = 2, \ldots J$, cf. [Glasserman, 2004, Section 6.4]. For each time point u_j and time-to-maturity t_i, we determine $\Lambda_{t_i + u_j} - \Lambda_{u_j}$ by computing the deterministic part and then the expectation using the distribution L_{t_i}.[5] In this way we obtain a term structure of default probabilities, which serves as the basis for computing credit spreads according to Equation (1). In the algorithm we use the credit triangle (see Appendix A) compute credit spreads.

[4] For notational simplicity we compute an $r \times n$ matrix of default probabilities; other setups of time points and time-to-maturities are possible.

[5] Actually, the deterministic part need not be computed for every simulation, hence for efficiency the computation of Line 19 should take place outside the loop $k = 1, \ldots, K$.

5 Calibration

Calibration is the process of assigning the parameters of the model such that the model reproduces market prices. One set of market prices is the term structure of credit spreads (or default probabilities). Further market prices, such as prices of default swaptions, provided they are available and liquid, may be suitable for calibrating the dynamics. In the absence of a liquid market for such claims, calibrating the dynamics via historical data may be a feasible alternative. We focus here on calibration to a given term structure and illustrate the attainable range of dynamics in the following section.

In the LOU model of Proposition 3.5, the deterministic function θ and the initial variance σ_0^2 will be chosen to reproduce a given term structure. The remaining parameters – mean reversion constant a, jump intensity λ, jump size distribution F, barrier b – are chosen to determine the dynamics. It should be noted, however, that the deterministic function θ influences the dynamics and that the parameters for the dynamics influence the calibration of θ. It is also the case that calibration to a given term structure imposes some restrictions on the dynamics parameters – in other words, given a set of dynamics parameters, it is not possible to achieve satisfactory calibration to an arbitrary term structure; this is outlined in detail below. The overall calibration process is to assign parameters for the dynamics first and then to calibrate to the spot curve.

The allocation of the parameters to spot curve calibration and dynamics calibration is justified as follows: in a model with a jump intensity of zero, the resulting time-change process is deterministic, which corresponds to the Overbeck-Schmidt model. In this case, the only parameters that are relevant for calibration to a given spot curve are the initial variance σ_0^2 and the deterministic function θ, and the dynamics are fixed by the deterministic time-change. Only when the jump intensity is greater than zero do the dynamics change, in which case all parameters allocated to the dynamics calibration become relevant for the dynamics.

5.1 Calibration to a term structure of default probabilities

Assume given a set of default probabilities $P(T_i) := \mathbf{P}(\tau \leq T_i)$, $T_1 < \cdots < T_n$, derived from market-given credit spreads (together with a recovery assumption). For fixed mean reversion a, barrier b, jump intensity λ and jump size distribution F, the objective is to determine σ_0^2 and θ to match the given default probabilities. Since default probabilities in the LOU-model are expectations, cf. Equation (10), there is in general no analytic method to calibrate. Moreover, it turns out that it is not even guaranteed that an exact solution of the calibration problem exists. We require that $P(t,T)$, $T \geq t$, be strictly increasing, capturing the fact that a risky entity may default at any time, for every $t \geq 0$, $\mathbf{P}$–a.s.. Clearly, by inspection of Equation (10), this condition is met if the time-change Λ is strictly increasing $\mathbf{P}$–a.s., or, equivalently, if $\sigma_t^2 > 0$, $t \geq 0$, $\mathbf{P}$–a.s..

Although an exact solution to the calibration problem with a certain set of dynamics parameters may not exist, satisfactory calibration quality to a given term-structure may always be obtained. Indeed, a model without jump component is equivalent to the OS-model, where analytic and exact calibration is possible. By choosing suitably moderate jump dynamics, an arbitrary calibration quality may be achieved, as we shall see below.

We calibrate numerically by minimising the error between market-given and model-computed default probabilities. In the following, we shall always assume θ to be piecewise constant,

$$\theta(t) = \sum_{i=1}^{n} \theta(T_i)\mathbf{1}_{(T_{i-1},T_i]}(t), \quad t > 0, \tag{20}$$

with the convention $T_0 = 0$. We define the *root mean square error (RMSE)* between market default probabilities and model default probabilities as

$$\delta(\sigma_0^2, \theta; P, a, b, \lambda, F) := \sqrt{\sum_{i=1}^{n} \frac{T_i - T_{i-1}}{T_n} \left(P(T_i) - 2\mathbb{E}\mathrm{N}\left(b/\sqrt{\Lambda_{T_i}}\right)\right)^2}, \tag{21}$$

where the expectation denotes the model-given default probability for maturity T_i, cf. Equation (12) and Λ_{T_i} is given by (cf. Equation (16))

$$\Lambda_{T_i} = \left(1 - \mathbf{e}^{-a\,T_i}\right) \frac{\sigma_0^2}{a} + \sum_{j=1}^{i} \theta(j) \left[T_j - T_{j-1} - \frac{\mathbf{e}^{-a(t-T_j)} - \mathbf{e}^{-a(t-T_{j-1})}}{a}\right] + \frac{1}{a} \sum_{0<u\le T_i} \left(1 - \mathbf{e}^{-a(T_i-u)}\right) \Delta Z_u. \tag{22}$$

In order for $P(t,T)$, $T \geq t$, to be strictly increasing for all t, we require that (cf. Equation (15))

$$\sigma_t^2 = \mathbf{e}^{-at}\,\sigma_0^2 + \sum_{i=1}^{n} \theta(T_i)\,\mathbf{e}^{-at} \left(\mathbf{e}^{a(t\wedge T_i)} - \mathbf{e}^{a(t\wedge T_{i-1})}\right) + \sum_{0<u\le t} \mathbf{e}^{-a(t-u)}\,\Delta Z_u > 0, \quad t \geq 0.$$

Taking into account that jumps are positive, the condition is satisfied if θ satisfies

$$\theta(T_i) > -\frac{\sigma_0^2 + \sum_{j=1}^{i-1} \theta(T_j)\left(\mathbf{e}^{aT_j} - \mathbf{e}^{aT_{j-1}}\right)}{\mathbf{e}^{aT_i} - \mathbf{e}^{aT_{i-1}}}, \quad i = 1, \ldots, n. \tag{23}$$

Define the set

$$\Theta = \{(\theta(T_1), \ldots, \theta(T_n)) \in \mathbb{R}^n : (\theta(T_1), \ldots, \theta(T_n)) \text{ satisfies } (23)\}.$$

For the model-given probabilities to be well-defined requires additionally that $\lambda \geq 0$, $a > 0$, $b \leq 0$ and $F(0) = 0$. Under these conditions, the solution to the calibration problem is then given by

$$(\sigma_0^{\star 2}, \theta^\star(T_1), \ldots, \theta^\star(T_n)) := \underset{\left\{\sigma_0^2 \in \mathbb{R}_+, (\theta(T_1),\ldots,\theta(T_n)) \in \Theta\right\}}{\arg\min} \delta(\sigma_0^2, \theta; P, a, b, \lambda, F). \tag{24}$$

Example 5.1. Assume given default probabilites $P(T_i) = 1 - \mathbf{e}^{-hT_i}$, at times $T_i = i$ (years), $i = 1, \ldots, 10$, with $h = 0.03$. We calibrate the model to these default probabilities, for different jump size distributions, jump intensities λ and mean reversion constants a.

The distribution of L_{0,T_i}, $i = 1, \ldots, 10$, is computed on an equidistant grid of 5000 points in the interval $[0, 120]$. The barrier is $b = -3$. The mean reversion a and the jump intensity λ are chosen from the set $\{1, 2, 3, 5, 10\}$ and the following jump size distributions are considered:

(i) The jump size is 1/4.

(ii) The jump size is 1/2.

(iii) The jump size distribution is exponential with parameter $\nu = 4$, i.e., $F(x) = 1 - \mathbf{e}^{-\nu x}$.

(iv) The jump size is 0.1 with probability 0.95 and 20 with probability 0.05. Here, we enlarged the grid for computing the distributions of $(L_{0,T_i})_{i=1,\ldots,10}$ to 11000 points on the interval $[0, 264]$.

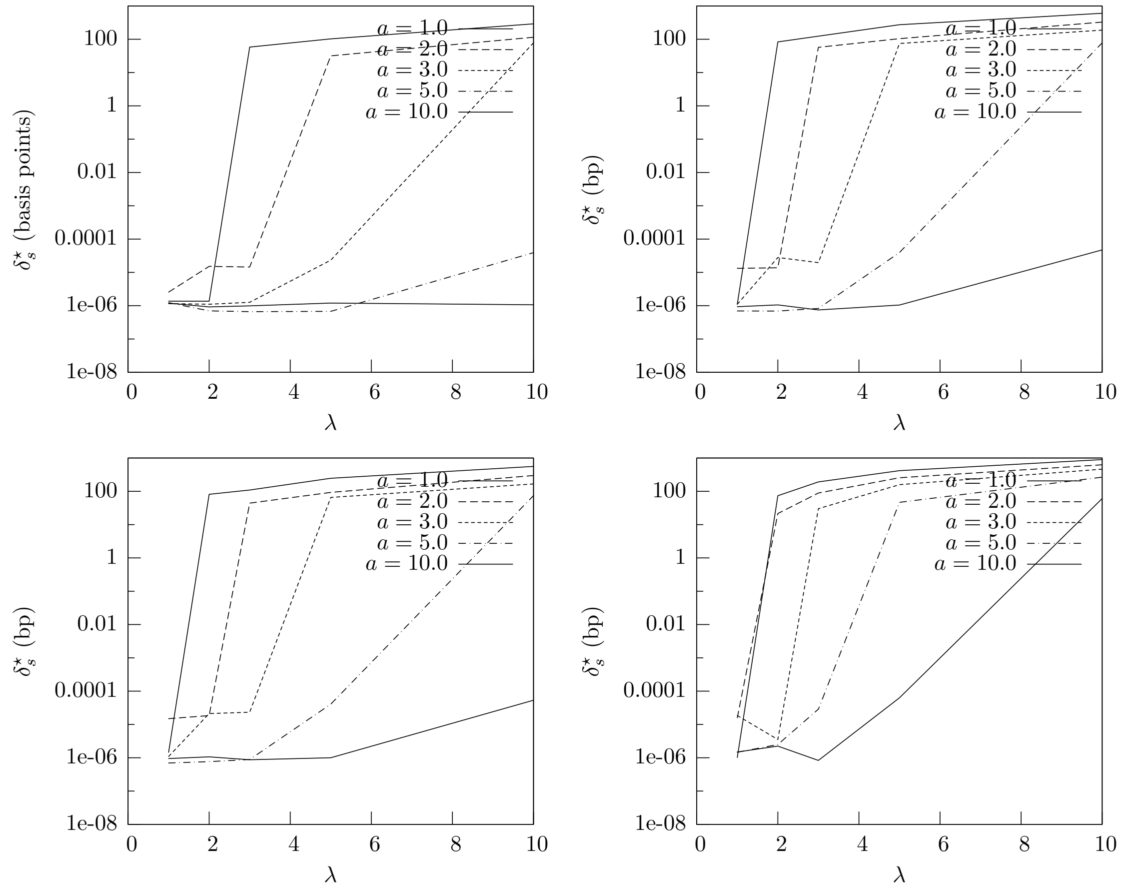

Figure 3: RMSE's of credit spreads for different parameter sets obtained by calibrating to default probabilities $P(T_i) = 1 - \mathbf{e}^{-0.03\,T_i}$, $i = 1, \ldots, 10$. Each figure contains six lines that correspond to parameter $a = \{1, 2, 3, 5, 10\}$ (ordered from top to bottom at $\lambda = 10$). Top left, top right, bottom left and bottom right: jump size distribution as in (i)-(iv), respectively, of Example 5.1.

