Journal of Agricultural and Resource Economics 24(2):398–423
Copyright 1999 Western Agricultural Economics Association

Reliability of Options Markets for Crop Revenue Insurance Rating

David Buschena and Lee Ziegler

Revenue insurance, only recently introduced for major crops in the U.S., has captured a considerable share of the multiple-peril insurance market. This study evaluates the predictive reliability of using price distributions inferred from options markets to rate revenue insurance products. We find for periods early in the crop growing season that price distributions inferred from options trades offer greater reliability than distributions based on historical futures trades. Options-based price distributions should receive further consideration in crop revenue insurance rating, but current administrative constraints must be considered.

Key words: crop insurance, options markets, probability assessment calibration

Introduction

Recent introductions of three separate revenue-based crop insurance products have substantially changed the opportunities for producers to reduce risk. These revenue products are Crop Revenue Coverage (CRC) developed under contract for a private insurer, Revenue Assurance (RA) developed under contract for the Iowa Farm Bureau, and Income Protection (IP) developed under contract for the Risk Management Agency (formerly the Federal Crop Insurance Corporation). In only the second year of their implementation, these three new crop insurance revenue policies together captured 14% of the total acres insured under federally underwritten multiple-peril contracts in 1997 [U.S. General Accounting Office (U.S. GAO)]. Historically, producers could insure crop yields of generally up to 75% of average historic *yield* using traditional Multiple-Peril Crop Insurance instruments. The new revenue insurance products allow these same producers to insure up to 75% of the crop's expected *revenue*, based on historic price and yield data. Higher coverage levels of up to 85% are being explored in some areas under Risk Management Agency pilot for both yield and revenue insurance.

Establishing actuarially fair rates for revenue insurance products is more complicated than establishing rates for yield insurance. Rates for crop yield insurance products require only a distribution over yield, while revenue insurance rating requires a distribution over the product of yield and price. For example, consider this estimation under

David Buschena is assistant professor and Lee Ziegler is former master's student, both in the Department of Agricultural Economics and Economics, Montana State University, Bozeman.

The authors wish to thank two anonymous journal referees and Paul Fackler, Joe Atwood, Myles Watts, and Gary Brester for their valuable input. We are also grateful to James Driscoll, Bruce Babcock, and Art Barnaby who were helpful in clarifying the various revenue products. Julie Searle and Donna Kelly provided able word processing assistance. Funding for this analysis was provided by the Montana Agricultural Experiment Station and the Risk Management Agency. However, the opinions and conclusions expressed here are those of the authors and do not necessarily reflect the views of the funding agencies.

joint lognormality of yield and price—revenue insurance rating requires estimation of a normal distribution over the log of yield, estimation of a normal distribution over the log of price, and also the correlation between them. Two of the three existing revenue insurance products consider the negative price/yield correlation (U.S. GAO). Consideration of a significant negative correlation would decrease actuarially based rates for revenue insurance. However, this negative correlation would need to be large in magnitude for revenue insurance rates to be lower than yield insurance rates.

The recent U.S. GAO report on crop revenue insurance specifically cites the use of options markets as "more appropriate" than historic prices in developing price distributions for revenue insurance contracts. However, only one of the three crop revenue insurance products (Revenue Assurance) currently uses options to develop price distributions for contract rating. The primary objective of this analysis is to determine how distributions from options trades compare to distributions based on historic futures prices for predicting harvest period prices, where distributions are estimated given the information available before planting.

Evaluating the potential reliability of these options-based distributions is the first and necessary step if they are to be incorporated into revenue insurance rating procedures. A subsequent operational step—how such options-based distributions might be feasibly incorporated into revenue insurance policies given current administrative rate-making and approval constraints—is discussed below, and will be addressed in depth in a future paper.

A Review of Current Literature and This Study's Contribution

Most of the theoretical basis for our analysis is formed by the seminal works of Fackler (1986), and Fackler and King (1987, 1990). These papers apply the concept of calibration from the Bayesian forecasting literature to the options pricing model developed by Cox and Ross to assess the reliability of distributions inferred from options trades. Fackler, and Fackler and King (1987) use two of the three distributional forms we apply in our estimations, while Sherrick, Garcia, and Tirupattur apply the third.

Reliability and calibration are closely related and are used interchangeably; a reliable distribution is one that is calibrated. The advantage of this concept is that it allows a very general evaluation of a distribution's performance over its entire domain—we are not restricted to only the first few moments of these distributions. As in Fackler and King (1990), we find the description in Lichtenstein, Fischoff, and Phillips to be quite useful. They note that a distribution "is calibrated if, over the long run, for all propositions assigned a given probability, the proportion that is true equals the probability assigned" (p. 307). We use this definition later to empirically test the relative reliability of price change distributions from various sources.

How, in general, do price distributions estimated from options markets relate to changes in the underlying futures price? And how do these distributions differ in predictive reliability from those based on historic futures prices? Empirical evidence supporting the use of options-based distributions for agricultural prices over the growing season is thin, in part due to the newness of these contracts.

The most relevant work regarding this issue was carried out by Fackler and King (1990), who studied probability distributions from options trades four and eight weeks prior to expiration for soybeans, hogs, corn, and live cattle. They found significant reliability problems for soybeans and hogs, while corn and live cattle showed no such reliability problems. Additionally, Fackler found no significant reliability problems for options markets for futures on Treasury bonds. Other test results made under a restrictive, i.i.d. lognormal price change assumption for grains and oilseeds (Heifner), and also generally for stock options (Beckers; Chiras and Manaster; Canina and Figlewski), found options markets to have imperfect predictive power for a more limited test of price variance.

Fackler and King's (1990) results showing reliability problems in options trades with relatively short (two-month) times to maturity raise the question of how prevalent these departures from reliability are for options trades with longer times to maturity. Taken at face value, their findings call into question the use of options markets in developing price distributions for soybeans. If price distributions estimated from options trades before planting exhibit the type of reliability problems as in Fackler and King, they may be inappropriate for crop insurance rating.

To evaluate this possibility, we assess the distributions implied by options markets on the harvest futures contract from the period prior to planting and throughout the growing season: (*a*) to evaluate how they may or may not be useful for crop revenue insurance rating, (*b*) to understand the nature of the miscalibration reported in Fackler and King, and (*c*) to explain the potential market imperfections occurring when trade volumes are low. This evaluation is also useful for assessing the use of options-based distributions for rating crops such as feed barley that have shorter growing seasons.[1]

The primary empirical contributions of this study are twofold. First, we find corn and soybeans options markets provide reliable (calibrated) distributions for trades throughout most of the growing season; for corn, however, these estimated distributions are not reliable when options contracts have few remaining trading weeks (less than five). Therefore, we find no sufficient evidence that distributions from options markets incorrectly estimate the location or scale of the underlying distribution; they do not exhibit significant miscalibration. Our results further suggest that the calibration problems in Fackler and King (1990) may to some degree be due to the relatively short time to maturity of the options contracts used in their analysis.

Second, we introduce likelihood functions to assess the reliability of options distributions relative to distributions from historic futures prices. We find that distributions from options offer slightly greater reliability than those from historic futures prices when estimated during the early and middle months of the corn and soybean growing season. Put another way, these likelihoods support the use of distributions estimated from options trades over historic futures markets distributions for corn and soybeans during most of the crop planning and growing season—this is the key result for our crop insurance application.

[1] IP is the only approved revenue insurance for feed barley, and currently uses the corn futures contract for price prediction at 85% of the average settlement prices for planting (February average on the September contract) and harvest (August average on the September contract).

Price Distributions in Revenue Contracts and Current Administrative Constraints

All three revenue insurance contracts develop an ex ante distribution for price changes from sign-up (pre-planting) to harvest. Note that these rating procedures are undergoing changes as each type of revenue insurance develops. We briefly describe key features of the price distribution for each product in its current form, following the discussion in the 1998 U.S. GAO report, a 1999 working paper by Goodwin, Roberts, and Coble, and discussions with the developers of these products. These descriptions focus on the estimation and use of price change distributions from pre-planting to harvest in each product and discuss how distributions from options trades are or might be incorporated into their rating procedures.

Crop Revenue Coverage (CRC)

CRC is a privately developed, rated, and sold revenue insurance that is approved as a pilot product under Section 508(h) of the Federal Crop Insurance Act (FCIA) as amended in 1996.[2] The Risk Management Agency approves the rating *process* for CRC, not the rates themselves, allowing CRC to incorporate new information into the rates up until shortly before the sales closing (pre-planting) date.

CRC includes an "upper price risk component" that applies when the farmer's yield is lower than the insured level but the output price is higher at harvest than before planting (the farmer is eligible for a payment equal to the difference between the insured and the actual yield at the harvest price). The planting price for corn, soybeans, and spring wheat is the February average of the futures settlement prices (Chicago for corn and soybeans, and Minneapolis for spring wheat). The harvest period price is the monthly average during the month prior to the end of trading for the futures contract (November for corn, October for soybeans, and August for spring wheat).