The RMSE's for credit spreads, denoted by $\delta_s^\star$, are given in Figure 3. Here, the spread (in basis points, 1bp = 0.01%) was computed from market-given default probabilities by

$$s(T_i) = 10^4 \cdot (1 - R) \frac{P(T_i)}{\sum_{j=1}^{i} (1 - P(T_j))(T_j - T_{j-1})}, \quad i = 1, \ldots, n,$$

and accordingly for model-given default probabilities. The recovery rate was $R = 0.4$. Observe that for some parameter sets, the RMSE between market-given and model-given credit spreads is as small as 10^{-6}. It turns out that the error is large when the jump intensity λ is high and mean reversion a is small. Increasing the frequency of jumps, increases the volatility of the credit quality process, which in turn increases the likelihood of the credit quality process hitting the barrier. A too high jump intensity may thus inhibit satisfactory calibration to a given term structure. To understand the effect of a low mean reversion, one must study Equation (16). First of all, the jump size in the time-change Λ is scaled by the mean reversion. Secondly, a low mean reversion "dampens" the function θ at the short end of the term structure, which is then amplied for longer time-to-maturity. Thus, for low mean reversion, calibrating the short term default probabilities selects high values for θ, whose effect is intensified for long term default probabilities. It follows that under many circumstances a low mean reversion fails to calibrate well to either the short end or the long end of a given term structure.

5.2 Shape of credit-spread term structure

Examples of credit spread term structures with different parameters are given in Figure 4. Recall from the stylised empirical facts stated in Section 2.3 that a term structure of credit spreads may assume different shapes. Typically, an investment grade company's term structure is upward sloping, reflecting lower default risk in the near future compared to higher uncertainty in the long term. A speculative-grade company may have an inverted term structure, indicating that the firm faces higher short-term default risk, but is more likely to survive in the long-term conditional on survival in the short-term.

A common observation is that credit spreads are strictly positive as time-to-maturity tends to zero, indicating that default may happen suddenly and unexpectedly. In Section 3.5 we established that the model is not capable of producing this property - credit spreads vanish as time-to-maturity tends to zero, and the default time is predictable. However, the possibility of large jumps in the variance may allow for "near-jump-to-default" events (see also case (d) of the examples presented in Section 6.1). We would then expect the spread term structure to be very steep at the short end.

By choosing extreme values for either the barrier or the initial variance, we obtain sharply humped term structures that approximate inverted term structures. Both cases reflect a low credit quality: default becomes more likely as either the credit quality process approaches the barrier or as the variance increases.

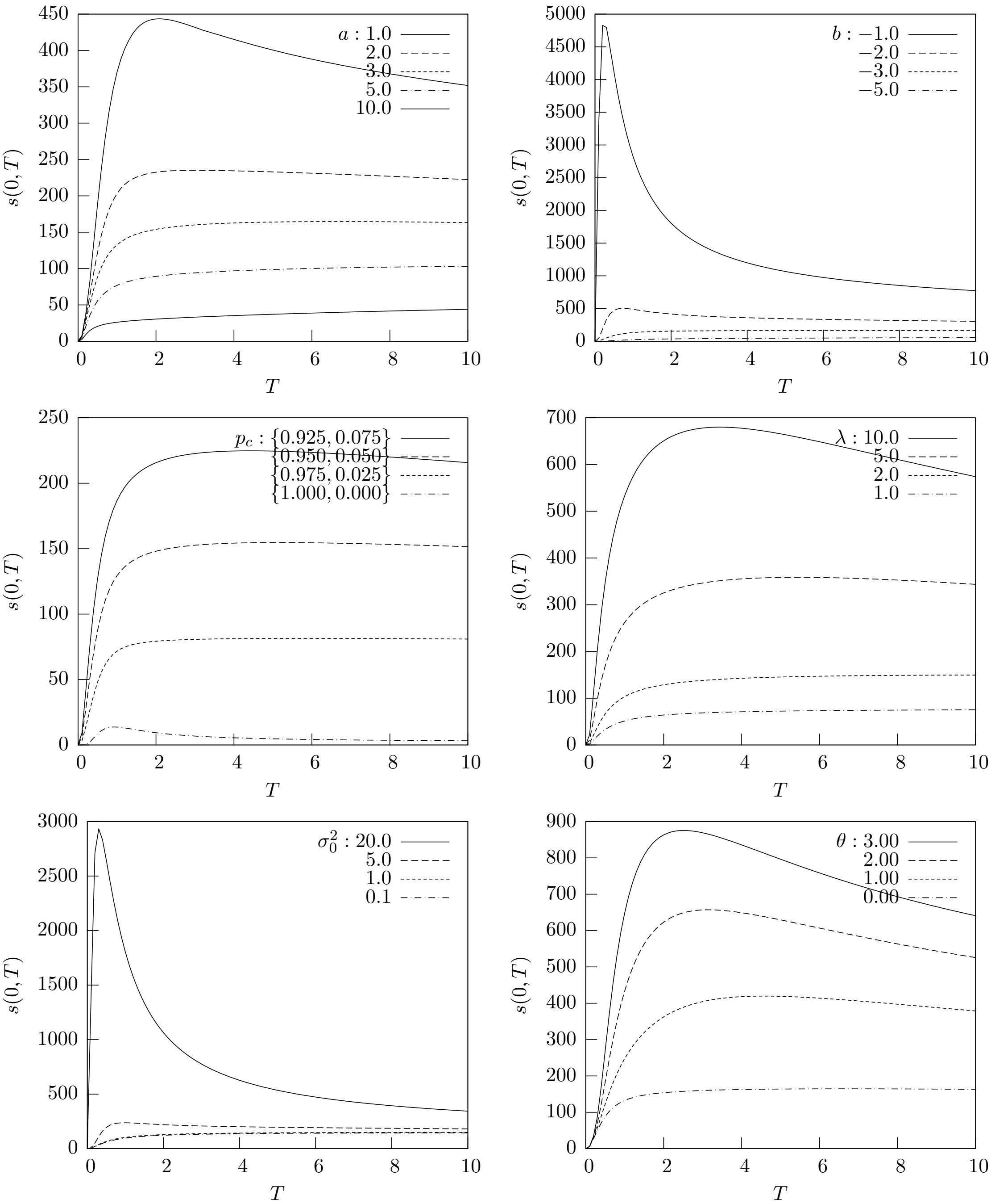

Figure 4: Impact of individual parameters on credit spread term structure. The standard parameter set is $a = 3$, $b = -3$, $\lambda = 2$, $\sigma_0^2 = 3$, $\theta \equiv 0$, jump size in $\{0.1, 20\}$ with probabilities $\{0.95, 0.05\}$. p_c denotes the jump size probabilities. In each case, the curves are ordered from top to bottom at $T = 10$.

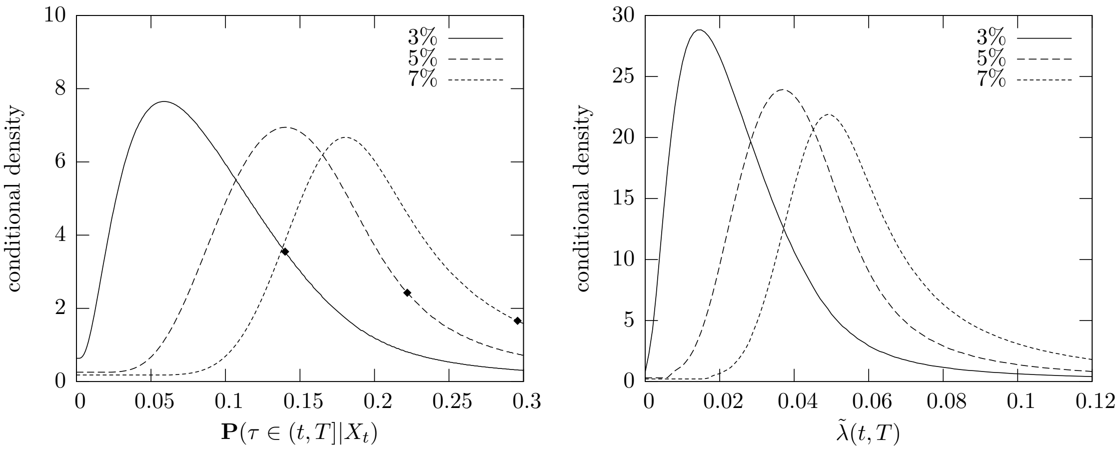

Figure 5: Distributions of $P(t,T)$ (left) and $\tilde{\lambda}(t,T)$ (right) conditional on $\{\tau > t\}$. The term hazard rate $\tilde{\lambda}(t,T)$ is an approximation of the credit spread, i.e., $s(t,T) \approx (1-R)\tilde{\lambda}(t,T)\cdot 10^4$ (in basis points). We have $t = 1, T = 5$ and initial hazard rates of $3\%, 5\%, 7\%$. The diamonds mark the initial 5-year default probability $\mathbf{P}(\tau \le T)$. The model parameters are mean reversion $a = 3$, recovery rate $R = 0.4$, barrier $b = -3$; σ^2 is driven by compound Poisson processes with jump intensity $\lambda = 2$ and jump size distributions $\{0.1, 20\}$ ($h = 0.03$), $\{0.2, 50\}$ ($h = 0.05$), $\{0.4, 100\}$ ($h = 0.07$) with probabilities $\{0.95, 0.05\}$. In each case, the the initial variance σ_0^2 and the function θ were chosen to fit the given term structure.