For CRC's upper price risk component, the following specification is the price component for the expected loss to the insurer from an upward price change if the actual yield is less than the insured yield:

$$EL = \int_{P_p}^{\textit{upper bound}} (P_p - P_h) f(P_h)\, dP_h. \tag{1}$$

In this expected loss equation, P_p is price at planting, P_h is the price at harvest, $f(P_h)$ is the probability density for price changes, and the price increase is limited to an upper bound (e.g., \$1.50 above the planting price for the corn contract).

The CRC procedure assumes normality for the probability density $f(P_h)$ (see Goodwin, Roberts, and Coble for an empirical evaluation of this assumption) and estimates a polynomial function for the integration of this truncated normal density (as developed in Botts and Boles) using historic futures price data (1973–present). Price and yield are assumed to be uncorrelated.

---

[2] U.S. Congress, § 1501, Sec. 508(h), Federal Crop Insurance Act (amended April 4, 1996). See especially items (1)–(5) of subsection (h).

CRC could theoretically incorporate information from options trades into the density $f(P_h)$, but this would require a different distributional form under most options pricing methods. A more general distributional form for $f(P_h)$ would likely require the use of numerical integration methods to estimate the expected loss in equation (1).

Revenue Assurance (RA)

As with Crop Revenue Coverage, RA is approved as a pilot by the Risk Management Agency under FCIA [Section 508(h), (1)–(5)], and as such requires only approval of the rate-making process, not the rates themselves. As a pilot, it is possible for RA rates to be adjusted using new options market information at the sign-up (pre-planting) date. However, existing restrictions on maximum rate movements (20% of the previous year's rates) and complexities associated with combining this new information with historical price/yield correlations will need to be addressed as RA moves from the pilot stage.

RA currently uses options prices to develop its loss function. The current year's pre-planting period (February average) futures price on the harvest contract (P_p) and the price volatility (σ) from Black's options pricing formula in this period are used under a lognormality assumption for price changes from planting to harvest, through:

(2) $$F(P_h) = \left(P_h\sqrt{\pi\sigma}\right)^{-1}\exp\left[\frac{-0.5(\log(P_h - \mu))}{\sigma^2}\right].$$

In equation (2), μ is the mean and σ is the standard deviation of the lognormal distribution estimated from options trades. Local basis and the price/yield correlation are included in RA's revenue guarantee. Yields are assumed to follow a scaled beta distribution, and Johnson and Tenenbein's approach is used to estimate the joint revenue distribution. Use of a distribution other than the lognormal in equation (2) would likely require numerical integration methods for rating RA.

Income Protection (IP)

IP is approved as a Risk Management Agency pilot under FCIA Section 508(h), (9). Because this product is produced under contract for the Risk Management Agency, the actual rates are subject to U.S. Department of Agriculture (USDA) approval, i.e., comparable to what would be required if IP were a standard, rather than a pilot, program. As such, the rating must be completed well in advance (approximately six months) of the sales closing (pre-planting) date. Under this bureaucratic structure, it may be more difficult to effectively include information from options markets. Because these bureaucratic constraints are likely to apply to other revenue insurance products as they move from their pilot stage, they are discussed further below in the context of rating IP.

IP uses an empirical distribution for price changes from planting to harvest employing bootstrap methods. Historic price ratios (P_h/P_p) from 1960 to the present are related to county-adjusted regional-level yields through nonlinear estimation. The dates and contract months used to define the planting and harvest prices are the same as those for CRC. The errors in the price ratio estimation (given the county-adjusted yields), farm-level error around the county trend, and errors from a yield-trend evaluation for the county are drawn in the bootstrap procedure (10,000 draws) to form empirical distributions for revenue (U.S. GAO; Atwood, Baquet, and Watts).

Introducing an options-based price distribution into the IP bootstrap procedure would require additional modeling effort because no distributional form is assumed in IP's rating model and subsequent estimation. Empirical bootstrap procedures could proceed using random draws from a price distribution estimated from options trades. Additionally, and if desired, Bayesian probability updating methods could be used to combine this new information with historic price data, where the appropriate weights would be determined by empirical evaluation.

Current Administrative Constraints for IP

The submission date to the Risk Management Agency for the year 2000 spring crop IP rates for approval was July 1999. This submission date reflects the time required for internal auditing and USDA approval of the rates. The July 1999 early submission date is well before the start of trading for the harvest period (year 2000) options contracts; options contracts generally begin trading approximately 12 months before contract close. This lack of availability for options contracts precludes their direct use in rate-making under these anticipated program time lines. Furthermore, the rates are not allowed to increase by more than 20% for any single area.

One potential method of including options information in rate setting would be to use the options contract nearest to the harvest contract for trades made at the submission deadline (e.g., use the trades during July 1999 for the July 2000 options contract) to develop a proxy distribution prior to rate submission in July 1999. Another potential adjustment is to incorporate an across-the-board change (increase or decrease) in rates before planting (March 2000) to reflect a change in the revenue distribution from options trades after the base rates have been submitted and approved. In our view, neither adjustment is very attractive, but actual rate-making must take these administrative considerations into account. The relative usefulness of these adjustments requires a good deal of additional study and discussion with the Risk Management Agency administrators.

Before development of alternative ways to include distributions from options prices into rate-making, however, we will focus on questions that must be addressed first. Are price distributions from options markets reliable estimates of changes in futures prices from planting to harvest? Furthermore, are these options-based price distributions superior in reliability to those from historic futures markets? Is there evidence—as in Fackler and King (1990) for options with short times to maturity—of miscalibration for distributions based on harvest contract options prices for trades near planting?

Commodity Futures, Options, and Options Pricing

Two types of uniformly defined, standardized, and widely traded contracts exist for agricultural commodities in the U.S.—futures and options. Both types are being used in rating crop revenue contracts (see the 1998 U.S. GAO report). Define the unknown and random price of a commodity in the future as C_{T+m}, where $T + m$ is the last trading day for futures, and m is approximately one month. A futures contract calls for delivery of a specified quantity and quality of the commodity to specific locations at or before time $T + m$. Because of these contracts' standardization and because of the reputation of the exchange guaranteeing their performance, a large number of these contracts trade daily (t) at prices F_t in highly liquid markets that have existed since well before the turn of the 20th century.

Crop revenue insurance policies take some form of the futures price as a proxy for the cash price, a view that has consistent empirical support (for example, see recent tests in Heifner, and in Ziegler). Although only one of the revenue insurance products makes an adjustment for local basis (U.S. GAO), we abstract from the basis for clarity.

Options on futures contracts began trading in 1984 (October) for soybeans, 1985 (March) for corn, and 1986 (November) for wheat on the Chicago Board of Trade (CBOT). These contracts give a buyer the right, but not the obligation, to enter into a futures transaction at a specified price at any time before the option expires at time T. A *call* option is a contract, written (sold) by a seller, that gives the buyer the right to *purchase* a futures contract at a strike price X_c. A *put* option is also a contract, written by a seller, that gives the buyer the right to *sell* a futures contract at a strike price X_p. Because their underlying commodity is a futures contract, options contracts are standardized through the specifications of the futures contract.

Options contracts trade on the same exchanges as futures contracts, but are generally less liquid (trade at lower volumes). These options contracts are valuable, trading at prices P_p for puts and P_c for calls. Prices for puts are expected to increase with the strike price (X_p) and with increases in the spread of the estimated (at time t) density for the futures price that will be realized at T. Define this underlying futures price density as $h_t(F_T)$. Prices for calls should decrease with the strike price (X_c) and increase as the spread of $h_t(F_T)$ increases. Under certain assumptions (e.g., Black), spread, and thus options price, increases proportionally with the time remaining ($T - t$) for the option. (For further discussion of current research into these contracts, see Hull; Copeland and Weston; and Campbell, Lo, and MacKinlay.)

The price an option commands depends on what market participants believe at time t about the underlying and unknown density, $h_t(F_T)$, for the upcoming changes in the futures price from now (t) until option expiration (T). Under assumptions of risk-neutral valuation and no transactions costs, Cox and Ross have developed equations defining the value of puts (V_p) and calls (V_c) at time t as follows:

(3)
$$V_{pt} = e^{-r(T-t)} \int_0^\infty \max(X_p - F_T, 0) g_t(F_T)\, dF_T;$$
$$V_{ct} = e^{-r(T-t)} \int_0^\infty \max(F_T - X_c, 0) g_t(F_T)\, dF_T.$$

These valuation formulas depend on the discount factor, $e^{-r(T-t)}$, for a risk-free real interest rate r, the observed strike prices (X_c and X_p), and an unknown density function $g_t(F_T)$. The density $g_t(F_T)$ is an artificial density at time t for F_T, defined under the assumptions of risk neutrality and no transactions costs, not the underlying $h_t(F_T)$ per se. Cox and Ross refer to $g_t(F_T)$ as a risk-neutralized pricing density (RNPD), and it is also called the risk-neutralized valuation measure (e.g., Fackler and King 1990).

Because options are rights, not obligations, the value to a holder of an option is bounded below by $0. We use the formulas from equation (3) in our empirical analysis to recover $g_t(F_T)$, the markets' consensus of the unknown RNPD at time t for F_T.

The general expressions in equation (3) apply to any distributional form. Also important for our analysis, Cox and Ross's formulation does not require a specification for the price change in each small time period within the time remaining $(T - t)$. In this sense, Cox and Ross's method differs from applications of ARCH or GARCH methods (e.g., Guo) that pool data and require assumptions over the time-series nature of price changes.

Lognormality tests over daily changes in settlement futures prices (Sherrick, Garcia, and Tirupattur; Ziegler) show the importance of considering distributions that allow for a wide range of skewness and kurtosis in modeling futures price distributions.[3] We use two general distributional forms from the Burr family of three parameter distributions (Burr 1942, 1968, 1973; Burr and Cislak) in our estimation of the densities in equation (3). Rodriguez; Singh and Maddala; Tadikamalla; and McDonald provide a thorough evaluation of these distributions' properties—a family that numerically includes the Pearson type (I–VII), gamma, normal, lognormal, exponential, logistic, Weibull, and other distributions, yet allows for a greater range of skewness and kurtosis than they do. Fackler; Fackler and King (1987); and Sherrick, Garcia, and Tirupattur have also used Burr distributions for modeling options prices. We also estimate the lognormal specification; this lognormal distribution still receives wide use by academics (e.g., Stokes, Nayda, and English; Tirupattur, Hauser, and Boyle) and apparently by market participants and information services (e.g., Data Transmission Services).

Although these Burr distributions can exactly match the skewness and kurtosis properties of many common distributions in a numerical sense, many of these distributions (in particular, for our purposes, the lognormal) are not analytically nested within the Burr distributions (see McDonald). Therefore, our empirical efforts focus on model fit criteria, since tests for the rejection of one model in favor of another is inappropriate in this framework. We can predict that (subject to estimation error) the more general Burr distributions should have lower sum of squared errors provided that the lognormal does not completely specify the RNPD.[4]

The formulas given in equation (3) are defined for European options, a common assumption for options pricing models because of its added tractability. The actual traded contracts are American options that allow early, and theoretically valuable, exercise. Only a small amount of early exercise occurs, typically at levels of the nature of 0.1% of the daily open interest. Therefore, the properties of the densities inferred from market trades, $g_t(F_T)$, will depend on how closely the European options formula matches the value of these American options.

We carried out a sensitivity analysis for corn and soybean options trades during 1997 that was based on the findings in Ramaswamy and Sundaresan. This sensitivity

[3] Ziegler's annual tests for excess skewness and kurtosis of daily changes in log futures prices (1960–96) for the harvest futures contract showed significant nonnormality of these log prices in 21 of 37 years (57%) for corn, 25 of 37 years (68%) for soybeans, and 18 of 37 years (49%) for spring wheat.

[4] Note that Fackler and King (1990) report that the Burr XII did not greatly improve model fit, while Sherrick, Garcia, and Tirupattur report some potential advantages in fit from the Burr III.

analysis showed no substantive effect on selection of the optimal distribution from measures of the early exercise value of these options.[5]

Data and Methods

Our analysis of options-based and historic futures distributions reflects components of the estimation methods used in each product, but was designed to address generally the predictive value of these methods. We consider a number of distributional forms, including the lognormal as in Revenue Assurance (RA), for estimating distributions from options trades prior to planting and throughout the growing season. The other distributional forms are considerably more general than lognormality, and we compare the relative fit of all these distributions. We compare these options-based distributions with historic distributions using futures [as in Crop Revenue Coverage (CRC) and Income Protection (IP)] using both an empirical cumulative distribution function (CDF) as in IP, and a fitted lognormal distribution (note that CRC uses normality, but see Goodwin, Roberts, and Coble). The trading dates used begin with the February average as in IP and RA.

We fit and compared three alternative distributions for the risk-neutralized pricing density (RNPD), $g_t(F_T)$, in the Cox-Ross model from equation (3). The Burr III, Burr XII, and lognormal distributions were estimated using traded option prices on identical days for all three crops. The CDFs that correspond with the estimated RNPDs at time t for the final period's futures price (F_T) are:

(4)
$$G_t(F_T;\, \alpha, c, k) = \left[1 + \left(\frac{F_T}{\alpha}\right)^{-c}\right]^{-k} \qquad \text{(Burr III)},$$

$$G_t(F_T;\, \beta, d, m) = 1 - \left[1 + \left(\frac{F_T}{\beta}\right)^{d}\right]^{-m} \qquad \text{(Burr XII)},$$

$$G_t(F_T;\, \mu, \sigma) = N\left[\frac{(\ln(F_T) - \mu)}{\sigma}\right] \qquad \text{(Lognormal)}.$$

All parameters (α, c, k, β, d, and m) in the Burr distributions are nonnegative. $N(\cdot)$ is the normal CDF. Differentiation of each CDF yields probability density functions (PDFs), $g_t(F_T)$, for each distributional form, and the results are used to estimate the RNPDs in equation (3).

The RNPDs were estimated for a number of periods throughout the year. Each period's data consisted of options settlement prices for a single day's trades at all traded strike prices, reflecting available information through the market's consensus on that

---

[5] Upper bounds for early-exercise premia developed in Chaudhury and Wei (see also the summary article by Söderlind and Svennson, and approximation methods in Plato, and in Barone-Adesi and Whaley) suggest that the premia estimates in Ramaswamy and Sundaresan are somewhat overstated for calls, but are very close for puts. The Chaudhury and Wei results offer additional support for our use of the European option valuation models to estimate the RNPD for corn and soybean options.

day.[6] Consistent with Fackler; Fackler and King (1987, 1990); and Sherrick, Garcia, and Tirupattur, our objective function minimizes the sum of squared differences between observed option prices (P_c and P_p, respectively) and the theoretical option values (V_p and V_c, respectively) from equation (3). The objective function for each RNPD, $g_t(\cdot)$, is indexed by $i = \{1, \ldots, n\}$ for calls and $j = \{1, \ldots, m\}$ for puts over the strike prices traded on day t, and is given by:

$$(5) \qquad \sum_{i=1}^{n} \left[P_{cti} - e^{-r(T-t)} \int_{X_{ci}}^{\infty} g_t(F_T)(F_T - X_{cti})\, dF_T \right]^2 + \sum_{j=1}^{m} \left[P_{ptj} - e^{-r(T-t)} \int_{0}^{X_{pj}} g_t(F_T)(X_{ptj} - F_T)\, dF_T \right]^2 .$$

We used corn, soybean, and wheat options prices from the Chicago Board of Trade for options on the harvest futures contracts (December for corn, November for soybeans, and September for spring wheat). The observation years begin at the start of options trading on the CBOT—1985 for corn and soybeans, and 1987 for wheat—and end in 1997. All prices are real, using the chain-type GDP deflator with 1992 as the base year. (We will later carry out tests over a time series for options RNPDs.) The real interest rate is given by six-month T-bill less inflation.

Our ending period T was the closest day on or before the 15th of the final month in which an options contract on the harvest futures was traded; this period is just prior to the expiration date for these options. The final options trading month was November for corn, October for soybeans, and August for wheat. For each crop and under each model, we estimated the RNPD for F_T from a single day's options trades at t, $g_t(F_T)$.

Although our crop revenue insurance rating application calls only for estimated distributions for price changes from just prior to planting to harvest, we assess distributions from many periods to better understand options market behavior during the growing season. RNPDs were estimated at roughly two-week intervals, with the first estimation taken on the first trading day in February. The parameters defining the RNPD were then estimated for each crop for the day closest to February 15th. Two days were selected in the same manner for every month (March, April, May, ...) up to and including the first day in the last month of options trading for the harvest contract (e.g., the first trading day in November for corn). The two days estimated during each month are henceforth referred to as the beginning day (nearest the 1st) and the middle day (nearest the 15th). We selected two-week intervals to give inclusive yet tractable coverage during each year.

Strike prices violating monotonicity were discarded, consistent with Sherrick, Garcia, and Tirupattur.[7] We used the GAUSS (Breslaw) optimization package to minimize equation (5). (Details of this gradient search procedure are available from the authors upon request.) Numerical grid search routines were not used because, for three parameters and the more than 2,000 estimations we carried out, they quickly become

[6] The average number of strike prices traded (both puts and calls) was approximately 10 in early months (e.g., February) and approximately 20 in later months (e.g., June).