6 Dynamics

In general, the prices generated by the model should be consistent with any liquid market prices. For example, option prices, if available, provide a source of information for calibrating risk-neutral dynamics. In the absence of such information, one may be forced to resort to information from historical time series.

6.1 Examples

To illustrate the range of dynamics, Figure 5 shows the distributions (conditional on no default) of the random variable $\mathbf{P}(\tau \le 5|\mathcal{F}_1)$, i.e., of the 5-year default probability in one year, for four different parameter sets, and the corresponding corresponding credit spread approximations (term hazard rates, see Appendix A). The parameters are given in Table 2. In each case, the initial variance σ_0^2 and the function θ are calibrated to match default probabilities corresponding to an initial hazard rate of 3%. For details on calculating the distribution of the random variable $\mathbf{P}(\tau \le T|\mathcal{F}_t)$ see [Packham et al., 2009].

Case (a) is a model with a deterministic time-change, and hence corresponds to the OS-model. Case (d) was chosen such that $\sigma_0^2 = 10^{-6}$ and $\theta \equiv 0$, so that the variance is very small until the first jump occurs. The jump size was chosen to be very large relative to the default barrier so that, heuristically, a single jump leads to default very quickly. Loosely speaking, case (d) can be considered an approximation of a reduced-form model with a deterministic and constant intensity: the credit quality process exhibits practically no movement, until the first jump occurs, which leads to default. This is also reflected in the jump intensity $\lambda = 0.0305$, which is approximately the initial hazard rate, and in $\mathbf{P}(\tau \in (1,5)|X_1, \sigma_1^2) \approx 1 - \mathbf{e}^{-0.03\cdot 4} = 0.11308$ conditional on no default until time 1.[6] These

[6]If the initial hazard rate is not constant, then a calibration where the variance moves purely by jumps

	(a)	(b)	(c)	(d)
		Parameters		
a	3	3	1	1
b	-3	-3	-3	-2
λ	0	2	1	0.0305
F		$\begin{bmatrix} 0.1 & (0.95) \\ 20 & (0.05) \end{bmatrix}$	$\begin{bmatrix} 0.1 & (0.95) \\ 10 & (0.05) \end{bmatrix}$	$\mathbf{1}_{\{[25000,\infty)\}}$
σ_0^2	3.16	4.59	3.25	10^{-6}
θ	$\in [0.23, 1.32]$	$\in [-0.23, 0.04]$	$\in [-1.10, 0.37]$	0
		RMSE's		
$\delta^\star$	$< 10^{-8}$	$< 10^{-8}$	$< 10^{-9}$	$< 10^{-3}$
$\delta_s^\star$	$< 10^{-5}$	$< 10^{-4}$	$< 10^{-5}$	0.56
		Characteristics of term hazard rate distributions		
mean	0.0521	0.0304	0.0199	0.0311
std. dev.	0.0794	0.0341	0.0315	$9.45 \cdot 10^{-5}$
skewness	3.39	6.98	9.27	n/a
exc. kurtosis	16.52	83.41	129.12	n/a

Table 2: Parameters of dynamics examples. The jump sizes are given by the first column and the corresponding probabilities in the second column of each matrix in the row of parameter F.

two cases illustrate that the parameters can be classified into parameters that govern the jump part of the variance process, namely the jump intensity and jump size distribution, and parameters that control the continuous behaviour of the process in the sense that the level of the function θ determines the minimum "default speed" of the credit quality process at any time. By calibration to a term structure, a low level of jump activity leads to a higher minimum "default speed" and vice versa.

The characteristics of cases (b) and (c) are "in-between" cases (a) and (d): in both cases, the variance process exhibits jumps. However, the jump dynamics are moderate enough for the level of the variance process induced by θ and σ_0^2, both of which are obtained by calibration to the given term structure, to be significantly above zero. In other words, the variance processes of both cases feature jumps and a significant minimum "default speed".

6.2 Parameters and dynamics

We discuss each parameter that influences the dynamics. These are the parameters of the variance process, i.e., mean reversion a, jump intensity λ, jump size distribution F, initial variance σ_0^2 and deterministic function θ. Additionally, as will be outlined below, we include the barrier b in our discussion. The initial variance σ_0^2 and deterministic function θ, although determined by calibration to a given term structure, influence the dynamics, implying that the choice of dynamics and the calibration to the spot term structure cannot be separated.

Mean reversion a The mean reversion parameter a determines the "speed" at which the variance reverts to its mean, cf. Equation (14). As can be seen from Equation (16) (resp. Equation (22)), the initial variance and jump size are scaled by $1/a$. Additionally, the factors $(1-\mathbf{e}^{-aT})$ and $(1-\mathbf{e}^{-a(T-u)})$ "dampen" the impact of θ and Z for short maturities, which are then amplified for longer maturities. The smaller a is chosen, the stronger this

cannot be attained. This is due to the fact that the jump intensity of the variance's compound Poisson process is constant, whereas a non-constant, deterministic hazard rate requires the jump intensity to be non-constant and deterministic. The former can be incorporated by specifying the jump process as an additive process, which is more general than a Lévy process.

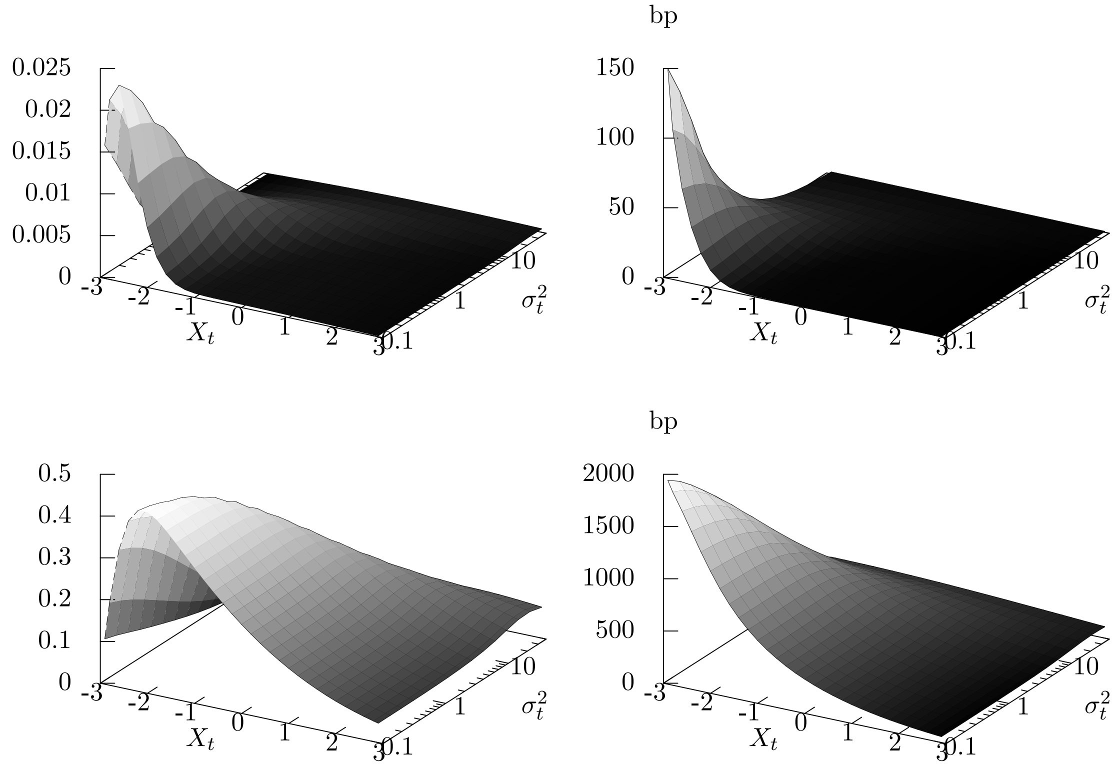

Figure 6: Jump size of 5-year default probability (left) and credit spread (right) when a jump of size 0.1 (top), resp. 20 (bottom) occurs. Parameters correspond to case (b) of Table 2.

effect of amplification as maturity increases. As outlined in Section 5.1, a small value of a frequently leads to poor calibration: Large values of θ are determined for a satisfactory calibration at the short end, which are then too large for a proper calibration to longer maturities. On the other hand, a small mean reversion makes the impact of jumps in the variance process more long-lasting.

Barrier b Strictly speaking, the inclusion of the barrier as a parameter is redundant: For any two barriers b and b' we may obtain the same spot curve and dynamics by proper scaling of σ_0^2, θ and the jump size. On the other hand, one can see in Figure 4 that a change in the barrier affects the shape of the curve. To obtain the desired shape, it may be more straightforward to adjust the barrier instead of adjusting the set of other parameters.

Jump intensity λ **and jump size distribution** F The jump intensity determines the average number of jumps to occur per year. The results from Section 3.5 imply that the jump intensities of the variance process and the term structures of default probabilities and credit spreads are all equal. The jump size of Z propagates to the credit spread jump size in a monotone way: the larger the jump of Z, the larger the jump in the credit spread – this is easily seen by the by monotonicity of default probabilities in σ^2 and extended to credit spreads via the CDS valuation formula (1).

However, the jump size also depends on the level of X and σ^2. For default probabilities, the jump size is not monotone in X. This is easily seen as follows: Assume that at time

t there is a jump, $\Delta\sigma_t^2 > 0$. For any maturity $T > t$, $\lim_{X_t \downarrow b} \Delta P(t,T) = 0$, as the jump size is bounded by $1 - P(t-,T)$ and $\lim_{X_t \downarrow b} P(t-,T) = 1$. On the other hand, $\lim_{X_t \uparrow \infty} \Delta P(t,T) = 0$ since $\Delta\sigma_t^2 < \infty$ $\mathbf{P}$–a.s.. By Proposition 3.6, $\Delta P(t,T) > 0$ for any fixed X_t, so $\Delta P(t,T)$ cannot be monotone in X_t.

For credit spreads, the situation is not so clear. Examples indicate that the jump size of credit spreads is monotone in X; Figure 6 provides an example of the jump size as a function of X_t and σ_t^2.

Initial variance σ_0^2 and deterministic function θ Although σ_0^2 and θ are chosen by calibration to a given term structure, their effect on the dynamics are significant. Intuitively, a decrease in the Lévy measure of Z (i.e., decrease in jump intensity or jump size), decreases the probability of default, which is compensated by a higher choice of θ when calibrating. Consequently, the variance process will maintain a higher deterministic level, causing the credit quality process to evolve in a more volatile fashion in order to hit the default barrier with a certain probability. This is well illustrated by comparing cases (a) and (d) of the previous example, Section 6.1.

6.3 Evolution of the term structure shape

In Section 5.2 it was shown that the shape of the term structure becomes inverted (more precise: sharply humped) with increasing barrier b and with increasing initial variance σ_0^2, cf. Figure 4. By inspection of the formula for conditional default probabilities we see that default probabilities at time t depend on the distance-to-default $b - X_t$. Both a decreasing distance-to-default and an increasing variance imply a decrease in credit quality, which eventually results in an inverted credit spread term structure.

7 Valuation examples

7.1 Information flow and pricing filtration

So far, we have made no assumptions about the filtration $(\mathcal{F}_t)_{t\geq 0}$, other than that it is rich enough for the credit quality process (X, σ^2) to be $(\mathcal{F}_t)_{t\geq 0}$-adapted. For a market model to be consistent with arbitrage theory requires that the filtration used for pricing is generated by observable information. For building trading strategies, the underlying filtration must even be generated by the observable prices of traded assets, see e.g. Chapter 7 of [Hunt and Kennedy, 2004]. In general, a credit quality process (X, σ^2) is neither directly observable nor a traded asset.

Suppose now that we wish to price financial claims derived from credit spreads (or, equivalently, default, resp. survival probabilities). Application of a risk-neutral valuation formula with conditional default probabilities given by the model via Equation (10) (or credit spreads derived thereof) is justified only if $(\mathcal{F}_t)_{t\geq 0}$ is generated by some observable information and if X and σ^2 are $(\mathcal{F}_t)_{t\geq 0}$-adapted. Otherwise, valuation of assets requires that prices are computed using a different – possibly coarser – filtration. One may think of a coarser filtration as the inavailability in the market of complete information about a company's state.

Assuming independence of risk-free interest rates and the default indicator process, the filtration generated by risk-free zero-coupon bonds $(B(t,T))_{T\geq t}$ and conditional default probabilities $(P(t,T))_{T\geq t}$, $t \geq 0$, will be sufficient for this purpose; owing to the valuation formula for risky zero-coupon bonds, given by

$$\mathbb{E}\left(\mathbf{e}^{-\int_t^T r_s \, \mathrm{d}s} \mathbf{1}_{\{\tau > T\}} | \mathcal{F}_t\right) = B(t,T)\, \mathbf{P}(\tau > T | \mathcal{F}_t), \quad T \geq t,$$

this filtration is equivalent to the filtration generated by risk-free and risky zero-coupon bonds (of all maturities).

The assumption that there is indeed a process that drives a company's credit quality via the information available about the company may be justified by the stylised facts recorded earlier, namely that the arrival of news about a company affects CDS spreads of all maturities in a similar fashion. We shall assume that the credit quality of a firm is indeed driven by a process (X, σ^2) as in Proposition 3.5 with σ^2 an LOU process driven by a compound Poisson process (with respect to the filtration $(\mathcal{F}_t)_{t\geq 0}$). Furthermore, we assume that the parameters of the credit quality process are known, i.e., θ, σ_0^2, a, b, c, λ, F are $\mathcal{F}_0$-measurable. The following result establishes that, under these assumptions, the formula for conditional default probabilities is suitable for valuation.

Proposition 7.1. *Let (X, σ^2) be a credit quality process as in Proposition 3.5, with σ^2 an LOU process driven by a compound Poisson process. Let $\mathcal{F}_t^P = \sigma((P(s,T)_{T>s}), 0 \leq s \leq t)$. Then, $\sigma(X_t, \sigma_t^2) \subseteq \mathcal{F}_t^P$. Moreover, $\mathbf{P}(\tau \leq T|\mathcal{F}_t^P) = \mathbf{P}(\tau \leq T|X_t, \sigma_t^2)$.*

For the proof see [Packham et al., 2009].

7.2 Leveraged credit-linked note

As an example application of the model we value a leveraged credit-linked note using the pricing formula (3), with pricing done via Algorithm 1. The note has a maturity of 5 years and a notional amount of €100. The leverage factor is $k = 5$, so that the payoff amount and time are linked to the mark-to-market value of a CDS position with nominal €500 on CDS with a maturity of 5 years at inception. The trigger level is $K =$ €60. The initial CDS spread term structure is flat at 180 basis point, the recovery rate is 40%. The note is monitored weekly, i.e., at time points $t_1 < t_2 < \cdots < t_{260}$, with $\Delta t_i = 1/52$. Denote by V_t^k the mark-to-market value at time t of the CDS position. The note is unwound at $S = \inf\{t_i : V_{t_i}^k \leq -K, i = 1, \ldots, 260\}$. At S, the investor receives $\max($€$100 + V_S^k, 0)$ and the issuer pays $\max(-V_S^k -$€$100, 0)$ (the gap option payoff). The premium of the gap option is $(k - \tilde{k})s(0,T)$. The risk-free interest rate is constant at 5%.

For each dynamics example we generated 10 times 1000 simulations. From each batch of 1000 simulations, we computed the fair factor $\tilde{k}$, the spread $(k - \tilde{k})s(0,T)$ (the total of which, over the lifetime of the note, is the gap option premium, cf. Equation (4)) and the spread on the note (for the investor) $s_{\text{inv}} = \tilde{k}s(0,T)$ for each of the four example models exhibiting different dynamics from Section 6.1. Additionally, denoting by L_{inv} the discounted loss to the investor, we computed the expected loss $\mathbb{E}(L_{\text{inv}})$, the probability that a loss occurs, the probability of a total loss $L_{\text{inv,tot}}$, the expected discounted earnings from the spread payments (excluding the default-free interest of the coupon payment), $\mathbb{E}(E_{\text{inv}})$, and the expected trigger time S conditional on a trigger event. The values obtained are given in Table 3; here, each table entry consists of the mean value taken over all runs and (in parentheses) the standard deviation with respect to the 10 simulation runs (the 10 simulation scenarios are simulations of the estimator, each of which is approximately normally distributed by the usual Central Limit Theorem).

Recall that in Section 2.4 we already determined the fair factor $\tilde{k}$ for some models via no-arbitrage arguments. Specifically, in the case where the mark-to-market value of a CDS evolves continuously, and when there is no jump-to-default risk, the fair factor is $\tilde{k} = k$, as there is no gap risk involved. This corresponds to case (a). Now consider the case where the mark-to-market value is constant and the note is exposed to default risk only by a jump-to-default event. Then $\tilde{k} = 1/(1-R)$ as the investor's payoff is equivalent to selling protection on $1/(1-R)$ CDS. This corresponds to case (d). Here, $\tilde{k}$ is slightly larger than $1/(1-R) = 1.67$ as there is still some, albeit small, volatility that drives

	(a)		(b)		(c)		(d)	
$\tilde{k}$	5.000	(0.005)	4.674	(0.058)	4.865	(0.036)	1.943	(0.203)
s_{issuer} (bp)	0.60	(0.90)	58.60	(10.44)	24.31	(6.52)	550.19	(36.54)
s_{inv} (bp)	899.40	(0.90)	841.40	(10.44)	875.69	(6.52)	349.81	(36.54)
$\mathbb{E}(L_{\text{inv}})$ (€)	26.51	(1.57)	27.60	(0.73)	29.95	(10.66)	11.93	(0.75)
$\mathbf{P}(L_{\text{inv}})$	0.40	(0.024)	0.38	(0.01)	0.44	(0.015)	0.134	(0.008)
$\mathbf{P}(L_{\text{inv,tot}})$	0.002	(0.002)	0.067	(0.006)	0.030	(0.006)	0.134	(0.008)
$\mathbb{E}(E_{\text{inv}})$ (€)	26.88	(0.76)	27.28	(0.56)	25.20	(0.51)	14.44	(1.58)
$\mathbb{E}S$ (yrs)	0.88	(0.062)	1.46	(0.09)	0.93	(0.054)	2.41	(0.08)

Table 3: Valuation examples of leveraged credit-linked note. The cases (a)-(d) correspond to the models of Section 6.1.

the credit quality process, and consequently the underlying CDS's mark-to-market value is not constant. Also note that in this case the probabilities of a loss and of a total loss to the investor are approximately equal and correspond to the 1-year default probability $1 - \mathbf{e}^{-0.03 \cdot 5} = 0.139$. Finally, note that in this case, the expected trigger time conditional on a trigger event is roughly half of the note's maturity. In the other cases the expected trigger time is significantly earlier. There are apparently two reasons for this: first of all, the mark-to-market value of a CDS position vanishes as maturity is approached. Hence, the trigger event is less likely to occur for shorter remaining time-to-maturity. Secondly, in the absence of the possibility of large jumps in the variance process, calibration to the short end of the term structure may require rather high volatility in the credit quality process; this is a consequence of the fact that in the model short-term credit spreads vanish, whereas in reality they do not. This leads to high values of σ_0^2 and θ for short maturities. The resulting high volatility in the credit quality process affects the whole term structure, so that not only short-term, but also long-term credit spreads exhibit higher volatility in the short term.

Finally, observe that the efficiency of the simulation can be increased significantly as follows. Most of the mark-to-market value computations are used only to check whether the trigger level has been hit, and only in this case the actual mark-to-market value is needed. Compared to the number of total observations this is a rare event. Now, observe that the mark-to-market value is monotone in both X and σ^2. For each time step, and for a set of five values of the variance process σ_t^2 we computed the corresponding value of X_t for the mark-to-market value to be at the trigger level (actually, to be on the safe side, we computed X for a slightly smaller mark-to-market value). In each simulation, we then first checked against these computed valued whether the mark-to-market value needs to be computed at all. It turned out that in more than two thirds of evaluation steps the computation of credit spreads and mark-to-market value could be skipped.

7.3 Default swaptions

As a second example, let us consider the valuation of options on CDS. For a more detailed description than given here, see e.g. [O'Kane, 2008, Chapter 9].

A *(default) swaption* features the right to enter into a CDS at a later point in time at a spread that is fixed today, the so-called *strike spread.* The maturity of the option is called the *expiry date.* We distinguish *receiver swaptions*, giving the option holder the right to sell protection and receive the strike spread, and *payer swaptions*, giving the holder the right to buy protection and pay the strike spread.

The underlying of a swaption is a *forward starting CDS* that starts at the option expiry. For a protection seller of a forward starting CDS with start t and maturity T, the payoff on the protection leg is equal to selling CDS protection with maturity T and buying CDS protection with maturity t. In particular, if there is a default event before option expiry, then the contract cancels at no cost to either counterparty.