[7] The bulk of these monotonicity violations occurred in the months close to the last trading day for options with strike prices that are far out of the money, and were likely the result of closing out outstanding contracts of little value.

inordinately time consuming for any degree of reasonable refinement. In general, convergence under our initial fit criteria occurred quickly for the Burr XII and the lognormal distributions. The parameter k in the Burr III occasionally took on negative or very large values (e.g., k = 200,000), and gave long convergence times or failed to converge. This lack of convergence was robust to changes in our fitting criteria and suggests that this Burr III form was for some periods particularly susceptible to estimation problems.

Properties of the Implied Distributions

Overall Model Fit

Using equation (5) for each distribution, the sum of squared errors (SSE) was estimated for the options traded. Summary statistics are given in table 1. Note that the number of observations differs somewhat by crop because there were some days for which a given distribution was not successfully estimated. The mean SSE criteria favors the Burr XII for corn and spring wheat, while the Burr III is favored for soybeans. The Burr XII distribution has the lowest median SSE for all three crops. For spring wheat, there were a number of periods early in the growing season where there were too few contracts traded to estimate the three-parameter Burr distributions; these periods are omitted from the sample.

In addition to showing superiority in the overall summary statistics in table 1, the Burr XII distribution in particular was superior in fit for all crops both across years and across biweekly periods.[8] Because of the overall support for the Burr XII, and since there was at times some estimation difficulty for the Burr III, the remainder of the analysis evaluates only the Burr XII and the lognormal distribution.

Note that the superiority in SSE of the Burr distributions relative to the lognormal is not completely unexpected, since they are more flexible, allowing for more skewness and kurtosis. Numerical fitting error gave rise to some instances (10.8% for corn, 27.8% for soybeans, and 5% for spring wheat) where the SSE from the Burr distributions was greater than that for the lognormal. This suggests that the benefit from these more general Burr distributions is lowest for soybeans.[9]

Calibration of the Estimated Distributions

Crop revenue insurance requires an understanding of the price distribution's entire range. Rating these revenue contracts further requires methods to assess the predictive usefulness of the entire distribution, not only of the first two moments. One assessment approach is the concept of calibration that is well known in the physical sciences and in psychology (see Lichtenstein, Fischoff, and Phillips; and Curtis, Ferrell, and Solomon). Calibration was applied to options prices by Fackler, and by Fackler and King (1987,

---

[8] Exhaustive tables showing this superiority are available from the authors upon request.

[9] One might argue that the SSE from the lognormal serves as a bound on the Burr formations' SSE and that this bound should be imposed on the estimation, but then clearly by definition the Burr is always nondominated under a criterion of model fit, and a clear comparison of the relative performance of each form is impossible.

Table 1. Summary Statistics for Sum of Squared Errors for Implied Distributions from Options

A. Chicago Board of Trade December Corn
Twice monthly, February–November, 1985–97 (n = 258)
(Note: February 1985 not traded)

Distribution	Burr III	Burr XII	Lognormal
Periods Estimated	249	255	254
Mean SSE	0.0844	0.0740	0.1579
Median SSE	0.0157	0.0079	0.0493
Minimum SSE	0.7E-06	0.4E-06	0.0002
Maximum SSE	0.8498	0.8505	3.2690

B. Chicago Board of Trade November Soybeans
Twice monthly, February–October, 1985–97 (n = 234)

Distribution	Burr III	Burr XII	Lognormal
Periods Estimated	231	231	222
Mean SSE	0.1745	0.1960	0.3420
Median SSE	0.0196	0.0144	0.0615
Minimum SSE	0.0001	< 0.1E-08	0.0017
Maximum SSE	6.6010	2.9330	19.6580

C. Chicago Board of Trade September Wheat
Twice monthly, February–October, 1985–97 (n = 154)

Distribution	Burr III	Burr XII	Lognormal
Periods Estimated	130	128	119
Mean SSE	0.0364	0.0316	0.0752
Median SSE	0.0012	0.0009	0.0123
Minimum SSE	< 0.1E-08	< 0.1E-08	< 0.1E-08
Maximum SSE	0.8362	0.8819	0.8637

1990); to our knowledge these have been the only such applications to date of calibration to options. (For discussion of the calibration concept, see Aitchison and Dunsmore; Morris; DeGroot and Fienberg; and Bunn.)

The use of calibration methods to assess options RNPDs is appealing since, by design, traders in these open-outcry auctions carry out the behavioral aggregation evaluated in Curtis, Ferrell, and Solomon to achieve a consensus judgment of the underlying RNPD. Indeed, allowing sufficient arbitrage, this behavioral aggregation argues that the market's consensus RNPD should approach the price density of the most accurate risk-neutral trader. Lichtenstein, Fischoff, and Phillips further discuss calibration studies for subjective judgments in various settings with small samples. These small sample settings are more comparable to our application than are those with thousands of observations such as weather forecasting and mechanical or human measurement devices.

We follow portions of the discussion in and make use of the notation from Fackler and King (1990) to describe key concepts for calibration. Calibration tests are carried out over the predicted value of the CDF for the realized level of the futures price at T, where the prediction is made in year i, at biweekly period t, and using distributional assumption j. This prediction is a random variable at t given as $U_{itj} = G_{itj}(F_{iT})$ with realization $u_{itj} = G_{itj}(f_{iT})$. Also, f_{iT} is the realization of the random variable F_T. The u_{itj} terms are independent, as are the realizations f_{iT}. The random variable U_{itj} has a CDF of $C(U)$. The inferred RNPD from options trades under the distributional assumption j is calibrated if the random variable U_{itj} is uniformly distributed on [0, 1], or alternatively if $C(u_{itj}) = u_{itj}$ for all the realized u_{itj}'s on [0, 1]. Comparing the estimated u_{itj} terms for a given (relative to a uniform) distribution indicates how reliably each distribution predicts the actual ending period futures prices at T. Furthermore, in the event that a distribution is miscalibrated, the test statistics discussed below can be used to develop a transformation of the miscalibrated distribution into a calibrated one.

As discussed in Fackler and King (1990), the uniform distribution's CDF, $C(U)$, is the 45-degree line in the U cumulative probability space—this is the behavior under a calibrated distribution. The realized u_{itj} terms at time T in each year could indicate an understatement of the location (figure 1A) of the ending period's futures price, giving too much mass on the [0.0, 0.5] interval for the u_{itj} terms. These u_{itj}'s could also indicate an overstatement of the dispersion (figure 1B) of F_T if there is too much weight in the tails, such as in the [0.0, 0.25] and in the [0.75, 1.00] intervals.

We tested the realized u_{itj} terms against their hypothesized behavior under uniformity (calibration) in three ways, following Fackler and King (1990). Since the wheat options markets have as few as three observations over our sample period for twice-monthly periods early in the year (e.g., February and March), only the calibration for corn and soybeans is assessed. Our samples of annual observations for the u_{itj}'s over all options' trading years (t) usually give a sample size of $n = 13$. We assume for these calibration tests that the random process affecting predictive performance is identical and independent across years.

The first statistic, the sign test, uses a binomial test for the counts of the u_{itj} terms within the [0.0, 0.5] and the [0.25, 0.75] intervals. If the u_{itj}'s are well calibrated, the expected value and variance of these counts for both intervals are $n(0.5)$ and $n(0.5^2)$, respectively. This statistic does not rely on asymptotic results, an important concern given our small sample size.

The second test was parametric, using the beta distribution to fit $C(\cdot)$ for the u_{itj} terms:

$$C(u_{itj}) = u_{itj}^{p-1}(1 - u_{itj})^{q-1}/\beta(p, q). \tag{6}$$

In equation (6), $\beta(p, q)$ is the beta function, with $p = q = 1$ under uniformity. In the event of significant miscalibration, the shape of the beta function indicates how the CDF, $G_{itj}(F_{iT})$, obtained under distributional assumption j, could be adjusted (recalibrated)