Denote by $A(t,T) = \int_t^T \mathbf{e}^{-r(u-t)}\,(1 - P(t,u))\,\mathrm{d}u$ the *risky present value of a basis point at t for the term* $(t,T]$. The quantity $A(0,T) = \int_0^T \mathbf{e}^{-ru}\,(1 - P(0,u))\,\mathrm{d}u$ is just today's *risky present value of a basis point until T*. The present value of the position in the two CDS satisfies

$$s(0,T)A(0,T) - s(0,t)A(0,t) = (1-R)\int_t^T \mathbf{e}^{-ru}\,\mathbf{P}(\tau \in \mathrm{d}u),$$

The *forward CDS spread* $s(0,t,T)$ is the spread at time 0 for a CDS contract starting at t and maturing at T (by definition, $s(t,t,T) = s(t,T)$). Using the fact that $\mathbf{1}_{\{\tau>t\}}$ is $\mathcal{F}_t$-measurable, the present value of the premium leg is

$$s(0,t,T)\,\mathbb{E}\left(\mathbf{e}^{-rt}\mathbf{1}_{\{\tau>t\}}\int_t^T \mathbf{e}^{-r(u-t)}(1-P(t,u))\,\mathrm{d}u\right) = s(0,t,T)(A(0,T) - A(0,t)).$$

Solving for the fair forward spread yields,

$$s(0,t,T) = \frac{s(0,T)A(0,T) - s(0,t)A(0,t)}{A(0,T) - A(0,t)}. \tag{25}$$

Turning back to swaptions, the payoff at expiry of a payer (resp. receiver) swaption with expiry t and strike spread K on a CDS with maturity T is

$$(s(t,T) - K)^+\,A(t,T) \qquad \text{(resp. } (K - s(t,T))^+\,A(t,T)\text{)}. \tag{26}$$

Valuation of default swaptions is conveniently done using as numeraire $N(s) = \mathbf{1}_{\{\tau>s\}}(A(s,T) - A(s,t)),\ s \geq 0$.[7] Under the corresponding martingale measure $\mathbf{Q}$, called *forward survival measure* or *forward risky annuity measure*, the values of the payer and receiver swaptions at time 0 are

$$\begin{aligned} V_{0,\text{payer}} &= (A(0,T) - A(0,t))\,\mathbb{E}_{\mathbf{Q}}\left((s(t,T) - K)^+\right) \\ V_{0,\text{receiver}} &= (A(0,T) - A(0,t))\,\mathbb{E}_{\mathbf{Q}}\left((K - s(t,T))^+\right). \end{aligned}$$

Observe that the spread $s(t,T)$ is the price of a claim expressed in units of $N(t)$, hence a martingale under the forward survival measure, so that

$$\mathbb{E}_{\mathbf{Q}}(s(t,T)) = s(0,t,T).$$

A put-call parity[8] that relates the value of a forward starting CDS with the prices of payer and receiver swaptions follows easily. It is given by

$$V_{0,\text{payer}} - V_{0,\text{receiver}} = (s(0,t,T) - K)\,(A(0,T) - A(0,t)).$$

Finally, assuming a lognormal model for the forward spread under the forward survival measure yields a Black formula for option prices, where

$$\begin{aligned} V_{0,\text{payer}} &= (A(0,T) - A(0,t))\,(s(0,t,T)\,\mathrm{N}(d_1) - K\,\mathrm{N}(d_2)) \\ V_{0,\text{receiver}} &= (A(0,T) - A(0,t))\,(K\,\mathrm{N}(-d_2) - s(0,t,T)\,\mathrm{N}(-d_1)), \end{aligned}$$

[7]Technically, it is required that the numeraire be strictly positive, which is not the case for N. However, in the case when $N(s) = 0$, then the option cancels and its value is 0.

[8]Put $\equiv$ payer, call $\equiv$ receiver

K	(a)		(b)		(c)		(d)	
0	566.23	(12.11)	561.27	(7.61)	551.08	(10.25)	556.33	(1.78)
50	468.23	(11.92)	414.99	(7.68)	410.81	(10.37)	404.91	(1.52)
100	403.64	(11.55)	297.42	(7.52)	322.77	(10.15)	263.50	(1.32)
150	354.14	(11.14)	215.33	(7.20)	270.57	(9.80)	102.08	(1.20)
182.73	327.04	(10.88)	177.29	(6.97)	245.71	(9.56)	2.97	(1.20)
200	314.11	(10.74)	161.37	(6.85)	234.38	(9.43)	1.77	(1.20)
250	281.07	(10.35)	126.92	(6.52)	207.35	(9.08)	1.73	(1.19)
300	253.48	(9.98)	104.05	(6.22)	186.09	(8.74)	1.68	(1.18)

Table 4: Valuation examples of default swaptions; payer option prices (standard error).

with

$$d_1 = \frac{\ln(s(0,t,T)/K) + 1/2\,\sigma^2 t}{\sigma\sqrt{t}} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{t}.$$

There is a one-to-one relationship between the price and the volatility σ of options in the Black model. Instead of quoting prices, it is custom in the market to quote the *implied volatility*, which is the volatility σ in the Black formula that yields the option price.

As an example, we consider valuation of default swaptions in the example models (a)-(d) of Section 6.1. We assume options of different strike spreads with expiry $t = 1$ (year) to enter into a CDS of 4 year maturity (i.e., $T = 5$). Suppose given survival probabilities $1-P(0,t_i) = \mathbf{e}^{-0.03\cdot t_i}$, $t_i = i$, $i = 1,\ldots,5$, a recovery rate of $R = 0.4$ and a constant interest rate of $r = 5\%$. We compute CDS spreads by

$$s(0,t_i) = \frac{(1-R)\sum_{j=1}^{i} \mathbf{e}^{-r\,t_j}\,\Delta P(0,t_j)}{\sum_{j=1}^{i} \mathbf{e}^{-rt_j}\,(1-P(0,t_j))\Delta t_j},$$

with $\Delta P(0,t_j) = P(0,t_j) - P(0,t_{j-1})$ and $\Delta t_j = t_j - t_{j-1}$. We obtain a flat credit term structure of 182.73 basis points. By Equation (25), the forward CDS rate is $s(0,t,T) = 182.73$ and the present value of the forward CDS's premium leg is 554.62 basis points.

For each of the example models (a)-(d), we generated 5000 simulations to value payer and receiver swaptions of different strikes. We simulated $(X_{t_1}, \sigma^2_{t_1})$ and then computed the term structure of default probabilities and the 4-year CDS spread as described in Section 4.3, with CDS spreads computed according to

$$s(t_1,t_i) = \frac{(1-R)\sum_{j=2}^{i} \mathbf{e}^{-r\,(t_j-t_1)}\,\Delta P(t_1,t_j)}{\sum_{j=2}^{i} \mathbf{e}^{-r\,(t_j-t_1)}\,(1-P(t_1,t_j))\Delta t_j}, \quad i = 2,\ldots 5,$$

and the risky present value of a basis point according to

$$A(t_1,t_i) = \sum_{j=2}^{i} \mathbf{e}^{-r\,(t_j-t_1)}\,(1-P(t_1,t_j))\Delta t_j, \quad i = 2,\ldots 5.$$

The resulting prices for payer swaptions, i.e., the discounted means of the option payoff, cf. Equation (26), are given in Table 4, with the corresponding standard error in parentheses. Figure 7 shows the prices (left) and implied volatilities (right) of receiver and payer swaptions in the cases (a)-(d) (top to bottom). Note that the payer option price with strike spread 0 is just the forward CDS price.

The implied volatility for case (a), which corresponds to the model with the continuous, deterministic variance process, is very high. On the other hand, for case (d), where the credit quality process' variance is very small unless a very large jump occurs, there is virtually no time value in the option prices and volatility is comparably low. In fact, in a pure jump-to-default model with no spread dynamics, the imlied volatility is zero. Theoretically, implied volatilities for payer and receiver swaptions are equal by put-call parity. The simulation error is due to not using put-call parity for computing receiver option prices from payer option prices.

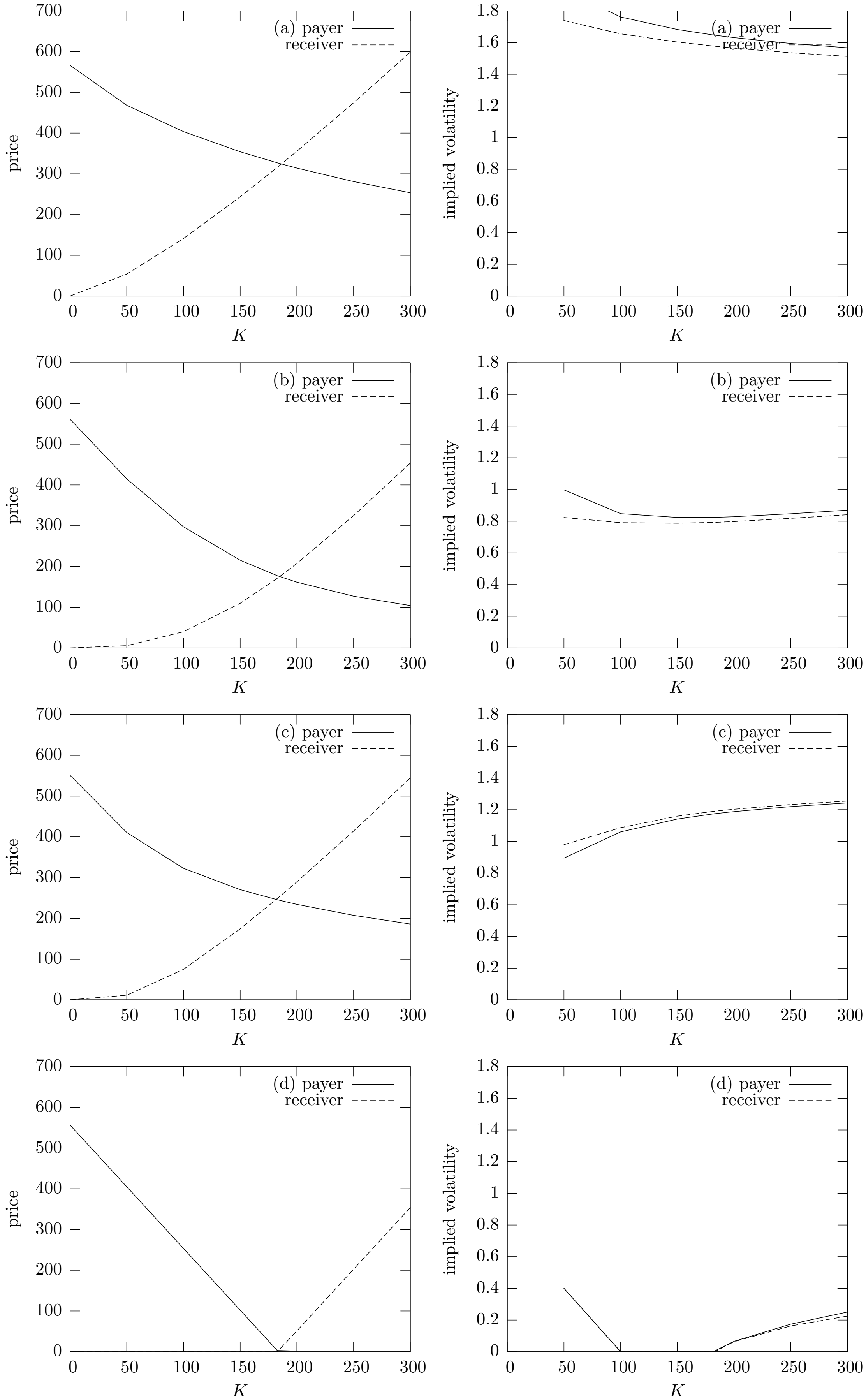

Figure 7: Prices (left) and implied volatilities (right) of payer and receiver swaptions of different strikes in the models (a)-(d) (top to bottom).

A Term hazard rate

We sometimes consider the *term hazard rate* defined by

$$\tilde{\lambda}(t,T) = -\frac{\ln(1-P(t,T))}{T-t}, \quad T > t, \quad P(t,T) < 1, \tag{27}$$

as a proxy for the credit spread $s(t,T)/(1-R)$. The use of the term hazard rate is motivated by the fact that $\tilde{\lambda}(t,T)$ is a function of $P(t,T)$ instead of $(P(t,T))_{T\geq t}$ as is the case for the spread $s(t,T)$. That it may be considered a proxy for the credit spread is explained as follows: If the default time admits a (conditional) density, then the hazard rate at time t is the mapping $T \mapsto \lambda(t,T)$ defined by

$$\lambda(t,T) = -\frac{\mathrm{d}}{\mathrm{d}T}\ln(1-P(t,T)), \quad T \geq t.$$

It follows easily that

$$1 - P(t,T) = \mathbf{e}^{-\int_t^T \lambda(t,u)\,\mathrm{d}u},$$

which, together with Equation (1) yields the relationship

$$\frac{s(t,T)}{1-R}\int_t^T \mathbf{e}^{-r(u-t)}(1-P(t,u))\,\mathrm{d}u = \int_t^T \lambda(t,u)\,\mathbf{e}^{-r(u-t)}(1-P(t,u))\,\mathrm{d}u.$$

An approximation of the right-hand side is

$$\tilde{\lambda}(t,T)\int_t^T \mathbf{e}^{-r(u-t)}(1-P(t,u))\,\mathrm{d}u,$$

with

$$\tilde{\lambda}(t,T) = \frac{\int_t^T \lambda(t,u)\,\mathrm{d}u}{T-t},$$

which yields the well-known credit triangle

$$\frac{s(t,T)}{1-R} \approx \tilde{\lambda}(t,T).$$

References

Ole E. Barndorff-Nielsen and Neil Shephard. Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial econometrics. *J. R. Statist. Soc. B*, 63:167–241, 2001.

Martin Baxter. Gamma process dynamic modelling of credit. *RISK*, October:98–101, October 2007.

Tomasz R. Bielecki and Marek Rutkowski. *Credit Risk: Modeling, Valuation and Hedging*. Springer, 2002.

Fischer Black and John C. Cox. Valuing corporate securities: some effects of bond indenture provisions. *The Journal of Finance*, 31(2):351–367, May 1976.

Jessica Cariboni and Wim Schoutens. Pricing credit default swaps under Lévy models. *Journal of Computational Finance*, 10(4):71–91, 2007.

Rama Cont and Peter Tankov. *Financial Modelling with Jump Processes*. Chapman & Hall/CRC, 2004.

Qiang Dai and Kenneth Singleton. Term structure dynamics in theory and reality. *The Review of Financial Studies*, 16(3):631–678, 2003.

S.R. Das. The surprise element: jumps in interest rates. *Journal of Econometrics*, 106(1):27–65, 2002.

Darrell Duffie and David Lando. Term structure of credit spreads with incomplete accounting information. *Econometrica*, 69:633–664, 2001.

Darrell Duffie and Kenneth J Singleton. Modeling term structures of defaultable bonds. *Review of Financial Studies*, 12(4):687–720, 1999.

Darrell Duffie and Kenneth J. Singleton. *Credit Risk: Pricing, Measurement, and Management*. Princeton University Press, 2003.

J.S. Fons. Using default rates to model the term structure of credit spreads. *Financial Analysts Journal*, 50:25–32, 1994.

Paul Glasserman. *Monte Carlo Methods in Financial Engineering*. Springer, 2004.

J. Helwege and C.M. Turner. The slope of the credit yield curve for speculative-grade issuers. *The Journal of Finance*, 54(5):1869–1884, 1999.

P. J. Hunt and J. E. Kennedy. *Financial Derivatives in Theory and Practice*. Wiley, revised edition, 2004.

Robert A. Jarrow and Stuart M. Turnbull. Pricing derivatives on financial securities subject to credit risk. *The Journal of Finance*, 50(1):53–85, 1995.

M. S. Johannes. A nonparametric view of the role of jumps to interest rates. Working paper, Columbia Business School, 2000.

Ioannis Karatzas and Steven E. Shreve. *Brownian Motion and Stochastic Calculus*. Springer, 2nd edition, 1998.

Rüdiger Kiesel and Matthias Scherer. Dynamic credit portfolio modelling in structural models with jumps. Working paper, December 2007.

David Lando. On Cox processes and credit risky securities. *Review of Derivatives Research*, 2: 99–120, 1998.

David Lando. *Credit Risk Modeling*. Princeton University Press, 2004.

Francis A. Longstaff and Eduardo S. Schwartz. A simple approach to valuing risky fixed and floating rate debt. *The Journal of Finance*, 50(3):789–819, July 1995.

Alexander J. McNeil, Rdiger Frey, and Paul Embrechts. *Quantitative Risk Management*. Princeton University Press, 2005.

Robert C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29(2):449–470, May 1974.

Ragnar Norberg. Vasicek beyond the normal. *Mathematical Finance*, 14(4):585–604, 2004.

Dominic O'Kane. *Modelling Single-name and Multi-name Credit Derivatives*. Wiley, 2008.

Ludger Overbeck and Wolfgang M. Schmidt. Modeling default dependence with threshold models. *Journal of Derivatives*, 12(4):10–19, Summer 2005.

Natalie Packham, Lutz Schlögl, and Wolfgang M. Schmidt. Credit dynamics in a first-passage time model with jumps. Working paper, 2009.

H.P. Panjer. Recursive evaluation of a family of compound distributions. *ASTIN Bull.*, 12:22–26, 1981.

Philip E. Protter. *Stochastic Integration and Differential Equations*. Springer, 2nd edition, 2005. Version 2.1.

K. Sato. *Lévy Processes and Infinitely Divisible Distributions*. Cambridge University Press, 1999.

Paul Schneider, Leopold Sögner, and Tanja Veza. The economic role of jumps and recovery rates in the market for corporate default risk. Working Paper (Journal of Financial and Quantitative Analysis; forthcoming), November 2007.

Wim Schoutens. *Lévy Processes in Finance*. John Wiley & Sons, 2003.

George Tauchen and Hao Zhou. Realized jumps on financial markets and predicting credit spreads. FEDS Working Paper No. 2006-35 (Journal of Econometrics, forthcoming), August 2006.

Benjamin Yibin Zhang, Hao Zhou, and Haibin Zhu. Explaining credit default swap spreads with the equity volatility and jump risks of individual firms. FEDS Discussion Paper No. 2005-63, BIS Working Paper No. 181 (Review of Financial Studies, forthcoming), August 2008.

Chunsheng Zhou. The term structure of credit spreads with jump risk. *Journal of Banking and Finance*, 25:2015–2040, 2001.

FRANKFURT SCHOOL / HFB – WORKING PAPER SERIES

No.	Author/Title	Year
133.	Bannier, Christina E. / Behr, Patrick / Güttler, Andre Rating opaque borrowers: why are unsolicited ratings lower?	2009
132.	Herrmann-Pillath, Carsten Social Capital, Chinese Style: Individualism, Relational Collectivism and the Cultural Embeddedness of the Instituti-ons-Performance Link	2009
131.	Schäffler, Christian / Schmaltz, Christian Market Liquidity: An Introduction for Practitioners	2009
130.	Herrmann-Pillath, Carsten Dimensionen des Wissens: Ein kognitiv-evolutionärer Ansatz auf der Grundlage von F.A. von Hayeks Theorie der „Sensory Order“	2009
129.	Hankir, Yassin / Rauch, Christian / Umber, Marc It’s the Market Power, Stupid! – Stock Return Patterns in International Bank M&A	2009
128.	Herrmann-Pillath, Carsten Outline of a Darwinian Theory of Money	2009
127.	Cremers, Heinz / Walzner, Jens Modellierung des Kreditrisikos im Portfoliofall	2009
126.	Cremers, Heinz / Walzner, Jens Modellierung des Kreditrisikos im Einwertpapierfall	2009
125.	Heidorn, Thomas / Schmaltz, Christian Interne Transferpreise für Liquidität	2009
124.	Bannier, Christina E. / Hirsch, Christian The economic function of credit rating agencies - What does the watchlist tell us?	2009
123.	Herrmann-Pillath, Carsten A Neurolinguistic Approach to Performativity in Economics	2009
122.	Winkler, Adalbert / Vogel, Ursula Finanzierungsstrukturen und makroökonomische Stabilität in den Ländern Südosteuropas, der Türkei und in den GUS-Staaten	2009
121.	Heidorn, Thomas / Rupprecht, Stephan Einführung in das Kapitalstrukturmanagement bei Banken	2009
120.	Rossbach, Peter Die Rolle des Internets als Informationsbeschaffungsmedium in Banken	2009
119.	Herrmann-Pillath, Carsten Diversity Management und diversi-tätsbasiertes Controlling: Von der „Diversity Scorecard“ zur „Open Balanced Scorecard	2009
118.	Hölscher, Luise / Clasen, Sven Erfolgsfaktoren von Private Equity Fonds	2009
117.	Bannier, Christina E. Is there a hold-up benefit in heterogeneous multiple bank financing?	2009
116.	Roßbach, Peter / Gießamer, Dirk Ein eLearning-System zur Unterstützung der Wissensvermittlung von Web-Entwicklern in Sicherheitsthemen	2009
115.	Herrmann-Pillath, Carsten Kulturelle Hybridisierung und Wirtschaftstransformation in China	2009
114.	Schalast, Christoph: Staatsfonds – „neue“ Akteure an den Finanzmärkten?	2009
113.	Schalast, Christoph / Alram, Johannes Konstruktion einer Anleihe mit hypothekarischer Besicherung	2009
112.	Schalast, Christoph / Bolder, Markus / Radünz, Claus / Siepmann, Stephanie / Weber, Thorsten Transaktionen und Servicing in der Finanzkrise: Berichte und Referate des Frankfurt School NPL Forums 2008	2009
111.	Werner, Karl / Moormann, Jürgen Efficiency and Profitability of European Banks – How Important Is Operational Efficiency?	2009
110.	Herrmann-Pillath, Carsten Moralische Gefühle als Grundlage einer wohlstandschaffenden Wettbewerbsordnung: Ein neuer Ansatz zur erforschung von Sozialkapital und seine Anwendung auf China	2009
109.	Heidorn, Thomas / Kaiser, Dieter G. / Roder, Christoph Empirische Analyse der Drawdowns von Dach-Hedgefonds	2009