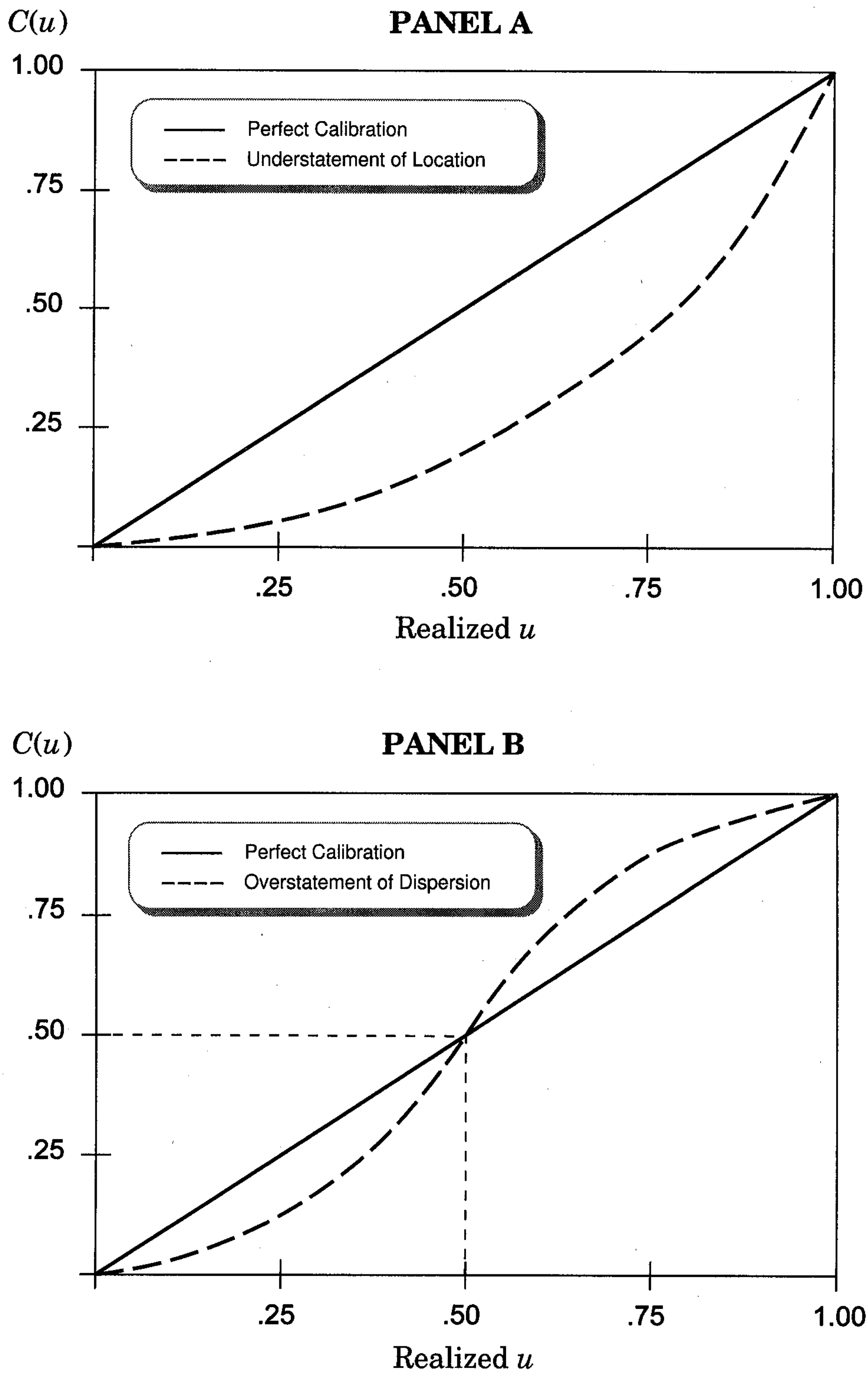

Figure 1. RNPD distributions and calibration

to give a reliable (calibrated) distribution through $C(G_{itj}(F_T)) = H_{it}(F_T)$. There are a number of interpretations for estimated values of p and q that differ from 1:

- CASE 1, $p \approx q > 1$: Reflects the flat, steep, then flat pattern in figure 1B, overestimation of variability;

- CASE 2, $p \approx q < 1$: Reflects a steep, flat, steep pattern that is the analog of figure 1B, corresponding with an underestimation of spread (variability);

- CASE 3, p and q differ substantially: Reflects underestimation in location as in figure 1A, with $p > 1$ and $q < 1$.

An asymptotic chi-squared test statistic with two degrees of freedom $(\chi^2_{(2)})$ is defined relative to uniformity ($p = q = 1$) for these parameters. We carried out a numerical search to maximize the log-likelihood function (LLF) over p and q in equation (6). The log-likelihood function is zero under uniformity, so the test statistic is $2*$LLF for the estimated beta LLF.

A third test used the nonparametric Kolmogorov-Sminov, Cramer-von Mises, and Watson test statistics (see Stephens, and Fackler for an evaluation of these statistics for small samples and their critical values).[10] Given our small sample size, there are important considerations for the power of these test statistics. Fackler carried out power tests for the beta likelihood-ratio statistic comparable to those for the nonparametric statistics in Stephens for samples of 10, 20, and 40 observations. He found that the beta statistic compared favorably in power to these nonparametric forms for most alternative (to uniformity) functional forms. However, for some alternative forms, the power for any of the tests, given small samples of $n = 10$ or $n = 20$, is quite low. Our comparable (in sample size, number of replications, and functional forms) Monte Carlo power tests for the binomial sign tests show even more severe power problems when these binomial tests are carried out separately from one another. (Results are available from the authors upon request.) Note that there is no clear way to evaluate the joint power of these two binomial statistics.

The results from all calibration tests are given in table 2 for corn and table 3 for soybeans. All of the tests—the sign test, parametric tests for a beta specification, and nonparametric tests—indicate that the CDFs estimated at t for ending-period futures price F_T behave well if t is early in the growing season. Under the lognormal specification, there is some indication of miscalibration during June and July for corn (only) under the parametric beta test. The Burr XII distribution shows no corresponding miscalibration under the beta test.[11]

All three testing methods reveal significant miscalibration for both corn distributions near the end of the options trading period. The estimated CDFs for both distributions exhibit some overestimation of the dispersion for the binomial test and the beta test

---

[10] Fackler and King (1990) suggest a small sample multiplier for this statistic of $(1 + 1/n)$, and Fackler evaluates another set of small sample adjustments. Calculations show that such adjustments would not influence our results.

[11] Evidence of miscalibration may be due to a number of factors. Calibration tests are joint, depending on the underlying distributional form used and on the assumptions of the Cox-Ross formulation (risk-neutral valuation and no transactions costs). Evidence of miscalibration could indicate that the underlying distribution differs from the one assumed, that there are important market imperfections violating the Cox-Ross assumptions, that there is systematic misjudgment by the market participants, or that a combination of these factors exists.

near the end of corn options trading. There is little evidence of miscalibration in soybeans. The results differ from those in Fackler and King (1990), who found no evidence of significant miscalibration in corn for contracts with short (four- and eight-week) time to maturity under lognormality, but significant miscalibration in soybeans under lognormality for these short time-to-maturity levels.

Distributions from Historic Futures Prices

It remains to be determined whether or not options markets provide superior distributional estimates to those from other sources. Power questions aside, the results above do not indicate significant calibration problems for RNPDs implied by options trades estimated in the early and middle periods of the growing season, though there is some evidence for miscalibration near the close of contract trading for corn.

The RNPDs implied by options trading may not be the only ones that pass the calibration tests. For example, densities constructed using historic futures prices may also be well calibrated. Alternatively, these historic densities may be miscalibrated if underlying price densities have changed due to changes over time in government price support and acreage restriction policy (U.S. GAO).

We created CDFs for corn and soybeans defined for each year i, and twice each month t, using the set of ratios for the change in historic futures prices from t until harvest (T). The elements of this set are defined as $r_{ts_i} = F_{Ts_i}/F_{ts_i}$, using the index $s_i = \{1960, \ldots, S_i\}$, where $S_i = i - 1$. For example, in 1985, there are 25 of these historic ratios, where $S_{1985} =$ 1984.

Two methods were used to define an ex ante CDF for the F_T by combining the historic futures change ratios and the futures price (F_{it}) in period t for year i, forming the set $\Phi_{it} = \{F_{it} * r_{tk_i}\}$. The first method uses the discrete CDF given by arranging the elements of Φ_{ti} in magnitude and assuming a uniform distribution over them. Each element is given a probability of occurrence $1/n_{it}$, where n_{it} is the number of ex ante observations for period t in year i. Using our 1985 example, each Φ_{ti} is given a probability of occurrence of 1 in 25. The second method fits a lognormal CDF to the elements of Φ_{ti}. These historic distributions exhibited no significant miscalibration using the binomial, beta, and the nonparametric tests defined previously.

Comparing Distribution Reliability: Quartile Likelihood

The binomial distribution used for the calibration sign tests in tables 2 and 3 can be extended to form a multinomial likelihood function for distributional fit under the null hypothesis of uniformity. These multinomial likelihoods are appealing because they are always estimable and do not rely on asymptotic results. For tractability and given our small sample size, multinomial likelihood tests using quartiles are employed.