108. Herrmann-Pillath, Carsten
Neuroeconomics, Naturalism and Language — 2008

107. Schalast, Christoph / Benita, Barten
Private Equity und Familienunternehmen – eine Untersuchung unter besonderer Berücksichtigung deutscher Maschinen- und Anlagenbauunternehmen — 2008

106. Bannier, Christina E. / Grote, Michael H.
Equity Gap? – Which Equity Gap? On the Financing Structure of Germany's Mittelstand — 2008

105. Herrmann-Pillath, Carsten
The Naturalistic Turn in Economics: Implications for the Theory of Finance — 2008

104. Schalast, Christoph (Hrgs.) / Schanz, Kay-Michael / Scholl, Wolfgang
Aktionärsschutz in der AG falsch verstanden? Die Leica-Entscheidung des LG Frankfurt am Main — 2008

103. Bannier, Christina E./ Müsch, Stefan
Die Auswirkungen der Subprime-Krise auf den deutschen LBO-Markt für Small- und MidCaps — 2008

102. Cremers, Heinz / Vetter, Michael
Das IRB-Modell des Kreditrisikos im Vergleich zum Modell einer logarithmisch normalverteilten Verlustfunktion — 2008

101. Heidorn, Thomas / Pleißner, Mathias
Determinanten Europäischer CMBS Spreads. Ein empirisches Modell zur Bestimmung der Risikoaufschläge von Commercial Mortgage-Backed Securities (CMBS) — 2008

100. Schalast, Christoph (Hrsg.) / Schanz, Kay-Michael
Schaeffler KG/Continental AG im Lichte der CSX Corp.-Entscheidung des US District Court for the Southern District of New York — 2008

99. Hölscher, Luise / Haug, Michael / Schweinberger, Andreas
Analyse von Steueramnestiedaten — 2008

98. Heimer, Thomas / Arend, Sebastian
The Genesis of the Black-Scholes Option Pricing Formula — 2008

97. Heimer, Thomas / Hölscher, Luise / Werner, Matthias Ralf
Access to Finance and Venture Capital for Industrial SMEs — 2008

96. Böttger, Marc / Guthoff, Anja / Heidorn, Thomas
Loss Given Default Modelle zur Schätzung von Recovery Rates — 2008

95. Almer, Thomas / Heidorn, Thomas / Schmaltz, Christian
The Dynamics of Short- and Long-Term CDS-spreads of Banks — 2008

94. Barthel, Erich / Wollersheim, Jutta
Kulturunterschiede bei Mergers & Acquisitions: Entwicklung eines Konzeptes zur Durchführung einer Cultural Due Diligence — 2008

93. Heidorn, Thomas / Kunze, Wolfgang / Schmaltz, Christian
Liquiditätsmodellierung von Kreditzusagen (Term Facilities and Revolver) — 2008

92. Burger, Andreas
Produktivität und Effizienz in Banken – Terminologie, Methoden und Status quo — 2008

91. Löchel, Horst / Pecher, Florian
The Strategic Value of Investments in Chinese Banks by Foreign Financial Insitutions — 2008

90. Schalast, Christoph / Morgenschweis, Bernd / Sprengetter, Hans Otto / Ockens, Klaas / Stachuletz, Rainer / Safran, Robert
Der deutsche NPL Markt 2007: Aktuelle Entwicklungen, Verkauf und Bewertung – Berichte und Referate des NPL Forums 2007 — 2008

89. Schalast, Christoph / Stralkowski, Ingo
10 Jahre deutsche Buyouts — 2008

88. Bannier, Christina E./ Hirsch, Christian
The Economics of Rating Watchlists: Evidence from Rating Changes — 2007

87. Demidova-Menzel, Nadeshda / Heidorn, Thomas
Gold in the Investment Portfolio — 2007

86. Hölscher, Luise / Rosenthal, Johannes
Leistungsmessung der Internen Revision — 2007

85. Bannier, Christina / Hänsel, Dennis
Determinants of banks' engagement in loan securitization — 2007

84. Bannier, Christina
"Smoothing" versus "Timeliness" - Wann sind stabile Ratings optimal und welche Anforderungen sind an optimale Berichtsregeln zu stellen? — 2007

83. Bannier, Christina E.
Heterogeneous Multiple Bank Financing: Does it Reduce Inefficient Credit-Renegotiation Incidences? — 2007

82. Cremers, Heinz / Löhr, Andreas
Deskription und Bewertung strukturierter Produkte unter besonderer Berücksichtigung verschiedener Marktszenarien 2007

81. Demidova-Menzel, Nadeshda / Heidorn, Thomas
Commodities in Asset Management 2007

80. Cremers, Heinz / Walzner, Jens
Risikosteuerung mit Kreditderivaten unter besonderer Berücksichtigung von Credit Default Swaps 2007

79. Cremers, Heinz / Traughber, Patrick
Handlungsalternativen einer Genossenschaftsbank im Investmentprozess unter Berücksichtigung der Risikotragfähigkeit 2007

78. Gerdesmeier, Dieter / Roffia, Barbara
Monetary Analysis: A VAR Perspective 2007

77. Heidorn, Thomas / Kaiser, Dieter G. / Muschiol, Andrea
Portfoliooptimierung mit Hedgefonds unter Berücksichtigung höherer Momente der Verteilung 2007

76. Jobe, Clemens J. / Ockens, Klaas / Safran, Robert / Schalast, Christoph
Work-Out und Servicing von notleidenden Krediten – Berichte und Referate des HfB-NPL Servicing Forums 2006 2006

75. Abrar, Kamyar / Schalast, Christoph
Fusionskontrolle in dynamischen Netzsektoren am Beispiel des Breitbandkabelsektors 2006

74. Schalast, Christoph / Schanz, Kay-Michael
Wertpapierprospekte: Markteinführungspublizität nach EU-Prospektverordnung und Wertpapierprospektgesetz 2005 2006

73. Dickler, Robert A. / Schalast, Christoph
Distressed Debt in Germany: What´s Next? Possible Innovative Exit Strategies 2006

72. Belke, Ansgar / Polleit, Thorsten
How the ECB and the US Fed set interest rates 2006

71. Heidorn, Thomas / Hoppe, Christian / Kaiser, Dieter G.
Heterogenität von Hedgefondsindizes 2006

70. Baumann, Stefan / Löchel, Horst
The Endogeneity Approach of the Theory of Optimum Currency Areas - What does it mean for ASEAN + 3? 2006

69. Heidorn, Thomas / Trautmann, Alexandra
Niederschlagsderivate 2005

68. Heidorn, Thomas / Hoppe, Christian / Kaiser, Dieter G.
Möglichkeiten der Strukturierung von Hedgefondsportfolios 2005

67. Belke, Ansgar / Polleit, Thorsten
(How) Do Stock Market Returns React to Monetary Policy ? An ARDL Cointegration Analysis for Germany 2005

66. Daynes, Christian / Schalast, Christoph
Aktuelle Rechtsfragen des Bank- und Kapitalmarktsrechts II: Distressed Debt - Investing in Deutschland 2005

65. Gerdesmeier, Dieter / Polleit, Thorsten
Measures of excess liquidity 2005

64. Becker, Gernot M. / Harding, Perham / Hölscher, Luise
Financing the Embedded Value of Life Insurance Portfolios 2005

63. Schalast, Christoph
Modernisierung der Wasserwirtschaft im Spannungsfeld von Umweltschutz und Wettbewerb – Braucht Deutschland eine Rechtsgrundlage für die Vergabe von Wasserversorgungskonzessionen? – 2005

62. Bayer, Marcus / Cremers, Heinz / Kluß, Norbert
Wertsicherungsstrategien für das Asset Management 2005

61. Löchel, Horst / Polleit, Thorsten
A case for money in the ECB monetary policy strategy 2005

60. Richard, Jörg / Schalast, Christoph / Schanz, Kay-Michael
Unternehmen im Prime Standard - „Staying Public“ oder „Going Private“? - Nutzenanalyse der Börsennotiz - 2004

59. Heun, Michael / Schlink, Torsten
Early Warning Systems of Financial Crises - Implementation of a currency crisis model for Uganda 2004

58. Heimer, Thomas / Köhler, Thomas
Auswirkungen des Basel II Akkords auf österreichische KMU 2004

57. Heidorn, Thomas / Meyer, Bernd / Pietrowiak, Alexander
Performanceeffekte nach Directors´Dealings in Deutschland, Italien und den Niederlanden 2004

56. Gerdesmeier, Dieter / Roffia, Barbara
The Relevance of real-time data in estimating reaction functions for the euro area 2004

55. Barthel, Erich / Gierig, Rauno / Kühn, Ilmhart-Wolfram
Unterschiedliche Ansätze zur Messung des Humankapitals 2004

54. Anders, Dietmar / Binder, Andreas / Hesdahl, Ralf / Schalast, Christoph / Thöne, Thomas
Aktuelle Rechtsfragen des Bank- und Kapitalmarktrechts I :
Non-Performing-Loans / Faule Kredite - Handel, Work-Out, Outsourcing und Securitisation 2004