The multinomial likelihood function for quartile counts of the u_{itj} terms under uniformity (again, the criteria for a calibrated distribution) is given by:

$$L_{tj} = \left(\frac{n_t!}{k_{1tj}!k_{2tj}!k_{3tj}!k_{4tj}!} \right) (0.25)^{k_{1tj}}(0.25)^{k_{2tj}}(0.25)^{k_{3tj}}(0.25)^{k_{4tj}}. \tag{7}$$

Table 2. Results of Calibration Tests for Distributions from Corn Options

		Burr XII Distribution							Lognormal Distribution						
		Sign Test			Beta Estimates			Sign Test			Beta Estimates		
Date[a]		n	Count [0, 0.5]	Count [0.25, 0.75]	p	q	$\chi^2_{(2)}$ Statistic[b]	n	Count [0, 0.5]	Count [0.25, 0.75]	p	q	$\chi^2_{(2)}$ Statistic[b]
Feb	*b*	12	6	6	0.94	0.89	0.09	11	6	5	0.88	0.92	0.12
	m	12	6	5	0.49	0.64	4.90*	11	6	6	0.97	0.95	0.02
Mar	*b*	13	8	6	0.81	0.95	0.48	13	8	7	0.93	0.93	0.04
	m	13	7	6	0.75	0.88	0.74	11	5	6	1.10	1.10	0.10
Apr	*b*	13	8	5	0.72	0.72	1.20	13	9	5	1.30	1.70	0.98
	m	13	7	5	0.96	1.10	0.18	13	8	5	0.20	0.33	3.10
May	*b*	13	6	5	0.74	0.71	1.20	11	5	5	1.40	2.40	4.50
	m	13	8	3*	0.80	0.76	0.68	13	8	6	1.30	1.80	2.00
Jun	*b*	13	7	4	0.99	1.10	0.16	13	7	7	0.60	1.30	5.80*
	m	13	8	5	0.90	1.10	0.50	13	7	8	2.70	4.80	10.80***
Jul	*b*	13	7	6	0.62	0.84	2.50	12	6	5	2.40	4.20	9.50***
	m	12	7	5	0.79	0.67	1.40	13	7	5	1.70	2.80	5.60*
Aug	*b*	12	5	7	1.10	0.79	1.20	13	6	9	1.70	1.60	1.50
	m	13	4	9	1.20	0.81	1.70	12	5	9	1.30	0.97	1.00
Sep	*b*	13	5	7	2.50	1.50	4.80*	13	5	6	2.10	2.00	3.50
	m	13	4	8	4.20	2.20	9.60***	13	3*	9	1.90	1.70	2.30
Oct	*b*	13	6	9	3.20	1.90	7.00**	12	6	9	2.70	2.70	5.20*
	m	13	5	9	2.70	2.30	4.80*	12	6	10*	11.10	8.50	21.00***
Nov	*b*	12	2**	10**	5.60	3.70	11.50***	13	4	11**	5.90	3.90	13.10***

Note: Single, double, and triple asterisks (*) denote significance at the 10%, 5%, and 1% levels, respectively.

[a] The *b* following each month denotes "beginning," and indicates the first trading day of the month; *m* denotes "middle," and indicates the trading date closest to the 15th.

[b] Fackler and King suggest a small sample correction for the χ^2 statistics of $\{1 + 1/n\}$ based on Monte Carlo estimates. Such an adjustment would not affect the number of occasions when these statistics were significant for the three critical levels considered here.

Table 2. Extended

		Burr XII Distribution			Lognormal Distribution		
Date[a]		Kolmogorov-Smirnov Test Statistic	Cramer-von Mises Test Statistic	Watson Test Statistic	Kolmogorov-Smirnov Test Statistic	Cramer-von Mises Test Statistic	Watson Test Statistic
Feb	*b*	0.554	0.026	0.048	0.657	0.036	0.057
	m	0.580	0.046	0.067	0.706	0.025	0.041
Mar	*b*	0.665	0.061	0.051	0.703	0.039	0.056
	m	0.608	0.023	0.034	0.835	-0.007	0.014
Apr	*b*	0.778	0.108	0.089	0.794	0.113	0.109
	m	0.744	0.088	0.073	0.683	0.086	0.076
May	*b*	0.803	0.068	0.074	0.609	0.028	0.030
	m	1.020	0.121	0.120	0.698	0.083	0.092
Jun	*b*	0.702	0.039	0.043	0.495	0.012	0.033
	m	0.703	0.072	0.037	0.635	0.062	0.073
Jul	*b*	0.597	0.017	0.025	1.020	0.173	0.163
	m	0.522	0.022	0.037	0.436	-0.003	0.017
Aug	*b*	0.544	0.022	0.010	0.679	0.059	0.041
	m	0.772	0.106	0.029	0.778	0.120	0.042
Sep	*b*	1.070*	0.273	0.082	1.150	0.268	0.092
	m	1.340**	0.504**	0.178*	1.440**	0.484**	0.165*
Oct	*b*	1.440**	0.399*	0.224**	1.490**	0.366*	0.214**
	m	0.823	0.100	0.104	1.450**	0.347*	0.080
Nov	*b*	1.330**	0.400*	0.286**	1.530**	0.457*	0.178*

Note: Single and double asterisks (*) denote significance at the 10% and 5% levels, respectively.

[a] The *b* following each month denotes "beginning," and indicates the first trading day of the month; *m* denotes "middle," and indicates the trading date closest to the 15th.

Table 3. Results of Calibration Tests for Distributions from Soybeans Options

		Burr XII Distribution						Lognormal Distribution					
		Sign Test			Beta Estimates			Sign Test			Beta Estimates		
Date[a]		n	Count [0, 0.5]	Count [0.25, 0.75]	p	q	$\chi^2_{(2)}$ Statistic[b]	n	Count [0, 0.5]	Count [0.25, 0.75]	p	q	$\chi^2_{(2)}$ Statistic[b]
Feb	*b*	13	7	4	0.88	0.91	0.20	13	7	6	0.92	0.90	0.08
	m	13	8	7	0.78	1.00	6.60**	13	7	6	0.94	0.91	0.07
Mar	*b*	13	7	5	0.76	0.91	0.66	12	6	5	0.90	0.91	0.09
	m	13	8	5	0.70	0.91	1.90	12	7	6	1.00	1.10	0.03
Apr	*b*	13	7	6	0.97	1.30	2.20	13	7	7	1.20	1.50	1.30
	m	13	6	4		Not Estimated		12	6	6	1.50	1.70	1.70
May	*b*	13	9	6	1.00	1.50	2.00	13	8	8	1.30	2.10	3.70
	m	13	9	6		Not Estimated		12	8	5	1.30	1.60	1.40
Jun	*b*	12	7	6	0.92	1.70	2.20	13	8	5	0.67	1.30	4.40
	m	13	9	5		Not Estimated		12	8	7	1.80	2.80	5.20*
Jul	*b*	13	9	6	1.20	1.50	1.10	13	9	6	1.10	1.30	0.68
	m	13	9	6	1.10	1.70	2.20	13	9	6	1.60	2.60	5.30*
Aug	*b*	13	9	8	1.70	2.30	3.80	11	6	6	1.70	1.70	1.90
	m	12	7	8	1.50	1.20	1.10	12	7	8	1.10	0.77	1.50
Sep	*b*	13	5	6	0.75	0.42	9.50	13	6	8	1.40	1.20	0.78
	m	13	10	10	1.90	2.20	3.40	13	8	9	1.10	0.75	1.60
Oct	*b*	13	7	3*	0.73	0.71	1.20	11	6	5	1.20	1.20	0.29

Note: Single and double asterisks (*) denote significance at the 10% and 5% levels, respectively.

[a] The *b* following each month denotes "beginning," and indicates the first trading day of the month; *m* denotes "middle," and indicates the trading date closest to the 15th.

[b] Fackler and King suggest a small sample correction for the χ^2 statistics of $\{1 + 1/n\}$ based on Monte Carlo estimates. Such an adjustment would not affect the number of occasions when these statistics were significant for the three critical levels considered here.

Table 3. Extended

Date[a]		Burr XII Distribution			Lognormal Distribution		
		Kolmogorov-Smirnov Test Statistic	Cramer-von Mises Test Statistic	Watson Test Statistic	Kolmogorov-Smirnov Test Statistic	Cramer-von Mises Test Statistic	Watson Test Statistic
Feb	*b*	0.794	0.093	0.112	0.966	0.104	0.100
	m	1.030	0.138	0.076	0.783	0.073	0.060
Mar	*b*	0.750	0.073	0.074	0.691	0.058	0.071
	m	0.858	0.120	0.072	0.633	0.015	0.030
Apr	*b*	0.958	0.102	0.054	0.858	0.107	0.055
	m	0.657	0.075	0.095	0.433	0.003	0.016
May	*b*	1.090	0.209	0.073	1.030	0.236	0.073
	m	1.140	0.287	0.100	0.655	0.042	0.010
Jun	*b*	0.710	0.072	0.051	0.615	0.085	-0.071
	m	0.955	0.168	0.101	0.771	0.134	-0.020
Jul	*b*	0.994	0.194	0.106	1.060	0.239	0.154*
	m	0.897	0.182	0.055	1.200*	0.323	0.108
Aug	*b*	1.030	0.181	0.113	0.733	0.075	0.093
	m	0.867	0.155	0.169*	0.982	0.182	0.075
Sep	*b*	0.592	0.041	0.028	0.685	0.057	0.220**
	m	1.150*	0.293	0.261**	0.959	0.205	0.044
Oct	*b*	0.715	0.068	0.088	0.488	0.028	0.214**

Note: Single and double asterisks denote significance at the 10% and 5% levels, respectively.

[a] The *b* following each month denotes "beginning," and indicates the first trading day of the month; *m* denotes "middle," and indicates the trading date closest to the 15th.

In equation (7), k_{qtj} is the quartile count (number of annual observations of the u_{itj}'s falling into quartile q) during biweekly period t for the CDF estimated through method j. The four methods assessed include the two inferred from options (one using the Burr XII and the other using the lognormal distribution), and two distributions based on historic futures prices (one discrete and the other defined under lognormality). The total number of observations across years and within a given biweekly period is given by n_t. Under a calibrated distribution reflecting uniformity, the probability that each observation falls within a quartile is 0.25.[12] Note that the likelihoods for these exactly observed quartile counts may appear to be somewhat small because many patterns of quartile counts comprise the range of potentially observed outcomes.

Tables 4 and 5, respectively, list for corn and soybeans the estimated quartile likelihoods over all four estimation methods. The method with the highest quartile likelihood in each biweekly period is also listed.[13] Keep in mind that the options distributions are inferred under the RNPD assumptions and also assuming that the European options pricing formula closely approximates the actual options trades, while those for the historic futures markets require no such assumptions. Because the options distribution under lognormality (and to a lesser degree the Burr XII) was not always successfully estimated, these periods were omitted from the sample in our assessment. Virtually all such periods occurred in the first half of the data set (1985–90), a period of development for these options markets. The total number of periods throughout the year for which a distribution had the highest likelihood is summarized at the bottom of each table. Distributions giving equal and nondominated likelihoods were rated as "ties." There were seven ties in corn and five in soybeans.

For every period before the middle period in August, corn price distributions inferred from options prices were either superior in quartile likelihood or tied with historic futures distributions. Historic futures distributions were superior in likelihood for periods in the closing months of this harvest contract. Both options-based distributions from the Burr XII and from the lognormal have support in the early and middle months of the growing season.

For soybeans, options-based distributions were superior in quartile likelihood to historic distributions for periods before mid-July. The options distributions under lognormality were superior to those under the Burr XII early in the year (before May 1).

Interestingly, given our previous results for both crops, the CDFs estimated from options trades under the lognormal distribution performed quite well in reliability relative to the Burr XII. The results in tables 4 and 5 (in light of those in table 1) suggest that, although there are gains to using the Burr distributions instead of the lognormal for modeling options trades, the RNPD inferred under the lognormal performs relatively well in estimating futures prices. This support for the lognormal is particularly evident for soybeans, and is consistent with the previously discussed result for soybeans—i.e., in approximately 28% of the observations, the SSE for the lognormal was lower than that for either the Burr XII or the Burr III, while for corn this occurred for only about 11% of the observations.

---

[12] Likelihoods formed by splitting the range into thirds gave comparable orderings.

[13] There is no clearly appropriate method to address the significance of these likelihood differences in these nonnested models. We focus only on their predictive power through likelihood differences.

Table 4. Estimated Quartile Likelihoods for Corn, Over Four Estimation Methods

Date[a]		n	Inferred from Options		Inferred from Historic Futures		Maximum-Likelihood Distribution
			Burr XII	Lognormal	Lognormal	Discrete	(multiple entries are ties)
Feb	*b*	11	0.018	0.024	0.024	0.024	Options Log, Fitted Log Historical, and Discrete Historical
	m	11	0.022	0.017	0.017	0.011	Options Burr XII
Mar	*b*	13	0.013	0.011	0.004	0.001	Options Burr XII
	m	11	0.022	0.022	0.007	0.007	Options Burr XII and Options Log
Apr	*b*	13	0.011	0.005	0.005	0.004	Options Burr XII
	m	13	0.011	0.011	0.005	0.005	Options Burr XII and Options Log
May	*b*	11	0.011	0.022	0.003	0.011	Options Log
	m	13	0.005	0.013	0.004	0.011	Options Log
Jun	*b*	12	0.008	0.018	0.011	0.008	Options Log
	m	13	0.011	0.011	0.005	0.011	Options Burr XII and Options Log
Jul	*b*	13	0.017	0.017	0.017	0.017	All Four Methods
	m	13	0.017	0.017	0.010	0.017	Options Burr XII, Options Log, and Discrete Historical
Aug	*b*	13	0.017	0.007	0.008	0.012	Options Burr XII
	m	13	0.005	0.011	0.013	0.011	Fitted Log Historical
Sep	*b*	13	0.005	0.003	0.008	0.008	Fitted Log Historical and Discrete Historical
	m	13	0.001	0.001	0.018	0.005	Fitted Log Historical
Oct	*b*	13	0.009	0.001	0.004	0.013	Discrete Historical
	m	13	0.005	0.002	0.017	0.022	Discrete Historical
Nov	*b*	13	0.0002	0.001	0.001	0.003	Discrete Historical

Summary Counts:	Options Burr XII	= 4 maximum and 5 ties
	Options Log	= 3 maximum and 6 ties
	Discrete Historical	= 3 maximum and 4 ties
	Fitted Log Historical	= 2 maximum and 3 ties

[a] The *b* following each month denotes "beginning," and indicates the first trading day of the month; *m* denotes "middle," and indicates the trading date closest to the 15th.

Conclusions and Future Directions

Crop revenue insurance rating requires ex ante price distributions. How can information from options trades be used to develop this distribution? And how do the resulting distributions compare in predictive value with those constructed using other methods?

In this study, we evaluated distributions implied by options over corn, soybeans, and spring wheat. A number of distributional specifications were considered, and their fit was evaluated twice monthly over the growing season for each crop. These fit criteria support a general form, the Burr XII, that allows for considerable skewness and kurtosis to represent distributions to fit options prices.

Table 5. Estimated Quartile Likelihoods for Soybeans, Over Four Estimation Methods

Date[a]		n	Inferred from Options		Inferred from Historic Futures		Maximum-Likelihood Distribution (multiple entries are ties)
			Burr XII	Lognormal	Lognormal	Discrete	
Feb	*b*	11	0.005	0.018	0.001	0.005	Options Log
	m	11	0.013	0.018	0.008	0.004	Options Log
Mar	*b*	13	0.007	0.017	0.010	0.003	Options Log
	m	11	0.017	0.017	0.007	0.010	Options Burr XII and Options Log
Apr	*b*	13	0.009	0.013	0.004	0.004	Options Log
	m	13	0.008	0.022	0.001	0.003	Options Log
May	*b*	11	0.008	0.007	0.003	0.001	Options Burr XII
	m	13	0.012	0.008	0.033	0.003	Options Burr XII
Jun	*b*	12	0.017	0.011	0.002	0.040	Options Burr XII
	m	13	0.008	0.008	0.000	0.007	Options Burr XII and Options Log
Jul	*b*	13	0.008	0.008	0.007	0.003	Options Burr XII and Options Log
	m	13	0.008	0.008	0.007	0.013	Discrete Historical
Aug	*b*	13	0.017	0.017	0.001	0.011	Options Burr XII and Options Log
	m	13	0.007	0.007	0.001	0.011	Discrete Historical
Sep	*b*	13	0.013	0.013	0.001	0.005	Options Burr XII and Options Log
	m	13	0.000	0.002	0.007	0.005	Discrete Historical
Oct	*b*	13	0.003	0.022	0.017	0.008	Options Log

Summary Counts:
Options Burr XII = 3 maximum and 5 ties
Options Log = 6 maximum and 5 ties
Discrete Historical = 3 maximum and 0 ties
Fitted Log Historical = 0 maximum and 0 ties

[a] The *b* following each month denotes "beginning," and indicates the first trading day of the month; *m* denotes "middle," and indicates the trading date closest to the 15th.

Calibration tests assessing the reliability of these distributions inferred from options for corn and soybeans showed no significant miscalibration during most of the growing season. There was some miscalibration evident for corn options during periods quite close to expiration, suggesting overestimation of the spread of the futures prices. Note for both crops that distributions from historical futures markets did not exhibit significant miscalibration in any time period.

Our final analyses allowed us to answer a critical question for crop insurance rate setting: To what degree do options markets provide useful information for pricing distributions early in the growing season beyond that available from other sources? Our criterion was distribution reliability across the entire range of the distribution using maximum-likelihood methods.