53. Polleit, Thorsten
The Slowdown in German Bank Lending – Revisited 2004

52. Heidorn, Thomas / Siragusano, Tindaro
Die Anwendbarkeit der Behavioral Finance im Devisenmarkt 2004

51. Schütze, Daniel / Schalast, Christoph (Hrsg.)
Wider die Verschleuderung von Unternehmen durch Pfandversteigerung 2004

50. Gerhold, Mirko / Heidorn, Thomas
Investitionen und Emissionen von Convertible Bonds (Wandelanleihen) 2004

49. Chevalier, Pierre / Heidorn, Thomas / Krieger, Christian
Temperaturderivate zur strategischen Absicherung von Beschaffungs- und Absatzrisiken 2003

48. Becker, Gernot M. / Seeger, Norbert
Internationale Cash Flow-Rechnungen aus Eigner- und Gläubigersicht 2003

47. Boenkost, Wolfram / Schmidt, Wolfgang M.
Notes on convexity and quanto adjustments for interest rates and related options 2003

46. Hess, Dieter
Determinants of the relative price impact of unanticipated Information in 2003
U.S. macroeconomic releases

45. Cremers, Heinz / Kluß, Norbert / König, Markus
Incentive Fees. Erfolgsabhängige Vergütungsmodelle deutscher Publikumsfonds 2003

44. Heidorn, Thomas / König, Lars
Investitionen in Collateralized Debt Obligations 2003

43. Kahlert, Holger / Seeger, Norbert
Bilanzierung von Unternehmenszusammenschlüssen nach US-GAAP 2003

42. Beiträge von Studierenden des Studiengangs BBA 012 unter Begleitung von Prof. Dr. Norbert Seeger
Rechnungslegung im Umbruch - HGB-Bilanzierung im Wettbewerb mit den internationalen 2003
Standards nach IAS und US-GAAP

41. Overbeck, Ludger / Schmidt, Wolfgang
Modeling Default Dependence with Threshold Models 2003

40. Balthasar, Daniel / Cremers, Heinz / Schmidt, Michael
Portfoliooptimierung mit Hedge Fonds unter besonderer Berücksichtigung der Risikokomponente 2002

39. Heidorn, Thomas / Kantwill, Jens
Eine empirische Analyse der Spreadunterschiede von Festsatzanleihen zu Floatern im Euroraum
und deren Zusammenhang zum Preis eines Credit Default Swaps 2002

38. Böttcher, Henner / Seeger, Norbert
Bilanzierung von Finanzderivaten nach HGB, EstG, IAS und US-GAAP 2003

37. Moormann, Jürgen
Terminologie und Glossar der Bankinformatik 2002

36. Heidorn, Thomas
Bewertung von Kreditprodukten und Credit Default Swaps 2001

35. Heidorn, Thomas / Weier, Sven
Einführung in die fundamentale Aktienanalyse 2001

34. Seeger, Norbert
International Accounting Standards (IAS) 2001

33. Moormann, Jürgen / Stehling, Frank
Strategic Positioning of E-Commerce Business Models in the Portfolio of Corporate Banking 2001

32. Sokolovsky, Zbynek / Strohhecker, Jürgen
Fit für den Euro, Simulationsbasierte Euro-Maßnahmenplanung für Dresdner-Bank-Geschäftsstellen 2001

31. Roßbach, Peter
Behavioral Finance - Eine Alternative zur vorherrschenden Kapitalmarkttheorie? 2001

30. Heidorn, Thomas / Jaster, Oliver / Willeitner, Ulrich
Event Risk Covenants 2001

29. Biswas, Rita / Löchel, Horst
Recent Trends in U.S. and German Banking: Convergence or Divergence? 2001

28. Eberle, Günter Georg / Löchel, Horst
Die Auswirkungen des Übergangs zum Kapitaldeckungsverfahren in der Rentenversicherung auf die Kapitalmärkte 2001

27.	Heidorn, Thomas / Klein, Hans-Dieter / Siebrecht, Frank Economic Value Added zur Prognose der Performance europäischer Aktien	2000
26.	Cremers, Heinz Konvergenz der binomialen Optionspreismodelle gegen das Modell von Black/Scholes/Merton	2000
25.	Löchel, Horst Die ökonomischen Dimensionen der ‚New Economy'	2000
24.	Frank, Axel / Moormann, Jürgen Grenzen des Outsourcing: Eine Exploration am Beispiel von Direktbanken	2000
23.	Heidorn, Thomas / Schmidt, Peter / Seiler, Stefan Neue Möglichkeiten durch die Namensaktie	2000
22.	Böger, Andreas / Heidorn, Thomas / Graf Waldstein, Philipp Hybrides Kernkapital für Kreditinstitute	2000
21.	Heidorn, Thomas Entscheidungsorientierte Mindestmargenkalkulation	2000
20.	Wolf, Birgit Die Eigenmittelkonzeption des § 10 KWG	2000
19.	Cremers, Heinz / Robé, Sophie / Thiele, Dirk Beta als Risikomaß - Eine Untersuchung am europäischen Aktienmarkt	2000
18.	Cremers, Heinz Optionspreisbestimmung	1999
17.	Cremers, Heinz Value at Risk-Konzepte für Marktrisiken	1999
16.	Chevalier, Pierre / Heidorn, Thomas / Rütze, Merle Gründung einer deutschen Strombörse für Elektrizitätsderivate	1999
15.	Deister, Daniel / Ehrlicher, Sven / Heidorn, Thomas CatBonds	1999
14.	Jochum, Eduard Hoshin Kanri / Management by Policy (MbP)	1999
13.	Heidorn, Thomas Kreditderivate	1999
12.	Heidorn, Thomas Kreditrisiko (CreditMetrics)	1999
11.	Moormann, Jürgen Terminologie und Glossar der Bankinformatik	1999
10.	Löchel, Horst The EMU and the Theory of Optimum Currency Areas	1998
09.	Löchel, Horst Die Geldpolitik im Währungsraum des Euro	1998
08.	Heidorn, Thomas / Hund, Jürgen Die Umstellung auf die Stückaktie für deutsche Aktiengesellschaften	1998
07.	Moormann, Jürgen Stand und Perspektiven der Informationsverarbeitung in Banken	1998
06.	Heidorn, Thomas / Schmidt, Wolfgang LIBOR in Arrears	1998
05.	Jahresbericht 1997	1998
04.	Ecker, Thomas / Moormann, Jürgen Die Bank als Betreiberin einer elektronischen Shopping-Mall	1997
03.	Jahresbericht 1996	1997
02.	Cremers, Heinz / Schwarz, Willi Interpolation of Discount Factors	1996
01.	Moormann, Jürgen Lean Reporting und Führungsinformationssysteme bei deutschen Finanzdienstleistern	1995

Frankfurt School / HfB – Working Paper Series
Centre For Practical Quantitative Finance

No.	Author/Title	Year
21.	Packham, Natalie / Schlögl, Lutz / Schmidt, Wolfgang M. Credit dynamics in a first passage time model with jumps	2009
20.	Reiswich, Dimitri / Wystup, Uwe FX Volatility Smile Construction	2009
19.	Reiswich, Dimitri / Tompkins, Robert Potential PCA Interpretation Problems for Volatility Smile Dynamics	2009
18.	Keller-Ressel, Martin / Kilin, Fiodar Forward-Start Options in the Barndorff-Nielsen-Shephard Model	2008
17.	Griebsch, Susanne / Wystup, Uwe On the Valuation of Fader and Discrete Barrier Options in Heston's Stochastic Volatility Model	2008
16.	Veiga, Carlos / Wystup, Uwe Closed Formula for Options with Discrete Dividends and its Derivatives	2008
15.	Packham, Natalie / Schmidt, Wolfgang Latin hypercube sampling with dependence and applications in finance	2008
14.	Hakala, Jürgen / Wystup, Uwe FX Basket Options	2008
13.	Weber, Andreas / Wystup, Uwe Vergleich von Anlagestrategien bei Riesterrenten ohne Berücksichtigung von Gebühren. Eine Simulationsstudie zur Verteilung der Renditen	2008
12.	Weber, Andreas / Wystup, Uwe Riesterrente im Vergleich. Eine Simulationsstudie zur Verteilung der Renditen	2008
11.	Wystup, Uwe Vanna-Volga Pricing	2008
10.	Wystup, Uwe Foreign Exchange Quanto Options	2008
09.	Wystup, Uwe Foreign Exchange Symmetries	2008
08.	Becker, Christoph / Wystup, Uwe Was kostet eine Garantie? Ein statistischer Vergleich der Rendite von langfristigen Anlagen	2008
07.	Schmidt, Wolfgang Default Swaps and Hedging Credit Baskets	2007
06.	Kilin, Fiodor Accelerating the Calibration of Stochastic Volatility Models	2007
05.	Griebsch, Susanne/ Kühn, Christoph / Wystup, Uwe Instalment Options: A Closed-Form Solution and the Limiting Case	2007
04.	Boenkost, Wolfram / Schmidt, Wolfgang M. Interest Rate Convexity and the Volatility Smile	2006
03.	Becker, Christoph/ Wystup, Uwe On the Cost of Delayed Currency Fixing	2005
02.	Boenkost, Wolfram / Schmidt, Wolfgang M. Cross currency swap valuation	2004
01.	Wallner, Christian / Wystup, Uwe Efficient Computation of Option Price Sensitivities for Options of American Style	2004

HFB – SONDERARBEITSBERICHTE DER HFB - BUSINESS SCHOOL OF FINANCE & MANAGEMENT

No.	Author/Title	Year
01.	Nicole Kahmer / Jürgen Moormann Studie zur Ausrichtung von Banken an Kundenprozessen am Beispiel des Internet (Preis: € 120,--)	2003

Printed edition: € 25.00 + € 2.50 shipping

Download:
Working Paper: http://www.frankfurt-school.de/content/de/research/Publications/list_of_publication0.html
CPQF: http://www.frankfurt-school.de/content/de/research/quantitative_Finance/research_publications.html

Order address / contact

Frankfurt School of Finance & Management
Sonnemannstr. 9 – 11 ▪ D – 60314 Frankfurt/M. ▪ Germany
Phone: +49 (0) 69 154 008 – 734 ▪ Fax: +49 (0) 69 154 008 – 728
eMail: m.biemer@frankfurt-school.de
Further information about Frankfurt School of Finance & Management
may be obtained at: http://www.frankfurt-school.de