Corn and soybean options markets provided the most reliable price distributions estimated during the first half of the growing season. The distributions inferred from options trades using both the Burr XII and the lognormal distribution were most reliable during the early months of the growing season for corn, while for soybeans those inferred from options under lognormality were superior in reliability.

The reliability likelihood methods for price distributions developed here could be refined to address specific parts of the probability distribution. For example, the lower tail of the distribution may be of particular interest for crop insurance rating and put option purchases. Rather than selecting the distribution that best fits the entire outcome range, a distribution could be selected based on its reliability of fit over the lower tail (e.g., the first two quartiles). Input from producers, policy makers, and underwriters can further define the distributional range that is most appropriate for the problem at hand.

Additional questions remain for the incorporation of options markets into rating crop revenue insurance. One important issue is to determine in what way distributions from such sources can be combined with other (historic) information on the crop yield distribution and on the correlation between price and yield to form a complete revenue distribution. Another issue to be addressed is in what way information from options trades can be incorporated into rate-making given the current administrative time constraints anticipated for these products. Alternatively, how might the administrative time constraints be adjusted to allow incorporation of options-based distributions? Such issues require input from academic researchers, Risk Management Agency administrators, private insurers, agency heads, and others.

Results reported here point to the potential predictive value of including information from options trades in future revenue insurance rating. The next step is to evaluate the predictive properties of the *complete* revenue distribution estimated using price distributions inferred from options markets combined with historic yield information. The performance of such a distribution could then be compared with alternative revenue distributions, such as one developed using only historic price and yield data.

[*Received December 1998; final revision received July 1999.*]

References

Aitchison, J., and I. R. Dunsmore. *Statistical Prediction Analysis.* New York: Cambridge University Press, 1975.

Atwood, J. A., A. E. Baquet, and M. J. Watts. "Income Protection." Staff Pap. No. 97-9, Dept. of Agr. Econ. and Econ., Montana State University, Bozeman, 1997.

Barone-Adesi, G., and R. E. Whaley. "Efficient Analytic Approximation of American Option Values." *J. Finance* 42(June 1987):301–20.

Beckers, S. "The Constant Elasticity of Variance Model and Its Implications for Option Pricing." *J. Finance* 35(June 1980):661–73.

Black, F. "The Pricing of Commodity Contracts." *J. Finan. Econ.* 3(March 1976):167–79.

Botts, R. R., and J. N. Boles. "Use of Normal-Curve Theory in Crop Insurance Rate Making." *J. Farm Econ.* 39(August 1958):733–40.

Breslaw, J. *GAUSSx Version 3.6 User's Manual.* Montreal: Econotron Software, 1997.

Bunn, D. W. *Applied Decision Analysis.* New York: McGraw-Hill, 1984.

Burr, I. W. "Cumulative Frequency Functions." *Annals Math. Statis.* 13(June 1942):215–32.

———. "On a General System of Distributions, III: The Simple Range." *J. Amer. Statis. Assoc.* 63(June 1968):636–43.

———. "Parameters for a General System of Distributions to Match a Grid of α_3 and α_4." *Communications in Statis.* 2(January 1973):1–21.

Burr, I. W., and P. J. Cislak. "On a General System of Distributions, I: Its Curve-Shaped Characteristics; II: The Sample Median." *J. Amer. Statis. Assoc.* 63(June 1968):627–35.

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay. *The Econometrics of Financial Markets.* Princeton NJ: Princeton University Press, 1997.

Canina, L., and S. Figlewski. "The Informational Content of Implied Volatility." *Rev. Finan. Stud.* 6(Spring 1993):659–81.

Chaudhury, M., and J. Wei. "Upper Bounds for American Futures Options: A Note." *J. Futures Mkts.* 14(February 1994):111–16.

Chiras, D., and S. Manaster. "The Informational Content of Option Prices and a Test of Market Efficiency." *J. Finan. Econ.* 6(June/September 1978):213–34.

Copeland, T. E., and J. F. Weston. *Financial Theory and Corporate Policy*, 3rd ed. Reading MA: Addison-Wesley, 1992.

Cox, J. C., and S. A. Ross. "Some New Results in Financial Option Pricing Theory." *J. Finance* 31(May 1976):145–66.

Curtis, P. E., W. R. Ferrell, and I. Solomon. "A Model of Calibration for Subjective Probability Distributions and Its Application to Aggregation of Judgments." *J. Forecasting* 4(September 1985): 349–61.

Data Transmission Services. *Weekly Options Review* [Omaha NE]. Various issues, 1996–99.

DeGroot, M. H., and S. E. Fienberg. "The Comparison and Evaluation of Forecasters." *Statistician* 32 (January 1983):12–21.

Fackler, P. L. "The Informational Content of Options Premiums." Unpub. Ph.D. diss., Dept. of Agr. and Appl. Econ., University of Minnesota, 1986.

Fackler, P. L., and R. P. King. "Calibration of Option-Based Probability Assessments in Agricultural Commodity Markets." *Amer. J. Agr. Econ.* 72(February 1990):73–83.

———. "Future Directions in Risk Analysis for Agricultural Production Firms." Paper presented at S-180 Regional Project Workshop titled "An Economic Analysis of Risk Management Strategies for Agricultural Production Firms," San Antonio TX, 22–25 March 1987.

Goodwin, B. K., M. C. Roberts, and K. Coble. "Measurement of Price Risk in Revenue Insurance: Implications of Distributional Assumptions." Work. Pap., Dept. of Agr. and Resour. Econ., North Carolina State University, Raleigh, 1999.

Guo, D. "The Predictive Power of Implied Stochastic Variance from Currency Options." *J. Futures Mkts.* 16(December 1996):915–92.

Heifner, R. "Price Analysis for Determining Revenue Insurance Indemnities and Premiums." Report to Office of Risk Management, U.S. Department of Agriculture, Washington DC, 1996.

Hull, J. C. *Options, Futures, and Other Derivative Securities*, 3rd ed. Upper Saddle River NJ: Prentice-Hall, 1997.

Johnson, M. E., and A. Tenenbein. "A Bivariate Distribution Family with Specified Marginals." *J. Amer. Statis. Assoc.* 76(March 1981):198–201.

Lichtenstein, S., B. Fischoff, and L. D. Phillips. "Calibration of Probabilities: The State of the Art in 1980." In *Judgement Under Uncertainty: Heuristics and Biases*, eds., D. Kahneman and A. Tversky, pp. 306–34. New York/London: Cambridge University Press, 1982.

McDonald, J. B. "Some Generalized Functions for the Size Distribution of Income." *Econometrica* 52 (May 1984):647–63.

Morris, P. A. "Combining Expert Judgements: A Bayesian Approach." *Manage. Sci.* 23(March 1977): 679–93.

Plato, G. "Valuing American Options on Commodity Futures Contracts." *Agr. Econ. Res.* 37(Spring 1985):1–14.

Ramaswamy, K., and S. M. Sundaresan. "The Valuation of Options on Futures Contracts." *J. Finance* 40(December 1985):1319–40.

Rodreiguez, R. N. "A Guide to the Burr Type XII Distributions." *Biometrika* 64(April 1977):129–34.

Sherrick, B., P. Garcia, and V. Tirupattur. "Recovering Probabilistic Information from Options Markets: Tests of Distributional Assumptions." *J. Futures Mkts.* 16(August 1996):545–60.

Singh, S. K., and G. S. Maddala. "A Function for the Size Distribution of Incomes." *Econometrica* 44 (September 1976):963–70.

Söderlind, P., and L. E. O. Svennson. "New Techniques to Extract Market Expectations from Financial Instruments." Work. Pap. No. 5877, National Bureau of Economic Research, Cambridge MA, 1997.

Stephens, M. A. "EDF Statistics for Goodness of Fit and Some Comparisons." *J. Amer. Statis. Assoc.* 69(September 1974):730–37.

Stokes, J. R., W. I. Nayda, and B. C. English. "The Pricing of Revenue Insurance." *Amer. J. Agr. Econ.* 79(May 1997):439–51.

Tadikamalla, P. R. "A Look at the Burr and Related Distributions." *Internat. Statis. Rev.* 48(December 1980):337–44.

Tirupattur, V., R. J. Hauser, and P. P. Boyle. "Theory and Measurement of Exotic Options in U.S. Agricultural Support Programs." *Amer. J. Agr. Econ.* 79(November 1997):1127–39.

U.S. General Accounting Office. "Crop Revenue Insurance: Problems with New Plans Need to Be Addressed." Pub. No. GAO/RCED-98-111, Washington DC, April 1998.

Ziegler, L. "Test of Distributional Assumptions and the Informational Content of Agricultural Futures Options." Unpub. M.S. thesis, Dept. of Agr. Econ. and Econ., Montana State University, Bozeman, 1997.